

# **Default Costs, Willingness to Pay and Sovereign Debt Buybacks**

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## **Default Costs, Willingness to Pay and Sovereign Debt Buybacks**

### **Abstract**

The arguments put forward by Bulow and Rogoff (1988, 1991) against sovereign debt buybacks are re-examined in a willingness-to-pay framework. This paper argues that the Bulow-Rogoff framework treats default by a debtor as an event with no dead-weight loss, and, as such, underestimates the potential gains from a buyback. The willingness-to-pay framework allows dead-weight costs of default to be introduced in a consistent and simple fashion into the buybacks calculus. Two versions of this framework are considered. First, a model in which the default costs induce an all-or-nothing default decision is analysed. In this case, an ambiguous result is derived in which the variability of the debtor's income determines whether (small) buybacks are beneficial to the debtor, even though expected total transfers to the creditor increase, consistent with Bulow-Rogoff. In a second version, default costs are modelled so as to induce at most a partial default. This model corresponds most closely, in terms of the repayment behaviour of the sovereign debtor, to the models used by Bulow and Rogoff. It is shown that small buybacks are always beneficial to the debtor in this case. The second version is extended to include an investment opportunity. Only if the country has sufficiently scarce resources when the investment can be made, will a buyback be harmful to the interests of the debtor.

Keywords: Sovereign Debt, Buybacks, Sanctions, Willingness to Pay

JEL Classification: F34

## I. INTRODUCTION

Market buybacks of sovereign debt appear an attractive course of action for a heavily indebted country. Secondary market prices may offer deep discounts, and substantial amounts of debt can be retired at a fraction of the costs that would be incurred if the debt was to be fully serviced. A number of commercial debt buyback operations have been carried out in the middle income indebted countries in recent years, and buybacks also have been or are being executed in the heavily indebted low income countries (e.g., Uganda in 1993, Zambia in 1994, Tanzania 1998/99). Moreover, multilateral institutions have become increasingly involved, for example through the World Bank financed IDA Debt Reduction Facility. Bulow and Rogoff (1988, 1991) have however argued that buybacks are likely to be a poor deal for the debtor (or third party donor), though a good one for creditors. Their argument is based on the observation that deeply discounted secondary market prices reflect an expectation that the debt is unlikely to be fully repaid, in which case reducing outstanding obligations may have little impact on the eventual transfer to creditors, while using up scarce current resources in the process of the buyback (see also Dooley (1988)).

Their arguments have proved persuasive.<sup>1</sup> Moreover the question remains an important one. It has been pointed out by a number of authors that a variety of debt-reduction schemes are essentially “buybacks in disguise” (Rotemberg, 1991). They can be decomposed into a debt buyback plus another operation; debt-equity swaps and debt-for-nature swaps can be seen from this perspective, for example. If buybacks are not advantageous from the point of view of the debtor, then such “composite” operations will be dominated by other operations. Likewise the assessment of the best use of money donated by third parties such as the IMF or World Bank depends to some extent on the solution to the buyback question. Negotiated settlements can also usefully be evaluated using the market buyback as a benchmark: see Bulow and Rogoff (1991) who evaluate the 1990 Mexican Brady deal using this approach.

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<sup>1</sup> Though see below for a brief discussion of some dissenting views.

Despite the persuasive argument they give that buybacks are unlikely to benefit debtor countries in terms of the expected transfer of funds to creditors, an initial reaction to Bulow and Rogoff (1988) was that their model omitted important costs associated with default.<sup>2</sup> It was pointed out that failure to repay debt in full can lead to costs such as difficulty in achieving future finance, trade sanctions, conditionality for aid packages, and more generally the costs associated with no longer being an accepted member of the international financial system. Buybacks may be a cheap way of avoiding these costs. Such costs are of course very difficult to quantify, and this may be one reason why formal treatments of the buyback issue usually suppress them. In this paper, by contrast, an attempt will be made to include an (ad hoc) specification of such costs by using a model in which the default decision is a voluntary one. The fact that the country is trading off the costs and benefits of default means that the magnitude of the default costs must be consistent with the repayment behaviour of the country. It is shown that even the simplest and most traditional version of the willingness to pay model can upset the Bulow-Rogoff result.

In the Bulow and Rogoff model, as in much of the literature relating to the 1980s debt crisis, repayments are modelled by means of an “output tax”. A given proportion  $q$  (or more generally, function) of output (or exports) is assumed to be available for repayment of outstanding debt. Actual repayments are then the minimum of this amount and debt. Bulow and Rogoff argue that provided  $q$  is not too high, buybacks will be to the detriment of the debtor country. (They suggest that  $q$  may in fact be no higher than 0.05.) In this paper, the output tax model will be replaced by a “willingness-to-pay” (WTP) model, in which the sovereign determines repayments as the solution to an explicit welfare maximization problem. This is more in accordance with the traditional sovereign debt literature following on from Eaton, Gersowitz and Stiglitz (1986), Bulow and Rogoff (1989), Eaton and Gersowitz (1981), Kletzer (1984), Cohen and Sachs (1986) and Grossman and Van Huyck (1988),<sup>3</sup> amongst others. What is shown below is that the output tax assumption is by no means an innocuous modelling approach. In the willingness-to-pay models which are closest to the Bulow/Rogoff models in

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<sup>2</sup> See the comments of John Williamson and others in the Brookings panel discussion of their paper, and also those of Rudiger Dornbusch, and the paper by Sachs (1988a) in the same volume.

<sup>3</sup> The latter four references stress the exclusion from the credit market as the default cost.

terms of repayment behaviour, it is shown that including the default costs in the calculus of buybacks reverses their conclusions: small market buybacks become beneficial to the debtor.

As already stated, the approach taken here is that repayments made by a heavily indebted country are not satisfactorily modelled by the assumption that when a country defaults, there is a costless transfer to the creditor which is increasing in its available resources. Eaton (1990) considers to what extent creditors can tax the income of debtors in this fashion. He argues that empirically there is only a weak positive association between GDP and transfers to private creditors. Ultimately the transfer will depend on the enforcement mechanism available to creditors: "nonpayment reflects a perception that avoiding the costs of default is not worth the burden of repayment, not that the country or the government has insufficient resources. Forecasting debt service then becomes a problem of identifying the perceived costs and benefits of paying." This is of course a statement of the classical willingness-to-pay approach, and it is the basis of the analysis of the current paper. For the most part I shall be interested in a willingness-to-pay model that predicts at most partial default, as opposed to an all-or-nothing default decision. For partial default to be a rational decision, it is necessary that the default costs are increasing in the size of the default (as opposed to exhibiting a discontinuous jump from zero to a maximum level when repayments fall even slightly short of the contractual levels).

The crucial distinction between the different repayment mechanisms can be put in a unified framework as follows. The Bulow-Rogoff approach posits an increasing function of country income ( $y$ ), say  $q(y)$ , which represents the cost in terms of current income to the country of defaulting, i.e., setting  $R < D$ , where  $R$  is repayments and  $D$  is debt owing. In case of default, the creditors receive all of  $q(y)$ , so there is no dead-weight loss. In Farazli (1998), who demonstrates in an interesting paper that buybacks may be beneficial under certain circumstances, this latter amount is modified to  $\alpha q(y)$  where  $0 < \alpha < 1$ , so there is a loss. In both cases, however, the country will default whenever  $q(y) < D$ , and will pay the penalty of amount  $q(y)$ ; otherwise it pays  $D$ .<sup>4</sup> In my model, there is a sanction function  $S(D-R)$ , which represents the

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<sup>4</sup> The paper by Farazli is difficult to compare with the current model in that it is a two period model with terms of trade shocks. At the end of the first-period, due to learning about the future distribution of shocks, it may become apparent that default is likely in the second period. A renegotiation of the loan contract in the form of a *negotiated* buyback (as opposed to a market buyback as considered here and in Bulow-Rogoff) can eliminate that possibility, and as there is an inefficiency involved in transferring resources to the creditor in case of default, there are mutual

cost to the debtor of defaulting, as a function of the size of default. This may be a step function with two values, zero for  $D-R=0$  and some positive value for  $D-R>0$ , or it may be some continuously increasing function of the payment shortfall. The creditors receive nothing from the sanctions. Thus sanctions are to be interpreted as something other than a direct claim on the country's current resources; they may for example represent the future utility loss, not modelled explicitly, due to a diminished reputation (which presumably would not benefit the creditors in any way). In line with this interpretation,  $S$  will be taken to be a utility loss to the country rather than a current consumption loss, and enters the utility function in a linear fashion.

The critical assumption being made here is that the default costs depend only on the size of the default, and not explicitly on the circumstances of the country. This may be a reasonable assumption given the asymmetric information which exists concerning the prospects of a sovereign country. Not only is it very difficult for outside parties to verify the true ability of a sovereign to repay its debt, but any enforcement mechanism that depends upon the circumstances of the country is likely to be subject to moral hazard both in the reporting of its circumstances by the country and also possibly in the country undertaking actions which will produce circumstances which diminish the default costs.

Other authors have come up with conclusions which favour buybacks under certain circumstances. For example Krugman (1988) and Froot (1989) have models in which buybacks can be beneficial. According to Bulow and Rogoff (1988) their conclusions follow because they implicitly assume  $q=1$ . Cline (1995) argues that because the Bulow-Rogoff one-period model is an abstraction of a multi-period reality,  $q$  should be estimated not from data concerning repayments during a single year, but rather from a discounted stream of payments over a longer horizon. This can lead to a high estimate which reverses the Bulow-Rogoff conclusions. Cohen and Verdier (1991), in a model with a continuum of debtors and asymmetric information, argue that if buybacks are secret, so that holders of debt cannot observe buyback operations, large buybacks will be undertaken. Eaton (1993), in a model with tax distortions and capital flight argues that there may be circumstances under which shifting repayments forward in

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gains to be had. A Rubinstein bargaining solution allows these gains to be realised. Nevertheless, the essential point being made has similarities to the one here.

time by means of a buyback can reduce distortions and benefit the debtor. In a similar vein, Rotemberg (1991) explicitly models bargaining costs between debtors and creditors. He uses the idea that the bargaining costs are increasing in the level of debt outstanding: when there is more at stake, both sides of the bargaining process have a greater incentive to act tough. It is then possible that buybacks, by reducing debt and hence bargaining costs, can benefit the debtor as well as creditors.<sup>5</sup> In the sense that his paper dispenses with the assumption of costless bargaining implicit in the output-tax approach, there is similarity of spirit with the approach adopted here.

An outline of the paper is as follows. In section II, an all-or-nothing default decision is modelled. In section III this is modified to a model of partial default. In section IV, an investment decision is introduced into the model of partial default. Finally, section V considers implications for third party donations and contains concluding comments.

## **II. A “WILLINGNESS TO PAY” MODEL OF BUYBACKS**

Rather than assume that the creditors can “tax “ output at a fixed rate, a model will be developed in which the amount of repayment by the debtor country is determined endogenously by an optimizing choice of the debtor. This allows us to include default costs as part of the buyback calculus. We start by looking at a model which is based on the standard model of willingness to pay, which is the simplest and probably least realistic case. We shall then develop a willingness to pay model which corresponds, observationally, to the output tax model.

### **A. The All-or-Nothing Default Model**

Going back to the Eaton and Gersowitz (1981) model, it has been standard in a strand of the sovereign debt literature to specify some fixed penalty for default, no matter how large the size of default. It follows that a rational debtor, if it decides to default at all, will default on all repayments.

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<sup>5</sup> This idea was also informally presented in Sachs and Huizinga (1987).

We shall assume a simple one-period model. Suppose the sovereign, which is an expected utility maximizer, has a utility function  $u(c)$ , where  $c$  is its consumption of the single good, and it is assumed that  $u'(c) > 0$ ,  $u''(c) \leq 0$  for all  $c \geq 0$ . If the country defaults on its debt, its utility is  $u(c) - S$ , where  $S$  is the loss due to sanctions.<sup>6</sup> It is assumed that when the loss  $S$  is suffered by the sovereign, there is no corresponding gain to the creditors.  $S$  may be thought of as losses due to disruption of trade, which would not benefit the creditors, as opposed to seizure of assets, for example, which would benefit them. Strict concavity of the utility function is needed in this model to obtain a default probability strictly between zero and one (thus the present model differs from the Bulow-Rogoff one in this respect as they effectively assume risk neutrality).

There is an inherited level of debt  $D$  due for repayment to a collection of risk-neutral creditors. In order to have a secondary market value of debt strictly between 0 and  $D$  it is necessary to introduce uncertainty, either in the form of a random income, or in the form of a random penalty. As we shall see, the desirability for the sovereign of a buyback may depend upon which form of uncertainty is assumed.

## **B. Subcase 1 : Random Income**

The sovereign is assumed to have a random exogenous income  $y$ , where  $y$  has continuous density  $f(y)$  on  $[\underline{y}, \bar{y}]$ , and it is assumed that  $\underline{y} > 0$ . In addition  $u(\cdot)$  is assumed to be strictly concave. In the absence of any buybacks, the country will default if

$$u(y) - S > u(y-D) . \quad (1)$$

Let  $y^*$  be such that

$$u(y^*) - S = u(y^*-D) , \quad (2)$$

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<sup>6</sup> It is important for this section that  $S$  is outside the utility function, and hence the utility loss is independent of  $y$ . This implies that the incentive to renege is greater in low income states when consumption is more valuable. Since the implications of default are likely to be felt in the future, it is reasonable to assume that  $S$  is not a perfect substitute for current consumption even when the ultimate costs are in the form of reduced consumption.

so the sovereign is indifferent about defaulting or not. We shall assume that there is an interior solution for  $y^*$  (ie.  $y < y^* < \bar{y}$ ) to allow for the default probability to lie strictly between zero and one. Then by the assumed strict concavity of  $u(\cdot)$ , the sovereign will default whenever  $y < y^*$ . Let  $\pi_d$  be the probability of a default, and  $\pi_{nd} = 1 - \pi_d$  be the probability of no default in the initial situation.

Suppose that the sovereign is able to spend  $x$  on a debt buyback at the beginning of the period before the uncertainty is resolved. It is assumed that the country has a certain amount of income ( $\leq \bar{y}$ ) at the beginning of the period before the final level is known. Net income available for consumption and repayment at the end of the period is then  $y - x$ . In terms of Bulow and Rogoff's (1988) analysis, money spent on buybacks does reduce by an equal amount the end of period resources available for repaying debt, but now the expected change in repayment depends on the endogenous default decision, rather than on an exogenously specified parameter  $q$ .

Let  $p(x)$  be the price at which the buyback takes place. The buyback will thus reduce outstanding debt by  $x/p(x)$ . It is assumed that the buyback is publicly known and hence the price must reflect the post-buyback value of one unit of debt, which is simply the probability there is no default:

$$p(x) = \int_{y^*(x)}^{\bar{y}} f(y) dy, \quad (3)$$

where  $y^*(x)$  is the level of income below which the sovereign defaults after a buyback of size  $x$ ; it solves

$$u(y^*(x) - x) - S = u(y^*(x) - x - D + \frac{x}{p(x)}) . \quad (4)$$

The functions  $y^*(x)$  and  $p(x)$  solve (3) and (4) simultaneously. Note that for small  $x$ ,  $p(x)$  is approximately  $\pi_{nd}$ , the probability that no default takes place at  $x=0$ .

The expected utility of the sovereign is

$$E[u] = \int_{\underline{y}}^{y^*(x)} [u(y-x) - S]f(y)dy + \int_{y^*(x)}^{\bar{y}} u(y-x-D + \frac{x}{p(x)})f(y)dy$$

$$\begin{aligned} \text{and } \frac{dE[u]}{dx} \Big|_{x=0} &= - \int_{\underline{y}}^{y^*} u'(y)f(y)dy + \int_{y^*}^{\bar{y}} u'(y-D)(\frac{1}{p(0)} - 1)f(y)dy \\ &= -E[u'] + \frac{1}{p(0)} \int_{y^*}^{\bar{y}} u'(y-D)d(y)dy \\ &= -E[u'] + E[u' \mid \text{no default}] , \end{aligned} \quad (5)$$

where  $u'$  is marginal utility considered as a random variable:

$$u' = u'(y) \text{ for } y < y^* \text{ and } u' = u'(y-D) \text{ for } y \geq y^* , \text{ and recall that } p(0) = \pi_{nd}.$$

Equation (5) implies that a small buyback has an ambiguous effect on the sovereign's welfare. This is because  $u'$  is declining everywhere except at  $y^*$ , where it jumps upwards from  $u'(y^*)$  to  $u'(y^*-D)$ , and so the conditional expectation  $E[u' \mid \text{no default}]$  can be greater or smaller than the unconditional expectation. If, for example, the distribution of income was concentrated between  $y^* - D/2$  and  $y^* + D/2$ , it would follow that  $E[u' \mid \text{default}] < u'(y^*-D/2) < E[u' \mid \text{no default}]$ . Hence

$$\begin{aligned} E[u'] &= \pi_d E[u' \mid \text{default}] + \pi_{nd} E[u' \mid \text{no default}] \\ &< E[u' \mid \text{no default}] . \end{aligned}$$

In this case, a small buyback increases the sovereign's welfare. Intuitively one dollar spent on a buyback implies that the face value of debt falls by  $1/\pi_{nd}$ , hence in the no default states consumption rises by  $(1/\pi_{nd} - 1)$ , while in the default states, consumption falls by 1. Suppose that the sovereign decided to default in exactly the same states as before; then the expected change in consumption would be

$$\pi_{nd} \left( \frac{1}{\pi_{nd}} - 1 \right) - \pi_d (1) = 0.$$

So the average transfer to the creditors would be unchanged, but the distribution of the transfer has improved for the sovereign since the marginal utility of consumption in the no default states is higher than in the default states. In fact it is easy to see that the sovereign will choose to default in fewer states ( $y^*(0) < 0$ ) - choosing  $y^*(x)$  optimally cannot reduce welfare - but the envelope theorem applies (treating  $x$  as a parameter) and consequently the gain from reducing  $y^*$  is only second-order, and this can be ignored. This argument also implies that there is a gain to the creditors ( $p(x)$  rises), since if  $y^*$  was unchanged, we have seen that expected repayments are constant; in fact since  $y^*$  is reduced, expected repayments increase (this is first order in  $x$ ).<sup>7</sup> Hence a small buyback leads to a Pareto improvement in this case. The buyback has no impact on the expected repayment under the old default rule, but improves the distribution of repayments for a risk-averse debtor; the change in the default rule has only a second-order effect on the debtor as fewer penalties received cancel out greater repayments, but the creditors receive a first-order benefit. If however, income is more widely dispersed than  $(y^* - D/2, y^* + D/2)$ , the conclusion can be reversed and even small buybacks will be undesirable for the sovereign.<sup>8</sup> We can summarise this discussion in:

**Proposition 1.** Under the all-or-nothing default rule with random income, a small buyback always has a positive effect on average repayments, but its effect on the sovereign's welfare is ambiguous. It may be positive when repayments are shifted into states where the marginal utility of consumption is lower. In such cases, small buybacks are Pareto improving.

In the case where a small buyback is favourable, it is clear, though, that sufficiently large buybacks will be undesirable.<sup>9</sup> Suppose that all outstanding debt is repurchased (assuming beginning of period resources are sufficient). Then  $p(x) = 1$  and

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<sup>7</sup> Assuming that the density  $f$  is not zero at  $y^*$ . If the distribution is, for example, discrete with no probability at  $y^*$ , then expected repayments are unchanged.

<sup>8</sup> Consider for example a two-point distribution concentrated at  $y^* - D/2$  and  $y^* + D/2$ . Here the first-order effect on welfare is zero. If income is more widely dispersed, the first-order effect is negative.

<sup>9</sup> This conclusion would change if the buyback operation were "secret", as in Cohen and Verdier (1991). It is true, however, that as in their paper, the operation should then be anticipated by rational creditors, and so the equilibrium would have to take this effect into account.

$x = D$ . This is equivalent to no buyback and the sovereign not defaulting in any state. Since by assumption some default is desirable, welfare is unambiguously lower under a complete buyback.

To conclude, small buybacks always have a positive effect on average repayments,<sup>10</sup> but they may also improve the sovereign's welfare by shifting repayments into states where the marginal utility of consumption is lower. In such cases, buybacks are Pareto improving.

### C. Subcase 2 : Random Sanctions

An alternative assumption to random income sometimes made in the literature (eg. Krugman (1985)) is that of random sanctions. Hence  $S$  is now assumed to be a random variable, and income is fixed at  $y$ . The default decision is still determined by the inequality in (1), but now default takes place in states where sanctions are low, rather than income being low. Equation (5) can be derived in this case as well, but now because  $y$  is non-random,  $u'$  is unambiguously higher in the no default states ( $= u'(y-D)$ ) and consequently there *must* be a first-order gain to the sovereign.

## III. AN OUTPUT TAX - WTP MODEL

In the previous section the default decision was an all or nothing matter, as in the standard willingness to pay model. In contrast, as discussed in the Introduction, the standard model used in the buyback literature has been the output tax model. It is assumed that creditors are able to claim some increasing function of output (or exports) which may be linear with slope less than unity as in Bulow and Rogoff (1988), or with slope equal to one ("gunboat technology"), as in Froot (1989). This form may be justified on the basis of being the outcome of bargaining between the sovereign and creditors after the output realisation is observed (e.g., Detragiache (1994)). A key feature of this class of models is that failure to repay the debt in full has no adverse consequences for the sovereign (other than the repayment that is actually made). While this seems highly unrealistic, the advantage of this approach is that it conforms to the observation that outright default on all obligations is nowadays rarely observed.

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<sup>10</sup> Including  $x$  of course. Since  $y^{*'}(0) < 0$ , the probability of repayment rises with the buyback and consequently the value of one dollar of debt increases. This implies that average repayments increase.

Nevertheless the decision to even partially repay is ultimately a voluntary decision of the sovereign.<sup>11</sup> To model this in a willingness to pay framework, I shall assume that the cost of sanctions no longer takes the form of a discontinuous loss which is fully imposed as soon as repayments fall short of outstanding debt. Instead, it will be assumed that the ‘discomfort’ or ‘disutility’ of failing to meet debt obligations is an increasing continuous function of the payment shortfall. Such a sanctions function can lead to an optimal repayment schedule which has the output tax properties that repayments are strictly positive and increasing in output up to the level of outstanding debt. I refer to this as an output tax - WTP (willingness-to-pay) model. Deriving such a repayments function from an explicit optimization problem of the debtor has the additional, important implication that, at a given level of income, repayments are increasing in the level of debt even when the debtor is partially defaulting. This is consistent with regression results in Eaton (1990), but not with the standard output-tax model.

Assume then that the default cost is  $S(D-R)$ , where  $R$  is repayments. It is further assumed that  $S(0) = 0$ ,  $S'(D-R) > 0$  and  $S(\cdot)$  is continuous (and differentiable). Otherwise the model is assumed to be as in the random income model of the previous section. Denote by  $R(y,x)$  the optimal repayment schedule when a buyback of amount  $x$  is undertaken at price  $p(x)$ ; it solves for each  $y$

$$\max_R \left\{ u(y - x - R) - S\left(D - \frac{x}{p(x)} - R\right) \right\}. \quad (7)$$

Suppose that  $R(y,x)$  has the general form assumed in the output tax models:  $R(y,x)$  is positive and increasing and continuous in  $y$  up to a level  $y^*(x)$  at which  $R(y^*(x),x) = D-x/p(x)$  and above  $y^*(x)$  entails full repayment of  $D$ . In this case it will be shown that a small buyback unambiguously benefits the sovereign.

The price of one unit of debt,  $p$ , as a function of the amount spent on buybacks,  $x$ , is now given by (assuming equal sharing of partial repayments amongst claimants)

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<sup>11</sup> Rogoff (1992, footnote 2) himself argues that it is willingness to pay rather than ability to pay which is generally the binding constraint on repayments.

$$p(x) = \frac{1}{D(x)} \left\{ \int_{\underline{y}}^{y^*(x)} R(y,x)f(y)dy + \int_{y^*(x)}^{\bar{y}} D(x)f(y)dy \right\}, \quad (8)$$

where  $D(x) = D - x/p(x)$ , the post-buyback face value of debt. For a given value of  $x$ ,  $p(x)$  and the function  $R(y,x)$  are found by solving (7) and (8) simultaneously.

The welfare of the sovereign is given by

$$E[u] = \int_{\underline{y}}^{y^*(x)} [u(y-x-R(y,x)) - S(D(x)-R(y,x))]f(y)dy \\ + \int_{y^*(x)}^{\bar{y}} u(y-x-D(x))f(y)dy .$$

To find the effect of a small buyback on the sovereign's welfare, the above expression is differentiated with respect to  $x$  holding the repayment decision rule fixed at  $R(y,0)$  for  $y < y^*(0)$  by the envelope theorem:

$$\frac{dE[u]}{dx} \Big|_{x=0} = - \int_{\underline{y}}^{y^*(0)} [u'(y-R(y,0)) - \frac{1}{p(0)} \cdot S'(D-R(y,0))]f(y)dy \\ + \int_{y^*(0)}^{\bar{y}} (\frac{1}{p(0)} - 1)u'(y-D)f(y)dy \quad (9)$$

From (7), for  $y < y^*(0)$ ,  $u'(y-R(y,0)) = S'(D-R(y,0))$ , and using this in (9):

$$\frac{dE[u]}{dx} \Big|_{x=0} = (\frac{1}{p(0)} - 1)E[u'] \quad (10)$$

which is unambiguously positive since  $1/p(0) > 1$ , assuming there is some initial risk of default. This is highly intuitive. Since there is always some repayment, one dollar spent on a buyback allows the country to keep the size of the default constant in each state by reducing repayments by  $1/p(0)$ , and so increase consumption by  $(1/p(0)-1)$ . Hence there would be a gain in each state of this amount multiplied by marginal utility. While keeping the size of the default  $(D(x) - R(y,x))$  constant may not be optimum, it is approximately so for small buybacks by the envelope theorem and so  $(1/p(0)-1)E[u']$  is the actual effect on welfare. This can be contrasted with the

all-or-nothing default case considered above. There, in the default states, it is not possible to cut repayments by the reduction in debt since repayments are already zero. Consequently there is simply a one dollar reduction in consumption - a loss - in such states. We can summarize this in:

**Proposition 2.** In the model with a continuous strictly positive repayment function, small buybacks are beneficial to the sovereign but not to the creditor.

As before, the result does not imply that large buybacks are beneficial; a complete buyback would be equivalent to always honouring debt, which is by assumption not an optimal course of action.<sup>12</sup>

A more plausible sanctions function than that considered so far might combine the discontinuity at the point of default with the feature that default is at most partial. This requires that  $S(\cdot)$  is discontinuous at zero, but then increasing - possibly continuously - in the size of default. Provided repayments are always positive, the above argument can be repeated to show that the debtor gains from a small buyback. The effect on the creditor is now ambiguous, being a combination of the effects present in the all-or-nothing default model (positive) and that in the continuous sanctions model.

#### **IV. AN OUTPUT TAX - WTP MODEL WITH INVESTMENT**

Much of the debate in the literature concerning the efficacy of buybacks sees the role of investment as being crucial. This is related to the “debt overhang” argument (Krugman, 1988, Sachs, 1988b) which suggests that excess debt acts as a tax on the returns to investment and will consequently lead to an inefficiently low level of investment. The reduction in debt due to a buyback, it is argued, will reduce the overhang and stimulate investment. This implies that the game between the debtor and creditors is no longer zero-sum (as it is in the original Bulow and Rogoff (1988) model) even ignoring the sanctions effect that we analyse here; if creditors gain from a buyback it is possible that debtors might gain too.

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<sup>12</sup> Though in this case we must be more careful. It is possible that the secondary market price does not go to unity as all the debt is bought up since the country might still choose to default on the last unit of debt. Introducing a small discontinuity in the sanctions function, as suggested below, will rule out this rather perverse case.

Bulow and Rogoff (1991) have, however, argued that this mechanism is unlikely to work because creditors will reap more than 100 percent of the benefits of increased investment: they will anticipate such gains and consequently only sell debt at a correspondingly high price. Their model is essentially a modification of the output tax model considered in the previous section, but where the repayment function is exogenously given, and the model incorporates an investment decision after the buyback decision. Output depends stochastically on investment. It is straightforward to modify our earlier analysis to construct a model corresponding closely to their model but in which repayments are endogenously determined, and explicit account is taken of default costs.

#### A. A One-Period Model with Investment

The country has an initial endowment  $W$  which it can split between buybacks  $x$  of the inherited debt  $D$ , investment in a safe technology  $H$  with zero rate of return, and investment in a risky technology  $I$ . After the investment is made, uncertainty  $\theta$  is resolved and output is known, being given by  $H+y(I,\theta)$ .<sup>13</sup>  $H$  can be interpreted as holding foreign exchange reserves. It is assumed that  $\partial y/\partial I > 0$ ; and  $\theta$  has continuous density  $g(\theta)$  on  $[\underline{\theta}, \bar{\theta}]$ ; higher values of  $\theta$  correspond to higher output:  $\partial y/\partial \theta > 0$ . At the end of the period the country decides how to divide  $H+y(I,\theta)$  between repayments  $R$  of remaining debt and consumption. As before, a welfare loss of  $S(D(x)-R)$  is suffered in the case of default, where  $D(x)$  is the post-buyback face value of debt.

Assume again that  $S(\cdot)$  is such that the sovereign never chooses to fully repudiate its debt. Moreover assume that there is initially (i.e., in the no buyback scenario) an interior solution for  $H$  (i.e.,  $H > 0$ ), to allow for the possibility of a buyback increasing investment in the risky technology. Essentially the same argument in favour of small buybacks goes through as before. I shall give an informal presentation. A small buyback reduces debt by  $1/p(0) > 1$ ,<sup>14</sup> and this can

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<sup>13</sup> In Bulow and Rogoff's analysis  $H$  is called "direct consumption" and it may not be perfectly substitutable with output from the risky technology from the point of view of what the creditors can extract, though it is from the point of view of the welfare of the country. Our approach therefore is a specialisation of theirs in this respect, though not in others.

<sup>14</sup> Assuming some initial default risk of course.

be achieved at the cost of a one-unit reduction of  $H$ ; keeping the difference between debt obligations and repayments constant implies that consumption increases in each state by  $(1/p(0) - 1)$ , so a small buyback increases the sovereign's welfare.

This argument depended on  $H > 0$  in the initial (no buyback) scenario, which was the case Bulow and Rogoff (1991) were interested in.<sup>15</sup> If  $H = 0$  then the buyback is no longer necessarily beneficial to the sovereign, since the opportunity cost of one dollar spent on the buyback is the marginal productivity of investment in the risky technology, which might be greater than  $1/p(0)$ , in which case consumption will fall in each state.<sup>16</sup>

The Bulow-Rogoff argument fails here in the  $H > 0$  case because the riskless technology is equivalent in the earlier analysis to holding cash: cash which is used for the buyback has a one-for-one negative effect on resources available for consumption/repayment at the end of the period. Hence the earlier argument in favour of a small buyback goes through. Nevertheless this is essentially a one-period model and as such does not give a full picture of the trade-offs involved. I turn then to a two-period model.

## **B. A Two-Period Model with Investment**

While so far it has been argued that incorporating the costs of default into the type of model studied by Bulow and Rogoff can reverse their negative conclusions concerning the desirability of buybacks, there is a sense in which their basic model actually tends to overstate the desirability of buybacks. Their model is essentially a one-period model (see the above remarks on their 1991 paper). This has the implication that cash used for buybacks at the beginning of the period is treated equally with cash available for consumption at the end of the period after repayments to creditors have been made. A more realistic treatment, however, would recognise that a highly indebted country is very likely to discount the future more heavily than would be the

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<sup>15</sup> Since it corresponded to an initial inefficient level of investment. It can be objected to our willingness-to-pay model that the usual argument for debt overhang induced inefficiencies does not apply since the country does receive the full benefit of future output increments.

<sup>16</sup> If the technology shock is not additive, so that the marginal product is stochastic, it would be necessary to check whether the expectation of marginal utility times  $(1/p(0) - \partial y/\partial I(I, \theta))$  was positive for a small buyback to be beneficial.

case for a country which can freely borrow at the world interest rate. This means that money spent on buybacks has a higher opportunity cost than suggested by the one-period approach. It will be shown that taking this into account can reverse our favourable conclusions concerning buybacks. This two-period model is also comparable to that of Froot (1989).

The one-period investment model analysed above will be amended slightly. There are two periods  $t = 1, 2$ . The country's consumption in period  $t$  is given by  $c_t$ . Investment  $I$  takes place in period 1. The country has an initial endowment in period 1 of  $W$ , which it divides between  $c_1$ ,  $I$  and buybacks,  $x$ . The country's welfare is given by

$u(c_1) + \delta u(c_2) - S$ . There is an inherited debt,  $D$ , at the start of period 1 which is due for repayment in period 2. The model is otherwise as before. Notice that the riskless technology  $H$  ("direct consumption" in Bulow and Rogoff's terminology) has now been replaced by  $c_1$  and this is no longer a perfect substitute for consumption  $c_2$ .

In this case the expected utility of the sovereign is

$$E[u] = u(W-x-I) + \delta \int_{\underline{\theta}}^{\bar{\theta}} [u(y(I, \theta) - R(\theta, x)) - S(D(x) - R(\theta, x))] g(\theta) d\theta$$

which, for a given amount of resources devoted to buybacks,  $x$ , is assumed to be maximized by choice of  $I$  and repayment function  $R(\theta, x)$ . I suppose that there is an interior solution for  $c_1$ ,  $c_2$  and  $I$ ,<sup>17</sup> and again it is assumed that  $R(\theta, 0) > 0$  for all  $\theta$ . In this two-period model, the secondary market price for debt in period 1,  $p(x)$ , will reflect the discounted value of anticipated repayments:

$$p(x) = \frac{1}{d(x)(1+r)} \int_{\underline{\theta}}^{\bar{\theta}} R(\theta, x) g(\theta) d\theta \quad (11)$$

where  $r$  is the world interest rate and  $D(x) = D-x/p(x)$  as before.

Repeating the steps of section III for this case, we get

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<sup>17</sup> This is guaranteed under Inada conditions on  $u(\cdot)$  and  $y(\cdot, \theta)$ .

$$\begin{aligned} \frac{dE[u]}{dx} \Big|_{x=0} &= -u'(W-I) + \frac{\delta}{p(0)} \int_{\underline{\theta}}^{\theta^*(0)} S'(D-R(\theta,0))g(\theta)d\theta \\ &+ \frac{\delta}{p(0)} \int_{\theta^*(0)}^{\bar{\theta}} u'(y(I,\theta)-D)g(\theta)d\theta, \quad (12) \end{aligned}$$

where  $\theta^*(0)$  is the critical state below which the sovereign partially defaults. Using  $S'(D-R(\theta,0)) = u'(y(I,\theta)-D)$  in the partial default states, (12) simplifies to

$$\frac{dE[u]}{dx} \Big|_{x=0} = -u'(W-I) + \frac{\delta}{p(0)} E[u'] . \quad (13)$$

To simplify the following discussion, I assume that the effect of uncertainty is additive so that  $\partial y/\partial I(I,\theta)$  is independent of  $\theta$ . Then since the initial allocation is an optimum for  $x=0$ ,

$$u'(W-I) = \delta \cdot \frac{\partial y}{\partial I} \cdot E[u'] . \quad (14)$$

Putting (13) and (14) together, a small buyback is beneficial if and only if

$$\frac{1}{p(0)} > \frac{\partial y}{\partial I} . \quad (15)$$

To interpret (15), suppose first that investment is at an efficient level in the sense that  $\partial y/\partial I$  is equal to one plus the world interest rate  $(1+r)$ . Since some default is anticipated, from (11)  $p(0) < (1+r)^{-1}$  and consequently inequality (15) is satisfied: a small buyback is beneficial if investment is initially efficient.

Efficient investment is, however, an unlikely scenario for a heavily indebted country. It requires the initial endowment  $W$  to be large. If  $W$  is sufficiently small, then the conclusion will be reversed and buybacks will not prove desirable. Nevertheless suppose that the country is able to borrow in period 1 by offering debt which is senior to the existing debt and which carries no default risk.<sup>18</sup> The country will borrow up to the point where  $\partial y/\partial I = 1+r$ . It would, then, also engage in a buyback (of junior debt) by the earlier argument.

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<sup>18</sup> There is evidence that certain types of debt, notably bonds, have been treated more favorably by indebted countries, and consequently such a strategy may not be impossible. Likewise foreign direct investment has not been included in debt writedowns.

## V. IMPLICATIONS AND CONCLUDING COMMENTS

The models considered above all assume that buybacks are undertaken with resources owned by the debtor country. In fact in practice buybacks are often financed by third party donations. Nevertheless the Bulow-Rogoff argument can be applied equally to such cases: it concludes that most of the donated money will end up in the hands of the banks, rather than in improving the lot of the debtor country, which was the intention. Again, taking the willingness to pay perspective of this paper, these conclusions are turned on their head.

Again, consider a small buyback, of value  $x$ , but this time financed by a third party. Assume that the debtor country has no beginning of period resources, and so cannot undertake any buyback out of its own resources. For purposes of comparison, I assume that the opportunity cost of this buyback operation is a donation of equal value to the country in a form which cannot be used to finance buybacks nor repayments of debt (for example, some form of development projects). For simplicity assume that this is perfect substitute for the consumption good in the original utility function. In the Bulow-Rogoff framework, if this alternative donation cannot be partially claimed by the creditors, then it is preferable to the buyback option. In the framework of this paper, whenever a small buyback is desirable if the country were to have sufficient funds, the country will also prefer the buyback option with donated money when it does not have funds (and this conclusion is strengthened if, as may reasonably be assumed, the alternative option has a lower utility value than cash with no strings attached).

In conclusion, while the specification of the sanctions function in this paper could be considered to be ad hoc (in particular, recall that it was assumed that sanctions were a function of degree of default but not of available resources), the paper demonstrates (at least) that a change from one set of ad hoc assumptions to another set (both of which are common in the sovereign debt literature) can radically change the conclusions concerning the desirability of buyback operations.

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