

# The Forward Premium Puzzle in a Model of Imperfect Information: Theory and Evidence

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## Abstract

This paper studies the forward premium puzzle in an environment where private agents do not perfectly observe the shocks that drive monetary policy. Private agents optimally update their conditional expectations by means of the Kalman filter. The transition dynamics associated with Kalman filtering lead to fixed time-effects and conditional heteroskedasticity in the forward premium regression. I provide evidence for the presence of time-effects in the forward premium regression and find that the forward premium puzzle is significantly weakened. In particular, a 1 percent increase in the 1-month interest differential is expected to be accompanied by an additional 0.34 percent depreciation of the currency in the following month.

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# 1 Introduction

The forward premium puzzle is the empirical finding that the forward exchange rate is a biased predictor of the future spot exchange rate.<sup>1</sup> Numerous studies have attempted, with limited success, to produce models in which this forward bias is a consequence of risk premia in exchange rate markets.<sup>2</sup> This paper examines the implications for the forward premium of assuming that private agents have imperfect information about the shocks that buffet the economy. As in most of the literature I use Lucas' (1982) model as the backdrop for the investigation. Agents have rational expectations and solve a signal extraction problem to infer the underlying shocks to the economy from the signals they receive.<sup>3</sup> They employ the Kalman filter to optimally update their conditional expectations about the state of the economy.

A simple example of an economy in which there is a signal extraction problem is one in which shocks to monetary policy follow a moving average process whose innovations are not publicly observed. This example, which I study in detail, has implications that are similar to those of environments in which there are signal extraction problems with respect to other variables (e.g. dividend flows, government spending) or in which there are interactions between monetary policy at home and abroad that are imperfectly observed. It is also a realistic assumption in light of the work of Christiano, Eichenbaum, and Evans (1998) who argue that the growth rates of the monetary base and M1 for the US can be well approximated by moving averages of order 2.

The main theoretical result of the paper is that the information structure of the model implies conditionally heteroskedastic forecast errors and fixed time-effects in the forward premium regression. These features arise exclusively due to the signal extraction problem and do not require the existence of a positive risk premium. Hence, the paper provides a rationale for the empirical findings in Mayfield and Murphy (1992). Mayfield and Murphy (1992) justified introducing fixed time-effects in the forward premium regression based on the existence of a deterministic time varying risk premium. This, however, is inappropriate since the risk premium is a stochastic variable in theoretical models (e.g. Hodrick and Srivastava (1984)). Complementing the results in Mayfield and Murphy, I estimate the model using instrumental variables and panel data regressions with fixed time-effects. Both procedures bring the data closer to the expectations hypothesis than previous results in the literature have suggested. In contrast to Mayfield and Murphy,

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<sup>1</sup>For surveys see Hodrick (1987), Lewis (1995), and Engel (1995).

<sup>2</sup>Models of the risk premium with and without complete asset markets have been studied in conjunction with conditional heteroskedasticity of the forcing variables, and non-standard preferences (displaying features such as habit persistence, or first-order risk aversion). Examples are Hodrick (1989), Bekaert and Hodrick (1993), Canova and Marrinan (1993), Telmer (1993), Bekaert (1993, 1996), Bekaert, Hodrick, and Marshall (1997). The most successful study by far is the last one cited which reports a theoretical slope coefficient of 0.8 in the standard forward premium regression. In the context of models of real exchange rate determination Serrat (1997) provides an explanation of the forward premium puzzle based on the consumption of nontradables.

<sup>3</sup>This differs from the 'signal extraction' problem associated with learning about regime changes.

in my results time-effects alone generate the improvement in the estimation of the slope parameter in the forward premium regression.

The paper is organized as follows. Section 2 outlines the theoretical model. Section 3 presents some results concerning the signal extraction problem, and discusses the theoretical implications of the model for the forward premium regression. Section 4 evaluates empirically the main prediction of the model and section 5 concludes. The appendix contains a description of the data set.

## 2 The Model

I use the two-country model with national currencies of Lucas (1982) augmented to incorporate imperfect information about the economy's underlying shocks. This model is the work horse of the literature on time-varying exchange rate risk premia, which facilitates the comparison of my results with previous work. The basic elements of the model are well known so I omit the description of the problems solved by each of the two representative agents to conserve on space and notation.

From the Lucas model I borrow the pricing formulas for the depreciation rate occurring at time  $t + 1$ ,  $ds_{t+1}$ , and for the forward premium,  $fp_t$ , associated with a contract maturing at time  $t + 1$ .<sup>4</sup> Recall that a forward contract is an obligation to trade currencies at a future date at prices and quantities chosen when the contract is written. Here, the spot rate is measured in units of domestic currency per units of foreign currency and a positive  $ds_{t+1}$  is a depreciation of the domestic currency. In the perfect pooling equilibrium of the model these are given by the expressions:

$$fp_t = \ln[E_t(\mu_{Y_{Mt+1}}^{1-\gamma}\mu_{M_{t+1}}^{-1})^{-1}E_t(\mu_{Y_{Nt+1}}^{1-\delta}\mu_{N_{t+1}}^{-1})] \quad (1)$$

$$ds_{t+1} = \ln[(\mu_{Y_{Mt+1}}^{1-\gamma}\mu_{M_{t+1}}^{-1})^{-1}\mu_{Y_{Nt+1}}^{1-\delta}\mu_{N_{t+1}}^{-1}]. \quad (2)$$

The notation and the assumptions on the utility function are as follows. The variables of the domestic country are indexed by  $M$ , and those of the foreign country by  $N$ . Momentary utility is assumed to be separable in the domestic and foreign good, and to display constant relative risk aversion, with  $\gamma$  and  $\delta$  being the coefficients of relative risk aversion associated with each good. The variables  $\mu_{Y_i}$ , and  $\mu_i$  are the gross growth rates of output and money in country  $i$ . Finally,  $E_t$  is the time  $t$  expectation operator.

For simplicity of presentation I restrict attention to the case in which  $\mu_M$  and  $\mu_N$  are stochastic and assume that output is constant ( $\mu_{Y_i} = 1$ ). The later simplification helps to make clear that my discussion does not rely on the existence of a risk premium, so that I can set  $\gamma = \delta = 0$ .

### *Domestic and Foreign Money Supply Growth*

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<sup>4</sup>See Lucas' equation (4.5) for the nominal exchange rate, and equations (4.17) and (4.18) for the nominal interest rates, and the fact that covered interest parity holds in this model.

Money growth is subject to both persistent shocks (with an innovation  $\epsilon_i$ ) and to transitory shocks ( $\nu_i$ ). Transitory shocks to country  $i$ 's money supply do not affect (but may be correlated to) country  $j$ 's current money growth. Country  $j$ 's money supply can potentially respond to persistent disturbances to money growth in country  $i$ . The process describing money creation in country  $M$  is:

$$\ln \mu_{Mt} = \ln \mu_M + h_{M0}\epsilon_{Mt} + h_{M1}\epsilon_{Mt-1} + h_{M2}\epsilon_{Mt-2} + \nu_{Mt}, \quad (3)$$

where  $\ln \mu_M$  is the mean growth rate of money. I assume that country  $N$  follows a similar monetary policy rule. This money growth rule is motivated by the work of Christiano, Eichenbaum, and Evans (1998). These authors argue that the stochastic processes for the growth rates of the monetary base and M1 for the US can be well approximated by moving averages of order 2. The leading assumption of the paper is that:

**Assumption A:** The shocks  $\epsilon_{it}, \nu_{it}$  cannot be perfectly inferred from the signals  $\mu_{it}$ ,  $i = M, N$ . Except for the shocks  $\epsilon_{it}, \nu_{it}$ , all other information is common knowledge.

Assumption A characterizes the information structure of this economy. It implies that the representative agent has to infer the underlying shocks  $\epsilon_{it}, \nu_{it}$  from the sequence of signals  $\{\mu_{js}\}_{s=1}^t$ . Since information is symmetric all private agents receive the same signals, and use the same information to update the conditional distribution function needed to evaluate (1). The central banks have superior information in that they observe the shocks  $\{\epsilon_{js}, \nu_{it}\}_{s=1}^t$ . In practice it is very hard to identify shocks to monetary policy. I view the efforts of the empirical literature in the identification of monetary shocks as attesting to the plausibility of assumption A (surveys to the literature include Canova (1995), Leeper, Sims, and Zha (1996) and Christiano, Eichenbaum and Evans (1999)).

### 3 The Signal Extraction Problem

In this section I show how agents construct and update their conditional expectations given assumption A. Because of imperfect information agents are unaware of the nature or magnitude of the shocks that drive monetary policy. For this reason they will potentially make incorrect assessments about the conditional distributions of future money growth.<sup>5</sup> In deriving the conditional distribution of future growth rates of money agents use all the information available in a way that is consistent with the rational expectations paradigm.

#### 3.1 Modeling Signal Extraction

Agents filter information by means of the Kalman filter (see Hamilton (1994, Chapter 13) and Anderson and Moore (1979)). The Kalman filtering technique is especially useful

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<sup>5</sup>Other models of signal extraction include, for example, economies in which agents learn about regime changes (see Lewis (1989)), or about permanent versus transitory shocks (see Gourinchas and Tornell (2003)).

because of its recursive representation. To obtain this recursive representation rewrite (3) with the more general notation:

$$y_t = x + H'\xi_t + \nu_t,$$

with

$$H' = \begin{bmatrix} h_{M0} & h_{M1} & h_{M2} & 0 & 0 & 0 \\ 0 & 0 & 0 & h_{N0} & h_{N1} & h_{N2} \end{bmatrix},$$

$$\xi_t' = \begin{bmatrix} \epsilon_{Mt} & \epsilon_{Mt-1} & \epsilon_{Mt-2} & \epsilon_{Nt} & \epsilon_{Nt-1} & \epsilon_{Nt-2} \end{bmatrix},$$

$$\nu_t' = \begin{bmatrix} \nu_{Mt} & \nu_{Nt} \end{bmatrix},$$

$$x = \begin{bmatrix} \ln \mu_M \\ \ln \mu_N \end{bmatrix}, \quad y_t = \begin{bmatrix} \ln \mu_{Mt} \\ \ln \mu_{Nt} \end{bmatrix}$$

and

$$\xi_{t+1} = F\xi_t + \eta_{t+1},$$

$$\eta_{t+1} = \begin{bmatrix} \epsilon_{Mt+1} & 0 & 0 & \epsilon_{Nt+1} & 0 & 0 \end{bmatrix}',$$

where  $F$  is defined implicitly and has all its eigenvalues inside the unit circle, and primes denote transposition. The innovations in the economy obey the following restrictions: (i)  $(\nu_{M,t+1}, \nu_{N,t+1}) \sim N(0, R)$ ,  $R$  is positive definite, and  $\nu_t$  is uncorrelated with  $\nu_\tau$ ,  $\tau \neq t$ ; (ii)  $(\epsilon_{M,t+1}, \epsilon_{N,t+1}) \sim N(0, \tilde{Q})$ ,  $\tilde{Q}$  is related to  $Q = E(\eta_t \eta_t')$  in an obvious way, and  $\eta_t$  is uncorrelated with  $\eta_\tau$ ,  $\tau \neq t$ .

Let  $\Omega_t$  be the information set at time  $t$ . All relevant information to the agents as of time  $t$  is contained in  $\Omega_t$ . The information set  $\Omega_t$  is the  $\sigma$ -algebra generated by the random variables  $\{(\mu_{j,i})_{i=1}^t, j = M, N\}$ . The following system of equations gives the expectation of  $y_{t+1}$  conditional on  $\Omega_t$  ( $\hat{y}_{t+1|t}$ ):

$$\hat{y}_{t+1|t} = x + H'\hat{\xi}_{t+1|t}, \quad (4)$$

$$\hat{\xi}_{t+1|t} = F\hat{\xi}_{t|t-1} + FP_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - x - H'\hat{\xi}_{t|t-1}). \quad (5)$$

The matrix  $P_{t|t-1}$  is the conditional covariance matrix of the forecast errors and is updated using the Riccati equation:

$$P_{t+1|t} = F \left[ P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}H'P_{t|t-1} \right] F' + Q. \quad (6)$$

The initial conditions for this process are given by a positive semi-definite symmetric matrix  $P_{t_0|t_0-1}$  for some  $t_0 < t$ . Notice that the sequence  $\{P_{t|t-1}\}_t$  does not depend on any particular realizations of the signals, but only on  $t - t_0$ , and the fixed matrices  $H$ ,  $F$ ,  $R$ ,  $Q$ , and  $P_{t_0|t_0-1}$ .<sup>6</sup>

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<sup>6</sup>Hamilton (1994, Proposition 13.2) shows that the sequence  $\{P_{t|t-1}\}_{t=t_0}^\infty$  has a unique limit  $P$ , which is positive definite, and for any real vector  $a$ ,  $a'P_{t|t-1}a \downarrow a'Pa$ .

Hamilton (1994, Chapter 13) shows that the forecast errors behave according to:

$$H'\xi_{t+1}|\Omega_t \sim N\left(H'\hat{\xi}_{t+1|t}, H'P_{t+1|t}H\right). \quad (7)$$

Under rational expectations the forecast errors of  $y_{t+1}$ ,  $y_{t+1} - \hat{y}_{t+1|t} = H'(\xi_{t+1} - \hat{\xi}_{t+1|t}) + \nu_{t+1}$ , are uncorrelated over time.

### 3.2 Theoretical Properties of the Forward Premium Regression

What are the implications of the signal extraction problem analyzed here for the forward premium regression? The next proposition gives the answer to this question.

**Proposition 1** *The forward premium regression in the model reads:*

$$ds_{t+1} = \varphi_t + fp_t + \zeta_{t+1}, \quad (8)$$

where  $\zeta_{t+1} = [1, -1] \left( H' \left( \xi_{t+1} - \hat{\xi}_{t+1|t} \right) + \nu_t \right)$ , with its implied conditional probability density function obtained from (7),  $\varphi_t = \frac{1}{2} (\sigma_{Mt}^2 - \sigma_{Nt}^2)$ , and  $\sigma_{Mt}^2$ , and  $\sigma_{Nt}^2$ , are the first and second diagonal elements of  $H'P_{t+1|t}H + R$ , respectively.

**Proof.** From (7) write

$$\ln \mu_{Mt+1}|\Omega_t \sim N\left(\ln \mu_{Mt+1|t}, \sigma_{Mt}^2\right),$$

with  $\ln \mu_{Mt+1|t} = \ln \mu_M + [1 \ 0]H'\hat{\xi}_{t+1|t}$ , and  $\sigma_{Mt}^2 = [1 \ 0](H'P_{t+1|t}H + R)[1 \ 0]'$ , and similarly for  $\ln \mu_{Nt+1}$ . Thus,

$$\begin{aligned} fp_t &= \ln \mu_{Mt+1|t} - \ln \mu_{Nt+1|t} - \frac{1}{2}\sigma_{Mt}^2 + \frac{1}{2}\sigma_{Nt}^2 \\ &= \ln \mu_M - \ln \mu_N + [1, -1]H'\hat{\xi}_{t+1|t} - \frac{1}{2}\sigma_{Mt}^2 + \frac{1}{2}\sigma_{Nt}^2 \end{aligned}$$

and

$$\begin{aligned} ds_{t+1} &= \ln \mu_{Mt+1} - \ln \mu_{Nt+1} \\ &= \ln \mu_M - \ln \mu_N + [1, -1] \left( H'\xi_{t+1} + \nu_t \right) \end{aligned}$$

using (3). Combining these expressions, equation (1) becomes:

$$ds_{t+1} = \frac{1}{2} (\sigma_{Mt}^2 - \sigma_{Nt}^2) + fp_t + [1, -1] \left( H' \left( \xi_{t+1} - \hat{\xi}_{t+1|t} \right) + \nu_t \right),$$

proving the result. ■

The forward premium regression contains a deterministic sequence ( $\varphi_t$ ) which could in principle be a highly nonlinear function of time. Estimation procedures that fail to take this into account are misspecified because  $\varphi_t$  is correlated with the forward premium. This is the main prediction of the model that I highlight next.<sup>7</sup>

**Implication 1:** The forward premium regression includes a deterministic time dependent variable.

The absence of perfect information alone justifies the presence of the time-effects  $\varphi_t$  in the forward premium regression. To see this note that the true risk premium in the model is zero. To make this point clear I have fixed the endowments over time and set the coefficient of relative risk aversion to zero. On the other hand, if it weren't for the signal extraction problems the matrix  $P_{t|t-1} = H'QH$  and hence  $\sigma_{Mt}^2$ , and  $\sigma_{Nt}^2$  would be constant over time. Further, note that  $\varphi_t$  is constant if the steady state of the Kalman filter  $P$  has been achieved. However,  $\varphi_t$  would not reach the steady state and would still be a deterministic function of time in a more general model with non-anticipated regime switches which occur at finite dates and are known to agents perfectly once they occur.

The omission of  $\varphi_t$  from the forward premium regression could help explain the parameter instability of the estimates of the slope of the forward premium that have been reported in the literature. This parameter instability is best documented by the wide variability of the estimates of the coefficient associated with  $fp_t$  in the forward premium regression (equation (9) below). Specific tests of parameter instability have also been conducted. Bachman (1992) analyzes the effects of political risk by examining the effects of election results on the forward premium puzzle. He runs separate regressions for each election and finds that for half of the elections in his sample the hypothesis of coefficient stability is rejected. Bekaert and Hodrick (1993) also show evidence of parameter instability by estimating a model of multiple regime switching for the bivariate system of the depreciation rate and the forward premium. If, as I argue, the finding of parameter instability is due to the forward premium being correlated with the variable  $\varphi_t$ , use of ordinary least squares (OLS) in econometric models which omit  $\varphi_t$  is bound to produce biased estimates.

A second prediction of the model is that the forecast errors in the forward premium regression (8) display conditional heteroskedasticity. This prediction has wide empirical support (see, for example, Cumby and Obstfeld (1984)). As I have shown in the previous subsection this feature follows immediately as a consequence of the Kalman filter and does not hinge on explicitly assuming conditional heteroskedasticity in the driving stochastic processes of the economy—in the model economy the conditional variance of the rate of money growth is constant if the shocks  $\epsilon_{it}, \nu_{it}$  are public information. Traditionally in the literature conditional heteroskedasticity is the outcome of imposing conditional

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<sup>7</sup>In a previous version of the paper I also analyzed empirically the prediction that  $\varphi_t$  approaches a unique steady state over time. The potential for this model to generate strange behavior in the forward premium estimates is increased if  $\{P_{t|t-1}\}$  is a non-convergent sequence.

heteroskedasticity in the processes for money creation, output growth, or government expenditures (see Domowitz and Hakkio (1985), Hodrick (1989), and Canova and Marrinan (1993)). Obstfeld (1987) argues that the peso problem can also give rise to conditional heteroskedasticity in the forecast errors. In contrast with other models of imperfect information with learning (see Stulz (1987), Obstfeld (1987), and Lewis (1989)) the errors in the theoretical model are not autocorrelated.

## 4 Estimating The Forward Premium Regression

Using exchange rate data for nine countries (see the appendix for a description of the data), estimation of

$$ds_{t+1} = \alpha_0 + \alpha_1 fp_t + \omega_{t+1} \quad (9)$$

with OLS produces  $\alpha_1$  negative with the  $R^2$  of these regressions quite low (results available from the author upon request). Implication 1 suggests that these regressions are flawed because the variable  $\varphi_t$  has been omitted. I follow two alternative approaches to incorporate time variation in  $\varphi_t$ . First, I use instrumental variables. Second, I directly estimate time fixed-effects.

### 4.1 Estimating the Forward Premium Regression Using Instrumental Variables

The choice of instruments is generally difficult. I use lagged values of the forward premia and interest rates. These instruments are valid only if they are orthogonal to  $\varphi_t$ . Because  $\varphi_t$  may display serial correlation induced by the Kalman filter into  $P_{t|t-1}$ , I expect that using long lags of the instrument eliminates any systematic relation between  $\varphi_t$  and the instruments.

Table I reports the results from estimating equation (9) with two-stage least squares. The set of instruments consisting of  $k$ -months lagged forward premia is labeled ‘Set 1’. ‘Set 2’ includes the  $k$ -months ahead forward premium at time  $t + 1 - k$  as an instrument for the 1-month ahead forward premium at time  $t$ . These are forward contracts with longer periods maturing at the same time as the current forward contract. I consider  $k = 3, 6$ , and 12. Finally, ‘Set 3’ of instruments includes the  $k$ -months ahead interest rate in country  $N$ , and, as a separate instrument, the  $k$ -months ahead interest rate in the US both lagged  $k$  periods. Set 2 is therefore equivalent to Set 3 once one imposes the restriction of covered interest parity, but Set 3 was used with the intention of trying to capture some additional information in the yield curve. A similar choice of instruments is used in Korajczyk (1985).

Table I shows a wide dispersion in coefficient estimates with some of them being bigger than 1. Most coefficients are statistically insignificant, in particular those whose value is bigger than one.<sup>8</sup> Also, the sign of the estimates is robust to the choice of

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<sup>8</sup>Standard errors are corrected using the Newey-West (1987) procedure to account for conditional

instruments for each country. Figure 1 displays an histogram of the absolute frequency of the entries from Table I. This histogram displays slope estimates more frequently positive and greater than 1 than usually found in the literature (see Froot and Thaler (1990)). Given the large number of positive slopes the histogram resembles what one might expect from a small sample (see Lewis (1988)).

To better assess the performance of my specification I increase the sample size by estimating a pooled version of the regression model. Table II presents the results of a pooled regression estimated with three-stage least squares to account for potential cross-correlation in the residuals; this cross-correlation is likely to be an issue since all exchange rates are priced with respect to the US Dollar. In the pooled regression results I estimate the system with, and without, the restriction that the constant term be identical across countries. I also experiment with excluding the Belgium Franc which is a clear outlier. This yields similar estimates of  $\alpha_1$  to the case in which I allow for a different constant only for the Belgium Franc (not reported). The estimates in Table II are almost all positive as well as statistically significant and different from 1. The average of the estimates in the first two panels is 0.37, and the average in the third panel is 0.65.<sup>9</sup>

## 4.2 Panel Data Estimation with Time Fixed-Effects

In this subsection I estimate the time-fixed effect directly with time dummies. I estimate the model in a panel of currencies, due to the lack of degrees of freedom. Table III presents the results with and without country and time fixed effects for forward contracts with duration  $\iota = 1, 3, 6,$  and 12 months. For each of the cases I eliminate the contract-overlapping observations. I use Feasible Generalized Least Squares estimation to account for conditional heteroskedasticity and within panel cross-correlations of the residuals. Correction for heteroskedasticity on its own increases both the magnitude of the estimates, and their significance, but these estimates still do not account for the presence of the time-effects and should be biased downwards. In fact, by introducing time fixed-effects the estimated  $\alpha_1$  increases with its value being positive and statistically significant although still somewhat below unity. The average estimate of  $\alpha_1$  with time effects alone is 0.57 (line 4 in Table III) and  $\hat{\alpha}_1 = 0.34$  for the time fixed-effects regression with 1-month forward contracts, which is somewhat more comparable to the results in Tables I and II. Finally, country effects lower the estimates of  $\alpha_1$  when introduced additively. My results thus complement those in Mayfield and Murphy (1992) who use a much smaller data set. They show that estimates of  $\alpha_1$  become positive in the regression of the  $k$ -month depreciation rate on the  $k$ -month forward premium, with

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heteroskedasticity and autoregressive errors as I ignore in the empirical specification the presence of  $\varphi_t$ .

<sup>9</sup>Korajczyk (1985) also runs three-stage least squares of the forecast error on the real interest differential and on lagged forecast errors. His results, like mine, imply a weakening of the forward premium puzzle. Note however that seemingly unrelated regression estimates typically yield negative estimates for  $\alpha$  (e.g. Flood and Rose (1995)).

$k = 3, 6$ , when using time *and* country effects.<sup>10</sup>

## 5 Conclusion

This paper examines the implications of modeling signal extraction in the context of fixed monetary policy rules for the analysis of the forward premium regression. The model's predictions are that the forward premium regression should account for time varying fixed effects, and, as suggested by vast empirical evidence, for conditional heteroskedasticity of the errors. These properties do not require a risk premium but rely only on the transition dynamics of the Kalman filter.

When I estimate the model I find that a 1 percent increase in the 1-month forward premium is expected to be accompanied by an additional 0.34 percent depreciation of the currency in the next month (over periods of 3, 6, and 12 months, the elasticities are .58, .73, and .62, respectively). I view the empirical results on the estimated time effects from these regressions as providing evidence in favor of the signal extraction hypothesis formulated here.

The estimates of  $\alpha_1$  (from line 4 in Table III) are significantly different than 0 *and* 1. Therefore, the theoretical model cannot be viewed as providing a complete resolution of the forward premium puzzle. The results suggest that understanding the origins of a time-varying risk-premium is still necessary if one wishes to understand why estimates of  $\alpha_1$  tend to lie between 0 and 1. Marston (1997) finds that both the risk premium, and systematic forecast errors account for deviations from uncovered interest parity. However, Frankel and Froot (1989), using survey data, have shown that the component of the forward premium puzzle explained by the risk premium has a tight upper-bound. Recent work points to the importance of interest rate and exchange rate smoothing by monetary authorities in explaining the forward premium puzzle (McCallum (1994)) and to extreme sampling (Lothian and Wu (2003) and Sercu and Vandebroek (2003)).

## 6 Appendix: Data

The data set is borrowed from Backus, Foresi, and Telmer (1996). There are 9 spot and forward (1, 3, 6, and 12 month) exchange rates vis-à-vis the US Dollar: Canadian Dollar, Belgium Franc, British Pound, French Franc, German Marc, Italian Lira, Japanese Yen, Netherlands Guilder, and Swiss Franc. The observations are taken from the last Friday of the month from July 1974 through November 1994. Missing observations for the 3, 6, and 12 month forward rates of the Italian Lira were constructed assuming covered interest parity (8 observations in total). A missing observation for the 12-month forward

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<sup>10</sup>Schotman, Straetmans, and de Vries (1997) introduce time effects *multiplicatively* through a time-varying slope parameter of the forward premium on the forward premium regression and find an average estimate of  $\alpha_1$  equal to 0.45.

rate of the Yen in Aug/74 was extrapolated using the adjacent observations. The results are not sensitive to the inclusion of the observations for which I had to extrapolate.

To construct ‘set 3’ of instruments in Tables II and III I replaced all missing observations from the interest series using Datastream’s data on Eurodeposits. The following 9 observations were still missing and were replaced by extrapolating using the adjacent observations: the 3-month interest rate for the Belgium Franc on Nov/75 and Dec/80, and the 1 year interest rate on May/78; the 3, 6, and 12 month interest rate for the Italian Lira on Nov/75, and Nov/76.

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Table I. Estimates of the Slope of the Forward Premium Regression by Instrumental Variables

$ds_{t+1} = \alpha_0 + \alpha_1 fp_t + w_{t+1}$				
X	k	Estimates of $\alpha_1$		
		Set 1	Set 2	Set 3
British P.	3	-1.2102 (1.0355)	-1.3569 (1.0658)	-1.4594 (1.0791)
	6	-1.3713 (1.5364)	-0.79 (1.509)	-0.8006 (1.5218)
	12	-1.229 (3.3794)	-1.0145 (2.7034)	-1.3483 (2.5889)
Can Dollar	3	-0.4384 (0.7557)	-0.5393 (0.7316)	-0.7511 (0.7309)
	6	0.3173 (1.0089)	0.2474 (0.9342)	0.1311 (0.9473)
	12	2.12 (2.5766)	0.347 (1.6)	-0.6125 (1.4272)
FFranc	3	0.8375 (1.5427)	0.3796 (1.2517)	0.6062 (1.2621)
	6	-0.3851 (2.198)	0.6264 (1.7722)	0.7059 (2.0243)
	12	1.6795 (2.6911)	2.6567 (3.4046)	3.0135 (3.9085)
DM	3	-0.0804 (0.8746)	-0.0811 (0.8845)	-0.075 (0.8813)
	6	-0.4225 (1.0152)	-0.2332 (0.9451)	-0.0985 (0.9482)
	12	-0.4024 (0.9543)	0.0114 (0.9116)	0.3179 (0.9067)
Lira	3	0.6421 (0.5859)	2.0288 (1.3688)	0.8139 (0.6061)
	6	1.558 (1.388)	2.2567 (1.5218)	2.3922 (1.5184)
	12	0.8391 (1.8357)	1.4641 (2.1476)	2.7443 (3.2004)
Swz F	3	-0.7777 (0.8761)	-0.7208 (0.8651)	-0.6977 (0.8636)
	6	-0.8215 (0.9185)	-0.615 (0.9005)	-0.5503 (0.8932)
	12	-0.6078 (0.9664)	-0.5448 (0.9533)	-0.3409 (0.9473)
Yen	3	-2.0678 (0.8656)	-2.1432 (0.8662)	-2.4795 (0.9656)
	6	-2.4091 (1.2088)	-2.4278 (1.1349)	-2.9208 (1.244)
	12	-2.9352 (1.669)	-3.3519 (1.5589)	-4.3504 (1.8143)
BelgF	3	0.7946 (1.2408)	0.3795 (1.2409)	1.1371 (1.4963)
	6	0.8064 (2.326)	0.3769 (2.1425)	0.649 (2.2403)
	12	-1.2111 (4.6199)	0.0762 (3.6931)	-0.437 (4.895)
Neth G	3	-0.8295 (0.9235)	-0.8459 (0.9448)	-0.9493 (0.9421)
	6	-0.9732 (1.1865)	-0.7161 (1.1014)	-1.021 (1.0965)
	12	-0.7924 (1.2528)	0.0135 (1.2088)	-0.8336 (1.1559)

Notes: Entries are 2SLS estimates. Set 1={constant, one period forward premium at time  $t+1-k$ }, and Set 2={constant, k-month forward premium lagged k periods}. Set 3={constant,  $\ln(1+US$  interest rate for k months) lagged k periods,  $\ln(1+N$ 's interest rate for k months) lagged k periods}. The number of observations is 243, 240, and 234, for  $k=3, 6$ , and 12, respectively. For Japan, with 'set 3' the no. of obs. is 228, 225, 219. All standard errors are corrected by the Newey-West method.

Table II. Pooled Estimates of the Slope of the Forward Premium Regression by Instrumental Variables

$ds_{t+1} = \alpha_0 + \alpha_1 fp_t + w_{t+1}$				
Estimates of $\alpha_1$				
	k	Set 1	Set 2	Set 3
All countries (unrestricted $\alpha_0$ )	3	0.13 (0.135)	0.1343 (0.1579)	0.2061 (0.1488)
	6	0.0781 (0.1674)	0.3295 (0.1891)	0.3289 (0.1688)
	12	-0.0489 (0.1806)	0.2177 (0.1969)	0.0844 (0.168)
All countries (restricted $\alpha_0$ )	3	0.5305 (0.089)	0.4659 (0.1039)	0.5368 (0.0922)
	6	0.6133 (0.0954)	0.5931 (0.1093)	0.6084 (0.0956)
	12	0.6614 (0.0984)	0.607 (0.1105)	0.6059 (0.0963)
All countries but Belgium (restricted $\alpha_0$ )	3	0.5479 (0.1088)	0.4714 (0.131)	0.5768 (0.1127)
	6	0.6506 (0.1176)	0.6563 (0.1418)	0.6918 (0.1184)
	12	0.7753 (0.1214)	0.7455 (0.1447)	0.7108 (0.1194)

Notes: Entries are 3SLS estimates, and standard errors are not corrected. Set 1= {constant, one period forward premium at time  $t+1-k$ }, and Set 2={constant,  $\ln((1+US \text{ int. rate } k/(1+X \text{ int. rate } k))$  both lagged  $k$  periods)}. Set 3={constant,  $\ln(1+US \text{ interest rate for } k \text{ months})$  lagged  $k$  periods,  $\ln(1+N's \text{ interest rate for } k \text{ months})$  lagged  $k$  periods}. For sets 1 and 2 the number of observations is 2187, 2160, 2106, for  $k=3, 6$ , and 12, respectively. For set 3 the no. of obs. is 2172, 2145, and 2091, for  $k=3, 6$ , and 12. When we exclude the BFranc we remove 243,240, and 234 obs (in all sets).

Table III. Pooled Estimates of the Slope of the Forward Premium Regression with Fixed Effects

		$ds_{t+1} = \alpha_0 + \alpha_1 fp_t + w_{t+1}$			
		Estimates of $\alpha_1$			
		Duration of the Forward Contract (k=months)*			
		1	3	6	12
All countries	Stacked OLS	-0.1722 (0.1992)	0.2389 (0.2649)	0.4019 (0.2708)	0.2951 (0.2773)
	FGLS correction (restricted $\alpha_0$ )	0.1154 (0.0814)	0.4207 (0.0915)	0.5073 (0.0939)	0.2829 (0.1015)
	Country effects (FGLS)	-0.2626 (0.1054)	0.1037 (0.1287)	0.1428 (0.1424)	-0.4257 (0.0977)
	Time effects (FGLS)	0.3354 (0.0834)	0.5772 (0.093)	0.7337 (0.1023)	0.618 (0.1148)
	Time & country effects (FGLS)	-0.0242 (0.1199)	0.4051 (0.1383)	0.7499 (0.1732)	0.2996 (0.2031)
Excluding Belgium	Time effects and FGLS	0.4124 (0.108)	0.5681 (0.108)	0.7274 (0.1197)	0.5738 (0.1358)

Notes: The number of obs is: 2205, 729, 360, 171, for k=1,3,6,12, since overlapping observations were eliminated. When we exclude the BFranc we remove 245,81,40, 19 obs. FGLS corrects for conditional heteroskedasticity and correlation within panels. Standard errors for the stacked OLS regressions are corrected using Newey-West's procedure. For k=3, the data runs from 10/74-10/94, for k=6, the data runs from 1/75-7/94, and for k=12, the data runs from 1/76-1/94.

(\*) The time period t, is given by the length of the forward contracts.

Figure 1. Histogram of entries in Table I.

