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System identification in noisy data environments: an application to six Asian stock markets

Abstract

This paper analyzes the systematic relationship between the stock market valuations, the nominal GDPs and the interest rates of six Asian countries, using not "single equation regression," but an alternative methodology based on complete, multidirectional, least squares projections. We compare the results with the spectral analysis of the information matrices and determine the noise levels. The objective is to extract the multidimensional economic system structures from the noisy empirical observations. This complete methodology sharply contrasts with the incomplete methodology of Fama (1990), Schwert (1990), etc., who presume planal relations, fit them to the multidimensional data by only one prejudiced unidirectional projection, thereby ignoring between 75% - 92% of the available covariance information and not publishing the absolute majority of all possible model projections. The results in this paper show that the analyzed countries are better analyzed using such complete multidirectional LS projections, even though the analysis is combinatorially much more complex. All six Asian financial-economic systems are high data noise environments, in which it is very difficult to separate the systematic signals from the noise. Because of these high noise levels, spectral analysis is very unreliable. We identify Taiwan's stock market, economy and financial market to be rationally coherent. In contrast, Malaysia, Singapore, Philippines and Indonesia show only partially coherent systems, while no coherent system can be identified among Japan's data.

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1 INTRODUCTION

The Asian stock markets have seen a substantial capitalization in the past two decades. In 1982, the 32 emerging stock markets surveyed by the International Finance Corporation had a total market capitalization of US\$67 billion, representing about 2.5% of the world market capitalization. Already by the end of 1994, the market capitalization of these emerging stock markets had grown nearly 27-fold, exceeding US\$1.8 trillion, or 11.8% of the world market capitalization.¹ Despite the slowdown of the Asian countries in the late nineties in the wake of the Asian Financial Crisis of 1997, many global investors and foreign fund managers, flush with cash due to low interest rates in the US and Europe, are again beginning to pour substantial funds into those same markets.²

Asia is again viewed as the new frontier which promises substantial returns on investments, proportional to the financial risks taken and related to the growth potential in the region.

There is little doubt that the region's Gross Domestic Product (GDP) growth rates are positively correlated to the equity market returns. But the more important question for the strategic global investors is: what is the systematic relationship between these countries' stock market valuations, their nominal GDPs and their (short term) interest rates? A similar question was asked by Roll (1988) about 15 years ago about the US and European economies. Since the same question is now asked by global investors, its correct answer has gained in importance

Guided by the methodologies exhibited in the financial literature, current (certified) financial analysts use conventional *unilateral* LS projections, *i.e.*, single equation least squares (LS) regression models to answer Roll's question. See, for example, Fama (1990), Schwert, 1990; Bittingmayer, 1992; Lee, 1992; Gallinger, 1994; and Canova and Nicolo, 1995. But their results are never conclusive. Their inconclusiveness is often blamed on the lack of sufficiently large data sets for macroeconomic and financial variables. But this paper demonstrates that this inconclusiveness

¹ *The NYU Salomon Center Report 1996-97*, p.10.

² *Cf.* the article "Hedge Funds Bet on Asia: U.S. hedge funds are flocking to Asia in search of bigger gains. Their return raises important questions for the region's financial markets," *Far Eastern Economic Review*, September 2, 2004, pp. 40 - 41.

has more to do with (1) the incomplete set of unilateral LS projections used and (2) the possible non-stationarity of the available data sets. The first analytic deficiency can be overcome by using complete LS projections, the second, perhaps, by computing rates of return for all variables.

This paper demonstrates the new system identification methodology of a *complete* set of *multilateral* least squares (CLS) projections. Our CLS "noisy system" identification methodology extracts the structure of (linear) financial economic systems from the empirical observations without theoretical presumptions. The emphasis is on the completeness of the system identification, since our projections are executed in all possible orthogonal directions in the available multidimensional data space and not in one *a priori* chosen direction. It also demonstrates that such a complete analysis is much more complex and difficult than the simple approaches illustrated in the financial literature.

This CLS analysis, which follows in the footsteps of the critical covariance ("confluence") analysis of Frisch (1933) and (somewhat less) in the footsteps of that of the Cowles Commissions of the 1940s and 1950s, improves upon the methodologies of both. I've discussed it in detail in Chapters 4 and 5 of my book *Computational Finance: A Scientific Perspective* (Los, 2001, pp. 57 - 98). Moreover, an earlier application of the same methodology to the bivariate CAPM, with a discussion of a tri-variate discriminatory bank performance scoring case, can be found in Los (1999, pp.1793 - 1829).

The reason for our insistence on a *complete* analysis of the covariance matrices is that, in principle, many single equation model planes can always be indiscriminately fitted to multidimensional data by Least Squares (LS)-based projection schemes. The number of such possible LS projections depends on the number of variables in the data set. However, it is the *raison d'être* of the scientific analysis of empirical data that the system to be identified must fit the multidimensional correlation structure of the data *uniquely*, even when the data are uncertain or "noisy," and, consequently, the computed parameter values lie in (finite) ranges of uncertainty.³

This can only be done, when the data are analyzed in a complete fashion, *i.e.*, by all possible LS projections. It cannot be done by *a priori* selection of one LS projection direction out of the many possible, since that would violate the scientific requirement of a *complete* analysis.

Our new CLS methodology is independent from the size of a data set. The size of a data set, *i.e.*, the number of observations T , does not appear anywhere in our analytic projection formulas, in contrast to the number of variables in the data set n . But, like any correlation analysis such as Granger's (1969) spectral analysis, our results are dependent on the stationarity of the data set, *i.e.*, the (relative) invariance of the data covariance matrices and thus of the information matrices. following Engle and Granger (1987), all authors who responded to Roll's (1988) call are concerned about the possible lack of covariance stationarity, and introduce logarithmic differencing of their data variables to introduce stationarity in their analyses, even at the cost of financial theory.

Such invariance of covariance matrices can, for example, be detected by "windowing," *i.e.*, by subdividing the data set and comparing the data covariance matrices of the consecutive or partially overlapping subsets. This is, indeed, easier when the number of observations T is larger. Because T does not appear in our methodology, it still operates correctly if the distributions of each data set are nonstationary but stable, *i.e.*, if such data distributions time-scale in a similar fashion, or can be Kalman-filtered to such an extent that the remaining distributions are stable.⁴

For the purpose of demonstration of the new system identification methodology, and because of the importance of the research question for global investors intending to invest in Asia, we imple-

³ Statistical "significance" testing is subject to the same critique of the subjective *a priori* choice of the projection direction into which the proposed multivariate probability distribution of the data is measured. Therefore, statisticians have little or nothing to say about "system structure identification," although it is the first thing on the minds of physical scientists and signal processing engineers. For example, biologists identify the geometric structure of any protein from a complete crystallographic 3D Fourier analysis, with three orthogonal spectral frequencies.

⁴ Stable distributions are more general than stationary distributions, since the only requirement is that they maintain their shape, even when their size varies over time. For a discussion of this important empirical research issue, see Chapters 3 and 4 in Los, 2003. For example, the Geometric Brownian Motion, popular in derivatives pricing, can describe a non-stationary pricing process that can be made (weakly) stationary by proper time-scaling, *i.e.*, by dividing the standard deviation of the rates of return, or volatility, by the square root of time: $\sigma = \sigma(T)/T^{0.5}$. This process can be further generalized to the non-stationary (homogeneous) Fractal Brownian Motion, where we find a constant $\sigma = \sigma(T)/T^H$, where H is the Hurst exponent describing the degree of persistence, or long memory, of the underlying pricing process.

ment the CLS identification to six Asian countries: Taiwan, Malaysia, Singapore, the Philippines, Indonesia and Japan. Each country's major stock market index is used to track the valuation of its stock market. Its nominal GDP and short term interest rate are the two other macroeconomic variables. The Asian quarterly data used for the particular analysis in this paper range from 1986 first quarter to 1995 third quarter.

In line with established business practices and fundamental financial valuation theory, we combine some elements of the hypothesized (constant and constant growth) Dividend Discount Model (DDM), following Peavy (1992), and of modern Asset Pricing Theory (APT), following Ariff and Johnson (1990), Chan, Roll and Ross (1986) and Ross (1976).⁵ However, taking account of some earlier critical discussions between Dhrymes (1984), Roll and Ross (1984) and Ross (1984) regarding the restrictiveness of the original APT model parametrization, we now allow for greater parameter freedom.

Along the way, we demonstrate the shortcoming of principal component (PC) analysis versus our new CLS analysis. We also demonstrate how the PC analytic scheme is encompassed by our noisy system identification methodology and can be improved. In the process, we'll gain a somewhat better understanding of the relative coherence of the financial economic systems of the six Asian countries and identify the various (uncertain) valuation systems of the their stock markets. But we do not find clear evidence of financial-economic system coherence in all six countries. For example, Taiwan's has a definite identifiable coherent financial-economic system, but our CLS cannot identify a coherent system from Japan's data set, despite the completeness of our projection analysis.

The paper is basically structured in four parts. Using a few examples in the literature related to our data set, Section 2 provides the motivation for the original paper. It discusses why single equation unidirectional projections ("regressions") as used by all authors who rose to Roll's (1988)

⁵ These valuation modeling practices were once taught, *i.a.* by the Association for Investment Management and Research (now called the CFA Institute) as part of its celebrated "Body of Knowledge" in its annual Certified Financial Analyst study and examination program (Cf. the *1999 CFA Level II: Study and Examination Program*).

call, provide a severely incomplete covariance analysis of multivariate data sets and why such incompleteness leads to bad science, even when it is in the form of VAR systems. This is followed by the three parts of our new complete analysis. Section 3 briefly discusses the simple tri-variate data sets used and, very summarily, the new noisy system identification methodology. Section 4 treats the data analysis using the conventional spectral decomposition of the data covariance matrices, the bivariate data scatter plots and bivariate noise/data and noise/signal ratios, and discusses their pros and cons. Section 5 discusses implementation of the new CLS methodology to each of the six Asian countries and the resulting 3D noise/data and noise/signal ratios. These three main parts are followed by Section 6 summarizing the identifiable coherence of the Asian systems and their measured parameter ranges. It also discusses the limitations of any covariance matrix-based methodology and provides some recommendations for future research.

2 SCIENTIFIC INCOMPLETENESS: CASES IN THE FINANCIAL LITERATURE

In this section five representative examples in the financial literature related to our CLS demonstration are discussed to show the prevalence of scientific incompleteness of the analyses reported. First, all five articles ignore the absolute majority of the available covariance information in their respective data sets. Second, all five articles don't report all possible LS projections based on the complete covariance data. We quantify the percentage of analytic incompleteness and the under-reporting of the possible number of projections in Table 1 using the following two measures.

First, the information matrix Σ^{-1} is the inverse of the covariance matrix Σ of all n variables in the data set, where a lagged variable counts as a separate variable. Each row of the information matrix is an elementary regression or $(n, 1)$ unidirectional LS projection. Since only one of these elementary regressions is reported in each of the following articles, the

$$\text{Percentage of analytic incompleteness} = 100.(n - 1)/n\%. \quad (1)$$

Second, the complete number of projections of the invariant number q of possible linear rela-

tions among n variables is given by

$$\text{Number of possible LS projections} = \sum_{q=1}^{n-1} \binom{n}{q} \quad (2)$$

The under-reporting in Table 1 is this number minus the one $(n, 1)$ unidirectional projection that is reported.

[TABLE 1 about here]

Let's start the discussion when Richard Roll (1988) in his presidential address to the American Financial Association (AFA) challenged the discipline to research the predictability of changes in asset prices. Many authors responded to this challenge by publishing the results of single equation multivariate unidirectional projections (= "reduced form, single equation regressions"), *i.a.* Fama (1990) and Schwert (1990).

Schwert (1990) analyzes the relationship between real stock returns and real activity from 1889-1988. His paper replicates Fama's (1990) results for the 1953-1987 period using an additional 65 years of data. Fama (1990) purports to show by single equation multivariate unidirectional projection that monthly, quarterly, and annual stock returns are highly correlated with future growth rates for 1953-1987. As Fama (1990, pp. 18-19) explicitly states "The variables used to explain returns are chosen largely on the basis of goodness-of-fit rather than the directives of a well-developed theory. . . ." Both papers suffer from a complete disregard for the geometric covariance structure of the data by prejudicially fitting planes to them and by ignoring thereby most of the information contained in the covariance matrices of their own sophisticated data sets. Accordingly, Schwert (1990, Table II, pp. 1242 - 1245 and Table III, pp. 1247 - 1248) shows the results for two 9-variable single equation unidirectional projections, thereby ignoring 89% of the available covariance information and 509 possible additional LS projections. Schwert (1990, Table V, pp. 1252 - 1253) then replicates Fama's (1990) results by a 5-variable single equation unidirectional projection, ignoring 80% of the available covariance information and 29 possible

additional LS model projections. Finally, Schwert (1990, Table VI, pp. 1254 - 1255) attempts to “improve” Fama’s 5-variable results by showing the parameter estimates of a 12-variable single equation unidirectional projection, ignoring 92% of the available covariance information and not reporting the 4,093 possible additional LS model projections.

On the basis of these two 1990 papers by Fama and Schwert, Bittlingmayer (1992, p. 1701) erroneously concludes: "The statistical relationship between stock prices and real variables such as GNP and industrial production is well established." It escapes this author how that can be the case, since their covariance analyses are incomplete and thus biased. Bittlingmayer then extends the Fama-Schwert equations by adding another variable - antitrust case filings - to their equation(s) to "explain" the quarterly returns of the Dow industrial average, since "Periodic antitrust attacks on corporations may have influence stock prices." His regressions are "an attempt to provide a robust test of the effect of antitrust by asking whether antitrust case filings affected quarterly industrial stock prices..." (Bittlingmayer, 1992, p. 1702). But also his test is not robust, since his 8-variable regressions ignores 88% of the available covariance information (Bittlingmayer, 1992, Table III on pp. 1714 - 1715, and Table IV on pp. 1718 - 1719) and he does not report the 253 additional LS projection results.

Next, Canova and De Nicolo (1995) attempt to put these same "reduced form regressions" analyses of Fama (1990) and Schwert (1990) within a dynamic general equilibrium, multi-country model of the business cycle using a set of leading indicators to provide an explicit "structural" behavioral theory," to provide a rationale for reduced form evidence à la Fama" (Canova and De Nicolo, 1995, p. 985). They examine the relationship between domestic output growth, stock returns and dividend yields using quarterly returns for the period 1973 - 1991 for five different countries, the US, the UK, Germany, France and Italy and for a European aggregate and claim to have found *i.a.* that future European GNP growth explains domestic European stock returns.

They report some empirical evidence in the form of several single equation unidirectional projections with the number of variables in each data set varying: $n = 4, 6, 9$ and 11 (Canova and

De Nicolo, 1995, Table 1, pp. 988-989), ignoring again between 75% and 91% of the covariance information and between 13 and 2,045 possible LS projections (*Cf.* Table 1).⁶ Next, they proceed to develop a theoretical three-country simulation model with three consumption goods, with each country specializing in the production of one good. They want to know under which generation mechanism (technology or government disturbances) their theoretical simulation model, by *parameter calibration*, comes closest reproducing their reduced form "evidence" (Canova and DeNicolo, 1995, p. 994). But what good is such a calibration when their own parameter information is so incomplete?

What about a proper study of the dynamic, causal, linkages in the valuation of asset prices? Canova and DeNicolo, 1995, p. 985) state: "Ideally, one would like to run VAR or multivariate systems in order to study the linkages between financial markets and real activity." Unbeknownst to them, this had already been done by Lee (1992), who uses Sims' (1980) multivariate vector-autoregression (VAR) approach and finds his results compatible with Fama's (1981), whose focus had been on the observed negative correlations between inflation and real asset returns.

The main problem with the VAR approach is that each variable in the VAR "system" is "explained" by a reduced form regression, *i.e.* by a single equation ($n, 1$) unidirectional LS projection. For example, Lee (1992, p. 1595) employs a four-variable "system" - real stock returns, real interest rates, growth in industrial production, and the rate of inflation - with 6 lags. This implies that he uses $n = 7$ variables for each ($n, 1$) LS projection for each of the four variables. His VAR uses thus four out of seven ($7, 1$) LS projections (= and ignores the three other ($7, 1$) LS projections, which Lee does not report. He only reports the explained percentage of 24-month forecast error variance in his Tables I and II (Lee, 1992, pp. 1597 and 1601). But the situation is actually worse, since his available data set contains $n = 4 \times 7 = 28$ separate variables to be correlated and his VAR

⁶ In Panel C of Table 1 of Canova and De Nicolo (1995, p. 989) some contemporaneous bivariate correlation coefficients and standard deviations are reported, but, unfortunately, the set is incomplete so that their original covariance matrix Σ cannot be reconstructed for a complete projection analysis following this paper.

information matrix is block diagonal.⁷ Consequently, Lee (1992) ignores 96% of the available covariance information and ca. 268 million additional LS model projections (*Cf.* Table 1).

A final example of the incompleteness of the covariance analysis reported in the financial literature is the causality analysis/stationarity test paper by Gallinger (1994), who refers also to the crucial Fama (1990) paper. Gallinger attempts to disentangle cause and effect from the stock return-real activity correlations within the framework of the theory of co-integration of Engle and Granger (1987), which tries to make sure that the covariance matrix of the data set is stationary by computing logarithmic differences (= rates of return) of the data series. Gallinger uses several variables from the monthly survey by the National Association of Purchasing Managers (NAPM) in the USA, as well as the S&P 500 index and the Federal Reserve Board Index of Industrial Production.

Engle and Granger (1987) define two scalar nonstationary time series x_{1t} and x_{2t} , measured as first differences of log levels (*i.e.*, as rates of return), as being cointegrated if x_{1t} and x_{2t} are both stationary (*i.e.*, co-integrated of order 1, or $I(1)$), so that there is a linear combination $\hat{x}_{1t} = A\hat{x}_{2t}$, with A the so-called co-integrating constant (to be identified). Then one tests if the residual noise $\tilde{x}_{1t} = x_{1t} - Ax_{2t}$ is white noise, $\tilde{x}_{1t} \sim i.i.d.(0, \sigma_{\tilde{x}_1}^2)$, by using various unidirectional LS projections of \tilde{x}_{1t} on a series of k of its lagged values.⁸ For both the Durbin-Watson test and the Dickey-Fuller test, the lag length $k = 1$, and thus bivariate LS projections are used, in which 50% of the covariance information is never reported (*Cf.* Los, 1999). The augmented Dickey-Fuller (ADF) test uses lag length $k = Integer\{12(T/100)^{0.25}\}$.⁹ All three white noise tests are based only on covariance analysis of the residual noise.¹⁰

Since Gallinger (1994, Table 1, p. 277) uses unidirectional LS projections of $n = k + 1 =$

⁷ Sims (1980) claims that his VAR scheme does not impose any restrictions, but this is not true, since his information matrix Σ^{-1} is block-diagonal.

⁸ Gallinger (1994) also analyzes the stationarity of the reverse residual noise $\tilde{x}_{2t} = x_{2t} - Bx_{1t}$.

⁹ This lag length criterion is Schwert's (1987).

¹⁰ Los (2003) has demonstrated that such collinearity tests to establish the "whiteness" of the noise are insufficient, since they ignore the existence of nonlinear long-term dependencies (implying fractional instead of integer differentiation), which can color the noise.

13, 15, 17, 18 and 19 variables, he ignores for the ADF test between 92% and 95% of the covariance information of the residuals and between 8,189 and 524,825 possible LS projections (*Cf.* Table 1).

So, in summary, the many eminent financial-economic researchers, who responded to Roll's (1988) call, have not responded in a scientifically complete and unbiased fashion. To assist future generations of researchers, my didactic paper is a simple demonstration why such unidirectional $(n, 1)$ LS projections of multivariate data sets with at least $n = 3$ variables (including their lagged values) is necessarily incomplete, scientifically deficient and prejudiced and why $n > 3$ data sets are more difficult to *completely* covariance analyze than is reported in the financial literature, even though such an analysis is clearly computationally possible. This demonstration uses a simpler, 3-variable data set with only one dynamic lag, for which the model and data set are theoretically justified by fundamental financial valuation theory, and for which the complete projection results can be easily visualized.

3 DATA AND METHODOLOGY

3.1 Data Sources

The selected stock market indices (S) include the Nikkei 225 Stock Average (Japan), Straits Times Industrials Index (Singapore), Weighted Price Index (Taiwan), Kuala Lumpur Composite Index (Malaysia), Jakarta Composite Index (Indonesia) and Philippines SE Composite Index (Philippines). The observations on these stock market indices were collected from the respective issues of the *Asian Wall Street Journal* and *The Business Times*.

Nominal GDP ($NGDP$) is used for all countries, except for Indonesia due to the non-availability of its quarterly $NGDP$ data. For Indonesia, the available Petroleum Production (PP) Index is the closest proxy we had available for its GDP. This is in the spirit of the statement by Canova and De Nicolo (1995, p. 986) that: "Fama shows that the qualitative features of the relationship are not altered when monthly data are used with industrial production replacing GNP."

For the short-term interest rate (IR), the 90-days Money Market Rate is used for Taiwan, while the respective bank lending rates are used for the other five countries. These $NGDP$, PP , and IR figures were extracted from the *International Financial Statistics*, *Asia Pacific Economic Outlook*, *United Overseas Bank* and *Financial Quarterly* publications for the appropriate quarters.¹¹

The overall behavior of S , $NGDP$, PP , and IR were pre-analyzed for each country. The PP series for Indonesia had an obviously erroneous outlier data point in the last quarter of 1993. Since the validity of this outlier is very doubtful, this one data point is excluded from the analysis for Indonesia. Consequently, the analysis for Indonesia is based on one quarter less than the other five countries, *i.e.*, on 38 instead of 39 quarterly observations.

3.2 Hypothetical Model Structures

The selection of the data sets is motivated by simple hypothetical financial models. For example, as suggested by *i.a.*, Peavy (1992) of the Association for Investment Management and Research (AIMR),¹² we can use the following simple hypothetical model for a country's system for stock market valuation:¹³

$$S_t = \frac{E\{NGDP_{t+1}\}}{IR_t} \quad (3)$$

The valuation of a country's stock market at time $t = 1, \dots, T$, is the infinitely discounted value of the *expected value* $E\{\dots\}$ of the one-quarter-ahead nominal GDP_{t+1} , representing the expected market value of all goods and services domestically produced during the next quarter.¹⁴

The discounting is in the fashion of a console, since countries are "forever ongoing concerns."

The infinite discounting is by the most readily available current cost of capital, *i.e.*, the domestic

¹¹ *Cf.* the Appendix for the specific sources and the discussion of the various data conversions, which were required to make the data of the six Asian countries mutually comparable.

¹² On May 9, 2004, the AIMR has been renamed the CFA Institute. The CFA Institute is an international, non-profit organization of more than 70,000 investment practitioners and educators in over 100 countries. It administers the annual Chartered Financial Analyst (CFA) exams. Dr. Peavy was its Research Director in the late 1980s and early 1990s and he incorporated this model in the required "Body of Knowledge" of the CFA exams.

¹³ This macro approach to a country's stock valuation was first suggested to me in 1990 by the Director of Financial Research of Smith-Barley, Inc. in New York City, who attempted to produce a comparable framework for global investment valuation. This valuation model was quite popular among many Wall Street's strategists and CFAs at the time when I was a Chief Economist of ING Bank in New York City (1991-93).

¹⁴ This one-quarter lag is as in Fama (1990), Schwert (1990) and Canova and DeNicolò (1995).

short-term interest rate IR_t . In the six Asian countries obtaining long term capital for stock market investment is still very difficult and short-term cash is usually the only source available for investment. This difficulty necessarily restricts us to the short-term capital access of an average stock investor in these countries. It also exposes these Asian countries to a very serious asset-liability duration mismatch, as we have observed in the 1997 Asian Financial Crisis.

Since we deal with historical data only, the implicit, and admittedly somewhat unrealistic, assumption of this hypothetical model is that there was, historically, *perfect foresight* and the expected $E\{NGDP_{t+1}\} = actual\{NGDP_{t+1}\}$ at all times t . Thus, for the model:

$$S_t = \frac{NGDP_{t+1}}{IR_t} \quad (4)$$

Alternatively one could have assumed a simple extrapolation, *i.e.*, the best expectation for next quarter's $NGDP_{t+1}$ is the current $NGDP_t$ multiplied by one plus the constant trend growth rate. By taking logarithms of this model, the following single linear equation results:

$$\ln S_t = \ln NGDP_{t+1} - \ln IR_t \quad (5)$$

Since this model is overly restrictive, we introduce some extra modeling flexibility. We do this in two steps. First, inspired by the financial multivariate Arbitrage Pricing Theory (APT) of Ross (1976; *Cf.* also Ariff and Johnson, 1990) and by the economic Cobb-Douglas (CD) Production Function Theory (Cobb and Douglas, 1928), we postulate the following single equation $(n, 1)$ system, or plane, for the "modified" DDM

$$\ln S_t = a. \ln NGDP_{t+1} + b. \ln IR_t + \ln D \quad (6)$$

Effectively, we introduce two elasticities, whose (uncertain) values are not predetermined by theory, but which are to be determined from the empirical data:

a = nominal GDP ("expected income") elasticity, which financial economists expect to be positive; and

b = interest rate elasticity, which financial economists expect to be negative.

Theoretically, the added $\ln D$ term may represent the deterministic value innovations in the stock market introduced by technological advances. By exponentiation the modified DDM model transforms then back into the elastic nonlinear CD structure

$$S_t = D.(NGDP_{t+1})^a.(IR_t)^b \quad (7)$$

Notice that this structure encompasses the original restrictive DDM structure, when the theoretical parameters are $a = 1$, $b = -1$ and $D = 1$. These parameters have thus empirically all testable value ranges.

Second, we introduce further structural flexibility by allowing for two independent linear equations ($q = 2$), *i.e.*, a ray, or cross-section of two planes, by postulating the following model system of two simultaneous, independent planes:

$$\ln S_t = c. \ln NGDP_{t+1} + \ln F \quad (8)$$

$$\ln S_t = d. \ln IR_t + \ln G \quad (9)$$

where c and d are the new expected income and interest rate elasticities respectively. By exponentiation, this linear CD type model transforms to a system of two simultaneous *nonlinear* equations:

$$S_t = F.(NGDP_{t+1})^c \quad (10)$$

$$S_t = G.(IR_t)^d \quad (11)$$

It is important to emphasize for geometric structure analysis, that, although we can create a unique single equation system from such a two-equation system, the reverse does *not* hold true. Thus, single equation $(n, 1)$ and two-equation $(n, 2)$ systems are structurally *not* equivalent. Note that $r + q = n$ where n is the number of *variables*, q is the number of *independent equations* and r is the number of *factors*. In terms of PC analysis, a two-equation system $(3, 2)$ behaves like a one-factor ($r = 1$) system, since all variables move simultaneously as a bundle along a ray in the same (or opposite) direction. In contrast, in a true single equation system $(3, 1)$ there are two

independent factors ($r = 2$) moving in a plane, since two variables move independently from each other.

In our simple demonstration, $n = 3$ for each of the six Asian countries. Since the $(n, 2)$ system encompasses the $(n, 1)$ system, the $(n, 2)$ system forms the basis for our system identification procedure. A $(n, 1)$ system shows up as a collapsed or degenerate $(n, 2)$ system. This procedure needs to determine which of the two possible system structures provides the best and simplest explanation of the observed noise-contaminated empirical data covariances of each of the six countries.

The covariance analysis to determine the geometric structure of the linear system (is $q = 1$ or 2 ?) and the finite parameter ranges for the elasticity parameters, $c_* \leq c \leq c^*$ and $d_* \leq d \leq d^*$ is executed on laterally shifted frames of logarithmic data reference, as follows.¹⁵ We define three new variables:

$$x_{1t} = \ln S_t - \overline{\ln S_t} \quad (12)$$

$$x_{2t} = \ln NGDP_{t+1} - \overline{\ln NGDP_{t+1}} \quad (13)$$

$$x_{3t} = \ln IR_t - \overline{\ln IR_t} \quad (14)$$

The averages, indicated by bars over the original data, are taken over all $T = 39$ observations ($T = 38$ for Indonesia). After our complete covariance analysis, the (projected and non-projected) deviations are transformed into the original variables by adding back these averages. For example,

$$\ln S_t = x_{1t} + \overline{\ln S_t} \quad (15)$$

By taking averages, the "residual technology" terms, $\ln F$ and $\ln G$ are of no substantial importance for the system identification. Once the parameter ranges for the income elasticity c and the interest elasticity d are computed, the parameter ranges for the $\ln F$ and $\ln G$ terms (respectively for F and G) can immediately be inferred.

¹⁵ The linearity of the system refers always to the linearity of the coefficients and not to the linearity of the variables!

3.3 Identification in Noisy Environments

The analysis follows the same recipe for each of the six Asian countries. We compute the data covariance matrices Σ (one matrix per country) of the logarithmic data. These six 3×3 logarithmic data covariance matrices form the empirical data set for this paper's system identification.

For demonstration purposes, we'll use several standard procedures to identify the system invariant q from this set of covariance data and to compute the finite parameter ranges $c_* \leq c \leq c^*$ and $d_* \leq d \leq d^*$. Our identification recipe consists of the following five steps:

1. *Spectral decomposition* of the six data covariance matrices Σ attempts to determine the number of factors r and thus, indirectly, the number of equations $q = n - r$ that support each country's covariance data. However, spectral or PC analysis fails to identify the significant factors, because it can't objectively differentiate between systematic and unsystematic information in high noise environments. Therefore, more discerning, algebraic geometric methods like CLS are needed.
2. *Information matrices* Σ^{-1} are used for (i) analysis, *e.g.*, for *visual inspection of the (a, b)-coefficient plots*, using the Elementary Regression Theorem of Kalman (1991) that each row of an information matrix is an elementary $(n, 1)$ regression, and for (ii) computing the $q = 2$ projectors in the 3D data space.¹⁶
3. *3D Complete Least Squares (CLS) analysis*, using the computed CLS noise projectors $\tilde{\mathbf{P}}^{CLS}$ and CLS signal projectors $\hat{\mathbf{P}}^{CLS}$ of Kalman (1991) and Los (1994, 2001), provides the complete and best LS system identification to extract the systematic shape of the 3D data covariance ellipsoid of each country.
4. Classical *bivariate noise/data and noise/signal ratios* provide 2D measurement of the relative noise levels. We prove in Appendix C of Los (2001) that these bivariate

¹⁶ The algebraic expressions for these $(n, q) = (3, 2)$ projectors can be found in Chapter 5 in Los, 2001. Such (a, b) -coefficient plots are also examined by Klepper and Leamer (1984).

ratios form, indeed, the constituents of the 3D $q = 2$ CLS system projectors of Kalman and Los.

5. *Three-dimensional noise/data and noise/signal ratios* provide an overall quantitative assessment of the systems' relative noise levels. To compute such 3D Noise/Data ratios, we use a simple Theorem of Linear Algebra to compute the "volume" of the noise covariance matrix $\tilde{\Sigma}^{LS}$ relative to the "volume" of the data covariance matrix Σ (Los, 2001, p. 289) This provides us with a standard for measurement and comparison of the relative noise levels in all six countries.¹⁷

3.4 3D Complete Least Squares Plots

We can combine the bivariate covariance projection information in the form of the 3D CLS projections for both $q = 2$ and $q = 1$. The $q = 2$ 3D plots are *rays*, representing the projected $(n, 2)$ systems, while the $q = 1$ 3D plots are *planes*, representing the projected $(n, 1)$ systems. The three $(n, q) = (3, 2)$ CLS systematic projector matrices are as follows:¹⁸

$$\hat{\mathbf{P}}_i = \hat{\Sigma}^{CLS} \Sigma^{-1} \quad (16)$$

When $\hat{\mathbf{P}}_1$ is post-multiplied by the $n \times T = 3 \times 39$ data matrix $\mathbf{x}' = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$, so that the 3×39 systematic, or modeled, signal is

$$\hat{\mathbf{x}}_1' = \hat{\mathbf{P}}_1^{CLS} \mathbf{x}' \quad (17)$$

the first data series in this signal $\hat{\mathbf{x}} = (\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3)$, remains non-projected, $\hat{\mathbf{x}}_1 = \mathbf{x}_1$, since it is the series on which we project, *i.e.*, the one which is assumed to have no noise, $\tilde{\sigma}_{11} = 0$. The other two series, $\hat{\mathbf{x}}_2$ and $\hat{\mathbf{x}}_3$ result from the simple bivariate orthogonal projections on \mathbf{x}_1 . Thus we have

¹⁷ Statisticians are familiar with bivariate noise/signal ratios $= R^2/(1 - R^2)$, where R is the bivariate correlation coefficient. But they are still unfamiliar with these new multidimensional noise/signal ratios of systems that do not consist of a single equation.

¹⁸ Hats $\hat{\cdot}$ above variables denote systematic signals and waves $\tilde{\cdot}$ denote unsystematic noise. For simple derivations of the $(3, 2)$ and $(3, 1)$ systematic projector matrices $\hat{\mathbf{P}}_i$, see Los, 2001a, Chapters 4 and 5. It is easy to check that these are all projector matrices, since $(\hat{\mathbf{P}}_i^{CLS})^2 = \hat{\mathbf{P}}_i^{CLS}$, $i = 1, 2, 3$.

the first CLS_1 projected system

$$\begin{aligned}\widehat{\mathbf{x}}_1 &= \mathbf{x}_1 \\ \widehat{\mathbf{x}}_2 &= \frac{\sigma_{12}}{\sigma_{11}} \widehat{\mathbf{x}}_1 \\ \widehat{\mathbf{x}}_3 &= \frac{\sigma_{13}}{\sigma_{11}} \widehat{\mathbf{x}}_1\end{aligned}\tag{18}$$

which is equivalent to the normalized system

$$\begin{aligned}\widehat{\mathbf{x}}_1 &= \mathbf{x}_1 \\ \widehat{\mathbf{x}}_1 &= \frac{\sigma_{11}}{\sigma_{12}} \widehat{\mathbf{x}}_2 = c \cdot \widehat{\mathbf{x}}_2 \\ \widehat{\mathbf{x}}_1 &= \frac{\sigma_{11}}{\sigma_{13}} \widehat{\mathbf{x}}_3 = d \cdot \widehat{\mathbf{x}}_3\end{aligned}\tag{19}$$

Notice that this projection system consists of two simultaneous planes which cross-sect and thereby form a ray through the origin of the data frame of reference. Each such ray provides a bit of information about the true underlying financial economic system, but only when viewed together with the rays from the other two orthogonal projections. In fact, an infinite number of projections - a *complete projection cone* - can be computed from linear combinations from these three extreme orthogonal CLS projections. The other two extreme $(n, q) = (3, 2)$ systematic projectors for the CLS_2 and CLS_3 projections and the $(n, q) = (3, 1)$ CLS projections are all similar.

In Section 4, we use these complete sets of $3D$ CLS projections to infer the empirical model structure of the true underlying financial economic valuation systems of the six Asian countries: Taiwan, Malaysia, Singapore, Philippines, Indonesia and Japan. But we start with the conventional PC analysis.

4 COMPLETE DATA ANALYSIS

4.1 Spectral Decomposition of Data Covariance Matrices

The spectral decomposition of a (3×3) data covariance matrix $\Sigma = \mathbf{U}\mathbf{\Lambda}\mathbf{U}$ produces the (3×3) diagonal matrix $\mathbf{\Lambda}$ with three eigenvalues λ_1 , λ_2 and λ_3 , while the matrix \mathbf{U} contains the corresponding orthonormal eigenvectors ($\mathbf{U}\mathbf{U} = \mathbf{I}$). Such a spectral decomposition of a positive

definite matrix Σ is *always* possible (Los, 1989a).

The scree plots of the λ_i for each of the six Asian countries are presented in Table 2. The determinant of the data covariance matrix of each country, $|\Sigma| = \lambda_1\lambda_2\lambda_3$, is an indication of the overall level of structural independence, or incoherence, of each of the variables. When the three variables x_1 , x_2 and x_3 are strictly linearly dependent, $|\Sigma| = 0$, since at least one of the λ_i equals exactly zero. The more independent the variables, the larger the value of the determinant. But, unfortunately, there exists no natural objective standard for comparison of the various data determinants, since the magnitudes of the λ_i depend on the empirical data variances.¹⁹ However, we can create comparative 3D Noise/Data Ratios by dividing by the country data determinants by the product of the variances, as we'll see in Section 5.

[TABLE 2. about here]

From Table 2 it seems that for five of the six countries - Taiwan, Malaysia, Singapore, Philippines and Indonesia - the number of principal components is one ($r = 1$), since each of these countries exhibits one dominant eigenvalue, $\lambda_1 \gg \lambda_2, \lambda_3$. For these five countries spectral decomposition suggests that there are at least two $q = 2$ linear combinations identifiable from the three variables, so that the systematic components of the three variables move simultaneously. A minimal level of coherence between the respective stock markets, economies and the financial markets of these five countries appears to exist. Japan is clearly an exception. For Japan it is not possible to distinguish between the situation of one ($r = 1, q = 3 - 1 = 2$) or two ($r = 2, q = 3 - 2 = 1$) dominant eigenvalues. There appears to be no identifiable linear combination among the three variables, ($r = 3, q = 0$), suggesting that the stock market, the economy and the financial market in Japan are disjointed, or, at least, that such a simple coherent system is not uniquely identifiable from the available data set.

However, extreme caution with such a loose interpretation of spectral decompositions is re-

¹⁹ We don't want to use the normalization of correlation matrices, since such diagonal-wise normalization distorts the information content, as explained in Los, 1989a, p. 1278.

quired, since such decompositions contain notoriously arbitrary convolutions of noise and signal elements.²⁰ A scree plot may be characteristic for a particular country, like it is for a particular chemical element, but it does not assist in the identification of the underlying (linear) system structure, since it does not provide an unambiguous decision on what part of the data variation is systematic and what is unsystematic.²¹

4.2 Information Matrices and (a, b) Parameter Plots

The Elementary Regression Theorem states that each row of the information matrix Σ^{-1} is an elementary ("ordinary") regression or unidirectional LS projection, *e.g.*, the first row of Σ^{-1} is the Least Squares (LS) projection, of variable x_{1t} on variables x_{2t} and x_{3t} , the second row is the LS projection of variable x_{2t} on x_{1t} and x_{3t} , etc.²²

The information matrices of all six countries are collected in Table 3, where the three elementary projections are normalized on the first variable x_{1t} , *i.e.*, its coefficient is always one, so that all a (income) and b (interest) elasticities are comparable:

$$\hat{x}_{1t} = a.\hat{x}_{2t} + b.\hat{x}_{3t} \tag{20}$$

²⁰ Since $\mathbf{U}\Sigma\mathbf{U} = \mathbf{\Lambda}$ diagonal, we can write $\mathbf{\Lambda} = \lambda_1\mathbf{E}_1 + \lambda_2\mathbf{E}_2 + \lambda_3\mathbf{E}_3$ where \mathbf{E}_i is a zero matrix with unity in the i, i diagonal position. Since also $\Sigma = \mathbf{U}\mathbf{\Lambda}\mathbf{U}'$ we can spectrally decompose $\Sigma = \lambda_1\mathbf{U}\mathbf{E}_1\mathbf{U}' + \lambda_2\mathbf{U}\mathbf{E}_2\mathbf{U}' + \lambda_3\mathbf{U}\mathbf{E}_3\mathbf{U}' = \lambda_1\mathbf{R}_1 + \lambda_2\mathbf{R}_2 + \lambda_3\mathbf{R}_3$ where $\mathbf{R}_i = \mathbf{U}\mathbf{E}_i\mathbf{U}'$ for $i = 1, 2, 3$. There is no clear conceptual separation between signal and noise. For example, does $\lambda_1\mathbf{R}_1$ constitute the signal and $\lambda_2\mathbf{R}_2 + \lambda_3\mathbf{R}_3$ the noise, or is $\lambda_2\mathbf{R}_2$ the signal and $\lambda_1\mathbf{R}_1 + \lambda_3\mathbf{R}_3$ the noise? In both cases $r = 1$ and $q = 2$. Or does $\lambda_1\mathbf{R}_1 + \lambda_2\mathbf{R}_2$ constitute the signal and $\lambda_3\mathbf{R}_3$ the noise, or is $\lambda_1\mathbf{R}_1 + \lambda_3\mathbf{R}_3$ the signal and $\lambda_2\mathbf{R}_2$ the noise? In both these last cases $r = 2$ and $q = 1$. Thus spectral analysis alone cannot determine what constitutes a signal and what noise and therefore it cannot unambiguously identify the system invariant q , particularly not in high noise environments.

²¹ This is not an uncommon situation in science. Until 1953 spectral analysis could determine which chemical elements are present in the protein molecule of DNA, but it could not determine the geometric structure of that protein. In 1953 Crick and Watson identified the helical system structure of DNA by algebraic geometric methods that inspired a new form of multi-dimensional spectral analysis based on *complete*, rotational, crystallographic analysis.

²² This Theorem is easily proved by elementary partitioned matrix algebra. The misleading term "regression" is Galton's, as explained in Los (1999). We prefer the mathematically explicit and more accurate term "unidirectional LS projection."

which is equivalent to the linear combination:

$$\widehat{x}_{1t} - a.\widehat{x}_{2t} - b.\widehat{x}_{3t} = 0$$

or, in vector-matrix notation, (21)

$$\mathbf{A}'\widehat{\mathbf{x}}_t = \begin{bmatrix} 1 & -a & -b \end{bmatrix} \begin{bmatrix} \widehat{x}_{1t} \\ \widehat{x}_{2t} \\ \widehat{x}_{3t} \end{bmatrix} = 0 \quad (22)$$

The hats over the variables x_{it} denote the systematic parts of the data. Other normalizations of this relationship on variable x_2 (by dividing the preceding equation by a), or on x_3 (by dividing the equation by b), are completely equivalent.²³

[TABLE 3. about here]

Notice that only the (a, b) coefficients of Singapore, Malaysia and Philippines show sign consistency: each of the three projections delivers the same sign for both the expected income elasticity $a > 0$ and interest rate elasticity $b < 0$. We conclude that for these three countries the hypothesis $q = 1$ cannot be rejected, in the *logical* Popperian sense of "it cannot be falsified" This is not "no rejection" in the subjective *statistical* sense of "significance" "testing, of which the results are dependent on the projection direction.²⁴ In these three economies at least two out of the three variables are mutually independent.

For example, the financial market and the economy may be disjoint, but they both influence the stock market; or the financial market and the stock market may be disjoint, but they both influence the economy. The (a, b) coefficients for Taiwan, Indonesia and Japan show sign inconsistency and we conclude that for these three countries the hypothesis of $q = 1$ is logically falsified. *Ergo*, for these countries it *must* be that $q = 2$.

²³ $(a, b)_{i,j}$ is the set of income (a) and interest elasticities (b) from the $q = 1$ CLS projection on variables \mathbf{x}_i and \mathbf{x}_j .

²⁴ Different unidirectional LS projection directions result in different levels of noise = "residuals," and thus in different "significance" tests.

There is considerable system uncertainty. In the conventional economic representation for Singapore, the measured (expected) income elasticity lies in the finite range $0.6570 \leq a \leq 1.1789$. This range includes the unit elasticity $a = 1$ of the original, restricted, DDM. Similarly, Singapore's measured interest rate elasticity b ranges from -4.5692 to -0.2112 , which includes the unit elasticity $b = -1$. Therefore, the original DDM cannot be rejected for Singapore. A different conclusion holds for Malaysia, where the expected income elasticity a ranges from 1.3414 to 1.5653 , which does *not* include $a = 1$, and the interest rate elasticity $-0.7766 \leq b \leq 2.0187$, which does *not* include $b = -1$. Therefore, the hypothesis of the original DDM is falsified for Malaysia. Similarly for the Philippines, where the expected income elasticity is in a narrow range $1.6058 \leq a \leq 1.8920$, which does *not* include $a = 1$, and the interest rate elasticity $-2.5332 \leq b \leq -0.7515$, which includes the unit elasticity $b = -1$. Also for the Philippines the classic $(n, 1)$ DDM is rejected, although not our more flexible, modified $(n, 1)$ DDM. Notice that the conclusion of $q = 1$ for Singapore, Malaysia and Philippines *contradicts* the apparent results of the earlier spectral analysis in Section 4.1, where the conventional interpretation of the scree plot found $q = 2$ for these countries.

The other three countries, Taiwan, Indonesia and Japan show much more system coherence than Malaysia, Singapore and the Philippines, since the elementary $q = 1$ projections show sign inconsistency. Sign inconsistency occurs when the expected income elasticity a and the interest rate elasticity b are both positive and negative for the same covariance data, depending on which projection direction is chosen. This is an indication that a single $(n, 1)$ linear equation cannot properly be identified from the data. The financial-economic literature refers to this condition as "coefficient instability." A system of $q = 2$ simultaneous independent linear equations exists. Thus, analysis of the information matrices of Taiwan, Indonesia and Japan suggests that the stock market, the economy and the financial market in these countries are mutually dependent on each other.

This logical information matrix analysis based on sign consistency of the projected elasticities, or lack of it, does not reveal the relative noise levels or uncertainty in the systems, since it is based

only on the adjoint of the data covariance matrix.²⁵ Because of high noise levels, this information matrix analysis may produce a spurious systematic correlation, giving the appearance of system coherence, when there actually isn't any.

However, a graphical presentation of the information matrix analysis, *i.e.*, of the magnitudes of the (a, b) coefficients, does provide a further indication of the relative noise levels. The theory of such (a, b) plots has already been explained in Klepper and Leamer (1984) and Los (1989a & b). Compare, for example, the first (a, b) plot for Taiwan in Fig. 1 with the one for the Philippines in Fig. 2. In both cases the information matrices are normalized on x_1 .²⁶

[FIGURE 1. about here]

[FIGURE 2. about here]

The expected income elasticity (with sign reversed!) is plotted on the horizontal axis while the interest rate elasticity (again, with sign reversed!) is plotted on the vertical axis. Sign consistency results only when the (a, b) plots all cluster in the same orthant. There are four orthants in the bivariate (a, b) space: $(+, +)$, $(+, -)$, $(-, +)$ and $(-, -)$, but the even sign combinations $(+, +)$ and $(-, -)$ are mirror-equivalent and so are the odd sign combinations $(+, -)$ and $(-, +)$.²⁷ It is clear that Taiwan has a $q = 2$ system, since its (a, b) plots show sign inconsistency and the Philippines a $(n, 1)$ system, since its (a, b) plots show sign consistency.

4.3 2D Data Scatter Plots and Noise/Signal Ratios

In the preceding sections we have seen that the conventional forms of multivariate analysis are deficient, in the sense that they deliver no unambiguous separation of data into systematic and

²⁵ The determinant $|\Sigma|$ indicates the system's noise level. The adjoint $Adj(\Sigma)$ reflects the system's structure. The data covariance matrix $\Sigma = Adj(\Sigma) \cdot |\Sigma|$.

²⁶ There are three (a, b) dots per country. The normalization does affect the (a, b) dots, because of the relative noise distribution in each picture, *i.e.*, the "volume," of the triangle formed by the three dots, but not the sign (in-)consistencies. All (a, b) plots of all countries were graphed and analyzed and the results are available upon request from the author. Lack of space prevents the publication of all (a, b) plots in this paper.

²⁷ Los (1989b) provides a complete analysis of such (a, b) coefficient plots. Compare, in particular, the geometric analysis of Fig. 1 of Los (1989b, p. 1291), which improves on the earlier model specification analysis by Klepper and Leamer (1984).

unsystematic parts, or, as signal engineers say, into "signal" and "noise" (Los, 1989a). Therefore, we use an alternative approach in the form of the CLS projections made possible by Kalman (1991, 1993) and Los (1989a & b, 1992, 1994, 1999 and 2001). These CLS projections exploit all bivariate covariance information in the $n \cdot (n - 1) / 2 = 3 \cdot 2 / 2 = 3$ bivariate covariance pictures per country.

In Section 4, the country data x_i for $i = 1, 2, 3$ are plotted in $3D$ data spaces as ellipsoid scatter in the center of the $3D$ logarithmic data reference frames. Each black dot represents a $3D$ measurement of the three respective variables - stock valuation, expected nominal GDP and interest rate (after logarithmic transformations and lateral shifts). The side panels of the $3D$ data frames show the orthogonal projections of these scatter plots as grey dots, forming three bivariate scatter plots. We computed the bivariate noise/data and noise/signal ratios, collected in Table 4. for the respective bivariate (x_1, x_2) , (x_1, x_3) and (x_2, x_3) scatter plots.²⁸

Table 4 shows that the noise/signal ratios of Taiwan are between $1.63\times$ and $2.96\times$, but they are evenly distributed. In other words, although Taiwan is clearly a high noise environment, some information can be extracted from the data by looking at all three bivariate scatter plots simultaneously, as in a simultaneous $q = 2$ system. Despite the high noise level, Taiwan functions as a coherent financial economic system: its economy, and its stock and cash markets all depend on each other.

[TABLE 4. about here]

We found earlier that Japan's data covariance structure is somewhat similar to Taiwan's ($q = 2$), except that Japan's data noise swamps its linear signals: Japan's noise/signal ratios are between $8.12\times$ and $46.47\times$! Therefore even our CLS methodology cannot identify a unique coherent linear financial-economic structure from its data set.

The noise signal ratios of Malaysia, Singapore, Philippines and Indonesia show close relation-

²⁸ The bivariate Noise/Data Ratio is $(N/D)_{ij} = (1 - \rho_{ij}^2)$ and the corresponding Noise/Signal Ratio is $(N/S)_{ij} = (1 - \rho_{ij}^2) / \rho_{ij}^2$ for $i, j = 1, 2$, where ρ_{ij} is the bivariate correlation coefficient between variables \mathbf{x}_i and \mathbf{x}_j and ρ_{ij}^2 is the familiar bivariate coefficient of determination.

ships between x_1 and x_2 and very uncertain bivariate relations between x_1 and x_3 and between x_2 and x_3 , respectively. We tentatively conclude that in those countries there is a close relationship between the stock market and the expected economy, but no relationship between the stock market and the financial market or between the expected economy and the financial market. These countries exhibit only $q = 1$ systematic covariance. The financial markets in these four countries appear to operate independently from their respective economies and stock markets.

5 SYSTEM IDENTIFICATION BY COUNTRY

5.1 Taiwan

In Fig. 3. we have plotted Taiwan's measured data \mathbf{x} in the $3D$ data space: x_1 is measured on the first horizontal axis, x_2 on the second horizontal axis and x_3 on the vertical axis. From the $3D$ scatter plot it is difficult to obtain a definite conclusion, although some information may be gleaned from the three $2D$ scatter-plots in the side panels. However, when we plot the three $q = 2$ projection systems $\hat{\mathbf{x}}'_i = \hat{\mathbf{P}}_i^{CLS} \mathbf{x}'$, $i = 1, 2, 3$, three rays are produced in the center of the $3D$ scatter plot, which all lie in a similar direction in Fig. 4.

[FIGURE 3. about here] [FIGURE 4. about here]

With appropriate visualization software,²⁹ we can rotate these $q = 2$ systems and observe that the three CLS rays are lying close together in a fairly tight cone. There is visible a positive systematic relationship between x_1 and x_2 as observed in the bottom (x_1, x_2) grid, a positive relationship between x_1 and x_3 as observed in the (x_1, x_3) grid, and consequently, there is also a positive relationship between x_2 and x_3 as observed in the (x_2, x_3) grid. Thus, we find that Taiwan logically has a $q = 2$ financial economic system, represented by the two simultaneous independent equations:

$$\begin{aligned} \hat{x}_1 - c.\hat{x}_2 = 0 & \quad \text{or, equivalently} & \quad \hat{x}_1 = c.\hat{x}_2 \\ \hat{x}_1 - d.\hat{x}_3 = 0 & & \quad \hat{x}_1 = d.\hat{x}_3 \end{aligned} \tag{23}$$

²⁹ For example, with Stanford Graphics, MathCad 6.0 PLUS, Matlab, Mathematica, IBM Visual Data Explorer, or MathWare's Cyclone.

with both $c > 0$ and $d > 0$. The stock market has a positive income elasticity and a *positive* interest rate elasticity, thereby empirically contradicting conventional economic theory, which presumes a negative interest rate elasticity, $d < 0$. When Taiwan's interest rate rises raised, its stock market valuation increases simultaneously.³⁰ One plausible "cost-push" explanation for this contradictory result may be that when cash becomes expensive in Taiwan, the domestic banks, which are deeply invested in the Taiwanese stock market, bid up the stock market to prevent their balance sheets from deteriorating.

The three finite boundaries of these elasticities from the corresponding bivariate elements in the $\widehat{\mathbf{P}}_i^{CLS}, i = 1, 2, 3$ systematic projector matrices are given in Table 5.³¹

[TABLE 5. about here]

Notice that, first, the wide ranges of modeling uncertainty reflect the high data noise environments, and, second, theoretically expected unit elasticities are mostly outside these empirical ranges, contradicting conventional financial (DDM) theory. The comparable 3D noise/data and noise/signal ratios are defined as follows (Los, 2001, Chapter 5):

$$3D \text{ Noise/Data Ratio} = \sqrt{|(\widehat{\mathbf{P}}_1^{CLS} + \widehat{\mathbf{P}}_2^{CLS} + \widehat{\mathbf{P}}_3^{CLS})'(\widehat{\mathbf{P}}_1^{CLS} + \widehat{\mathbf{P}}_2^{CLS} + \widehat{\mathbf{P}}_3^{CLS})|} \quad (24)$$

$$= \begin{vmatrix} 1 & \frac{\sigma_{12}}{\sigma_{22}} & \frac{\sigma_{13}}{\sigma_{33}} \\ \frac{\sigma_{12}}{\sigma_{11}} & 1 & \frac{\sigma_{23}}{\sigma_{33}} \\ \frac{\sigma_{13}}{\sigma_{11}} & \frac{\sigma_{23}}{\sigma_{22}} & 1 \end{vmatrix} \quad (25)$$

$$= \frac{\sigma_{11}\sigma_{22}\sigma_{33} - \sigma_{33}\sigma_{12}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{11}\sigma_{23}^2 + 2\sigma_{12}\sigma_{13}\sigma_{23}}{\sigma_{11}\sigma_{22}\sigma_{33}} \quad (26)$$

$$= \frac{|\boldsymbol{\Sigma}|}{\sigma_{11}\sigma_{22}\sigma_{33}} \quad (27)$$

$$= \frac{\prod_{i=1}^3 \lambda_i}{\prod_{i=1}^3 \sigma_{ii}} \geq 0 \quad (28)$$

³⁰ No causality is implied, since this is a correlation analysis only.

³¹ $(c, d)_i$ is the set of income (c) and interest elasticities (d) from the $q = 2$ CLS projection on variable \mathbf{x}_i .

The first of this series of equations demonstrates the completeness of the CLS projections, since they exhaust in sum all the systematic information available in the data covariance matrix. When the determinant of the data covariance matrix in the fourth equation $|\Sigma| = 0$, the data exhibit exact linear relations and, as shown in the fifth equation, at least one of the eigenvalues must be $\lambda_i = 0$. We have also the

$$3D \text{ Noise/Signal Ratio} = \frac{|\Sigma|}{\sigma_{11}\sigma_{22}\sigma_{33} - |\Sigma|} \quad (29)$$

Both information quality ratios are presented in Table 6.

[TABLE 6. about here]

In Taiwan the noise cone spanned by the three projected rays occupies 37% of the data space. This means that 37% of the 3D variation of the Taiwanese data is unsystematic, while the remaining 63% is systematic. Note that in the case of complete certainty, *i.e.*, when all variation is certain, the three projected rays would coincide on one ray and the 3D noise/data ratio would be zero.

Taiwan's 3D noise/data ratio of 37% implies a 3D noise/signal ratio of 58%, *i.e.*, the unsystematic variation in its financial economy is slightly larger than half of its systematic variation. Taiwan forms thus a fairly coherent financial-economy.

5.2 Malaysia and Singapore

Malaysia's financial economic system is very different from Taiwan's, as is immediately clear from the $(n, q) = (3, 2)$ plots in Fig. 5. and Fig. 6. One of the $q = 2$ rays in Fig. 6 is almost orthogonal to the other two rays, as observed from their reflections in the two side grids for (x_1, x_3) and (x_2, x_3) , indicating that two planes are crossing almost vertically. This is corroborated by rotating the 3D plot. This indicates that the interest rate variable x_3 varies almost independently from the stock market index x_1 and the *NGDP* variable x_2 . At the same time the reflections in the bottom (x_1, x_2) grid are bundling closely together, suggesting a close bivariate relationship

between the stock market S and the economy $NGDP$. Malaysia's projected (c, d) coefficients do have the correct sign according to conventional economic theory, but not the correct financial magnitude, as can be seen in Table 5.

[FIGURE 5. about here] [FIGURE 6. about here]

The 2D noise/signal ratios in Table 4 in Section 4.3 point to a similar conclusion, which should not surprise us since the $(n, q) = (3, 2)$ systematic projectors take complete account of all systematic bivariate grid information. While the noise/signal ratio between the stock market x_1 and the expected economy x_2 is only 0.14, the noise/signal ratio for the bivariate relationship between the stock market x_1 and the financial market x_3 is 5.79, and the noise/signal ratio between the expected economy and the financial market is a very large 32.38.

However, when we look at 3D noise/data and noise/signal ratios of 7% and 8% respectively in Table 6 in Section 5.1, the financial economy of Malaysia is clearly not a high noise environment, since the single $(n, 1)$ relationship is very tight. From a system point of view, only 7% of all observed 3D data variation is noise, but 93% is systematic! There is a very tight relationship between the expected economy and the stock market, even though Malaysia's cash market behaves independently from both.

The magnitude of data noise does not depend on large variation of the variables, as is sometimes believed. The stock and financial market and the expected economy can all show substantial variation and yet the financial economic system can still be knitted together by rational economic calculations and behave (almost) like a machine. In the case of Malaysia, only the stock market and the economy appear to be in such a tight rational relationship.

A comparison of the $(n, q) = (3, 2)$ plots of Malaysia and Singapore in Figures 6. and 8, respectively, show an almost identical configuration. This conclusion reinforces the impression that our new system identification method identifies similarities and dissimilarities between the structures of financial-economic systems, which cannot be found via the conventional statistical

approaches.

[FIGURE 7. about here] [FIGURE 8. about here]

While there is a tight relationship between the expected economy and the stock market in Singapore, its financial market operates virtually independently from these two variables. At the same time Singapore's stock market is closely linked to its economic prospects.

5.3 The Philippines

The $(n, 1)$ financial economic system of the Philippines operates similarly to the ones of Malaysia and Singapore, as can be seen by comparing Fig. 10 with Figures 6 and 8. But there are two aspects which make the Philippines somewhat different from Malaysia and Singapore.

[FIGURE 9. about here] [FIGURE 10. about here]

First, the relationship between the Philippine's stock market and the expected economy is somewhat looser than in Singapore. In Table 4, the bivariate (x_1, x_2) noise/signal ratio of Philippines is 0.25 instead of Singapore's 0.17 and there is absolutely no relationship between the expected economy x_2 and the financial market x_3 : the bivariate noise/signal ratio is 54721.40!

This still doesn't mean that the Philippines' economy is completely "disjointed." Surprisingly, it is almost as coherent as Singapore's economy. The noise/data and noise/signal ratios of the Philippines in Table 6 are 14% and 16%, respectively, which is very similar to Singapore's 13% and 14%, respectively. This is again due to the close relationship between the stock market valuation and the expected economy. Although the cash market is almost disconnected, expected economic growth and wealth creation are tightly connected.

5.4 Indonesia

Indonesia follows the pattern discovered for Malaysia, Singapore and the Philippines.

[FIGURE 11. about here] [FIGURE 12. about here]

However, Indonesia distinguishes itself by:

(i) having a considerably looser relationship between the stock market x_1 and the expected economy (represented by petroleum production) x_2 than Malaysia, Singapore or the Philippines. According to Table 4, Indonesia's $2D$ (x_1, x_2) noise/signal ratio is 0.73, which is higher than Malaysia's 0.14, Singapore's 0.17 or Philippine's 0.25.³²

(ii) having very weak relationship between its financial market and both its stock market and its expected economy, very much like Singapore. The bivariate relationship between the stock market and financial market and the expected economy and financial market, measured by (x_1, x_3) and (x_2, x_3) bivariate noise signal ratios of 11.43 and 16.47 (high noise) respectively. This is very similar to Singapore's 9.49 and 18.34.

(iii) an overall more integrated economy than Malaysia, Singapore or the Philippines, and more like Taiwan's, since Indonesia's $3D$ noise/data and noise/signal ratios are 39% and 63%, respectively, as seen in Table 6. That is roughly three times as large as for the Philippines, four times as for Singapore and six times as for Malaysia. These figures are similar to those for Taiwan, which shows $3D$ noise/data and noise/signal ratios of 37% and 58%, respectively.

The $q = 2$ system identification using two consecutive data windows and the time series data suggest that Indonesia's data set is very non-stationary. Since the first quarter of 1990 Indonesia's financial market has had some negative relationship with the expected economy, which did not exist before. Perhaps this was because before 1990 the variability of the interest rates in Indonesia was virtually nil: the deviation was no more than 5 basis points in the pre-1990 period. When the variability of the interest rate increased after 1990, the economy, as very crudely measured by Indonesia's petroleum production, correlated somewhat negatively with the interest rate movements.

³² This conclusion is subject to the observation that we use Indonesia's petroleum production as a proxy for its GDP. The additional system uncertainty may very well be attributable to that substitution. A strict comparison would therefore be invalid. This remark also holds for the next point (ii).

5.5 Japan

From Japan's data set we could not identify a coherent financial- economic system. In first instance we seem to encounter a financial economy similar to that of Taiwan, *i.e.*, a $(n, q) = (3, 2)$ financial economic system, with high noise levels. But the spectral analysis already showed a major difference in the structure: in Table 2 Japan shows a very gradual fall-off of the scree plot of its data covariance matrix ($r = 3, q = 0$), while Taiwan shows one dominant eigenvalue ($r = 1, q = 2$).

[FIGURE 13. about here] [FIGURE 14. about here]

The $(n, q) = (3, 2)$ plot in Fig. 14 shows a financial economic system that is almost completely disjointed. The $q = 2$ CLS projection on the interest rate x_3 is virtually orthogonal to the projections on the stock market value x_1 and the expected economic earnings x_2 . Simultaneously the $q = 2$ projection on x_2 is also almost orthogonal to the $q = 2$ projection on x_1 as observed in the various side grids. In addition, the projected (c, d) elasticities show signs opposite of those expected by conventional economic and financial theory, *i.e.*, a negative income elasticity and a positive interest rate elasticity, both larger than unity in absolute value.

The $2D$ noise/signal ratios in Table 4 tell the same story. The bivariate noise/signal ratio of the (x_1, x_2) relationship between the stock market and the expected economy is an extremely high of 46.47; the (x_1, x_3) noise/signal ratio between the stock market and the financial market is a high 8.12; and the (x_2, x_3) noise/signal ratio between the expected economy and the financial market is again extremely high 44.15. In addition, the $3D$ noise/data and noise/signal ratios in Table 6 does indicate almost complete disjointedness: they are a large 86% and an explosive 619%, respectively.

A clue for a possible economic explanation of this surprising result can be found in comments like the one by Roche, the Chief Strategist for the London-based company Independent Strategy. Roche wrote an op-ed article "Japans Contradictory Economy" in *The Asian Wall Street Journal*

of Wednesday, March 12, 1997, in which he stated: "It is irrelevant to say that Japan is, or is not, a centrally planned economy. What is relevant is that it is not an economically rational one." Japan's financial economic system is disjointed because its various markets were possibly not linked by rational economic calculations during the period over which our data are collected.

6 CONCLUSION

6.1 Summary

In this paper we investigate the financial-economic systems of six Asian countries - Taiwan, Malaysia, Singapore, Philippines, Indonesia and Japan - over the period of first quarter of 1986 through third quarter of 1995, to determine how their stock markets, expected economies and financial (cash) markets are interrelated and if these economies show similarities in their structure. The purpose of this exercise is to demonstrate the applicability of the new Complete Least Squares (CLS) system identification methodology to investigate how financial systems and economies interact in various countries. We apply CLS to six Asian economies, because of the renewed interest of global investment and hedge funds in their financial-economic structures.

We compare the results of this new $3D$ system identification based on $q = 1$ and $q = 2$ system projections with the results of some conventional methodologies. Thus we apply spectral analysis of the data covariance matrices, analysis of the elementary regressions in the information matrices, and bivariate noise/signal determinations.

Our algebraic-geometric methods of CLS projections provide the best (= complete, with least residual noise) identifications of the Grassmanian invariants of the structures of these national financial economic systems, as far as the noise levels of these data sets allow. The Grassmanian invariant q of a system is the minimum number of independent linear equations required by the covariance structure to model the systematic variation of the data. Thus, for three variables, $q = 1$ or 2, *i.e.*, a single equation system or a two-equation system.

Once we have identified this invariant of each financial economic system, we compute the

observable parameter ranges for the systems (*Cf.* Tables 2 and 4). We also checked our conclusions against the time series data by computing the relevant time series projections and by comparing them against the actual historical data. However, no in-sample or out-of-sample testing of the homogeneity of the data series and the integrity of the identified systems has been done, due to the scarcity of the available quarterly data. In other words, we could not test well the stationarity of the distributed systems. This would require several more years of relatively scarce comparable observations. However, we find that around 1990 the structure of Indonesia's financial economic system changed: its financial market became linked to its economy and its stock market. In formal system terms: around 1990 Indonesia's system probably changed from a $(n, 1)$ to a $(n, 2)$ structure. It became more coherent.

Our identification results for the crucial financial-economic system invariant q are summarized in Table 7, where we also present our conclusions about structure and relative noise levels.

First, we found that all six financial economic data sets are high noise data environments, in which it is difficult to discern the model systems from the data noise, no matter what analytic methodology is used. Because of these high noise levels, spectral analysis is not very discriminatory, since this form of analysis tends to mix the signal with the noise in the same way as coffee and cream.

Second, the analysis of the information matrix (= inverse of the data covariance matrix) and the (a, b) plots of the elementary regressions concludes that the countries can be classified into two groups. Malaysia, Singapore and the Philippines show $(n, 1)$ system structures, while Taiwan, Indonesia and Japan *appear* to show $q = 2$ system structures. The (a, b) plots also confirm that both Indonesia and Japan are very high noise environments.

Our CLS analysis shows that only Taiwan has a clearly identifiable $q = 2$ system. Of the six Asian countries, only Taiwan has a financial economic system where the stock market, the economy and the financial market are rationally integrated. Its relevant parameter ranges are to be computed from the two identified independent equations.

In addition to Malaysia, Singapore, Philippines, Indonesia has a $(n, 1)$ system. While the stock markets and expected GDPs of these countries are closely related and vary together, their respective domestic financial markets have virtually no relationship with their stock markets or their expected GDPs.

These four economies differ only in the degree of tightness of the relationship between their stock markets and respective expected economies. Malaysia shows the tightest relationship between these two variables, while Indonesia appears to have the loosest relationship. However, analysis of the relevant computed elasticities in Table 5 reveals that Indonesia's expected petroleum production has a closer relationship with the stock market in Indonesia, than the respective expected nominal GDPs in Malaysia, Singapore and the Philippines.

6.2 Limitations of the Analysis

Due to the limited available data resources in Asia, much time and effort is spent on data collection. Indeed, Hing argued in the December 19, 1996 issue of the *Far Eastern Economic Review* (p. 31) that there should be "Clear, Swift Market Data For All." He complained that "Many of Asia's emerging markets are infamous for their opaque nature and inefficiencies" and provided the examples of Malaysia and Indonesia. Most of our data were sourced and extracted from hard copies of newspapers and magazines (*Cf.* Appendix), and only a perfunctory quality check of the data was possible.

All countries, except Indonesia, produce quarterly nominal *GDP* figures. Indonesia does not have a policy of regularly publishing them, since it remains thoroughly uninformative about its imports and exports.³³ Therefore, we use its quarterly petroleum production index as a crude proxy for its economic growth rate. This proxy ignores also the overall inflation component of nominal *GDP*.

Furthermore, in some instances, nominal quarterly *GDP* had to be calculated and converted

³³ Perhaps, this huge country of 16,000 islands, and a fractal, very long coastline, is fundamentally unable to monitor all its imports and exports?

from its quarterly real *GDP* to suit the purpose of our study. In doing this conversion, some loss of systematic information may also have resulted. To ensure compatibility and consistency, we've tried, where possible, to use comparable data sets in all our computations. For instance, we have chosen the lending rate to be the interest rate, representing the financial markets for all countries. However, this type of interest rate is not available for Taiwan where we used the money market rate instead.

World stock markets are usually driven by many other quantitative and qualitative factors. Using our modified version of DDM to identify the systematic relationship between the valuation of the stock market, the measurement of the GDP and the interest rate is not sufficient to fully understand all driving forces behind Asian stock markets. We have identified very simple *open* systems.

Therefore, perhaps more variables should be included in the data sets: $n > 3$. In particular, we could include the countries' exchange rates to distinguish the Asian countries which peg their exchange rates to the US dollar from the ones which float theirs.³⁴ However, as seen in Section 2, the complexity of such financial economic system identification increases combinatorially with the number of variables n . For example, if one proposes to identify linear financial-economic systems from $n = 5$ variable data sets, like in Fama (1990), a complete system identification should produce 30 LS model projections for each data set. Fama (1990) shows only one of those 30 projections, namely that of a simple $(n, q) = (5, 1)$ single equation projections!³⁵ Computers are of course very good at such complete system identification, when properly programmed. It should now be obvious why this paper didactically presents only $n = 3$ applications, since they can be visualized in *3D*.

³⁴ A starting point could be Kholdy and Sohrabian's finding that the asset-market approach to exchange rate determination has not performed well empirically. They attribute that to the testing of the model by conventional ordinary (single equation) least squares econometric techniques, which they deem inappropriate if variables are nonstationary and co-integrated (Kholdy and Sohrabian, 1995). We concur with their assessment. Single equation least squares "regression" provides incomplete system identification.

³⁵ In addition, there are 10 $(n, q) = (5, 2)$ LS projections, 10 $(5, 3)$ LS projections and 5 $(5, 4)$ LS projections.

We also don't know how stable these financial-economic systems actually are over time. Our analytical methodology operates correctly when the distributions of the underlying data series are stable (which is a more general requirement than stationarity) and the data distributions within each country time-scale in a similar fashion. It is only required that the shape of these distributions doesn't vary over time. Thus, our new methodology allows that the data distributions change their amplitude or dispersion in a systematic fashion.

Finally, for didactic purposes, this paper has not accounted for the possible dynamics and causality of the relationships, other than a one-quarter expectation lag. Moreover, global investors make their asset allocation decisions faster than the available quarterly data frequency and such a higher data frequency for comparative data in Asia is not (yet) available.

Although "correlation is not causality," Bittlingmayer (1992, p. 1704-1705) erroneously asserts that "Statistical correlation cannot establish whether x causes y ." Granger (1969) demonstrates that, in principle, it is possible to apply causality analysis using the measured correlations of a dynamic data generating system and Sims (1980) developed his (incomplete) VAR approach from it. Of course, such a Granger causality analysis increases the dimensionality of the identification of the linear financial-economic system, since each added time-lag adds a separate variable to the data set, thereby increasing n and thus the number of possible system identification projections. Moreover Granger's causality analysis and Sim's VAR only measures correlations = linear dependencies between current and lagged variables with finite lags. It does not measure nonlinear dependencies between current and lagged variables with possible "infinite" lags. Los (2003) looks into the difficult issues of measurement and analysis of such long term, non-serial, nonlinear dependencies.

7 APPENDICES

7.1 Data Sources

The following data sources are used for the analysis in this paper:

- *Asia Pacific Economic Outlook* (4th quarter 1992, pp. 47 - 48 and 4th quarter 1996, pp. 67, 69).
- *The Asian Wall Street Journal* (March 1986 - December 1995).
- *The Business Times* (March 1986 - December 1995).
- *International Financial Statistics* (April 1988, April 1992, April 1996).
- *United Overseas Bank Economic and Financial Quarterly* (1987 II, pp. 13 - 14; 1988 II, pp. 18 - 19; 1989 II, pp. 22 - 23; and 1990 III, pp. 14 - 15).

7.2 Data Conversions

Each country's stock market index is based on the last trading day of each quarter. As the other two variables, *NGDP* and *IR*, are quoted as period averages, the stock market indices are converted to comparable period averages using geometric averaging of consecutive observations

$$S_t = [P_t.P_{t-1}]^{0.5} \quad (30)$$

where

S_t = stock market index as quarter t average

P_t = index on the last trading day of quarter t

P_{t-1} = index on the last trading day of the preceding quarter $t - 1$.

Since only real *GDP* figures for Singapore and Malaysia are quoted in quarterly year-on-year percentage change, their respective quarterly *NGDP* figures have to be estimated. No proper *GDP* implicit deflators are published, consequently we used their Consumer Price Indices (CPI) as proxies. First, their CPI figures were converted to a common base year 1985 using the same method of conversion as used for the Indonesian PP Index. Next, we use Fisher's Equation to estimate the quarterly *NGDP* rate of change

$$g_t^{NGDP} = (1 + g_t^{GDP}).(1 + f_t) - 1 \quad (31)$$

where

g_t^{NGDP} = rate of change of nominal *GDP*

g_t^{GDP} = rate of change of real *GDP*

f_t = rate of inflation.

As the Petroleum Production *PP* index of Indonesia is published with different base years, it is converted to a common base year 1985 using the next two formulas. Values with base year before 1985 are converted using the rebasing formula 1 while those with base year after 1985 are converted using the concatenation formula 2.

$$\text{Formula 1: } PP_t^{1985} = \frac{PP_t}{PP_t^{1985A}} \cdot 100 \quad (32)$$

where

PP_t^{1985} = Petroleum Production Index for quarter t with base year 1985

PP_t = Petroleum Production Index for quarter t with base year before 1985

PP_t^{1985A} = Annualized Petroleum Production Index of base year 1985

$$\text{Formula 2: } PP_t^{1985} = \frac{PP_{mt}}{100} \cdot PP_{mA} \quad (33)$$

where

PP_t^{1985} = Petroleum Production Index for quarter t with base year 1985

PP_{mt} = Petroleum Production Index for quarter t with base year before m , where m is a period after 1985

PP_{mA} = Annualized Petroleum Production Index for year m with base year 1985.

TABLE 1. SCIENTIFIC INCOMPLETENESS			
Article (Type of Analysis)	# of Variables	Analytic incompleteness %	# of unreported LS p
Fama (1990) (LS)	5	80	29
Schwert (1990) (LS)	5	80	29
	9	89	509
	12	92	4,093
Bittlingmayer (1992) (LS)	8	88	253
Canova & De Nicolo (1995) (LS)	4	75	13
	6	83	61
	9	89	509
	11	91	2,045
Lee (1992) (VAR)	28	96	268,000,000
Gallinger (1994) (ADF)	13	92	8,189
	15	93	32,765
	17	94	131,060
	18	94	262,141
	19	95	542,285

TABLE 2. SCREE PLOT OF EIGENVALUES				DETERMINANTS
	λ_1	λ_2	λ_3	$ \Sigma = \lambda_1 \cdot \lambda_2 \cdot \lambda_3$
TAIWAN	0.532	0.075	0.047	$1.90E - 03$
MALAYSIA	0.357	0.022	0.005	$3.75E - 05$
SINGAPORE	0.201	0.012	0.007	$1.63E - 05$
PHILIPPINES	0.533	0.050	0.013	$3.39E - 04$
INDONESIA	0.574	0.014	0.002	$1.91E - 05$
JAPAN	0.073	0.033	0.016	$3.98E - 05$

TABLE 3. RANGES OF INCOME (a) AND INTEREST RATE (b) ELASTICITIES			
From $(n, 1)$ CLS Projections	$(a, b)_{2,3}$	$(a, b)_{1,3}$	$(a, b)_{1,2}$
TAIWAN	[+0.9096, +0.7262]	[+4.5042, -0.6887]	[-0.8626, +3.4887]
MALAYSIA	[+1.4320, -0.8291]	[+1.5653, -0.7766]	[+1.3414, -2.0187]
SINGAPORE	[+1.0033, -0.3225]	[+1.1789, -0.2112]	[+0.6570, -4.5692]
PHILIPPINES	[+1.6103, -0.7515]	[+1.8920, -0.7494]	[+1.6058, -2.5332]
INDONESIA	[+7.3722, -0.6586]	[+13.3330, +0.2081]	[-2.3291, -25.2981]
JAPAN	[-0.1875, +0.3858]	[-17.7667, -1.2813]	[+0.6229, +3.8562]

TABLE 4. 2D NOISE/DATA AND NOISE/SIGNAL RATIOS						
	$(N/D)_{12}$	$(N/S)_{12}$	$(N/D)_{13}$	$(N/S)_{13}$	$(N/D)_{23}$	$(N/S)_{23}$
TAIWAN	0.62	1.63	0.62	1.60	0.75	2.96
MALAYSIA	0.12	0.14	0.85	5.79	0.97	32.38
SINGAPORE	0.14	0.17	0.90	9.49	0.95	18.34
PHILIPPINES	0.20	0.25	0.94	15.43	1.00	54721.40
INDONESIA	0.42	0.73	0.92	11.43	0.94	16.47
JAPAN	0.98	46.47	0.89	8.12	0.98	44.15

TABLE 5. RANGES OF INCOME (c) AND INTEREST RATE (d) ELASTICITIES			
From ($q = 2$) CLS Projections	$(c, d)_1$	$(c, d)_2$	$(c, d)_3$
TAIWAN	[+3.623, +2.817]	[+1.375, +2.145]	[+2.751, +1.084]
MALAYSIA	[+1.706, -9.433]	[+1.495, -13.1578]	[+3.541, -1.392]
SINGAPORE	[+1.205, -10.000]	[+1.030, -12.561]	[+1.513, -0.959]
PHILIPPINES	[+2.012, -12.500]	[+1.612, -537.333]	[+104.560, -0.763]
INDONESIA	[+13.210, -21.505]	[+7.632, -19.384]	[+11.902, -1.731]
JAPAN	[-13.158, +3.676]	[-0.278, +1.188]	[-4.253, +0.040]

TABLE 6.	INFORMATION	QUALITY RATIOS
	NOISE/DATA RATIO	NOISE/SIGNAL RATIO
TAIWAN	0.37	0.58
MALAYSIA	0.07	0.08
SINGAPORE	0.13	0.15
PHILIPPINES	0.14	0.16
INDONESIA	0.39	0.63
JAPAN	0.86	6.19

TABLE 7.	IDENTIFICATION OF SYSTEM INVARIANT: RANK q				
From:	Spectral Analysis of Σ	Information Matrix Σ^{-1}	Inspection of (a, b) Plots	3D CLS Plots	3D N/D Ratio
TAIWAN	$r = 1, q = 2$	2	2	2	0.37
MALAYSIA	$r = 1; q = 2$	1	1 ($\mathbf{x}_3 = \text{noise}$)	1	0.07
SINGAPORE	$r = 1; q = 2$	1	1 ($\mathbf{x}_3 = \text{noise}$)	1	0.13
PHILIPPINES	$r = 1; q = 2$	1	1 ($\mathbf{x}_3 = \text{noise}$)	1	0.14
INDONESIA	$r = 1; q = 2$	2	2 ($\mathbf{x}_3 = \text{noise}$)	1	0.39
JAPAN	$r = 3; q = 0$	2	2 ($\forall \mathbf{x}_i = \text{noise}$)	0	0.86

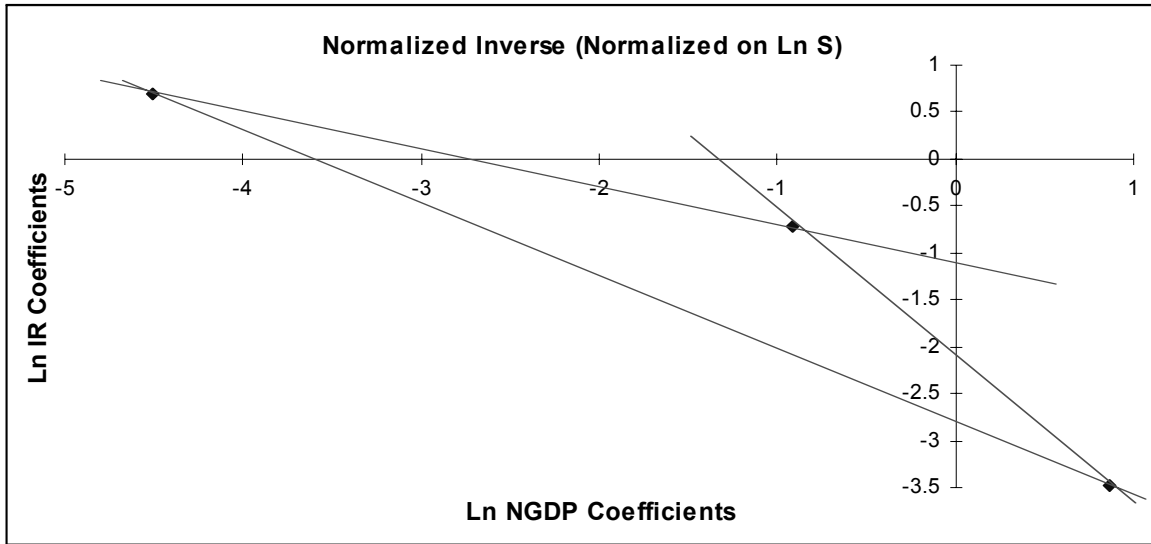


Fig. 1. Taiwan (a, b) plot: $q = 2$

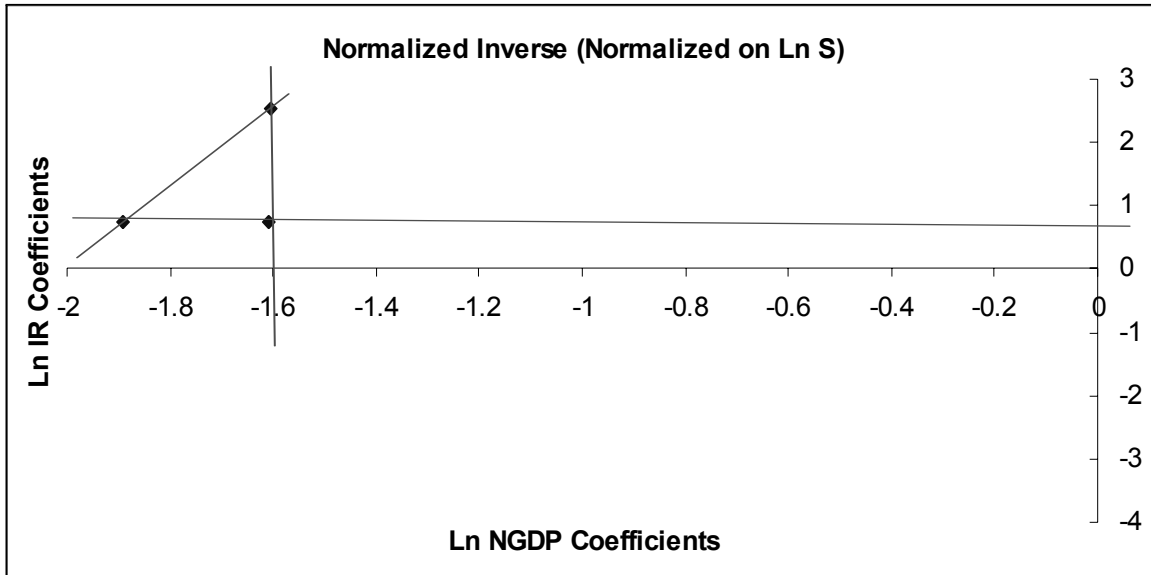


Fig. 2. Philippines (a, b) plot: $q = 1$

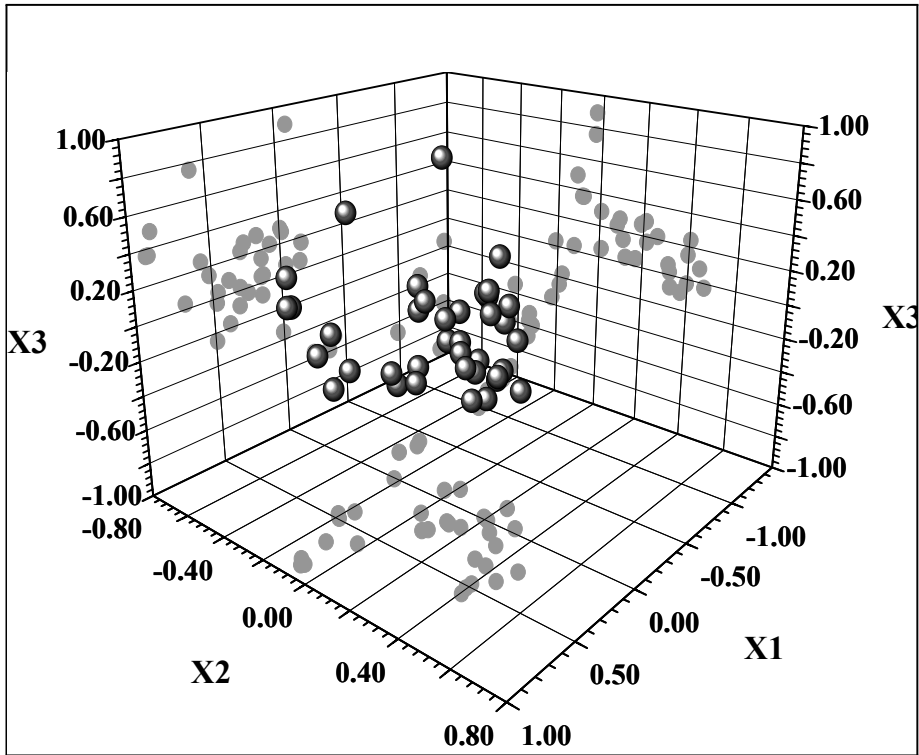


Fig. 3. Taiwan 3D Scatterplot

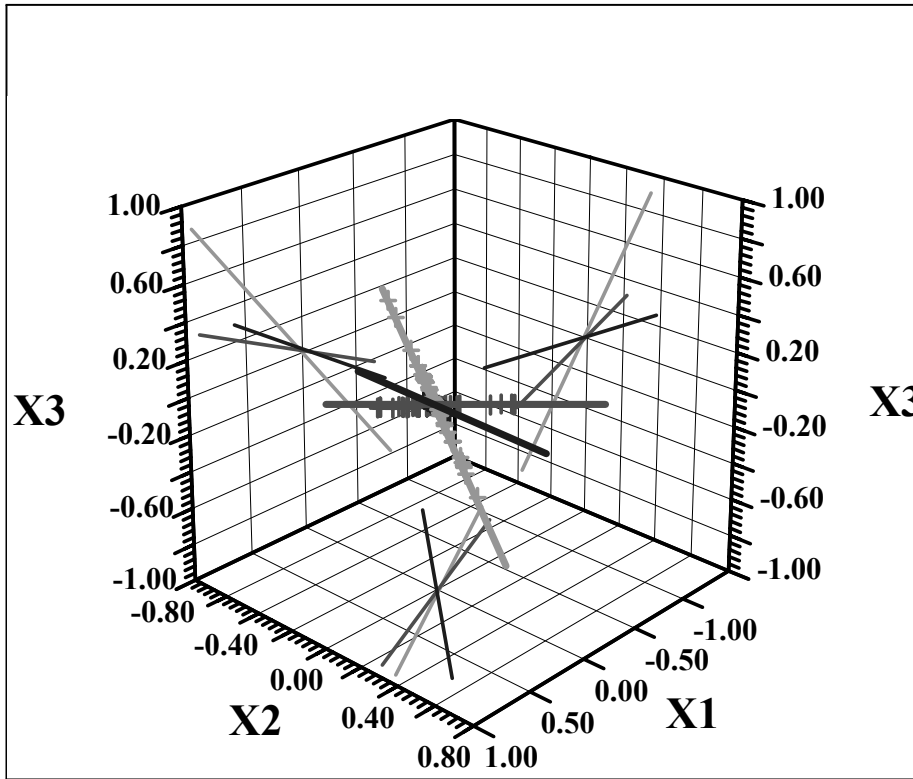


Fig. 4. Taiwan 3D CLS Plot

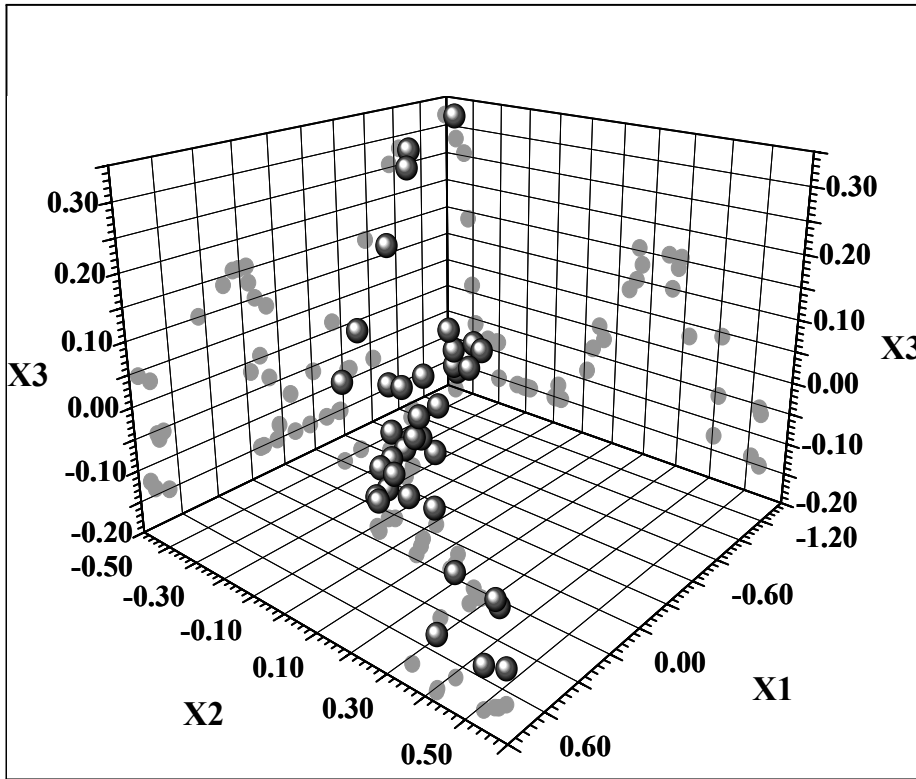


Fig. 5. Malaysia 3D Scatterplot

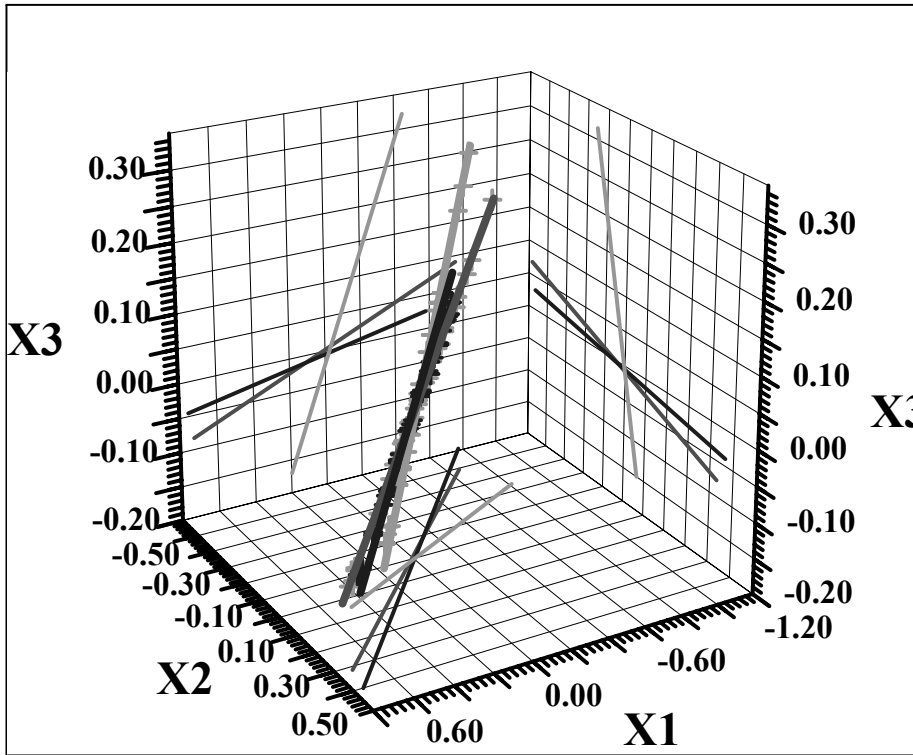


Fig. 6. Malaysia 3D CLS Plot

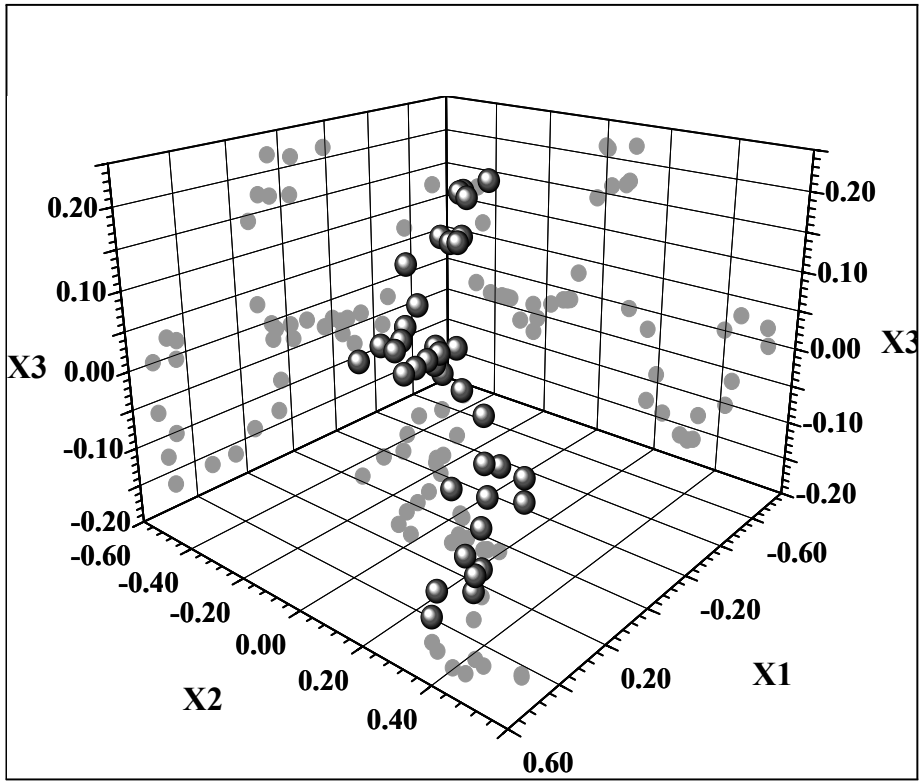


Fig. 7. Singapore 3D Scatterplot

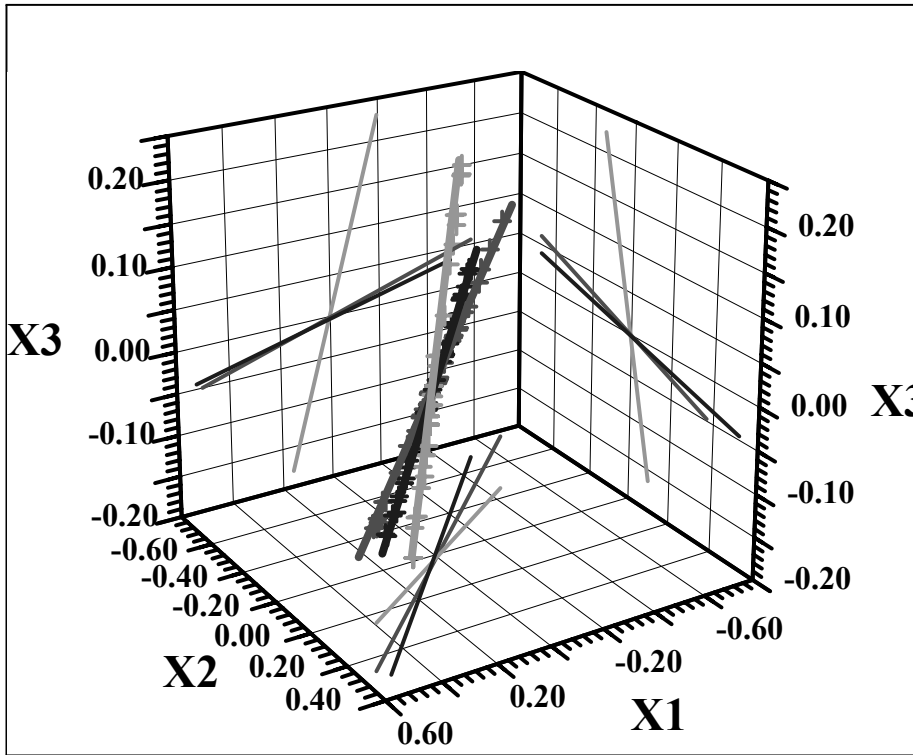


Fig. 8. Singapore 3D CLS Plot

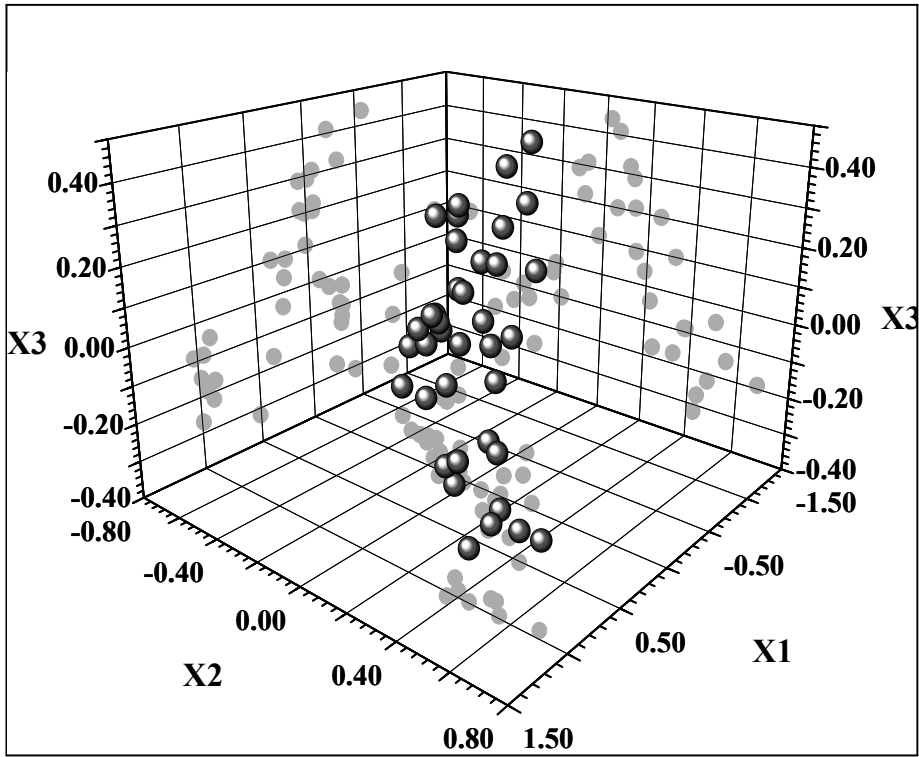


Fig. 9. Philippines 3D Scatterplot

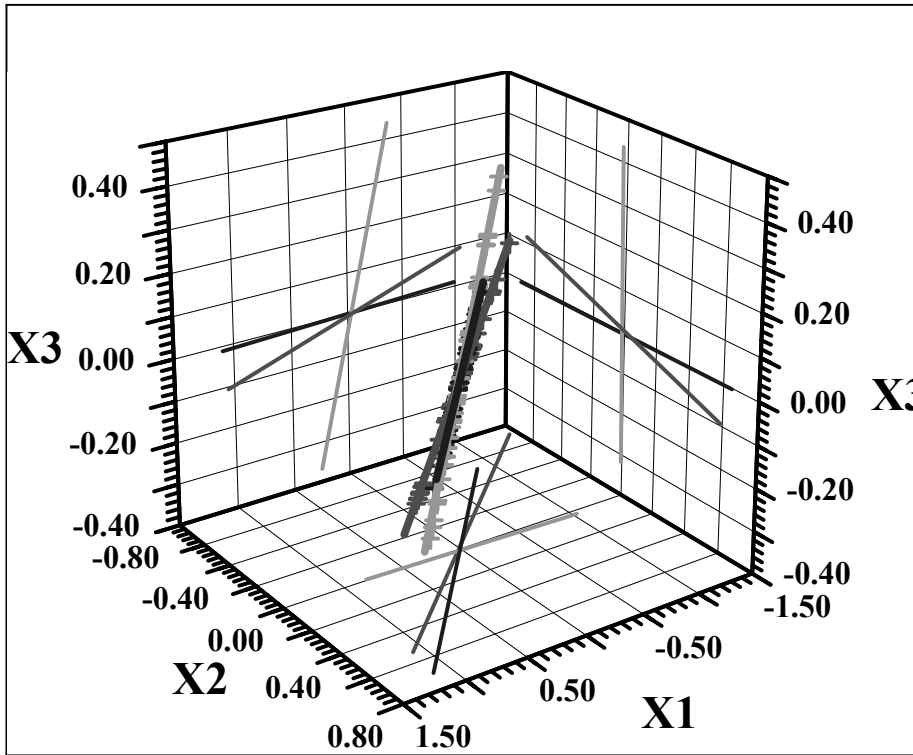


Fig. 10. Philippines 3D CLS Plot

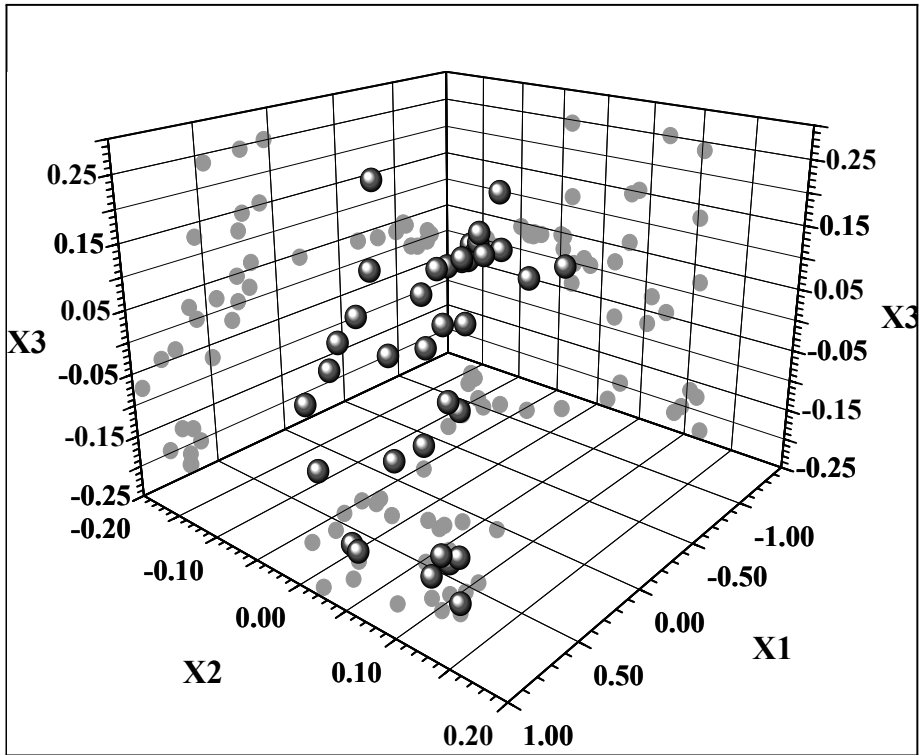


Fig. 11. Indonesia 3D Scatterplot

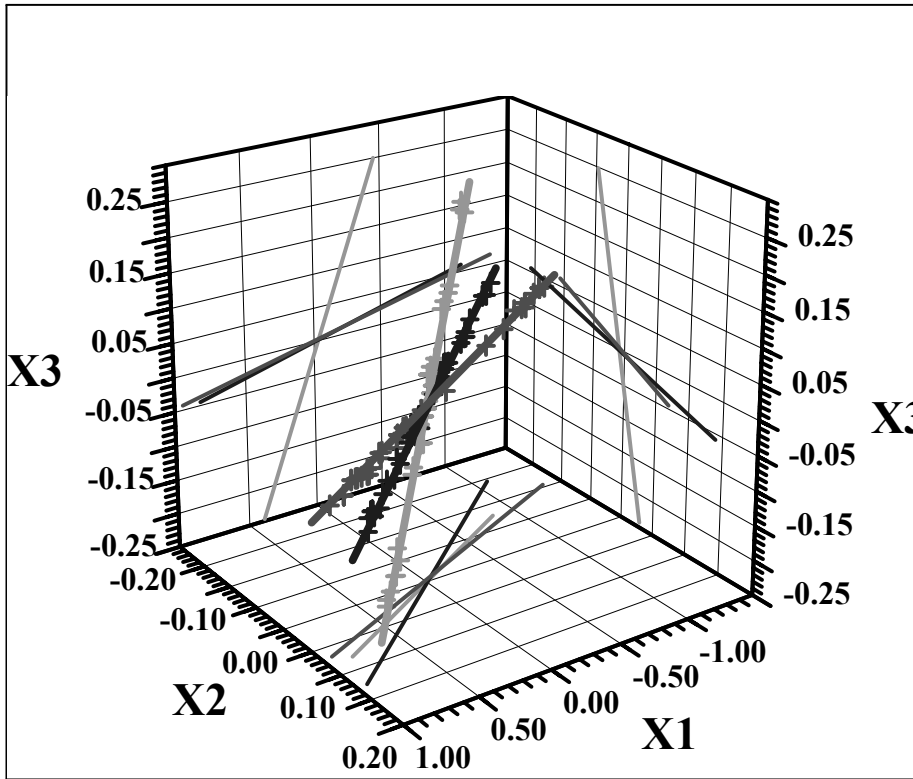


Fig. 12. Indonesia 3D CLS Plot

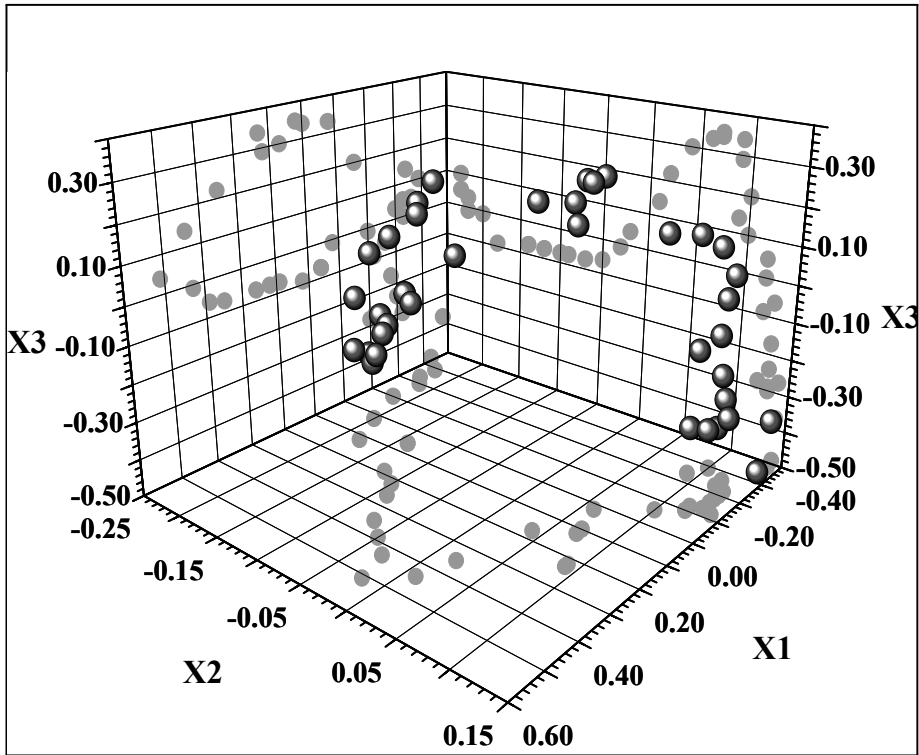


Fig. 13. Japan 3D Scatterplot

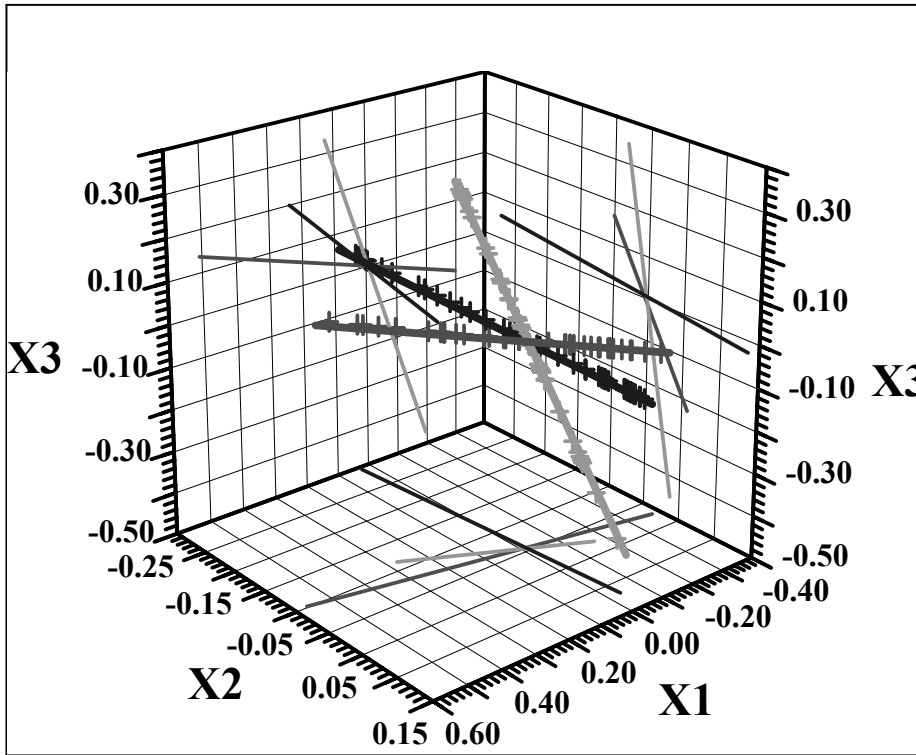


Fig. 14. Japan 3D CLS Plot

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