The Equilibrium Exchange Rate in a Bayesian State-Space Model: An Application to Australia*

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Abstract

The equilibrium exchange rate is a closely scrutinized variable in international finance and monetary economics. A model to estimate an equilibrium exchange rate is proposed in this paper. It consists of several building blocks: a state-space structure, uncovered interest parity and the equilibrium exchange rate determinants. Prior information about the impact of the determinants is used when Bayesian estimation of the model is carried out. The estimates reveal that on average the Australian dollar was overvalued by about 3 percent during the period 1984-2004. The major overvaluations of the AUD/USD exchange rate took place during the period 1986-1988, and around the years 1990, and 2003. The information content of the acquired estimates of exchange rate misalignments appears to be superior to the purchasing parity equivalent.

Keywords: Exchange-Rate Modeling, Equilibrium Exchange Rate, State-Space Model, Bayesian Estimation and Inference, Australian Dollar.

JEL Classification: F31, F37, F41, C11

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1 Introduction

The equilibrium exchange rate is a closely scrutinized variable in international economics as it is used in forecasting models of exchange rates (see e.g. Cheung at al., 2002), as a basis for determining optimal central parity for exchange rates' fixing (see ECB, 2003), and to estimate the size of misalignments that become an obstacle to economic growth (see e.g. Razin and Collins, 1997). However, the equilibrium exchange rate is yet another unobserved (latent) variable, comparable to potential output or equilibrium velocity of money.

Estimations of equilibrium exchange rates are conducted mostly in frameworks developed for real exchange rates rather than nominal exchange rates which are of greater interest to financial markets and in some instances to policy makers as well. There is extensive literature dealing with theoretical foundations for real exchange rates and estimation of implied econometric models. For a comprehensive summary of this literature see Driver and Westaway (2005) and MacDonald (2000). The turn towards modeling real, rather than nominal, exchange rates was perhaps brought about by the fact that fundamentals, in general, fail to successfully explain the movements in nominal exchange rates (e.g. Cheung at al., 2002). By the same token, inflation volatility is sufficiently low so that over short horizons information about the outlook for the real exchange rates also sheds light on their nominal counterparts.

In this paper I propose a simple model to estimate equilibrium nominal exchange rates based on the behavioral approach to equilibrium exchange rates (see e.g. Clark and MacDonald, 1998). The proposed model has a structure of a state-space model that proved to be efficient in estimation of potential outputs and equilibrium money velocities. This setup is a novelty to the literature dealing with estimation of equilibrium exchange rates. It starts from the uncovered interest parity which is the basis of the behavioral approach to equilibrium exchange rate. Next, some well established determinants of the equilibrium exchange rates are included in the model to be the driving force for the fundamental value of the nominal exchange rate. In addition, theoretical implications and empirical findings related to the selected determinants of the equilibrium exchange rate are employed as prior information in Bayesian estimation and inference on the model. In next paragraphs I briefly discuss some of the recent findings concerning the building blocks of the proposed model – uncovered interest parity (UIP) and selected determinants of equilibrium exchange rates.

Chinn and Meredith (2003) provide some explanation of the troublesome empirical performance of UIP. Based on their estimation and simulation of an open economy policy model they conclude that the failure of UIP over short horizons can be attributed to endogenous reactions of monetary policy to stochastic disturbances in exchange rate markets (see also Engle and West, 2005). On the other hand, over the long term, exchange rate movements are driven by fundamentals resulting in a relationship between interest rates and exchange rates rather consistent with UIP. Nelson and Young-Kyu (2001) also find some evidence that UIP is more consistent with long-run dynamics of the exchange rate when using several methods of exchange rate de-trending. Their findings are especially favorable for the UK pound when purchasing power parity (PPP) is used as the terminal condition in their model.

The theory and empirics offer several determinants of the equilibrium exchange rate. Ones of the most prominent are the productivity differential and the terms of trade which will be used in this paper. The theory is that an improvement in the terms of trade of the domestic economy and an increase in productivity of the domestic economy relative to the foreign economy result in a domestic currency appreciation relative to the foreign currency.

Theoretical exposition of the impact of the terms of trade on the real exchange rate can be found for instance in De Gregorio and Wolf (1994) or Benigno and Thoenissen (2002). The empirical findings of a domestic currency appreciation in response to an improvement in the terms of trade include e.g. Begchi et al. (2004), Djoudad et al. (2000), Clark and MacDonald (1998), Gruen and Kortian (1996), Amano and van Norden (1995) and Gruen and Wilkinson (1994). The theory behind the impact of the productivity differential on the exchange rate is associated with the well-known Balassa-Samualson effect (Balassa, 1964; Samuelson, 1964). The effect of the productivity differential on the exchange rate was examined by many researchers including a recent panel data study for selected OECD countries of Maeso-Fernandez et al. (2004) and MacDonald's (1997) study for the major currencies.

2 Model and Estimation Methodology

A theoretically appealing starting point for exchange rate modeling is uncovered interest parity (UIP) which has also been used extensively as a building block in policy models of open economies. UIP has also a considerable intuitive appeal as an approximate no-arbitrage principle under risk neutrality. The general form of UIP is:

$$E_t \Delta_k s_{t+k} = \left(i - i^*\right)_{k,t} \tag{1}$$

where s_t is the spot exchange rate, and $\left(i - i^*\right)_{k,t}$ is the interest rate differential. k stands for the number of periods over which the change in the s_t variable is calculated. It also specifies the maturity of the interest-bearing security. Δ_k is then the long-difference operator.

Along the lines of the behavioral equilibrium exchange rate (BEER) approach (see e.g. Clark and MacDonald, 1998) equation (1) can be written as:

$$s_t = E_t s_{t+k} - \left(i - i^*\right)_{k,t} + \eta_t \tag{2}$$

where we allow for a time-varying risk premium, η_t, and for simplicity assume that η_t is a white noise process[1]. The $E_t s_{t+k}$ term, i.e. the conditional expectation of the future exchange rate, represents the equilibrium exchange rate (EqER) to which the current spot rate should converge. It is a well known fact in international finance that UIP does not hold in the short-run. However, UIP has gained some support from long-run regressions and simulations of open economy macro models (see e.g. Chinn and Meredith, 2003; Nelson and Young-Kyu, 2001).

One can think of $f_t \equiv E_t s_{t+k}$ as a latent variable that bears certain characteristics and has a known (or assumed) data generating process (DGP). The first feature that has to be included in the underlying DGP is the order of

[1] Alternativelly, one can think of the UIP condition as of an Euler equation and when allowing for the time-varying risk premium one can write it as:

$$E_t \left[s_{t+k} - s_t - \left(i - i^*\right)_{k,t} + \sum_{i=0}^{k} \eta_{t+i} \right] = 0$$

which produces (2) under the assumption that η_t is a white noise process.

integration. Namely, the f_t variable has to be integrated to order I(1) so that the unit root accounting implied by equation (1) holds. Hence, f_t is assumed to follow a random walk process with an auto-correlated disturbance term, say, $f_{t+1} = f_t + e_{t+1}$. The systematic part of the process driving the disturbance term is then explained by fundamentals that proved to have explanatory power for EqER behavior, such as the productivity differential, prd_t, and the terms of trade, tot_t. The model as a whole can be written as:

$$\begin{aligned} s_t &= f_t - \left(i - i^*\right)_{k,t} + \sigma_\eta \eta_t \\ f_{t+1} &= f_t + \delta_1 prd_t + \delta_2 tot_t + \sigma_\nu \nu_{t+1} \end{aligned} \tag{3}$$

where $\begin{bmatrix} \eta_t \\ \nu_{t+1} \end{bmatrix} \sim NID\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$. The non-diagonal covariance matrix of the residuals allows for the possibility that the shock to the current spot rate is correlated with the permanent shock to the EqER. Further, the variables prd_t and tot_t are assumed to be weakly exogenous to the system considered. This is a plausible assumption since the model is set up for a small open economy which is a price taker in international markets and the productivity shock is traditionally seen as the primary (exogenous) driving force, e.g. in real business cycle models.

The observation equation of the state-space model in (3) specifies the conditional distribution of the observables given the unknown states, in this case the latent fundamental value of the exchange rate, and the correlation of the shocks, i.e. $s_t \mid f_t, \left(i - i^*\right)_{k,t}, \sigma_\eta^2, \rho$. The unknown states are assumed to follow a Markovian transition over time and are affected by the movements in the productivity differential and the terms of trade as described by the state dynamics equation of the model in (3). The initial state is distributed as $f_0 \sim N\left(0, \sigma_\nu^2\right)$. The aim here is to incorporate some prior information one can get from relevant theory and existing empirical findings to estimate the fundamental value of an exchange rate. A convenient way how to handle such a task is to make use of Bayesian estimation and inference. For notational simplicity the vector X_t is defined as $X_t \equiv \left[\left(i - i^*\right)_{k,t} \quad prd_t \quad tot_t \right]'$.

The model in (3) implies a bivariate Normal distribution for the conditionals $s_t \mid f_t, \left(i - i^*\right)_{k,t}, \sigma_\eta^2, \rho$ and $f_{t+1} \mid f_t, prd_t, tot_t, \delta_1, \delta_2, \sigma_\nu^2, \rho$. Alternative specification of the model in (3) can be achieved by writing this bivariate Normal density as the product of the density of $f_{t+1} \mid f_t, prd_t, tot_t, \delta_1, \delta_2, \sigma_\nu^2$ and the conditional density of $s_t \mid f_{t+1}, f_t, X_t, \delta_1, \delta_2, \sigma_\eta^2, \sigma_\nu^2, \rho$:

$$\begin{aligned} &f_{t+1} \mid f_t, prd_t, tot_t, \delta_1, \delta_2, \sigma_\nu^2 \sim N\left(f_t + \delta_1 prd_t + \delta_2 tot_t, \sigma_\nu^2\right) \\ &s_t \mid f_{t+1}, f_t, X_t, \delta_1, \delta_2, \sigma_\eta^2, \sigma_\nu^2, \rho \sim N\begin{pmatrix} f_t - \left(i - i^*\right)_{k,t} + \rho \dfrac{\sigma_\eta^2}{\sigma_\nu^2}\left[f_{t+1} - f_t - \delta_1 prd_t - \delta_2 tot_t\right], \\ \sigma_\eta^2\left(1 - \rho^2\right) \end{pmatrix} \end{aligned} \quad (4)$$

A full Bayesian model contains the joint prior distribution of all unobservables, i.e. the parameters, $\delta_1, \delta_2, \sigma_\eta^2, \sigma_\nu^2, \rho$, and the unknown states, $f_0, f_1, ..., f_T$, and the joint distribution of the observables. Bayesian inference is then based on the posterior distribution of the unobservables given the data. Using the idea of successive conditioning, one can express the joint prior density as:

$$p\left(\delta_1, \delta_2, \sigma_\eta^2, \sigma_\nu^2, \rho, f_0, f_1, ..., f_T\right) = p\left(\delta_1, \delta_2, \sigma_\eta^2, \sigma_\nu^2, \rho\right) p\left(f_0 \mid \sigma_\nu^2\right) \prod_{t=1}^{T} p\left(f_{t+1} \mid f_t, \delta_1, \delta_2, \sigma_\nu^2, \rho\right) \quad (5)$$

the parameters $\delta_1, \delta_2, \sigma_\eta^2, \sigma_\nu^2, \rho$ are assumed to be independent and the following priors are imposed. Uninformative conjugate inverse-gamma priors are chosen for σ_η^2 and σ_ν^2 i.e. $\tau \equiv 1/\sigma^2 \sim G(0.1, 0.1)$. The prior for the correlation coefficient is given as $\rho \sim U(-1,1)$. Following the theory and empirical findings discussed in the introduction the priors for δ_1 and δ_2 are chosen to be truncated uninformative

Normal distributions such that both coefficients are constrained to zero or a negative value. Namely, informative priors are chosen such that $\delta_1 \sim N(0,1\mathrm{e}{+}06)I(-10,0)$ and $\delta_2 \sim N(0,1\mathrm{e}{+}06)I(-10,0)$. When carrying out the estimation and inference in BUGS (Bayesian Estimation and Inference Using Gibbs Sampling)[2] the latter non-logconcave full conditional distributions require the use of the Metropolis-Hastings step.

The likelihood is specified by the observation equation in (3) and the independence assumption:

$$p\left(s_t, | X_t, f_0, \ldots, f_T, \delta_1, \delta_2, \sigma_\eta^2, \sigma_\nu^2, \rho\right) = \prod_{t=1}^{T} p\left(s_t \mid \left(i - i^*\right)_{k,t}, f_t, \sigma_\eta^2, \rho\right) \tag{6}$$

Applying the Bayes' theorem, the joint posterior distribution of the unobservables given the data is proportional to the prior times the likelihood:

$$\begin{aligned} p\left(\delta_1, \delta_2, \sigma_\eta^2, \sigma_\nu^2, \rho, f_0, \ldots, f_T \mid s_1, \ldots, s_T, X_1, \ldots, X_T\right) \propto p\left(\delta_1\right) p\left(\delta_2\right) p\left(\sigma_\eta^2\right) p\left(\sigma_\nu^2\right) p\left(\rho\right) p\left(f_0 \mid \sigma_\nu^2\right) \times \\ \times \prod_{t=1}^{T} p\left(f_{t+1} \mid f_t, \delta_1, \delta_2, \sigma_\nu^2, \rho\right) \times \prod_{t=1}^{T} p\left(s_t \mid \left(i - i^*\right)_{k,t}, f_t, \sigma_\eta^2, \rho\right) \end{aligned} \tag{7}$$

When this Bayesian model is estimated we are especially interested in the estimates of the fundamental value of the equilibrium exchange rate, f_t, the parameter estimates characterizing the influence of the productivity differential and the relative terms of trade on the fundamental exchange rate, i.e. δ_1 and δ_2, respectively. Another estimate of interest is the forward risk premium, η_t.

3 Data Description

The data used for estimation span the period of 1984 quarter 1 to 2004 quarter 3 and were obtained from International Financial Statistics. The series are expressed

[2] Detailed exposition of Gibbs sampling for state-space models can be found in e.g. Carter and Kohn (1994).

in logarithms except for the interest rate differential which is untransformed. Referring to equation (3) the first variable (observable) is the beginning-of-period spot exchange rate, s_t, quoted as AUD/USD. The beginning-of-period spot rate is the one-period lagged value of the corresponding end-of-period spot rate so that the timing is in line with the UIP definition. The interest rate differential, $\left(i-i^*\right)_{k,t}$, is approximated here by the difference between the two-year Treasury bond yields in Australia and the US. The $k=8$ has been chosen in accord with the findings on UIP set forth in the introduction and data availability. An attempt has been made to use similar underlying assets for Australia and the US, i.e. treasury bonds, even though such an effort is likely to be disrupted by different treatment of the securities, by e.g. the taxation law, in each country.

The applied proxy for the productivity differential, prd_t, is similar to that used by MacDonald (1997). Namely, it is the ratio of the domestic consumer price index to the wholesale price index relative to the equivalent foreign ratio[3]. The relative terms of trade, tot_t, are constructed as the ratio of domestic export unit value in US dollars to import value in US dollars as a proportion of the equivalent foreign ratio[4]. A plot of the range scaled data series is available in figure 1 as an illustration of their relative development over time:

******* Figure 1 Here *******

[3] Even though another suitable proxy would be the difference between the growth rates of real manufacturing production at home relative to the foreign equivalent, such a proxy is not adopted due to data unavailability.

[4] We are aware of the argument by Obstfeld and Rogoff (2000) that a certain component of the terms of trade may move mechanically with the exchange rate as a result of wage and price rigidities interacting with at least partial pass-through. However, we stick to the common practice in the existing literature and policy discussions and use the terms of trade *per se*.

4 Estimation Results

The estimations are carried out in BUGS using the Bayesian model described in section 2. In addition to the priors displayed above, the initial state f_0 is generated from $f_0 \sim N\left(0, \sigma_\nu^2\right)$ and two chains of iterates are run simultaneously to check on the convergence properties of the processes. The warm-up sample is 5000 and the posterior means are calculated using 10000 iterates. To initialize the two chains we use two sets of starting values:

$$\text{Set 1: } \delta_1 = -0.2,\ \delta_2 = -0.5,\ \rho = 0,\ \sigma_\eta^2 = 100,\ \sigma_\nu^2 = 100$$

$$\text{Set 2: } \delta_1 = -0.7,\ \delta_2 = -0.5,\ \rho = 0.2,\ \sigma_\eta^2 = 100,\ \sigma_\nu^2 = 10$$

A plot of iterates of the five basic parameters of the model is displayed in figure A1 in the Appendix. The convergence seemed to be achieved as the two chains of iterates overlap substantially. Further, figure A2 documents efficiency of the sampling scheme as the autocorrelation functions (ACFs) of the analogous iterates die out rapidly. Estimates of the parameters and their Bayesian confidence intervals are reported in table 1:

******* Table 1 Here *******

Given the prior information imposed upon the model, the results suggest that movements in the productivity differential have a relatively higher impact on the development of the EqER compared to the changes in the terms of trade. Chen and Rogoff (2003) argue that the terms of trade are significantly more volatile than other fundamentals and their inclusion in exchange rate equations may help to solve the problem of relatively low volatility of fundamentals compared to exchange rates. The relatively lower magnitude of the estimated coefficient on the terms of trade is thus along the line of such an argument. Similarly, the lower variation in the productivity differential corresponds to the coefficient estimate higher than one.

Further, the estimated standard deviations of the shocks to the spot rate and the EqER are similar, although one would expect the former to be higher as trading in exchange rate markets is likely to generate more volatility (noise) then shocks to the fundamental value. This may be due to the fact that the model proposed in this paper tends to generate more variation in the fundamental value of the exchange rate that takes out more of the variation in the actual spot rate.

Finally, the estimated correlation of the shocks is positive and significant. One is however constrained from interpreting the direction in which volatilities get disseminated through the system. An attempt to do so would require different specification and this will be considered in future research. The reported statistics at the bottom of table 1 will be discussed shortly when alternative model specifications are introduced to inspect robustness of the acquired results.

4.1 Misalignment Calculation

To get a good grasp of the intuition behind the BEER, we rewrite equation (1) here using $f_t \equiv E_t s_{t+k}$:

$$f_t - s_t = \left(i_{k,t} - i^*_{k,t}\right) + \eta_t \tag{8}$$

Note first that the fundamental value f_t is effective as from period t owing to the fact that it is conditional on information available to agents at time t, as stated by the expectation operator $E\left(s_{t+k} \mid I_t\right)$. From date t on, the spot rate adjusts toward such equilibrium. Equation (8) further states that in absence of the shocks, η_t, the current spot rate can be at its fundamental value only if the interest rate differential, $\left(i_{k,t} - i^*_{k,t}\right)$, is zero. It is crucial to bear in mind that the interest rate differential does not have an (direct) impact on the fundamental value and if so, it is assumed that:

$$\left|\frac{\partial f_t}{\partial\left(i_{k,t}-i_{k,t}^*\right)}\right| \ll \left|\frac{\partial s_t}{\partial\left(i_{k,t}-i_{k,t}^*\right)}\right| \tag{9}$$

This postulates that monetary policy has no direct effect on the fundamental (equilibrium) value of the exchange rate as it is determined by the real-economy variables. For instance, if $\eta_t = 0$ and the interest rate differential is positive, $\left(i_{k,t}-i_{k,t}^*\right) > 0$, the current spot rate is overvalued, i.e. $f_t - s_t > 0$ (recall that we use a direct quotation in this paper, i.e. AUD/USD).

The estimated EqER and the actual spot rate are plotted in figure 2 together with the resulting exchange rate misalignment.

******* Figure 2 Here *******

It is important that one bears in mind the structure of the model in (3) when interpreting the results. Namely, we have assumed that movements in the productivity differential and the terms of trade are of an equilibrium nature.

The estimates of the EqER suggest that the Australian dollar was on average overvalued during the considered period 1984-2004. The average overvaluation relative to the EqER was about 3 per cent. The first major overvaluation appeared in 1987 due to an adverse movement in the relative terms of trade, the decreasing productivity differential and the significantly positive interest rate differential with respect to the U.S (refer to figure 1). Next overvaluation period occurred during 1989 as a result of again the significantly positive interest rate differential relative to the U.S. and a rather temporary decline in the productivity differential. This overvaluation pressure was reduced by favorable developments in Australia's relative terms of trades. The most recent overvaluation emerged during 2003 mainly due to the positive interest rate differential combined with a significant decline in the productivity differential, while the relative terms of trade were gradually improving.

The two major undervaluation of the Australian dollar occurred in 1984 and during 2004. The former undervaluation is attributable to the substantially positive productivity differential, the low interest rate differential and the favorable relative terms of trade during that period. Similarly as the 1984 undervaluation, the 2004 undervaluation emerged as a consequence of strong improvements in the relative terms of trade and the productivity differential accompanied by a moderate decline in the interest rate differential.

4.2 Tests of Robustness

To test robustness of the EqER estimates we run two more models. In the first one, we impose only uninformative priors on all the parameters. In the second one we use slightly informative priors for the precisions corresponding to σ_{η}^{2} and σ_{ν}^{2}, namely, $\tau \equiv 1/\sigma^{2} \sim G(2,10)$, in addition to the priors used for the primary model. This will place a relatively small probability mass on implausibly small variances. The estimates from the two alternative Bayesian models are reported in table 2 and 3:

******* Table 2 Here *******

******* Table 3 Here *******

The posterior means of σ_{η}, σ_{ν} and ρ are almost unchanged for both alternative models. Only the posterior mean of the correlation coefficient for the second model decreased lightly from about 0.4 to 0.3. The estimates of δ_1 and δ_2 for the model with flat priors are rather insignificant judging by the span of the confidence intervals. This suggests that the probability of the coefficients being negative and positive is similar. However, this does not invalidate the priors we have used in the primary model given the indications from the model selection criteria reported at the bottom of both tables. All the criteria should be maximized to arrive at the best model specification. When comparing the primary model with the first alternative model one can observe that all the selection criteria indicate better performance of the primary model as compared to the first alternative.

The magnitude of δ_1 and δ_2 estimates from the second alternative model are substantially higher compared to the primary model. The coefficients of about -3.9 and -1.2 attached to the productivity differential and the relative terms of trade respectively, indicate that the variation in the fundamentals is substantially magnified before passed onto the fundamental value of the exchange rate and the spot exchange rate itself. However, imposing the additional priors on the relevant precisions took the model considerably away from the primary model such that the model selection criteria are hard to compare. The *Dp* criterion stating the effective number of estimated coefficients is however still suggestive in favor of the second alternative model. Overall, in some cases it may be hard to judge relative performance of models just based on statistical measures, especially, when latent variables such as the EqER, potential output or equilibrium money velocity are in question. Justification of the acquired estimates of the latent variables should come from the economic perspective. Such justification is attempted in the next section when we investigate the information content of the acquired estimates in relation to the exchange rate dynamics.

5 Information Content of the Estimates

In this section we strive to test validity of our estimates by looking at whether the estimates serve the economic purpose for which they are required. As mentioned in the introduction one can use the estimates of the EqER to forecast the dynamics of the spot exchange rate, by building suitable error (equilibrium) correction model (ECM). Economic and econometric theory provides rather strict guidance about the expected sign of the coefficients on the variables constructed from the estimates of the EqER.

To get an idea about how the prior information (economics) employed in the estimation of the EqER changes performance of the EqER in the selected application we work with all acquired estimates of the fundamental exchange rate. These are the estimates from the primary model, f_t, the first alternative model, f_t^1, and the second alternative model, f_t^2. In addition, we will compare the results to a benchmark of purchasing power parity, f_t^{ppp}. The latter is constructed using CPI indexes for Australia and the US obtained from International Financial Statistics. The resulting misalignments, i.e. deviation of s_t from the four estimates of the EqER are plotted in figure 3:

******* Figure 3 Here *******

As can be seen from the plots the alternative misalignments of the spot exchange rate coming from different specifications of the Bayesian model follow a similar pattern. The misalignment according to PPP is significantly different in some periods, an example being the recent overvaluation of the spot rate according to PPP that contradicts the misalignment estimates from the Bayesian models.

5.1 Exchange Rate Dynamics and Implied Misalignments

Consider a simple ECM of the spot exchange rate, s_t:

$$\triangle s_t = c + \alpha\left(\left(f_{t-1} - s_{t-1}\right) - E\left(f_{t-1} - s_{t-1}\right)\right) + \sum_{j=1}^{m} \phi_j \triangle s_{t-j} + \upsilon_t \tag{10}$$

where $E\left(f_{t-1} - s_{t-1}\right)$ is estimated using the sample mean of $\left(f_t - s_t\right)$, $\hat{\mu}_{f_t - s_t}$, c is an intercept and m is set to 1 after inspection of the partial autocorrelation function to ensure that the residual υ_t is uncorrelated. The coefficient of interest here is α which is expected to be positive and significant. This postulates that the exchange rate converges to its equilibrium value and thus eliminates the existing misalignment. The α coefficient is only significant if the estimate of the EqER is *reasonable*. Equation (10) is estimated subsequently for all four misalignments by OLS. The results are reported in table 4:

******* Table 4 Here *******

The adjusted R-squares for the ECM's based on the estimates of equilibrium exchange rates for which some prior information has been used are considerably higher than the adjusted R-squared for the ECM involving PPP. On the other hand, the ECM based on the estimates of the fundamental exchange rate using flat priors is not any better than the ECM with PPP. More importantly, the estimates of the α's associated with any of the f_t^X bear opposite sign to what one would expect from the conventional error-correction model. Nevertheless, this outcome is quite plausible given the structure of $\left(f_t - s_t\right)$ as described in equation (8). More specifically, this term can be decomposed into the interest rate differential and some estimate of the forward risk premium from the Bayesian state-space model. We estimate equation (10) again applying such decomposition and allowing the coefficients on the interest rate differential and the risk premium to differ, i.e.:

$$\Delta s_t = c + \alpha_1 \left(i_{t-1} - i^*_{t-1} \right) + \alpha_2 \hat{\eta}_{t-1} + \phi \Delta s_{t-1} + \xi_t \tag{11}$$

We also run the same regression with no estimate of the risk premium to get some further insights. The estimates are reported in table 5.

******* Table 5 Here *******

The results suggest that the interest rate differential by itself does not deliver any information for the future development of the exchange rate. However, once accompanied by some estimate of the risk premium the resulting regression shows a decent fit[5]. More specifically, the regressions with a risk premium produce significantly negative coefficients attached to the interest rate differential and either estimate of the risk premium. The magnitude of the coefficients on the risk premium is higher than that on the interest rate differential. Albeit the structure of the estimated equation (11) resembles an augmented UIP condition the impact of the interest rate differential is rather consistent with the *portfolio story*[6] than UIP. It would be illuminating to know more about the interaction between the interest rate differential and the estimates of the risk premium. For this reason, the relative developments of the interest rate differential and the estimates of the risk premium are shown in figure 4.

******* Figure 4 Here *******

[5] Although the model with flat priors shows the best in-sample fit when estimated unconstrained the best out-of-sample performance is a privilege of our primary model. This observation is derived from the one-step-ahead forecast errors. Namely, the relative RMSE's of the primary model and the model with flat priors with respect to a random-walk are 0.8271 and 0.8790 respectively.

[6] An increase in interest rate differential between the domestic and foreign economies induces rearrangements of portfolios and a capital inflow into the domestic economy. The resulting increase in demand for the domestic currency causes appreciation of the domestic currency.

Figure 4 reveals that the three estimates of the risk premium move quite closely together and that they are likely negatively correlated with the interest rate differential. The corresponding correlation coefficients appear to be significantly negative of the magnitude -0.59, -0.46 and -0.63, for the risk-premium estimates associated with f_t, f_t^1 and f_t^2 respectively. This suggests that the risk premium counteracts the predicted future appreciation due to the positive interest rate differential. Further investigation of the interaction of the interest rate differential and appropriate estimates of the risk premium can shed some light on the poor performance of UIP. This is however left for future research.

6 Conclusion

A model to estimate an equilibrium exchange rate has been proposed in this paper. It possesses several building blocks. First, it follows a state-space structure which enables one to estimate latent variables such as the equilibrium exchange rate. Second, it starts from a theoretically appealing concept of uncovered interest parity and perceives the equilibrium exchange rate as a permanent component of the spot exchange rate driven by fundamental variables, here the relative terms of trade and the productivity differential. Prior information about the impact of the terms of trade and the productivity differential is used when Bayesian estimation of the model is carried out.

Estimates of the primary model presented in this paper reveal that on average the Australian dollar was overvalued by about 3 percent during the period 1984-2004. The major overvaluation of the AUD/USD exchange rate took place during the period 1986-1988, and around the years 1990, and 2003. We compare the economic content delivered by the acquired estimates of exchange rate misalignments to misalignments based on purchasing power parity. The estimations of the implied models for exchange rate dynamics suggest that the estimates of equilibrium exchange rates from the proposed Bayesian model deliver superior information when compared to the purchasing parity equivalent. It would be interesting to consider a richer structure of the proposed model in future research. This may include more determinants of the equilibrium exchange rate, introduction of a more sophisticated approximation of the risk premium and consideration of alternative distributions for the shock to the spot exchange rates.

Appendix

******* Figure A1 Here *******

******* Figure A2 Here *******

References

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Tables

Table 1 Estimation Results - Priors on δ_1 and δ_1

node	posterior mean	Bayesian 95 % Conf. Interval	
δ_1	-1.2650	[-3.5100; -0.0527]	
δ_1	-0.3739	[-0.9905; -0.0159]	
σ_η	0.0892	[0.1138; 0.0722]	
σ_ν	0.0825	[0.1056; 0.0665]	
ρ	0.4132	[0.2768; 0.5627]	
Dbar	**Dhat**	**Dp**	**DIC**
-155.6	-208.889	53.213	-102.464

Dbar is the posterior mean of -2logL, Dhat is -2logL at the posterior mean of the parameters, Dp is the number of effective parameters, and DIC is a Bayesian model selection criterion.

Table 2 Estimation Results – No Priors

node	posterior mean	Bayesian 95 % Conf. Interval	
δ_1	0.1241	[-2.9050; 3.1430]	
δ_1	0.0181	[-0.8155; 0.8437]	
σ_η	0.0891	[0.1134; 0.0663]	
σ_ν	0.0822	[0.1064; 1.7857]	
ρ	0.4122	[0.2773; 0.5600]	
Dbar	**Dhat**	**Dp**	**DIC**
-177.876	-231.089	53.213	-124.663

Dbar is the posterior mean of -2logL, Dhat is -2logL at the posterior mean of the parameters, Dp is the number of effective parameters, and DIC is a Bayesian model selection criterion.

Table 3 Estimation Results - Priors on δ_1, δ_1, σ_η and σ_ν.

node	posterior mean	Bayesian 95 % Conf. Interval	
δ_1	-3.8510	[-9.2380; -0.2010]	
δ_1	-1.1970	[-3.2110; -0.0461]	
σ_η	0.6770	[0.8203; 0.5675]	
σ_ν	0.8811	[1.1612; 0.6988]	
ρ	0.3005	[0.1024; 0.4811]	
Dbar	**Dhat**	**Dp**	**DIC**
155.948	115.884	40.064	196.012

Dbar is the posterior mean of -2logL, Dhat is -2logL at the posterior mean of the parameters, Dp is the number of effective parameters, and DIC is a Bayesian model selection criterion.

Table 4 Spot-Rate ECM's with different Misalignment Measures

Model/ Variable	$\triangle s_t = g(f_t, \cdot)$	$\triangle s_t = g(f_t^1, \cdot)$	$\triangle s_t = g(f_t^2, \cdot)$	$\triangle s_t = g(f_t^{ppp}, \cdot)$
$\left(f_{t-1}^X - s_{t-1}\right)$	-0.4368 (0.1063)***	-0.2621 (0.1068)**	-0.3358 (0.0992)***	0.1071 (0.0429)**
$\triangle s_{t-1}$	-0.2891 (0.1243)**	-0.1602 (0.1272)	-0.2643 (0.1314)**	-0.0738 (0.1123)
c	0.00162 (0.0023)***	0.0014 (0.0025)**	0.0016 (0.0024)	0.0013 (0.0025)
$R^2 adj.$	0.1538	0.0467	0.1034	0.0495

The dependent variable is the change in the spot exchange rate. f_t^X are different measures of fundamental exchange rate $f_t^X = f_t, f_t^1, f_t^2, f_t^{ppp}$. The estimation method is OLS, standard errors are in parentheses. *,**,*** - assigns significance on the 10 %, 5 % and 1 % level, respectively.

Table 5 Unconstrained Spot-Rate ECM's with different Risk-Premium Estimates

Model/ Variable	no $\hat{\eta}_t$	$\hat{\eta}_t = h\left(\hat{f}_t\right)$	$\hat{\eta}_t = h\left(\hat{f}_t^1\right)$	$\hat{\eta}_t = h\left(\hat{f}_t^2\right)$
$\left(i_{t-1} - i_{t-1}^*\right)$	-0.0848 (0.0824)	-0.4181 (0.1047)***	-0.5034 (0.1132)***	-0.3458 (0.1131)***
$\hat{\eta}_{t-1}$	NA	-0.6123 (0.1361)***	-1.4300 (0.2957)***	-0.3305 (0.1037)***
$\triangle s_{t-1}$	-0.0068 (0.1119)	-0.3826 (0.1306)***	-0.7760 (0.1873)***	-0.2619 (0.1328)**
c	0.0040 (0.0037)	0.0149 (0.0041)***	0.0103 (0.0035)***	0.0132 (0.0046)***
$R^2 adj.$	-0.0116	0.1846	0.2096	0.0924

The dependent variable is the change in the spot exchange rate. $\hat{\eta}_t$ are different measures of fundamental exchange rate $f_t^X = f_t, f_t^1, f_t^2, f_t^{ppp}$. The estimation method is OLS, standard errors are in parentheses. *,**,*** - assigns significance on the 10 %, 5 % and 1 % level, respectively.

Figures

Figure 1 Plot of the Data Series

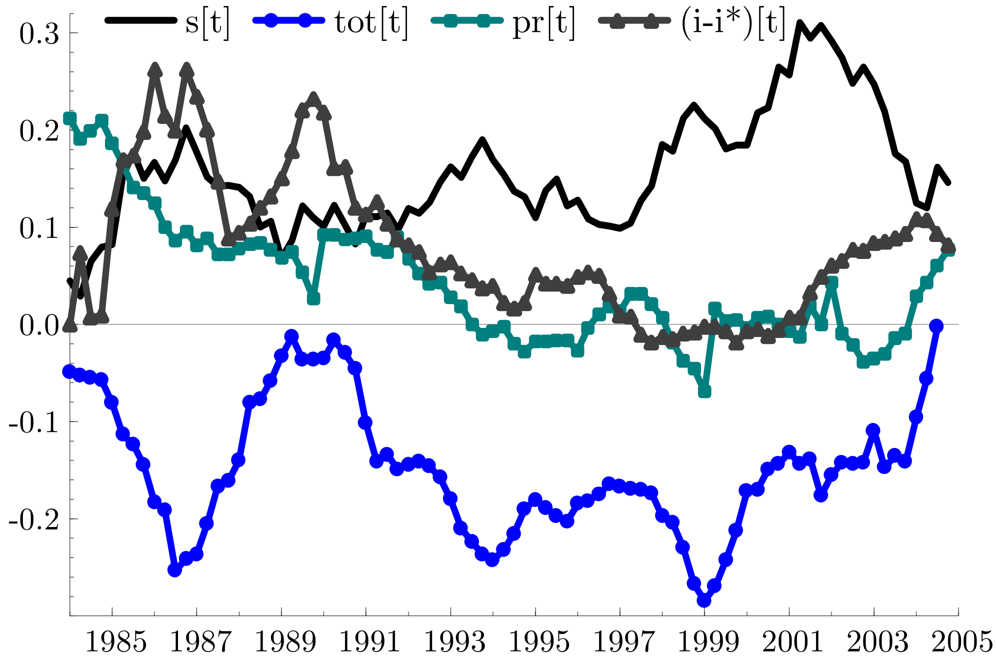


Figure 2 Spot and Equilibrium Spot Exchange Rates and the Misalignment

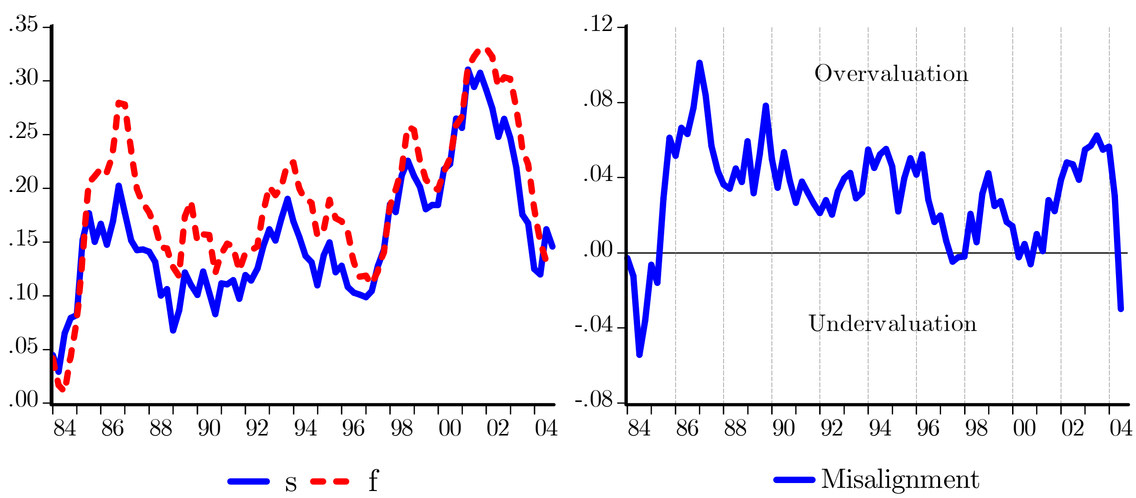


Figure 3 Misalignment Measures According to Alternative Models of EqER

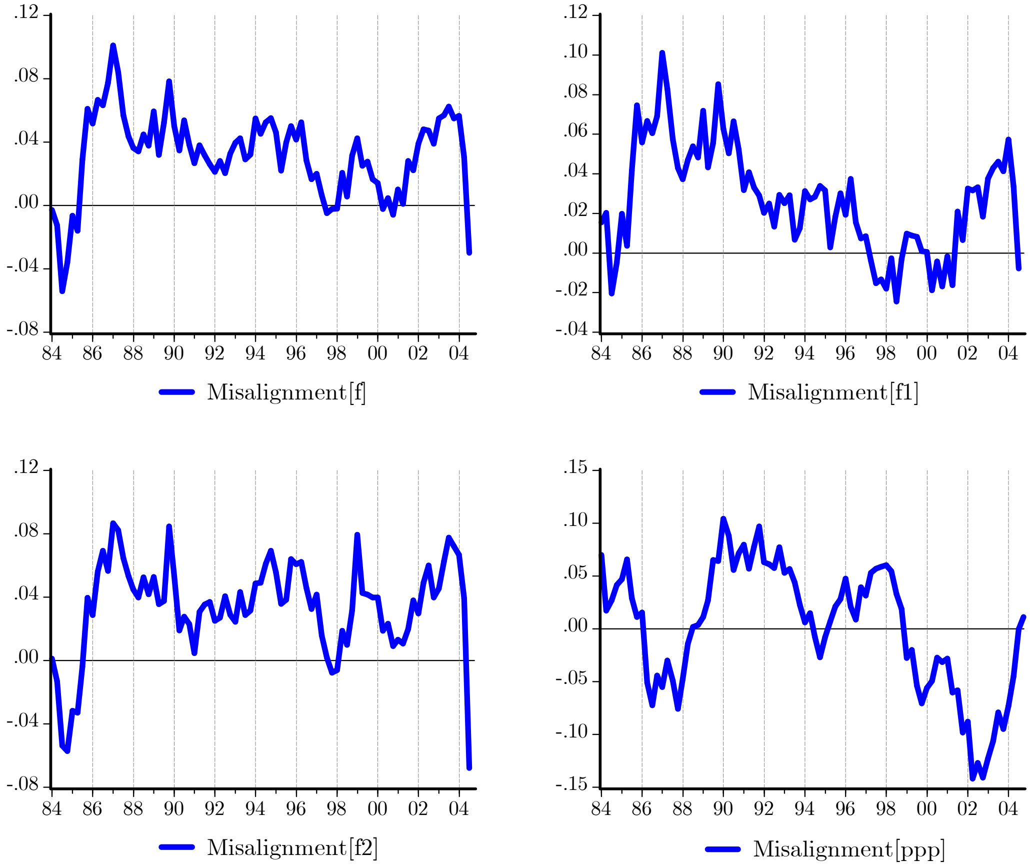


Figure 4 The Interest Rate Differential and the Estimated Risk Premia

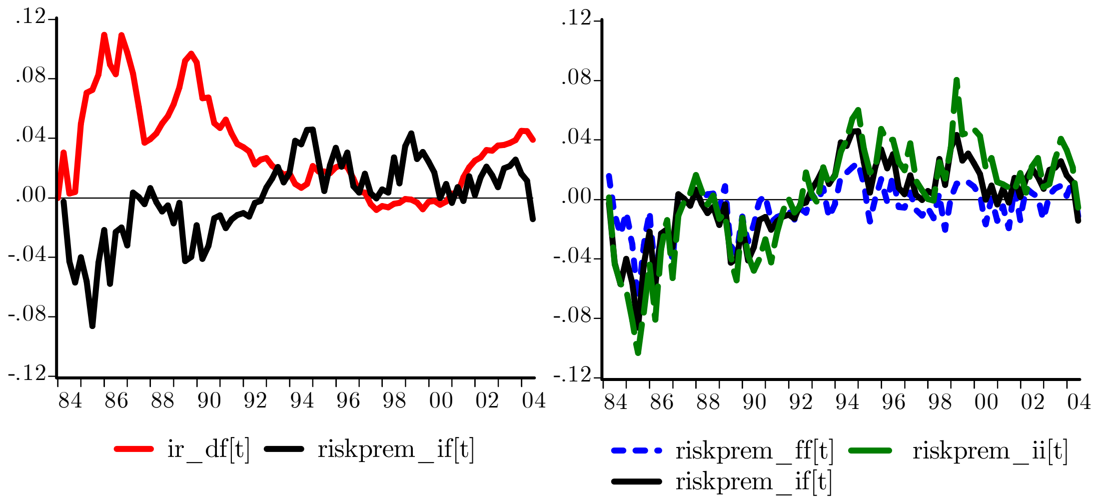
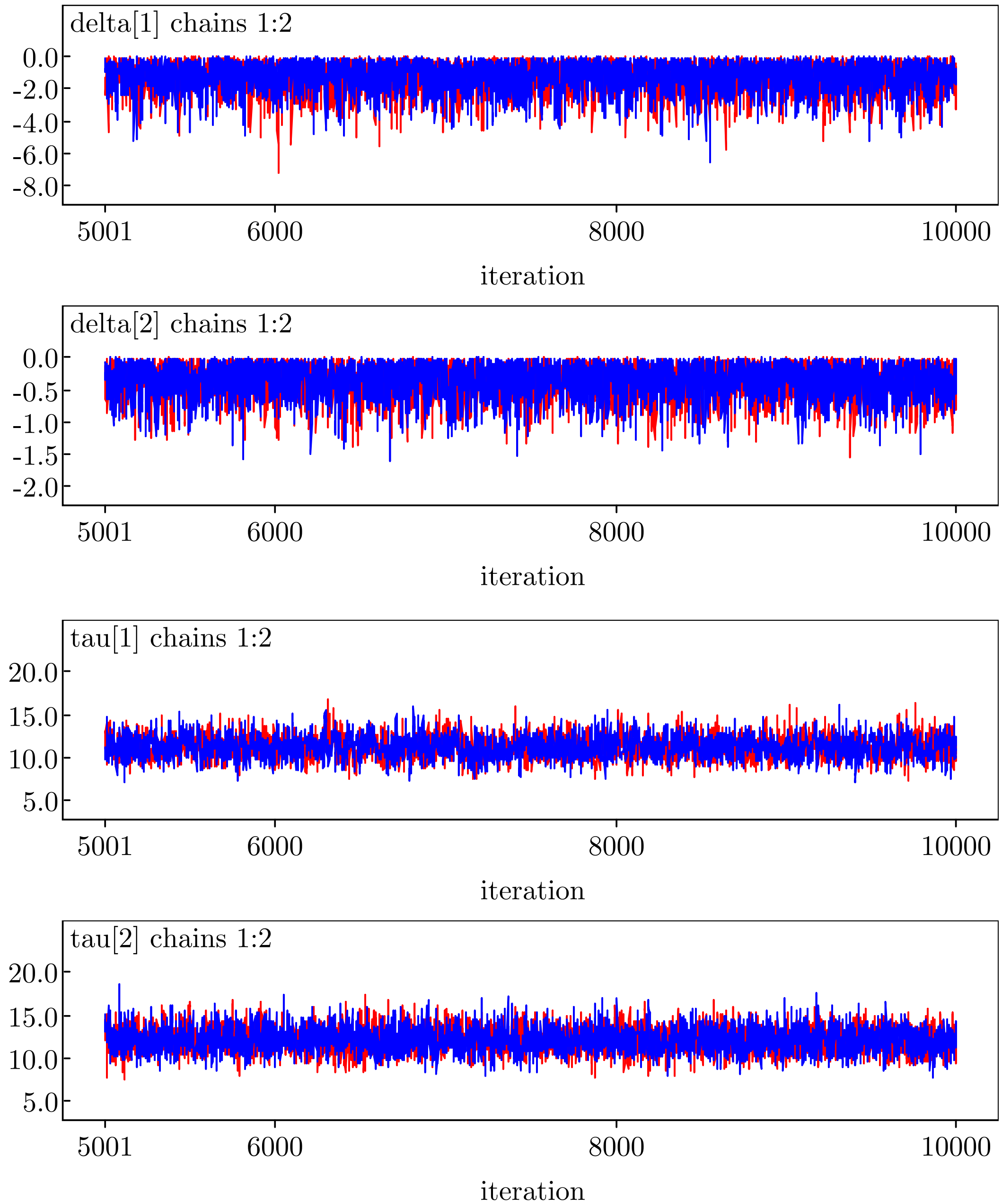


Figure A1 History of Coefficients' Iterates



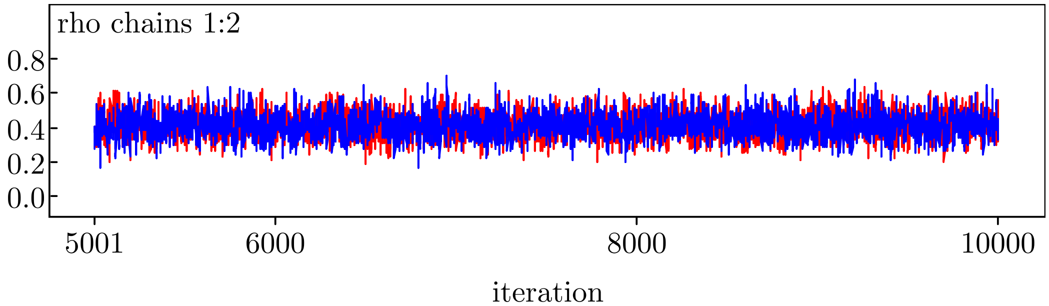


Figure A2 Autocorrelation Function of the Coefficients' Iterates