Persistent misalignments of the European exchange rates : some evidence from nonlinear cointegration*

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Abstract

This paper investigates the asymmetric and persistent adjustment of the European real exchange rates using the framework of nonlinear cointegration. We explain the episodes of slow mean-reversion dynamics over the period from 1979 to 1999. A test of unit root against STAR cointegration is proposed and we present some complete estimations and stochastic simulations of ESTAR models. We conclude to the presence of effective nonlinear adjustment during the moving of the currencies to their long-run fundamental equilibrium exchange rate value.

Keywords: Nonlinear cointegration - Equilibrium exchange rates - STAR models - Asymmetry

JEL classification: C32, C51, F31, F49

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1 Introduction

The backbones of equilibrium exchange rate economics have been for decades the theories of purchasing power and interest rates parity, the monetary model and the more or less sophisticated variants of the fundamental equilibrium exchange rate models. Up until the eighties, the widespread intuition was that nominal and real exchange rates could not be misaligned over long periods, given the mean-reversion mechanisms of the economic fundamentals. Excessive wide movements were interpreted as bubbles that originated from speculative behaviors or market rigidities (such an interpretation was for instance the core message of Dornbusch (1976)'s model).

Still, the comprehension of the adjustment mechanisms of the exchange rates remains a challenge for governments. Indeed, over the eighties and the nineties many empirical works have emphasized the poorly econometric performance of the traditional theories in explaining the observed movements of the exchange rates. A specific question was their long lasting misalignments with the economic fundamentals, especially during periods of floating regimes. In order to deal with what appears to be an "exchange rate puzzle" two competing approaches are generally used.

A first category of authors argue that the rejection of the traditional theories tends to indicate that the initial deviations are often cumulative so that the misalignments are likely to last forever. People sharing this view report evidence of a unit root behavior in the deviations of exchange rates, or argue that the fundamental themselves sometimes evolve in a "pathological" way. Papers regarding the dynamics of exchange rates as random walks are numerous and include the works of Darby (1983), Huizinga (1987), Corbae and Ouliaris (1988), Meese and Rogoff (1988), Taylor (1988), Edison and Fisher (1991), Engel, Hendrickson and Rogers (1997). Papers on the non-attracting character of the economic fundamentals provide some evidence of multiple equilibria (Villa (1997)), intrinsic bubbles (Ikeda and Shibata (1995)) or episodes of crises like hyperinflation. These studies, however, cast some doubts about their validity. Firstly, several studies provide a strong evidence against the hypothesis of a unit root (see Grilli and Kaminsky (1991), Pedroni (1995), Frankel and Rose (1996), Lothian and Taylor (1996), Lothian (1997), Papell and Theodoridis (1998), Higgins and Zakrajsek (1999)) Secondly, the observation of unstable fundamentals is in general episodic. .

The exchange rate puzzle paved the way to a second line of arguments. There is a growing recognition that the introduction of nonlinearities in the modelling approach made it possible to explore the slowness of the adjustment process toward the long-run equilibrium. This allows a finer analysis of the persistence properties of the misalignments regarding the numerous causes of nonlinear dynamics. To name just a few, the following factors are at play : the transaction costs (Dumas (1992), Sercu, Uppal and Van Hulle (1995), O'Connel and Wei (1997), Obstfeld and Taylor (1997)), the heterogeneity of buyers and sellers (Taylor and Allen (1992)), speculative attacks on currencies (Flood and Marion (1998)), the presence of target zones (Krugman (1991)), noisy traders (De Long, Shleifer, Summers and Waldmann (1990)). All these factors imply, either a nonlinear relationship between the exchange rates and the economic fundamentals, or a nonlinear adjustment mechanism with time-dependence properties.

Applied econometric studies along this second line of arguments have been developed in two directions. Some authors argue that we need not to find the exact nonlinear structures underlying the exchange rates deviations. It is enough to show that there are nonlinearities at work during the adjustment process. Regarding this argument, one can estimate nonlinear nonparametric long-run relationships (see for instance, Ma and Kanas (2000), Bec, Ben Salem and MacDonald (1999), Dufrénot and Mignon (2002)). In some circumstances, some nonlinearities do, however, give rise to specific reduced forms. For instance, if we think that target zones or transaction costs play a crucial role in the adjustment mechanism, then a possible reduced form model should include threshold variables. Such an argument has led to use threshold autoregressive models (TAR) and smooth threshold autoregressive models (STAR) (see, MacDonald (1997), Michael, Nobay and Peel (1997), Pippenger and Goering (1998), Baum, Barkoulas and Caglayan (2001), Taylor, Peel and Sarno (2001)).

This paper is a further contribution to the issue of the nonlinear adjustment of the real exchange rates. We show some evidence of persistent mean-reverting effects for some currencies of the members of the European Monetary Union (EMU) over the period from 1979 to 1999. Clearly, we find that strong nonlinearities do account for sustained periods of overvaluation and undervaluation. Our long-run equilibrium concept is richer than the PPP or PPI theories. We use a behavioral equilibrium exchange rate *a la* Clark and MacDonald (1999). This allow to model the economic fundamentals as a complete set of macroeconomic variables reflecting the internal and external balances (interest rate differential, net foreign assets, the ratio of governments' deficit over GDP, the terms of trade, the prices of tradable over non tradable goods). Our methodology for modelling the nonlinear adjustment of the European real exchange rates involves the following steps.

(i) Using a behavioral equilibrium exchange rate as a long-run concept for the exchange rates poses some problems. Indeed, the regressors in our static equation, neither have the same order of integration, nor exhibit the same

type of nonstationary behavior. Some variables are $I(0)$ (for instance, the interest rate differentials for some countries), while others are $I(1)$. Moreover, the unit root and stationarity tests contradict each others. In some cases, the ADF and Phillips and Perron tests conclude in favor of the unit root hypothesis, while KPSS or Zivot-Andrews tests reject this assumption. In the circumstances, the standard cointegration tests based on Engle-Yoo's or Johansen 's approaches are not valid. So, the first problem we face is to use an appropriate approach for testing the hypothesis of linear cointegration and see whether the assumption of a fast long-run convergence is still rejected. This issue is important to avoid that the finding of a nonlinear dynamic appears as factious and caused by an inappropriate testing approach[1]. In this paper, we use a bounds testing approach following Pesaran, Shin and Smith (2001). We show that, even when an appropriate approach is used, there are some countries for which the null hypothesis of nonlinear cointegration is strongly rejected, thereby suggesting a failure to detect a mean-reverting dynamics in the linear context.

(ii) As evidenced in the recent empirical literature, the presence on nonlinear components can strongly affect the conclusions obtained in terms of linear cointegration (for a survey, see Dufrénot and Mignon (2002)). The rejection of the linear cointegration hypothesis in the PSS approach does not mean that z_t is necessarily $I(1)$. Indeed, if z_t is $I(1)$, then using the PSS approach yields to reject the hypothesis of linear cointegration. But, the reciprocal is not true. However, we assume that the unit root hypothesis is a possible origin of the rejection of linear cointegration, as this case is usually examined in the literature.Accordingly, we run procedures that test the null hypothesis of unit root in the residuals of our long-run equations against the alternative of nonlinear adjustment of the deviations of the real exchange rates. We consider a specific parametric form, namely STAR models. Such models can be envisaged as reduced forms of structural models of fundamental exchange rate accounting for transaction costs, changing-regimes fluctuations, asymmetric deviations due to policy interventions,... We show that our test has a good power for large sample and conclude in favor of a STAR adjustment for several European currencies.

(iii) In regard to the results of our nonlinearity tests, we then estimate exponential smooth autoregressive models (ESTAR). The results point out that the transition to the long-run equilibrium is slow over some periods, yielding persistent mean-reverting dynamics. Additionally, we compute generalized impulse response functions (GIRF) to show the asymmetric patterns

[1]Although never discussed in the literature, the same question is posed for standard models such as the monetary model, PPI and PPP models.

in the response of the real exchange rates to different innovations.

The paper is structured as follows. In section 2, we briefly sketch out the long-run equilibrium model (the behavioral equilibrium exchange rate model) and show that the linear cointegration hypothesis is rejected for some countries. Section 3 presents a test, of unit root against stationary STAR alternatives. Monte Carlo simulations are done, that show the power and size of the tests. Section 4 contains the estimates of the STAR models (as shown, in all cases the best specifications are ESTAR models). This allows us to construct indicators of the degree of misalignments and the length of mean-reversion. In section 5, we use stochastic simulations to compute GIRF in order to provide some evidence of asymmetric adjustments. Section 6 concludes the paper.

2 Rejecting the mean-reversion hypothesis from linear cointegration models

2.1 The theoretical model

In regard to the stylized facts, most of the currencies of the countries participating in the ERM have been characterized by repetitive episodes of wide movements together with a varying volatility over the period from the beginning seventies to the end nineties. Such movements were observed despite the existence of a managed floated regime. They mirrored several factors at play in the European countries. Firstly, many countries often experienced periods of speculative attacks as a test of the governments' credibility in controlling the movements of the exchange rates (the empirical studies strongly reject the hypothesis of a target zone over the eighties and the nineties (see Lindberg and Soderlind (1994)). Secondly, the failure to predict fast realignments was certainly due to the sharp deterioration of the macroeconomic fundamentals in some countries over the mid eighties and the nineties (see Artis and Nachane (1990)).

The stylized facts thus provide an intuition against the idea of quick realignments and mean-reversion dynamics. This would mean that the European currencies do not move in line with the economic fundamentals. To test this conjecture, we adopt a framework based on the behavioral equilibrium exchange rate approach (BEER), which in its spirit is very close to the fundamental equilibrium exchange rate concept (FEER) (for a survey on FEER, see Williamson (1994)). A simple formulation of a BEER equation has been proposed by Clark and MacDonald (1999, 2000)[2] :

$$\log(q_t) = \alpha_0 + \alpha_1 \log(TOT_t) + \alpha_2 \log(TNT_t) + \alpha_3(r_t - r_t^*) + \alpha_4\lambda_t + \alpha_5 NFA_t + \alpha_6 FISCAL_t + \varepsilon_t, \quad (1)$$

where ε_t is an *iid* process. q_t is the real effective exchange rate, TNT is the ratio of tradable goods over non-tradable goods, TOT is an indicator of the terms of trade, $(r - r^*)$ is the interest rate differential, λ is the ratio of the government deficit to GDP, NFA is the ratio of the net foreign assets to GDP and $FISCAL$ is the fiscal wedge. This equation is a BEER equation in the sense that the parameters are not calibrating in order to satisfy some normative structural requirements as is usually the case in FEER models. One simply say that the equilibrium exchange rate must be compatible with the internal and external balances. Further, the regressors are thought of as driving both short-medium term and long-term components in the exchange rates. The short-medium term variables are the interest rate differential (which captures the influence of risk premium) and the ratio of government deficit to GDP (which is included as a proxy of the cyclical variations of the activity). The other variables influences the exchange rates in the long-term. The terms of trade capture the Balassa-Samuelson effect and the ratio of net foreign assets to GDP accounts for the influences of the determinants of national saving and investment[3].

A standard approach in testing the validity of the BEER as a long-run equilibrium concept is to see whether the exchanges rates and their fundamentals are cointegrated. Our variables show different types of nonstationary behavior, not necessarily corresponding to the unit root hypothesis[4]. In this case, the classical methods would yield misleading conclusions. An alternative cointegration testing approach is needed, which relies here upon Pesaran, Shin and Smith (2001)'s methodology (henceforth, PSS).

2.2 Data sources and construction of the variables

We consider the following countries : Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Por-

[2] Alternative approaches for studying the misalignments of the exchange rates are surveyed by Driver and Westaway (2001)

[3] These regressors are standard in the modelling of real exchange rates (for surveys, the reader may refer to Alberola, Susana, Humberto and Ubide (1999), Stein (1999), Lane and Milesi-Ferretti (2000).

[4] To avoid too many tables, the results of the ADF, PP, KPSS, ERS and mixing tests are not reported here, but they are available upon request to authors.

tugal, Spain, Sweden and the United Kingdom. The data are obtained from three sources: the IMF's International Financial Statistics, the OECD Analytical Database and Eurostat Database. The data are quarterly and span the period 1979:1-1999-2 giving a total of 82 observations[5]. The construction of the variables follows Clark and McDonald (1999).

- The real effective exchange rate (REER) is an indicator of competitiveness between two countries. It is defined as a price ratio of domestic nontraded goods to foreign countries (CPI-based). We compute a multilateral CPI-based REER as the ratio of the domestic REER to its European partners. The weights (ω) are based on Zanello and Desruelle (1997). This variable is noted q and we work with its logarithm ($\log(q)$)

$$q_i = \frac{REER_i}{\sum_{j=1}^{14} \omega_{i,j} REER_j} \qquad i = 1, ..., 15. \tag{2}$$

- The terms of trade of a country is defined as the ratio of the export unit value (EXP) to the import unit value (IMP). Here, we compute for each country the ratio of the domestic terms of trade to the weighted terms of trade of the partners (the same weights as described above are used). The variable is noted TOT and is log transformed ($\log(TOT)$):

$$TOT_i = \frac{EXP_i/IMP_i}{\sum_{j=1}^{14} \omega_{i,j} EXP_j/IMP_j} \qquad i = 1, ..., 15. \tag{3}$$

- The price of traded goods to nontraded goods for a country is defined as the ratio of the Consumer Price Index (CPI) to the Producer Price Index (PPI). We computed the ratio of this index for each country to its equivalent weighted foreign average. The variable is noted TNT and is log transformed ($\log(TNT)$):

$$TNT_i = \frac{CPI_i/PPI_i}{\sum_{j=1}^{14} \omega_{i,j} CPI_j/PPI_j} \qquad i = 1, ..., 15. \tag{4}$$

- The ratio of net foreign assets over GDP is noted NFA.

[5] We did not succeed to get a whole set of the same variables for all the countries. Consequently, some regressors are included in some regressions while missing in others. For instance, the variable λ is missing for Finland, Greece and Sweden. The variable $r - r^*$ is not included in the regressions for Denmark, Finland and Greece. The variable $fiscal$ is missing for Finland.

- The stock of government debt was not available for all the countries, so we used as a proxy the government deficit (DEF) relative to the GDP. We then computed the ratio of the domestic deficit to the foreign deficit according the same methodology. The variable is noted (λ):

$$\lambda_i = \frac{DEF_i/GDP_i}{\sum_{j=1}^{14} \omega_{i,j} DEF_j/GDP_j} \qquad i = 1, ..., 15. \tag{5}$$

- The real interest rates of the domestic country (noted r) is computed as the difference between the nominal ten year government bond yield and the change in the CPI from the previous year. The foreign real interest rates (noted r^*) is a weighted average of the partners real interest rates. In the regressions, we used the real interest rates differential: $r - r^*$.

- The fiscal wedge (noted fw) is computed for each country as:

$$fw_i = \sum_{k=1}^{3} \log(1 + t_k) \qquad i = 1, ..., 15. \tag{6}$$

  where t_1 is the employers' insurance contribution, t_2 is the employees' insurance contribution and t_3 is the income tax ratio. Once the fiscal wedge obtained, we computed the ratio of the domestic fiscal wedge to the weighted average of the fiscal wedges of the partners. This variable is noted $FISCAL$:

$$FISCAL_i = \frac{fw_i}{\sum_{j=1}^{14} \omega_{i,j} fw_j} \qquad i = 1, ..., 15. \tag{7}$$

2.3 Testing for linear cointegration using the PSS approach

As discussed in the introduction, the first step when analyzing a long-term equilibrium relationship is to test whether the series are $I(0)$ or $I(1)$. To save place, we do not report the results of the unit root and stationarity tests, but briefly sketch out our main conclusions. Several tests were used, based on both the unit root and stationarity null hypotheses.(ADF, Phillips-Perron, KPSS, Zivot-Andrews). They yielded contradictory conclusions. For instance,for some countries the series $r - r^*$ was considered as $I(0)$ using ADF and PP tests and as $I(1)$ using ZA test. Similarly, the series $\log(TOT)$ sometimes appeared to be $I(1)$ with ADF and PP tests but $I(0)$ according to ZA test. So, the classical procedures were inconclusive

To solve this problem, we use a bound testing approach, as suggested by proposed by Pesaran, Shin and Smith (2001). Their approach is quite suited for testing the stationarity properties of a long-run relationship involving one dependent variable and independent variables for which an uncertainty exists regarding the degree of integration of the regressors, the case we are concerned with. Over the five cases considered by the authors, only three cases correspond to our data: case III, IV, and V along the terminology of the authors. Using the same notations as in Pesaran, Shin and Smith (2001), the endogenous variable is noted $\Delta \log(q_t) = \Delta y_t$ and the tth-row of the regressors' matrix is noted:

$$\left(\mathbf{x}_t^{'} = log(TOT_t),\, log(TNT_t),\, NFA_t,\, \lambda_t,\, (r_t - r_t^{*}),\, FISCAL_t\right). \quad (8)$$

We also define $\mathbf{z}_t = (y_t,\, \mathbf{x}_t^{'})^{'}$. The conditional ECM equations corresponding to the three cases are the following :

- *Case III* (unrestricted intercepts, no trends):

$$\begin{aligned} \Delta y_t = \;& c_0 + \pi_{yy} y_{t-1} + \pi_{yx,x}\mathbf{x}_{t-1} + \\ & \textstyle\sum_{i=1}^{p-1} \psi_i^{'} \Delta \mathbf{z}_{t-i} + \delta^{'} \Delta \mathbf{x}_t + u_t, \quad u_t \rightsquigarrow i.i.d.(0, \sigma^2). \end{aligned} \quad (9)$$

- *Case IV* (unrestricted intercepts, restricted trends):

$$\begin{aligned} \Delta y_t = \;& c_0 + \pi_{yy}\left(y_{t-1} - \gamma_y t\right) + \pi_{yx,x}\left(\mathbf{x}_{t-1} - \gamma_x t\right) + \\ & \textstyle\sum_{i=1}^{p-1} \psi_i^{'} \Delta \mathbf{z}_{t-i} + \delta^{'} \Delta \mathbf{x}_t + u_t, \quad u_t \rightsquigarrow i.i.d.(0, \sigma^2). \end{aligned} \quad (10)$$

- *Case V* (unrestricted intercepts, unrestricted trends):

$$\begin{aligned} \Delta y_t = \;& c_0 + c_1 t + \pi_{yy} y_{t-1} + \pi_{yx,x}\mathbf{x}_{t-1} + \\ & \textstyle\sum_{i=1}^{p-1} \psi_i^{'} \Delta \mathbf{z}_{t-i} + \delta^{'} \Delta \mathbf{x}_t + u_t, \quad u_t \rightsquigarrow i.i.d.(0, \sigma^2). \end{aligned} \quad (11)$$

The procedure amounts to test the assumption of *no* relationship (in level) between the dependent variable y and the independent variables $\mathbf{x}_{t-1}$ in the regressions (9) to (11). So, the test is formulated as follows:

$$H_0 : \pi_{yy} = 0 \quad \text{and} \quad \pi_{yx,x} = \mathbf{0}^{'}$$

against

$$H_1 : \pi_{yy} \neq 0 \quad \text{or} \quad \pi_{yx,x} \neq \mathbf{0}^{'}$$

The authors construct F- and t-statistics under H_0 and derive some critical value bounds. To apply their methodology to our data, we proceed in two steps:

- **Step 1**

One decides whether to include a deterministic trend in the ECM, and the number of lags p to include in the regression (an appropriate choice of p is needed to check that the disturbance terms are serially uncorrelated)[6]. We checked for the absence of correlation in the errors at orders 1 and 4 by computing the Breusch and Godfrey statistics $\chi^2_{BG}(1)$ and $\chi^2_{BG}(4)$. Secondly, we retained the model with the minimum Akaike (AIC) and Schwarz's Bayesian Information Criteria (BIC)[7].

- **Step 2**

We compute the values of the F- and t-statistics for testing the existence of a level relationship between $\log(q_t)$ and its regressors. We consider the three cases mentioned above and compare the values of the statistics to the critical values defined by Pesaran, Shin and Smith (2001). Three features are clearly evidenced in tables 1 to 4.

- For six countries, namely Austria, France, Germany, the Netherlands, Portugal and the United Kingdom, the values of the test statistics lie below the 5% critical values. We accordingly do not reject the null hypothesis that there is no cointegration relation between $\log(q_t)$ and its regressors, irrespective of the nature of the regressors ($I(0)$, $I(1)$ or mutually cointegrated).
- For four countries, namely Greece, Ireland, Italy and Spain, the values of the test statistics lie within the 5% critical values. The bound test is therefore inconclusive.
- For five countries, namely Belgium, Denmark, Finland, Luxembourg and Sweden, the values of the test statistics lie above the 5% critical values. We thereby conclude in favor of the alternative hypothesis that there exists a cointegration relationship.

[6] The F- and t-statistics are computed under the assumption of serially uncorrelated errors.

[7] The results are available upon request to the authors.

As a conclusion, we thus observe that the failure or ability to detect a mean-reversion dynamics in real exchange rates data heavily depends upon the methodology used to test for linear cointegration. The fact that the linear cointegration hypothesis is clearly rejected for only 6 countries out of 15 is in sharp contrast with the results obtained when classical tools are used[8]. However, we reject this hypothesis for the core countries of Europe in regard to the per capita incomes or trading volumes. Some of these countries were participating in the EMR during the period under study : France, Germany, the Netherlands and Portugal. For these countries, the rejection of linear cointegration is in accordance with a result found in other papers, that is the failure of the ERM target zones as stabilizing factors (see Rose and Svensson (1991), Bertola and Svensson (1993)). For the UK, the rejection of linear cointegration may be attributable to factors operating on the side of monetary fundamentals. Artis and Taylor (1994) report that periods of overvaluation and undervaluation in the UK are periods of excess credibility (or pessimism) of the exchange rate markets about monetary policy announcements.

Given these results, our question now is whether the rejection of the hypothesis of linear cointegration comes from the presence of nonlinear dynamics in the adjustment mechanism.

3 Testing for nonlinear adjustment

To see whether the rejection of cointegration between the real exchange rates and their economic fundamentals is due to the inappropriateness of the linear framework, we propose a test for detecting possible nonlinear relationships. We do not investigate general nonlinearity, but rather concentrate on testing for a STAR adjustment type towards the long-run equilibrium (see our justification in the introduction).

Define z_t as the deviation at time t of the real exchange rate from its fundamental value. For purpose of simplicity, suppose that z_t follows a $STAR$ process with one lag (the autocorrelation functions of the estimated residuals of the BEER equations show, at most, one significant lag for the five currencies).

$$z_t = \rho_1 z_{t-1} + \rho_1^* z_{t-1} F(x_{t-d}, \theta) + \upsilon_t, \tag{12}$$

where υ_t is an iid process. x_{t-d} is a transition variable that involves a

[8] For instance, a strong rejection of linear cointegration is found for the European currencies by Baum, Barkoulas and Caglayan (2001).

switching-regime dynamics in the adjustment process and F is either a logistic or an exponential function :

$$F(x_{t-d}, \theta) = \{1 + \exp[-\gamma(x_{t-d} - c)]\}^{-1}, \quad \gamma > 0, \quad \theta = (\gamma, c), \tag{13}$$

$$F(x_{t-d}, \theta) = 1 - \exp\left[-\gamma(x_{t-d} - c)^2\right], \quad \gamma > 0, \quad \theta = (\gamma, c). \tag{14}$$

γ is a transition parameter that controls the length of transition between the regimes. c is a threshold value. In our applications, the selected transition variables are either z_{t-d} or Δz_{t-d}. z_{t-d} allows to consider the implications of the present deviation on the future deviations. Δz_{t-d} captures the influence of the variability of the deviations on the size of the deviations from the long-run equilibrium. The variability tells us whether the exchange rates becomes increasingly ($\Delta z_{t-d} > c$) or decreasingly ($\Delta z_{t-d} < c$) misaligned with the economic fundamentals. Assume that $\rho_1 = 1$ and $-2 < \rho_1^* < 0$. In this case equation(12) with (13) or (14) describe a process that is locally nonstationary (given the unit root in the linear term), but globally stationary. A possible economic explanation of the transition functions are the following. If F is the logistic function, we have two extreme regimes. Some periods, the exchange rates become increasingly misaligned with their fundamentals and this may correspond to periods of strong influence of the market forces (such as the chartists' behavior). In such a situation, we may have $z_t = z_{t-1} + \upsilon_t$. In the opposite case, there are periods characterized by stronger effects of the economic fundamentals. Therefore, in these cases the exchange rates show a mean-reverting behavior. We may therefore have $z_t = (1 + \rho_1^*)\ z_{t-1} + \upsilon_t$, with $-1 < 1 + \rho_1^* < 1$ (or equivalently, $-2 < \rho_1^* < 0$). If F is an exponential function, the locally unit root behavior corresponds to the inner regime, where the deviations of the exchange rates show no variability: they tend to remain close to the same value.

We now present a testing procedure that allows to say whether the non mean-reverting behavior of the exchange rates is punctual (implying for instance a slow transition towards the long-run equilibrium) or permanent (implying perpetual deviations).

(12) can be rewritten as

$$\Delta z_t = [\widetilde{\rho}_1 + \rho_1^* F(x_{t-d}, \theta)]\, z_{t-1} + \upsilon_t, \quad \widetilde{\rho}_1 = \rho_1 - 1, \tag{15}$$

where $\upsilon_t \sim iid(0, \sigma_\upsilon^2)$. We want to test :

$$H_0 : \ \widetilde{\rho}_1 = \rho_1^* = 0 : \quad \text{random walk},$$

against

$$H_1 : \widetilde{\rho}_1 = 0 \text{ and } -2 < \rho_1^* < 0 : \quad \text{nonlinear mean-reversion}$$

To circumvent the nuisance parameter problem (the parameters of the $STAR$ models are not identified under the null), we replace the logistic and exponential functions by polynomials that sum up their main properties[9]. The auxiliary regressions are the following.

(a) In the logistic case:

$$\Delta z_t = \phi_0^1 z_{t-1} + \phi_1^1 z_{t-1} x_{t-d} + \phi_3^1 z_{t-1} x_{t-d}^3 + \omega_t^1, \quad \omega_t^1 \sim iid(0, \sigma_\omega^2), \tag{16}$$

with $\phi_0^1 = 0$, $\phi_1^1 < 0$ and $\phi_3^1 > 0$[10].

(b) In the exponential case:

$$\Delta z_t = \phi_0^2 z_{t-1} + \phi_2^2 z_{t-1} x_{t-d}^2 + \phi_4^2 z_{t-1} x_{t-d}^4 + \omega_t^2, \quad \omega_t^2 \sim iid(0, \sigma_\omega^2), \tag{17}$$

with $\phi_0^2 = 0$, $\phi_2^2 < 0$ and $\phi_4^2 > 0$[11].

Testing H_0 against H_1 implies testing for exclusion restrictions in the above equations. We now have two null hypotheses to test :

$$\begin{array}{ll} H_0: & \phi_1^1 = \phi_3^1 = 0, \quad \text{(unit root against stationary } LSTAR \text{ process)}, \\ H_0^{'}: & \phi_2^2 = \phi_4^2 = 0 \quad \text{(unit root against stationary } ESTAR \text{ process)}. \end{array}$$

We use two statistics for each test:

$$\begin{gathered} STAT1 = T(SSR_0 - SSR_1)/SSR_0 \\ \text{and} \\ STAT2 = [(SSR_0 - SSR_1)/SSR_1] \times [nd_1/(nd_0 - nd_1)] \end{gathered} \tag{18}$$

where T is the number of observations, SSR_0 is the sum of squared residuals of the auxiliary regression under H_0, SSR_1 is the sum of squared residuals under H_1, nd_0 and nd_1 are the number of degrees of freedom under H_0 and H_1.

The testing approach can be extended by adding some lags to the variable z. However, since the autocorrelation functions of our exchange rate deviations show only, at most, one significant lags, the size and power of the test are computed for the case $p = 1$. The empirical distributions of the statistics $STAT1$ and $STAT2$ are obtained using Monte Carlo simulations. Under H_0, the data generating process (DGP) is a random walk model $z_t = z_{t-1} + \upsilon_t$,

[9] This comes from the local equivalence theorem and was firstly suggested for $STAR$ models by Saikkonen, Lüükkonen and Teräsvirta (1988).

[10] These conditions are induced by the restrictions on the parameters $\widetilde{\rho}_1$ and ρ_1^* under H_0 and H_1. Using the third-order Taylor expansion of the logistic function, we obtain $\phi_1^1 = \rho_1^* \gamma/4$ and $\phi_3^1 = -\rho_1^* \gamma^3/48$. The sign of both ϕ_1^1 and ϕ_3^1 depends on the sign of ρ_1^*.

[11] The preceding remark also applies here. The fourth-order Taylor expansion of the exponential function yields $\phi_2^2 = \rho_1^* \gamma$ and $\phi_4^2 = -\rho_1^* \gamma^2/2$.

where $v_t \sim N(0,1)$ and $z_0 = 0$. We simulate 2000 series of respective lengths $T = 50$, 100, 500. Under the alternative hypothesis, we simulate different $LSTAR$ and $ESTAR$ models for different values of the parameters γ, ρ_1^* (we assume that $c = 0$ since choosing $c \neq 0$ does not change the conclusions). To avoid too many tables, we have selected the results on the power and the size in table 6. The critical values corresponding to the statistics $STAT1$ and $STAT2$ are shown in table 5. The latter are computed as the percentiles of the empirical distribution.

From the simulations, the main conclusions are the following. When the nonlinearities in the $STAR$ components are stationary, the tests generally behave well in terms of power. This means that when nonlinearities of the $STAR$ type exist, they are detected. However, as ρ_1^* moves towards the nonstationary area (ρ_1^* is close to 0), the tests behave badly and the power decreases slightly, but only for the logistic model. Therefore, the tests seem to be more robust to the presence of a linear unit root when the nonlinear model is an $ESTAR$ model. The computed sizes always lie above the nominal sizes. The tests thus tend to overreject the null hypothesis when it is true. This calls for cautious when interpreting the results of these tests, because they are "biased" towards the different $STAR$ alternatives.

Tables 7 and 8 contain the results of the application to five currencies: France, Germany, Portugal, the Netherlands and the UK[12]. To save place, we report only the results based on the statistics $STAT1$. Using $STAT2$ yields similar results. We choose several possible transition variables. As is clearly seen, in many cases, the computed statistics lie above the critical values, which can be interpreted as an evidence of nonlinear cointegration. But what does the expression "nonlinear cointegration" means? In general, there are two possible interpretations. One can consider, either that the long-run relationship is nonlinear, or that the exchange rates adjust nonlinearly when they move to their long-run value. All the subsequent sections relies upon the second interpretation. Before examining the modelling approach, another important remark is in order.

The nonlinear cointegration test gives us an indication about possible lags of the transition variable for which there exists a stationary $STAR$ model under the alternative, *when the linear component of the model contains a unit root*. But, this is only an indication. Remember that the rejection of the linear cointegration hypothesis does not necessarily mean that z_t is $I(1)$. When z_t is $I(0)$ the tests based on the statistics $STAT1$ and $STAT2$ also reject the

[12] As explained in the next sections, the mean-reversion mechanism are linked to the membership to EMU. We do not consider the case of Austria, since this country entered the EMU lately (in 1995), in comparison of the other five countries.

null hypothesis of a unit root against a stationary $STAR$ alternative. Further, as noticed in the above simulations, if the linear component contains a unit root and if the coefficient ρ_1^* is negative but small, then the tests tends to accept the unit root hypothesis. More generally, the probability to conclude in favor of the unit root hypothesis increases when $\widetilde{\rho}_1 + \rho_1^* \to a$, where a is a value for which the $STAR$ model becomes nonstationary. So, accepting H_0 means, either that we have a true unit root behavior with no nonlinear components of the $STAR$ type, or that we have a $STAR$ model with $\widetilde{\rho}_1 + \rho_1^* \to a$. Before entering into the modelling procedure, we thus need an additional test which is the standard test of linearity against a $STAR$ alternative (see the next section).

Consider the following two tests :

Test 1 : H_0 : unit root against H_1: $STAR$ process
Test 2 : $H_0^{'}$: linear process against $H_1^{'}$: $STAR$ process

The acceptation of H_0 means that the process under study is, either a true random walk model, or a $STAR$ model with $\widetilde{\rho}_1 + \rho_1^* \to a$. The rejection of H_0 is simply an indication that we have no unit root. The acceptation of $H_0^{'}$ implies that we have a linear process and its rejection implies that we have a $STAR$ process. The combination of the two tests thus yields the following conclusions.

	H_0 accepted	H_0 rejected
$H_0^{'}$ accepted	*case1* random walk	*case 2* stationary linear model
$H_0^{'}$ rejected	*case 3* $STAR$ model with $\widetilde{\rho}_1+\rho_1^* \to$ a	*case 4* stationary $STAR$ model

4 STAR modelling of the adjustment process

4.1 The modelling procedure

The modelling procedure involves the following steps[13]

- **Step1**

[13] The main steps of the STAR modelling procedure are based upon the methodology suggested by Lüükkonen, Saikkonen and Teräsvirta (1988), Granger and Teräsvirta (1993), Eitrheim and Teräsvirta (1996), Escribano and Jorda (1999)).

We select the best linear model among all possible estimations obtained by the regression of Δz_t on z_{t-1} and lagged values of Δz_t. The optimal highest lags is obtained by combining an information criterion such as AIC and the results of misspecification tests on the residuals of the regression. In our case the selected lags were either $p = 0$, or $p = 1$.

- **Step 2**

We apply several linearity tests against $STAR$ alternatives. The procedures consist in testing nullity restrictions on the following regressions, depending upon the alternative hypothesis:

Regression 1 : test of a linear process against a $STAR$ alternative (either $LSTAR$ or $ESTAR$)[14]:

$$\Delta z_t = \alpha_o + \sum_{i=0}^{4} [\alpha_{i1} z_{t-1}] (x_{t-d})^i + v_{1t}, \quad v_{1t} \sim iid(0, \sigma^2_{v_1}), \tag{19}$$

Regression 2 : test of a linear process against an $LSTAR$ alternative

$$\Delta z_t = \alpha_o + \sum_{i=0,1,3} [\alpha_{i1} z_{t-1}] (x_{t-d})^i + v_{1t}, \quad v_{1t} \sim iid(0, \sigma^2_{v_1}), \tag{20}$$

Regression 3 : test of a linear process against an $ESTAR$ alternative

$$\Delta z_t = \alpha_o + \sum_{i=0,2,4} [\alpha_{i1} z_{t-1}] (x_{t-d})^i + v_{2t}, \quad v_{2t} \sim iid(0, \sigma^2_{v_2}). \tag{21}$$

The corresponding LM statistics are noted LM_{NL}, LM_{LS}, LM_{ES} in tables 9 to 13[15]. The results show that for all countries, we are able to find a transition variable that yields to the rejection of the linearity hypothesis against a $STAR$ alternative at 5% or 10% level of significance (see the p-values under 0.05 and 0.1). Moreover, there is a clear evidence that the deviations of the exchange rates are mainly described by an $ESTAR$ model. This is in accordance with the same finding in a broad range of papers dealing with $STAR$ cointegration on exchange rates data. Even, there are some cases where both the $ESTAR$ and $LSTAR$ hypotheses are accepted simultaneously. Such a situation means that when asymmetric dynamics are at play during the adjustment process towards the long-run equilibrium, there

[14] We also estimated these regression with lagged values of Δz_{t-d}, but none were statistically significant in the linear regressions.

[15] The testing procedures are now quite standard in the literature. The interested reader can refer to van Dijk, Teräsvirta and Franses (2001) for a survey.

may be better approximated by an exponential transition function than by a logistic function. We finally remark that the delay parameter in many cases is high, thereby reflecting the plausibly persistent effects of the nonlinear adjustment (note for instance that for Germany, the linearity hypothesis is rejected for $d = 11$).

- **Step 3**

We estimate the best $STAR$ model for the deviations of the real exchange rates. We try many models for each our five currencies and finally select the more performing models in regard to several criteria:

1) the ratio of the sum of squared residuals of the best linear model to the sum of squared residuals of the estimated $STAR$ models. Clearly, we found that $ESTAR$ models performed better than $LSTAR$ models;

2) the stationarity restrictions on the parameters, in order to obtain a nonlinear model that is globally stationary, despite the existence of possible locally explosive or nonstationary dynamics;

3) the value of the parameter c which must lie in the interval $[\underline{c}, \overline{c}]$ where $\underline{c}$ and $\overline{c}$ are respectively the lowest and highest values of the transition variable;

4) some misspecification tests on the residuals of the estimated models : the white test, heteroskedasticity tests (ARCH and Jarque-Bera), Box-Pierce tests to detect high order autocorrelation, tests of remaining nonlinearities, tests of parameter constancy.

To facilitate the estimation, we use an estimating approach based on the concentration of the quasi-maximum likelihood estimator as suggested by Leybourne, Newbold and Vougas (1998). This is a grid search method. Indeed for a range of possible values for the parameters γ and c, we estimate the other parameters of the model. The regression that is finally selected corresponds to the values of γ and c yielding the smallest sum of squared residuals. It must be noted that if such an approach avoids to face the usual problems of finding local maximum in the estimation of the likelihood function, it is time consuming.

The regression results are shown in table 14. The economic implications of the regressions are discussed in the next paragraph. Here, we give some brief comments on the econometric aspects.

For some countries, the adjustment mechanisms exhibit both a locally "explosive" dynamics and a global mean-reversion effect. For instance, in the regressions of the Netherlands, Portugal and the United Kingdom, we see that the coefficient of the linear lag for z_{t-1} is not statistically significant, while the coefficient of the $STAR$ component is significant and negative. The cases of France and Germany show another feature. The sum of the coefficients of z_{t-1} and $z_{t-1}F$ are negative but small. For these countries, the deviations thereby show a kind of high persistence. These estimations are in accordance with the conclusions obtained when the results of the unit root and linearity tests are combined. Indeed for France and Germany, the unit root hypothesis is accepted (at 5% significance level) and the linearity hypothesis is rejected against the $ESTAR$ alternative (compare the tables 8, 9 and 10 when $x_{t-d} = z_{t-2}$ for France and z_{t-11} for Germany). The deviations thereby have a dynamics corresponding to the case 3 (stationary $STAR$ model with $\widetilde{\rho}_1 + \rho_1^* \to 0$). Similarly, for the United Kingdom, when $x_{t-d} = \Delta z_{t-4}$, both the unit root and the linear models are rejected against the $STAR$ alternative (in the present case, we have an $ESTAR$ model that captures the asymmetric dynamics of an $LSTAR$ model). The deviations behave in a way corresponding to the case 4 (e.g. a stationary $STAR$ model with no persistent dynamics. This is confirmed by the regression, since it is seen that the sum of the lagged coefficients corresponding to z_{t-1} and $z_{t-1}F$ are significantly negative. If we look at the results for the Netherlands, the tests lead us to conclude that we are in the case 1 (compare the tables for $x_{t-d} = \Delta z_{t-3}$). Indeed the null hypotheses are accepted in both tests. This would mean that the deviations behave like a random walk. The regression gives a similar information. Indeed, it is seen that z_{t-1} is non significant and further that the coefficient of $z_{t-1}F$ is not statistically significant at 5% significance level. In the case of Portugal, we have a contradiction between the results of the tests and of the regression. Indeed, according to the conclusion of the tests, the dynamics of the deviations of the exchange rate should correspond to the case 3. However, the regression suggest that we rather have a behavior corresponding to the case 4.

All the models are satisfactorily estimated in regard to the different misspecification tests, except the ARCH tests. The presence of such effects is frequent in many studies that use the $STAR$ model to describe the exchange rate adjustment mechanism. One way to remove such effects would be to estimate $STAR - GARCH$ models, but this is beyond the scope of this paper[16].

4.2 The ESTAR models and the stylized facts on the European misalignments

To get more insights into the implications of the estimated $ESTAR$ models, it is interesting to construct indicators of both the degree of misalignments

[16] The reader interested by such models may refer to Franses, Neele and van Dijk (1998), Lundbergh and Teräsvirta (1998), Chan and McAleer (2002).

and the degree of mean-reversion. In this view, we use the measure suggested by Taylor and Peel (2000).

To gauge the degree of undervaluation and overvaluation, the authors suggest to use the following transformation of the transition function

$$G(x_{t-d}) = 100 \left\{ \widehat{F}(x_{t-d}) \right\} sgn(x_{t-d}), \quad sgn(x_{t-d}) = x_{t-d}/ \left| x_{t-d} \right| . \tag{22}$$

This measures the importance of the degree of misalignment when the exchange rate is above and under its long-term equilibrium. The degree of mean-reversion is measured by the function

$$H(x_{t-d}) = 1 - \widehat{F}(x_{t-d}) \tag{23}$$

The computation of the above function relies upon the fact that the transition function captures the misalignment of the exchange rates. Consequently, the "complement" can be interpreted as an indication of mean-reversion. Similarly, the function G measures the importance of the misalignments taking into account the sign and the size of the deviations.

Figures 1 to 5 contain the plots of these functions for our five countries. They allow a strong appreciation of the economic history of the European misalignments over the period from 1980 to 1999. Firstly, we observe a difference between the plots corresponding to France and Germany and those of the three other countries. For the first two countries, the graphs show some evidence of a succession of periods with persistent dynamics in the deviations of the real exchange rates (when z_t enters in one extreme regime it stands in this regime during a few periods before moving back to the other extreme regime). Say it another way, the duration between two extreme peaks or troughs lasts several quarters, especially after 1984:3. On the opposite, the plots of Portugal and the UK are characterized by frequent fluctuations of the real exchange rates, showing no persistent dynamics at all. The case of the Netherlands is intermediate. These graphs reproduce some of the economic facts of the European misalignments. On the plots of France and Germany, we globally identify three periods : 1980:1-1984:3, 1984:4-1993:2 and the period after 1993:2.

The first sub-period is characterized by frequent adjustments of the exchange rates in comparison to the other sub-periods. The observed movements can be explained as follows. France and Germany were both participating in the EMR and faced the so-called impossibility trinity principle : a fixed exchange rate, independent monetary policies, and free capital mobility. Over the first sub-period the government have privileged the second objective. The non-coordinated monetary policies were justified by differences in the national inflation rates. This resulted in frequent modifications of the exchange rates to cope with competitiveness and external imbalance. The same argument hold for the Netherlands which chose the Deutsche Mark as an anchor for its currency. For this sub-period, we observe opposite dynamics between for France and Germany. The former is characterized by marked undervaluations, while the latter experiences marked overvaluation periods. This reflects some differences in the macroeconomic fundamentals : a low inflation rate in Germany and a higher inflation rate in France.

A second sub-period extends approximately over 1984:4-1993:2. This period is characterized by more persistent deviations and mean-reversion dynamics, meaning that the realignments were less frequent. This corresponds to the era of competitive disinflation in France and Germany : both countries decided to coordinate their economic policies and to dismantle the control of the capital market.

For the periods after 1993:2, some persistent dynamics remain for France, whereas the plots for Germany show more frequent realignments. One possible explanation for France may be that factors of rigidities were still at play, notably on the supply side : stickiness of wages and prices due to the labor and good markets imperfections. These factors induced some rigidities for the adjustment of the real exchange rates. Such kind of rigidities were not observed in Germany.

The plots of the other three countries are quite different from those of France and Germany. They show no evidence of strong persistence, or simply over a short period (as is the case for the Netherlands over 1984:4-1993:2). The case of Portugal may reflected the fact that this country did not participate in the ERM before 1992 and the Escudo was frequently subject to speculative attacks. As for the UK, its economy is highly disconnected with those of the other countries participating in the ERM, hence the difference in the adjustment dynamics of the exchange rates.

5 Generalized impulse response functions

The preceding results give a support to the idea that for the five European countries, the rejection of linear cointegration may be attributable to the presence of a nonlinear mean-reverting effect, except Portugal where a seemingly random-like process seems to govern the exchange rate deviations. However, there are some questions that are still unanswered. For instance, the $ESTAR$ models are not necessarily "true" $ESTAR$ models, but they may mimic some asymmetries that are present in $LSTAR$ models. Although the best estimated models are of an $ESTAR$ type, the tests however sometimes concluded to the rejection of a linear model against the alternative of an $LSTAR$ model. It is interesting to see whether such asymmetries exists. Another interesting question concerns the persistence properties of the misalignments. A slow mean-reverting mechanism implies that the absorption of the external shocks by the fundamental is very progressive. An easy way to get some insights about the asymmetric effects of shocks and to gauge their persistent effects is to compute generalized impulse response functions (GIRF) using stochastic simulations.

The GIRF of a shock $\varepsilon_t = \widetilde{\delta}$ applied to the estimated $ESTAR$ models is defined as:

$$GIRF(\varepsilon_t = \widetilde{\delta}) = E\left[\Delta z_{t+n}/h_{t-1}, \varepsilon_t = \widetilde{\delta}\right] - E\left[\Delta z_{t+n}/h_{t-1}\right], \quad n = 1, 2, ..., N, \tag{24}$$

where h_{t-1} is the "history" of the $ESTAR$ model up until time $t-1$, represented here by the set of variables $z_{t-1}, \Delta z_{t-1}, x_{t-d}$. The $GIRF$ is thus a difference between two conditional forecasts, one based on a model with an initial shock and the other without the shock. The forecasts are estimated using random simulations as suggested by Potter (1995) and Koop, Pesaran and Potter (1996).

Suppose that the estimated $ESTAR$ model is written

$$z_t = \widehat{\rho}_0 + \widehat{\rho}_1 z_{t-1} + \widehat{\rho}_1^* z_{t-1}\widehat{F}(x_{t-d}) + \upsilon_t. \tag{25}$$

We generate 1000 vectors $\upsilon_t^{\prime}s$ from a Normal law $N(0, \widehat{\sigma}_\upsilon^2)$, where $\widehat{\sigma}_\upsilon^2$ is the estimated residual variance of the estimated $ESTAR$ model. From these vectors, we iterate the above equation and obtain 1000 $GIRF's$ using the formula (24). Then the latter are averaged to form an estimation of our final $GIRF$.

We repeat this experience 100 times in order to construct a distribution of the estimator of the $GIRF$, instead of a single realization. The distribution is obtained using a Normal kernel. We do this for $n = 1, 2, ..., 12$. For our purpose, we use the distributions of the $GIRF's$ to show two important features of the misalignments of the exchange rates. We firstly have some indication about the persistent or non persistent effects of the shocks by comparing the dispersion of the distributions for increasing values of n. In case of a strong mean-reverting effect, the densities should show less dispersion when n is increased. In the limit case, when $n \to +\infty$, the densities reduce to a vertical line at $GIRF = 0$. We can also study the asymmetric properties of the exchange rates deviations, looking at the density of the following random variable :

$$ASY = GIRF(\varepsilon_t = \widetilde{\delta}) + GIRF(\varepsilon_t = -\widetilde{\delta}). \tag{26}$$

The above formula tells us whether positive and negative shocks have a similar impact on the dynamics of the misalignments. If this is the case, then the density of the above variable should be symmetric around 0. Otherwise, the distribution would be skewed to the left or to the right depending upon whether negative or positive shocks have the predominant effects.

Figure 6 plots the $GIRF's$ for a given shock $(3\widehat{\sigma}_v^2)$ and different forecast horizons $(n = 2, 4, 8)$. The simulations were done by considering many more shocks $(\widetilde{\delta} = \delta\widehat{\sigma}_v^2$, where $\delta = 1, 2, 3, ...)$ and forecasts. To avoid too many figures, we had to select some of the results. The graphs reveal several interesting features. An initial shock on the real exchange rate is rapidly absorbed by the economy, in the case of UK only. For this country, we clearly see that the higher the value of the forecast horizon, the lesser the dispersion of the $GIRF's$. For the other countries, the shape of the dispersion of the distribution functions clearly demonstrated the persistent dynamic of the shocks, an extreme case being Portugal. For the latter, it seems that an initial shock on the exchange rate would have a seemingly permanent effect. This comes from the random-like dynamics of the Escudo, as shown in the estimation of the $ESTAR$ model. Comparing the Netherlands with France and Germany, we observe that in these latter countries the mean-reversion dynamics occurs less rapidly than in the former. Indeed, it is seen on the axis of the abscissas that the length of the interval of the $GIRF's$ is smaller for the Netherlands than for the other two countries. Finally, it is noteworthy that the nature of the misalignments are very different in Germany and France. The Deutsche Mark tends to be overvalued when a shock occurs (the $GIRF's$ are positive), whereas the French Franc seems on the opposite to be undervalued (the $GIRF's$ are negative). This last observation is in accordance with the stylized facts over the period under consideration in this study.

Figures 7, 8 and 9 show the distributions corresponding to the asymmetric impulse response functions. We still consider different forecast horizons $(n = 2, 4, 12)$. We further introduce shocks of different magnitudes. This allows us to contrast the persistence of large and small shocks. To avoid misinterpretations of the graphs, it is important to briefly say what a "positive" and a "negative" shock means for the exchange rates deviations. Note that since our variable of interest is the misalignment of the currencies in regard to their fundamental value, a positive shocks leads the exchange rate to become more distant from its fundamental trajectory, while a negative shock has the opposite effect. The economic interpretation of a positive shock could be for instance a degradation of the internal or external balance (a higher trade deficit, a higher inflation rate, an economy recession). Negative shocks, since they reduces the distant of the exchange rates from their long-run value, can be viewed as any factor yielding a currency near its fundamental value (for instance a policy decision that reduces the inflation rate).

With this remark in mind, our main conclusions are the following. Firstly, it is seen that the magnitude of the shocks strengthens the type of asymmetric response observed for small shocks (this is true for all forecast horizons). For instance, if positive shocks (resp. negative shocks) have a higher impact when δ is small, this dominance is increased when δ is augmented. So, the size of the initial shocks is an important element to be consider when studying the response of the exchange rate misalignments. Secondly, taking 0 as a reference on the axis of the abscissas, we see that in the long-run (this correspond to $n = 9$) negative shocks dominate, except for Portugal (the peaks of the distribution are on the left-hand side). This corroborates the fact that in the long-run, the exchange rate deviations should dampen. We further observe that the length of the mean-reversion effect depends upon the size of the initial shock. The smaller the shock, the more rapid the convergence towards the equilibrium (the curve with the weaker dispersion correspond to the case $\delta = 1$.

If we consider the short-term and medium effects of the shocks, the graphs clearly show a kind of time dependence effect, which is very frequent when one computes $GIRF's$ from nonlinear models. The exceptions are Germany and Portugal. For the latter, when there are initial detrimental effects on the exchange rates at a given period, they always predominate in the next periods (the curves are always peaked at the right of 0, thereby showing a predominance of positive shocks). For the other countries, in the cases $n = 2, 4$, the nature of the dominance changes (if positive shocks are predominant when $n = 2$, then negative shocks dominates when $n = 4$, and the reciprocal is true).

6 Conclusion

This paper has provided a further evidence of the presence of nonlinearities in the mean-reverting adjustment of some European exchange rates. These nonlinearities account for both the turbulent episodes of repetitive speculative crises in the 80's and 90's and the rigidities on the demand and supply sides of the economies. Although it might be argued that transition functions only give a visual impression of the type of nonlinear dynamics occurring, by contrast the $GIRF's$ clearly show that the deviations of the real exchange rates from their fundamental value are persistent and exhibit asymmetric patterns over time. A natural extension of this work would be to use more elaborated forms of transition functions. For instance, $MRSTAR$ models would help to distinguish between the impact of monetary or financial fundamentals and real fundamentals. It is an interesting question as whether they have the same contribution in explaining the long lasting misalignments periods. $MRSTAR$ models have been successfully estimated in other contexts, and it would be interesting to see whether they yield interesting insights into the analysis of the real exchange rates (for some illustrations of the use of $MRSTAR$ models, see van Dijk and Franses (1999), Dufrénot, Mignon and Péguin-Feissolle (2002)).

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Table 1: PSS cointegration test:
Belgium, Finland, Denmark

	Belgium					Denmark					
p	F_{IV}	F_V	t_V	F_{III}	t_{III}	p	F_{IV}	F_V	t_V	F_{III}	t_{III}
1	4.17[c]	4.60[c]	4.74[c]	-4.64[c]	-4.76[c]	2	3.67[b]	4.15[b]	-4.58[c]	4.16[c]	-4.59[c]
	Finland					Austria					
p	F_{IV}	F_V	t_V	F_{III}	t_{III}	p	F_{IV}	F_V	t_V	F_{III}	t_{III}
1	4.92[c]	5.50[c]	-4.0[b]	5.39[c]	-3.96[c]	0	1.67[a]	1.69[a]	-2.92[a]	1.68[a]	-2.91[b]

Note: F_{IV} is the F-statistic for testing $\pi_{yy} = 0$, $\pi_{xy,x} = \mathbf{0}'$ in (10). F_V is the F-statistic for testing $\pi_{yy} = 0$ and $\pi_{xy,x} = \mathbf{0}'$ in (11). F_{III} is the F-statistic for testing $\pi_{yy} = 0$ and $\pi_{xy,x} = \mathbf{0}'$in (9). t_{III} and t_V are the t-ratios for testing $\pi_{yy} = 0$ respectively in (9) and (11). [a] means that the test statistics lies below the 5% lower bound. [b] indicates that the statistic is within the 5% bound. [c] means that the statistic is above the 5% upper bound. The optimal value of p is obtained using AIC and BIC criteria as well as the LM test for testing no serial correlation at orders 1 and 4.

Table 2: PSS cointegration test:
France, Germany, Greece, Ireland

France						Germany					
p	F_{IV}	F_V	t_V	F_{III}	t_{III}	p	F_{IV}	F_V	t_V	F_{III}	t_{III}
0	2.25[a]	2.52[a]	-2.66[a]	2.54[b]	-2.67[a]	0	2.62[a]	2.82[a]	-2.06[a]	2.82[b]	-2.05[a]
Greece						Ireland					
p	F_{IV}	F_V	t_V	F_{III}	t_{III}	p	F_{IV}	F_V	t_V	F_{III}	t_{III}
0	3.32[b]	3.90[b]	-3.29[a]	3.93[b]	-3.31[b]	1	3.54[b]	3.15[b]	-4.37[b]	2.88[b]	-4.18[b]

Note: see note of table 1.

Table 3: PSS cointegration test:
Italy, Luxembourg, Netherlands, Portugal

Italy						Luxembourg					
p	F_{IV}	F_{V}	t_{V}	F_{III}	t_{III}	p	F_{IV}	F_{V}	t_{V}	F_{III}	t_{III}
2	3.48[b]	3.14[b]	-3.06[a]	2.86[b]	-2.92[b]	3	8.11[c]	8.43[c]	-5.57[c]	7.53[c]	-5.27[c]
Netherlands						Portugal					
p	F_{IV}	F_{V}	t_{V}	F_{III}	t_{III}	p	F_{IV}	F_{V}	t_{V}	F_{III}	t_{III}
1	1.14	1.30[a]	-2.56[a]	1.32[a]	-2.58[a]	0	1.97[a]	1.28[a]	-3.41[a]	1.17[a]	-3.26[b]

Note: see note of table 1.

Table 4: PSS cointegration test:
Spain, Sweden, United Kingdom

Spain						Sweden					
p	F_{IV}	F_{V}	t_{V}	F_{III}	t_{III}	p	F_{IV}	F_{V}	t_{V}	F_{III}	t_{III}
0	3.08[b]	3.52[b]	-3.96[b]	3.58[b]	-3.99[b]	2	4.72[c]	5.18[c]	-3.88[b]	5.09[c]	-3.85[b]
United Kingdom											
p	F_{IV}	F_{V}	t_{V}	F_{III}	t_{III}						
0	2.60[a]	2.64[a]	-3.41[a]	2.59[b]	-3.37[b]						

Note: see note of table 1.

Table 5: Critical values for the statistics $STAT1$ and $STAT2$ ($p = 1$)

T		5%	10%	25%	50%	90%	95%	99%
	STAT1LOG	0.0116	0.0440	0.2226	0.8418	3.5744	4.8406	7.4430
50	STAT2LOG	0.0054	0.0206	0.1050	0.4020	1.8098	2.5212	4.1194
	STAT1EXP	0.0216	0.0790	0.3778	1.1773	4.6106	5.8146	8.5853
	STAT2EXP	0.0101	0.0371	0.1787	0.5662	2.3889	3.0968	4.8859
	STAT1LOG	0.0127	0.0455	0.2235	0.8000	3.5862	4.8849	7.7309
100	STAT2LOG	0.0061	0.0220	0.1086	0.3910	1.8041	2.4914	4.0658
	STAT1EXP	0.0293	0.0979	0.4298	1.3010	4.8093	6.3902	9.5233
	STAT2EXP	0.0142	0.0475	0.2093	0.6392	2.4508	3.3121	5.1088
	STAT1LOG	0.0147	0.0480	0.2268	0.8324	3.6864	4.8692	7.8850
500	STAT2LOG	0.0073	0.0238	0.1127	0.4144	1.8457	2.4438	3.9817
	STAT1EXP	0.0427	0.1240	0.5143	1.4832	5.2568	6.8273	10.2861
	STAT2EXP	0.0212	0.0616	0.2558	0.7393	2.6404	3.4402	5.2197

Note: These critical values are obtained from 10000 simulations. $STAT1LOG$ and $STAT1EXP$ are the statistic $STAT1$ in the logistic and exponential cases. Similarly, $STAT2LOG$ and $STAT2EXP$ correspond to the statistic $STAT2$.

Table 6: Simulations results : power and size

$\rho_1^* = -1.5$

T			N1LOG	N2LOG	N1EXP	N2EXP
		90%	21.75	21.75	18.55	18.55
	(1)	95%	11.95	11.95	9.10	9.20
		99%	3.00	3.00	2.30	2.30
		90%	67.95	67.95	99.85	99.85
100	(2)	95%	54.95	55.00	99.75	99.75
		99%	34.40	34.40	98.65	98.65
		90%	41.65	41.65	100.00	100.00
	(3)	95%	31.75	31.75	100.00	100.00
		99%	18.85	18.85	100.00	100.00
		90%	21.05	21.05	16.65	16.65
	(1)	95%	11.90	11.90	9.10	9.10
		99%	2.80	2.80	2.00	2.00
		90%	98.15	98.15	100.00	100.00
500	(2)	95%	96.65	96.65	100.00	100.00
		99%	90.65	90.65	100.00	100.00
		90%	38.80	38.80	100.00	100.00
	(3)	95%	32.25	32.25	100.00	100.00
		99%	21.55	21.55	100.00	100.00

Table 6: Simulation results : power and size (continued)

$\rho_1^*= -1.0$

T			N1LOG	N2LOG	N1EXP	N2EXP
		90%	20.15	20.15	17.55	17.55
	(1)	95%	10.10	10.10	9.05	9.05
		99%	2.60	2.60	2.10	2.10
		90%	51.80	51.80	98.90	98.90
100	(2)	95%	38.80	38.85	97.55	97.65
		99%	20.05	20.05	88.05	88.10
		90%	46.35	46.35	100.00	100.00
	(3)	95%	36.10	36.10	100.00	100.00
		99%	19.25	19.30	100.00	100.00
		90%	20.80	20.80	18.80	18.80
	(1)	95%	11.40	11.40	9.35	9.35
		99%	2.65	2.65	2.20	2.20
		90%	89.90	89.90	100.00	100.00
500	(2)	95%	84.35	84.35	100.00	100.00
		99%	67.35	67.35	100.00	100.00
		90%	47.45	47.45	100.00	100.00
	(3)	95%	38.75	38.75	100.00	100.00
		99%	22.00	22.00	100.00	100.00

Table 6: Simulation results : power and size (continued)

$\rho_1^*= -0.5$

T			N1LOG	N2LOG	N1EXP	N2EXP
		90%	23.10	23.10	18.00	18.00
	(1)	95%	12.70	12.70	8.35	8.35
		99%	2.60	2.60	1.80	1.85
		90%	37.30	37.30	86.40	86.40
100	(2)	95%	24.10	24.10	74.25	74.25
		99%	9.35	9.35	45.85	46.10
		90%	36.45	36.45	99.60	99.60
	(3)	95%	25.15	25.15	98.55	98.55
		99%	10.70	10.75	92.45	92.45
		90%	20.05	20.05	18.55	18.55
	(1)	95%	12.15	12.15	9.95	9.95
		99%	2.85	2.85	2.45	2.45
		90%	67.40	67.40	100.00	100.00
500	(2)	95%	55.80	55.80	100.00	100.00
		99%	32.60	32.60	100.00	100.00
		90%	36.80	36.80	100.00	100.00
	(3)	95%	27.00	27.00	100.00	100.00
		99%	13.35	13.35	100.00	100.00

Note: These results are based on 2000 simulations. we show the power and size for $\gamma = 0.5$ (the value of γ has no influence on the results). The values in the tables are the percentage of rejections of the null hypothesis is rejected. Different cases are considered : (1) the data are generated under the null hypothesis, and the reported numbers thereby describe the size of the tests; (2) and (3) correspond to the cases where the data are generated under the alternative hypothesis (logistic model (2) and exponential model (3)). the number thus show the power of the tests. N1LOG, N1EXP, N2LOG, N2EXP are the test statistics $STAT1$ and $STAT2$ based on the Taylor expansions of the logistic and exponential models.

Table 7: Testing for unit root against LSTAR - Transition variable : Δz_{t-d}

d		France	Germany	Netherl.	Portugal	U-K
1	STAT1LOG	6.073**	1.339	0.343	2.304	2.699
	STAT1EXP	7.089**	2.714	7.193**	1.633	12.418***
2	STAT1LOG	10.147***	0.076	0.000	2.871	3.795*
	STAT1EXP	12.276***	14.235***	0.926	0.265	4.654
3	STAT1LOG	5.073**	X	1.716	1.189	2.429
	STAT1EXP	7.465**	5.663*	2.998	1.066	1.540
4	STAT1LOG	X	4.736*	1.070	5.421**	1.366
	STAT1EXP	5.682	3.028	1.047	2.343	8.514**
5	STAT1LOG	1.758	1.176	0.871	0.906	0.385
	STAT1EXP	7.134**	4.437	7.633**	0.003	3.158
6	STAT1LOG	0.029	0.508	0.424	7.951***	0.492
	STAT1EXP	8.021**	3.222	9.380**	0.881	2.211
7	STAT1LOG	0.554	1.257	2.783	0.455	0.568
	STAT1EXP	5.387*	2.087	8.834**	2.028	4.886*
8	STAT1LOG	1.376	0.216	1.186	0.672	7.770***
	STAT1EXP	4.390	0.754	1.895	1.289	5.117*
9	STAT1LOG	0.001	4.810*	0.592	0.173	0.978
	STAT1EXP	2.823	2.861	3.124	2.528	11.277***
10	STAT1LOG	1.034	2.900	0.572	1.595	1.443
	STAT1EXP	7.406**	8.346**	2.241	4.594	4.978*
11	STAT1LOG	0.147	5.604**	X	0.309	0.135
	STAT1EXP	2.249	2.525	4.672	4.967*	1.150
12	STAT1LOG	0.821	3.917*	0.507	0.264	0.302
	STAT1EXP	9.257**	2.004	2.389	0.547	4.572

Note: The critical values are:in the following table:

	90%	95%	99%
STAT1LOG	3.586	4.884	7.730
STAT1EXP	4.809	6.390	9.523

*, ** and *** mean that the null hypothesis is significantly rejected respectively at 90%, 95% and 99%. "X" means that a maximization problem happened.

Table 8: Testing for unit root against LSTAR - Transition variable : z_{t-d}

d		France	Germany	Netherl.	Portugal	U-K
1	STAT1LOG	0.061	0.066	3.477	7.676*	0.451
	STAT1EXP	13.679***	5.308	7.330*	1.297	11.784***
2	STAT1LOG	6.430	0.155	2.613	1.294	0.00078
	STAT1EXP	22.121***	6.789	9.110**	3.048	9.294**
3	STAT1LOG	7.093*	0.068	0.944	0.305	0.269
	STAT1EXP	14.914***	7.773*	13.037***	2.104	12.843***
4	STAT1LOG	2.333	0.582	0.044	X	3.235
	STAT1EXP	13.911***	7.822*	11.251**	2.857	18.633***
5	STAT1LOG	0.312	5.662	0.024	1.983	0.816
	STAT1EXP	11.316**	10.766**	5.015	4.265	10.619**
6	STAT1LOG	0.190	3.616	0.591	1.351	1.485
	STAT1EXP	8.478**	5.771	4.229	5.661	10.620**
7	STAT1LOG	0.220	1.872	0.225	1.338	6.542
	STAT1EXP	4.888	1.955	4.420	3.273	2.766
8	STAT1LOG	1.381	1.930	0.893	0.765	1.716
	STAT1EXP	5.602	2.654	3.335	1.325	0.982
9	STAT1LOG	0.154	0.710	0.163	1.568	1.035
	STAT1EXP	5.466	1.163	7.923*	2.031	6.031
10	STAT1LOG	1.016	2.303	1.115	1.664	0.580
	STAT1EXP	4.916	0.485	4.252	1.667	3.095
11	STAT1LOG	0.406	3.773	3.664	1.571	2.045
	STAT1EXP	2.819	0.049	3.694	3.722	7.136*
12	STAT1LOG	0.093	0.020	3.064	2.929	4.318
	STAT1EXP	3.335	0.432	3.431	1.619	4.306

Note: the critical values are:in the following table:

	90%	95%	99%
STAT1LOG	6.722	8.252	11.714
STAT1EXP	6.889	8.303	11.783

*, ** and *** mean that the null hypothesis can be significantly rejected respectively at 90%, 95% and 99%. "X" means that a maximization problem happened.

Table 9: Testing for linearity against STAR alternatives: France

FRA	z_{t-d}			Δz_{t-d}		
d	LM_{NL}	LM_{ES}	LM_{LS}	LM_{NL}	LM_{ES}	LM_{LS}
1	0.27 (0.90)	0.46 (0.63)	0.086 (0.92)	1.59 (0.19)	0.88 (0.42)	2.99 (0.057)
2	4.23 (0.004)	5.46 (0.006)	0.18 (0.84)	2.47 (0.05)	0.37 (0.69)	0.99 (0.38)
3	1.42 (0.24)	0.50 (0.61)	0.48 (0.62)	1.37 (0.25)	0.26 (0.77)	1.67 (0.20)
4	1.45 (0.23)	2.32 (0.11)	1.32 (0.27)	0.13 (0.97)	0.16 (0.85)	0.18 (0.84)
5	0.43 (0.78)	0.68 (0.51)	0.22 (0.80)	0.31 (0.87	0.03 (0.97)	0.31 (0.74)
6	0.31 (0.87)	0.09 (0.91)	0.52 (0.59)	0.38 (0.82)	0.75 (0.48)	0.44 (0.64)
7	0.43 (0.79)	0.31 (0.73)	0.49 (0.61)	0.19 (0.94)	0.03 (0.97)	0.35 (0.71)
8	1.04 (0.39)	0.47 (0.63)	1.75 (0.18)	0.53 (0.71)	0.43 (0.65)	0.52 (0.60)
9	0.76 (0.55)	1.34 (0.27)	0.27 (0.77)	0.77 (0.55)	0.33 (0.72)	1.12 (0.33)
10	0.34 (0.85)	0.42 (0.66)	0.32 (0.73)	1.23 (0.31)	1.91 (0.16)	0.79 (0.46)
11	1.82 (0.14)	3.55 (0.034)	0.16 (0.85)	0.71 (0.59)	1.31 (0.28)	0.18 (0.84)
12	1.89 (0.12)	3.73 (0.03)	0.35 (0.71)	0.90 (0.47)	1.26 (0.29)	0.149 (0.82)

Note : LM_{NL}, LM_{LS}, LM_{ES} are the LM statistics corresponding to the linearity tests based on the regressions (19), (20) and (21). The table gives the value of the statistics and between brackets the corresponding p-values. The linear model is rejected when the p-values lie under 10% or 5%.

Table 10: Testing for linearity against STAR alternatives: Germany

GER	z_{t-d}			Δz_{t-d}		
d	LM_{NL}	LM_{ES}	LM_{LS}	LM_{NL}	LM_{ES}	LM_{LS}
1	0.47 (0.76)	0.25 (0.78)	0.83 (0.44)	1.22 (0.31)	1.22 (0.30)	1.39 (0.26)
2	1.13 (0.35)	0.19 (0.83)	1.29 (0.28)	2.76 (0.034)	4.30 (0.017)	0.28 (0.76)
3	1.51 (0.21)	2.02 (0.14)	1.99 (0.14)	1.59 (0.19)	0.66 (0.52)	1.43 (0.25)
4	1.29 (0.28)	2.1 (0.13)	0.71 (0.49)	1.31 (0.28)	0.11 (0.90)	2.04 (0.14)
5	2.17 (0.08)	3.57 (0.03)	0.13 (0.88)	0.46 (0.76)	0.65 (0.53)	0.21 (0.81)
6	1.75 (0.15)	2.26 (0.11)	1.46 (0.24)	0.11 (0.98)	0.087 (0.92)	0.090 (0.91)
7	0.88 (0.48)	1.56 (0.22)	1.68 (0.19)	1.96 (0.11)	3.37 (0.04)	1.046 (0.36)
8	1.14 (0.35)	1.06 (0.35)	0.22 (0.80)	0.51 (0.73)	0.87 (0.42)	0.05 (0.95)
9	1.79 (0.14)	2.34 (0.10)	0.78 (0.46)	4.3 (0.004)	5.74 (0.005)	4.38 (0.016)
10	1.65 (0.17)	3.07 (0.053)	0.89 (0.42)	1.44 (0.23)	0.30 (0.74)	0.39 (0.68)
11	3.66 (0.01)	5.49 (0.006)	4.8 (0.01)	0.96 (0.43)	0.58 (0.56)	1.60 (0.21)
12	1.05 (0.39)	2.08 (0.13)	0.18 (0.83)	1.08 (0.38)	0.68 (0.51)	0.56 (0.57)

Note: see note of table 9.

Table 11: Testing for linearity against STAR alternatives: the Netherlands

NET	z_{t-d}			Δz_{t-d}		
d	LM_{NL}	LM_{ES}	LM_{LS}	LM_{NL}	LM_{ES}	LM_{LS}
1	0.57 (0.68)	0.56 (0.57)	0.025 (0.97)	0.56 (0.69)	0.85 (0.43)	0.05 (0.95)
2	0.77 (0.55)	0.81 (0.45)	0.62 (0.54)	0.53 (0.71)	0.93 (0.40)	0.36 (0.70)
3	2.05 (0.096)	3.84 (0.026)	0.16 (0.85)	1.16 (0.34)	0.83 (0.44)	2.30 (0.11)
4	1.37 (0.25)	2.61 (0.08)	0.21 (0.81)	2.89 (0.029)	5.09 (0.0088)	5.17 (0.008)
5	0.65 (0.63)	0.57 (0.57)	1.24 (0.30)	0.98 (0.42)	0.58 (0.56)	0.076 (0.93)
6	0.59 (0.67)	1.11 (0.34)	0.94 (0.40)	0.61 (0.66)	0.91 (0.41)	0.25 (0.78)
7	1.20 (0.40)	1.61 (0.21)	1.49 (0.23)	1.74 (0.15)	1.47 (0.24)	2.82 (0.067)
8	0.017 (0.99)	0.013 (0.99)	0.02 (0.98)	0.20 (0.94)	0.045 (0.96)	0.39 (0.68)
9	0.93 (0.45)	1.24 (0.30)	0.64 (0.53)	0.42 (0.77)	0.30 (0.74)	0.38 (0.69)
10	0.23 (0.92)	0.09 (0.91)	0.40 (0.67)	0.52 (0.72)	0.059 (0.94)	0.99 (0.38)
11	0.83 (0.51)	0.03 (0.97)	1.15 (0.32)	0.49 (0.74)	0.71 (0.49)	0.017 (0.98)
12	1.16 (0.34)	0.09 (0.91)	2.04 (0.14)	0.06 (0.99)	0.09 (0.91)	0.09 (0.91)

Note: see note of table 9.

Table 12: Testing for linearity against STAR alternatives: Portugal

POR	z_{t-d}			Δz_{t-d}		
d	LM_{NL}	LM_{ES}	LM_{LS}	LM_{NL}	LM_{ES}	LM_{LS}
1	3.88 (0.006)	6.72 (0.002)	6.27 (0.003)	2.23 (0.074)	0.75 (0.48)	4.40 (0.016)
2	0.82 (0.52)	0.41 (0.66)	1.43 (0.25)	1.42 (0.24)	2.12 (0.13)	1.34 (0.27)
3	1.06 (0.38)	1.76 (0.18)	1.66 (0.20)	0.16 (0.96)	0.056 (0.95)	0.25 (0.78)
4	1.00 (0.41)	0.35 (0.70)	0.86 (0.43)	1.95 (0.11)	2.28 (0.11)	2.82 (0.067)
5	0.91 (0.46)	0.26 (0.78)	1.12 (0.33)	0.18 (0.95)	0.30 (0.74)	0.09 (0.91)
6	1.09 (0.37)	1.28 (0.29)	0.82 (0.45)	0.60 (0.66)	0.014 (0.99)	0.99 (0.38)
7	0.45 (0.77)	0.51 (0.60)	0.17 (0.84	0.048 (0.99)	0.016 (0.98)	0.068 (0.93)
8	0.09 (0.98)	0.16 (0.85)	0.046 (0.95)	0.48 (0.75)	0.075 (0.93)	0.95 (0.39)
9	0.19 (0.94)	0.26 (0.77)	0.12 (0.89)	0.074 (0.99)	0.09 (0.91)	0.091 (0.91)
10	0.18 (0.95)	0.09 (0.91)	0.29 (0.75)	0.82 (0.52)	0.61 (0.55)	0.37 (0.69)
11	0.59 (0.67)	0.76 (0.47)	0.23 (0.79)	1.70 (0.16)	5.17 (0.008)	0.98 (0.42)
12	0.81 (0.52)	0.77 (0.47)	0.064 (0.94)	0.53 (0.71)	0.63 (0.54)	0.49 (0.61)

Note: see note of table 9.

Table 13: Testing for linearity against STAR alternatives: the United Kingdom

UK	z_{t-d}			Δz_{t-d}		
d	LM_{NL}	LM_{ES}	LM_{LS}	LM_{NL}	LM_{ES}	LM_{LS}
1	1.52 (0.20)	1.53 (0.22)	1.08 (0.34)	3.55 (0.01)	3.30 (0.04)	5.42 (0.006)
2	1.55 (0.20)	0.13 (0.88)	2.14 (0.13)	0.31 (0.87)	0.13 (0.88)	0.54 (0.58)
3	0.78 (0.54)	1.52 (0.23)	0.59 (0.56)	1.42 (0.24)	2.19 (0.12)	0.66 (0.52)
4	2.22 (0.075)	3.17 (0.048)	0.44 (0.65)	1.33 (0.27)	0.86 (0.43)	2.45 (0.093)
5	1.73 (0.15)	1.48 (0.23)	0.75 (0.48)	0.26 (0.90)	0.26 (0.77)	0.37 (0.69)
6	1.71 (0.16)	1.08 (0.34)	1.71 (0.19)	0.60 (0.67)	0.75 (0.48)	0.075 (0.93)
7	3.35 (0.015)	1.97 (0.15)	6.54 (0.0026)	0.26 (0.90)	0.04 (0.96)	0.03 (0.97)
8	2.27 (0.07)	0.14 (0.87)	2.64 (0.079)	1.97 (0.11)	0.067 (0.94)	3.09 (0.052)
9	0.55 (0.70)	0.49 (0.61)	0.99 (0.38)	2.04 (0.1)	2.90 (0.06)	1.37 (0.26)
10	0.12 (0.97)	0.02 (0.98)	0.09 (0.91)	0.89 (0.47)	0.69 (0.51)	1.42 (0.25)
11	0.47 (0.76)	0.79 (0.46)	0.11 (0.89)	0.54 (0.70)	0.38 (0.69)	0.82 (0.44)
12	0.76 (0.55)	0.62 (0.54)	1.48 (0.24)	0.48 (0.75)	0.32 (0.73)	0.69 (0.50)

Note: see note of table 9.

Table 14: Estimated $STAR$ models

Observation Period	1979:01-1999:02 (Quarterly Data)				
Model	$\Delta z_t = \alpha_0+\alpha_1 z_{t-1}+\alpha_2\Delta z_{t-1}+[\beta_0+\beta_1 z_{t-1} +\beta_2\Delta z_{t-1}]F(x_{t-d};\gamma;c)+\varepsilon_t$				
Country	GER	FRA	NET	POR	UK
F	ESTAR	ESTAR	ESTAR	ESTAR	ESTAR
x_{t-d}	z_{t-11}	z_{t-2}	Δz_{t-3}	Δz_{t-1}	Δz_{t-4}
α_0	$*$	$*$	0.002 (0.86)	−0.002 (−0.212)	0.0058 (0.722)
α_1	0.744 (2.80)	−2.56 (−4.59)	−0.004 (−0.033)	0.114 (1.078)	−0.0145 (−0.107)
α_2	$*$	$*$	0.425 (1.732)	0.143 (0.389)	0.109 (0.508)
β_0	$*$	$*$	−0.032 (−1.417)	0.007 (0.0862)	−0.085 (−0.824)
β_1	−1.00 (−3.61)	2.3 (3.97)	−2.141 (−1.862)	−1.204 (−2.158)	−4.185 (−2.218)
β_2	$*$	$*$	−3.031 (−1.936)	−1.179 (−1.876)	−1.867 (−0.647)
γ	80.0	365.0	80.0	95.0	5.00
c	−0.057	−0.0467	0.0369	0.04	0.112
GB	0.975	0.918	0.606	0.819	0.863
JB	0.947	0.2×10^{-4}	0.210	0.201	0.912
NL	0.66	0.10	0.517	0.118	0.33
$CONST$	0.91	0.006	0.624	0.344	0.59
$ARCH(1)$	0.3×10^{-5}	0.8×10^{-4}	0.4×10^{-5}	0.4×10^{-5}	0.17×10^{-6}
$ARCH(4)$	0.1×10^{-3}	0.4×10^{-3}	0.9×10^{-4}	0.2×10^{-3}	0.1×10^{-4}

Note: Under the values of the coefficients, we indicate, in parenthese, the Student statistics. Since $\gamma$and $c$were obtained through a grid search method, they have no Students. GB is the Godfrey-Breusch statistic for q^{th}-order serial correlation (we tried $q = 1, ..., 4$. The indicated p-value is for $p = 4$. JB is the Jarque-Bera test for Normality (p-value). NL and $CONST$ are tests of no remaining nonlinearity and constancy of parameters (p-values), as suggested by Lundbergh and Teräsvirta (1998). $ARCH$ is a test for no remaining $ARCH$(p-values).

Figure 1. Mean-reversion dynamics and degree of misalignment : France

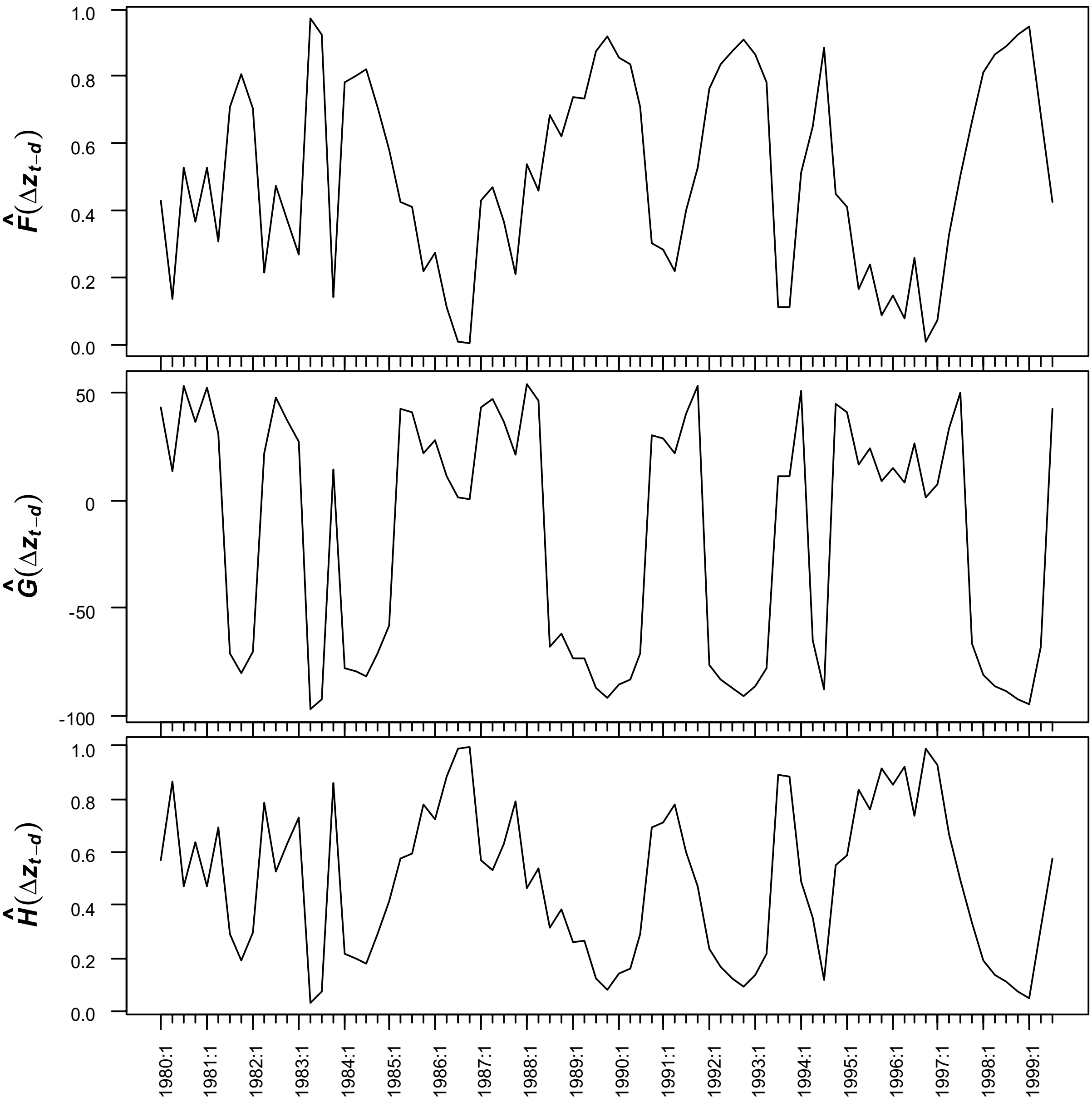


Figure 2. Mean-reversion dynamics and degree of misalignment : Germany

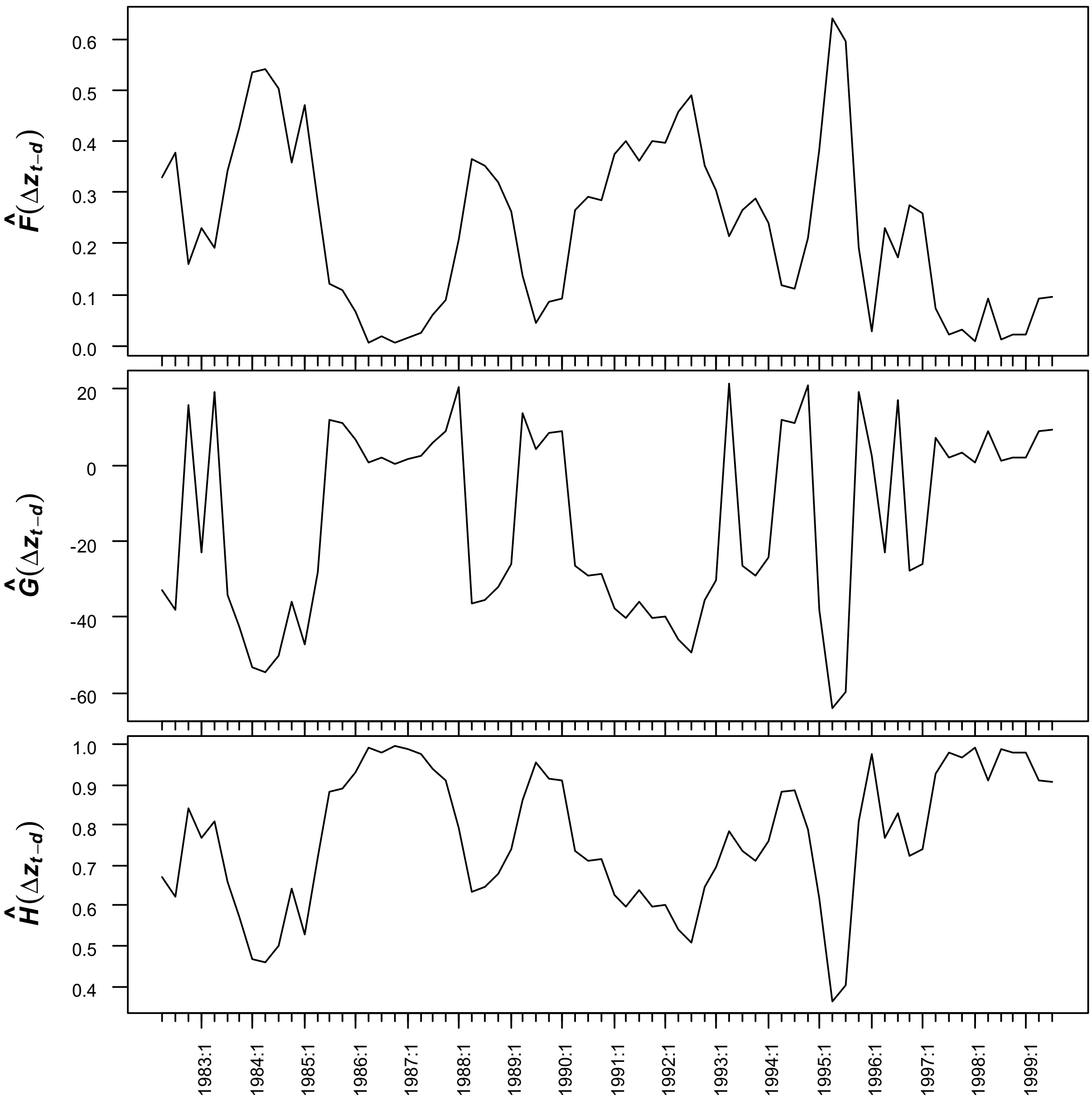


Figure 3. Mean-reversion dynamics and degree of misalignment : Netherlands

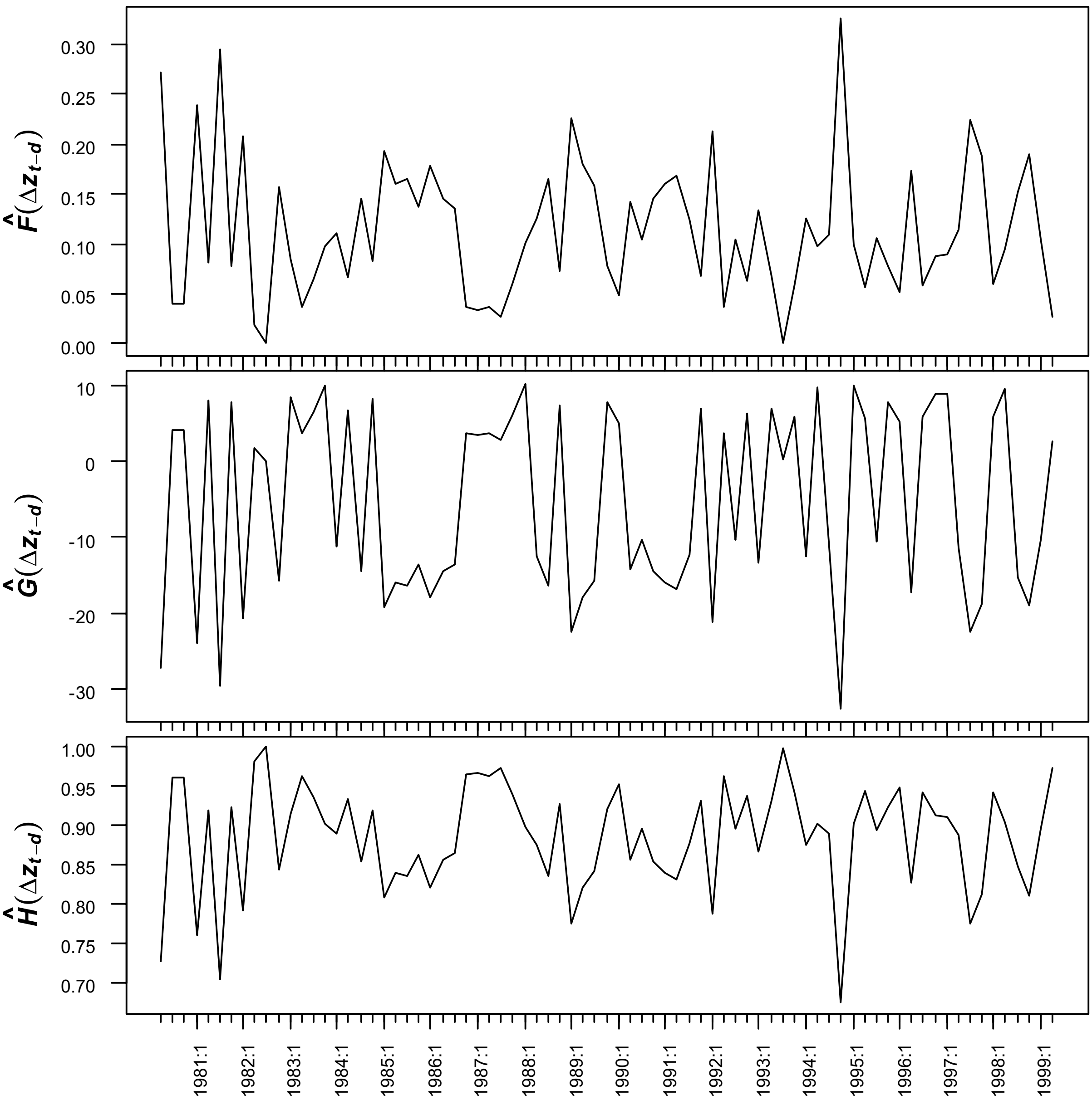


Figure 4. Mean-reversion dynamics and degree of misalignment: Portugal

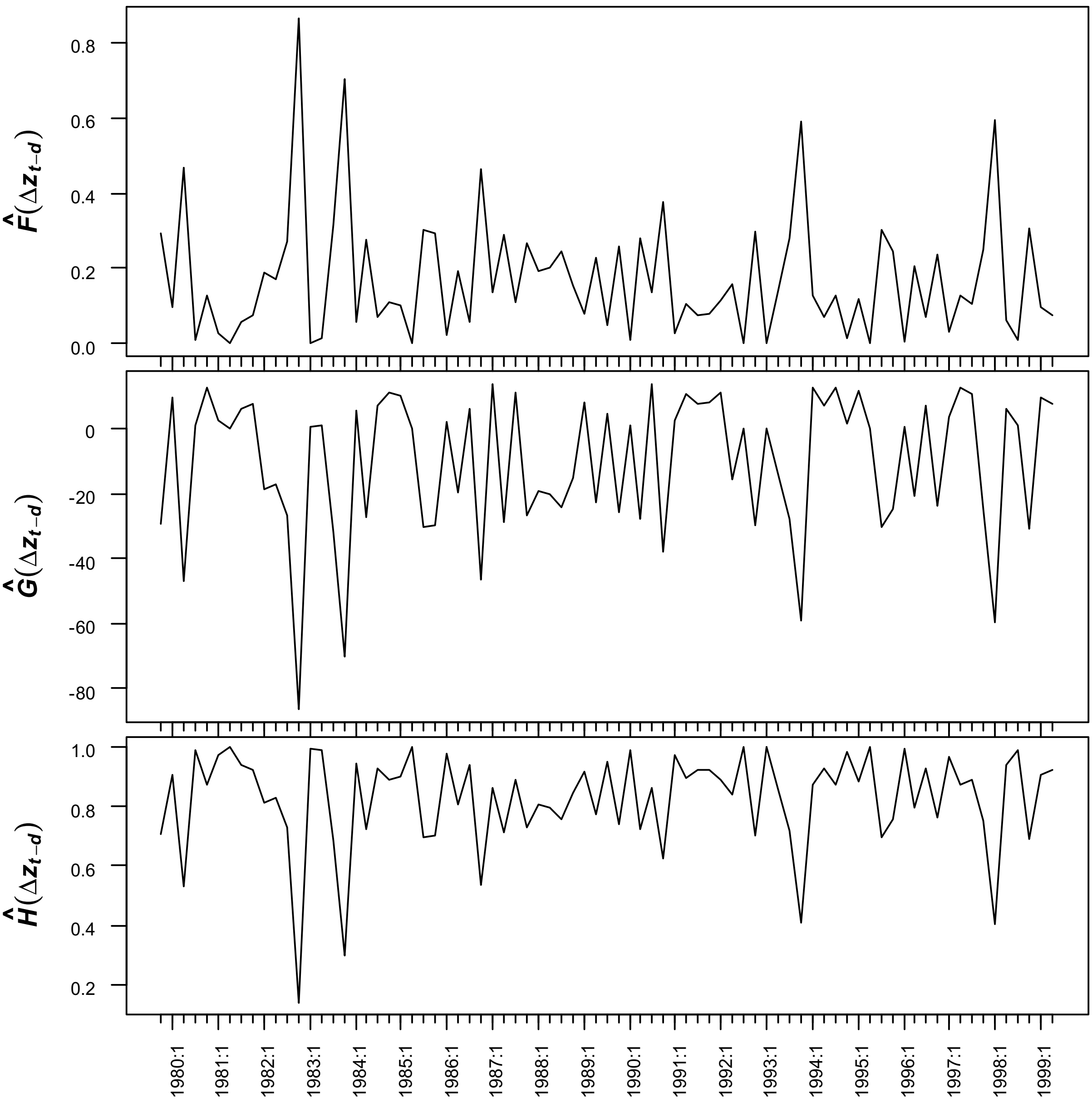


Figure 5. Mean-reversion dynamics and degree of misalignment : UK

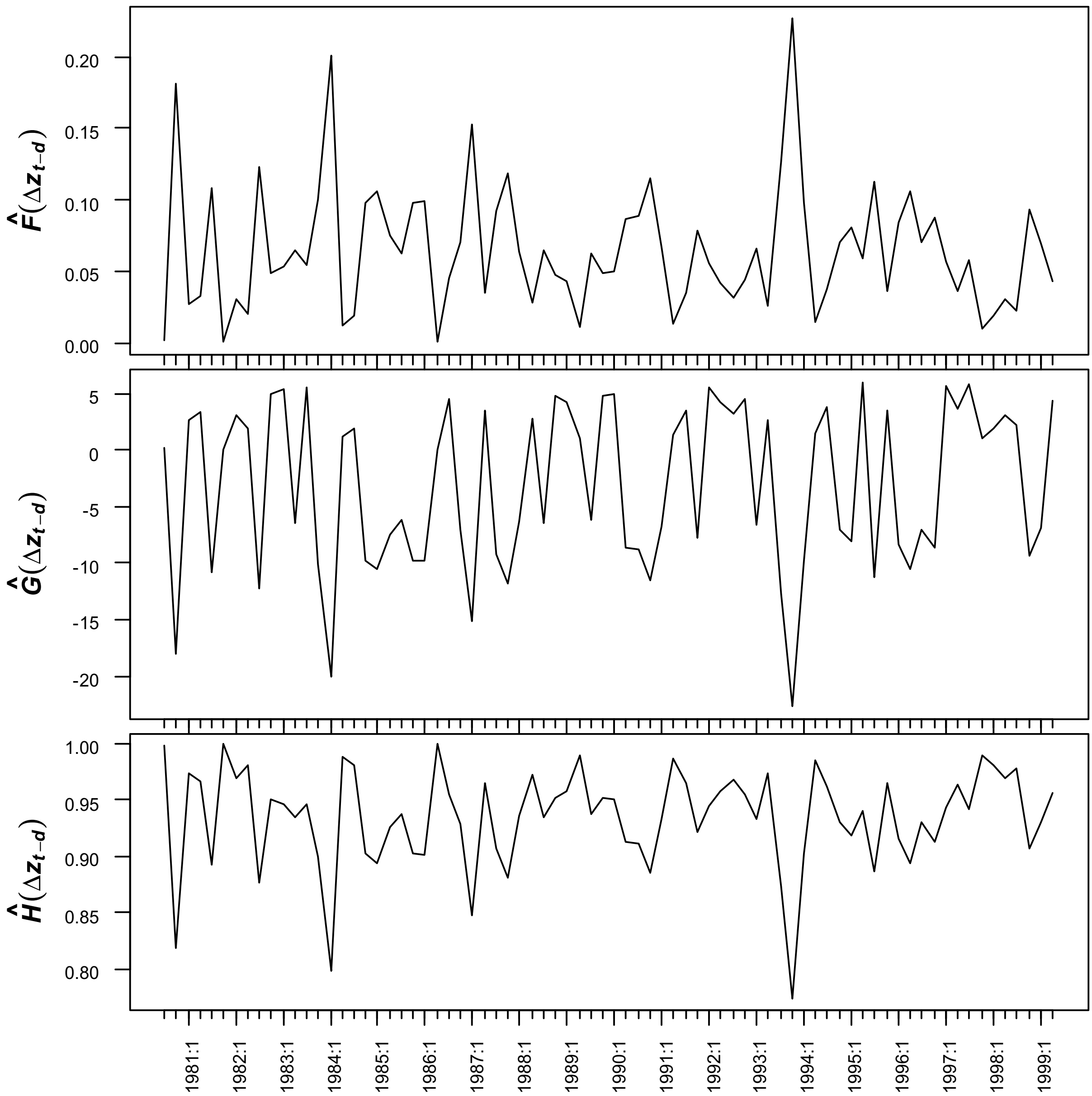


Figure 6. Generalized impulse response functions

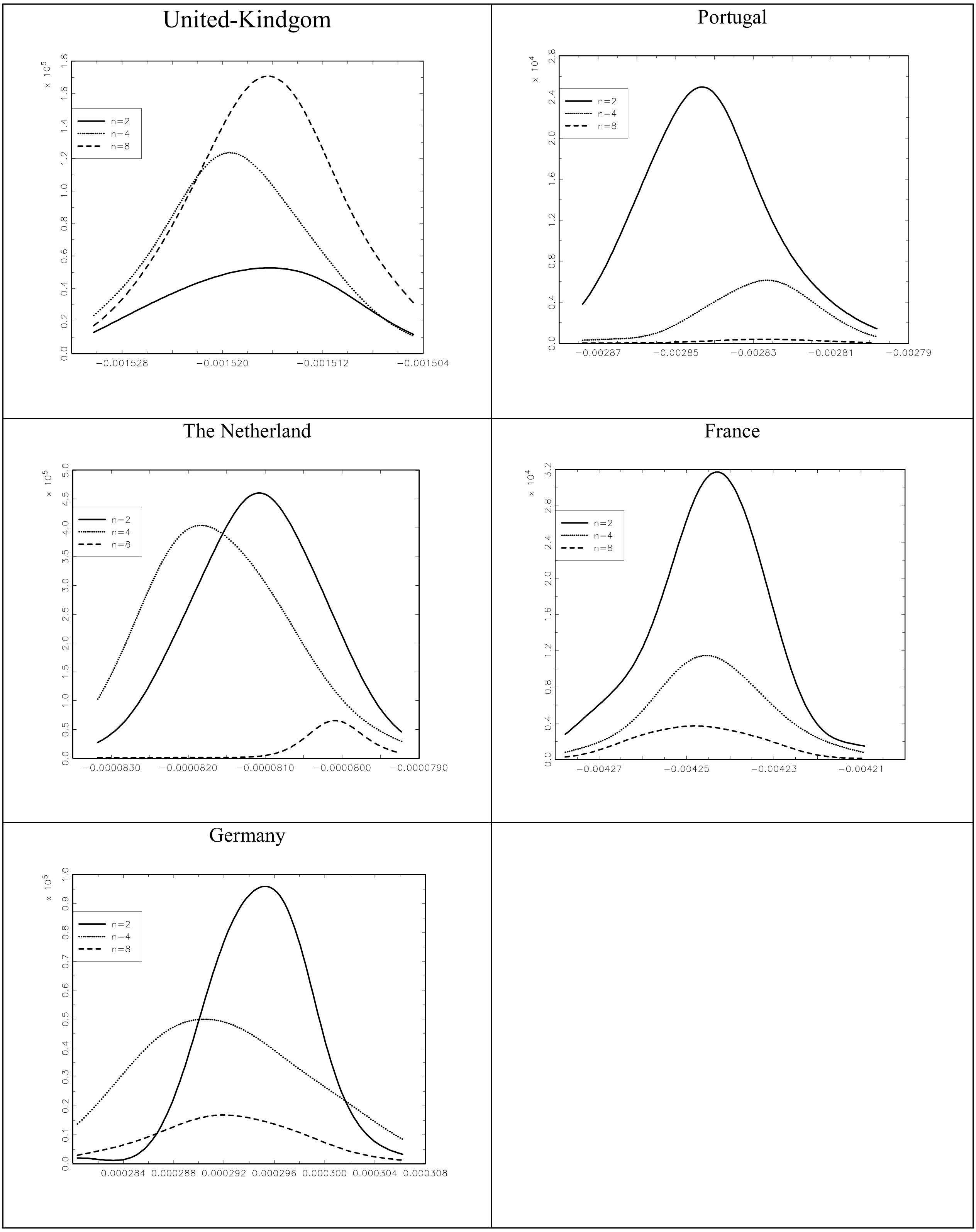


Figure 7. Asymmetric impulse response functions – n=2

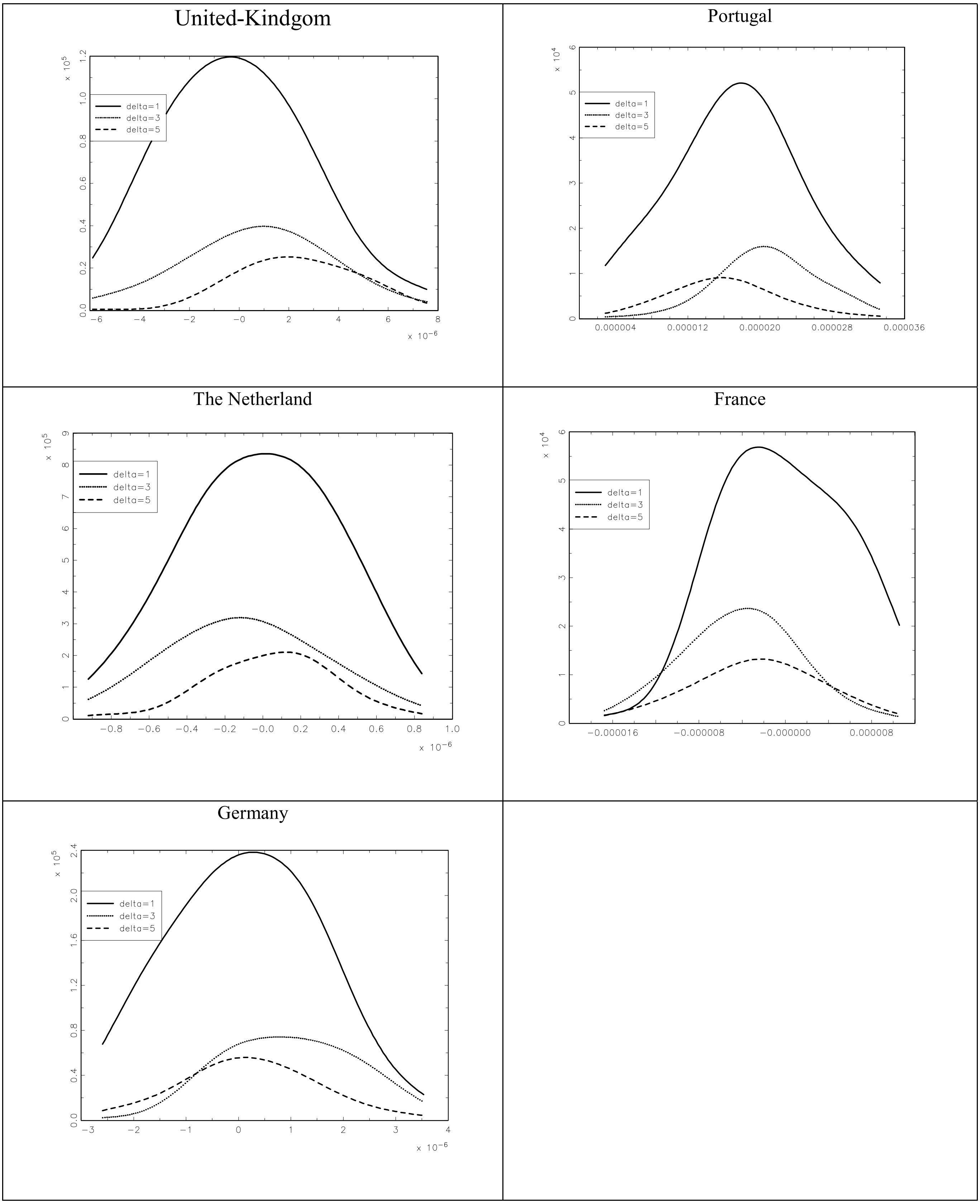
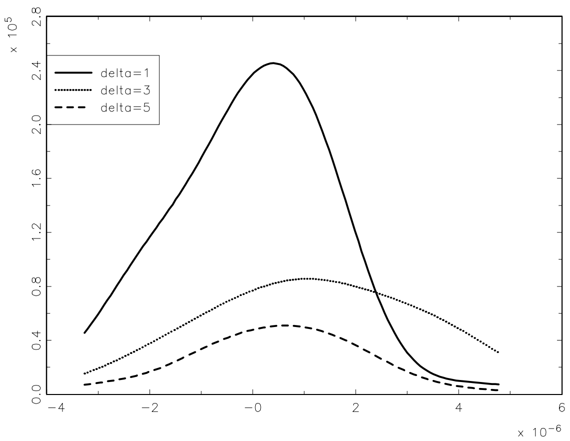
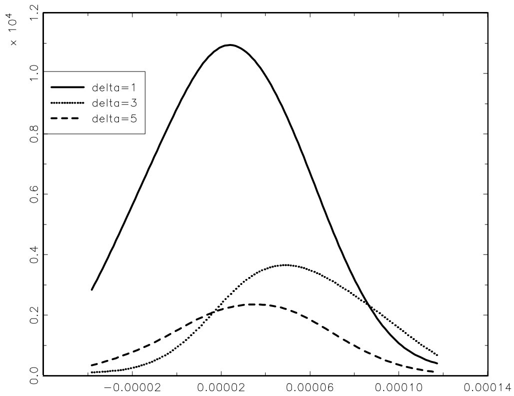


Figure 8. Asymmetric impulse response functions – n=4

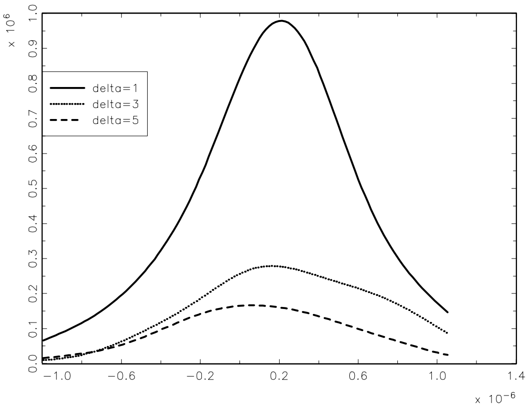
United-Kindgom



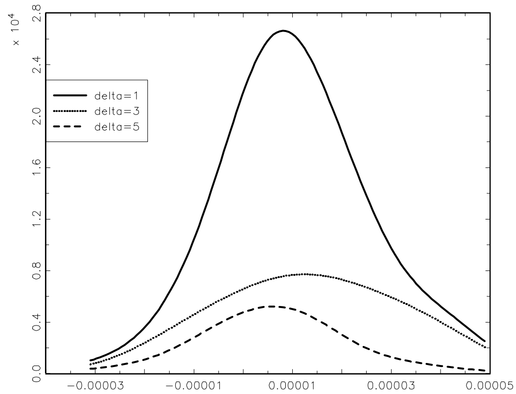
Portugal



The Netherland



France



Germany

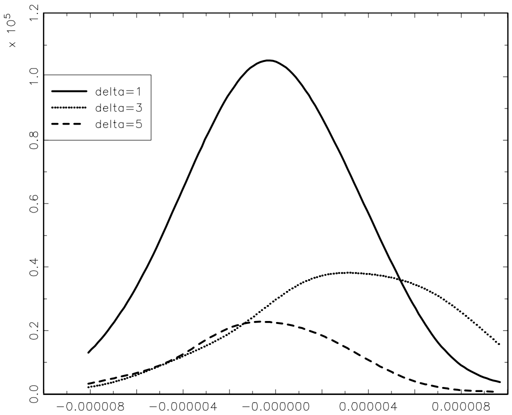


Figure 9. Asymmetric impulse response functions – n=8