Open Capital Account: Concrete Wealth or Paper Wealth*

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Abstract

Empirical evidence shows that capital inflows are often used by developing countries to finance excessive consumption. The existing literature explains these phenomena as resulting from institutional imperfections. In contrast, we argue that they can be fundamental outcomes of open capital account, under which ineffectiveness in using foreign savings for investments tends to result in capital inflows being channeled to consumption through wealth effect. Our analysis shows that, while risk aversion causes low investment elasticity and hence reduces the total benefit of capital account liberalization for society over time, it nevertheless tends to increase the benefit enjoyed by current generations and hence drive consumption booms. We show that the proportion of capital inflows used for financing consumption is negatively correlated with investment elasticity. We show that a positive yet uncertain future productivity shock is likely to cause consumption booms because of sluggish investment reactions. Our analysis shows that, the greater the expected future productivity is; or the greater the uncertainty is; the stronger the consumption booms will be. (JEL F21 F32 F41 F43)

Keywords: open capital account; wealth effect; consumption boom; investment elasticity.

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1. Introduction

One major motivation for developing countries to open capital account is to let free capital inflows facilitate domestic capital formation (Calvo et al. 1996). However, empirical evidence shows that unfettered (net) capital inflows are often used by developing country recipients to finance excessive consumption (Calvo et al., 1996; Ffrench-Davis and Reisen, 1998; Galvin et al., 1997). We call such saving-crowd-out phenomena as “foreign-capital-financed” consumption booms, which are destabilizing in the short run and detrimental to national wealth accumulation in the long run. Particularly, foreign-capital-financed consumption booms tend to happen in countries undergoing macroeconomic reforms such as exchange-rate-based (ERB) stabilization programs (Montiel, 2000; Nazmi, 1997).

In explaining foreign-capital-financed consumption booms, the existing literature focuses on how “institutional imperfections” in developing countries can trigger excessive consumption demands, which with the aid of free capital inflows will easily turn into consumption booms. One such imperfection is credit overexpansion (due to moral hazard) by underdeveloped and ill-regulated financial systems, which (by itself and through its impacts) tends to cause consumption booms (McKinnon and Pill, 1998; Reisen, 1998). Another imperfection is inconsistent or incredible government policies—consumption booms can be the result of inconsistency between monetary (fiscal) policies and ERB (stabilization) programs (Helpman and Razin, 1987); besides, incredible (or perceived as temporary) ERB programs per se can cause consumption booms (Calvo, 1986; Dornbush, 1985). Furthermore, ERB programs failing to stabilize inflation quick

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1 Capital inflow recipients in this paper are small developing countries; and capital inflows are net inflows.
enough (due to of inertia in expectations) can also lead to consumption booms (Rodriguez, 1982).

In summary, although open capital account certainly plays a role in financing consumption booms, the existing literature generally views consumption booms as a macroeconomic “side” effect of open capital account due to institutional imperfections.

In contrast, this paper investigates a wealth effect mechanism by which consumption booms can be a fundamental outcome of open capital account.

The essence of the mechanism is as follows. Open capital account tends to attract foreign capitals into developing countries for high-yielding opportunities. Such capital inflows can increase the stock of productive capital ($K$), its price (Tobin’s $q$), or most likely both. While the increase in $K$ is the result of capital inflows being “properly” channeled to investments, the $q$ appreciation tends to stimulate consumption demand (through wealth effect) and hence essentially channel capital inflows to consumption.

A key yet underappreciated point is that the magnitude of $q$ appreciation and hence the corresponding consumption booms are negatively correlated to the $q$-elasticity of investments. Thus, a conjecture is that, when investment “impediments” make it difficult to turn foreign savings into investments (as “concrete” wealth), “paper” wealth will nevertheless be created (through asset price appreciation) and result in foreign savings being channeled to consumption.

The remainder of the paper attempts to examine this conjecture and its implications. Section 2 analyzes the influence of investment elasticity on the aftermath of capital account liberalization. Section 3 examines foreign-capital-financed consumption booms driven by productivity shocks. Section 4 concludes.
2. Capital account liberalization: “concrete” vs. “paper” wealth creation

For a developing country with domestic interest rates higher than the world interest rate, capital account liberalization tends to increase foreign demands on domestic assets, which can facilitate productive capital formation as “concrete” wealth. However, if investments are less than perfectly elastic, increases in asset demands will also result in “paper” wealth formation through asset price appreciation, which tends to enrich current consumers (as a whole) and thus encourage consumption. In a word, that foreign savings are used to finance consumption can simply be the result of their not being effectively channeled to investments. We examine this conjecture in the following.

The Model

Consumption

The modeling of consumption follows the finite-horizon model in Blanchard (1985). Each of many identical consumers throughout her lifetime faces a constant probability of death $\pi$. At any instant of continuous time, a cohort with size $\pi$ is born. Thus, the population size ($\int_{-\infty}^{t} \pi e^{-\pi(t-s)} ds = 1$) is constant (at unity) over time.

In every period, a living consumer supplies one unit of labor inelastically and maximizes (log) utility from consumption:

$$\text{Max} \int \log c(s, v) e^{-\pi(v-t)} dv,$$

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2 While increases in productive capital are “concrete” wealth because they imply more future incomes, asset price appreciation induced by increases in asset demands is “paper” wealth in the sense that it is a revaluation of the same amount of future earnings represented by assets. Nevertheless, such “paper” wealth does make current consumers as a whole wealthier—see Cai (2003) for detailed discussion.
where \( c(s, t) \) denotes the period-\( t \) consumption of a consumer born in period \( s \)—apply this \((s, t)\) notation rule to other variables as well. Note that for simplicity we assume zero time preference. The consumer faces a lifetime budget constraint

\[
\int_{s}^{\infty} [c(s, v)] e^{-\int_{t}^{v} [r(u) + \pi] du} dv = a(s, v) + \int_{t}^{\infty} w(s, v) e^{-\int_{t}^{v} [r(u) + \pi] du} dv,
\]

and transversality condition

\[
\lim_{v \to \infty} e^{-\int_{t}^{v} [r(u) + \pi] du} a(s, v) = 0,
\]

where variables \( a, w, \) and \( r \) are, respectively, asset, wage, and the rate of return to asset (i.e., the interest rate). Note that the “effective” rate of return to asset is \( r + \pi \) because the consumer can use her asset as a stake to “bet” on her own death—see Blanchard (1985, p.226) for detail.

The solution to the consumer’s maximizing problem gives

\[
c(s, t) = \pi [a(s, t) + h(s, t)]
\]

(1)

where

\[
h(s, t) = \int_{t}^{\infty} w(s, v) e^{-\int_{t}^{v} [r(u) + \pi] du} dv
\]

represents the consumer’s human wealth.

Aggregating equation (1) gives the aggregate consumption function:

\[
C_t = \pi (A_t + H_t)
\]

(2)
where variables $C$, $H$ and $A$ are the aggregate consumption, human wealth and non-human wealth respectively; the dynamics of which are as follows:

$$\dot{C}_t = r_t C_t - \pi^2 A_t$$  \hspace{1cm} (3)

$$\dot{H}_t = (r_t + \pi)H_t - W_t$$  \hspace{1cm} (4)

$$\dot{A}_t = r_t A_t + W_t - C_t$$  \hspace{1cm} (5)

Production

In every period, identical, profit-maximizing, and perfectly competitive firms hire capital and labor to produce consumption goods with the standard Cobb-Douglas technology.

With inelastic unit labor supply, the aggregate production function is

$$Y_t = F(K_t) = \lambda K_t^\alpha,$$  \hspace{1cm} (6)

where variables $K$ and $Y$ are capital stock and output respectively; parameters $\lambda$ and $\alpha$ are respectively technical coefficient and capital share. Profit maximization under perfect competition makes firms pay factors by their marginal products:

$$R_t = F'(K_t),$$  \hspace{1cm} (7)

$$W_t = F(K_t) - KF'(K_t),$$  \hspace{1cm} (8)

where $R_t$ and $W_t$ are income per unit of capital and labor respectively.

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3 The aggregate counterpart of a variable $x(s, t)$ is given by $X(t) = \int_{-\infty}^{t} x(s, t) p e^{p(s-t)} ds$; see Blanchard (1985, pp.228-229) for detail.
Investment

A variety of investment impediments can make investments less than perfectly elastic: e.g., investment adjustment costs, risk-averse entrepreneurs, investment uncertainty and irreversibility—to name a few fundamental ones; let alone those caused by institutional imperfections.

Considering the (arguable) lack of entrepreneur spirit in developing countries, we in the following model risk-averse investing behaviors as one example of investment impediments; whereas investment adjustment costs will give the same results⁴—we will look into the case of uncertainty and irreversibility in the next section.

In every period, identical entrepreneurs engage in investing activities that transform consumption goods into new capital.⁵ Individual entrepreneur \( j \) chooses the amount of investment \( I_j^t \) to maximize expected utility: ⁶

\[
\text{Max } EU(\Pi_j^t)
\]

where \( \Pi_j = q_j I_j - c(I_j) \) represents investment profits—\( c(I) \) is the investment cost (function) in terms of consumption.

Investments are risky with a stochastic cost function:

\[
c(I_j^t) = I_j^t (1 + z_j^t),
\]

where \( z_j \sim N(0, \sigma^2) \) is a normally distributed random variable. Entrepreneurs are risk-averse with utility function:

⁴ See Cai (2003) for a case of investment adjustment costs as an investment impediment.
⁵ To clearly examine investment behaviors, we model the production and investment decision makings separately—see Abel (2003) for a similar framework.
⁶ The utility rather than profit maximization is for the purpose of modeling risk-averse investment behaviors; otherwise, utility and profit maximizations are equivalent
\[ U(\Pi) = -e^{-\phi \Pi}, \] 

where parameter \( \phi \) measures (constant) absolute risk aversion.

According to equations (9) and (10), entrepreneur \( j \)'s maximizing problem becomes

\[
\text{Max } \int \mathbb{E} U(\Pi^j) = -\int e^{-\phi \Pi^j} f(\Pi^j) d\Pi^j = -e^{-\phi(q,-1/2 - \phi \sigma^2 / 2)} ,
\]

the solution to which gives individual investment function: \( q_i = 1 + \phi \sigma^2 l^j_i \). Then, the aggregate investment function (with \( n \) identical entrepreneurs) would be

\[ q_i = 1 + \eta l^i , \]

where coefficient \( \eta = \phi \sigma^2 / n \) is negatively related to the \( q \)-elasticity of investments ("investment elasticity" in short); and \( l^i = n l^j_i \) represents the aggregate investment.

Equation (11) implies that under risky investments (\( \sigma > 0 \)) and risk-averse entrepreneurs (\( \phi > 0 \)), the aggregate investment is less than perfectly elastic; and the elasticity is negatively correlated with the riskiness of investments or the risk aversion of entrepreneurs.

Given a large number of entrepreneurs (\( n >> 0 \)) and according to the law of large numbers, the aggregate investment cost function is

\[ c(I_i) = \sum_j c(l^j_i) = l^j_i (n + \sum_{j=1}^{n} z^j_i) = n l^j_i = I_i , \]

which implies constant marginal cost of investment in aggregate. Note that the less-than-perfectly-elastic aggregate investment notwithstanding constant aggregate marginal investment cost is because of increasing marginal risk premia demanded by risk-averse entrepreneurs.
The Close Economy

In autarky, the perishable consumption goods is either consumed or invested; thus the goods market equilibrium condition implies

\[ Y_t = C_t + I_t \]  \hspace{1cm} (13)

Capital is the only store of value; thus the aggregate non-human wealth is equal to the value of the capital stock

\[ A_t = q_t K_t \]  \hspace{1cm} (14)

The return to capital is equal to capital income plus capital gain; thus,

\[ r_t = \frac{F'(K_{t+1}) + E_t \dot{q}_t}{q_t} \]  \hspace{1cm} (15)

For simplicity, assume zero depreciation in capital; thus,

\[ \dot{K}_t = I_t \]  \hspace{1cm} (16)

The dynamics of the close economy is described by the simultaneous system composed of equations (2), (4), (6), (8), and (11)-(16) with endogenous \( A, H, K \) (as stock variables), \( Y, W, C, I \), (as flow variables), plus \( r \) and \( q \) (as prices). A little inspection reveals the steady-state values (denoted with asterisks) of several key variables: \( r^* = \pi \alpha^{1/2}, \quad q^* = 1, \)

\[ C^* = \lambda^{(a-1)^{-1}} \left( \pi \alpha^{-1/2} \right)^{a(a-1)^{-1}}, \quad \text{and} \quad K^* = \lambda^{(a-1)^{-1}} \left( \pi \alpha^{-1/2} \right)^{(a-1)^{-1}}. \]

The Open Economy

With open capital account, the interest rate is exogenously determined by the world interest rate. For comparison, assume the world interest rate is equal to the steady-state interest rate \( (r^* \) of the close economy, under which the close and open economies converge to the same steady state, but tend to have different convergent paths.
In the open economy, the goods market equilibrium condition becomes
\[ Y_t = C_t + I_t + TB_t, \]
where \( TB_t \) represents trade balance. With open capital account, the non-human wealth becomes
\[ A_t = q_t K_t + B_t, \]
where \( B \) represents net foreign asset; the dynamics of which, i.e., the current account (CA) dynamics, is characterized by
\[ CA_t = \frac{\dot{B}_t}{B_t} = TB_t + rB_t. \]
The other aspects of the open economy are the same as those in the close economy.

In summary, the dynamics of the open economy can be described by the simultaneous system composed of equations (2), (4), (6), (8), (11), (12), and (15)-(19) with endogenous \( A, H, K, B \) (as stock variables), \( Y, W, C, I, TB \) (as flow variables), and \( q \) (as price). It is not difficult to verify that the steady state of the open economy is identical to that of the close economy.

Post-liberalization scenario comparison under different investment elasticity.

Intuitively, countries with high investment elasticity, \textit{ceteris paribus}, will have high post-liberalization investments and hence high gross national products (GNP = \( Y + r*B \)); thus they should accordingly enjoy high post-liberalization consumption. However, we will show that countries with low post-liberalization GNP over time (due to low investment elasticity) can nonetheless have high post-liberalization consumption for some time.
The post-liberalization dynamics of an economy (opening up at time \( t = 0 \) with \( K_0 < K^* \) and \( B_0 = 0 \)) can be characterized by the following differential equations:  

\[
\dot{C}_t = r^* C_t - \pi^2 (B_t + q_t K_t), \quad (20) 
\]

\[
\dot{B}_t = r^* B_t + \lambda K_t^\alpha - C_t - \eta^{-1} (q_t - 1), \quad (21) 
\]

\[
\dot{K}_t = \eta^{-1} (q_t - 1), \quad (22) 
\]

\[
\dot{q}_t = r^* q_t - \alpha \lambda [K_t + (q_t - 1) \eta^{-1}]^{\alpha-1}. \quad (23) 
\]

Linearizing the differential equation system (20)-(23) around steady state gives

\[
\begin{bmatrix}
\dot{C} \\
\dot{B} \\
\dot{K} \\
\dot{q}
\end{bmatrix} = 
\begin{bmatrix}
r^* - \pi^2 - \pi^2 - \pi^2 K^* \\
-1 & r & r & -\eta^{-1} \ni

\begin{bmatrix}
C - C^* \\
B - B^* \\
K - K^*
\end{bmatrix}
\end{bmatrix}
\]

(24)

where \( m = r^* + \alpha (1 - \alpha) \lambda (K^*)^{\alpha-2} \eta^{-1} \) and \( n = \alpha (1 - \alpha) \lambda (K^*)^{\alpha-2} \). The solution to which gives the growth paths of \( C, B, K \) and \( q \):

\[
C_t = (K_0 - K^*) [(1 + \beta) e^{(r^* - \pi) t} + \beta (\varepsilon - r^*) e^\sigma] + C^* \quad (25)
\]

\[
B_t = (K_0 - K^*) (1 + \beta) [e^{(r^* - \pi) t} - e^\sigma] \quad (26)
\]

\[
K_t = (K_0 - K^*) e^\sigma + K^* \quad (27)
\]

\[
q_t = (K_0 - K^*) \varepsilon \eta e^\sigma + 1 \quad (28)
\]

where \( \varepsilon = m / 2 - (m^2 / 4 + n \eta^{-1})^{1/2} < 0 \) and \( \beta = \pi^2 K^* \varepsilon \eta (\pi - \varepsilon + r^*)^{-1} (\pi + \varepsilon - r^*)^{-1} \).

Equations (25)-(28) describe the post-liberalization dynamics of several key variables; based on which comparative statics can be used to illustrate the impacts of \( \eta \).

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7 Equation (20) is derived from equations (3) and (18); equation (21) from equations (6), (11), (17) and (19); equation (22) from (11) and (16); and equation (23) from (11), (15) and (16).

8 See Mathematical Appendix for detail.
(or essentially investment elasticity) on each variable. However, due to mathematical complications, the signs of some comparative statics are hard to be determined analytically. Thus, we choose to use numerical simulations to compare post-liberalization scenarios under different $\eta$.

The parameters in the simulation model composed of equations (25)-(28) are set as: $\alpha = 1/3$ (as usual); $\pi = 1/60$ (i.e., the average life expectancy is 60); and $\lambda = \sqrt{3}/60$ (for normalizing the steady-state capital stock to unity, i.e., $K^* = 1$). Initial capital stock is set as $K_0 = 0.9$. Results based on this setting are qualitatively robust for other parameter settings.

Based on this simulation model, we first compare post-liberalization GNP under different $\eta$. For easy visualization, Figure 1 (a) compares GNP under $\eta = 10, 20, 40, 60$ respectively; whereas Figure 1 (b) provides more complete GNP comparison for $\eta \in [0, 600]$. Figure 1 indicates the following result.

Remark 2.1 *Ceteris paribus, high $\eta$ (i.e. low investment elasticity) tends to result in permanent low post-liberalization GNP.*

This result is not surprising and can be explained as follows. Note that part of the capital stock of a country with negative net foreign asset is essentially owned by foreign residences. Suppose the total and domestic-owned capital stock are $\bar{K}$ and $K_d$ respectively; then $\text{GNP} = \bar{K}K_d + \bar{w} = \alpha \bar{K}^{\alpha - 1} K_d + (1 - \alpha) \lambda \bar{K}^\alpha$. It is not difficult to verify that, given $K_d < \bar{K}$, $\partial \text{GNP}/\partial \bar{K} > 0$, which implies that, given domestic-owned capital
stock \( (K_d) \), the lower the total capital stock \( (K) \) is, the smaller the GNP will be. Therefore, given the initial capital stock, the smaller the (post-liberalization) investments are, the smaller the GNP will be. As \( \eta \) negatively affects post-liberalization investments, high \( \eta \) will lead to low post-liberalization GNP over time.

While the impact of \( \eta \) on post-liberalization GNP is as expected, that on consumption is puzzling. As shown in Figure 2, the impact of \( \eta \) on post-liberalization consumption can be summarized as follows.

**Remark 2.2** *Ceteris paribus, high \( \eta \) (i.e. low investment elasticity) tends to result in high consumption for some time immediately after the liberalization; notwithstanding the effect of \( \eta \) on (future) consumption will eventually become negative in the long run.*

Base on the aggregate consumption function represented by equation (2), this result is not difficult to explain mathematically. Although high \( \eta \) tends to result in low post-liberalization capital stock, hence low labor income, and hence low human wealth \( (H) \), it also tends to cause high capital price (Figure 3) and hence high non-human wealth \( (A) \). For some time after the liberalization, the latter effect tends to outweigh the former; thus the total wealth \( (A+H) \) will be positively affected by \( \eta \). Then, according to equation (2), so will be the aggregate consumption. As \( \eta \) has a negative impact on \( A+H \) in the long run, the impact of \( \eta \) on future consumption will eventually become negative.

However, a puzzling issue is how to reconcile Remarks 2.1 and 2.2, which together imply that, given the constant (world) interest rate, an economy with low GNP in the entire post-liberalization period can nonetheless have high consumption for some time.
More fundamentally, given the interest rate, how can permanently low GNP be consistent with large total wealth, which is supposed to embody the total (present) value of GNP over time?

The key to this puzzle is that the total wealth of current consumers does not include future labor incomes beyond their (finite) horizons; in another word, the total wealth \(A+H\) does not embody the entire GNP over time.\(^9\) Thus, while high \(\eta\) makes society as a whole worse off by lowering post-liberalization GNP over time, it can nevertheless make current consumers as a whole better off by increasing their wealth \(A+H\). We explain this point in detail in the following.

Ceteris paribus, low capital stock will result in low labor income \((W_t)\) but high (per unit) capital income \((R_t)\). Since the positive effect of capital stock on \(W_t\) tends to dominate its negative effect on \(R_t\), the net effect of low post-liberalization capital stock over time (due to high \(\eta\)) will be low GNP over time. However, a key point is that, while the gains from high \(R_t\) (over time) are completely reaped by current consumers through \(q\) appreciation,\(^{10}\) the losses from low \(W_t\) will be mostly burdened by future unborn consumers. Therefore, current consumers (as a whole) can nevertheless be better off from high \(\eta\) (i.e. low investment elasticity), even though the total benefit for the society as a whole is lowered. In a sense, current consumers can benefit from the failure of capital account liberalization in accomplishing its presupposed mission.

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\(^9\) In Blanchard’s (1985) framework adopted here, finite horizon is modeled as a constant probability \((\pi)\) of death. Thus, while an infinite-living outlier individual is theoretically possible, current consumers as a whole is expected to have a horizon equal to the mean life expectancy (i.e., \(1/\pi\)). The implication of this finite-horizon feature on human wealth is mathematically captured by a higher discount rate (i.e., \(r+\pi\)) for labor income.

\(^{10}\) Whatever \(R_t\) is, the rate of return to future asset ownerships will be fixed at the world interest rate.
Remark 2.3 Low investment elasticity (i.e. high $\eta$) tends to result in low post-liberalization GNP over time and hence reduce the total benefit (for society as a whole) of capital account liberalization. However, low investment elasticity can nonetheless increase the wealth of current generations and hence lead to high consumption for some time immediately after the liberalization.

Figure 2 (a) indicates that the high-consumption era lasts for around 60 periods, i.e., the mean lifespan of a generation.

The (transitory) negative relationship between investment elasticity and post-liberalization consumption suggests that post-liberalization consumption booms can result from low investment elasticity. To confirm, we need to compare between post-liberalization and autarky scenarios.

Investment elasticity and post-liberalization consumption booms
Suppose an autarky economy opens at $t = 0$ with $K_0 < K^*$ and $B_0 = 0$, foreign capitals will flow in; the amount of which can be measured by current account (CA) deficits. Part of the capital inflows will be used to financed extra investments $\Delta I_0 = I^o_0 - I^a_0$—note that $I^o$ and $I^a$ represent the open and autarky aggregate investments respectively—and the rest will essentially be used for extra consumption.

The proportion of CA used to finance consumption can be measured by

$$\rho = 1 - \Delta I_0 / (-CA_0),$$

which can be taken as an indicator of the extent of post-liberalization consumption booms.
A comparison between $I_0^o$ and $I_0^a$ is necessary to reveal $\rho$. Unfortunately, while $I^o$ (and CA as well) can be solved from equations (25)-(28), the analytical solution to $I^a$ is hard to obtain, because the autarky interest rate is endogenous.

To provide some basic insights as to the impact of investment elasticity on $\rho$, we choose to conduct autarky-open comparison in a tractable model designed to approximate the rational-expectation (RE) model presented above.

In the “proxy” model, we assume aggregate consumption function as

$$C_r = \pi (A_r + \bar{H}_r)$$

(2')

where $\bar{H}_r = (r^* + \pi)^{-1} W_r$, as compared to $H_r = \int W(t)e^{-\int_{t}^{\infty} [r(u) + \pi] du} dv$ in the RE aggregate consumption function represented by equation (2). The difference between $\bar{H}_r$ and $H_r$ is that, while $H_r$ implies that consumers have perfect foresights over future wage incomes and interest rates, $\bar{H}_r$ implies that, in calculating human wealth, consumers use the current wage income and steady-state interest rate to approximate future wage incomes and interest rates respectively. With equation (2) replaced by (2'), the proxy model is tractable for both open and autarky scenarios, and hence allows us to conduct the autarky-open comparison.

In the proxy model, the autarky economy will have the same steady state as the RE model; yet the dynamics may be different. On the one hand, by using the current wage (as a proxy for increasing wage incomes over time) to calculate human wealth, the proxy model tends to “underestimate” the autarky consumption $C_0^a$ (relative to the RE consumption as a benchmark). On the other hand, by using the steady-state interest rate
as a proxy for the autarky decreasing interest rates over time, the proxy model tends to “inflate” human wealth and hence “overestimate” \( C^a_0 \). If the balance of these two opposite effects is neutral, the proxy model provides a good approximation of \( C^a_0 \) in the RE model. Unfortunately, the state of the balance is unclear. However, while the underestimation problem also happens to the post-liberalization consumption \( C^o_0 \), the overestimation problem will not, because the open-economy interest rate over time is indeed \( r^* \). Thus, the proxy model would in general have larger overestimation (or smaller underestimation) on \( C^a_0 \) than \( C^o_0 \), which, in light of the fact that \( C^o_0 > C^a_0 \), implies an underestimation of \( C^o_0 - C^a_0 \).

In sum, relative to the RE model, the proxy model tends to underestimate the open-autarky consumption difference and hence the severity of post-liberalization consumption booms. Technically, as (will be shown in a moment) we find a positive impact of \( \eta \) on \( \rho \) in the proxy model, we expect the impact will be stronger in the RE model.

In the proxy model, the autarky (aggregate) investment, the post-liberalization investment, and the (post-liberalization) current account can be solved analytically.\(^\text{11}\) The results are

\[
I^a_t = \omega(K_0 - K^*)e^{\omega t},
\]

\[
I^o_t = e(K_0 - K^*)e^{\omega t},
\]

and

\[
CA_t = \beta(K_0 - K^*)[(r^* - \pi)e^{(r^* - \pi)t} - \varepsilon e^{\omega t}],
\]

\(^\text{11}\) See Mathematics Appendix (A.2) for detail
where \( \omega = \left[ \Psi r^* - \pi + (\alpha - 1) \pi \eta \lambda (K^*)^\alpha \right] (1 + \pi \eta K^*)^{-2} < 0 \) and \( \Psi = 1 - (1 + \alpha^{1/2})^{-1} (1 - \alpha) \); 
\[ \tilde{\beta} = 1 - \left[ \pi r^* (1 - \alpha) (r^* + \pi)^{-1} + \varepsilon \eta \pi K^* \right] (r^* - \varepsilon - \pi)^{-1} \]

According to equation (30), the investment would be \( I^*_0 = \omega (K_0 - K^*) \) if the economy stays autarky at time \( t = 0 \). Yet, if it chooses to open up, equations (31) and (32) indicate that the investment would be \( I^*_0 = \varepsilon (K_0 - K^*) \); and the current account be \( CA_0 = \tilde{\beta} (K_0 - K^*) (r^* - \pi - \varepsilon) \). Substituting these results into equation (29) will give \( \rho = 1 + (\varepsilon - \omega) \tilde{\beta}^{-1} (r^* - \pi - \varepsilon)^{-1} \), which measures the extent of post-liberalization consumption boom.

Since mathematical complexity prevents us from determining the sign of \( d\rho / d\eta \) analytically, we choose to illustrate the impact of \( \eta \) on \( \rho \) through numerical simulations. The result of numerical simulations based on the parameters setting used above (i.e. \( \alpha = 1/3 \); \( \pi = 1/60 \); and \( \lambda = \sqrt{3}/60 \)) is illustrated in Figure 4, which indicates a positive relationship between \( \rho \) and \( \eta \). It should be noted that the positive relationship is qualitatively robust for other parameter settings we have tried.

Therefore, we have the following remark.

**Remark 2.4** The higher the \( \eta \) (or the lower the investment elasticity) is, the greater the proportion of post-liberalization capital inflows will be used for financing consumption booms.

In a word, post-liberalization consumption booms can simply be the results of sluggishness in transforming foreign savings into domestic investments.
3. Productivity shock, uncertainty, and consumption booms

One insight provided by the literature on investment uncertainty and irreversibility is that, uncertain yet profitable investment opportunities can nevertheless remain unexploited even when markets are efficient and entrepreneurs are risk neutral (Dixit and Pindyck, 1994). This is because “wait-and-see” can be a better strategy when the cost of waiting (i.e., profits unearned) is smaller than that of being stuck with underperformed yet irreversible investments.

Therefore, similar to entrepreneurs’ risk aversion, uncertainty and irreversibility together (as two common features of investments) can be another “investment impediment” responsible for foreign-capital-financed consumption booms.

Consider an open developing economy under structural reforms that are expected to increase future productivities. With easy access to low-cost foreign funds, high future productivities imply profitable investment opportunities. However, these opportunities may not be taken by entrepreneurs who prefer to postpone investment decisions till the outcomes of the reforms become more certain. If so, the influence of the (expected) high future productivity will be on asset price (appreciation), which tends to trigger consumption booms. Based on a simplified and discrete version of the model presented above, we examine this conjecture in the following.

Backdrop

At the beginning of period $t = 0$, a small open economy initiates a reform that will take one period to accomplish. If successful, the reform will lead to higher (than current) productivity from period $t = 1$ onward; whereas a failure will result in lower productivity.
Notwithstanding uncertain, the reform is promising; i.e., the expected future productivity is higher.

Against this backdrop, we are interested in the impacts of the reform on current economic activities including investments, asset price movements, and consumption.

**Productivity and capital income**

At the beginning of period $t = 0$, the period-zero productivity is known as $\lambda_0$. Yet, future productivities are uncertain as follows:

$$\lambda_t = \lambda_0 + z \ (t \geq 1), \quad (33)$$

where $z = \begin{cases} 
z^d < 0: \Pr(z = z^d) = p \\
\lambda_0 \geq z^m \geq z^d, \quad (33) \\
z^m > 0: \Pr(z = z^m) = 1 - p \end{cases}.$$

According to equation (33), the economic future (i.e., period one onward) can be either of a “miracle” or a “debacle” (with probability $p > 0$)—$z^m$ and $z^d$ are the miracle and debacle productivity shocks respectively. Despite uncertain, the future is promising, with expected future productivity higher than $\lambda_0$; i.e., $E(z) = pz^d + (1 - p)z^m > 0$.

The uncertainty is temporary—at the end of period zero, the nature of productivity shock ($z^m$ or $z^d$) is determined and observable.

Let $R_0$, $R^m$, and $R^d$ denote period-zero, “miracle” future, and “debacle” future income per unit of capital respectively. Then, given capital stock $K$ and according to equation (7),

$$R_0 = \alpha\lambda_0 K^{\alpha - 1}, \quad R^m = \alpha(\lambda_0 + z^m)K^{\alpha - 1} \quad \text{and} \quad R^d = \alpha(\lambda_0 + z^d)K^{\alpha - 1},$$

which give

$$R^m = \left(1 + \frac{z^m}{\lambda_0}\right)R_0, \quad (34)$$

and
$$R^d = (1 + \frac{z^d}{\lambda_0}) R_0 .$$  \hspace{1cm} (35)

**Investment**

Investments are of “putty-clay” nature. That is, one unit of consumption good can produce one unit of capital; yet, capital is irreversible.

With open capital account, the cost of fund is equal to the world interest rate \((r^*)\). Then, the expected profit per unit of investment at the beginning of period zero would be

$$EIT_{\text{invest}} = \frac{(R_0 - r^*)}{1 + r^*} + \frac{1}{1 + r^*} \left[ \frac{(1 - p)(R^m - r^*) + p(R^d - r^*)}{r^*} \right] ,$$ \hspace{1cm} (36)

with the first and second terms on the right hand side (RHS) representing the present values of period-zero and expected future profits respectively.

Entrepreneurs can choose not to invest at the beginning of period zero, but to postpone investment decisions till the end of it when \(R^m\) or \(R^d\) is observable. No investment at the beginning of period zero means zero profit during which. If the future turns out to be a debacle at the end of period zero, entrepreneurs will not invest, because the debacle capital income is less that the cost of capital (i.e., \(R^d < r^*\)).\(^{12}\) If the future is a miracle, entrepreneurs will invest. Since the probability of the miracle future is \(1 - p\), the present value of the expected profits from this wait-and-see strategy would be

$$EIT_{\text{wait}} = \frac{1}{1 + r^*} \frac{(1 - p)(R^m - r^*)}{r^*} .$$ \hspace{1cm} (37)

\(^{12}\) We ignore trivial equilibria where \(R^d > r^*\).
**Equilibrium**

**Investment**

Risk-neutral and profit-maximizing entrepreneurs will keep investing as long as $E\Pi_{\text{invest}} \geq 0$ and $E\Pi_{\text{invest}} > E\Pi_{\text{wait}}$. Therefore, one necessary condition for equilibrium is \(^{13}\)

$$E\Pi_{\text{invest}} = E\Pi_{\text{wait}},$$

which, according to (36) and (37), gives the equilibrium capital income in period zero,

$$R_0^e = r^* + \frac{p(r^* - R^d)}{r^*}. \tag{38}$$

According to equations (35) and (38),

$$R_0^e = r^* \left( \frac{r^* + p}{r^* + p(1 + z^d / \lambda_0)} \right). \tag{39}$$

Thus, according to equations (7) and (39), the equilibrium period-zero capital stock would be

$$K_0^e = \left( \frac{R_0^e / \alpha \lambda_0}{\alpha^{-1}} \right)^{\frac{1}{\alpha-1}} = \left( \frac{r^{*2} + r^* p}{\alpha \lambda_0 r^* + \alpha \lambda_0 p(1 + z^d / \lambda_0)} \right)^{\frac{1}{\alpha-1}}, \tag{40}$$

which implies $\partial K_0^e / \partial p < 0$ and $\partial K_0^e / \partial z^d > 0$. Thus,

**Remark 3.1** *The higher the debacle probability (or the lower the debacle productivity) is, the lower the current investments will be.*

---

\(^{13}\) We ignore trivial equilibria where $E\Pi_{\text{invest}} < E\Pi_{\text{wait}}$. 
Intuitively, the probability and severity of the future debacle are two “impediment” elements that keep entrepreneurs from taking profitable investment opportunities immediately.

Given \( p \) and \( z^d \), equation (40) implies \( \partial K^e_t / \partial z = 0 \). Thus,

**Remark 3.2** The expected future productivity (per se) has no influence over current investments.

This so-called “bad-news” (or “irrelevant-good-news”) principle (Bernanke, 1983) is because the wait-and-see strategy will not cost entrepreneurs the opportunity to invest in the miracle future.

**Capital Price**

The price of irreversible capital is determined by the present value of expected future incomes per unit of capital. Thus, the equilibrium period-zero capital price is given by:

\[
q^e_0 = \frac{R^e_0}{1 + r^*} + \frac{p R^d}{1 + r^*} + \frac{1 - p R^m}{1 + r^*} \\
= \frac{r^* + p}{r^* + p(1 + z^d / \lambda_0)} \left[ 1 + \frac{\bar{z}}{\lambda_0(1 + r^*)} \right]
\]

which implies that, if \( p > 0 \), \( q^e_0 > 1 \).\(^{14}\) Put plainly,

\[^{14}\text{Without uncertainty (i.e., } p = 0 \text{), } q^e_0 \text{ will be equal to one, but not } 1 + \bar{z} \lambda_0^{-1} (1 + r^*)^{-1} \text{ implied by equation (41). This is because, without uncertainty, firms will be active in investments; hence competitive market force will make equilibrium achieved only at } q^e_0 = 1.\]
Remark 3.3 Uncertainty over future productivities tends to result in (expected) profitable investment opportunities unexploited in equilibrium, even though markets are efficient; and entrepreneurs are risk-neutral.

This “inefficient” outcome is not the result of any market failure. Positive profits (in equilibrium) are necessary to compensate expected losses from being stuck with debacle investments.

Equation (41) implies that $\frac{\partial q^e}{\partial z} > 0$. Thus,

Remark 3.4 The greater the expected future productivity is, the higher the current equilibrium capital price will be.

As (high) future productivity has no influence over capital formation (Remark 3.2), its impact will be on asset price (appreciation).

Equation (41) implies that, given $\bar{z}$, $\frac{\partial q^e}{\partial p} > 0$ and $\frac{\partial q^d}{\partial z^d} < 0$. Thus,

Remark 3.5 Given expected future productivity, the higher the debacle probability (or the lower the debacle productivity) is, the higher the capital price will be.

Intuitively, high debacle probability (or low debacle productivity) makes it more costly to be stuck in the debacle future; thus, high asset prices (i.e., high investment profits) are necessary for entrepreneurs to invest.
**Consumption**

Given the inherited aggregate human-wealth $K_{-1}$ (at the beginning of period zero), and according to equation (2), the equilibrium aggregate consumption would be

$$C_0^e = \pi (q_0^e K_{-1} + H_0^e),$$

(42)

where

$$H_0^e = (1 - \alpha) F(K_0^e)(r^* + \pi)^{-1} + \bar{H},$$

(43)

in which the first term on the RHS represents the period-zero labor income; and $\bar{H}$ is equal to the present value of expected labor incomes from period one onward, which depend on the expected future productivity: $\partial \bar{H} / \partial \bar{z} > 0$.

According to equation (43), that $\partial K_0^e / \partial \bar{z} = 0$ and $\partial \bar{H} / \partial \bar{z} > 0$ imply $\partial H_0^e / \partial \bar{z} > 0$.

According to equation (42), that $\partial q_0^e / \partial \bar{z} > 0$, $\partial K_{-1} / \partial \bar{z} = 0$ and $\partial H_0^e / \partial \bar{z} > 0$ imply $\partial C_0^e / \partial \bar{z} > 0$. Note that the positive effect of future productivity ($\bar{z}$) on consumption ($C_0^e$) includes both human and non-human wealth effect.

The impact of $p$ on $C_0^e$ is two folded: That $\partial q_0^e / \partial p > 0$ implies a positive $p$-effect on $C_0^e$ through non-human wealth; whereas $\partial K_0^e / \partial p < 0$ implies a negative $p$-effect on $C_0^e$ through human wealth $H_0^e$. An analytical determination of the balance of the two effects is intractable in this simple model here. However, considering the fact that (given $\bar{z}$) $\partial \bar{H} / \partial p = 0$, $p$ will only influence the period-zero labor income but not beyond, the human wealth effect tends to be dominated by the non-human wealth effect—this conjecture is supported by simulations in the last section. Therefore, the case of $\partial C_0^e / \partial p > 0$ is more likely; and following the same logic, so is $\partial C_0^e / \partial z^d < 0$. 
The results $\partial C^e_0 / \partial z > 0$, $\partial C^e_0 / \partial \pi > 0$, and $\partial C^e_0 / \partial z^d < 0$ provide the following insights:

**Remark 3.6** *The greater the expected future productivity is; or the greater the debacle probability is; or the greater the severity of the debacle is, the higher the current consumption will be.*

Without uncertainty, the major impact of (high) expected future productivity will be on (high) investments; and capital price will be anchored by the marginal cost of investments (assumed constant at unity here). Consumption will increase because of the positive human wealth effect; yet, consumption booms driven by asset price appreciation will not happen.

With uncertainty (plus irreversibility), high expected future productivity will have little influence over current investments because of the “wait-and-see” atmosphere. Then its impact will be on asset price appreciation, which can trigger consumption booms.

Given expected future productivity, the magnitude of current asset price appreciation is also related to the chance and severity of future debacles. A high debacle probability (or a low debacle productivity) will result in low investments and hence strong asset price appreciation, which tends to increase the magnitude of consumption booms.

### 4. Concluding Remarks

The main message this paper attempts to convey is that, when developing countries have difficulty in turning foreign savings into domestic investments, they tend to use them for
consumption. Our analysis confirms this conjecture in the cases of capital account liberalization and productivity increase as two driving forces of capital inflows.

In general, when capital inflows, driven by any force, cannot be sufficiently absorbed into physical capital formation due to less than perfectly elastic investments, they tend to result in asset price appreciation. Through wealth effect, the appreciation tends to drive consumption booms (financed by foreign capital). In the sense that investments in reality can seldom be perfectly elastic even with perfect institutions, we claim that consumption booms tend to be fundamental outcomes of open capital account. In another word, institutional improvements may not be sufficient to prevent capital inflows from being channeled to consumption.

One may argue that, notwithstanding financed by capital inflows, consumption booms without institutional imperfections are not excessive but normal outcomes of efficient market mechanisms. In particular, post-liberalization high consumption can be viewed as welfare-improving consumption intertemporal allocation by an eternal representative agent (or dynasty) facing the low world interest rate.

However, the welfare implications from a finite-horizon perspective are more complicated. On the one hand, a ceteris paribus interest rate fall tends to benefit current non-human-wealth owners (through asset price appreciation) at the cost of current and future human-wealth owners (through a lowered rate of return to savings). On the other hand, high investments induced by the interest rate fall can lead to high labor incomes, which (if sufficiently high) can compensate (or outweigh) human-wealth owners’ losses. Therefore, the welfare implications of capital account liberalization to human wealth

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15 Under open capital account, a variety of domestic “pull” factors (such as favorable productivity shocks or reduction in country risk premium etc.) or foreign “push” factors (such as a fall in the world interest rate) can drive capital inflows.
owners depend on its effect on capital formation. When low investment elasticity makes the gains from high labor incomes dominated by the losses from low interest rate, human-wealth owners will be worse off from capital account liberalization. Therefore, post-liberalization consumption booms enjoyed by current consumers can be at the cost of low consumption suffered by future generations.

A somewhat surprising insight is that, while low post-liberalization investments (due to low investment elasticity) reduce the total benefit of capital account liberalization for society as a whole, they nonetheless tend to make current consumers as a whole better off through “paper” wealth created by asset price appreciation. Such wealth is not bubbles driven by speculation or credit overexpansion, but rather a result of asset revaluation under lower interest rates. Notwithstanding commonly viewed as “real” wealth, we call it “paper” wealth to emphasize the fact that it can be destroyed as easily as it is created. During the 1997 Asian financial crises, Mahathir Mohamad, Prime Minister of Malaysia at that time, complained that it took speculators only two weeks to destroy the wealth painstakingly accumulated by Malaysian people over decades. Wealth that can easily “evaporate” without physical resources being destroyed, notwithstanding “real”, may not be concrete enough. In this sense, a concrete-paper wealth tradeoff leaned to the “paper” side may not be really in the interest of current consumers.

Besides, foreign-capital-financed consumption booms can be the root of low savings, high current account deficits, and real exchange rate appreciation (if non-tradable goods taken into consideration), which combined tend to be a recipe for crises.

If taken as undesirable, what can be done to prevent foreign-capital-finance consumption booms? To increase investment elasticity through reducing investment
impediments (such as nurturing entrepreneur spirit or reducing uncertainties) will certainly help, but may not be easy or practical. Investment subsidies can be used to stimulate investments directly, but may not be practical and can have little influence on temporary investment sluggishness caused by “wait-and-see” strategies. Consumption credit controls can restrain consumption financed by borrowing but not those by asset holdings. Capital controls can avoid paper wealth creation by aligning the domestic interest rate level with the earning level of domestic assets; yet, its enforceability and “collateral damages” need to be taken into consideration.

All in all, sensible policy prescriptions for addressing foreign-capital-financed consumption booms tend to be case sensitive and belong to the scope of empirical studies. The main contribution of this paper is to provide a diagnosis underappreciated by the existing literature.
Mathematic Appendix (A.1-A.2)

A.1

The four eigenvalues of the coefficient matrix of the simultaneous system

\[
\begin{bmatrix}
\dot{C} \\
\dot{B} \\
\dot{K} \\
\dot{q}
\end{bmatrix}
= 
\begin{bmatrix}
1 & r & -\eta & 1 & 0 \\
-1 & r & r & -\eta & 0 \\
0 & 0 & 0 & \eta & 0 \\
0 & 0 & n & m
\end{bmatrix}
\begin{bmatrix}
C - C^* \\
B - B^* \\
K - K^* \\
q - 1
\end{bmatrix}
\]

are, respectively,

\[
\varepsilon_1 = r^* + \pi > 0, \quad \varepsilon_2 = \frac{m}{2} + \sqrt{\frac{m^2}{4} + \frac{n}{\eta}} > 0, \quad \varepsilon_3 = r^* - \pi < 0, \quad \text{and} \quad \varepsilon_4 = \frac{m}{2} - \sqrt{\frac{m^2}{4} + \frac{n}{\eta}} < 0.
\]

Assume \( \varepsilon_3 \neq \varepsilon_4 \). Then, with two negative eigenvalues and two initial conditions \( K(0) = K_0 \) and \( B(0) = 0 \), there exists a unique convergent path to the steady state. To solve for the path, we need the eigenvectors of \( \varepsilon_3 \) and \( \varepsilon_4 \), which are respectively

\[
\varepsilon_3 : \begin{bmatrix} \pi \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \varepsilon_4 : \begin{bmatrix} (\varepsilon - r^*)\beta \\ -(1 + \beta) \\ 1 \\ \varepsilon\eta \end{bmatrix} \quad (\text{denote } \varepsilon = \varepsilon_4),
\]

where \( \beta = \pi^2 K^* \varepsilon\eta (\pi - \varepsilon + r^*)^{-1} (\pi + \varepsilon - r^*)^{-1} \). Thus, the solution to equation (24) is

\[
\begin{align*}
C_t - C^* &= u_1 \pi e^{(r^* - \pi)t} + u_2 \beta (\varepsilon - r^*) e^{\alpha} \\
B_t - B^* &= u_1 e^{(r^* - \pi)t} - u_2 (1 + \beta) e^{\alpha} \\
K_t - K^* &= u_3 e^{\alpha} \\
q_t - 1 &= u_2 \varepsilon \eta e^{\alpha}
\end{align*}
\]

(A.1)
Given initial conditions $K(0) = K_0$ and $B(0) = 0$, according to the second and third equations in the simultaneous system (A.1), we have $0 = u_1 - u_2 (1 + \beta)$ and $u_2 = K_0 - K^*$. Thus, $u_1 = (1 + \beta) (K_0 - K^*)$. Substitute $u_1$ and $u_2$ back to (A.1) gives the solution to (24).

A.2

The Proxy Model (autarky)

The modified aggregate consumption function can be written as

$$C(t) = \pi q_t K_t + (1 + \alpha^{1/2})^{-1} (1 - \alpha) \lambda K_t^a$$

(A.2)

Equations (11) and (16) imply

$$q_t = 1 + \varphi \sigma^{-2} \dot{K}_t$$

(A.3)

Substituting equations (A.2), (A.3), and (6) into equation (13) gives

$$\dot{K}_t = \left( \Psi \lambda K_t^a - \pi K_t \right) \left( 1 + \pi \eta K_t \right)^{-1},$$

(A.4)

where $\Psi = 1 - (1 + \alpha^{1/2})^{-1} (1 - \alpha)$. Linearizing equation (A.4) around steady state gives

$$\dot{K}_t = \omega (K_t - K^*),$$

(A.5)

where $\omega = \left[ \Psi r^* - \pi + (\alpha - 1) \pi \eta \Psi \lambda (K^*)^2 \right] \left( 1 + \pi \eta K^* \right)^{-2} < 0$. Thus, according to (A.5) and the initial condition $K(0) = K_0$, we have $K_t = K^* + (K_0 - K^*) e^{\omega t}$, based on which we have equation (30), i.e., $I_t^a = \omega (K_0 - K^*) e^{\omega t}$.

A.3

The Proxy Model (open economy)
In the open-economy model, using the modified consumption function to substitute for $C$ will give the following dynamic system:

$$
\dot{B}_t = r^* B_t + \lambda K_t^\alpha - \pi q_t K_t - \pi B_t - \pi (r^* + \pi)^{-1} (1 - \alpha) \lambda K_t^\alpha - \eta^{-1} (q_t - 1),
$$

$$
\dot{K}_t = \eta^{-1} (q_t - 1),
$$

$$
\dot{q}_t = m(q_t - 1) + n(K_t - K^*),
$$

which, after linearization, gives

$$
\begin{bmatrix}
\dot{B} \\
\dot{K} \\
\dot{q}
\end{bmatrix} =
\begin{bmatrix}
r^* - \pi & \Delta & - (\pi K^* + \eta^{-1}) \\
0 & 0 & \eta^{-1} \\
0 & n & m
\end{bmatrix}
\begin{bmatrix}
B - B^* \\
K - K^* \\
q - 1
\end{bmatrix}
$$

(A.6)

where $\Delta = r^* - \pi - \pi (r^* + \pi)^{-1} (1 - \alpha) r^*$. Given initial conditions $K(0) = K_0$ and $B(0) = 0$, the solution to (A.6) is

$$
B_t = \tilde{\beta} (K_0 - K^*) (e^{(r^* - \pi)t} - e^{\alpha t})
$$

$$
K_t = (K_0 - K^*) e^{\alpha t} + K^*
$$

$$
q_t = (K_0 - K^*) \epsilon \eta e^{\alpha t} + 1
$$

where $\epsilon = m / 2 - (m^2 / 4 + n / \eta)^{1/2}$ and $\tilde{\beta} = 1 - [\pi r^* (1 - \alpha)(r^* + \pi)^{-1} + \epsilon \eta \pi K^*] \epsilon (r^* - \pi)^{-1}$.

Given initial conditions $K(0) = K_0$ and $B(0) = 0$, we can have equations (31) and (32), respectively, $I_t^{\alpha} = \epsilon (K_0 - K^*) e^{\alpha t}$ and $CA_t = \tilde{\beta} (K_0 - K^*) (r^* - \pi) e^{(r^* - \pi)t} - \epsilon e^{\alpha t}$. 
References


Figure 1
Figure 2