Real Exchange Rate Fluctuations, Endogenous Tradability and Exchange Rate Regime

Kanda Naknoi*

First draft: November 2003

This draft: July 2006

Abstract

This paper develops a stochastic dynamic general equilibrium model to explain variance decompositions of real exchange rates. Its key features are trade costs, heterogeneous productivity and sticky wages. Dynamics of comparative advantage amplify the expenditure switching. The model predicts that importance of the relative price of traded goods is increasing in the covariance between terms of trade and productivity differentials in the non-traded and export sectors. Given interest rate shocks, exchange rate stability reduces the covariance and importance of the relative price of the traded goods. Total factor productivity shocks raise the covariance and cause the relative price of traded goods to drive real exchange rates, regardless of exchange rate regimes.

JEL classifications: F41, F42

Keywords: Real exchange rate, exchange rate regime and trade costs.

*Department of Economics, Purdue University, West Lafayette, IN 47907-1310. Email: knaknoi@krannert.purdue.edu. An earlier version of this paper appears as Stanford Institute of Economic Policy Research Discussion Paper No. 03-017, Stanford University (January, 2004). I am indebted to comments from David Hummels, Michael Kumhof, Douglas Laxton, Slavi Slavov, Mark Wright, Asaf Zussman and seminar participants at Stanford, Purdue, Federal Reserve Bank of New York, International Monetary Fund, 2004 NBER Summer Institute (IFM) and 2005 SED Annual Meeting.

1 Introduction

The empirical literature on real exchange rate (RER) volatility has found that some RERs are driven by the relative price of traded goods (traded RER) and some by the relative, relative price of non-traded to traded goods (non-traded RER).[1] The question of which relative price dominates RER has important implications for international shock transmissions. The theoretical literature has proposed the following determinants: price stickiness and invoice currency, pricing to market, trade costs, and endogenous tradability.

This paper presents an alternative theory with an emphasis on endogenous tradability and exchange rate regimes. The role of exchange rate regimes was first documented by Mendoza (2000). He finds that the contribution of non-traded RERs to the variance of Mexico-U.S. RERs is over 30 percent higher in the period of fixed than the period of flexible exchange rates. He also finds that the correlation between traded and non-traded RERs is negative under the period of fixed but positive under the period of flexible exchange rates. Recently, Burstein et al. (2005) find that RERs after devaluations closely track the non-traded RERs.

Before describing the model, I confirm both Mendoza's (2000) and Burstein et al.'s (2005) findings using RERs of European and emerging economies. Although the contribution of the non-traded RER is quite high after some currency crises, it is still lower than it was before devaluations by 31 percent on average. I also document a similar pattern in a cross-section of 595 RERs. The country pairs of which the RER is relatively quite influenced by the non-traded RER have relatively stable nominal exchange rates.

My main contribution is a stochastic general equilibrium model explaining these stylized

[1] Engel (1999) finds that the U.S. RERs are driven by the traded RER. His finding is supported by Chari et al. (2002) and Betts and Kehoe (2001a). Mendoza (2000), Betts and Kehoe (2001a) and Burstein et al. (2005) document importance of the non-traded RER.

facts. Its key features are heterogeneous productivity, trade costs and sticky wages. The specialization pattern follows comparative advantage as in Dornbusch et al. (1977) and some goods are not traded. Aggregate shocks create dynamics of comparative advantage and cause firms to transit into or out of exporting. These transitions generate asymmetry in aggregate productivity variations across sectors and amplify the expenditure switching. In particular, productivity variations in the non-traded sector are positively correlated with but larger than those in the export sector, because firms in the non-traded sector are subject to both domestic and foreign competition.

I analytically show that, given a condition, the contribution of the traded RER is increasing in the covariance between terms of trade and productivity differentials in the non-traded and export sectors. A large covariance implies that wage inflation is offset by productivity gain more in the non-traded than in the export sector. The covariance measures the degree to which shocks are transmitted to prices in the export sector relative to those in the non-traded sector. The difference in the covariance across exchange rate regimes is essentially the expenditure switching effect of exchange rates generated by endogenous tradability.

To highlight the importance of endogenous tradability, I analytically show that the correlation between traded and non-traded RERs is perfect in the absence of endogenous tradability. The perfect correlation results from the fact that shocks are transmitted to both RERs only through terms of trade when the trade pattern is exogenous. However, Engel (1999) and Mendoza (2000) find that the correlation in the data is not perfect. Hence, my model offers a better explanation for the observed correlation than the existing framework.

In the quantitative part, I calibrate the model under fixed and flexible exchange rate regimes, to match the behaviors of RERs before and after devaluations in the data. Given interest rate shocks, exchange rate stability reduces the covariance and raises the contribution of the non-traded RER by 31 percent, as observed in the data. The predicted correlation between traded and non-traded RERs is positive when exchange rate is flexible but negative when exchange rate is fixed, as found by Mendoza (2000). Consistent with Mussa (1986) and Stockman (1989), exchange rate stability substantially reduces the overall RER volatility. The RER volatility under a flexible exchange rate regime is much larger than output volatility, in line with the exchange rate disconnect puzzle in Obstfeld and Rogoff (2000b).

However, shocks on total factor productivity (TFP) do not create significant differences across regimes, and the contribution of the traded RER exceeds 50 percent. Favorable TFP shocks prevent high-cost exporters from exiting and reduce the average productivity, besides reducing terms of trade as in standard models. Given that productivity variations are larger in the non-traded than the export sector, TFP shocks produce positive covariations between terms of trade and productivity differentials. My model suggests that TFP shocks could explain why the U.S. RERs are driven by the traded RER in Engel (1999).

Besides the mechanism in my model, exchange rate stability reduces volatility of the traded RER when exporters practice local currency pricing, as in Devereux and Engel (2002). My model should be viewed as complementary to local currency pricing, since its evidence is limited to developed countries.[2] On the other hand, the evidence for transitions in and out of exporting applies to both developed and developing countries.[3]

My study is related to recent work on endogenous tradability. In Betts and Kehoe (2001b) and Bergin and Glick (2003), trade pattern depends on heterogeneity of trade costs, and asymmetry in price variations arises from variations of aggregate trade costs. My work

[2] See Goldberg and Knetter (1997) and Campa and Goldberg (2005).
[3] See Aitken et al. (1997), Bernard and Jensen (2004), Besedes and Prusa (2003) and Das et al. (2001).

is more similar to Ghironi and Melitz (2005), in which asymmetric price variations also arise from productivity variations. These models are isomorphic in producing asymmetric price variations, and predict that the traded-RER drives RER in response to TFP shocks. My innovation is showing that nominal shocks raise the importance of the non-traded RER when exchange rate is fixed. My model implies that the assumption that the relative price of non-traded to traded goods drives RER, such as in Calvo (1986), is useful for studying stabilization policy in the wake of nominal shocks. However, deviations from the law of one price for traded goods are necessary for studying the effects of real shocks. Moreover, my study is the first one that shows that endogenous tradability is essential for generating the observed correlation between traded and non-traded RERs.

I present the stylized facts in the next section. The model is developed in Section 3. Section 4 discusses the calibration results. I conclude the study in Section 5.

2 Stylized facts and related literature

RER is defined as the relative price level of two economies,

$$Q_t = \frac{S_t P_t^\star}{P_t},$$

where S_t is nominal exchange rate and P_t is the consumer price index (CPI). The superscript * denotes the foreign variables. Let $P_{t,T}$ be the traded-goods price index. Define the traded RER as the relative price of traded-goods baskets, $Q_{t,T} = S_t P^\star_{t,T}/P_{t,T}$. The non-traded RER is the residual $Q_{t,N} = Q_t/Q_{t,T}$, as in Engel (1999) and Chari et al. (2002).

The log of RERs are filtered with the Baxter-King filter with 12 leads and lags, passing the cyclical components lasting from 6 to 32 quarters. Let lowercases denote the detrended series. We can decompose the RER variance as $\sigma^2(q_t) = \sigma^2(q_{t,T}) + \sigma^2(q_{t,N}) + 2\sigma(q_{t,T}, q_{t,N})$, where σ denotes standard deviation or covariance. Define the contribution of the traded RER, v_T, or that of the non-traded RER, v_N, as follows.

$$v_i = \frac{\sigma^2(q_{t,i}) + \sigma(q_{t,i}, q_{t,-i})}{\sigma^2(q_t)}, \quad i \in (T, N) \tag{1}$$

v_i may take negative values, since the covariance term is negative by construction.

First, I focus on the variance decomposition around the following currency crises: Brazil in 1999, Finland in 1992, Korea in 1997, Indonesia in 1997, Mexico in 1994, Philippines in 1997, Sweden in 1992, Thailand in 1997 and United Kingdom in 1992. I define the RERs of Finland, Sweden and United Kingdom against Germany, and the rest against the U.S.

I construct quarterly RERs from various databases. Exchange rate series are from the World Currency Report provided by Reinhart and Rogoff (2004) and International Financial Statistics (IFS). The Report has an advantage, since it tracks market rates while the IFS only records official rates. However, the IFS covers a longer period. I use two measures of traded-goods prices: producer price index (PPI) and a geometric average of import and export unit values (UV). Both series are from the IFS and have different drawbacks. The PPI is partly influenced by prices of local inputs, creating a downward bias in the contribution of the non-traded RER. However, it is available in a large sample. On the other hand, the UV series proxy prices of actual traded goods but we have no data on the expenditure share of imports and exports. I assume that the share is 0.5 each. These series produce 4 different datasets. Dataset 1 uses market exchange rates and PPI. Dataset 2 uses market exchange rates and UV. Dataset 3 uses official exchange rates and PPI. Dataset 4 uses official exchange rates and UV. The number of crises in each datasets is 3, 3, 9 and 6, respectively.

I decompose the RER variance over the course of 12 quarters before and after devaluations. Figure 1 plots the contributions of non-traded RERs after devaluations against those before devaluations. Letters indicate first initials of country names. The numbers index the dataset. Figure 1 presents two facts. First, almost all observations lie below the 45-degree line, meaning that the contributions of non-traded RERs fall following devaluations. The contribution rises following only two crises: Philippines and Thailand both in 1997. On average, the contribution before and after devaluations is 50 percent and 19 percent, respectively. The 31-percent difference is in the range in Mendoza (2000), which is from 30 to 70 percent. He uses disaggregated price series to decompose the variance of Mexico-U.S. RERs. The Mexico-U.S. RER in my dataset, however, is only marginally influenced by devaluations.

The other fact is that the use of UV in Datasets 2 and 4 results in higher contributions of non-traded RERs than the use of PPI in Datasets 1 and 3. This contrast is in line with Burstein et al. (2005), who argue that the UV-based price index is a better measure of traded-goods prices than the domestic price index. They use a similar UV-based index and find that the non-traded RER plays a more important role than the traded RER after devaluations. This is also the case for four out of six observations in Dataset 4. However, they do not compare the periods before and after devaluations. My comparison indicates that exchange rate flexibility reduces the importance of non-traded RERs.

Next, I expand all datasets to a broad cross-section of countries. Datasets 1 and 3 are extended to 35 countries and 595 RERs, and Datasets 2 and 4 to 22 countries and 231 RERs.[4] Datasets 1 and 2 covers from the first quarter of 1980 to the end of 1998. Datasets 3 and 4 covers from the first quarter of 1980 to the second quarter of 2005. I compute the standard deviation of quarterly depreciation as a measure of exchange rate volatility. In Figure 2, I plot the contribution of the non-traded RER against volatility of market exchange rates.

[4] Countries in the sample are listed in the appendix.

Panels A and B use the PPI and the UV, respectively.

Figure 2 summarizes three facts. First, as in Figure 1, the contribution of the non-traded RER is higher when calculated with the UV index than with the PPI. Also, the observations with relatively high exchange rate volatility have relatively small contributions by the non-traded RER. Lastly, the observations with relatively low exchange rate volatility are much more heterogeneous, and some of them have relatively large contributions of the non-traded RER. Some RERs are clearly driven by non-traded RERs. The standard deviation of depreciation of the observations with relatively large contributions of the non-traded RER is lower than 10 percent in both panels. When I switch the exchange rate series to official exchange rates in Figure 3, these patterns remain.

Figures 2 and 3 alone do not relate exchange rate regimes to the variance decomposition, since exchange rate may be stable even when a flexible exchange rate regime is adopted. However, Figure 1 strongly suggests the role of exchange rate regimes. It raises an important question: what mechanism could cause a flexible exchange rate system to suppress the role of the non-traded RER? Devereux and Engel (2002) propose that local currency pricing reduces the degree of exchange rate pass-through to import prices and let exchange rate fluctuations raise volatility of the traded RER. However, the evidence for local currency pricing is limited to developed countries (Goldberg and Knetter, 1997; Campa and Goldberg, 2005), while my datasets also include developing countries.

The literature has recently turned to questions that are relevant to both developed and developing countries. How do goods are classified as traded or non-traded? Does the composition of trade respond to shocks? The questions are motivated by the evidence for dynamics of exporting decision. Aitken et al. (1997) find that 10 percent of Mexican manufacturers change their export status in 3 years. In Das et al. (2001), the annual rate of entry into exporting in Columbia is 9 percent, and the exit rate is 7 percent. For the U.S., Bernard and Jensen (2004) estimate that the entry and the exit rates are 14 and 13 percent, respectively. Within 3 years 18 percent of non-exporters begin to export and 20 percent of exporters stop. Also, Besedes and Prusa (2003) finds the median duration that a country exports a product to the U.S. ranges from 2 to 4 years.

The evidence suggests that the dynamics of exporting decision are relevant to the dynamics of aggregate price and RER. This idea is not new, however. Baldwin (1988) has used a partial equilibrium model with monopolistic competition to show that sunk costs produce hysteresis in export and persistent RERs. However, the early studies that incorporate exporting decision into a general equilibrium analysis take a simpler approach.

Betts and Kehoe (2001b) assume a multi-sector economy with heterogeneous trade costs in a two-country model. Bergin and Glick (2003) assume heterogeneous trade costs in a small-open-economy model. In both papers, transitions in and out of exporting in response to TFP shocks act as another adjustment channel and damp price variations. They quantitatively show that the mechanism reduces the volatility of the non-traded RER. However, these models are at odds with trade theory in which demand or supply factors are the prime determinants of trade. Moreover, there is a limit to discussing interactions between trade and price in a small open-economy model taking export prices as given.

The first study incorporating a rigorous trade theory into a general equilibrium model is by Ghironi and Melitz (2005). They assume sunk costs, monopolistic competition and heterogeneous productivity as in Melitz (2003) and Baldwin (1988). Their model generates low volatility in the non-traded RER and persistent RERs in response to TFP shocks.

However, similar to the previous studies, it overlooks the role of exchange rate regimes.

3 The model

This section develops a stochastic dynamic general equilibrium model of endogenous tradability. The model adopts the theory of comparative advantage by Dornbusch et al. (1977). It is simpler than Ghironi and Melitz (2005), but retains the essence that productivity is crucial for exporting decision. Its other innovation is the introduction of sticky wages so that the exchange rate is relevant. There are two countries: home and foreign. There exists a continuum of goods indexed by z, where $z \in [0, 1]$. Residents in the two countries consume all goods. Although differences in the composition of consumption can play a role in RER fluctuations, I abstract from that possibility to focus on the role of composition of trade.

3.1 Firms and specialization pattern

There are a large number of homogeneous firms taking price as given in each industry z. Thus, invoice currency is irrelevant. Let the subscript t denote the period. Producer price, $\bar{p}_t(z)$, is in the seller's currency. The representative firm in each industry has the constant-returns-to-scale technology,

$$y_t(z) = X_t a_t(z) l_t(z). \tag{2}$$

$y_t(z)$ is output. X_t is TFP. $a_t(z)$ is the industry-specific productivity. $l_t(z)$ is labor input, of which unit cost is W_t. Cost minimization yields marginal-cost pricing.

$$\bar{p}_t(z) = W_t/(X_t a_t(z)) \tag{3}$$

Similar equations apply to the foreign firms. The superscript * denotes the foreign variables.

To prevent an unrealistically high rate of transitions in and out of exporting, I introduce a cost of beginning exporting, denoted by $\Phi_{t,a}(z)$. It represents additional costs such as marketing or distribution costs. I model it as an iceberg cost which reduces productivity and drives up price, $a_t(z) = (1 - \Phi_{t,a}(z))a_{ss}(z)$, where the subscript ss denotes the steady state. It is increasing in deviations of the steady-state relative productivity of the previously least-competitive industry from that of the currently least-competitive one. This assumption ensures that the cost is zero in the steady state. Let us define the industry-specific *relative productivity* as $A_t(z) = a_t(z)/a_t^\star(z)$, the set of new exporting industries at home as $\mathbf{Z}_t^n$, and the set of disappearing exporting industries at home as $\mathbf{Z}_t^d$. Let z_t^l and z_t^h denote the endogenously-determined least-competitive industry in each country. Let ϕ_a be a parameter and $\phi_a > 0$. Then, the entry cost in each country is given by the following.

$$\Phi_{t,a}(z) = \begin{cases} \phi_a \left[A_{ss}(z_{t-1}^l)/A_{ss}(z_t^l) - 1 \right] & : \quad z \in \mathbf{Z}_t^n \cup \mathbf{Z}_t^d \\ 0 & : \quad otherwise \end{cases}$$

$$\Phi_{t,a}^\star(z) = \begin{cases} \phi_a \left[A_{ss}(z_{t-1}^h)/A_{ss}(z_t^h) - 1 \right] & : \quad z \in \mathbf{Z}_t^{n\star} \cup \mathbf{Z}_t^{d\star} \\ 0 & : \quad otherwise. \end{cases}$$

International trade is subject to iceberg-type trade costs melting a fraction τ of goods. Define relative wage as $\omega_t = W_t/S_tW_t^\star$, and relative TFP as $\chi_t = X_t/X_t^\star$. Dornbusch et al. (1977) shows that if $A_t(z)$ is monotonic, $\partial A_t/\partial z < 0$ and $0 < \tau < 1$, then there is a unique solution for z_t^l and z_t^h such that $0 < z_t^l < z_t^h < 1$ and the following conditions hold.

$$A_t(z_t^h) = (1-\tau)\frac{\omega_t}{\chi_t} \tag{4}$$

$$A_t(z_t^l) = \frac{\omega_t}{\chi_t(1-\tau)} \tag{5}$$

z_t^l and z_t^h characterize the specialization pattern as follows. The home country produces the goods $z \in [0, z_t^h]$ and exports the goods $z \in [0, z_t^l]$. The foreign country produces the goods $z \in [z_t^l, 1]$ and exports the goods $z \in [z_t^h, 1]$. Both produce the non-traded goods $z \in (z_t^l, z_t^h)$ for domestic consumption. If there are new exporters at home, then $z_t^l > z_{t-1}^l$, $\mathbf{Z}_t^d = \emptyset$, and $\Phi_{t,a} > 0$ for $z \in \mathbf{Z}_t^n$. If some home exporters exit, then $z_t^l < z_{t-1}^l$, $\mathbf{Z}_t^n = \emptyset$ and $\Phi_{t,a} < 0$ for $z \in \mathbf{Z}_t^d$. Consequently, the entry cost raises the slope of the relative productivity schedule and productivity of the exporters relative to non-exporters.

I classify goods according to the specialization pattern, from the perspective of the home economy, into the import, the export and the non-traded sectors. Each is indicated by the subscript $i \in (F, H, N)$. Define $\mathbf{Z}_{t,F} = [z_t^h, 1]$, $\mathbf{Z}_{t,H} = [0, z_t^l]$, and $\mathbf{Z}_{t,N} = (z_t^l, z_t^h)$. The consumer prices of goods in each location are given as follows.

$$p_t(z) = \begin{cases} S_t W_t^\star / (X_t^\star a_t^\star(z)(1-\tau)) & : \quad z \in \mathbf{Z}_{t,F} \\ W_t / (X_t a_t(z)) & : \quad z \in \mathbf{Z}_{t,H} \cup \mathbf{Z}_{t,N} \end{cases} \tag{6}$$

$$p_t^\star(z) = \begin{cases} W_t^\star / (X_t^\star a_t^\star(z)) & : \quad z \in \mathbf{Z}_{t,F} \cup \mathbf{Z}_{t,N} \\ W_t / (S_t X_t a_t(z)(1-\tau)) & : \quad z \in \mathbf{Z}_{t,H} \end{cases} \tag{7}$$

If $\tau = 0$, then $\mathbf{Z}_{t,N} = \emptyset$ and $p_t(z) = S_t p_t(z)^\star$. Clearly, trade costs are crucial in this model because they create both non-traded goods and deviations from the law of one price.

3.2 Price indices and real exchange rates

There are a large number of wholesalers in each sector $i \in (F, H, N)$. The representative wholesaler bundles goods into a constant-elasticity-of-substitution (CES) composite.

$$C_{t,i} = \left[\left(\frac{1}{\delta_{t,i}} \right)^{\frac{1}{\theta}} \int_{z \in \mathbf{Z}_{t,i}} c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}},$$

where $\delta_{t,F} = 1 - z_t^h$, $\delta_{t,H} = z_t^l$, and $\delta_{t,N} = z_t^h - z_t^l$. $c_t(z)$ is demand for the good z, and θ ($\theta > 1$) is the elasticity of substitution. The CES aggregation is often used in the models of monopolistic competition with differentiated products. However, the aggregation here takes place across industries, not within an industry. Cost minimization gives the unit cost $P_{t,i}$,

$$P_{t,i} = \left[\frac{1}{\delta_{t,i}} \int_{\mathbf{Z}_{t,i}} p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

There are a large number of retailers who bundles the three baskets into final consumption in two steps. First, they bundle export and the import baskets into the traded-goods basket as a CES aggregate $C_{t,T}$.

$$C_{t,T} = \left[\left(\frac{\delta_{t,H}}{\delta_{t,H} + \delta_{t,F}} \right)^{\frac{1}{\theta}} C_{t,H}^{\frac{\theta-1}{\theta}} + \left(\frac{\delta_{t,F}}{\delta_{t,H} + \delta_{t,F}} \right)^{\frac{1}{\theta}} C_{t,F}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

This assumption is motivated by the evidence that the elasticity of substitution of traded goods is greater than one (Hummels, 2001; Anderson and van Wincoop, 2004). Cost minimization gives the traded-goods price index $P_{t,T}$ and expenditure share $s_{t,i}, i \in (F, H)$.

$$P_{t,T} = \left[\frac{\delta_{t,H}}{\delta_{t,H} + \delta_{t,F}} P_{t,H}^{1-\theta} + \frac{\delta_{t,F}}{\delta_{t,H} + \delta_{t,F}} P_{t,F}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

$$s_{t,i} = \frac{\delta_{t,i}}{\delta_{t,i} + \delta_{t,-i}} \left(\frac{P_{t,i}}{P_{t,T}} \right)^{1-\theta} s_T$$

The expenditure share depends on terms of trade defined as $\Omega_t = P_{t,H}/P_{t,F}$.

Next, the retailers bundle the traded and the non-traded baskets into final consumption as a Cobb-Douglas composite.

$$C_t = \frac{C_{t,T}^{s_T} C_{t,N}^{s_N}}{s_N^{s_N} s_T^{s_T}}$$

s_j, $j \in (N, T)$ is the exogenous expenditure share and $s_N = 1 - s_T$. I assume a constant expenditure share because Stockman and Tesar (1995) find that the expenditure share of non-traded goods is quite stable at high frequency. The CPI is the geometric average of the traded-goods and the non-traded-goods price indices.

$$P_t = P_{t,T}^{s_T} P_{t,N}^{s_N}$$

I decompose the RER into the traded and the non-traded RERs, $Q_t = Q_{t,T} Q_{t,N}$, where

$$Q_{t,T} = \frac{S_t P_{t,T}^\star}{P_{t,T}} = \left[\frac{s_{t,H}}{s_T} \left(\frac{1}{1-\tau} \right)^{1-\theta} + \frac{s_{t,F}}{s_T} (1-\tau)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (8)$$

$$Q_{t,N} = \left(\frac{P_{t,N}^\star}{P_{t,T}^\star} \right)^{s_N} \left(\frac{P_{t,T}}{P_{t,N}} \right)^{s_N}. \quad (9)$$

The traded RER is the relative price of the traded-goods basket. Its fluctuations depend on the intratemporal substitution between export and import goods, which in turn depends on terms of trade. Trade costs raise the fluctuations of the traded-RER when $\theta \neq 1$.[5] The non-traded RER is the relative, relative price of non-traded to traded goods.

3.3 Variance decomposition

Define the sector-level productivity such that the aggregate output is a monotonically increasing function of the product of TFP, productivity and labor input.[6]

$$A_{t,i} = \left[\frac{1}{\delta_{t,i}} \int_{\mathbf{Z}_{t,i}} a(z)^{\theta-1} dz \right]^{\frac{1}{\theta-1}}, i = H, N$$

For simplicity, assume that $a^\star(z) = 1$. Substitute equilibrium prices into (8) and (9) and log-linearize them around the steady state. Let $\hat{x}$ be percentage deviation of x_t from its steady state x_{ss}. Define $Z_t \equiv z_t^l/(1 - z_t^h + z_t^l) + (1 - z_t^h)/(1 - z_t^h + z_t^l)$. Define the steady-state home bias in traded-goods consumption as $hb_{ss} = s_{ss,H}/s_{ss,F}$, and the parameter

[5] If $\theta = 1$, then $Q_{t,T} = \left(\frac{P_{t,H}}{P_{t,F}}\right)^{s_H^\star/s_T^\star - s_H/s_T} (1-\tau)^{s_F^\star/s_T^\star - s_H^\star/s_T^\star} \left(\frac{s_T^{s}{}_T}{s_T^{\star s_T^\star}}\right)$, where s_i is given. Assume symmetric preferences. Then $Q_{t,T} = (1-\tau)^{(s_F - s_H)/s_T}$.

[6] See the appendix.

$\xi = hb_{ss}/(1+hb_{ss})^2[(1-\tau)^{1-\theta} - (1-\tau)^{\theta-1}] > 0$. Then,

$$\hat{\Omega}_t = \hat{\omega}_t - \hat{\chi}_t - \hat{A}_{t,H}, \tag{10}$$

$$\widehat{Q}_{t,T} = -\xi\widehat{\Omega}_t + \frac{\xi}{\theta-1}\widehat{Z}_t, \tag{11}$$

$$\widehat{Q}_{t,N} = s_N(\widehat{A_{t,N}} - \widehat{A_{t,H}}) - s_N\left[(1-\xi)\widehat{\Omega}_t, + \frac{\xi}{\theta-1}\widehat{Z}_t\right]. \tag{12}$$

ξ is the elasticity of traded RER appreciation with respect to the terms of trade. It plays a role in transmitting shocks from the terms of trade and the composition of trade to the two RERs. The correlation between the two RERs in (11) and (12) is

$$\begin{aligned} \rho_{TN} &= \frac{s_N\xi(1-\xi)\sigma^2(\widehat{\Omega}_t)}{\sigma(\widehat{Q}_{t,T})\sigma(\widehat{Q}_{t,N})} - \frac{s_N\xi\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\Omega}_t)}{\sigma(\widehat{Q}_{t,T})\sigma(\widehat{Q}_{t,N})} \\ &\quad - \frac{s_N\xi}{\theta-1}\left[\frac{\sigma(\widehat{\Omega}_t, \widehat{Z}_t) + \xi\sigma^2(\widehat{Z}_t)/(\theta-1) - \sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{Z}_t)}{\sigma(\widehat{Q}_{t,T})\sigma(\widehat{Q}_{t,N})}\right]. \end{aligned} \tag{13}$$

ρ_{TN} is governed by the terms of trade in the first term, and the composition of trade in the remaining terms. It has important implications for the variance decomposition.

Proposition 1. *If $\hat{z}_{t,l} = \hat{z}_{t,h} = 0$, then $|\rho_{TN}| = 1$.*[7]

Proposition 1 states that the two RERs are perfectly correlated when trade patterns are exogenous. The perfect correlation is intuitive, since terms of trade is the only variable transmitting shocks to both RERs in the absence of firm transitions in and out of exporting. On the other hand, when exporting decision is endogenous, the transition is the other channel for shock transmissions and breaks the perfect correlation.

Proposition 2. *If $\hat{z}_{t,l} = \hat{z}_{t,h} = 0$, then $v_N = s_N(1-\xi)/(\xi + s_N(1-\xi))$ and $\partial v_N/\partial hb_{ss} < 0$.*[8]

---

[7] *Proof*: If $\hat{z}_{t,l} = \hat{z}_{t,h} = 0$, then $\hat{Z}_t = 0$ and $\hat{A}_{t,i} = 0$. Substitute these into (11) and (12) and (13). Then $\rho_{TN} = 1$ if $\xi < 1$ and $\rho_{TN} = -1$ if $\xi > 1$. Q.E.D.

[8] *Proof*: From the proof of Proposition 1, if $\hat{z}_{t,l} = \hat{z}_{t,h} = 0$, then $\rho_{TN} = 1$ if $\xi < 1$ and $\rho_{TN} = -1$ if $\xi > 1$. Substituting these together with (11) and (12) into (1) gives v_N. Then $\partial v_N/\partial hb_{ss} = (\partial v_N/\partial\xi)(\partial\xi/\partial hb_{ss}) = -s_N[(1-\tau)^{1-\theta} - (1-\tau)^{\theta-1}](1-hb_{ss})^2/[(1+hb_{ss})^4(\xi + s_N(1-\xi))^2]$. Since $s_N > 0$ and $\tau > 0$, then $\partial v_N/\partial hb_{ss} < 0$. Q.E.D.

Proposition 2 states that when trade patterns are exogenous, the contribution of the non-traded RER is independent of transition dynamics and decreasing in the steady-state home bias in traded-goods consumption. A rise in home-bias increases the substitution effect among traded goods and raises importance of the traded RER. Propositions 1 and 2 are related. When the two RERs are perfectly correlated, shocks influence their variance without changing their relative importance. Proposition 2 highlights the role of endogenous tradability as the channel for transition dynamics to affect the variance decomposition.

We can derive the contribution of the non-traded RER for general cases from (11), (12) and (13). Define $\Sigma_j = \sigma^2(\widehat{Q}_{t,j}) + \rho_{TN}\sigma(\widehat{Q}_{t,j})\sigma(\widehat{Q}_{t,-j}), j \in (T, N)$. Then,

$$v_N = \frac{\Sigma_N}{\Sigma_T + \Sigma_N}. \tag{14}$$

Proposition 3. *If $\xi < 1$, then $\partial v_N / \partial\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\Omega}_t) < 0$, and vice versa.*[9]

Proposition 3 predicts that the contribution of the non-traded RER is decreasing in the covariance between terms of trade and productivity differentials in the non-traded and export sectors, when the terms of trade elasticity of traded RER appreciation is less than 1. The specialization pattern in (4) and (5) provides an intuition. A rise in terms of trade drives some low-productivity producers to quit exporting and thus creates productivity variations which absorb the effect of terms of trade on price. The covariance then measures the degree

[9] *Proof*: (11) and (12) and (13) imply that:

$$\begin{aligned}
\Sigma_T &= (s_N\xi(1-\xi)+\xi^2)\sigma^2(\widehat{\Omega}_t) + \frac{(1-s_N)\xi^2}{(\theta-1)^2}\sigma^2(\widehat{Z}_t) - \frac{s_N\xi(1-\xi)+\xi^2}{\theta-1}\sigma(\widehat{\Omega_t}, \widehat{Z}_t) + \frac{s_N\xi}{\theta-1}\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{Z}_t), \\
\Sigma_N &= s_N(1-\xi)(s_N(1-\xi)+\xi)\sigma^2(\widehat{\Omega}_t) + \frac{s_N(1+s_N)\xi^2}{(\theta-1)^2}\sigma^2(\widehat{Z}) - \frac{3s_N\xi(1-\xi)}{\theta-1}\sigma(\widehat{\Omega_t}, \widehat{Z}_t) \\
&\quad + \frac{3s_N\xi}{\theta-1}\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{Z}_t) - (2(1-\xi)s_N^2 + \xi s_N)\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\Omega}_t).
\end{aligned}$$

Then $\partial v_N/\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\Omega}_t) = -2((1-\xi)s_N^2 + \xi s_N)(1-\tilde{v}_N)/(\Sigma_N + \Sigma_T)$. Note that $s_N > 0$, $\Sigma_j > 0$, and $v_N > 0$. If $\xi < 1$, then $(1-\xi)s_N^2 + \xi s_N > 0$, and vice versa. Q.E.D.

of asymmetry in price variations across sectors. Proposition 3 has an implication for the stylized facts presented in the previous section. Specifically, the facts will be consistent with my model if exchange rate flexibility raises (reduces) the covariance when the terms of trade elasticity of traded RER appreciation is smaller (greater) than 1.

3.4 Sticky wages

I introduce sticky wages in order to model exchange rate. Following the open-economy macro models such as Obstfeld and Rogoff (2000a), I assume monopolistically-competitive labor market. Households are differentiated by the superscript k, $k \in [0,1]$. The set of home residents is $[0,\alpha], \alpha \in (0,1)$, and that of foreign residents is $(\alpha,1]$. I describe the home residents' optimization problem, and that in the foreign economy is the mirror image.

The household k chooses wages W_t^k, final consumption C_t^k and money balance M_t^k. Her labor supply l_t^k depends on the wages she set. She maximizes the following utility function.

$$U_t^k = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\sigma_c}{\sigma_c - 1} C_t^{k\,\frac{\sigma_c - 1}{\sigma_c}} + \frac{\chi_m}{1-\epsilon} \left(\frac{M_t^k}{P_t}_t \right)^{1-\epsilon} + \frac{\chi_l}{\mu} (1 - l_t^k(W_t^k))^{\mu} \right] \tag{15}$$

$0 < \beta < 1$, $\mu < 1$, $\sigma_c > 0$, $\epsilon > 0$. $\chi_m > 0$, and $\chi_l > 0$.

She can accumulate wealth by holding M_t^k or a one-period international bond F_t^k. The bond is denominated in the home currency and pays interest rate i_t. I assume that it is costly to adjust the bond holding, to prevent the bond holding from becoming infinitely large. Otherwise I cannot solve the model using log-linearization (Turnovsky, 1985). The portfolio adjustment cost is quadratic in deviations from the steady-state bond holdings, which is assumed to be zero, $\Phi(F_t^k/P_t) = \frac{1}{2}\phi(F_t^k/P_t)^2$, and $\phi > 0$.

The other critical assumption is that adjusting wages is also costly. The cost is analogous to the price-adjustment cost in Rotemberg (1982) and essential for generating sluggish wage adjustment. It is quadratic in deviations of wage inflation from its steady state, $h(\pi_t^{w^k}) = \phi^w\left(\pi_t^{w^k} - \pi_{ss}^{w^k}\right)^2/2$, where $\pi_t^{w^k} = W_t^k/W_{t-1}^k$ and $\phi^w > 0$. The other source of household income is government transfer T_t^k. Let Δ denote the first difference. The budget constraint requires that asset accumulation $\Delta M_t^k + \Delta F_t^k$ is the gap between income and expenditure:

$$\Delta M_t^k + \Delta F_t^k = i_t F_{t-1}^k + W_t^k l_t^k + T_t^k P_t - \left[C_t^k + \Phi(F_t^k/P_t) + h(\pi_t^{w^k})\right] P_t. \tag{16}$$

The aggregate labor supply is a CES index with elasticity η, $L_t = \left[(1/\alpha)^\eta \int_0^\alpha l_t^{k(1-1/\eta)} dk\right]^{1-1/\eta}$. Cost minimization gives the unit labor cost $W_t = \left[\frac{1}{\alpha}\int_0^\alpha W_t^{k1-\eta} dk\right]^{\frac{1}{1-\eta}}$ and labor demand facing the household k.

$$l_t^k = \frac{1}{\alpha}\left(\frac{W_t^k}{W_t}\right)^{-\eta} L_t. \tag{17}$$

The household k chooses the stochastic processes $\{F_t^k, M_t^k, W_t^k\}_{t=0}^\infty$ to maximize (15) subject to (17), (16), and the transversality condition $lim_{j\to\infty} E_t\left[F_{t+s}^k/\Pi_{s=0}^{j-1}(1+i_{t+s})\right] \geqslant 0$, taking as given the process of price $\{P_t\}_{t=0}^\infty$ and the initial conditions $(M_{-1}^k, F_{-1}^k, W_{-1}^k)$. All households face the same problem, so I drop the index k from the first-order conditions:

$$1 + \phi\frac{F_t}{P_t} = \beta(1+i_t)E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\frac{1}{\sigma_c}} \frac{P_t}{P_{t+1}}\right], \tag{18}$$

$$\left(\frac{M_t}{P_t}\right)^\epsilon = \chi_m C_t^{\frac{1}{\sigma_c}} \frac{1+i_t}{i_t}, \tag{19}$$

$$\eta\chi_l(1-l_t)^\mu = C_t^{-\frac{1}{\sigma_c}}\left((\eta-1)l_t\frac{W_t}{P_t} + \phi^w(\pi_t^w - \pi_{ss}^w)\pi_t^w\right) - E_t\left[C_{t+1}^{-\frac{1}{\sigma_c}}\phi^w(\pi_{t+1}^w - \pi_{ss}^w)\pi_{t+1}^w\right]. \tag{20}$$

Technically, we need a foreign-currency bond to discuss foreign interest rate $i_t^\star$. It is issued by the foreign government and available to only foreign residents. The bond stock is zero in equilibrium. We can use the Euler conditions to derive the interest rate parity.

$$(1+i_t^\star)\left(1 + \phi\frac{F_t^\star}{S_t P_t^\star}\right) = (1+i_t)E_t\left[\frac{S_t}{S_{t+1}}\right] + \beta(1+i_t)(1+i_t^\star)\sigma\left(\left(\frac{C_t^\star}{C_{t+1}^\star}\right)^{\frac{1}{\sigma_c}} \frac{P_t^\star}{P_{t+1}^\star}, \frac{S_t}{S_{t+1}}\right) \tag{21}$$

3.5 Exchange rate regime

Exchange rate stability may arise even when a flexible exchange rate regime is adopted. However, the stylized facts presented by Mendoza (2000), Burstein et al. (2005) and Figure 1 strongly suggest the role of exchange rate regimes. For this reason, I postulate exchange rate variability as an outcome of exchange rate regime characterized by the domestic interest rule. Under a fixed exchange rate regime, the home central bank adopts the following rule.

$$\hat{i}_t = \hat{i}^\star_t + \lambda_s \hat{S}_t + \lambda_f f^\star_t, \tag{22}$$

where $\lambda_s = 1$ and $\lambda_f = \phi(1+ss)/i_{ss}$. $\hat{S}_t$ is deviation from the target exchange rate and $f^\star_t = F^\star_t / S_t P^\star_t$. Endogenously (21) and (22) result in $\hat{S}_t = 0$. Apart from the debt term, the rule is similar to that in Benigno (2004) and Monacelli (2004). The debt enters the rule because of the technical assumption that it cannot explode. Whether it is empirically relevant has not been investigated.

Define the real gross domestic product (GDP) as $Y_t = P_t^{-1} \int_{z \in \mathbf{Z}_{t,H} \cup \mathbf{Z}_{\mathbf{t},\mathbf{N}}} p_t(z) y_t(z) dz$. Let π_t be the inflation rate. The following rule is adopted under a flexible exchange rate regime.

$$\hat{i}_t = \lambda_i \hat{i}_{t-1} + (1-\lambda_i) \left[\lambda_\pi E_t \hat{\pi}_{t+1} + \lambda_y \hat{Y}_t \right]. \tag{23}$$

I assume away shocks on the domestic interest rate rules, to control the source of shocks, so that a comparison of the effect of shocks across regimes is meaningful.

The foreign central bank adopts the same rule independent of exchange rate regime. I assume the rule adopted by Chari et al. (2002).

$$\hat{i}^\star_t = \lambda_i \hat{i}^\star_{t-1} + (1-\lambda_i) \left[\lambda_\pi E_t \hat{\pi}^\star_{t+1} + \lambda_y \hat{Y}^\star_t \right] + V^\star_t. \tag{24}$$

$V_t^\star$ is the shock on foreign interest rate which follows the AR(1) process:

$$log(V_t^\star) = \rho_i log(V_{t-1}^\star) + v_t^\star, \tag{25}$$

where $v_t^\star$ is normally distributed by $N(0, \sigma_v^2)$. The world economy is also subject to the following TFP shocks:

$$log(X_t) = \rho_x log(X_{t-1}) + \rho_x' log(X_{t-1}^\star) + u_t, \tag{26}$$

$$log(X_t^\star) = \rho_x log(X_{t-1}^\star) + \rho_x' log(X_{t-1}) + u_t^\star, \tag{27}$$

where u_t and $u_t^\star$ are jointly normally distributed by $N(0, \Sigma_u)$.

Since my focus is on exchange rate policy, I assume a simple fiscal policy that rebates seigniorage revenues to households, $T_t = (M_t - M_{t-1})P_t^{-1}$. All markets clear in equilibrium.[10] I quantitatively evaluate the model using calibration exercise in the next section.

4 Calibration

4.1 Parametrization

Table 1 summarizes the baseline parameter values. The most important ones concern the relative productivity schedule. Some studies have estimated productivities or relative productivities of manufacturing sectors (Harrigan, 1999; Yi, 2003; and Eaton and Kortum, 2002). However, specifying the schedule requires knowledge of *variations*, not levels, of productivity or variations of the set of export goods. Hummels and Klenow (2005) estimate that the long-run elasticity of the relative range of export goods with respect to relative per-capita income is 0.85. The elasticity can be driven by various factors besides comparative advantage. I assume that in the steady state $A_{ss}(z) = ne^{-\gamma z}$ and set n and γ so that the

[10]See the appendix for the market clearing conditions.

elasticity is less than 0.85. Its baseline value is 0.69. The entry-cost parameter is set to 9, so that the short-run elasticity is substantially lower than its long-run level. The short-run elasticity is 0.02 under flexible and less than 0.01 under fixed exchange rate regimes.

Except for the productivity structure, all parameters are symmetric. Hummels (2001) estimates that the freight rate varies between 4 and 13 percent of shipment value. I choose 0.15 to reflect other trade barriers such as tariffs and quotas. The expenditure share of non-traded goods is 0.5 and in line with Falvey and Gemmell (1995). The intratemporal elasticity of substitution varies from 5 to 10 in Anderson and van Wincoop (2004). I use 3 because the classification of goods in my model is quite broad.

The remaining parameters follow the business cycle literature, such as Mendoza (1999), Huang and Liu (2002) and Chari et al. (2002). The wage-adjustment parameter is chosen so that the labor contract period is 4 quarters. The interest rate rule follows Clarida et al. (2000). Standard deviation of shocks are set so that the foreign output has the same standard deviation as the Baxter-King-filtered series of the U.S. output. In the steady state, the aggregate allocation is symmetric. The home country exports 24 percent of goods, and imports 43 percent of goods. The trade-to-GDP ratio is 42 percent and close to that of the U.K. The terms of trade elasticity of traded RER appreciation is 0.16.

4.2 Baseline results

I simulate the log-linearized version of the model with the foreign interest rate shocks for 100 periods for 50 times and report the summary statistics in Table 2. The statistics are average of all simulations and statistically different from zero at the 1-percent level. The first two columns correspond to flexible and fixed exchange rate regimes, and the last one indicates their difference. I display the corresponding statistics from Section 2 as a comparison.

The first block reports the contribution of the non-traded RER. It is 64 percent under a flexible and 95 percent under a fixed exchange rate regime. The model matches the 31-percent difference reported in Section 2, although it overpredicts the level.

The second block summarizes volatility of key variables, which are RER, traded RER, non-traded RER, terms of trade, and the ratio of trade balance to GDP. The volatility is measured as the standard deviation relative to output. Consistent with Mussa (1986) and Baxter and Stockman (1989), exchange rate flexibility raises the volatility of RER. In line with the exchange rage disconnect puzzle in Obstfeld and Rogoff (2000b), the RER volatility under a flexible exchange rate regime is more than 5 times of output volatility, as in the data. The model underpredicts volatility of all variables except for the non-traded RER and the ratio of trade balance to GDP under a flexible exchange rate regime.

In the third block, consistent with Proposition 1, the correlation between traded and non-traded RERs is not perfect. Exchange rate stability reduces the correlation from 0.66 to -0.67. It is not meaningful to compare these numbers with those in my dataset, which are negative by construction. The study using disaggregated price series by Mendoza (2000) provides a usual comparison. The corresponding numbers in his study is 0.28 and -0.70, respectively. The model matches the sign and closely matches the magnitude under a fixed exchange rate regime. According to (13), the correlation is positively influenced by the volatility of terms of trade, which is reduced by exchange rate stability. That is why exchange rate stability reduces the correlation and turns it to a negative value.

The next block confirms Proposition 3, given that the terms of trade elasticity of traded RER appreciation is below 1. Exchange rate stability reduces the covariance between terms of trade and productivity differentials from 0.06 to -0.15. To gain an intuition, I rewrite the covariance using (10):

$$\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\Omega}_t) = \sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\omega}_t) - \sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{A}_{t,H}) - \sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\chi}_t). \quad (28)$$

The last term in (28) is zero given nominal shocks, so the covariance measures the difference between two covariances: that between productivity differentials and relative wage and that between productivity differentials and export-sector productivity. Both covariances are positive for the following reason. A rise in relative wage causes some home exporters to quit and some foreign firms to begin exporting. The quitting firms have lower productivity than the remaining ones, hence their exits raise the export-sector productivity. Their exits also raise the productivity in the non-traded sector in which producers are less productive. The new foreign exporters further raise the productivity of the non-traded sector by replacing low-productivity industries at home. Overall, a rise in relative wage raises both the export-sector productivity and productivity differentials between the non-traded and export sectors. [11]

Exchange rate flexibility raises volatility of relative wage and thus the covariance between terms of trade and productivity differentials. Shocks are transmitted more to prices in the export sector than those in the non-traded sector, as dynamics of comparative advantage amplify the expenditure switching between import and export goods. Fixing exchange rate reduces the effect and increases the substitution between non-traded and traded goods.

For the persistence of RER, the first-order autocorrelation of RER under flexible exchange rate regimes is 0.31. It is close to Chari et al.'s (2002) prediction of 0.4, but far below 0.71

[11] By the definition of covariance, we write the first two covariances in (28) as follows.

$$\begin{aligned} \sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\omega}_t) &= \sigma(\widehat{\omega}_t)[\sigma(\widehat{A}_{t,N})\rho(\widehat{A}_{t,N}, \widehat{\omega}_t) - \sigma(\widehat{A}_{t,H})\rho(\widehat{A}_{t,H}, \widehat{\omega}_t)] \\ \sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{A}_{t,H}) &= \sigma(\widehat{A}_{t,H})[\sigma(\widehat{A}_{t,N})\rho(\widehat{A}_{t,N}, \widehat{A}_{t,H}) - \sigma(\widehat{A}_{t,H})] \end{aligned}$$

Equations (4) and (5) imply that $\rho(\hat{z}^l_t, \widehat{\omega}_t) < 0$ and $\rho(\hat{z}^h_t, \widehat{\omega}_t) < 0$. Since $A'_t(z) < 0$, then $\rho(\widehat{A}_{t,N}, \widehat{\omega}_t) > \rho(\widehat{A}_{t,H}, \widehat{\omega}_t) > 0$ and $\rho(\widehat{A}_{t,N}, \widehat{A}_{t,H}) = 1$. From the definitions of $A_{t,H}$ and $A_{t,N}$, $\sigma(\widehat{A}_{t,N}) > \sigma(\widehat{A}_{t,H})$. Substituting these inequalities into the above expressions gives $\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\omega}_t) > 0$ and $\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{A}_{t,H}) > 0$.

in the data. The model overpredicts persistence under fixed exchange rate regimes.

To summarize, the model correctly predicts the sign of differences in summary statistics across exchange rate regimes, except for the persistence of RER. It successfully predicts the magnitude of differences across regimes in the contribution of the non-traded RER. Moreover, it matches the sign of correlation between traded and non-traded RERs.

4.3 Sensitivity analysis

This section compares the baseline model with the model of exogenous tradability in which trade pattern is given by the baseline steady state. I report the summary statistics in Table 3. They are in boldface letters when the difference across regimes is significant at the 1-percent level. I also explore the role of other dynamic properties: TFP shocks, a reduction of the wage adjustment parameter to 25 percent of its baseline value, and a reduction of the intertemporal elasticity of substitution by 25 percent.

Table 3 summarizes five results. First, when the trade pattern is exogenous, the type of shocks and the exchange rate regime are irrelevant to the variance decomposition, as predicted by Proposition 2. In this case, the traded and non-traded RERs are perfectly correlated, as predicted by Proposition 1. However, Engel (1999) and Mendoza (2000) find that the correlation in the data is not perfect. Hence, endogenous tradability is essential for generating the observed correlation. Moreover, the sign of correlation under each regime always matches with that in Mendoza (2000) when the trade pattern is endogenous.

Second, as predicted by Proposition 3, the contribution of the non-traded RER is decreasing in the covariance between terms of trade and productivity differentials. The difference in the covariance across regimes measures the expenditure switching effect of exchange rate generated by endogenous tradability, since it is zero when the trade pattern is exogenous.

Third, the traded RER dominates the non-traded RER in response to TFP shocks, regardless of exchange rate regime. From the trade pattern in (4) and (5), favorable TFP shocks have the opposite effects on productivity from relative wage inflation, because they encourage low-productivity firms to export. Hence, they reduce productivity and productivity differentials together with terms of trade. Consequently, TFP shocks create positive covariations between productivity differentials and terms of trade.[12] The covariance is statistically the same across regimes, meaning that the expenditure switching effect of exchange rate generated by endogenous tradability is small. Consistent with Betts and Kehoe (2001b), Bergin and Glick (2003) and Ghironi and Melitz (2005), my model suggests that TFP shocks could be a reason why the relative price of traded goods drives the U.S. RERs in Engel (1999).

Fourth, a reduction of the wage adjustment parameter or the elasticity of intertemporal substitution in the presence of endogenous tradability further increases the contribution of the non-traded RER under a fixed exchange rate regime. The reduction strengthens the expenditure switching through the intratemporal channel. Thus, the reduction raises the covariance between terms of trade and productivity differentials under a flexible but reduces it under a fixed exchange rate regime.

Finally, the assumption of endogenous tradability has a stabilization effect which reduces the volatility of RER and terms of trade by raising trade responses. However, the increase in the volatility of trade-balance-to-GDP ratio is not extreme and below 20 percent regardless of the type of shocks.

[12]By the definition of covariance,

$$\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\chi}_t) = \sigma(\widehat{\chi}_t)[\sigma(\widehat{A}_{t,N})\rho(\widehat{A}_{t,N}, \widehat{\chi}_t) - \sigma(\widehat{A}_{t,H})\rho(\widehat{A}_{t,H}, \widehat{\chi}_t)]$$

Equations (4) and (5) imply that $\rho(\widehat{A}_{t,N}, \widehat{\chi}_t) < 0$ and $\rho(\widehat{A}_{t,H}, \widehat{\chi}_t) < 0$. Since $A_t'(z) < 0$, then $\rho(\widehat{A}_{t,N}, \widehat{\chi}_t) < \rho(\widehat{A}_{t,H}, \widehat{\chi}_t) < 0$. From the definitions of $A_{t,H}$ and $A_{t,N}$, $\sigma(\widehat{A}_{t,N}) > \sigma(\widehat{A}_{t,H})$. Substituting these inequalities into the above expression gives $\sigma(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\chi}_t) < 0$.

Conceptually, the expenditure switching effect of exchange rate may not be increasing in the intratemporal substitution elasticity, since a high value may reduce price variations while raising quantity responses. To illustrate this point, I vary the elasticity θ from 1.5 to 5 and compute the expenditure switching effect of exchange rates generated by endogenous tradability. Figure 4 plots the effect and the difference between the contribution of the non-traded RER under flexible and fixed exchange rate regimes against the elasticity.

In Figure 4, the expenditure switching effect is increasing in θ for $\theta \in [1.5, 2.5)$, but decreasing for $\theta \in (2.5, 5]$. The non-monotonicity is in contrast with the monotonicity of the corresponding relationship in Monacelli (2004). His measure of the expenditure switching effect is the ratio of RER volatility under flexible and fixed regimes. However, even when I use the same measure, my model continues to predict the non-monotonicity. The main difference between our models is that he abstracts from non-traded goods. The existence of non-traded goods in my model produces a monotonic relationship between the expenditure switching effect and the difference in the variance decomposition across regimes, not the substitution elasticity. The exchange rate regime has a significant effect on the variance decomposition even when the substitution elasticity is as low as 1.5, which is often used in the business cycle literature such as Backus et al. (1994). With this value, exchange rate stability raises the contribution of the non-traded RER by 22 percent.

5 Concluding remarks

The model shows that the firm transitions in and out of exporting can amplify the expenditure switching effect of exchange rates in response to nominal shocks. The effect can be so large that its removal raises importance of fluctuations in the relative, relative price of non-traded to traded goods, as reported in the data. The insights in my model have three implications for open-economy macroeconomics. First, the assumption that the relative price of non-traded to traded goods drives RER, as in Calvo (1986), is useful for studying stabilization policy when shocks are predominantly nominal. However, deviations from the law of one price for traded goods are necessary for studying the effects of real shocks.

The second implication is related to the theoretical literature on trade and business cycles. Proposition 1 suggests that the correlation between traded and non-traded RERs in those models is perfect and contradicts the findings in Engel (1999) and Mendoza (2000). The imperfect correlation between the traded and non-traded RERs in my model implies imperfect correlation between relative traded consumption and relative non-traded consumption. Hence, my model could be useful for solving some puzzles related to trade and business cycles such as the consumption correlation puzzle in Backus and Smith (1993).

The final implication is related to the theory of optimum currency area. The theory views similarity of economic structure as a condition for forming a currency union, since that implies high cross-country correlation of real shocks. However, my model suggests that different specialization pattern has a stabilization effect which reduces the cost of losing monetary independence. We can extend the model to include capital to discuss the impact of exchange rate regimes on economic growth. I leave these extensions for future work.

A Appendix

A.1 Sample countries

Datasets 1 and 3: Australia, Austria, Belgium, Brazil, Canada, Chile, Colombia, Denmark, Egypt, El Salvador, Finland, Germany, Greece, India, Indonesia, Ireland, Israel, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Pakistan, Peru, Philippines, Singapore, Spain, Sri Lanka, Sweden, Switzerland, Thailand, U.K., U.S., Venezuela.

Datasets 2 and 4: Australia, Brazil, Canada, Colombia, Denmark, Finland, Germany, Greece, Ireland, Israel, Japan, Korea, Netherlands, New Zealand, Norway, Pakistan, Singapore, Spain, Sweden, Thailand, U.K. and U.S.

A.2 Aggregate Productivity and output

We can write the aggregate output in each sector $i \in (H, N)$ as a monotonic and increasing function of the effective unit of labor input defined as the product of TFP, the aggregate productivity and the aggregate labor input. First, define the aggregate output as $Y_{t,i} = P_t^{-1} \int_{z \in \mathbf{Z}_{t,i}} p_t(z) y_t(z) dz$. Next, define the world demand as $C_t^w = \alpha C_t + (1-\alpha)(1-\tau)^{\theta-1} Q_t^{\theta} C_t^{\star}$. Substitute the equilibrium prices and the definition of the aggregate productivity into the definition of the aggregate output. Then, for all i,

$$Y_{t,i} = \delta_{t,i} (X_t A_{t,i})^{\theta-1} \left(\frac{W_t}{P_t} \right)^{1-\theta} C_t^w \tag{A.2}$$

The constant-returns-to-scale technology implies the following zero-profit condition:

$$P_t Y_{t,i} - W_t l_{t,i} = 0. \tag{A.3}$$

Substitute the equilibrium wage implied by the zero-profit condition into (A.2) to get:

$$Y_{t,i} = \delta_{t,i} \frac{1}{\theta} \left(X_t A_{t,i} l_{t,i} \right)^{\frac{\theta-1}{\theta}} C_t^{w \frac{1}{\theta}}. \tag{A.4}$$

The product $A_{t,i} l_{t,i}$ is the effective labor input in the sector i.

A.3 Market clearing conditions

The market clearing conditions for goods markets and labor market in the home country are given by the following:

$$\begin{aligned} Y_{t,N} &= \alpha C_{t,N}, && \text{(A.5)} \\ Y_{t,H} &= \alpha C_{t,H} + (1-\alpha) C^{\star}_{t,H} / (1-\tau), && \text{(A.6)} \\ L_t &= l_{t,H} + l_{t,N}. && \text{(A.7)} \end{aligned}$$

Similar conditions hold in the foreign country.

Next, define the trade balance as $TB_t = P_t^{-1} \left[\int_{z \in \mathbf{Z_{t,H}}} p_t(z) c_t^{\star}(z) dz - \int_{z \in \mathbf{Z}_{t,F}} p_t(z) c_t(z) dz \right]$. Then, $TB_t = \alpha \left[Y_{t,H} - C_{t,H} P_{t,H} P_t^{-1} - C_{t,F} P_{t,F} P_t^{-1} \right]$. The budget constraint, the government budget constraint and the zero-profit conditions give the current account dynamics.

$$\Delta F_t = TB_t P_t + i_t F_{t-1} - \Phi(F_t / P_t) P_t \tag{A.8}$$

Finally, the bond market clears.

$$0 = \alpha F_t + (1-\alpha) F_t^{\star} \tag{A.9}$$

References

Anderson, J. E., and E. van Wincoop (2004): "Trade Costs," *Journal of Economic Literature*, 42(3), 691–751.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1994): "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve," *American Economic Review*, 84(1), 84–103.

Backus, D. K., and G. W. Smith (1993): "Consumption and Real Exchange Rates in Dynamic Economies with Non-traded Goods," *Journal of International Economics*, 35(3-4), 297–316.

Baldwin, R. (1988): "Hysteresis in Import Prices: The Beachhead Effect," *American Economic Review*, 78(4), 773–785.

Baxter, M., and R. G. King (1999): "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series," *Review of Economics and Statistics*, 81, 575–593.

Baxter, M., and A. Stockman (1989): "Business Cycles and the Exchange Rate Regime: Some International Evidence," *Journal of Monetary Economics*, 23, 377–400.

Bergin, P. R., and R. Glick (2003): "Endogenous Nontradability and Macroeconomic Implications," *NBER Working Papers*, 9739.

Besedes, T., and T. J. Prusa (2003): "On the Duration of Trade," *NBER Working Papers*, 9936.

Betts, C. M., and T. J. Kehoe (2001a): "Real Exchange Rate Movements and the Relative Price of Nontraded Goods," University of Minnesota and University of Southern

California, Working Paper.

——— (2001b): "Tradability of Goods and Real Exchange Rate Fluctuations," University of Minnesota and University of Southern California, Working Paper.

Burstein, A., M. Eichenbaum, and S. Rebelo (2005): "Large Devaluations and the Real Exchange Rate," *Journal of Political Economy*, 113(4), 742–784.

Calvo, G. A. (1986): "Temporary Stabilization: Predetermined Exchange Rates," *Journal of Political Economy*, 94(6), 1319–1329.

Campa, J. M., and L. Goldberg (2005): "Exchange Rate Pass-Through into Imports Prices," *Review of Economics and Statistics*, 87(4), 679–90.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2002): "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?," *The Review of Economic Studies*, 69, 533–563.

Clarida, R., J. Gali, and M. Gertler (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 115, 147–180.

Das, S., M. J. Roberts, and J. R. Tybout (2001): "Market entry costs, producer heterogeneity and export dynamics," *NBER Working Papers*, 8629.

Devereux, M. B., and C. Engel (2002): "Exchange Rate Pass-through, Exchange Rate Volatility and Exchange Rate Disconnect," *Journal of Monetary Economics*, 49, 913–940.

Dornbusch, R., S. Fischer, and P. A. Samuelson (1977): "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American*

Economic Review, 67(5), 823–839.

Eaton, J., and S. Kortum (2002): "Technology, Geography and Trade," *Econometrica*, 70(5), 1741–1779.

Engel, C. (1999): "Accounting for U.S. Real Exchange Rate," *Journal of Political Economy*, 107(3), 507–538.

Falvey, R. E., and N. Gemmell (1995): "Explaining International Differences in the Share of Services in Real Expenditure," *Economic Letters*, 47, 53–58.

Ghironi, F., and M. J. Melitz (2005): "International Trade and Macroeconomic Dynamics with Heterogeneous Firms," *Quarterly Journal of Economics*, 120(3), 865–915.

Harrigan, J. (1999): "Estimation of Cross-country Differences in Industry Production Functions," *Journal of International Economics*, 47(2), 267–293.

Huang, K. X., and Z. Liu (2002): "Staggered Price-Setting, Staggered Wage-Setting and Business Cycle Persistence," *Journal of Monetary Economics*, 49, 405–433.

Hummels, D., and P. Klenow (2005): "The Variety and Quality of a Nation's Trade," *American Economic Review*, 95(3), 704–723.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6), 1695–1725.

Mendoza, E. G. (1991): "Real Business Cycles in a Small Open Economy," *American Economic Review*, 81(4), 797–818.

——— (2000): "On the Stability of Variance Decompositions of the Real Exchange Rate Across Exchange-Rate Regimes: Evidence from Mexico and the United States," *NBER Working Papers*, 7768.

Monacelli, T. (2004): "Into the Mussa Puzzle: Monetary Policy Regimes and the Real Exchange Rate in a Small Open Economy," *Journal of International Economics*, 62(1), 191–217.

Mussa, M. (1986): "Nominal Exchange Rate Regimes and The Behavior of Real Exchange Rates: Evidence and Implications," *Carnegie-Rochester Conferences Series on Public Policy*, 25, 117–214.

Obstfeld, M., and K. Rogoff (2000a): "New Directions for Stochastic Open Economy Models," *Journal of International Economics*, 50(1), 117–53.

——— (2000b): "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," in *Macroeconomic Annual*, ed. by B. S. Bernanke, and K. Rogoff, pp. 339–412. NBER.

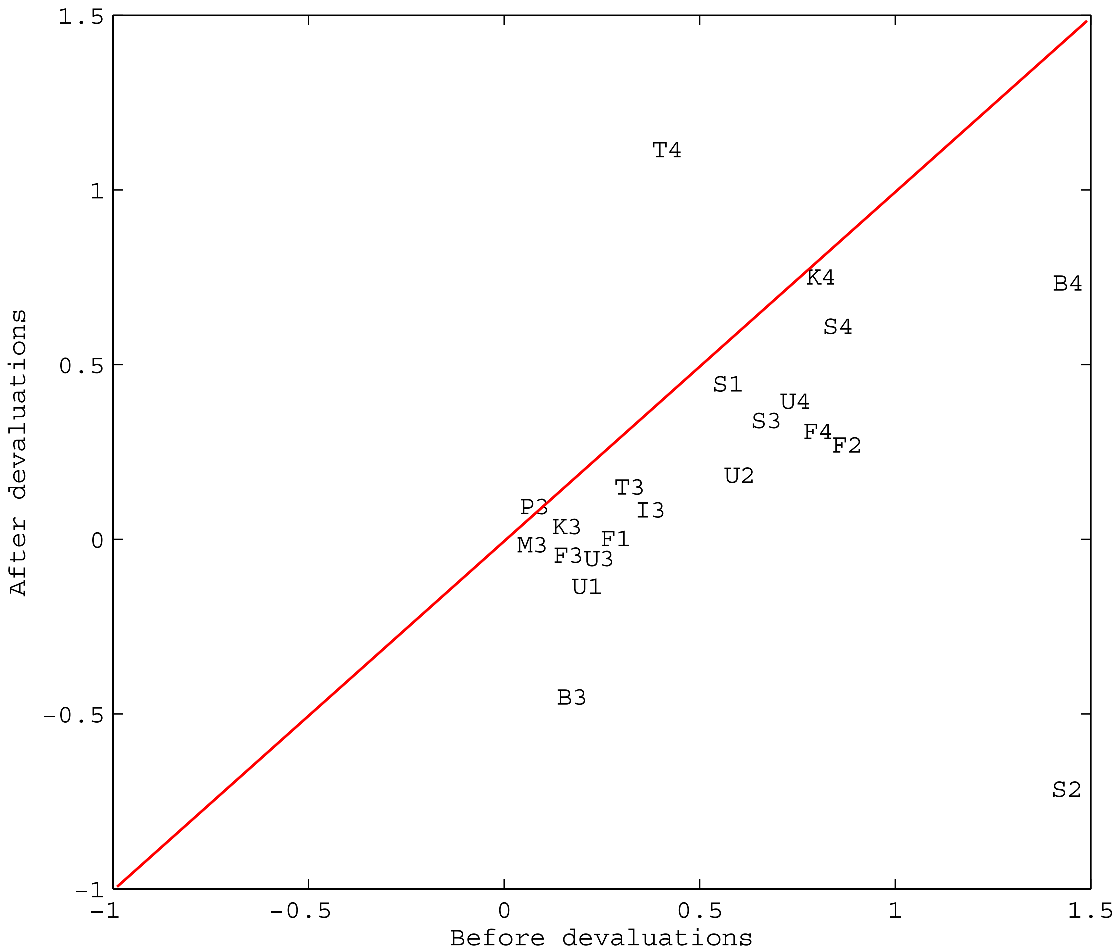
Reinhart, C., and K. Rogoff (2004): "The Modern History of Exchange Rate Arrangements: A Reinterpretation," *Quarterly Journal of Economics*, 119(1), 1–48.

Rotemberg, J. J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, 90, 1187–1211.

Stockman, A. C., and L. L. Tesar (1995): "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining Comovements," *American Economic Review*, 85(1), 168–185.

Yi, K. M. (2003): "Can Vertical Specialization Explain the Growth of World Trade?," *Journal of Political Economy*, 111(1), 52–102.

Figure 1: Contribution of non-traded RER and devaluations.



Notes: Letters are first initials of country name. The numbers index the dataset: 1 uses market exchange rate and PPI; 2 uses market exchange rate and UV index; 3 uses official exchange rate and PPI; and 4 uses official exchange rate and UV index.

Figure 2: Contribution of non-traded RER and volatility of market exchange rates

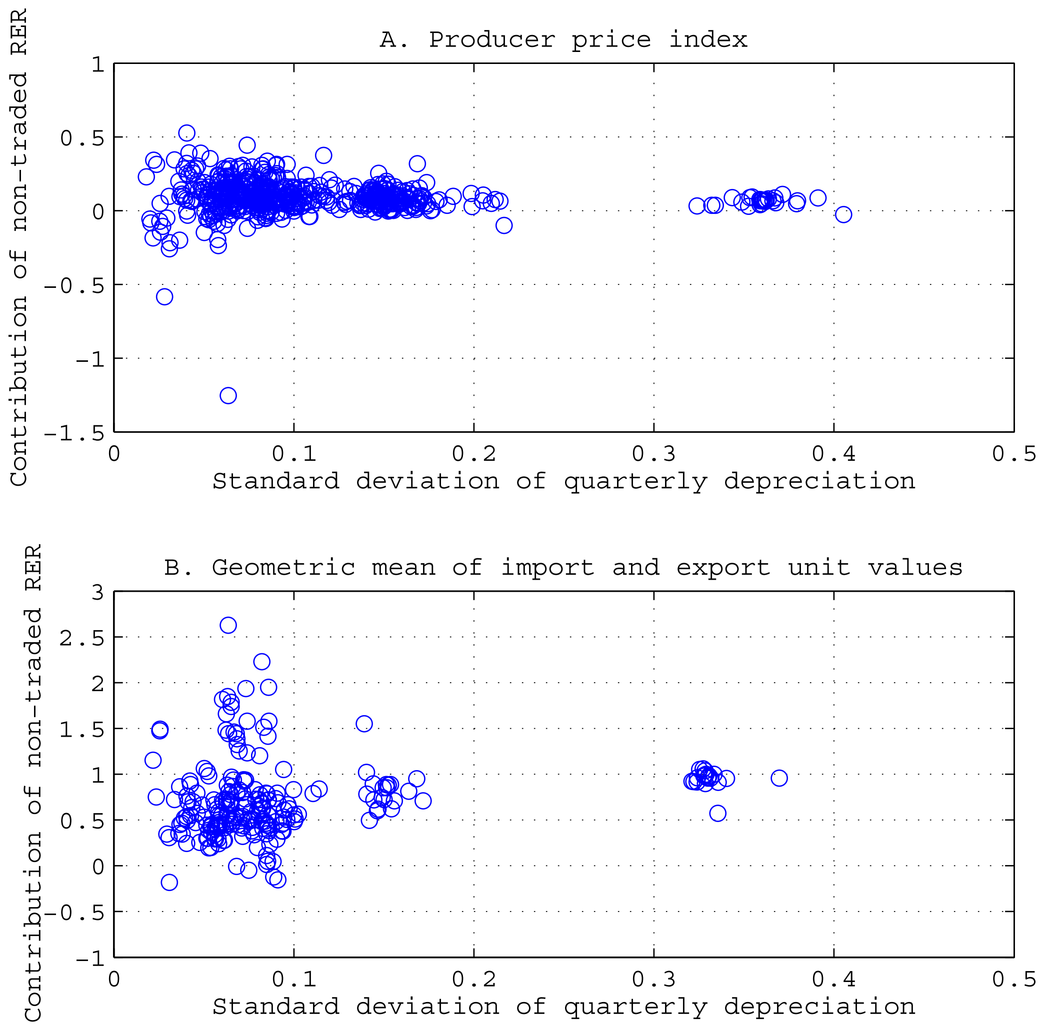


Figure 3: Contribution of non-traded RER and volatility of official exchange rates

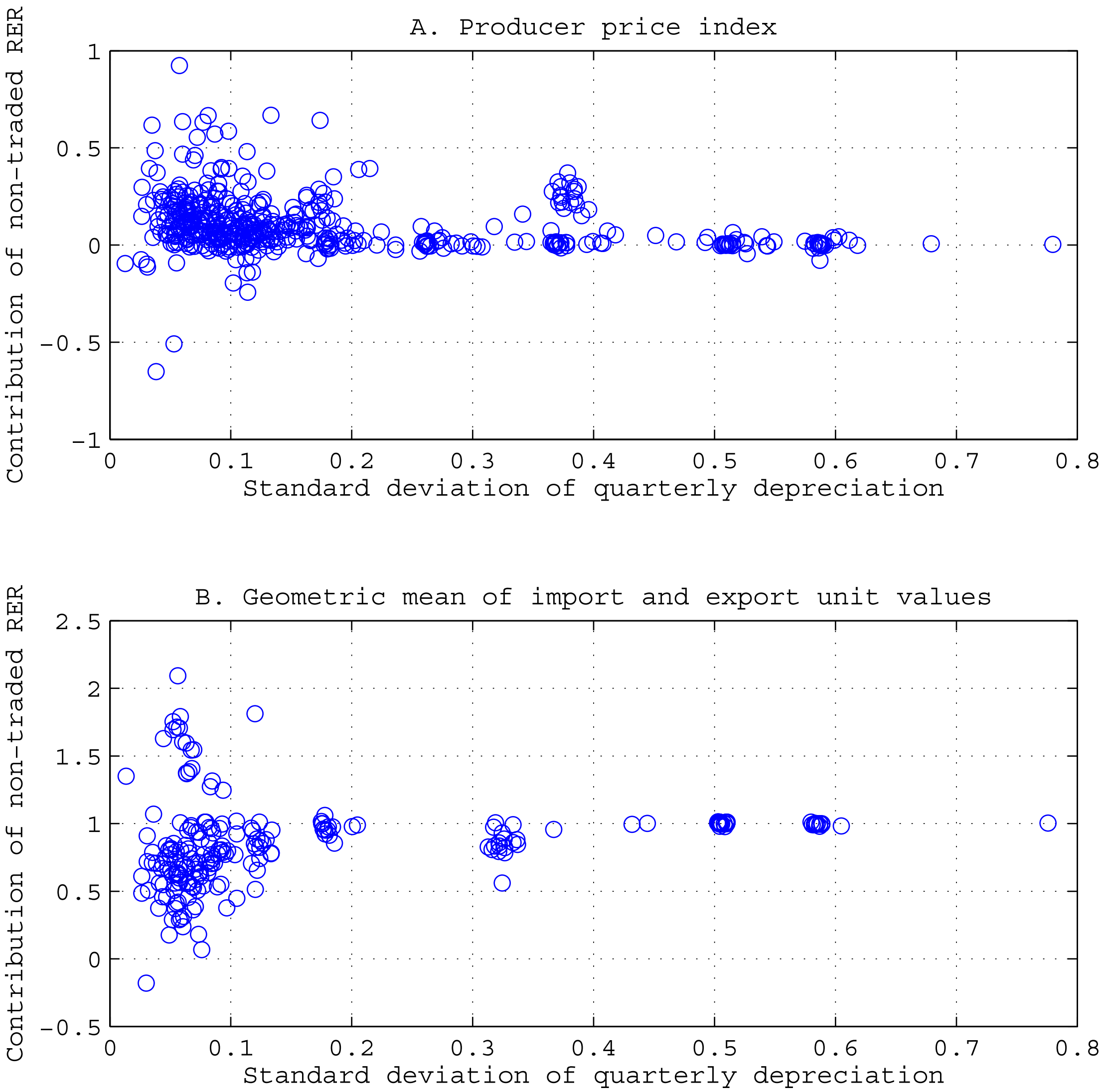
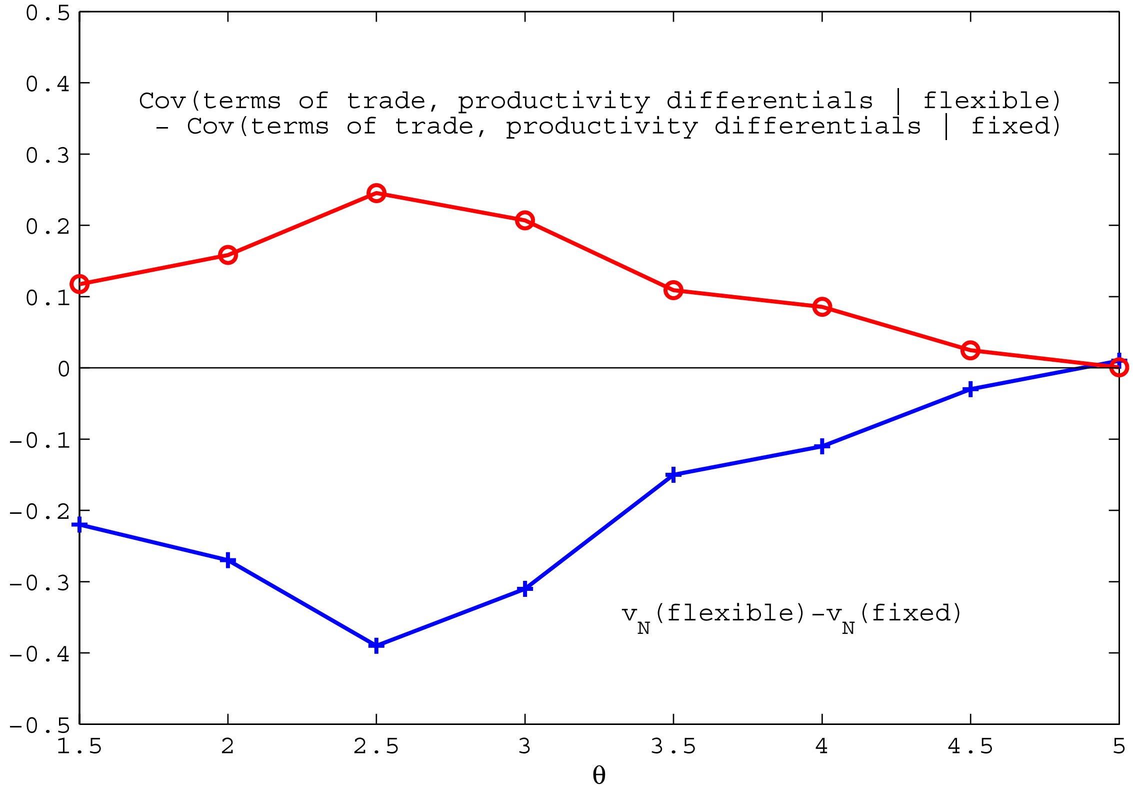


Figure 4: Expenditure switching effect of exchange rates and elasticity of substitution



Note: v_N denotes the contribution of the non-traded RER.

Table 1: Benchmark parameter values

Parameters	Value
International Trade	
Country size	$\alpha = 0.5$
Relative productivity	$n = 1.5, \gamma = 1$
Entry-cost parameter	$\phi_a = 9$
Trade costs	$\tau = 0.15$
Households	
Intratemporal elasticity of substitution	$\theta = 3$
Intertemporal elasticity of substitution	$\sigma_c = 0.2$
Discount factor	$\beta = 0.99$
Elasticity of labor supply	$\mu = 1 - 1/\sigma_c$
Interest semi-elasticity of money demand	$1/\epsilon = 0.39$
Portfolio adjustment cost	$\phi = 0.00074$
Elasticity of substitution of labor	$\eta = 2$
Wage adjustment cost	$\phi^w = 5.8935$
Monetary policy	
Steady-state inflation	$\pi_{ss} = \pi^{\star}_{ss} = 1.0358^{1/4}$
Interest rate rule	$\lambda_i = 0.79$, $\lambda_\pi = 2.15$, $\lambda_y = 0.93$
Interest rate shock	
Persistence	$\rho_v = 0$
Volatility	$\sigma_v = 0.02$
Productivity shock	
Persistence	$\rho_x = 0.95$
Spillover	$\rho'_x = 0$
Volatility	$\sigma_u = \sigma_{u^\star} = 0.01$, $\sigma_{u,u^\star} = (0.25)0.01^2$

Table 2: Baseline calibration

	Flexible	Fixed	Flexible - Fixed
Contribution of non-traded RER			
Model	0.64	0.95	-0.31
Data	0.19	0.50	-0.31
Standard deviation			
RER			
Model	5.19	0.11	5.08
Data	5.87	2.66	3.21
Traded RER			
Model	2.15	0.11	2.05
Data	5.95	2.61	3.34
Non-traded RER			
Model	3.52	0.15	3.37
Data	2.66	1.95	0.71
Terms of trade			
Model	5.68	0.12	5.56
Data	6.83	2.53	4.30
Trade balance to GDP			
Model	3.63	0.17	3.46
Data	2.05	0.59	1.46
Correlation of traded and non-traded RERs			
Model	0.66	-0.67	1.33
Data (Mendoza, 2000)	0.28	-0.70	0.98
Covariance of terms of trade and productivity differentials			
Model	0.06	-0.15	0.21
Persistence of RER			
Model	0.31	0.84	-0.53
Data	0.71	0.65	0.06

Notes: The statistics are averages of 50 simulations and statistically different from zero at the 1-percent level. Covariance between terms of trade and productivity differentials is measured relative to variance of terms of trade. Standard deviation is measured relative to that of output.

Table 3: Endogenous tradability vs. exogenous tradability

	Endogenous		Exogenous	
	Flexible	Fixed	Flexible	Fixed
Contribution of non-traded RER				
Baseline	**0.64**	**0.95**	0.72	0.72
TFP shocks	0.43	0.47	0.72	0.72
Low wage-rigidity	**0.65**	**1.04**	0.72	0.72
Low intertemporal elasticity	**0.65**	**1.03**	0.72	0.72
Correlation of the traded- and non-traded RERs				
Baseline	**0.66**	**-0.67**	1	1
TFP shocks	0.04	-0.04	1	1
Low wage-rigidity	**0.73**	**-0.62**	1	1
Low intertemporal elasticity	**0.69**	**-0.64**	1	1
Covariance of terms of trade and productivity differentials				
Baseline	**0.06**	**-0.15**	0	0
TFP shocks	0.21	0.18	0	0
Low wage-rigidity	**0.06**	**-0.20**	0	0
Low intertemporal elasticity	**0.07**	**-0.21**	0	0
Standard deviation of RER				
Baseline	**5.19**	**0.11**	6.97	0.27
TFP shocks	2.22	1.94	4.53	3.85
Low wage-rigidity	**5.17**	**0.22**	6.66	0.43
Low intertemporal elasticity	**5.02**	**0.18**	6.62	0.43
Standard deviation of terms of trade				
Baseline	**5.68**	**0.12**	7.62	0.29
TFP shocks	2.43	2.13	4.89	4.15
Low wage-rigidity	**5.66**	**0.24**	7.27	0.47
Low intertemporal elasticity	**5.48**	**0.20**	7.22	0.46
Standard deviation of trade-balance-to-GDP ratio				
Baseline	**3.63**	**0.17**	3.49	0.15
TFP shocks	2.36	2.23	2.18	1.86
Low wage-rigidity	**3.46**	**0.27**	3.34	0.25
Low intertemporal elasticity	**3.45**	**0.25**	3.35	0.25

Notes:
1. Boldface numbers indicate that the null that the statistics are the same across exchange rate regimes is rejected at the 1-percent level.
2. Standard deviation is measured relative to that of output.