CAN LONG HORIZON DATA BEAT RANDOM WALK UNDER ENGEL-WEST EXPLANATION?

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Abstract: Engel and West (2004a) provide an explanation to reconcile the random walk behavior of exchange rate and linear present value asset pricing models. In this paper, we study the long horizon property of exchange rate under Engel-West explanation. It is found that the long horizon data can not significantly improve our chance of beating random walk. This result is consistent with recent empirical studies on the long horizon exchange rate. Under E-W explanation, the change of exchange rate can be more serially correlated in the long horizon data, but this change in most cases is only marginal. Depending on the persistence of change in fundamentals, two patterns may exist between the autocorrelation of exchange rate change and the time horizon. Both of these two patterns are found existing in the real data of exchange rates. These results support E-W explanation for exchange rate puzzle.

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1. Introduction

A long standing puzzle in international finance is the disconnection between exchange rate and fundamentals. In Meese and Rogoff's (1983a, 1983b) seminal papers, they find that a simple random walk model can perform as well as various structural and time series exchange rate models based on the out-of-sample forecasting accuracy. This result suggests that exchange rate statistically follows a random walk. But most fundamental variables, which are believed in theories to determine exchange rate, are not following a simple random walk. Therefore, it is very puzzling for us to find exchange rate approximately follows a random walk process. This disconnection has been well documented in many succeeding papers and enormous attempts have been made to solve this puzzle. However, in a recent comprehensive study by Cheung, Chinn and Pascual (2003), they conclude, "No model consistently outperform a random walk, by a mean squared error measure...Overall, model/specification/currency combinations that work well in one period will not necessarily work well in another period."

Engel and West (2004a) explore a new direction of attacking exchange rate puzzle. They find sufficient conditions under which exchange rate approximately follows a random walk even if it is actually determined by fundamentals in a general present value asset pricing model. They also show in their paper that these sufficient conditions are generally satisfied in data. Since a wide range of exchange rate determination models can be written as a special case of this general present value asset pricing model in Engel and West's paper, the random walk behavior of exchange rate is just a natural outcome of the structural model and the failure of structural model in beating random walk is not a puzzle any more under Engel-West explanation (E-W explanation thereafter).

The E-W explanation could completely shift the direction of research on exchange rate puzzle if it is correct in explaining the exchange rate puzzle. At least, under this explanation the merit of theoretical models should not be solely judged by whether they can predict exchange rate more accurately than a random walk or not. Their results indeed suggest the predictability in the opposite direction: exchange rate can help to predict fundamentals. However, E-W explanation is among many other possible reasons in explaining why exchange rate follows a random walk. More studies on the implications of E-W explanation are required so as to see if they are consistent with previous studies and data.

In this paper we will connect E-W explanation with another cluster of papers studying long horizon predictability of exchange rate. The authors of these papers attack exchange rate puzzle by studying if long horizon exchange rate is more predictable. Intuitively, if exchange rate is connected with some fundamentals, it may deviate from its equilibrium level in the short run, but will converge to the long run level over time. So, the long horizon exchange rate is more predictable under this explanation. The early claims of success in this field include Mark (1995), Chinn and Meese (1995) and MacDonald and Taylor (1994). However, this success has been challenged recently due to the unrobustness of the results and the severe size distortion and small sample bias in the test. Kilian (1999) finds no evidence for the long horizon predictability after including recent observations into the dataset of Mark (1995). He also shows that previous evidence of long horizon predictability could also be caused by the incorrect bootstrap method. After correcting for inconsistencies and small sample bias, the long horizon predictability disappears.

In this paper, we are going to study how the random walk behavior of exchange rate varies with time horizon under E-W explanation. We find that long horizon change of log exchange rate is more serially correlated and might have better chance to beat a random walk model if the change of fundamentals is persistent. However, the increase in autocorrelation is still very limited and we should not be over-optimistic about beating random walk with long horizon data. Moreover, the autocorrelation decreases with time horizon in long run and when the time horizon grows too big, long horizon exchange rate behaves more like a random walk than the short horizon data. More interestingly, we find that with some reasonable calibration, the autocorrelation is not monotonic with time horizon. The autocorrelation increases in the beginning and goes down eventually with time horizon. This hump-shaped pattern is also found in the exchange rate of four countries: Australia, Canada, Japan and U.K. These results to some extent can be used to support the E-W explanation for the exchange rate puzzle. We also use a simple present asset pricing model to show that under E-W explanation, even if exchange rate converge to its long horizon value, the gain is small and therefore no evidence can be found to support long horizon predictability. This provides a possible theoretical explanation for Kilian's (1999) findings.

The remaining of the paper is organized in this way. Section 2 provides a brief introduction to the E-W explanation, which helps us to provide intuition for the long horizon property of exchange rate. In this section we also show analytically the relation between the random walk behavior of exchange rate and the time horizon. In section 3, we connect our results with the studies of long horizon regressions. In a Vector Error Correction (VEC) model, we show that the gain of long-run convergence is small when the discount factor is close to one. In section 4, we study the long horizon autocorrelation of log exchange rate change of Australia, Canada, Japan and U.K. and find some interesting patterns predicted in section 2. Section 5 summarize major findings and concludes.

2. Asset Pricing Model and E-W Explanation

In this section, we give a brief introduction to E-W explanation and derive the long horizon properties of exchange rate under this explanation. The asset pricing model can be generally written as[1]

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t(a_1' x_{t+j}) + b\sum_{j=0}^{\infty} b^j E_t(a_2' x_{t+j}) \tag{2.1}$$

where 0<b<1 is the discount factor, x_{t+j} is a $n \times 1$ vector containing fundamentals and a_1 and a_2 are $n \times 1$ parameter vectors. Equation (2.1) says an asset price is equal to the discounted sum of present and expected future fundamentals. In the exchange rate determination models, s_t is the log exchange rate at time t. Engel and West (2004) show that a wide range of exchange rate determination models can be written as a special case of equation (2.1). They show analytically that if the discount factor is close to unity and either (1) $a_1' x_{t+j} \sim I(1)$, $a_2 = 0$ or (2) $a_2' x_{t+j} \sim I(1)$, the log exchange rate will approximately follow a random walk. To develop intuition, we follow the same example as in Engel and West (2004). Suppose the logarithm[2] of exchange rate is determined by the equation (2.2).

$$s_t = (1-b) f_t + b\rho_t + bE_t(s_{t+1}) \tag{2.2}$$

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[1] For example, see Campbell and Shiller (1987, 1988) and West (1988).

[2] For simplicity, we will call s_t exchange rate instead of logarithm of exchange rate in the remaining of the paper when it raises no confusion.

where f_t and ρ_t are fundamentals that determine exchange rate in theoretical models. Applying the "non-bubble" condition to (2.2), it is easy for us to obtain

$$(2.3) \qquad s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t f_{t+j} + b\sum_{j=0}^{\infty} b^j E_t \rho_{t+j}$$

A simple explanation for why exchange rate follows a random walk is that the fundamentals are following a random walk. In this case, the expectation of f_{t+j} (ρ_{t+j}) at time t is just equal to f_t (ρ_t). So, it is easy for us to obtain from (2.3) that

$$(2.4) \qquad s_t = f_t + \frac{b}{1-b}\rho_t$$

It is obvious that the exchange rate is also a random walk when f_t and ρ_t are not cointegrated.

In the data however, most fundamentals are not following a simple random walk. The change of some fundamentals is actually very persistent. For instance, the interest rate differential and the price differential among industrial countries usually have a lag-one autocorrelation of more than 0.5. So, the above explanation is unlikely to be the reason why exchange rate follows a random walk. It is more reasonable to model the change of fundamentals with an AR(1) process as in (2.5).

$$(2.5) \qquad \Delta f_t = \phi \Delta f_{t-1} + \varepsilon_{mt} \qquad \Delta\rho_t = \gamma\Delta\rho_{t-1} + \varepsilon_{\rho t}$$

Substitute (2.5) to (2.3), the change of exchange rate can be written as

$$(2.6) \quad \Delta s_t = \frac{\phi(1-b)}{1-b\phi}\Delta m_{t-1} + \frac{1}{1-b\phi}\varepsilon_{mt} + \frac{b\gamma}{1-b\gamma}\Delta\rho_{t-1} + \frac{b}{(1-b)(1-b\gamma)}\varepsilon_{\rho t}$$

E-W explanation says that if the discount factor (b in the equation (2.6)) is close to unity and at lease one fundamental is I(1)[3], the exchange rate will statistically indistinguishable from a random walk process except we have a very large number of observations (Usually it requires thousands of observations to reject unit-roots hypothesis under a reasonable calibration of parameters).

Exchange rate follows a random walk if and only if the change of exchange rate is serially uncorrelated. In the equation (2.6), the change of exchange rate can be taken as a weighted average of fundamentals and "white noises". In our model the change of exchange rate is serially correlated because the changes of fundamentals (Δm_{t-1} and $\Delta \rho_{t-1}$) are serially correlated. However, as the discount factor (b in the equation) becomes bigger, more weight is given to the noise terms rather than the fundamentals. When the discount factor (b) approaches to unity, the coefficients of fundamental changes ($\frac{\phi(1-b)}{1-b\phi}$ and $\frac{b\gamma}{1-b\gamma}$) go to zero or a constant. However, the coefficients of error terms ($\frac{1}{1-b\phi}$ and $\frac{b}{(1-b)(1-b\gamma)}$) will go to a constant or infinity. Therefore the variance of exchange rate change will be dominated by the variance of serially uncorrelated error terms when the discount factor is close to unity. In this way, the autocorrelation of exchange rate change is statistically indifferent from zero when sample size is small. Empirical studies show that the discount factor is actually very close to unity for

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[3] This condition is generally satisfied since most fundamentals in exchange rate determination models are found to follow I(1) processes.

quarterly data. The estimates of quarterly discount factor in previous empirical studies are generally above 0.97[4].

Obviously the value of the discount factor is very crucial to E-W explanation. From their paper, we know that 55 observations will be enough to reject the null hypothesis of random walk if the discount factor is 0.5 or less[5]. In short horizon, the discount factor is very close to unity. However, with the increase of time horizon, discount factor becomes smaller (we refer it as discount factor effect from now on). In this aspect, long horizon data (for instance, annual data) could be more serially correlated and might increase our chance of beating random walk under E-W explanation. However, we must be cautious about this claim. First of all, the autocorrelation is not necessarily going up in the long horizon data. With the increase of time horizon, the autocorrelation of change of fundamentals declines too (we refer it as persistence effect). As we have just mentioned, if the change of fundamentals is serially uncorrelated, asset pricing model predicts exchange rate follows a random walk. It is not clear which effect (the persistence effect or the discount factor effect) will dominate in determining the autocorrelation of exchange rate change as time horizon increases. Secondly, an increase of autocorrelation in long horizon data is not sufficient for us to have better chance of rejecting random walk. As time horizon goes larger, the number of non-overlapping observations drops sharply. It will decrease the test power dramatically. Only when the increase in autocorrelation is large enough to offset this power loss, would we have better chance in rejecting random walk.

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[4] Some examples include Clarida, Gali and Gertler (1998) and Stock and Watson (1993). See Engel and West (2004a) for a more complete list of examples.

[5] ϕ is equal to 0.5 or higher.

Let’s first look at how the autocorrelation of long horizon data change with time horizon. As we have just mentioned, the change of autocorrelation depends on the tradeoff between the discount factor effect and the persistence effect. Under the effects of these two factors, the change of long horizon exchange rate would be more persistent if the discount factor dominates and less persistent if otherwise. But we can see that the persistence effect will eventually dominate the discount factor effect since as the time horizon gets larger, the persistence of fundamentals will go to zero very fast. If the changes of fundamentals are serially uncorrelated, the exchange rate follows a random walk process regardless of the value of discount factor. Depending on which effect dominates in the beginning, there are two possible scenarios. If the persistence effect dominates from the beginning, the autocorrelation of long horizon data decreases with the time horizon. Alternatively, the discount factor effect might exceed the persistence effect for some time before the latter takes over the power. In this scenario, the autocorrelation of long horizon data will increase with the horizon when the discount factor dominates and decrease later when the persistence factor dominates. So we will observe a hump-shaped pattern in the autocorrelation of long horizon data under this situation.

Following Engel and West (2004), we focus on the first order autocorrelation in this paper when testing the hypothesis that the changes of exchange are serially uncorrelated. The results for higher order autocorrelations follow in a similar way. We calculate the first order autocorrelation numerically for long horizon data and compare it with that of short horizon data to see how much the autocorrelation increases in the long horizon data. By changing values of parameters, we can also check if the hump-shaped pattern is possible under some reasonable calibration of parameters. To keep the calculation tractable, we set $\Delta\rho_t = 0$ for all t. The equation (2.6) reduces to

$$(2.7) \qquad \Delta s_t = \alpha \Delta m_{t-1} + \beta \varepsilon_t$$

where $\alpha = \frac{\phi(1-b)}{1-b\phi}$, $\beta = \frac{1}{1-b\phi}$. When b➔1, α ➔0 and $\beta \Rightarrow \frac{1}{1-\phi}$. Therefore, the change of exchange rate behaves in a similar way as ε_t when the discount factor is close to one.

The long horizon exchange rate change is just the sum of corresponding short horizon changes. We use capital letters to represent corresponding long horizon data. Suppose the horizon is equal to k. The first observation in the long horizon data will be

$$(2.8) \qquad \Delta S_1 = \Delta s_1 + \Delta s_2 + \ldots + \Delta s_k$$

The (k+1)th observation is

$$(2.9) \qquad \Delta S_{k+1} = \Delta s_{k+1} + \Delta s_{k+2} + \ldots + \Delta s_{2k}$$

It is easy to see that ΔS_1 and ΔS_{k+1} are the first pair non-overlapping observations in the long horizon data. To calculate the autocorrelation, we need first write ΔS_1 and ΔS_{k+1} in terms of Δf_0 and the error terms ($\varepsilon_1, \varepsilon_2 \ldots\ldots \varepsilon_{2k}$). Δf_0 is a AR(1) process with a mean of zero and a variance of $\frac{1}{1-\phi^2}$. Δf_0 is also uncorrelated with the error terms. From equation (2.7) and (2.5), we can obtain

$$
\begin{aligned}
\Delta S_1 &= \Delta s_1 + \Delta s_2 + \ldots + \Delta s_k \\
&= \alpha\left(\Delta m_0 + \Delta m_1 + \ldots + \Delta m_{k-1}\right) + \beta\left(\varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_k\right) \\
&= \alpha\begin{bmatrix} \left(1 + \phi + \phi^2 + \ldots + \phi^{k-1}\right)\Delta m_0 \\ + \left(\varepsilon_1 + \varepsilon_2 + \phi\varepsilon_1 + \varepsilon_3 + \phi\varepsilon_2 + \phi^2\varepsilon_1 + \ldots + \varepsilon_{k-1} + \phi\varepsilon_{k-2} + \ldots + \phi^{k-2}\varepsilon_1\right) \end{bmatrix} \\
&\quad + \beta\left(\varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_k\right) \\
&= \alpha\begin{bmatrix} \dfrac{1-\phi^k}{1-\phi}\Delta m_0 + \varepsilon_1\left(1 + \phi + \ldots + \phi^{k-2}\right) + \varepsilon_2\left(1 + \phi + \ldots + \phi^{k-3}\right) + \ldots \\ + \varepsilon_{k-1} \end{bmatrix} \\
&\quad + \beta\left(\varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_k\right) \\
&= \alpha\left[\frac{1-\phi^k}{1-\phi}\Delta m_0\right] + \left[\frac{\alpha\left(1-\phi^{k-1}\right)}{1-\phi} + \beta\right]\varepsilon_1 + \left[\frac{\alpha\left(1-\phi^{k-2}\right)}{1-\phi} + \beta\right]\varepsilon_2 + \ldots \\
&\quad + \left(\alpha + \beta\right)\varepsilon_{k-1} + \beta\varepsilon_k
\end{aligned}
\tag{2.10}
$$

Similarly, we can obtain the solution for ΔS_{k+1}

$$
\begin{aligned}
\Delta S_{k+1} &= \frac{\alpha\phi^k\left(1-\phi^k\right)}{1-\phi}\Delta m_0 + \alpha\left[\frac{\phi^k\left(1-\phi^{k-1}\right)}{1-\phi} + \phi^{k-1}\right]\varepsilon_1 \\
&\quad + \alpha\left[\frac{\phi^k\left(1-\phi^{k-2}\right)}{1-\phi} + \frac{\phi^{k-2}\left(1-\phi^2\right)}{1-\phi}\right]\varepsilon_2 + \ldots \\
&\quad + \alpha\left[\phi^k + \frac{\phi\left(1-\phi^{k-1}\right)}{1-\phi}\right]\varepsilon_{k-1} + \frac{\alpha\left(1-\phi^k\right)}{1-\phi}\varepsilon_k \\
&\quad + \left[\frac{\alpha\left(1-\phi^{k-1}\right)}{1-\phi} + \beta\right]\varepsilon_{k+1} + \left[\frac{\alpha\left(1-\phi^{k-2}\right)}{1-\phi} + \beta\right]\varepsilon_{k+2} + \ldots + \left(\alpha + \beta\right)\varepsilon_{2k-1} + \beta\varepsilon_{2k}
\end{aligned}
\tag{2.11}
$$

For given parameters (b, ϕ and k), we can calculate the first order autocorrelation of ΔS_1 and ΔS_{k+1}. As we have mentioned, a conservative estimate for the quarterly discount factor b is 0.97. So we set the discount factor to 0.97 and change the value of ϕ and k to see how the autocorrelation varies with time horizon and persistence of change of fundamentals. The first order autocorrelations are reported in table 1, where k is the horizon length. The first row in table 1 is the autocorrelation of short horizon data and the succeeding rows report the autocorrelations for corresponding long horizon data.

Table 1 Auto-correlation of long horizon data*

K	ϕ=0.4	ϕ=0.5	ϕ=0.6	ϕ=0.7	ϕ=0.8	ϕ=0.9
1 (short horizon)	**0.0120**	**0.0150**	**0.0180**	**0.0210**	**0.0240**	**0.0270**
2	0.0117	0.0167	0.0228	0.0299	0.0382	0.0612
3	0.0096	0.0150	0.0226	0.0327	0.0462	0.0722
4	0.0078	0.0129	0.0207	0.0326	0.0502	0.0819
5	0.0064	0.0110	0.0185	0.0310	0.0516	0.0896
6	0.0054	0.0094	0.0164	0.0289	0.0513	0.0954
7	0.0046	0.0082	0.0146	0.0266	0.0501	0.0994
8	0.0041	0.0072	0.0130	0.0244	0.0482	0.1020
9	0.0036	0.0064	0.0117	0.0224	0.0461	0.1035
10	0.0032	0.0058	0.0106	0.0206	0.0438	0.1040
11	0.0029	0.0052	0.0096	0.0190	0.0416	0.1038
12	0.0027	0.0048	0.0088	0.0176	0.0394	0.1030
13	0.0025	0.0044	0.0082	0.0163	0.0372	0.1018
14	0.0023	0.0041	0.0076	0.0152	0.0352	0.1002
15	0.0022	0.0038	0.0071	0.0142	0.0334	0.0984
16	0.0020	0.0036	0.0066	0.0134	0.0316	0.0963
17	0.0019	0.0034	0.0062	0.0126	0.0300	0.0942
18	0.0018	0.0032	0.0059	0.0119	0.0285	0.0919
19	0.0017	0.0030	0.0056	0.0113	0.0272	0.0896
20	0.0016	0.0029	0.0053	0.0107	0.0259	0.0873

*The discount factor is set to 0.97.

From table 1, we find that when the discount factor (0.97) is close to one, the autocorrelation of exchange rate change is very close to zero. This confirms E-W explanation: when the discount factor is close to unity, the change of exchange rate is approximately serially uncorrelated. It is intuitive to find that the autocorrelation increases with the value of ϕ. When the changes of fundamentals are more serially correlated, the fundamental determined exchange rate change is also more serially correlated. However, even if the persistence of fundamentals is set to a very high level (0.9), the autocorrelation of exchange rate change is still below 0.03 (0.027) and we need more than 5,000 observations to reject the null hypothesis of zero-autocorrelation ($\frac{0.027}{1/\sqrt{5200}} \approx 1.95 < 1.96$)[6].

It is more interesting to compare the autocorrelations of short and long horizon data. Under the usual persistence of fundamentals ($\phi > 0.4$), long horizon data is more serially correlated than the short horizon one for some horizon(s). In figure 1, we plot the autocorrelation of short and long horizon data. The solid line is the autocorrelation of long horizon data and dashed line is that of short horizon data. The long horizon data have higher autocorrelation when the solid line is above the dashed line. This is true when k is small in all of our plots except for $\phi = 0.4$. The difference is bigger when the change of fundamentals is more persistent (that is, when ϕ is larger). Also with the increase of ϕ, more and more long horizon data have higher autocorrelation than the short horizon data. This result is intuitive—when ϕ is bigger, the persistent effect is less significant since the persistence of long horizon fundamental changes goes down exponentially with ϕ [7]. However, it is worth of noticing that the increase in autocorrelation is still very limited. The maximum of autocorrelation is 0.1040 (when $\phi = 0.9$ and k=10). This means we still need approximately 370 observations to reject null hypothesis of no-autocorrelation ($\frac{0.104}{1/\sqrt{370}} \approx 2 > 1.96$). Since the time horizon is 10, it is equivalent to 3,700 observations in the short horizon data.

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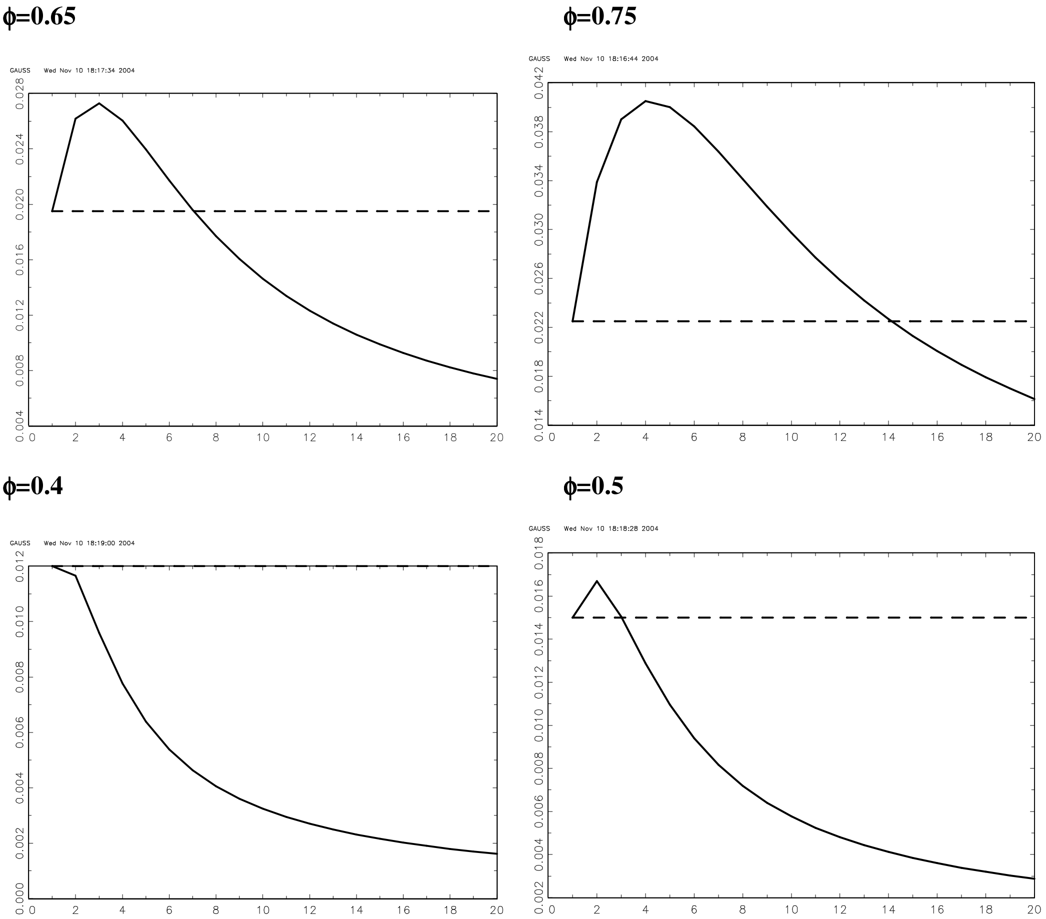
[6] We choose 95% confidence level in the test and do not consider sample errors.

[7] This relation holds just approximately.

Figure 1



ϕ=0.65 **ϕ=0.75**

ϕ=0.4 **ϕ=0.5**

Note: The solid line is the autocorrelation between ΔS_1 and ΔS_{k+1}. The dashed line is the first order autocorrelation of the short horizon data. The discount factor (b) is equal to 0.97 in all plots.

It is also very interesting that when ϕ is greater than 0.65, we do see the hump-shaped relation between the autocorrelation and the length of horizon. In figure 1, we can find the plots of autocorrelation against time horizon when ϕ is equal to 0.65 and 0.75. In these plots, the lag-one autocorrelation increases in the long horizon data for some periods and then declines with the time horizon. But this hump-shaped pattern does not exist when the change of fundamentals is less persistent (For instance, see the plots in figure 1 when ϕ is equal to 0.4). When the value of ϕ is small, the autocorrelation monotonically decreases with the time horizon. We can see this clearly from table 1 that when ϕ is equal to 0.4, the autocorrelation decreases monotonically with k. This result is intuitive and confirms our conjecture on the tradeoff between the persistence and discount factor effect. As we have mentioned, the smaller the ϕ is, the faster the persistence of fundamental change goes down as horizon increases. When ϕ is less than 0.6, the persistence effect dominates from k=3. So we see the autocorrelation goes down all the way for k is greater than or equal to 3. Actually, when ϕ is less than 0.5, the persistence effect dominates even when k=2. So in this case, all the long horizon data have smaller autocorrelation than the short horizon one.

As we have mentioned, the increase in autocorrelation is not sufficient for the long horizon data to have better chance of rejecting random walk. The gain in autocorrelation must be big enough to offset the loss of power due to the decrease of observations as time horizon grows. In table 2, we report the number of observations we need to reject null hypothesis at 95% confidence level. When the changes of fundamentals are persistent, long horizon data require fewer observations to reject null-hypothesis. For instance, when $\phi = 0.9$, we know long horizon data require 2,054 observations to reject null-hypothesis of zero-autocorrelation. Although this requirement is still too luxury in reality, it is much better than the requirement of more than 5,200 observations when we test short horizon data directly. In this sense, the long horizon data has better chance to beat random walk though we still have to depend on our luck to do so. When the change of fundamentals is less persistent, the chance of beating random walk is even worse for the long horizon data. This can be seen from the fact that when

$\phi = 0.6$ or less, the long horizon data generally require more observations than the corresponding short horizon data. Therefore, when the change of fundamentals is very persistent, the long horizon data may have better chance of rejecting random walk. However, the improvement usually is not very impressive and when the change of fundamentals is less persistent, the long horizon data even perform worse than the corresponding short horizon one.

Table 2 Number of observations required to reject h-null*

K	ϕ=0.4	ϕ=0.5	ϕ=0.6	ϕ=0.7	ϕ=0.8	ϕ=0.9
1 (short horizon)	**26,678**	**17,074**	**11,857**	**8,711**	**6,669**	**5,270**
2	56,539	27,551	14,835	8,599	5,268	2,054
3	125,205	50,933	22,630	10,752	5,392	2,214
4	254,690	92,625	35,818	14,494	6,095	2,290
5	469,864	159,986	56,149	20,008	7,218	2,391
6	795,243	260,954	85,768	27,683	8,750	2,534
7	1,254,148	403,451	126,997	38,002	10,728	2,722
8	1,868,970	595,125	182,214	51,506	13,211	2,953
9	2,661,657	843,333	253,778	68,765	16,271	3,229
10	3,654,050	1,155,207	343,985	90,370	19,990	3,551
11	4,868,040	1,537,755	455,065	116,916	24,455	3,921
12	6,325,605	1,997,934	589,186	148,994	29,761	4,343
13	8,048,805	2,542,694	748,469	187,184	36,005	4,820
14	10,059,756	3,179,000	935,006	232,058	43,286	5,356
15	12,380,609	3,913,836	1,150,870	284,172	51,708	5,954
16	15,033,541	4,754,211	1,398,129	344,074	61,372	6,621
17	18,040,736	5,707,149	1,678,849	412,301	72,382	7,362
18	21,424,387	6,779,687	1,995,098	489,382	84,839	8,180
19	25,206,693	7,978,873	2,348,950	575,841	98,845	9,082
20	29,409,851	9,311,759	2,742,482	672,198	114,502	10,074

*The discount factor is set to 0.97 and the confidence level of test is 95%.

3. Long-horizon Regressions and E-W Explanation

In the last section, we find that the increase of autocorrelation in long horizon data is very limited. Although we have better chance to reject the null hypothesis that the change of exchange rate is un-correlated when the change of fundamentals are persistent, the increase still marginal. In this section, we connect this result with the studies of long horizon exchange rate.

The long-horizon regressions have been widely used to test the efficiency of asset market[8]. Mark (1995) and Chinn and Meese (1995) use this method to test if exchange rate is more predictable in the long horizon data. In the long horizon regressions, two equations are compared.

$$s_{t+k} - s_t = \alpha_k + \beta_k z_t + \mu_{t+k} \tag{3.1}$$

$$s_{t+k} - s_t = \gamma_k + \mu_{t+k} \tag{3.2}$$

In the equation (3.1), $s_{t+k} - s_t$ is k-period change of exchange rate. z_t is the deviation of exchange rate from its long-run equilibrium level at time t ($z_t = s_t - f_t$). The equation (3.2) is just a random walk model for exchange rate. If β_k and R-square of equation (3.1) increase with time horizon k, it is taken as evidence that exchange rate converges to its long-run value over time. More formally, the null-hypothesis that (3.1) and (3.2) have the same forecast accuracy is tested against the alternative that (3.1) is more accurate in forecasting exchange rate.

Mark (1995) and Chinn and Meese (1995) find some evidence to support the claim that exchange rate is more predictable in the long horizon data. However, the results are found not robust and also subject to different econometric methods used in the test. Kilian (1999) finds that if we extend the sample to include more recent observations or employ a new (and more reasonable) bootstrap method, the long horizon data can not beat a driftless random walk anymore. He actually finds that the long horizon data even perform worse than the short horizon data. These results are consistent with what we

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[8] Some examples include Fama and French (1988) and Campbell and Shiller (1988).

have found in section 2. When we increase the time horizon, the increase in the autocorrelation of exchange rate change is very limited and the autocorrelation is even smaller for long horizon data when the horizon grows too big or when the short horizon change of fundamentals is not very persistent. In this sense, the long horizon data behave more like a random walk than the short horizon data. So in the long horizon regressions, it is very hard to find some statistical evidence in favor of model (3.1) even if the exchange rate is determined by present value asset pricing model.

Berkowitz and Giorgianni (2001) find that Mark's results are also sensitive to the assumption that there exists a long-run relation between exchange rate and fundamentals. They show that long horizon predictability also weakens when we release the restriction that exchange rate and fundamentals are cointegrated. As Kilian (1999), in our present value asset pricing model, we actually admit the cointegration between exchange rate and fundamentals. We will show that under E-W explanation, even if exchange rate and fundamentals are cointegrated, the gain for long run convergence is very limited. So it is unlikely to beat random walk with long horizon data.

We first look at the underlying reason for why we are expecting long horizon data to be more predictable. In the absent of speculative bubbles, the present value asset pricing model implies

$$s_t = (1-b)E_t\left[\sum_{j=0}^{\infty} b^j f_{t+j}\right] \tag{3.3}$$

From (3.3) we can obtain

$$s_t - f_t = E_t\left[\sum_{j=1}^{\infty} b^j \Delta f_{t+j}\right] \tag{3.4}$$

As we have mentioned, most fundamentals are I(1) in our data and we suppose the change of fundamental follows an AR(1) process.

(3.5) $$\Delta f_t = \phi \Delta f_{t-1} + \varepsilon_t$$

From (3.4) we know that exchange rate and fundamentals are cointegraged with a conintegrating vector of (1, -1). f_t can be interpreted as the long-run equilibrium level of exchange rate and $z_t = s_t - f_t$ as the deviation of exchange rate from its long-run value. Generally, we can write the cointegrated system in the vector error correction (VEC) form.

(3.6) $$\Delta y_t = \nu + \xi_1 \Delta y_{t-1} + \xi_2 \Delta y_{t-2} + \ldots + \xi_{p-1} \Delta y_{t-p+1} + \xi_0 y_{t-1} + \mu_t$$

where $y_t = (s_t, \quad f_t)'$ and $\xi_0 = -\mathrm{BA}'$. A' is the cointegrating vector (1, -1) and B is a 2X1 vector with elements b_1 and b_2. Substitute ξ_0 back to (3.6) and subtract the second equation from the first one in the equation system of (3.6), we can get an AR(1) form for the error correction term z_t.

(3.7) $$z_t = (1 - b_1 + b_2) z_{t-1} + \tilde{\mu}_t$$

Since z_t is stationary, $|1 - b_1 + b_2| < 1$, that is, the exchange rate converges to its long-run equilibrium level over time. The equation (3.1) exploits this long-run convergence by comparing how β_k changes with k. β_k measures the cumulative effect of fundamentals on exchange rate change and should increase with the time horizon k.

However, when $|1 - b_1 + b_2|$ is close to zero, the gain over time is negligible. In this case, the long horizon data do not have much advantage compared to the short horizon data. We find that when the discount factor is close to unity, $|1 - b_1 + b_2|$ is actually very close to zero. After some tedious algebra, we can write the exchange rate and fundamentals into error correction form by manipulating equation (3.3) and (3.5)[9].

$$\begin{aligned} &\Delta s_t = \frac{\phi(1-b)}{1-b\phi}\Delta m_{t-1} + \frac{\phi}{1-b\phi}\Delta f_{t-1} - \frac{1}{b} z_{t-1} + \frac{1}{1-b\phi}\varepsilon_t \\ &\Delta f_t = \phi \Delta f_{t-1} + \varepsilon_t \end{aligned} \tag{3.8}$$

From (3.8), we know $b_1 = \frac{1}{b}$ and $b_2 = 0$. When the discount factor is very close to one, $|1 - b_1 + b_2|$ is close to zero. For instance, under our conservative estimation of discount factor 0.97, $|1 - b_1 + b_2|$ is about 0.03. So the gain from long-run convergence is negligible. If we can not beat random walk at short horizon, increase in the time horizon should not help us. This result is consistent with Kilian's (1999) findings and can serve as a potential theoretical explanation for his results.

We have to make it clear however, that we are not claiming the exchange rate and fundamentals are cointegrated. The empirical research on the cointegration of exchange rate and fundamentals is still a controversy. Nonetheless, even under a more restrictive condition—exchange rate and fundamentals are cointegrated—the long horizon data do not have advantage in beating random walk when the discount factor is close to zero. An obvious reason for failing in detecting cointegration in the data is that some unobservable fundamentals may follows a random walk or be very persistent. Engel and West (2004b) consider a model with unobservable fundamentals and estimate that the variation of fundamentals can only explain about 40% exchange rate variation.

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[9] See appendix for more details.

4. Long Horizon Autocorrelation of Exchange Rate Data

In section 2, we find two interesting patterns in the autocorrelation of long horizon exchange rate change: 1.When the change of fundamentals is moderately persistent, the long horizon change of exchange rate is more serially correlated than that of short horizon data and declines with time horizon; 2. When the change of fundamentals are more persistent, there exists a hump-shaped pattern between autocorrelation and time horizon. This can be seen clearly from our plots in figure 1. It is interesting for us to see whether these two patterns also exist in the data or not. In this section, I choose four series of exchange rates: U.S. dollar prices of the currency of U.K., Canada, Japan and Australia and test how they change with time horizon. The quarterly data are from IFS during the period between 1974Q1 and 2004Q2 with 132 observations for each country. The exchange rates are the end of period U.S. dollar price for one unit national currency in these four countries.

If we test the null hypothesis that the first order autocorrelation is zero for k-period change of log exchange rate, the sample size decrease sharply with time horizon. For instance, when the time horizon k is equal to 5, we have only about 25 observations. Instead, we will allow the overlapping of the observations as in the literature of long horizon regressions. From the equation (2.8) and (2.9), we know under the null hypothesis that the change of log exchange rate follows a random walk, the kth order autocorrelation is zero for the data with time horizon of k. Let ρ_k^k be the k$^{\text{th}}$ order autocorrelation of the data with horizon k. Our null hypothesis is H0: $\rho_k^k = 0$ against the alternative hypothesis $\rho_k^k \neq 0$. We can construct a T-statistic as

(4.1) $$T_k = \frac{\hat{\rho}_k^k}{s.e.}$$

The problem is when the overlap is large relatively to the sample size, the distribution of this T-statistic is not the standard t-distribution any more. So I use the parametric bootstrap method to construction the distribution of T_k in order to find the p-value for each T_k.

We use p_k to denote the p-value associated with T_k in a two-tail test. That is, $p_k = \Pr ob(-|T_k| < x < |T_k|)$. p_k represents the confidence level at which we can reject H-null. We are expecting to find a larger p_k when the long horizon exchange rate change is more serially correlated. As for the two patterns we mentioned in the beginning of this section, we are expecting to find p_k decreases with time horizon when the change of fundamentals is moderately persistent and when it is more persistent, p_k should exhibit a hump-shaped pattern.

We use the following bootstrap method to find the small-sample distribution of T_k. Under the null hypothesis that long exchange rate follows a random walk, we use (4.2) to estimate the data generating process (DGP).

(4.2) $$\Delta s_t^i = \alpha^i + \varepsilon_t^i$$

Δs_t^i is the change of log exchange rate for country i at time t. α^i is a constant and ε_t^i is an i.i.d. random variable from a normal distribution with zero mean and variance of ${\sigma^i}^2$. We use the data of each country to estimate the parameters: $\hat{\alpha}^i$ and ${\hat{\sigma}^i}{}^2$ for i=1, 2, 3 and 4.

Table 3 Parameter Estimates

	U.K.	Canada	Japan	Australia
$\hat{\alpha}^i$	-0.002 (0.005)	2.96E-05 (0.003)	-0.002 (0.010)	-0.001 (0.007)
$\hat{\sigma}^i$	0.057	0.037	0.119	0.084

Then we simulate exchange rate with the estimated parameters.

$$\Delta\hat{s}_t^i = \hat{\alpha}^i + \varepsilon_t^i \tag{4.3}$$

We draw 131 observations to match the data available for each country[10]. The long horizon long exchange rate change is obtained from the sum of short horizon changes as we have mentioned in equation (2.8) and (2.9).

$$\Delta\hat{S}_1 = \Delta\hat{s}_1 + \Delta\hat{s}_2 + \ldots + \Delta\hat{s}_k \tag{4.4}$$

The (k+1)th observation is

$$\Delta\hat{S}_{k+1} = \Delta\hat{s}_{k+1} + \Delta\hat{s}_{k+2} + \ldots + \Delta\hat{s}_{2k} \tag{4.5}$$

It is easy for us to calculate the correlation between $\Delta\hat{S}_1$and $\Delta\hat{S}_{k+1}$ ($\hat{\rho}_k^k$), which is the k$^{\text{th}}$ autocorrelation for $\left\{\Delta\hat{S}_t\right\}_{t=1}^T$. The standard error of $\hat{\rho}_k^k$ can be calculated from Bartlett's approximation as in equation (4.6)

$$s.e. = \sqrt{\operatorname{var}[\hat{r}_k]} \cong \sqrt{\frac{1}{N-k}\left(1 + 2\sum_{v=1}^{k-1}\hat{\rho}_v^2\right)} \tag{4.6}$$

where $\hat{\rho}_v$ is the lag-v autocorrelation calculated from our simulated sample. Now we are ready to calculate the T-statistic just following (4.1). This process is repeated for 2,000 times to find the small sample distribution of T-statistic.

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[10] We have 132 observations of exchange rate for each country. After taking difference, one observation is lost. So the total number of observations for the change of exchange rate is 131.

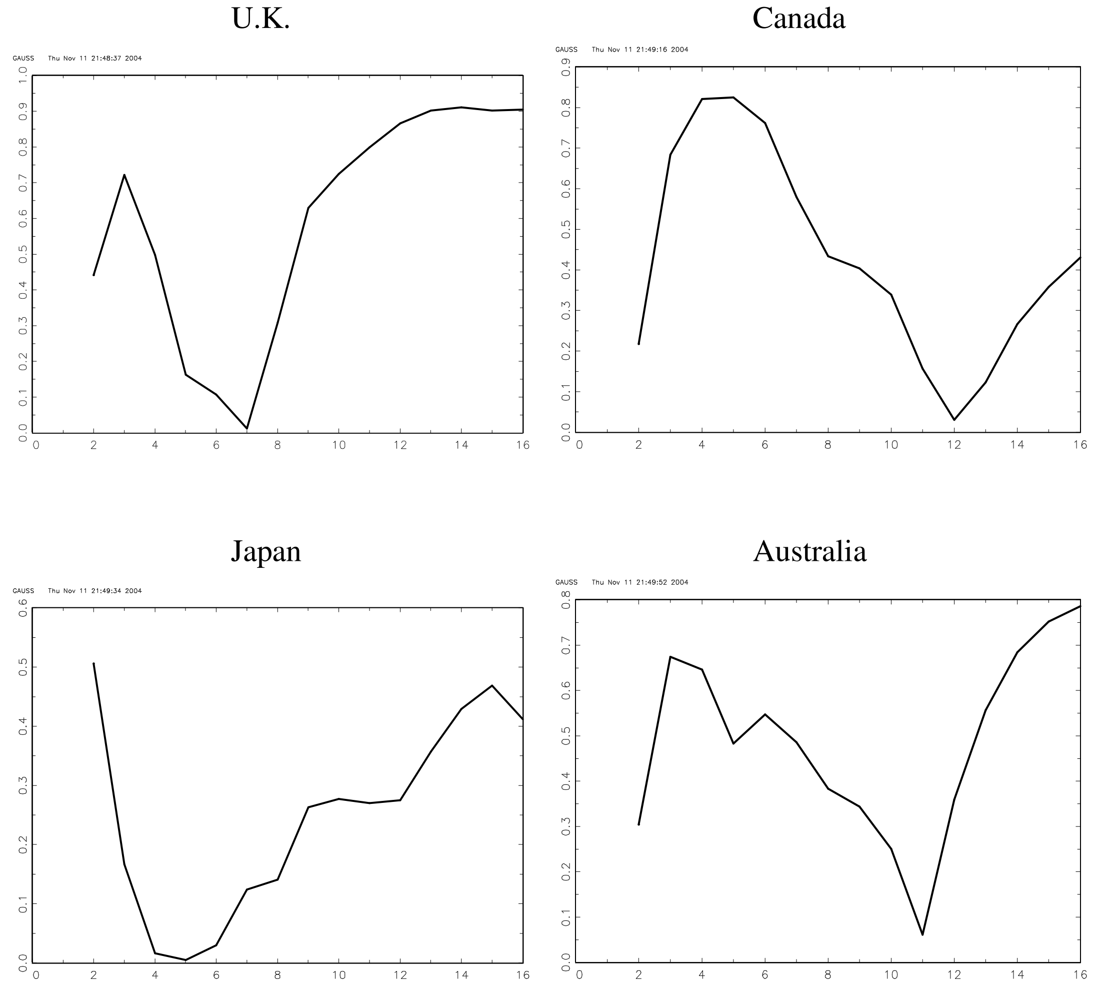
Following the same way, we can obtain the T-statistic for the data we get from IFS and compare this T-statistic with the distribution to find the p-value for each time horizon k and also for each country. The p-values are reported in table 4. From this table, we can see that all the p-values are less than 0.95. So we can not reject H-null hypothesis that the lag-one autocorrelation is equal to zero at 95% or higher level. This confirms the well documented random walk behavior of exchange rate.

Table 4 p-value across time horizon

k	U.K.	Canada	Japan	Australia
1	0.858	0.053	0.656	0.663
2	0.441	0.217	0.506	0.304
3	0.722	0.684	0.167	0.674
4	0.498	0.821	0.016	0.646
5	0.163	0.825	0.005	0.483
6	0.107	0.762	0.030	0.547
7	0.012	0.580	0.124	0.486
8	0.308	0.434	0.141	0.383
9	0.630	0.404	0.263	0.344
10	0.725	0.339	0.277	0.250
11	0.799	0.157	0.270	0.061
12	0.866	0.031	0.275	0.359
13	0.902	0.123	0.357	0.556
14	0.911	0.266	0.429	0.684
15	0.902	0.358	0.469	0.752
16	0.905	0.431	0.412	0.786

If we compare the autocorrelations between short and long horizon data, the increase in the p-values for the long horizon data is not impressive for all the countries except for Canada. For the case of Japan, no long horizon data actually has a higher p-value than the short horizon data. This confirms our results that the increase in time horizon usually can not significantly increase the serial autocorrelation of the change of log exchange rate under E-W explanation. So if we can not reject random walk hypothesis for the short horizon data, the increase in the time horizon can not significantly increase our chance of rejecting random walk under E-W explanation. It is more interesting for us to look at the pattern of change in the autocorrelation over time horizon. Starting with k=2, we find for U.K., Canada and Australia, the autocorrelation increases for some periods and then declines with time horizon k. This hump-shaped pattern can be seen more clearly from the plots in Figure 2.

Figure 2



The hump-shaped pattern is very clear for U.K., Canada and Australia. Compare this with the plots in Figure one, we find this pattern is very similar with the case when the change of fundamentals is very persistent (ϕ is greater than 0.6). The plot of Japan is more like the case where the change of fundamentals is less persistent (for instance, ϕ is equal to 0.4.) A deviation of Figure 2 from Figure 1 is that after some point of time horizon, the p-values in Figure 2 start to increase again. There is no a matching pattern in Figure 1. We think this may be caused by the decrease in the sample size when the time horizon gets too big. For example, when the time horizon is equal to seven, we have only about 15 non-overlapping observations, which may make our inference unreliable based on this small amount of observations.

We have noticed a second difference between our analytical results and the results from exchange rate data. In our analytical results, when we observe the hump-shaped patterns, the autocorrelation of k=2 is higher than that of k=1. However, among our 3 countries exhibiting the hump-shaped patter, only Canada is consistent with this result. For the U.K. and Australia, we can find in table 4 that short horizon p-value is much higher than that when time horizon is equal to two.

We do not have a clear explanation for these differences at the moment. We have to admit that the present-value asset pricing model we discussed in this paper is a very simple and limited one. The true mechanism determining exchange rate must be much more complicated. So we are not surprised to find these discrepancies between our analytical results and data. However, it is still very interesting to find some important patterns exist in the data. Our results here suggest that we should be more cautious in hoping long horizon data to solve exchange rate puzzle. From E-W explanation, if we can not beat random walk in short horizon, the long horizon data does not help much either.

5. Conclusion

It is very puzzling for economists to find that we can not statistically reject the random walk hypothesis for exchange rate while most fundamentals which theoretically determine exchange rate, do not follow a random walk. Similarly, it is also well documented that a random walk model performs as well as any theoretical model based on the out-of-sample forecast accuracy. Engel and West (2004a) find that under some conditions, this random walk property for exchange rate is consistent with the present value asset pricing models, which actually covers a large range of exchange rate determination models in the literature. In this sense the random walk behavior of exchange rate is just a natural outcome of the theoretical models and we should not be surprised for this finding.

In this paper, we investigate the long horizon properties of exchange rate based on E-W explanation. We found that E-W explanation predicts that long horizon data will not significantly increase our chance of rejecting random walk. The intuitive is simple, as the time horizon becomes very large, the change of fundamentals by itself is approximately serially uncorrelated. Therefore, the fundamental determined exchange rate will also follow a random walk. This result is consistent with the recent studies on long horizon exchange rate, which find that the previous success of beating random walk with long horizon data is not robust over different sampling periods and more likely to be caused by the mis-specified econometric methods. We also show analytically that the gain from the long-run convergence is small for the long horizon data when the discount factor is close to one.

Under E-W explanation, we find some interesting patterns for the change of autocorrelation of exchange rate change over time horizon. When the change of fundamentals is not very persistent, the lag-one autocorrelation of exchange rate change monotonically decrease with the time horizon. However, when the change of fundamentals is more persistent, the autocorrelation has a hump-shaped pattern over time horizon: the long horizon data is more serially correlated than the short horizon data when the time horizon is small, but will eventually become less serially correlated when the time horizon grows too big. We find both of these two pattern in our empirical part when studying the exchange rate of U.K., Canada, Japan and Australia against U.S. dollar. Japanese Yens show the first pattern we mentioned above; U.K., Canada and Australia show the hump-shaped pattern. These results provide support to E-W explanation in solving exchange rate puzzle.

The real mechanism that determines exchange rate must be much more complicated than the linear present value asset pricing model. We believe that is why we also observe some discrepancies between our theoretical predictions and the empirical results. However, we also believe some of these discrepancies, if not all of them, can be reconciled by enriching the model, for instance, including the unobservable fundamentals and/or considering nonlinearity between exchange rate and fundamentals. We leave this as our future research topics.

Appendix

In this section we derive the vector error correction (VEC) form of the present value asset pricing model we used in section 3. For reader's convenience, we reproduce the equation (3.3), (3.4) and (3.5) here.

$$s_t = (1-b)E_t\left[\sum_{j=0}^{\infty} b^j f_{t+j}\right] \tag{3.3}$$

$$s_t - f_t = E_t\left[\sum_{j=1}^{\infty} b^j \Delta f_{t+j}\right] \tag{3.4}$$

$$\Delta f_t = \phi \Delta f_{t-1} + \varepsilon_t \tag{3.5}$$

Equation (3.3) is from the present value asset pricing value model after imposing "no bubble" constraint. Equation (3.4) can be derived from (3.3). If we suppose the change of fundamentals is stationary and follows an AR(1) process (equation (3.5)), the equation (3.4) guarantees that exchange rate and fundamentals are cointegrated. The cointegrated system can generally be written into an Error-Correction representation (see page 580, Hamilton 1994.)

Substitute (3.5) into (3.4) we can obtain

$$\begin{aligned} z_t = s_t - f_t &= E_t\left[\sum_{j=1}^{\infty} b^j \Delta f_{t+j}\right] \\ &= E_t\left[\sum_{j=1}^{\infty} (b\phi)^j \Delta f_t\right] \\ &= \frac{b\phi}{1-b\phi}\Delta f_t \end{aligned} \tag{A.1}$$

From (A.1) and (3.5) we find

(A.2)
$$\begin{aligned} z_{t-1} &= s_{t-1} - f_{t-1} = \frac{b\phi}{1-b\phi}\Delta f_{t-1} \\ &= \frac{b}{1-b\phi}(\Delta f_t - \varepsilon_t) \end{aligned}$$

From (A.2) we have

(A.3)
$$\frac{b}{1-b\phi}\varepsilon_t = \frac{b}{1-b\phi}\Delta f_t - z_{t-1}$$

From (3.3) and (3.5) we can get

(A.4)
$$\Delta s_t = \frac{\phi(1-b)}{1-b\phi}\Delta f_{t-1} + \frac{1}{1-b\phi}\varepsilon_t$$

Substitute (A.3) into (A.4) it is easy for us to obtain

(A.5)
$$\Delta s_t = \frac{\phi(2-b)}{1-b\phi}\Delta f_{t-1} + \frac{1}{1-b\phi}\varepsilon_t - \frac{1}{b}z_{t-1}$$

(A.5) and (3.5) consist of the vector error correction (VEC) representation we have mentioned in section 3. In this error correction representation, we can write the error correction term into an AR(1) process.

(A.6)
$$z_t = \rho z_{t-1} + \tilde{\mu}_t$$

where $\rho = 1 - b_1 + b_2$. In our case, $b_1 = \frac{1}{b}$ and $b_2 = 0$, so when b is close to one, ρ is close to zero.

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