Private and Public Information in Self-Fulfilling Currency Crises*

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Abstract

This paper analyses the implications of information dissemination on currency crises in models with self-fulfilling expectations. Following Morris/Shin (1999, 2000), we introduce noisy private and public information, so that under certain conditions for the noise parameters a unique equilibrium is derived. Comparative statics then show that if the fundamental state of the economy is good, the probability of a currency crisis decreases in the precision of public information, but increases in the precision of private information. In case of bad fundamentals, however, more precise public information increases the likelihood of a crisis, whereas more precise private information leads to a lower crisis probability.

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1 Introduction

Following the currency crises of the last decade in Europe, Mexico and Asia, a growing literature has appeared trying to explain and formalize financial turmoil. First-generation models of currency crises (Krugman 1979) emphasize the role of deteriorating fundamentals for speculative attacks on the fixed currency peg, e.g. due to inconsistent governmental debt policies. Second-generation models (Obstfeld 1996), however, show that crises can occur even in the context of policies that are consistent with the fixed peg. In these models, crises follow a coordination problem due to mutually reinforcing actions on the part of the speculators. Since it is more attractive to attack the fixed exchange rate if others do so as well, whereas there is an incentive to refrain from attacking if one expects the other market participants to do the same, beliefs turn out to be self-fulfilling and multiple equilibria result. A major drawback of these models is the ambiguity of possible outcomes, as the sole prediction to be made is that there exists a range of fundamentals for which a crisis is possible but does not have to occur necessarily. Moreover, the multiple equilibria approach does not explain the shift in beliefs, which incites the economy to move from one equilibrium to the next. Consequently, there is no way of generating comparative statics and policy devices can hardly be derived.

The procedure of eliminating a number of equilibria in a currency-market setting was firstly taken up by Morris and Shin (1998). Drawing on a concept by Carlsson/vanDamme (1993), they show that the introduction of noisy private information into a simple currency crisis model with multiple equilibria leads to a unique equilibrium as long as the amount of noise is relatively small. Adding noise removes multiplicity since agents' beliefs and actions can no longer be perfectly coordinated. In later models, they allow for private as well as public information and show that under certain conditions for the precision of private and public signals uniqueness of equilibrium is guaranteed (Morris/Shin 1999, 2000).

In this paper we make use of the Morris/Shin method and analyse a second-generation currency crisis model with private and public information. Based on the existing results for the uniqueness of equilibrium, we are able to derive comparative statics. Our aim is to analyse the influence of different model parameters, especially of varying precision of private and public information, on the probability of a currency crisis. Moreover, we want to find out whether and in which way the influence of varying precision depends on the fundamental state of the economy.

In our model, public information is disseminated by the central bank through publishing economic data and statistics. However, the publicly available information can be quite noisy. This might be due to the fact that economic concepts, which are the basis of statistical measurements, are faulty, or due to preliminarily published statistics with some underlying data still missing. We presume that the central bank can control for the precision of public information, for instance by prohibiting the publishing of preliminary and incomplete data or in general by controlling for the amount of data that is to be published. Nevertheless, the central bank can only determine the precision of its signal[1] ex-ante, whereas from then on this parameter stays constant. This assumption is plausible since a central bank's communication policy involves institutions that cannot be altered easily and quickly. The commonly observed public signal is then used by economic agents to make inferences over the fundamental state of the economy, which is unknown to them. Additionally to the public signal, each market participant receives an individual private signal, which is observed only by him. Although the precision is the same for all private signals, they might and will differ from each other.

As Morris and Shin (1999) show, uniqueness of equilibrium is guaranteed if the precision of private information is large relative to the precision of public information. Taking this condition as met, we are able to analyse comparative statics, giving particular attention to the influence of private and public information on the probability of a currency attack. In the setting of our model, we find that in case of a bad fundamental state of the economy, the probability of a currency crisis is higher the more precise the public information and the less precise the private information is. In contrast, in a situation with good fundamentals, a higher precision of the public signal and a lower precision of the private signal lead to a lower likelihood of a crisis.

Our main finding thus complements the results of several other research papers. These are mainly concerned with only one form of information at a time, either private or public. Whereas Heinemann/Illing (1999) analyse a model with only private information and infer that increasing the precision of this information always decreases the danger of a crisis, Sbracia/Zaghini (2001) compare models with public information on the one hand and with private information on the other. They conclude that "providing public information seems to be more convenient when fundamentals are 'rather bad' than when fundamentals are 'rather good'." In using a model with both public and private information we see that this result has to be corrected for the interaction between the two types of information.

[1]The terms *information* and *signal* are used interchangeably.

The remainder of the paper is organized as follows. Section two presents the basic form of the currency crisis model to be used. Section three sketches the unique equilibrium result by Morris and Shin. We briefly review how the lack of common knowledge of speculators' beliefs and actions eliminates dominated strategies and give the condition for the unique equilibrium strategy. The main part of the paper in section four concentrates on comparative statics of the unique equilibrium, which will be derived analytically and explained thoroughly. Section five analyzes the importance of private and public information for equilibrium selection. Section six concludes.

2 The Basic Model

We consider a small open economy where the central bank has pegged the exchange rate at a certain parity. There is a continuum of risk-neutral speculators in the foreign exchange market, indexed by the unit interval $[0, 1]$. Each speculator disposes of one unit of the currency and can decide whether to short-sell this unit, i.e. attack the currency peg, or not to do so. If the attack is successful he gets a fixed payoff D, $D > 0$. Taking a speculative position in the market, however, also leads to costs of t, $t > 0$, which comprise both transaction costs and the interest rate differential between the considered countries. We assume that costs t are small relative to the available payoff D, i.e. $t < D$, so that there is a potential incentive to attack the currency peg in the first place. If a speculator refrains from selling the currency he is not exposed to any costs, but he does not gain anything either.

Since we abstract from welfare considerations, we simply assume that the central bank is willing to defend the peg as long as the international reserves that it is endowed with, are above a predetermined critical level. This critical level depends on the central bank's assessment of the fundamental state of the economy. If the fundamentals are good, the critical level is low, so that the central bank will be willing to use a large amount of international reserves to defend the exchange rate. However, if the fundamentals are bad, the central bank will only want to lose few reserves before giving in to the attack and devalue the peg. In our model, an index of the fundamental state of the economy is given by θ, with a high value of θ referring to good fundamentals and a low value of θ representing bad fundamentals. Let the proportion of attacking speculators be denoted by l. If θ is sufficiently high, the central bank is able to always defend the peg, irrespective of the number of attacking speculators. Nevertheless, if θ is sufficiently low, the central bank abandons the peg in favour of a devaluation even if none of the speculators sell the currency. More precisely,

- if $l < \theta$, the central bank keeps the peg: an attack is unsuccessful
- if $l \geq \theta$, the central bank devalues the peg: an attack leads to success

The game between speculators and central bank is then structured as follows: Nature chooses the value of the fundamental index θ according to a uniform distribution over the real line.[2] This can be interpreted as the limiting case where speculators have very diffuse (almost no) prior information about the distribution of θ, so that they take each possible value as equally likely.[3] Nature's choice of θ can be observed by the central bank, but not by the speculators. After having observed θ, the central bank disseminates a *public signal* $y = \theta + \nu$, with $\nu \sim \mathrm{N}(0, \frac{1}{\alpha})$, $\alpha > 0$ and $E(\nu\theta) = 0$, so that the noise parameter is independent of the truly chosen fundamental state. This signal is public in the sense that it is common knowledge to all market participants.[4] The precision α of the public signal is exogenous to the model, i.e. α is chosen before the central bank gets to know the true value of θ and stays constant throughout the course of the game. The distribution of the noise parameter ν is common knowledge as well.

Additionally to the public signal, each speculator i individually receives a *private signal* $x_i = \theta + \varepsilon_i$, with $\varepsilon_i \sim \mathrm{N}(0, \frac{1}{\beta})$, $\beta > 0$. The noise parameters of the private signals are assumed to be independent of each other, of the fundamental state and of the noise parameter in the public signal: $E(\varepsilon_i\varepsilon_j) = 0$ for $i \neq j$, $E(\varepsilon_i\theta) = 0$ and $E(\varepsilon\nu) = 0$. The distributional properties of the noise parameter in the private signal are again presumed to be common knowledge to all speculators. However, as long as the precision β of the private signals is finite, private signals might differ from each other and speculators cannot accurately establish the signals of their opponents.

Thus, the information set I of speculator i in this model consists of two parts: the common public signal and the individual private signal: $I_i = (y, x_i)$. Note, that the public signal y enters every agent's information set, so that y not only provides information about the fundamental state of the economy but also about what other agents observe. On the basis of their information sets, speculators simultaneously have to decide whether to attack the currency or to stay with the

[2] This improper prior distribution with infinite mass presents no difficulties as long as we are concerned with conditional beliefs only. See also Hartigan (1983).

[3] For a more thorough discussion of this point see Morris/Shin (2000).

[4] Something is common knowledge, if everybody knows it, everybody knows that everybody else knows it, and so on to infinity.

peg. The central bank then observes the proportion l of attacking agents and decides on maintaining the peg (if $l < \theta$) or abandoning it (if $l \geq \theta$).

In order to derive the unique equilibrium, it is crucial to correctly define which elements of the game are common knowledge. These are payoff D, cost t, the public signal y and the distributional parameters of noise ν and ε, α and β respectively. Whether the fundamental state θ is common knowledge as well, is endogenous to the model, since θ becomes common knowledge if the public signal is infinitely precise. Not only is then θ commonly known, but each speculator can infer his opponents' optimal equilibrium strategies, which invites multiple equilibria.

If the fundamental index θ becomes common knowledge, we get the typical tripartition of fundamentals of a complete information game as in the original multiple equilibria model by Obstfeld (1996):

- If $\theta > 1$, the currency peg is *stable*, since the economy is sound enough so that the central bank is always able to defend the peg.

- If $\theta \leq 0$, the central bank always abandons the peg, irrespective of the speculators' actions and the currency peg is *unstable*.

- For $0 < \theta \leq 1$, the currency peg is said to be *ripe for attack*. In this interval, if all speculators attack, the central bank will be forced to devalue, whereas the peg will be kept if the speculators do not attack. However, since the agents will only attack the currency if they believe in success and will refrain from attacking otherwise, their actions vindicate the initial beliefs so that expectations are self-fulfilling for this range of fundamentals.

3 Incomplete Information - Unique Equilibrium

In the model of incomplete information we assume that α and β take on finite values, so that θ is prevented from becoming common knowledge.

In accordance with Morris and Shin (1999) we can state the following condition for a unique equilibrium:
If the private signals are sufficiently precise, i.e. for $\beta > \frac{\alpha^2}{2\pi}$, there exists a unique equilibrium consisting of a unique value of the fundamental index, θ^*, up to which the central bank always abandons the peg, and a unique value of the private signal, x^*, such that every speculator who receives a signal lower than x^* attacks the currency peg.

The general intuition behind this proposition is the following: In the depicted model, there is a unique fundamental value, denoted θ^*, which generates a distribution of private signals, such that there is exactly one signal x^*, which makes a speculator receiving this signal indifferent between attacking and not-attacking, and which - if all speculators with signals smaller than x^* decide to attack - generates a proportion of exactly $l = \theta^*$ of attackers that is just sufficient to force a devaluation of the currency peg.

Before we turn to evaluating the influence of the model parameters on the outcome of the game between speculators and central bank, we derive the equilibrium and show that it is indeed unique if the above condition is met.

3.1 Derivation of the Unique Equilibrium

The two equilibrium values θ^* and x^* belong to two situations of indifference: for $\theta = \theta^*$ the government is indifferent between defending the currency peg and abandoning it, whereas speculators receiving a private signal of x^* are indifferent between attacking the peg and refraining from doing so[5].

θ^* and x^* can be obtained as follows: Due to the assumption of normally distributed noise parameters, the distribution of θ conditional on private and public information is normal as well, so that the expected value of the unknown fundamental value of the economy conditional on player i's information is given by:

$$E(\theta|I_i) = \frac{1}{\alpha + \beta}(\alpha \cdot y + \beta \cdot x_i) \tag{1}$$

with variance

$$\text{Var}(\theta|I_i) = \frac{1}{\alpha + \beta} \tag{2}$$

As can be seen, the posterior expectation of θ is a weighted average of the information the speculator possesses. The higher the precision of the public information, α, the more important the public signal y gets, whereas the private signal gains importance the higher the precision β of this signal is.

However, since the public signal y is common knowledge for all speculators and as such does not help to distinguish player i's behaviour from player j's, we will in the following skip the public signal y as conditional argument whenever possible and only use the private signal x_i (resp. x_j).

After receiving the private and public signal, each speculator has to decide whether to attack the currency, which leads to costs of t and an uncertain payoff

[5]For reasons of mathematical tractability we assume that after receiving the signal x^*, a speculator decides to attack rather than not-attack.

D, or not to sell the currency which is associated with a net profit of zero with certainty. Indifference between these two possible actions is achieved if both lead to the same expected net profit:

$$0 = D \cdot \text{Prob}(\text{attack successful}|x) - t \tag{3}$$

Since the central bank will abandon the peg for all fundamental indices smaller than or equal to θ^*, the probability of a successful attack equals the probability that θ is smaller than or equal to θ^*, given x. Thus, with Φ denoting the cumulated normal density:

$$\begin{aligned} t &= D \cdot \text{Prob}(\theta \leq \theta^*|x) \\ &= D \cdot \Phi\big(\sqrt{\alpha+\beta}(\theta^* - \frac{\alpha}{\alpha+\beta}y - \frac{\beta}{\alpha+\beta}x)\big) \end{aligned} \tag{4}$$

The central bank is indifferent between defending the currency peg and abandoning it, if the proportion of speculators attacking the peg, l, equals θ. The proportion of attacking speculators, however, is given by the proportion of speculators who observe a private signal smaller than or equal to x^*. Since ε is assumed to be independent of the true value of θ, this proportion corresponds to the probability with which one single speculator observes a signal smaller than or equal to x^*, given θ. l can thus be calculated as:

$$\begin{aligned} l &= \text{Prob}(x \leq x^*|\theta) \\ &= \Phi(\sqrt{\beta}(x^* - \theta)) \end{aligned} \tag{5}$$

Hence, the central bank is indifferent between defending the peg and abandoning it if:

$$\theta = \Phi(\sqrt{\beta}(x^* - \theta)) \tag{6}$$

From equations (**??**) and (**??**) we can thus derive the indifference curve for the speculators, denoted by $x^{SP}(\theta)$, and the central bank, denoted by $x^{CB}(\theta)$:

$$x^{SP}(\theta) = \frac{\alpha+\beta}{\beta}\theta - \frac{\alpha}{\beta}y - \frac{\sqrt{\alpha+\beta}}{\beta} \quad \Phi^{-1}\Big(\frac{t}{D}\Big) \tag{7}$$

and

$$x^{CB}(\theta) = \frac{1}{\sqrt{\beta}}\Phi^{-1}(\theta) + \theta \tag{8}$$

The equilibrium is then given as the intersection point of the two indifference curves which can be seen from the following figure.

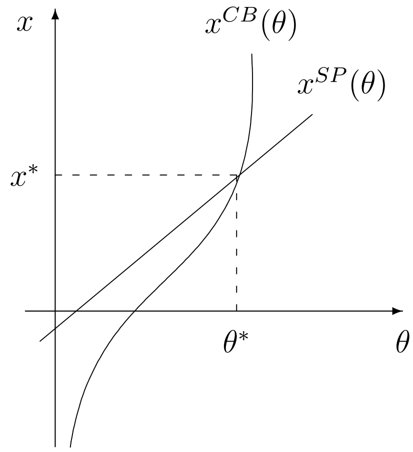

Figure 1: Determination of the unique equilibrium

The equilibrium value of θ is then determined as

$$\theta^* = \Phi\left(\frac{\alpha}{\sqrt{\beta}}\left(\theta^* - y - \frac{\sqrt{\alpha+\beta}}{\alpha} \quad \Phi^{-1}\left(\frac{t}{D}\right)\right)\right) \tag{9}$$

while x^* can be obtained from equation (**??**).

In the present model, (θ^*, x^*) forms a trigger-point equilibrium in the following respect: A speculator observing a private signal x lower than the switching signal x^* chooses *attack* as optimal action, whereas after observing a private signal higher than x^* *not attack* is the optimal action. In the same way, the central bank's optimal action is to *abandon the peg* whenever the observed fundamental value θ is lower than θ^*, but *keep the peg* is optimal if θ turns out to be higher than θ^*.

It is important to note, though, that the equilibrium values θ^* and x^* are given by the exogenous parameters of the model which are common knowledge to all players. Thus, the equilibrium can be determined before agents receive their signals and before they take any actions. However, the choice of the true fundamental state θ by nature determines whether there will be a crisis, by giving the distribution of public and private signals that incite the speculators to run on the currency peg or not to do so according to the above delineated decision process.

3.2 Condition for Uniqueness

To show that the equilibrium is unique, we have to prove that there can be only one value of the fundamental index and one value of the private signal which make both the central bank and the speculators indifferent, i.e. there is only one intersection point of $x^{SP}(\theta)$ and $x^{CB}(\theta)$. This condition for a unique equilibrium is satisfied if one of the indifference curves runs steeper than the other throughout the whole range of possible values. Since neither of the two indifference functions is limited to any range, the unique equilibrium then exists with certainty.

The slopes of the two indifference curves are equal to:

$$\frac{\partial x^{*SP}}{\partial \theta^*} = \frac{\alpha + \beta}{\beta} \tag{10}$$

and

$$\frac{\partial x^{*CB}}{\partial \theta^*} = \frac{1}{\sqrt{\beta}} \cdot \frac{\partial \Phi^{-1}(\theta^*)}{\partial \theta^*} + 1 \tag{11}$$

respectively.

Thus, the sufficient (but not necessary) condition[6] for a unique equilibrium is satisfied, if

$$\frac{\alpha + \beta}{\beta} < \frac{1}{\sqrt{\beta}} \quad {}_{min}\left(\frac{\partial \Phi^{-1}(\theta^*)}{\partial \theta^*} \right) \quad + \quad 1 \tag{12}$$

For the following derivation of the uniqueness condition note that the smallest value of $\frac{\partial \Phi^{-1}(\theta^*)}{\partial \theta^*}$ is equal to the reciprocal of the maximum value of the partial derivative of $\Phi(\theta^*)$ with respect to θ^*. This maximum value is given by the normal density $\phi(\theta^*)$ at its mean μ with $\phi(\mu) = \frac{1}{\sqrt{2\pi}}$. Thus, the above sufficient condition of uniqueness is fulfilled, if

$$\begin{aligned} \frac{\alpha + \beta}{\beta} \quad &< \quad 1 + \frac{1}{\sqrt{\beta}} \cdot \frac{1}{\frac{1}{\sqrt{2\pi}}} \\ \beta \quad &> \quad \frac{\alpha^2}{2\pi} \end{aligned} \tag{13}$$

Hence, for a given precision of the public signal, α, the depicted equilibrium is unique as long as the precision of the private signal, β, is high enough. If θ^* is unique, then x^* must be unique as well.

Additionally to proving that there is only one equilibrium trigger strategy around (θ^*, x^*), it can also be shown that this trigger strategy is the only strategy which survives the iterative elimination of dominated strategies. For a complete conduction of this proof we refer to Morris/Shin (1999)

[6] The two necessary conditions for the unique equilibrium to exist are $D > t$, and $E(\varepsilon_i \varepsilon_j) = 0$.

4 Comparative Statics

In the following, we assume that uniqueness of equilibrium is guaranteed, i.e. $\beta > \frac{\alpha^2}{2\pi}$, and examine the influence which the different parameters exert on the unique switching point (θ^*, x^*). Define the probability of a currency crisis as being proportional to the size of the interval $[-\infty, \theta^*]$, since for values of θ in this interval the fixed exchange rate will be devalued. Thus, the higher the switching point θ^* turns out to be, the higher is the danger of a currency crisis and vice versa.[7]

From equation (**??**), which gives the equilibrium value of the fundamental index, we can infer the following propositions:

Proposition 1 The probability of a currency crisis *rises* whenever t *decreases* and/or D *increases.*

Proof:
The partial derivatives of θ^* with respect to t and D are:

$$\frac{\partial \theta^*}{\partial t} = \phi(\cdot)\left(\frac{\alpha}{\sqrt{\beta}}\frac{\partial \theta^*}{\partial t} - \sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial \Phi^{-1}(\frac{t}{D})}{\partial t}\right) = \frac{-\sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial \Phi^{-1}(\frac{t}{D})}{\partial t}\phi(\cdot)}{1-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}} < 0$$

$$\frac{\partial \theta^*}{\partial D} = \phi(\cdot)\left(\frac{\alpha}{\sqrt{\beta}}\frac{\partial \theta^*}{\partial D} - (-1)\sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial \Phi^{-1}(\frac{t}{D})}{\partial D}\right) = \frac{\sqrt{\frac{\alpha+\beta}{\beta}}\frac{\partial \Phi^{-1}(\frac{t}{D})}{\partial D}\phi(\cdot)}{1-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}} > 0$$

The partial derivative of θ^* with respect to t (D) is always negative (positive), since due to the condition of uniqueness the denominator stays positive and nonzero. A rising t (D) thus decreases (increases) the switching value θ^* and thereby the probability of an exchange rate crisis. ■

Increasing costs t reduce the expected net profit of an attack for every probability of success. Consequently, controlling for the costs of international capital transactions might be a possibility to prevent speculative attacks on currency pegs. This result obviously favours the introduction of a tax on international capital transactions in order to avoid currency crises.

[7]Note, that due to the assumed improper prior distribution this definition of probability is, strictly taken, flawed. However, working with a more realistic prior distribution should lead to similar results. As such, it is justified to work with the simpler concept of an improper prior distribution and at least approximate the true probability of a currency crisis with the above definition.

Proposition 2 The public signal, y, influences the probability of a currency crisis *negatively.*

Proof:

$$\frac{\partial\theta^*}{\partial y} = \phi(\cdot)\left(\frac{\alpha}{\sqrt{\beta}}\frac{\partial\theta^*}{\partial y} - \frac{\alpha}{\sqrt{\beta}}\right) = \frac{-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}}{1-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}} < 0$$

The higher the public signal, the lower the switching point θ^* turns out to be, and thus the narrower gets the range of fundamentals for which an attack would be successful and vice versa. ■

Thus, the higher the public signal about the fundamental state of the economy, the lower is the probability of a crisis. Since the public signal y is symmetrically distributed around the realized fundamental index θ, $E(y|\theta) = \theta$, we can moreover state that y tends to be high if the realized fundamental index θ is high and vice versa. As such, the fundamental index θ has an - albeit indirect - negative effect on θ^*, which is the stronger the higher α, i.e. the more precise the public signal is. This is a desirable feature of our model which is not contained in the models of multiple equilibria. Whereas under common knowledge of θ the switch in beliefs about the outcome of the game does not depend on the fundamentals, there is a dependence in the model of incomplete knowledge through the public signal.

Proposition 3 If $\theta^* > y + \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$, the precision of the private signal β exerts a *negative* influence on the probability of a currency crisis.

If $\theta^* < y + \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$, the precision of the private signal β exerts a *positive* influence on the probability of a currency crisis.

Proof:

$$\begin{aligned}\frac{\partial\theta^*}{\partial\beta} &= \phi(\cdot)\left(-\frac{\alpha}{2\sqrt{\beta^3}}\theta^* + \frac{\alpha}{\sqrt{\beta}}\frac{\partial\theta^*}{\partial\beta} + \frac{\alpha}{2\sqrt{\beta^3}}y + \frac{\alpha}{2\beta^2}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)\right) \\ &= \frac{\phi(\cdot)\left(-\frac{\alpha}{2\sqrt{\beta^3}}\theta^* + \frac{\alpha}{2\sqrt{\beta^3}}y + \frac{\alpha}{2\beta^2}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)\right)}{1-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}}\end{aligned}$$

In the unique equilibrium, $\frac{\partial\theta^*}{\partial\beta}$ is negative if θ^* is larger than $y + \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$, so that the numerator becomes negative, whereas $\frac{\partial\theta^*}{\partial\beta}$ is positive if $\theta^* < y + \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$. ■

Hence, if the switching value θ^* exceeds the threshold $y+\frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$, a rising precision of the private signals decreases the probability of a currency crisis. In contrast, if θ^* falls short of the threshold, a higher precision of the private signals increases the likelihood of a crisis. Since the threshold-function $y+\frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$ increases in y, whereas θ^* decreases in y according to proposition 2, there must be a value of y, denoted as y_β, such that θ^* is exactly equal to the threshold:[8] $\theta^*(y_\beta)=y_\beta+\frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$. It is easy to see that for all public signals lower than y_β, a higher precision of the private signals is affiliated with a lower probability of a crisis, whereas for all public signals higher than y_β more precise private signals lead to a higher probability of a currency crisis. Since the public signal tends to be high if the realized fundamental index θ is high and vice versa, it follows that increasing the precision of the private signals influences the probability of a crisis negatively in case of bad fundamentals and has a positive influence in case of good fundamentals.

For interpreting the influence of β on the probability of a currency crisis, note that a speculator deciding on his optimal action has to take two aspects into account: on the one hand, he wants to choose an action that is appropriate to the realized but unknown fundamental state. On the other hand, he knows that for the relevant intermediate range of fundamentals it is possible to force a devaluation of the currency peg through sheer speculative pressure. As such, he wants to coordinate his own decision on his opponents' actions, so that even for good fundamentals a devaluation might be achieved.

Whereas both signals give information about the fundamental state of the economy, only the private signals have a direct effect on the coordination incentive described above. Speculators decide on their optimal action solely based on their specific information set. Equivalent information sets lead to the same actions. What makes speculator i's information set different from speculator j's is only the private part, since private signals might differ from each other. The more precise the private signals are, however, the more closely they will be distributed around the truly realized fundamental state θ and the more similar the information sets will be. This amounts to saying that varying the precision of the private signals foremost affects the coordination incentive: the higher β the easier it is to coordinate on a certain action. However, both private and public signal exert an indirect effect on the coordination incentive. The more precise one type of signal is, the higher is the respective weight that is attached to this signal in calculating the expected value of the unknown fundamental index θ. Since the

[8] y_β exists with certainty, since the public signal y is not restricted to any range of values.

weighing scheme of the signals is common knowledge, this indirect coordination effect is not negligible but quite important.

The interpretation of the effects of an increasing precision β is then quite intuitive. The more precise the private signals, the less weight will be given to the informational content of the public signal. Take the case of good fundamentals, where the public signal is high. With an intermediate precision of the private signals, speculators tend to refrain from attacking since they know that for good fundamentals a large proportion of agents has to coordinate on the attack-action in order to force a devaluation. If, in contrast, the precision of private signals is extremely high, speculators will simply neglect the content of the public signal, which tells them that the fundamental state of the economy is good. Consequently, they will become more aggressive in attacking the peg compared to a situation with less precise private signals, so that the probability of a currency crisis increases. In case of bad fundamentals, the reverse holds. Here, the public signal will be low, so that speculators should want to attack, since a devaluation can easily be achieved. If, however, the private signals are extremely precise, speculators neglect the informational content of the public signal and refrain from attacking, which leads to a lower crisis probability in case of bad fundamentals.

Proposition 4 The precision of the public signal α exerts a *positive* influence on the probability of a currency crisis if $\theta^* > y + \frac{1}{2\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$.

However, if $\theta^* < y + \frac{1}{2\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$, the precision of the public signal α exerts a *negative* influence on the probability of a crisis.

Proof:

$$\begin{aligned}\frac{\partial\theta^*}{\partial\alpha} &= \phi(\cdot)\left(\frac{1}{\sqrt{\beta}}\theta^* + \frac{\alpha}{\sqrt{\beta}}\frac{\partial\theta^*}{\partial\alpha} - \frac{1}{\sqrt{\beta}}y - \frac{1}{2\beta}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)\right) \\ &= \frac{\phi(\cdot)\left(\frac{1}{\sqrt{\beta}}\theta^* - \frac{1}{\sqrt{\beta}}y - \frac{1}{2\beta}\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)\right)}{1-\phi(\cdot)\frac{\alpha}{\sqrt{\beta}}}\end{aligned}$$

In the unique equilibrium, the partial derivative of θ^* with respect to the precision of the public signal α is positive if $\theta^* > y + \frac{1}{2\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$, whereas it is negative if θ^* is lower than $y + \frac{1}{2\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$. ■

Thus, if the equilibrium switching value of θ^* is high enough to exceed the threshold $y + \frac{1}{2\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{t}{D}\right)$, increasing the precision of the public signal raises the probability of a currency crisis. If, however, θ^* falls short of the threshold, increasing α decreases the likelihood of a crisis. Again, the threshold-function increases in the value of the public signal y, whereas θ^* decreases in y, so that there must be a value denoted by y_α, so that $\theta^*(y_\alpha) = y_\alpha + \frac{1}{2\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})$. Consequently, for public signals lower than y_α, a higher precision α of the public signal leads to a higher probability of a currency crisis. If, however, y is higher than y_α, an increased precision of public information leads to a lower crisis probability. Since the value of the public signal tends to be high if the realized fundamental state is good and vice versa, we find that a higher precision of public information leads to a higher danger of a crisis if the fundamentals of the economy are bad, whereas the probability of a crisis will be lower with very precise public information if this information is about good fundamentals.

These effects are again quite intuitive. In contrast to the private signal, the public signal only gives information about the fundamental state of the economy and is included in each speculator's information set. Since it cannot be used to differentiate between individual private signals, the public signal has no direct influence on possible coordination effects. Consequently, if the public signal is known to be very precise and if it indicates a bad fundamental state of the economy, this clearly tells speculators to attack the currency peg for two reasons: first, each speculator knows that in case of bad fundamentals the proportion of attacking agents necessary to force a devaluation is not very high, which increases the probability of a successful attack. Second, if the public signal is very precise, each agent knows that all other agents will put more weight on the public signal in calculating the expected value of θ, so that there is an indirect coordination effect. For good fundamentals, exactly the opposite effects occur.

The results of propositions 3 and 4 are summed up in the following figure:[9]

[9]It has been assumed here that $\frac{1}{2}D < t < D$ so that $y_\beta > y_\alpha$. If, instead, $0 < t < \frac{1}{2}D$ then y_β lies to the left of y_α and for all public signals in-between, both α and β exert a positive influence on θ^*.

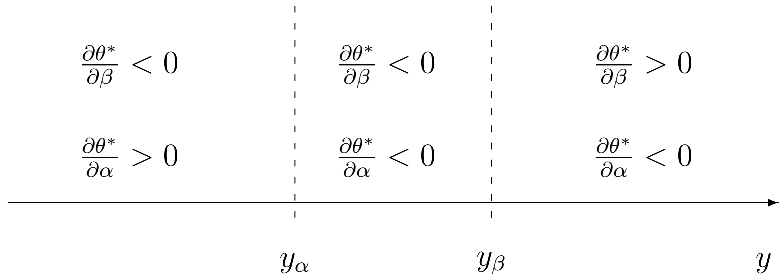

Figure 2: Influence of α and β

What can be seen from the discussion of comparative statics in this section is that *better* information, in the sense of *more precise* information, does not always lead to a lower probability of a currency crisis, as has been suggested by former models. Moreover, the mechanisms of a changing informativeness are very complex. The most important result, however, is that with the given structure of private and public signals, the fundamental state of the economy plays an important role in determining the influence of the signals' precision on the probability of a currency crisis. If the fundamental state of the economy is good, a higher precision of the public signal leads to a lower probability of a crisis, whereas very precise private signals tend to increase the likelihood of a crisis. For bad fundamentals, the influence of the signals' precision is exactly the reverse. As can be seen from the above figure, there is only a small range of public signals (and as such of fundamental values), where both signals exert the same influence on the probability of a crisis. However, this interval for y, respectively θ, vanishes with increasing α and/or β.[10]

5 Unique and Multiple Equilibria and the Importance of Private and Public Information

Since we know that a unique equilibrium can only be sustained for rather precise private signals relative to the public signal, a declining β, respectively a rising α, will eventually lead to multiple equilibria. This can easily be seen from the figure

[10]The length of the interval of equal influence of α and β is given by $\frac{1}{2\sqrt{\alpha+\beta}}\Phi^{-1}(\frac{t}{D})$, which decreases in α, β and D, but increases in t.

below, which depicts the sufficient condition for uniqueness of equilibrium.[11]

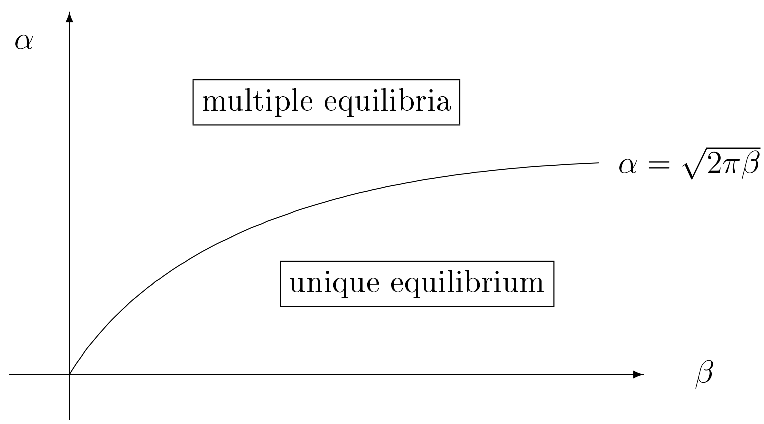

Figure 3: Regions of unique and multiple equilibria

If β declines, while α stays constant, speculators are not able to precisely establish the information received by the other agents, so that they are not able to coordinate on a specific action for a given θ. Since they cannot be sure that the necessary amount of coordination, i.e., the proportion of attacking agents, will be achieved, the optimal action for them at some point is again to attack if everyone else attacks and to refrain from short-selling if that is what they expect everyone else to do. With β going to zero, the private part in the information sets will simply be neglected, so that information sets are the same for all speculators which invites multiple equilibria. However, if the precision of public information α increases for a given β, the informational content of the private signal falls more and more behind so that it is eventually neglected and again, multiple equilibria arise.

Hence, in case of a bad fundamental state of the economy, both a very precise public signal and a very imprecise public signal, i.e. high as well as low α, can lead to an outcome of the game with *not-attack* being the chosen strategy: On the one hand, if α is sufficiently low relative to β so that a unique equilibrium is guaranteed, bad fundamentals lead to θ^* exceeding the above given threshold so that a low (lowering) α brings about a small (decreasing) probability of a currency crisis. On the other hand, if the public information's precision is extraordinarily high, so that the condition for uniqueness is violated, there is at least a certain,

[11] Note, however, that due to the depicted condition only being sufficient but not necessary for uniqueness, there might be a unique equilibrium above the line $\alpha = \sqrt{2\pi\beta}$ as well. Yet, generally, we will refer to the upper area as the multiple equilibria region.

however not calculable, probability that in the revived multiplicity of equilibria speculators will coordinate on the no-attack equilibrium, since this coordination will no longer depend on the fundamental state of the economy. However, it obviously stands to reason if an increased uncertainty of the realized equilibrium will ever be preferred, even with very bad fundamentals.

6 Conclusion

From the delineated model we are able to see that the introduction of noise to the information gathering process of speculators brings a further crucial condition (of a net expected profit equal to zero) into the model. Consequently, all equilibria but one are eliminated, if the precision of private information is high enough relative to the precision of public information.

In contrast to the earlier multiple equilibria models of currency crises we are now able to give directions for policy devices. First, the increase of transaction costs certainly reduces speculators' incentives to intervene on international financial markets and therefore reduces the probability of a speculative currency attack. Second, the better (i.e. the higher) the public signal about the fundamental state of the economy, the lower is the danger of a crisis. Third, the sheer increase in the amount of information in the market, i.e. increasing the precision of information, is obviously not enough to prevent currency crises. In particular, if the fundamental state of the economy is bad, disseminating very precise public information is crucial as it increases the probability of a currency crisis further, whereas a high precision of private signals will decrease it. In contrast, in case of good fundamentals, the precision of public information decreases the probability of a currency crisis, while the precision of private information raises it.

However, there is still a number of open questions. Sbracia and Zaghini (2001) for instance analyse further conditions for unique and multiple equilibria in a slightly different setting. Hellwig (2000) investigates into information structures in more detail and connects the resulting equilibrium with higher-order uncertainty. Morris and Shin (2001) are concerned with welfare effects of public information in models with strategic complementarities. Similarly, Chui/Gai/Haldane (2000) analyse the implications of sovereign liquidity crises for public policy. Another interesting aspect is to depart from simultaneous move games, in which speculators have to decide on their strategies at the same point in time, and to look at sequential move games. In these models, agents not only get a private signal but they can also observe the actions that earlier speculators decided on. As an example of such a model see for instance Dasgupta (2000). An again different aspect in current research is to allow for speculators of different sizes (Corsetti/Dasgupta/Morris/Shin (2000)).

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