The Structure Models for Futures Options Pricing and Related Researches

Feng DAI   *Dongkai ZHAI*   *Zifu QIN*
Department of Management Science
Zhengzhou Information Engineering University, Henan 450002, China
E-mail:fengdai@public2.zz.ha.cn; fengdai@126.com

Abstract: Based on the structure model of option pricing (Feng DAI, 2005) and the Partial Distribution (Feng DAI, 2001), this paper designs a new kind of expression of futures price, presents the structure pricing model for American futures options on underlying non-dividend-paying, and gives three put-call parities between American call and put option on spots, call and put option on futures, and spot options and futures options, they are different from put-call parity of European options. We prove analytically that an American call option on futures must be worth more than the corresponding American call option on spot and an American put option on futures must be worth less than the corresponding American put option on spot in normal market; and the oppositions in inverted market. The final empirical researches also support the conclusions in this paper.

Key words: structure pricing, American options on futures, non-dividend-paying, analytic formula, put-call parity

1 Introduction

In theoretical studies of international economics and finance engineering, options pricing is an important problem to which economists pay the exceptional attentions. In the studies of option pricing, there have been many significant results (Black and Scholes 1973, Merton 1976, Sharpe 1978, Whaley 1981, Gesk and Roll 1984), and approximation methods for American put option (MacMillan 1986, Stapleton and Subrahmanyam 1997). "Unfortunately, no exact analytic formula for the value of an American put option on a non-dividend-paying stock has been produced" [9]. The authors of this paper have solved the problem in reference [10]. And in this paper, author will present the structure pricing model for American futures options on underlying non-dividend-paying.

In addition, when the futures and options contracts have the same maturity, and "Suppose that there is a normal market with futures prices consistently higher than spot prices prior to maturity. ⋯.An American call futures option must be worth more than the corresponding American call option on the underlying assets. ⋯. Similarly, An American put futures option must be worth less than the corresponding American put option on the underlying assets. If there is an inverted market with futures prices consistently lower than spot prices, ⋯, the reverse must be true. American call futures options are worth less than the corresponding American call option on the underlying assets, whereas American put futures options are worth more than the corresponding American put option on the underlying assets" [9]. The real trade in market shows that the conclusions above are true. But, in this paper, we shall prove them in analytic way, and give computing method for the deference between the values of American futures option and the corresponding American option on the underlying assets. By the way, this paper will presents three kinds of put-call parity, i.e. put-call parity of call spot option and put spot option, put-call parity of call futures option and put futures option, and put-call parity of call spot option, put spot option, call futures option and put futures option. The former two of put-call parity here have small differences with those we have known in expression, and the later one is a new.

2 The Basic Assumptions for the Prices of Assets and the Partial distribution

2.1 The basic assumptions of prices of assets

The basic assumptions we use to define the price of an underlying assets (spot, stock and stock indices), regarded as the basis of the discussion in this paper are as follows:

Assumption 1.

1) The prices of an underlying assert includes the cost price and the market price. The cost price means the average value of all the prices paid by the market traders to produce or buy an underlying asset and the

market price is the current trading price of an underlying asset.
2) The prices (cost price and market price) have been fluctuating with time. Any price and the fluctuation extent (i.e., the variance) of price are non-negative.
3) The possibilities that the market price of underlying is much lower than the cost price, or is much higher than the cost price, will be very small.
4) All securities are perfectly divisible.
5) There are no transaction costs or taxes.

The assets we shall mention in the following include the spot, stock, stock indices, etc.

2.2 Partial distribution [11]-[12] and price behavior of underlying assets

Definition 1 (The Partial Distribution)**.** Let S be a non-negative stochastic variable, and it follows the distribution of density

$$f(x)=\begin{cases} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx & x\geq 0 \\ 0 & x<0 \end{cases}$$

then S is said to have a Partial Distribution, and denotes $S\in P(\mu, \sigma^2)$.

Definition 2 (The Partial Process). If stochastic variable S is related to time, i.e., $\forall\, t\in[0,\infty)$, we have $S(t)\in P(\mu(t),\sigma^2(t))$, then the $\{S(t),\, t\in[0,\infty)\}$ is called a partial process. Especially, if

$$S\,(t)\in P(\mu(t),\sigma^2(t)t)$$

then we call $\{\xi(t),t\in[0,\infty)\}$ a DF process.

Assumption 2. Let $\mu(t)$ be the cost price of stock at the time t, and $\sigma^2(t)$ be the variance of cost price at the time t. If the market prices of stock satisfy the assumptions 1, we suppose that the market price, $S(t)$, follows the partial distribution at time t, and denotes $S(t)\in P(\mu(t), \sigma^2(t))$.

According to references [11]-[12], we have the following theorem 1 and theorem 2:

Theorem 1. Let S, the market price of an underlying asset, follows the partial distribution $P(\mu, \sigma^2)$, thus

1) The expected value $E(S)$ of S is as follows

$$E(S) = \mu + \sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{\sigma^2}}\,dx$$

where, $R(S) = \sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$ is the average trading profit in the market.

2) The $D(S)$, variance of the market price S is as follows

$$D(S) = \sigma^2 + E(S)[\mu - E(S)]$$

where, μ is the cost price of underlying asset, σ^2 is the variance of the cost price.

Theorem 2. For any $x\in[0,\infty]$, we have the following equations:

1) $\int_0^x e^{-\frac{t^2}{2}}\,dt = \sqrt{\frac{\pi}{2}(1-e^{-\frac{2}{\pi}x^2})}$.

2) $\int_0^x e^{-\frac{(u-\mu)^2}{2\sigma^2}}\,du = \sqrt{\frac{\pi}{2}}\sigma\times(\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}} + +\operatorname{sgn}(x-\mu)\sqrt{1-e^{-\frac{2}{\pi}(\frac{x-\mu}{\sigma})^2}})$

where, $\operatorname{sgn}(x)=\begin{cases} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{cases}$.

3 The DF Structure Model for Futures Option Pricing

3.1 The probability distribution of futures prices

If $S(t)$ is the market price of an underlying asset, $S(t)\in P(\mu(t), \sigma^2(t))$, where, $\mu(t)$ is the cost price of underlying asset, $\sigma^2(t)$is the variance of the cost price. $F(t)$ is the market price of futures on $S(t)$. In general, the futures price is expressed as (see reference [9]):

$$F(t)=S(t)e^{\delta t} \tag{1}$$

In (1), $S(t)$ is the underlying asset price. For a non-dividend-paying asset, if it is an investment asset, $\delta=c$, i.e. $c=r$; if it is a consumption asset, $\delta=c-y$. Where, c is the cost of carry, r is the risk-free rate, y is the convenience yields.

In fact, if $F(t)=S(t)e^{\delta t}$, thus the distribution function

$$P_F\{F(t)<w\}= P_S\{S(t)<we^{-\delta t)}\}$$

$$=\int_0^{we^{-\delta t}} f(x)dx = \frac{\int_0^{we^{-\delta t}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx}{\int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx}$$

let $x=ve^{-\delta t}$, have

$$P_F\{F(t)<w\}=\int_0^{we^{-\delta t}} f(x)dx = \frac{\int_0^{w} e^{-\frac{(v-\mu e^{\delta t})^2}{2(\sigma e^{\delta t})^2}}dv}{\int_0^{\infty} e^{-\frac{(v-\mu e^{\delta t})^2}{2(\sigma e^{\delta t})^2}}dv}$$

namely $F(t)\in P(\mu(t)e^{\delta t}, [\sigma(t)e^{\delta t}]^2)$, $E(F(t))= E(S(t))e^{\delta t}$ and $D(F(t))= D(S(t))e^{2\delta t}$.

If T is the time of expiration, then the futures price $F(t)$ follows

$$F(t)\in P(\mu(t)e^{\delta(T-t)}, [\sigma(t)e^{\delta(T-t)}]^2) \tag{2}$$

From (2), we know:

1) When the time nears to expiration, the fluctuation of futures price becomes smaller, and futures price nears to the price of underlying asset.

2) When $t=T$, $F(T)\in P(\mu(T), \sigma^2(T))$ is equal to $S(t)\in P(\mu(t),\sigma^2(t))$, and considering that the underlying asset price becomes a constant to its futures contact when which is completing to business transaction, so we have $F(T)=S(T)$.

All of these inosculate to the real characteristics of the futures price. Comparing with (1), the expression in (2) describes the real behavior of futures price more delicately. Also we have

Theorem 3. If $S(t)\in P(\mu(t), \sigma^2(t))$ is the price of an underlying asset and $F(t)\in P(\mu(t)e^{\delta(T-t)}, [\sigma(t)e^{\delta(T-t)}]^2)$ is the price of the futures of $S(t)$, thus

1) The expected value: $E(F(t))=e^{\delta(T-t)}E(S(t))$.

2) The variance: $D(F(t))=e^{2\delta(T-t)}D(S(t))$.

where, T is the time of expiration.

3.2 Definitions and denotes

Definition 3 Let a and b be non-negative constants, If $a>0$, $b=0$, we define:

$$e^{-\frac{a}{b}} = \lim_{z\to 0^+} e^{-\frac{a}{z}} =0.$$

Definition 4(*DF* structure). Let X be the value of an asset related to an underlying asset $A(t)\in P(\mu(t), \sigma^2(t))$, if $\forall\ t\in[0,\infty)$ and $T>t$, $X_A(t,T)\in P(X, D[A(t)](T-t))$, then we call $X_A(t,T)$ the *DF* stochastic structure of X on $A(t)$. $X_A(t,T)$ is called a *DF* structure of X for short.

Where, $A(t)$ can be the price of an underlying asset or a futures contact.

For any $t\in[0,\infty]$, if $S(t)$ is the market price of an spot asset, T is the expiration time of derivatives on $S(t)$, and X is the strike price of the derivatives, thus the *DF* stochastic structure of strike price X on $S(t)$ is $X_S(t,T)\in P(X,D[S(t)](T-t))$. Similarly, $Y_F(t, T)$ is the *DF* stochastic structure of strike price Y on futures $F(t)$, thus $Y_F(t,T)\in P(Y, D[F(t)](T-t))$.

Although the futures has certain connections with its *DF* structure $Y_F(t, T)$ in changing, their stochastic movements may have no inevitable relation, so we could suppose that $Y_F(t, T)$ and $F(t)$ are independent of one each other.

Assumption 3.

1) There are no dividends during the life of the futures and derivatives.

2) The risk-free rate of interest, r, is constant.

3) The prices of trading is continuous.

Denoting:

t—the current time.

T—time of expiration of futures and options.

r—risk-free rate of interest to maturity T.

$S(t)$—market price of the spot asset at t.

$F(t)$—market price of the futures asset at t

X—strike price of option on $S(t)$.

Y—strike price of option on $F(t)$

$S(t)e^{r(T-t)}$—the current value of $S(t)$ which is as a forward.

$F(t)e^{r(T-t)}$—the current value of $F(t)$ which is as a forward.

$X_S(t,T)$—DF structure of X on $S(t)e^{r(T-t)}$.

$Y_F(t,T)$—DF structure of Yon $F(t)e^{r(T-t)}$.

C_S—value of call option to buy one spot share.

P_S—value of put option to sell one spot share.

C_F—value of call option to buy one futures share.

P_F—value of call option to sell one futures share.

So we have $S(t)\in P(\mu(t),\sigma^2(t))$, $F(t)\in P(\mu(t)e^{\delta(T-t)},[\sigma(t)e^{\delta(T-t)}]^2)$, and stochastic structure $X_S(t,T)\in P(X, D[S(t)e^{r(T-t)}](T-t))$ and $Y_F(t,T)\in P(Y,D[F(t)e^{r(T-t)}](T-t))$.

3.3 The DF structure pricing for American options on spot

If denoting:

$$d_1=\frac{S(t)-Xe^{-r(T-t)}}{\sqrt{D[S(t)](T-t)}},\quad d_2=\frac{Xe^{-r(T-t)}}{\sqrt{D[S(t)](T-t)}},\quad \varphi(x)=\int_0^x e^{-\frac{t^2}{2}}dt=\sqrt{\frac{\pi}{2}(1-e^{-\frac{2}{\pi}x^2})}.$$

Thus, according to [10], we have the structure pricing models for American options on spot as follows:

3.3.1 The current price of call option on spot is

$$C_S(t)=e^{-r(T-t)}E[\max(S(t)e^{r(T-t)}-X_S(t,T),0)]$$

$$=e^{-r(T-t)}\int_0^{S(t)e^{r(T-t)}}[S(t)e^{r(T-t)}-x]f_{X_S}(x)dx$$

$$=(S(t)-Xe^{-r(T-t)})\times\frac{\varphi(d_1)+\varphi(d_2)}{\varphi(\infty)+\varphi(d_2)}+\sqrt{D[S(t)](T-t)}\;\frac{e^{-\frac{d_1^2}{2}}-e^{-\frac{d_2^2}{2}}}{\varphi(\infty)+\varphi(d_2)}\qquad(3)$$

3.3.2 The current price of put option on spot is

$$P_S(t)=e^{-r(T-t)}E[\max(X_S(t,T)-S(t)e^{r(T-t)},0)]$$

$$=e^{-r(T-t)}\int_{S(t)e^{r(T-t)}}^{\infty}[x-S(t)e^{r(T-t)}]f_{X_S}(x)dx$$

$$=(Xe^{-r(T-t)}-S(t))\times\frac{\varphi(\infty)-\varphi(d_1)}{\varphi(\infty)+\varphi(d_2)}+\sqrt{D[S(t)](T-t)}\;\frac{e^{-\frac{d_1^2}{2}}}{\varphi(\infty)+\varphi(d_2)}\qquad(4)$$

3.4 The DF structure pricing for American options on futures

If denoting:

$$z_1=\frac{F(t)-Ye^{-r(T-t)}}{\sqrt{D[F(t)](T-t)}},\quad z_2=\frac{Ye^{-r(T-t)}}{\sqrt{D[F(t)](T-t)}},\quad \varphi(x)=\int_0^x e^{-\frac{t^2}{2}}dt=\sqrt{\frac{\pi}{2}(1-e^{-\frac{2}{\pi}x^2})}.$$

According to [10], we have the structure pricing models for American options on futures as follows:

3.4.1 The current price of call option on futures is

$$C_F(t)=e^{-r(T-t)}E[\max(F(t)e^{r(T-t)}-Y_F(t,T),0)]$$

$$= e^{-r(T-t)} \int_0^{F(t)e^{r(T-t)}} [F(t)e^{r(T-t)} - x] f_{Y_F}(x)dx$$

$$= (F(t) - Ye^{-r(T-t)}) \times \frac{\varphi(z_1) + \varphi(z_2)}{\varphi(\infty) + \varphi(z_2)} + \sqrt{D[F(t)](T-t)}\ \frac{e^{-\frac{z_1^2}{2}} - e^{-\frac{z_2^2}{2}}}{\varphi(\infty) + \varphi(z_2)} \qquad (5)$$

3.4.2 The current price of put option on futures is

$$P_F(t) = e^{-r(T-t)} E[\max(Y_F(t,T) - F(t)e^{r(T-t)}, 0)]$$

$$= e^{-r(T-t)} \int_{F(t)e^{r(T-t)}}^{\infty} [x - F(t)e^{r(T-t)}] f_{Y_F}(x)dx$$

$$= (Ye^{-r(T-t)} - F(t)) \times \frac{\varphi(\infty) - \varphi(z_1)}{\varphi(\infty) + \varphi(z_2)} + \sqrt{D[F(t)](T-t)}\ \frac{e^{-\frac{z_1^2}{2}}}{\varphi(\infty) + \varphi(z_2)} \qquad (6)$$

4 The Relations between the American Options on Spots and Futures

Based on (3), (4), (5) and (6), and denoting:

$I_1 = C_F(t) - C_S(t)$, $I_2 = P_S(t) - P_F(t)$

4.1 The put-call parity between the American Options on Spots and Futures

4.1.1 The put-call parity between the American call and put options on spots.

According to (3) and (4), and before the expiration, have $C_S(t)-P_S(t)=S(t)-Xe^{-r(T-t)}-R_S(t)e^{-r(T-t)}$, i.e.

$$C_S(t)+E(X_S)e^{-r(T-t)}=S(t)+P_S(t) \qquad (7)$$

Where, the expected value $E(X_S)=X+R_S(t)$, $R_S(t)=\dfrac{\sqrt{D[S(t)e^{r(T-t)}](T-t)}e^{-\frac{d_2^2}{2}}}{\varphi(\infty)+\varphi(d_2)}$.

Comparing with the put-call parity of European options [9], the $R_S(t)$ in expression (7) is an added part, This is special part just about American options. Because $R_S(t)$ is the value increment which is produced by the fluctuation of underlying asset price. When the American options is near to its maturity, this kind of value increment will become smaller and smaller, till disappeared. The final result is equal to the put-call parity of European options.

According to (5) and (6), and before the expiration, have

4.1.2 The put-call parity between the American call and put options on futures.

$$C_F(t)+E(Y_F)\ e^{-r(T-t)} =F(t)+P_F(t) \qquad (8)$$

Where, the expected value $E(Y_F)=Y+R_F(t)$, $R_F(t)=\dfrac{\sqrt{D[F(t)e^{r(T-t)}](T-t)}e^{-\frac{z_2^2}{2}}}{\varphi(\infty)+\varphi(z_2)}$.

4.1.3 The put-call parity between the American options on spots and futures.

By use of (7) and (8), have

$I_1+I_2=F(t)-Ye^{-r(T-t)}-R_F(t)e^{-r(T-t)}-[S(t)-Xe^{-r(T-t)}-R_S(t)e^{-r(T-t)}]$

$= F(t)-S(t)-[E(Y_F)-E(X_S)]e^{-r(T-t)}$

Namely the put-call parity between the American options on spots and futures is

$$C_F(t)+P_S(t)+S(t)+E(Y_F)e^{-r(T-t)}=C_S(t)+P_F(t)+F(t)+E(X_S)e^{-r(T-t)} \qquad (9)$$

4.2 The values relations between the options on spots and futures

In general, if the futures prices $F(t)$ are consistently higher than its underlying spot prices $S(t)$ prior to maturity, the market is called a normal market; and if futures prices are consistently lower than its underlying spot prices, the market is called an inverted market. From (3), (5), (4) and (6), we shall have the following theorems:

Theorem 4 In the normal market, i.e. the following inequation is correct

$$F(t)\text{-}S(t)>0 \qquad (10)$$

thus we have $I_1>0$ and $I_2>0$ at the same time. Namely, an American call futures option is worth more than the corresponding American call option on the underlying assets and an American put futures option is worth less than the corresponding American put option on the underlying assets.

Proof. Because

$\frac{\partial I_1}{\partial F}=\int_0^F f_{X_F}(x,t)dx>0$, I_1 is monotone increasing strictly about F, hence, we have $I_1>0=\lim_{F\to 0} I_1$ when expression (10) is correct. Again because

$\frac{\partial I_2}{\partial S}=-\int_S^\infty f_{X_S}(x,t)dx<0$, I_2 is monotone descending strictly about S, and then we have $I_2>0=\lim_{S\to\infty} I_2$ when (10) is correct.

Theorem 5 In the inverted market, i.e. the following inequation is correct

$$F(t)\text{-}S(t)<0 \qquad (11)$$

thus we have $I_1<0$ and $I_2<0$ at the same time. Namely, an American call futures option is worth less than the corresponding American call option on the underlying assets and an American put futures option is worth more than the corresponding American put option on the underlying assets.

The proof of **Theorem 5** is similar to **Theorem 4**. Here we do not depict it again.

5 Empirical Researches

5.1 Comparing analysis between the prices of options on spot and futures

The fitting of partial distribution for *DJX*(1/100DJ INDU). We take the close points of *DJX* as sample data.

Time: Jun. 19, 2002 -Dec. 24, 2002. The close points of Trading days are n=132. The estimated results of parameters in partial distribution $P(\mu,\sigma^2)$ are as follows:

$\hat{\mu}$=84.84577713; $\hat{\sigma}^2$=28.65615031

The close price of *DJX* at current date (Dec. 24, 2002) is 84.48$. The option contact of *DJX* would be expired after 19-Dec-2003 (Friday).

Suppose:

1) The current price of spot is $S(t)$ =84.84577713 and the current price of futures is $F(t)$= 84.84577713$e^{\delta(T-t)}$ ($\delta>0$) when t=0.

2) risk-free rate is r=0.07, the δ *in (2)* is equal to 0.01.

5.1.1 In the normal market, i.e. inequation (10) is correct, and when the strike price of option on spot X=88 and the strike price of option on futures Y=88, then the prices of American call and put option on spot and futures are compared separately in figure 1 and figure 2. In figure 1, we see that an American call option on futures is worth more than the corresponding American call option on spot. In figure 2, an American put option on futures is worth less than the corresponding American put option on spot.

When the strike price of option on spot X=80 and the strike price of option on futures Y=80, then the prices of call and put option on spot and futures are compared separately in figure 3 and figure 4. We also see that an American call option on futures is worth more than the corresponding American call option on spot and an American put option on futures is worth less than the corresponding American put option on spot.

5.1.2 In the Inverted market, i.e. inequation (11) is correct. Also suppose that the current price of spot is $S(t)$ =84.84577713 and the current price of futures is $F(t)$=84.84577713$e^{\delta(T-t)}$ ($\delta<0$) when t=0, where risk-free rate is r=0.07, δ=-0.01. When the strike price of option on spot X=88 and the strike price of option on futures Y=88, then the prices of call and put option on spot and futures are compared separately in figure 5 and figure 6. In figure 5, we see that an American call option on futures is worth less than the corresponding American call option on spot. In figure 6, an American put option on futures is worth more than the corresponding American put option on spot.

If the strike price X=80 and Y=80, We also see that an American call option on futures is worth less than the corresponding American call option on spot and an American put option on futures is worth more than the corresponding American put option on spot. See also in figure 7 and figure 8.

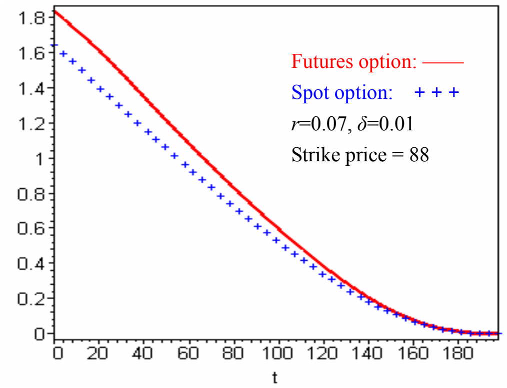

Figure 1. Comparing prices between American call options on futures and its spot. In normal market and current price is lower than the strike price, we see an American call option on futures is worth more than the corresponding American call option on spot.

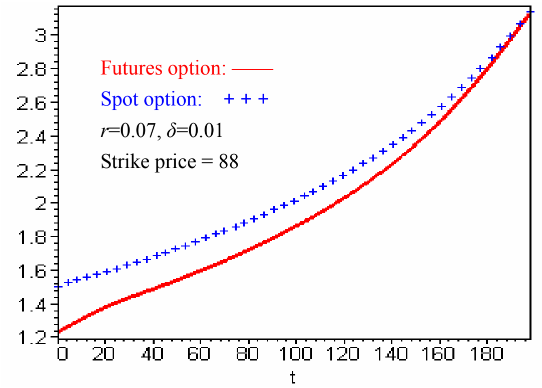

Figure 2. Comparing prices between American put options on futures and its spot. In normal market and current price is lower than the strike price, we see an American put option on futures is worth less than the corresponding American put option on spot.

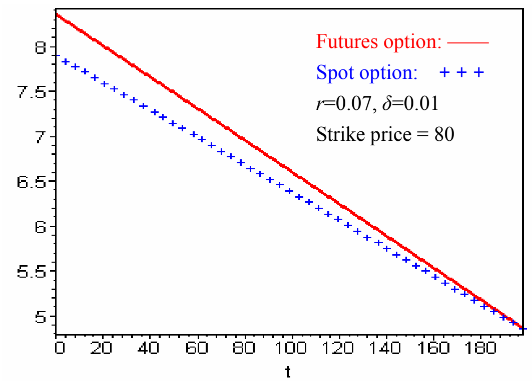

Figure 3. Comparing prices between American call options on futures and its spot. In normal market and current price is higher than the strike price, we see an American call option on futures is worth more than the corresponding American call option on spot.

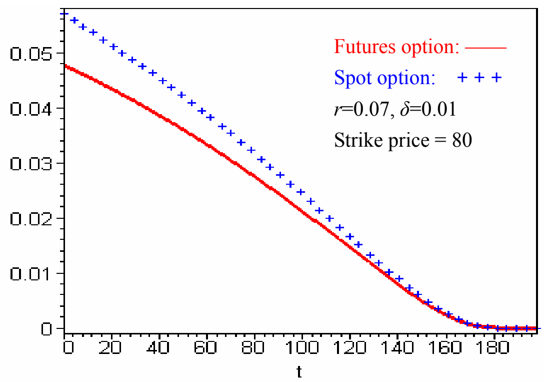

Figure 4. Comparing prices between American put options on futures and its spot. In normal market and current price is higher than the strike price, we see an American put option on futures is worth less than the corresponding American put option on spot.

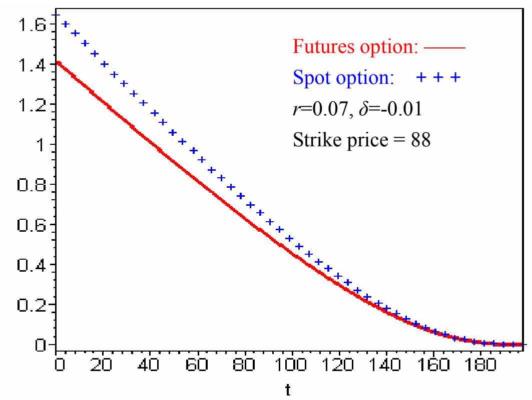

Figure 5. Comparing prices between American call options on futures and its spot. In inverted market and current price is lower than the strike price, we see an American call option on futures is worth less than the corresponding American call option on spot.

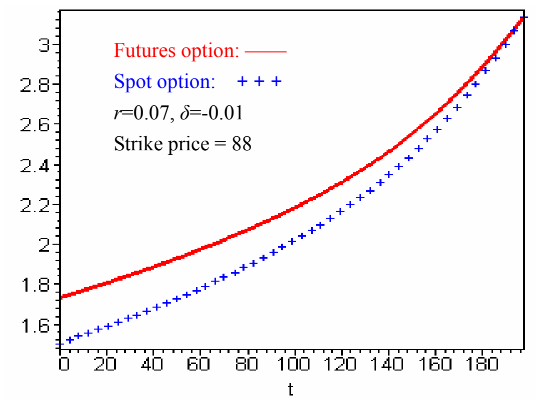

Figure 6. Comparing prices between American put options on futures and its spot. In inverted market and current price is lower than the strike price, we see an American put option on futures is worth more than the corresponding American put option on spot.

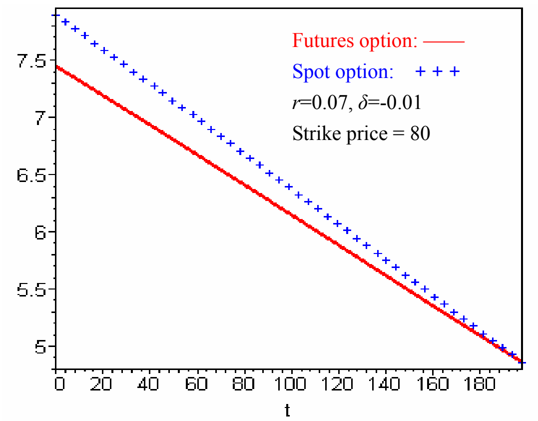

Figure 7. Comparing prices between American call options on futures and its spot. In inverted market and current price is higher than the strike price, we see an American call option on futures is worth less than the corresponding American call option on spot.

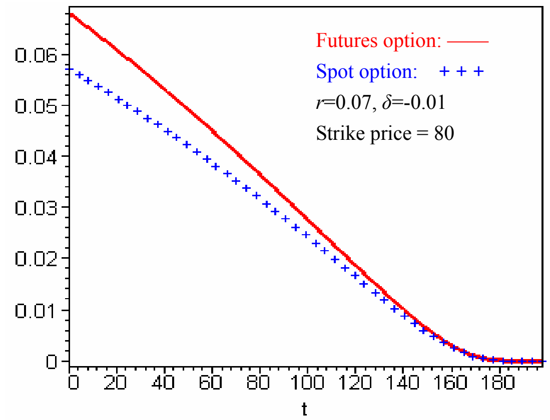

Figure 8. Comparing prices between American put options on futures and its spot. In inverted market and current price is higher than the strike price, we see an American put option on futures is worth more than the corresponding American put option on spot.

5.2 Fitting for the prices of futures

Product: WT407 (The futures contact of forced wheat in CZCE (China ZHENGZHOU COMMODITY EXCHANGE).

Expiration time: July, 2004.

Data: The balance prices of trading days from 2003-10-23 to 2004-5-13.

Taking 4.4 points as the length of fields of samples, and Fields number of samples is m=38.

Based on references [13]-[14], we have the parameters estimated in partial distribution $P(\mu_F,\sigma_F^2)$ and the results of statistic test as follow:

$\hat{\mu}_F$ =1662.141047, $\hat{\sigma}_F^2$ =3118.057796, and χ^2 =52.98752329< $\chi^2_{0.025}$ (35)=53.203.

According to the method of maximum likelihood, we have the parameters estimated in lognormal distribution $Ln(\mu_l,\sigma_l^2)$ and the corresponding statistic test results as follow:

$\hat{\mu}_l$ = 7.416018751, $\hat{\sigma}_l^2$ = 0.000669216686, and χ_l^2 =77.79868312> $\chi^2_{0.025}$ (35)=53.203.

The corresponding histogram and fitting curve of Partial Distribution and Lognormal Distribution are shown in the figure 9.

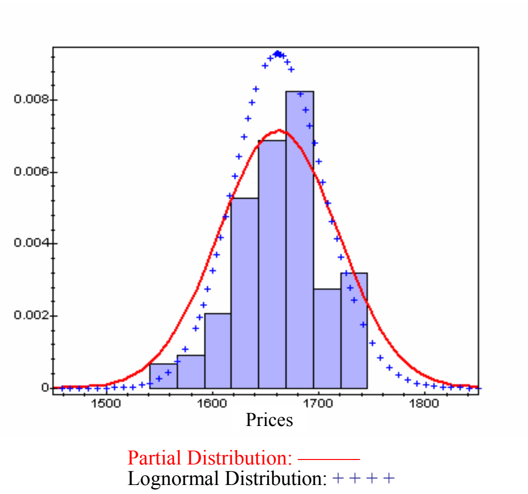

Figure 9. The fitting for prices of futures contact WT407 in Partial and Lognormal distribution. Because the χ^2 =52.98752329< χ_l^2 =77.79868312, the Partial is better than Lognormal in fitting the prices of WT407.

5.3 Comparing analysis between the DF prices and B-S prices of futures option

With the actual data of futures contact WT407 and suppose the expiration time of futures option is T=212, current price of futures is $F(0)$=1432, the strike price of option on futures is Y=1460. By use of *DF* structure pricing model for option on futures, (5) and (6), and Black-Scholes pricing model [1], we have the results of options pricing drawn in figure 10 and figure 11. Figure 10 and figure 11 show that whether the call or put option price given by *DF* structure model are larger than the prices by Black-Scholes model. We could interpret that DF model brings the fluctuations of futures price at any time into considerations, and these fluctuations may bring increment value to American option on futures.

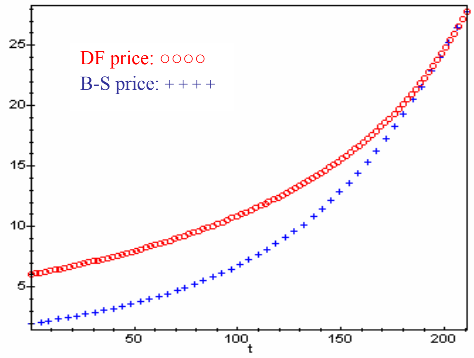

Figure 10. Comparing DF price with B-S price of call options on futures contact WT407

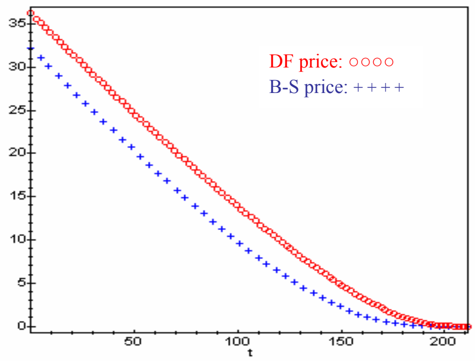

Figure 11. Comparing DF price with B-S price of put options on futures contact WT407

6 Conclusions and Remarks

The authors had discussed the reliability for American option pricing by partial distribution and DF structure model [10], and compare DF model with Black-Scholes model in pricing the American options by use of trading data from China Zhengzhou Commodity Exchange in this paper. The results by DF model are more suitable to reality.

Here we give the three put-call parities about underlying asset, future and their options, i.e. (7), (8) and (9). There is one part or two parts in the put-call parities, it or they will become smaller and smaller along with the time going, until they are disappeared finally, so these put-call parities are different from ones existed, especially the (9) is a new one. The one part or two parts in the put-call parities means that the fluctuation of prices should bring something more into consideration.

The pricing models given in this paper could be applied to both American and European options on futures. It is more significant, by the models given here, we can interpret and prove that an American call futures option is worth more than the corresponding American call option on the underlying assets and an American put futures option is worth less than the corresponding American put option on the underlying assets in a normal market with futures prices consistently higher than spot prices prior to maturity, and the reverses are true in an inverted market with futures prices consistently lower than spot prices.

Here we do not give the empirical research for expression (2), and this work would be done in the future if needed.

References

[1] F. Black, M. Scholes. The Pricing of Options and Corporate Liabilities. Journal of Political Economics, 1973, 81: 637-654

[2] R. C. Merton. Option Pricing when Underlying Stock Returns are Discontinuous. Journal of Financial Economics, 1976, 3: 125-144

[3] W. F. Sharpe. Investment. Prentice-Hall Inc.,1978. 118-130,145-152

[4] J. C. Cox, S. A. Ross. Valuation of Options for Alternative Stochastic Process. Journal of Political Economics, 1976, 3: 145-166

[5] R. Gesk, R. Roll. On Valuing of American Call Options with the Black-Scholes European Formula. Journal of Finance, 1984, 39: 443-455

[6] E. Briys. Options, Futures and Exotic Derivatives. Jhon Wiley & Sons, Inc.,1998.

[7] P. Ritchen, R. Trevor. Pricing Options Under Generalized GARCH and Stochastic Volatility Processes. Journal of Finance, 1999, 54: 377-402

[8] K. R. Miltersen, Schwartz. Pricing of Options on the Commodity Futures with Stochastic Term Structures of Convenience Yields and Interest Rates. Journal of Financial and Quantitative Analysis, 1998, 33(1): 33-59

[9] John C. Hull. Options, Futures, and Other Derivatives (4th ed). Prentice Hall Inc., 2000. 72,182, 251,296, 388

[10] F. Dai, Z. F. QIN. DF Structure Models for Options Pricing. International Journal of Applied Economics. 2005, accepted.

[11] F. Dai, G. P. Gi. A New Kind of Pricing Model for Commodity and Estimating Indexes System for Price Security. *Chinese Journal of Management Science*, 2001, 9(1): 62-69.

[12] F. Dai. The Market Value Analytic Process for Investment Based on the Partial Distribution. Proceedings of SCI 2001/ ISAS 2001, Orlando, USA.
[13] F. Dai, W. X. Xu, H. Liu, H.Xu. A New Kind of Method of Optimal Pricing for Commodity. *Chinese Journal of Management Science*, 2003, 11(1): 33-37
[14] F. Dai, X. Chen and K. Sun. The Pd-Fitness Analysis of Price Structures on Chinese Stocks Market. Proceedings of SCI 2003, Orlando, USA.