

# **Sovereign Debt as Intertemporal Barter**

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## **Abstract**

Borrowing and lending between sovereign parties is modelled as intertemporal barter that smooths the consumption of a risk-averse party subject to endowment shocks. The surplus anticipated in the relationship offers sufficient incentive for cooperation by all parties, including any other competitive agents who are potential lenders to the sovereign. The sole punishments consist of renegotiation-proof changes in the path of future payments. We show that intertemporal trade can be sustained in the absence of any exogenous enforcement of lending relationships whatsoever. That is, borrowing and lending are possible under anarchy, and are supported by punishments that consist of cheating any cheater. Long-term implicit relationships may be fulfilled as the continual renegotiation of simple incomplete short-term loans. The analysis suggests that the crucial role of the explicit loan contract is the identification of the relationship and the parties involved.

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## 1. Introduction

Respect for sovereign immunity has long been recognized as a crucial constraint facing lenders to sovereign states (see, for example, Keynes [1924]). Intertemporal exchange is restricted by the absence of a supranational legal authority to enforce the terms of agreements. The history of lending to sovereigns shows the consequences of lenders' inability to enforce repayments specified in loan contracts. Overall payments on sovereign loans during the 19th and 20th century have not come close to discharging the original explicit contractual obligations in an overwhelming number of cases, and many defaults have been documented.<sup>1</sup>

Although debt service has fallen far short of formal contractual obligations, the lack of collateral has not meant that lenders did not recover their principal on average. In fact, economic historians have shown that lending to sovereign nations has been very profitable overall, with average returns comparing favorably to those on contemporaneous domestic government debt issued in lender nations.<sup>2</sup> Even loans in default were frequently profitable *ex post*.<sup>3</sup>

It has been widely noted that when payment deviations or defaults occurred there was generally no abrupt termination of the borrower-lender relationship as often seen in domestic bankruptcy. Instead, most sovereign debt relationships have continued through renegotiation under the guise of reschedulings, partial payments, new loans, debt repurchases or similar financial arrangements. Agreement has been achieved in almost all instances on a case by case basis through bilateral negotiation.<sup>4</sup> Indeed, all parties may view a default as an implicit contingent outcome of the underlying international financial relationship.<sup>5</sup>

The literature on sovereign borrowing explains its persistence by identifying (explicitly or implicitly) some exogenous means of enforcement as an alternative to the allocation of collateral by a government capable of commitment. This paper presents a model that assumes no exogenous means of enforcement of intertemporal exchange whatsoever. Yet, it is shown that international intertemporal trade can be sustained in perfect equilibrium using punishment threats that are proof to renegotiation. In our model, long-term debt relationships are sustainable, although they are continually subject to renegotiation and the threat of potential entry by competing lenders. We demonstrate that intertemporal exchange that serves to smooth borrower consumption is possible under the anarchy that characterizes international relations.<sup>6</sup>

The presence of government as a means of contract enforcement is crucial in domestic credit markets, where loans are frequently collateralized. In a simple loan, a lender completes her obligation by making an initial payment to the borrower in exchange for contingent rights to claim collateral. These rights are valuable because of the credible commitment of the government to reallocate collateral across agents. Long-term relationships need not be an intrinsic feature of lending in this case.

The wide array of existing models of sovereign debt can similarly be characterized as dependent on positive or negative awards administered by a third party whose credibility is assumed.<sup>7</sup> For example, in the bargaining model of Bulow and Rogoff [1989a], it is implicit that a third party exists to protect the exporting country from interference in its trade. The exporting country sells this right to protection to the “lender” by taking a “loan.” In this case, the exporter and the lender Nash bargain each period over the amount paid as “protection money” to keep the lender from interfering with the country’s trade. Essentially, the lender buys a monopoly franchise to the country’s exports by making the initial payment, and “repayments” are the equilibrium surpluses going to the monopsonist each period. If the lender held this right initially, there would be no “loan” and subsequent transactions would be the same, but it would be more obvious that the relationship is one of repeated contemporaneous bilateral trade.

Some examples of contemporaneous trade of goods for sanctions are found by Bulow and Rogoff [1989a] in the history of sovereign borrowing. However, evidence of a marked reluctance on the part of lenders or their governments to interfere with a non-performing debtor’s trade is found by Eichengreen and Portes [1989b] in the historical record, by Sachs [1989] in the contemporary experience of Brazil, Ecuador and Peru, and in the recent handling of the Asian financial crisis.

Another motive for repayment, introduced to formal models by Eaton and Gersovitz [1981], is the possibility of interference with a country’s intertemporal trade through an embargo on further loans for smoothing its consumption given a fluctuating income stream.<sup>8</sup> In equilibrium, the agents play standard trigger strategies: any deviation from the equilibrium path of intertemporal trade triggers reversion to permanent autarky. These punishments, however, create losses for both the lender and the borrower in that they could be better off returning to a new equilibrium

foresaking punishment by mutual agreement.

When there are at least two potential lenders, all of them must participate in punishment of the borrower to maintain credibility of the threat. If intertemporal exchange can be sustained, this requires that other lenders forego a share of the gains from trade to punish a recalcitrant borrower. Bulow and Rogoff [1989b] argue that reputation alone cannot work when there are other potential lenders and, therefore, third party enforcement of lender seniority rights is necessary. Otherwise, the borrower can simply abandon her relationship with one lender when she is required to make a repayment and start up another, achieving more surplus, with a new entrant. But they implicitly assume the new lender can commit to insure the borrower, that is, to make future contingent payments that require external enforcement.<sup>9</sup>

We model intertemporal barter using an infinitely-repeated game with at least two participants. The borrower's preference for smooth consumption given a stochastic endowment stream generates the gains from intertemporal trade.<sup>10</sup> In equilibrium, agents make state-contingent unilateral payments to each other on different dates. No agent can force a payment from another, either directly or by appeal to a third party. Every payment is voluntary; the sole incentive for making a payment is the increase in surplus for the payee looking forward in the relationship given the prior behavior of others. This applies equally to all agents, whether identified as a borrower or lender.

In a perfect equilibrium, intertemporal trade is supported by punishment threats that reallocate the surplus in the long-term relationship. Agents can always renegotiate the terms of any relationship, including punishments. In contrast to other formal models of sovereign debt, our equilibria are renegotiation-proof so that future mutual gains from intertemporal trade are not foregone in punishment.<sup>11</sup> Strategies for all agents are shown to exist such that the equilibrium payments path and all punishment threats are Pareto efficient within the set of all perfect equilibrium payments paths for the repeated game. With a borrower and a single lender, punishments cannot be renegotiated to mutual benefit. Under potential entry by competing lenders, the punishments assure that the borrower cannot defect and successfully negotiate a new consumption-smoothing relationship with another lender. The equilibrium is proof to renegotiation by any coalition.

The punishments demonstrated have a simple and appealing interpretation. They imply that a short-lived payments moratorium is imposed on any participant who fails to make an equilibrium state-contingent payment. The long-term consumption-smoothing relationship resumes as soon as the punished agent makes a payment sufficient to give the surplus to the other party. These punishments are as effective as the threat of permanent loan autarky, but unlike trigger strategies, these threats are credible. If a competing lender does not respect such a moratorium imposed on the borrower, the strategies of the other lenders induce the borrower to cheat this new lender. These “cheat the cheater” punishments sustain intertemporal exchange under anarchy. This type of punishment has indeed been observed in trading relations, for example, by Greif [1993] for the Maghribi traders of the late Medieval period.

Borrowing for the purpose of consumption smoothing is sustainable without international legal enforcement of seniority privileges or other restrictions on the entry of lenders from different countries. Our analysis predicts international lending but not insurance in the absence of exogenous international contract enforcement. In Bulow and Rogoff [1989b], Atkeson [1991] and other models, “lenders” offer insurance contracts that require the external enforcement of explicit state-contingent payments. Equilibrium payments in our model follow an implicit contract that does not require interference with the sovereignty of either creditor or debtor nations. Our equilibrium is consistent with the observation that international lending with frequent renegotiation of short-term loan contracts rather than international insurance is the predominant form of financial flows to developing countries. We offer a new interpretation of the role of simple short-term loan contracts in international lending. In the absence of exogenous enforcement, the primary function of a contract in our model is to identify the parties to long-term financial relationships.

The next section of the paper presents the model. The analysis begins with the case of a single lender (two-player game) in Sections 3 and 4 and is extended to the case of many possible lenders in Section 5. We discuss the implementation of the long-term equilibrium relationship using short-term contracts in Section 6 and contrast the implications of our model with the literature. The last section concludes.

## 2. Model

We use a simple model of an infinite-horizon economy in discrete time in which there are gains from intertemporal exchange. There are two types of infinite-lived agents, and each receives an endowment of a single non-storable good each period. For simplicity, we assume that one agent is risk-averse and has a stochastic endowment stream. There are  $L - 1$  risk-neutral agents, each of whom receives a constant endowment stream. For convenience, the risk-averse agent is called the borrower, and each risk-neutral agent is called a lender. To make the exposition easier to follow, we refer to the borrower as she and to lenders as he. The endowment of each agent in any given period, as well as all past and present actions, are common knowledge.

An agent can give part or all of an endowment to others, but no agent can force another to make a payment either through direct confiscation or by appeal to an external authority. More generally and in contrast to previous literature, there is no third party to force the borrower or any lender to pay part of a future endowment to any other agent. At any time, an agent can always choose to consume his or her endowment.

The borrower's preferences over consumption streams are represented by the following function:

$$U_t^b = E_t \sum_{i=1}^{\infty} \beta^{-i} u(c_{t+i}); \quad (1)$$

where  $u(c_{t+i})$  is increasing, strictly concave and continuously differentiable, and  $0 < \beta < 1$ . The expectation is taken with respect to the distribution of consumption plans,  $(c_{t+1}; c_{t+2}; \dots)$ , conditional on information available on date  $t$ . The borrower's endowment is observed at the beginning of each period, before any payments or consumption take place. The preferences for each lender,  $l = 1; \dots; L$ , are represented by:

$$U_t^l = E_t \sum_{i=1}^{\infty} \beta^{-i} c_{t+i}^l; \quad (2)$$

where  $c_{t+i}^l$  equals consumption in period  $t + i$  by lender  $l$  and the expectation is again taken with respect to date  $t$  information. We assume that the discount factor,  $\beta$ , is the same for both the borrower and all lenders, to concentrate on borrowing for the purpose of consumption smoothing without consumption tilting.

For simplicity, the borrower's endowment is assumed to be independent and identically dis-

tributed each period over a finite number,  $N > 1$ , of realizations, labelled in increasing order,  $y^1 < y^2 < \dots < y^N$ . Each lender's endowment is constant and set equal to  $y^N$ , for simplicity. However, every result for the model goes through if the borrower's endowment follows a Markov chain displaying first-order stochastic dominance.<sup>12</sup> The borrower's endowment at date  $t$  is denoted  $y_t$ .

Because every payment is voluntary, at any time an agent can stop making payments and consume his or her endowment thereafter. We define the surplus for an agent under a given consumption plan as the difference between the utility realized under the plan and the utility achieved under permanent autarky. The borrower's surplus at time  $t$  is

$$V_t^b = [u(c_t) - u(y_t)] + E_t \sum_{i=1}^{\infty} \beta^i [u(c_{t+i}) - u(y_{t+i})] \quad (3)$$

from the state-contingent consumption plan,  $(c_t, c_{t+1}, \dots)$ , and each risk-neutral lender realizes the surplus

$$V_t^l = (c_t^l - y^N) + E_t \sum_{i=1}^{\infty} \beta^i (c_{t+i}^l - y^N): \quad (4)$$

The inability to commit future payments implies that the incentive constraints,  $V_t^b \geq 0$  and  $V_t^l \geq 0$ , for each  $l$ , hold at all times  $t \geq 1$ .

Interactions between the agents can be modelled as a repeated non-cooperative game with  $L + 1$  players. On each date a stage of the repeated game is played. In the single-period game, each agent can make a non-negative payment to any of the other agents. The borrower's payments cannot total more than her current endowment. Each lender's total payments cannot exceed his endowment in any period. The borrower's single-period payoff in the game is  $[u(c_t) - u(y_t)]$ , and the single-period payoff for lender  $l$  equals  $(c_t^l - y^N)$ . The resource constraint implies that  $(c_t - y_t) = \sum_{l=1}^L (y^N - c_t^l)$ . The payments made by an agent  $j$  to the other  $L$  agents on date  $t$  is denoted  $a_t^j$ , a vector of length  $L$ .

In the case of a single lender, the borrower pays the lender  $a_t^b, 0 \leq a_t^b \leq y_t$ , and the lender pays the borrower  $a_t^l, 0 \leq a_t^l \leq y^N$ . The borrower's and lender's single-period payoffs are  $[u(y_t + a_t^l - a_t^b) - u(y_t)]$  and  $[a_t^b - a_t^l]$ , respectively. Paying nothing to anyone else is a dominant strategy in the single-period game, so that payments autarky is the unique Nash equilibrium for

that game.

For the infinitely-repeated game, each agent makes a sequence of state-contingent payments to the other participants over time, and intertemporal exchange is possible. Payments can be conditioned on the current and past states of nature and on the past payments made by all agents. The history of the game is given by the sequences of past endowment realizations for the borrower and of past payments by everyone. Each possible history of the game up to any date  $t$  defines a subgame of the repeated game beginning at date  $t$ . The borrower's strategy specifies the payments she makes contingent on her endowment at all dates for every possible subgame, and similarly for each lender. An equilibrium for the game specifies the strategy for each agent.

In any period, the borrower's payoff for the remainder of the repeated game is her expected surplus, as defined by equation 3, from the consumption streams resulting from a sequence of state-contingent payments from date  $t$  forward. Lenders' payoffs are defined by equation 4. For the case of a single lender, these payoffs are

$$V_t^b = \mathbb{E}_t [u(y_t + a_t^l - a_t^b)] + \sum_{i=1}^{\infty} \beta^i \mathbb{E}_t [u(y_{t+i} + a_{t+i}^l - a_{t+i}^b)] \quad (5)$$

and

$$V_t^l = \mathbb{E}_t [a_t^b - a_t^l] + \sum_{i=1}^{\infty} \beta^i \mathbb{E}_t [a_{t+i}^b - a_{t+i}^l] \quad (6)$$

A sequence of state-contingent payments for all agents is called a path of the repeated game. We show below that in general, equilibrium payments are contingent on the current and past endowments of the borrower even though her endowment stream is i.i.d. Consumption may only be smoothed partially in equilibrium in this economy.

### 3. Equilibrium with a Borrower and a Single Lender

There are Pareto gains from risk sharing in this economy; state-contingent payments exist that give each agent higher expected utility than under permanent autarky. But the inability of the borrower and lender to commit may prevent efficient risk sharing. The incentive constraints hold at all dates for each participant, so we first require that an equilibrium of the repeated game be perfect. One perfect equilibrium is permanent repetition of the single-period Nash equilibrium, that is, permanent autarky. The theory of infinitely-repeated non-cooperative games implies that

there are perfect equilibria of this game that Pareto dominate permanent autarky if the discount rate is low enough.

Intertemporal exchange is enforced by punishment threats in a perfect equilibrium. Failure to make an equilibrium payment to the other party increases the current consumption of an agent. To be effective for supporting intertemporal trade, a punishment must reduce the discounted stream of future surpluses for the agent enough to offset the single-period gain. Refusing to pay the other agent is the most severe retaliation that the borrower or lender can make in this economy. A permanent payments embargo minimizes the maximum intertemporal surplus that each agent can attain. But permanent autarky is not immune to renegotiation between lenders and the borrower because it eliminates all future mutual gains from intertemporal exchange. In the absence of commitment opportunities, renegotiation of punishments to mutual benefit is natural to consider. Our second requirement is that an equilibrium be renegotiation-proof.

In the the case of a single lender, a perfect equilibrium will be renegotiation-proof if the borrower and the lender cannot agree to abandon the equilibrium for another perfect equilibrium after any history. There are many definitions of renegotiation-proofness in the literature on repeated games.<sup>13</sup> In this economy, a perfect equilibrium is shown to exist that satisfies all the existing notions and supports intertemporal exchange. In Section 5, renegotiation-proofness is extended to coalition-proofness for the case of many potential lenders by requiring that coalitions of two or more participants cannot mutually agree to abandon the equilibrium for another perfect equilibrium.

To find renegotiation-proof equilibria for the single-lender case, we first characterize the set of all perfect equilibria. This can be done by using permanent autarky as a punishment threat for any deviation from a proposed equilibrium payments path.<sup>14</sup> If the borrower fails to make the payment,  $a_t^b$ , required by the given path, then the punishment begins and her surplus starting the next period,  $V_{t+1}^b$ , equals zero; similarly, the lender realizes zero future surplus in equilibrium if he fails to make a payment. A perfect equilibrium path must satisfy the constraints

$$\max_{a_t^b} \mathbb{E}_t \left[ u(y_t + a_t^l - a_t^b) \right] + \mathbb{E}_t \sum_{i=1}^{\infty} \delta^i \mathbb{E}_t \left[ u(y_{t+i} + a_{t+i}^l - a_{t+i}^b) \right] - u(y_{t+i}) \quad (7)$$

and

$$\mathbb{E}_t \left[ a_t^b - a_t^l + \sum_{i=1}^{\infty} \beta^i \mathbb{E}_t \left[ a_{t+i}^b - a_{t+i}^l \right] \right] \leq \max_{a_t^l} \mathbb{E}_t \left[ a_t^b - a_t^l \right] \quad (8)$$

The inability to commit ensures that

$$\mathbb{E}_t \left[ \sum_{i=1}^{\infty} \beta^i \mathbb{E}_t \left[ u(y_{t+i} + a_{t+i}^l - a_{t+i}^b) - u(y_{t+i}) \right] \right] \leq 0$$

and

$$\mathbb{E}_t \left[ \sum_{i=1}^{\infty} \beta^i \mathbb{E}_t \left[ a_{t+i}^b - a_{t+i}^l \right] \right] \leq 0:$$

Inequalities 7 and 8 impose constraints on the stream of net payments made between the borrower and the lender on the left-hand side. The punishment payoffs on the right-hand side are minimized if only a one-way payment equal to the net payment is made at any time. Under a unilateral payments path, the highest single-period payoff to an agent who deviates is zero. This minimizes the present-value payoff either agent can achieve by deviating under any punishment threat (since punishments begin after deviation). Making simultaneous payments serves to reduce the efficacy of punishment threats. Because only net payments enter agents' utilities, restricting payments to be unilateral can increase the opportunities for consumption smoothing in perfect equilibrium. Therefore, the model predicts that we should not observe agreements to make two-way payments simultaneously.<sup>15</sup> Whenever the net payment to the borrower is positive, only the lender's constraint (inequality 8) can be binding, and conversely.

A perfect equilibrium that fully smooths the borrower's consumption exists if the following constraints are satisfied for all states of nature for some constant consumption  $c$ :

$$[u(c) - u(y_t)] + \mathbb{E}_t \left[ \sum_{i=1}^{\infty} \beta^i [u(c) - u(y_{t+i})] \right] \leq 0$$

and

$$(y_t - c) + \mathbb{E}_t \left[ \sum_{i=1}^{\infty} \beta^i (y_{t+i} - c) \right] \leq 0:$$

Because  $u(c)$  is concave, there is a solution to these inequalities for every state if  $\beta$  is larger than some  $\bar{\beta}$ , where  $1 > \bar{\beta} > 0$ . The appendix demonstrates that there is another value of the discount factor,  $\underline{\beta}$ , that is positive and less than  $\bar{\beta}$  such that the incentive constraints cannot all be satisfied except under autarky if  $\beta$  is less than  $\underline{\beta}$ . Our first proposition summarizes these observations so far.

**Proposition 1:** The borrower's consumption can be at least partially smoothed in perfect equilibrium if the discount factor is greater than  $\mathbf{b}$ . It can be fully smoothed if the discount factor is at least  $\bar{\beta}$ , where  $1 > \bar{\beta} > \mathbf{b} > 0$ .

If  $\beta > \mathbf{b}$ , then there are perfect equilibrium payment paths giving both the borrower and lender positive future expected surplus at any date. That is,

$$\beta E_t V_{t+1}^b > 0; \quad \beta E_t V_{t+1}^l > 0$$

and the incentive constraints,  $V_{t+1}^b \geq 0$ ,  $V_{t+1}^l \geq 0$ , hold for both agents in each state of nature for  $t + 1$ . This positive surplus gives an agent the incentive to make unilateral state-contingent payments to the other. At least some consumption smoothing is possible under perfection.

Values for  $\mathbf{b}$  and  $\bar{\beta}$  can be calculated given the standard deviation of output and an empirically sensible choice for the coefficient of relative risk aversion ( $\gamma$ ) for developing country debtors using a two-state example. For a coefficient of variation of the GDP equal to 4% (average for Latin America in the 1980s) and  $\gamma$  equal to four,  $\mathbf{b} = 0.92$  and  $\bar{\beta} = 0.96$ . For  $\gamma = 3$ ,  $\mathbf{b} = 0.955$  and  $\bar{\beta} = 0.978$ . Partial smoothing is still possible for a discount rate exceeding 3% when  $\gamma$  equals two and the coefficient of variation of GDP is 3% or more.

The set of all perfect equilibrium payment paths generates a utility possibility set for each possible value of the borrower's endowment,  $y^b$ . This is depicted in Figure 1 in terms of  $V_t^b$  and  $V_t^l$  for any  $y_t \in \{y^1, \dots, y^N\}$ . The set is convex and has a continuous frontier (see the proof of Proposition 2). The utility possibility frontier gives the surplus for each agent for equilibrium paths that are Pareto-efficient within the set of perfect equilibrium paths. These are constrained-efficient; they are not necessarily Pareto-optimal. Some efficient perfect equilibrium paths are Pareto-optimal when  $\beta \leq \bar{\beta}$ , since complete consumption smoothing is possible in this case. None are when  $\bar{\beta} > \beta > \mathbf{b}$ , although efficient perfect equilibrium paths partially smooth borrower consumption and Pareto-dominate permanent autarky in that case. An efficient perfect equilibrium path must produce surpluses for the two agents on the utility possibility frontier in every state for each date.<sup>16</sup> The payment paths associated with any distribution of welfare on the frontier are found by solving a dynamic programming problem, as discussed in the next section.

By the envelope theorem, the utility possibility frontier is downward-sloping: reducing the lender's surplus by decreasing the borrower's current net payment, raises the borrower's surplus.

As a consequence, perfect equilibrium paths exist that are efficient but give zero surplus to either the borrower or lender. The payoffs for these paths are the intercepts of the frontier with each axis, points A and B in Figure 1. In either of these paths, future payments smooth, at least partially, the borrower's consumption. Because expected future surplus ( $E_t V_{t+1}^l$  or  $E_t V_{t+1}^b$ ) can never be negative, an agent realizes positive surplus whenever he or she receives a payment. At A the borrower's surplus is zero, and the lender realizes all the surplus from the efficient perfect equilibrium in state  $y_t$ . The borrower cannot receive payment from the lender on date  $t$  if the allocation of surplus is given by point A. Because there is positive surplus in the intertemporal exchange relationship beginning at A on date  $t$ , the borrower will receive contingent payments in equilibrium in some future states. This implies that the borrower makes the first payment in the equilibrium path starting at time  $t$  with the division of surplus given by A.

The payment paths that correspond to points A and B can be used as punishments that are just as strong as permanent autarky, but unlike autarky, they are not Pareto-dominated by any other perfect equilibrium path. Any perfect equilibrium path can be supported by these punishments. Therefore, we can select any efficient perfect equilibrium path beginning in the first period for each possible state of nature as an equilibrium path. The associated punishments are the paths that provide surpluses at points A and B for any date and state. In every possible subgame, the equilibrium payoffs are on the Pareto frontier of all perfect equilibrium payoffs, so that this equilibrium is renegotiation-proof by all definitions in the literature. These arguments, proved in the appendix, are summarized as Proposition 2.

**Proposition 2:** A payments path that smooths the borrower's consumption, at least partially, can be sustained in a renegotiation-proof equilibrium for any  $\bar{\omega} > \mathbf{b}$ . This holds, in particular, for all efficient perfect equilibrium paths.

#### 4. Payments Dynamics with a Single Lender in and out of Equilibrium

The next step is to characterize payment dynamics in a renegotiation-proof equilibrium with a single lender. The payment dynamics in any equilibrium path or punishment path can be found by solving for the borrower's consumption dynamics in efficient perfect equilibrium. This is

done by solving a constrained dynamic programming problem. In every state for each date, renegotiation-proof punishments can be used to hold the borrower's or the lender's surplus to zero.

The constrained dynamic program solves for the utility possibility frontier treating the lender's surplus as a state variable in each state of nature in the solution path. This is denoted  $V_t^l(y_t)$ . The problem is stated as

$$V^b(V_t^l(y_t); y_t) = \max_f [u(c_t) - u(y_t)] + \beta E[V^b(V_{t+1}^l(y_{t+1}); y_{t+1})]g$$

with respect to the borrower's consumption,  $c_t$ , and  $V_{t+1}^l(y_{t+1})$  for  $y_{t+1} = y^1; \dots; y^N$ , subject to

$$[y_t - c_t] + \beta E[V^l(y_{t+1})] \leq V_t^l(y_t) \quad (9)$$

$$V^b(V_{t+1}^l(y_{t+1}); y_{t+1}) \leq 0; \quad \text{for each } y_{t+1} = y^1; \dots; y^N \quad (10)$$

$$V_{t+1}^l(y_{t+1}) \leq 0; \quad \text{for each } y_{t+1} = y^1; \dots; y^N; \quad (11)$$

$V^b(V_t^l; y_t)$  is the utility possibility frontier for date  $t$  when the borrower's endowment is  $y_t$ , expressed as a function of the lender's surplus.  $V_t^l$  must be chosen so that the borrower's surplus is non-negative in the maximum. A solution is an implicit contract specifying the borrower's current consumption and the promised surplus for the lender the next period.

This programming problem has been solved by Thomas and Worrall [1988] in their analysis of (non-renegotiation-proof) wage contracts.<sup>17</sup> Although all of our results and arguments hold if the borrower's endowment is generalized to follow a Markov chain displaying first-order stochastic dominance, we use the i.i.d. case to take advantage of the existing proofs of the dynamics for brevity's sake. The equilibrium path of state-contingent payments for any division of the initial total surplus,  $V_1^l$  and  $V_1^b$  is unique because  $u(c)$  is strictly concave.

Using the first-order and envelope conditions for an interior solution gives the Euler conditions, one for each state,  $y_{t+1} = y^1; \dots; y^N$ :

$$u^0(c_t) = u^0(c_{t+1}^n)(1 + \beta \lambda_{t+1}^n) - \tilde{A}_{t+1}^n; \quad (12)$$

where  $\lambda_{t+1}^n$  and  $\tilde{A}_{t+1}^n$  are non-negative Lagrange multipliers for each state,  $n = 1; \dots; N$ , and  $c_{t+1}^n$  is state-contingent consumption in  $t + 1$ . The multipliers are associated with each incentive constraint, 10 and 11, respectively.<sup>18</sup> In any particular state of nature, an incentive constraint can only be binding for one of the two agents, so that at most one of  $\lambda_{t+1}^n$  or  $\tilde{A}_{t+1}^n$  can be positive in

any given state (both can be zero).

The Euler condition implies that the borrower's consumption is completely smoothed between today and tomorrow if neither incentive constraint binds in tomorrow's state of nature. If  $\bar{A}_{t+1}^n > 0$  for state  $n$ , the lender's constraint binds and he realizes zero surplus in state  $n$ . In this case, the borrower's consumption falls ( $c_{t+1}^n < c_t$ ) because the lender is unable to pay any more to the borrower in that state in perfect equilibrium. If  $\bar{A}_{t+1}^m > 0$  for state  $m$ , then the borrower realizes zero surplus in state  $m$ , and her consumption rises ( $c_{t+1}^m > c_t$ ) because she is unwilling to make any larger payment to the lender in that state.

Because exogenous contract enforcement is unavailable, risk-sharing is limited to what can be sustained by endogenous punishments. The borrower's consumption is smoothed as much as possible in perfect equilibrium. The Euler conditions imply that the borrower's consumption at date  $t$  depends her endowment at  $t$  and on her consumption in the previous period. Therefore, payments follow a Markov chain even though the borrower's endowment is i.i.d. unless her consumption is fully smoothed.

Each point on the utility possibility frontier for  $y^n$  corresponds to a different current consumption for the borrower in equilibrium. Point A gives the borrower her lowest equilibrium consumption in state  $n$ ,  $\underline{c}^n$ . Moving along the utility possibility frontier toward B, her equilibrium consumption rises monotonically to a maximum,  $\bar{c}^n$ . Thomas and Worrall prove that the upper and lower bounds on the borrower's consumption are ordered as  $\underline{c}^1 < \underline{c}^2 < \dots < \underline{c}^N$  and  $\bar{c}^1 < \bar{c}^2 < \dots < \bar{c}^N$ . Along with the Euler conditions, these imply that if the borrower gets no surplus in the lowest state,  $y^1$ , she receives zero surplus (over her autarky utility) in every possible state the next period. Therefore,  $\underline{c}^1 = y^1$  and her consumption is never less than the lowest possible value of her endowment. Similarly, the borrower's consumption is never higher than  $y^N$ , and  $\bar{c}^N = y^N$ . Only the lender ever makes a payment in the lowest state, and only the borrower ever makes a payment in the highest. For intermediate states,  $\underline{c}^n < y^n < \bar{c}^n$  if  $\bar{c}^n > y^n$ .<sup>19</sup> If  $\bar{c}^n < y^n$ ,  $\underline{c}^n > y^n$  so that full smoothing of the borrower's consumption is possible in some equilibria.

Figure 2a portrays consumption intervals when there are three possible endowment levels and  $\bar{c}^n > y^n$ . The vertical bars show the range for the borrower's consumption in renegotiation-proof

equilibrium paths for each state. An equilibrium path is depicted by arrows. The borrower's endowment first equals  $y^2$ , and she gets all of the surplus from the equilibrium (anticipating free entry by lenders in the next section). The lender makes the first payment, equal to  $c^2$  minus  $y^2$ . The arrows show the borrower's consumption and net payments as her endowment rises to  $y^3$ , then falls to  $y^1$  and returns to  $y^2$ . In Figure 2a, complete smoothing is possible, but it is not immediately achieved for this initial division of surplus between the two agents.<sup>20</sup> In the steady state, the borrower's consumption is smoothed and equals  $c^1$ .

Figure 2b shows an example for  $\bar{w} > \bar{w} > \mathbf{b}$  starting in state  $y^1$ . The arrows depict a steady-state path beginning with payment by the lender. The borrower's endowment rises from  $y^1$  to  $y^2$ , then to  $y^3$  and back to  $y^2$ . Because consumption can only be partially smoothed, the borrower's steady-state consumption depends on her endowment and her past endowments via her past consumption level. She receives a payment in the middle state if her endowment is falling and makes a payment if it is rising. The four consumption levels of the sample path shown in Figure 2b are the steady-state realizations for this example. Both figures show that the division of surplus between the borrower and lender varies over time with the state of nature.

Suppose the borrower decides not to make her payment in her highest endowment state at some date  $t$  in the example of Figure 2b. The renegotiation-proof punishment is a new equilibrium path with all the surplus going to the lender. This is a payments path that begins in period  $t + 1$  with the borrower's consumption equal to  $c^n$  for whichever state  $y^n$  occurs. Punishment of the borrower beginning in state  $y^2$  is depicted in Figure 3a. In the borrower's punishment, the lender does not make a payment in any state until after the borrower has made a payment that gives the lender all the equilibrium continuation surplus. Note that in the example, the moratorium on payments to the borrower lasts only one period, but her surplus in punishment is as severe as it would be under the incredible threat of permanent autarky.

If the borrower fails to make her punishment payment at time  $t$ , the punishment simply restarts in period  $t + 1$ . The lender still makes no payments to the borrower until after she has made a payment that gives him all the surplus gained from renewing intertemporal exchange. It does not matter how long it takes for her to make this payment. As soon as she does, a new efficient perfect equilibrium path begins. Thus, the lender punishes the borrower by imposing a payments

moratorium that can be as short as one period. Following through on his punishment threat is both the lender's myopic and forward-looking best action. The borrower is just as well off making her payment in punishment as she is continuing to refuse.

If the lender fails to make an equilibrium payment, then he is similarly punished. The borrower makes no payments to the lender until after he has made a payment that gives all of the surplus from a new efficient perfect equilibrium path to her. The borrower imposes a payments moratorium that ends with the first payment by the lender. A punishment of the lender beginning in state  $y^3$  is depicted in Figure 3b.

The equilibrium payments path can be interpreted using a sequence of one-period loan contracts with state-contingent repayments. In the absence of commitment, long-term relationships can be supported using a sequence of one-period contracts, as shown by Fudenberg, Holmstrom and Milgrom [1990] and Rey and Salanie [1990] for long-term agency relationships. As they argue, longer maturity contracts are unenforceable when they specify behavior that cannot be supported by a sequence of one-period contracts.

Each contract consists of an amount lent,  $\ell_t(y_t)$ , at date  $t$  and state-contingent repayments,  $R_{t+1}(y_{t+1})$ , for date  $t + 1$  in each state of nature. Each contract earns zero expected profit,  $\ell_t = -E_t R_{t+1}(y_{t+1})$ . The loan principal,  $\ell_t(y_t)$ , equals  $c_t - y_t - R_t(y_t)$ , and the net payment to the borrower is  $\ell_t(y_t) - R_t(y_t)$ . The lender's surplus is

$$V_t^l = \ell_t + R_t(y_t) + E \left[ \sum_{i=t+1}^{\infty} -\delta^{i-t} (\ell_i - R_i(y_i)) \right] = R_t(y_t);$$

where the second equality is due to rearrangement.

The incentive constraint for the lender is equivalent to the restriction that every state-contingent repayment be non-negative. This rules out some insurance contracts by imposing an inequality constraint on  $R_{t+1}$ . This is another way of seeing why consumption-smoothing may be incomplete. We can think of the long-term relationship with state-contingent payments as guided by a series of simple standard loan contracts with repayments renegotiated ex post. The contract simply states a maximum repayment, at least as great as the highest actual repayment for the next period.<sup>21</sup> Our restriction that the lender cannot commit is equivalent to the constraint that renegotiation of a loan repayment does not result in a negative repayment. A negative repayment,  $R_t$ , implies that the lender makes a payment that is unprofitable in expectation when he

makes it. A negative value for  $R_t$  would make him worse off than if he left the relationship altogether; therefore, such a “renegotiation” would not be renegotiation-proof. Renegotiation of repayments may appear in accounting schemes as debt write-downs, reschedulings, or new concessionary loans without changing the equilibrium path of net payments in any way.<sup>22</sup>

It makes sense to distinguish between debt and insurance, but much of the literature on foreign lending does not do so. Bulow and Rogoff [1989b], Worrall [1990] and Atkeson [1991] all allow commitment by the lender. The solutions for the Worrall and Atkeson models explicitly show that the lender’s surplus is negative in some states in equilibrium.<sup>23</sup> Negative repayments play the key role in the argument of Bulow and Rogoff [1989b] against the possibility of reputational equilibria as discussed below.

## 5. Self-enforcement with Potential Entrants

In practice, markets such as that for international lending have more than two participants. In this section, we prove that intertemporal exchange can be sustained under anarchy in the presence of potential entry by other lenders. A renegotiation-proof equilibrium is constructed that maintains the bilateral consumption-smoothing relationship described above in the presence of more lenders. Our equilibrium is extended to one that is invulnerable to renegotiation within coalitions, and for clarity is called a coalition-proof equilibrium. The key to our demonstration is that all payments are made voluntarily looking forward in equilibrium.

The problem that motivates the extension of our equilibrium is as follows. Suppose that the borrower fails to pay her lender as required on some date. Then she also fails to make the subsequent state-contingent payment necessary to end the lender’s payments moratorium under the renegotiation-proof punishment for the two-agent economy. Instead, she starts a new consumption-smoothing path with a second lender. Both the borrower and second lender appear to be able to divide the positive surplus between them, while the first lender receives nothing.

The strategy for the first lender needs to include a reaction to interference in the borrower’s punishment by the second lender. The punishments we propose are a form of “cheating the cheater.” Suppose that the borrower and first lender follow an efficient plan for the two-agent economy beginning in the first period. Under free entry by lenders, the borrower will get all the

surplus at date 1. If she deviates, the borrower is punished as in the two-agent equilibrium: a moratorium on payments by any lender remains in force until she makes a payment to the first lender giving him all the surplus gained by starting a new intertemporal exchange path with him. If another lender makes a payment to the borrower while she is being punished, then he has deviated from the ongoing punishment. A new punishment, this time of the deviant lender, begins.

The second lender, like the first, cannot be forced by any exogenous punishment to make payments. He pays the borrower in a given state only if he thinks he will be repaid at some future date. Likewise, the borrower pays the second lender in a future state only if it raises her surplus in equilibrium at that time. In a state in which she is expected to repay the second lender, the original lender (or a third lender) can offer to start another (perfect equilibrium) intertemporal exchange path with her that begins with a smaller payment by the borrower. This gives her higher surplus, so she should not repay the second lender. The original lender can induce the borrower to abandon any new intertemporal exchange relationship she might form.

Under these strategies, the second lender never pays the borrower in perfect equilibrium of a subgame reached by the borrower cheating. Other lenders respect the punishment of the borrower in equilibrium. Using this type of strategy, if the borrower gives a new lender a payment, in any subgame, the new lender should cheat her by never repaying in equilibrium. Similarly, if a new lender cheats by paying the borrower, she maximizes her surplus by cheating him.

These strategies imply that a renegotiation-proof equilibrium path for the bilateral case can be made proof to possible renegotiations between the borrower and new lenders. Under “cheat the cheater,” a coalition of the borrower and a new lender, or group of new lenders, cannot attain for themselves payoffs superior to permanent autarky. The appendix formally defines the equilibrium and proves the following proposition:

**Proposition 3:** Intertemporal exchange can be sustained in the absence of all external enforcement even if there is more than one potential lender. An equilibrium exists that supports constrained-efficient smoothing of the borrower’s consumption and is immune to any mutually beneficial renegotiation between agents.

The proof demonstrates the existence of an equilibrium that generates an efficient perfect

equilibrium path in every possible subgame of the repeated game.<sup>24</sup>

This result contradicts the claim of Bulow and Rogoff [1989b] that the threat of non-cooperation alone cannot support lending and repayment in an infinitely repeated game of smoothing a sovereign's consumption. In their proof, however, they allow lenders to offer contracts that bind them to make future payments enforced by an external authority.<sup>25</sup> With exogenous enforcement of lenders' agreements, a new lender can offer an insurance contract to the borrower under which she pays first and he credibly commits to repay. In a model with asymmetric exogenous enforcement, intertemporal trade to smooth the risk-averter's consumption is possible, but it can be initiated only by a payment from the party that cannot commit to the party that can. Bulow and Rogoff show that under their assumptions international insurance is possible. In contrast, we show that international lending under anarchy is possible.<sup>26</sup>

## **6. Sovereignty, Debt Contracts and Renegotiation**

The equilibrium sequences of payments between the sovereign borrower and a lender can be interpreted as the equilibrium outcome of lending and repayment guided by crude debt contracts that are renegotiated without benefit of exogenous enforcement. The explicit terms of repayment on loans to sovereigns are not generally followed in equilibrium, in contrast with the case for many domestic lending relationships. The designation of collateral is typical for domestic lending in developed market economies. In a simple contract, the lender's obligation is discharged at the initiation of the loan. If a borrower who defaults loses collateral of greater value than the repayment, adherence to the explicit conditions of the loan is subgame perfect; renegotiation is not an issue. This requires the existence, capability and commitment of a third party to allocate collateral contingent on debtor performance. If these conditions are fulfilled, lender commitment is moot, and borrower performance is induced by the commitment of a third party to enforce contracts.

International loans are different because sovereign immunity limits the capacity of third parties to enforce explicit contracts through the international reallocation of collateral. Our model captures lending and repayment between sovereigns by assuming that payments are only made if doing so raises equilibrium surplus looking forward at the time of payment. Lenders are agents

who make payments voluntarily on the expectation of future repayments and not under force by a third party.

A common approach of the literature has been to reverse the balance of commitment in the collateralized loan contract by assuming that the lender always commits to fulfill any contractual obligations (as would be plausible if the lender's obligations were collateralized).<sup>27</sup> In two-agent models, such collateralization makes the threat of reverting to autarky a perfect equilibrium punishment for the borrower. This can support financial relationships of the type discussed by Grossman and van Huyck [1988] and interpreted with explicit dynamics by Worrall [1990] and Atkeson [1991]. In models with more parties, Bulow and Rogoff [1989b] show that asymmetric enforcement binding only on lenders can render this threat of punishment incredible.

Lender commitment requires an explicit contract to inform third parties of the commitments undertaken. In our model, there is no exogenous party to enforce any commitments, so that with a single lender and borrower, no explicit contract is needed; explicitness, after all, is for third parties. With potential free entry by other lenders and no exogenous enforcement, an explicit crude debt contract could play the modest role of identifying the lender making the initial loan and of disclosing the terms of repayment. In the equilibrium demonstrated for this economy, all lenders but the first simply need to know that an efficient bilateral relationship was formed. Then they recognize that there is nothing to be gained by making payments to the borrower. The other lenders do not need to participate in the negotiation of the implicit contract between the borrower and her lender. They just need to be informed of the bilateral relationship so that they cooperate in equilibrium.

A crude debt contract may serve to publicize the relationship between the borrower and her lender as a "tombstone" advertisement. This publicity facilitates cooperation by the other potential lenders by making prior relationships common knowledge. In common parlance, such cooperation is called "respect for seniority," meaning that other lenders will not deal with the borrower until her obligation to the initial lender has been discharged. In contrast, much of the earlier literature assumes exogenous enforcement of lender seniority by the lenders' governments: any payments to a junior lender by a borrower in default are reallocated by force of law to the senior lender. Under the asymmetric commitment opportunities assumed in Bulow and

Rogoff [1989b], seniority enforcement must extend to other financial transactions (in particular, the cash-in-advance insurance transactions) if international lending is to occur.

## 7. Conclusion

Borrowing and lending between sovereigns can be viewed as intertemporal barter without exogenous enforcement of commitments. The surplus in a consumption-smoothing relationship by itself provides sufficient incentive for cooperation by the borrower, lender and any potential lenders along the equilibrium path and in punishments if an agent has deviated. The explicit or implicit assumption in other models of sovereign debt that a third party is available to enforce commitments, including respect for lender seniority or monopoly rights in commodity trade, is inessential to sustain lending to sovereign states.

By modelling renegotiation-proof intertemporal exchange, our model captures the essence of credit transactions without collateral. We show that intertemporal barter under anarchy is feasible without appealing to the threat of exogenous force (Hirshleifer [1995], p. 28). Both sides of the market have symmetric lack of capacity for commitment, so we directly affirm that reputation alone can sustain intertemporal exchange including cases where the initial payment flows to the party whose consumption is smoothed, as observed in lending to sovereign states. Reputation here refers purely to the past actions of the participants; under common knowledge there is no need for an agent to signal her type via actions to sustain intertemporal barter. Incomplete information, introduced for example by assuming a borrower type that values honesty for its own sake (as assumed by Cole and Kehoe [1992]) is an alternative way to model reputations, but is not necessary to assure that reputations alone sustain sovereign borrowing.<sup>28</sup>

In contrast, the “constant recontracting model” of Bulow and Rogoff [1989a] portrays repeated simultaneous trade in a bilateral monopoly. In that model, a “loan” is the one-time payment for a monopoly franchise to the purchase of a country’s exports, and a “repayment” is the monopsonist surplus gained in simultaneous exchange each period. In our model, the lender and borrower trade intertemporally, making payments at different dates throughout the relationship. This relationship is permanent, even though it is formally manifest as sequence of short-term debt contracts that are frequently “violated” and “renegotiated”.

## Appendix

**Proof of Proposition 1:** Existence of  $\bar{\beta} < 1$  is immediate from the perfection constraints. That some gains from trade can be sustained in perfect equilibrium is an immediate application of general results for repeated games. The existence of a  $\beta > 0$  such that smoothing is not possible for  $\beta > \bar{\beta}$  is proved showing that small payments in the two extreme states of nature cannot satisfy the perfection constraints if  $\beta$  is small but positive. For  $y_t = y^1$ ,  $c^1 \leq y^1 > 0$ , for  $y_t = y^N$ ,  $c^N \leq y^N < 0$  and for every other state,  $c^n = y^n$ . The constraints that may bind are

$$\frac{\beta}{1-\beta} \left[ \frac{1}{4} u(c^1) - u(y^1) + \frac{1}{4} u(c^N) - u(y^N) \right] \leq u(y^N) - u(c^N)$$

and

$$\frac{\beta}{1-\beta} \left[ \frac{1}{4} u(y^1) - c^1 + \frac{1}{4} u(y^N) - c^N \right] \leq c^1 - y^1;$$

where  $\frac{1}{4}$  is the probability that state  $n$  occurs. Using a first-order Taylor expansion, these lead to

$$-\frac{1}{4} u'(y^1) (c^1 - y^1) \leq \frac{1}{4} [u'(y^1) - u'(y^N)] (y^N - c^N)$$

and

$$-\frac{1}{4} u'(y^N) (y^N - c^N) \leq \frac{1}{4} [u'(y^1) - u'(y^N)] (y^1 - c^1).$$

These are violated if

$$-2 < \frac{u'(y^N) - u'(y^1)}{\frac{1}{4} [u'(y^1) - u'(y^N)]} \leq \frac{1}{4} [u'(y^1) - u'(y^N)] \left( \frac{1}{4} [u'(y^1) - u'(y^N)] \right)^{-1}.$$

### Proof of Proposition 2:

Let  $W^n$  denote the utility possibility set in state  $n$ . Define  $h_t = (y_1, \dots, y_t)$  to be the history of nature. The proposition follows from the following two results: (a) For each state of nature,  $y^n$ , the set of all payoffs sustained by some perfect equilibrium,  $W^n$ , is non-empty, compact and convex. (b) Its Pareto frontier,  $V^b(V^l; y^n)$ , is decreasing in  $V^l$  and contains as its endpoints, two points given by  $(\hat{V}^b(y^n); 0)$  and  $(0; \hat{V}^l(y^n))$ , where  $\hat{V}^b(y^n)$  is the maximum of  $V^b$  over  $W^n$  and  $\hat{V}^l(y^n)$  is the maximum of  $V^l$  over  $W^n$ .

(a) The set  $W^n$  is non-empty because it always includes the origin. Compactness can be proven by application of Theorem 4 of Abreu, Pearce and Stacchetti [1990]. Their proof, based on the notion of self-generation, are written under the assumption that the action space is finite at each stage. Careful inspection of their proof reveals that the Theorem is valid for a discounted

game of perfect information when the action space for each player is a compact interval and the stage-game payoffs are continuous. Compactness of the set  $W^n$  for each  $n$  follows from Theorem 4.<sup>29</sup> An alternative proof of compactness follows from application of Tychonoff's Theorem.

Convexity could be proved by applying Theorem 5 of Abreu, Pearce and Stacchetti [1990], but it is simpler to show that the set of perfect equilibrium paths for any initial state  $y^n$  is convex. Suppose that  $s$  and  $s^0$  are two equilibrium paths. The convex combination of  $s$  and  $s^0$  is given by  $s^\alpha = \alpha a^\alpha(!_t)g_{t=1}^1$  where  $a^\alpha(!_t) = \alpha a(!_t) + (1 - \alpha)a^0(!_t)$  for every  $!_t$  and  $0 \leq \alpha \leq 1$ . Since  $u(c)$  is concave,  $V^b(s^\alpha; !_t) \geq \alpha V^b(s; !_t) + (1 - \alpha)V^b(s^0; !_t) \geq 0$ . Also,  $V^l(s^\alpha; !_t) = \alpha V^l(s; !_t) + (1 - \alpha)V^l(s^0; !_t) \geq 0$ . Therefore,  $s^\alpha$  is a perfect equilibrium path. Since  $u(c)$  is concave, convexity of  $W^n$  follows.

(b) That the Pareto frontier of  $W^n$  is decreasing is straightforward (simply increase  $\zeta(y^n) = a^l(y^n) - a^b(y^n)$  in any efficient path providing positive surplus to each agent). Together with compactness and non-emptiness, this assures that there are points in  $W^n$ ,  $(\bar{V}^b(y^n); 0)$  and  $(0; \bar{V}^l(y^n))$ , such that  $\bar{V}^i(y^n)$  is the maximum of  $V^i(y^n)$  over  $W^n$ :

### Proof of Proposition 3:

This proof demonstrates that the proposed equilibrium is a strong perfect equilibrium, as defined by Rubinstein [1980], which is a more restrictive criterion than coalition-proofness as defined by Bernheim, Peleg and Whinston [1987].

For the  $L + 1$  person repeated game, the set of payoffs sustainable using subgame perfect equilibria, given initial state  $y_1 = y^n$ , is given by

$$f(V^b; V^1; \dots; V^L) \in W^n \iff \begin{matrix} \exists \\ \text{for } (\alpha^b; \alpha^1) \in W^n \end{matrix} V^l = \alpha^l; \text{ and } V^l \geq 0; \text{ for all } l = 1; \dots; L;$$

where  $W^n$  is the utility possibility set for all perfect equilibria of the two-person repeated game. Let there be at least two lenders, that is,  $L > 1$ . To prove the proposition, we construct a strong perfect equilibrium that sustains an efficient perfect equilibrium payments path that gives all the surplus to the borrower. That is,  $V^b = \bar{V}^b(y^n)$ ,  $V^1 = 0, \dots, V^L = 0$ , where  $\bar{V}^b(y^n) = \max_{(\alpha^b; \alpha^1) \in W^n} f^b : (\alpha^b; \alpha^1) \in W^n$  (this maximal surplus depends on the state of nature for period 1). This path is

denoted by  $s = \{a_t^1\}_{t=1}^{\infty}$  where  $a_t^1$  is the vector of all payments made by every agent to each other agent in period  $t$  contingent on the history of nature through date  $t$ ,  $h_t = (y_1, \dots, y_t)$ . In the selected path  $s$ , only the borrower and lender 1 make any payments and only to each other. The sequence of state-contingent payments made by the borrower and lender 1 in this path are an efficient equilibrium path of unilateral payments for the single-lender case such that the borrower gets all the surplus in period 1.

Our proposed punishment of the borrower (if she deviates from any ongoing path at date  $t$ ) is the perfect equilibrium path starting in period  $t + 1$  that gives all the surplus for the repeated game starting in state  $y_{t+1}$  to lender 1. In this punishment, other lenders pay and receive nothing. The punishment of lender 1 for deviating in period  $t$  is a restart of the path  $s$  in period  $t + 1$ . Call these punishments,  $p_{t+1}^b$  and  $p_{t+1}^1$ , respectively, for all  $t$ . Note that  $p_{t+1}^b$  starts if the borrower deviates from  $p_t^b$  by not paying lender 1 in period  $t$ .

In the paths,  $s$ ,  $p_{t+1}^b$  and  $p_{t+1}^1$ , lenders 2 through  $L$  make no payments. Any one of them deviates by making a payment. No reaction is needed if one of these lenders makes a payment to any other lender. If one of them makes a payment to borrower during her punishment, then the agents' strategies need to react.

Consider a subgame beginning at date  $t + 1$  such that the borrower deviated in period  $t$ . Let lenders 1 through  $L - 1$  behave according to the punishment  $p_{t+1}^b$ . Suppose that the borrower and lender  $L$  negotiate some perfect equilibrium payments path for the two-agent game played between them from period  $t + 1$  onward other than permanent autarky. Label this path  $q_{t+1}$ . For  $q_{t+1}$  to be a non-autarkic perfect equilibrium path, there must be future events in which it specifies that lender  $L$  pays the borrower. If lender  $L$  makes a positive payment to the borrower at time  $t^0$ ,  $t^0 \geq t + 1$ , then there must be a subsequent event in which  $q_{t+1}$  requires the borrower to pay lender  $L$ .

Consider a sample sequence of states of nature from  $t + 1$  to some date  $t^{00} > t + 1$ , such that lender  $L$  pays the borrower at a time  $t^0 < t^{00}$  and the borrower's first subsequent payment occurs at  $t^{00}$  following  $q_{t+1}$ . In period  $t^{00}$ , lender 1 and the borrower can negotiate to start an efficient perfect equilibrium path with a payment by the borrower to lender 1 that is smaller than her payment to lender  $L$  would be under  $q_{t+1}$ . This gives the borrower a higher payoff than she

can realize by paying lender  $L$ . Therefore, she defects from  $q_{t+1}$ . The path  $q_{t+1}$  is not proof to renegotiation between the borrower and lender 1.

The borrower maximizes her surplus by never paying anyone besides lender 1 after she has defected from the initial payments path,  $s$ . This is also true if she has not defected: if lender  $L$  deviates from  $s$  by paying her, she can only reduce her surplus by paying him back. Autarky is the only two-agent perfect equilibrium path for the coalition of the borrower and lender  $L$  that is proof to renegotiation by the coalition of the borrower and lender 1. This holds for every lender 2 through  $L$ . Therefore, lenders 2 through  $L$  make no payments ever to the borrower in the equilibrium for any subgame that can be reached. Consequently, the only payments the borrower ever makes in this perfect equilibrium are to lender 1, and the proposed equilibrium is a strong perfect equilibrium and, therefore, is coalition-proof.

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## Notes

<sup>1</sup>Lindert and Morton [1989] examined 1552 external bonds of ten borrowing governments (approximately the top ten borrowers over the past thirty years) outstanding in 1850 or floated between then and 1970, following all through to settlement or the end of 1983. Defaults were not only common but widespread in their sample; most of the countries had some defaults in each of the periods 1820-1929 and the 1930s (p. 61). A detailed summary of experience by country is presented in their Table 2.8.

<sup>2</sup>Eichengreen and Portes [1989b] examined 125 London overseas issues and a sample of 250 United States foreign issues floated in the 1920's. (Nearly half of latter, by value, lapsed into default (p. 233)). In their samples, British bonds had an overall internal rate of return of 5 percent, higher than domestic investments (Eichengreen and Portes [1989a, p. 77], while United States loans to national governments had an internal rate of return of 4.6 percent, compared to the 4.1 percent yield on United States treasury bonds over the 1920s (pp. 35, 38). These yields were, however, substantially below those offered ex ante, which were generally between 7 and 8 percent (p. 27). Overall, the bonds in the Lindert and Morton [1989] sample proved profitable; the average 2 percent ex ante premium over domestic government bonds became a 0.42 percent premium ex post (p. 77). Further, they find (p. 59), that "there is no clear evidence of a systematic difference in realized returns" between the bonds of their ten borrower governments and United States domestic corporate bonds.

<sup>3</sup>Eichengreen and Portes [1989b, p. 234] report that, in their 1920s samples, "The typical default reduced the internal rate of return by 4.3 percent for dollar loans, but 1.4 to 2.3 percent for sterling loans." They note, for example, that all sterling loans to Brazil in that period went into default, but they yielded positive internal rates of return between 1.1 and 2.3 percent.

<sup>4</sup>See Eichengreen and Lindert [1989].

<sup>5</sup>Wallich [1943] expresses this view, and Grossman and van Huyck [1988] call such defaults "excusable". The idea that defaults might not always violate the underlying equilibrium relationship helps explain the findings of Lindert and Morton [1989] and Eichengreen [1989] that defaulters have not generally suffered subsequent discrimination in credit terms, and also the finding of Ozler [1993] for loans from 1968-81 that the average penalty for past defaults was only a small fraction of interest spreads.

<sup>6</sup>Our model of anarchy in international relations is related to Hirshleifer [1995]. In particular, see p.27. In contrast to Hirshleifer's generic model, we assume implicitly that fighting is ineffective for appropriating international resources, as is true if Hirshleifer's "decisiveness parameter" is zero.

<sup>7</sup>These include the enforcement of trade sanctions and of creditor seniority privileges either explicitly or implicitly assumed.

<sup>8</sup>The many papers that use a reputation for repayment to sustain international lending in infinite-horizon models with complete information include Eaton and Gersovitz [1981], Kletzer [1984, 1989], Eaton, Gersovitz and Stiglitz [1986], Grossman and van Huyck [1988], Wörrall [1990], Atkeson [1991] and Kehoe and Levine [1993]. Papers using using games of incomplete information include Cole and Kehoe [1992], Cole, Dow and English [1994] and Gale and Hellwig [1989]. All assume some form of exogenous enforcement.

<sup>9</sup>Eaton [1990] and Chari and Kehoe [1993] also point out that Bulow and Rogoff [1989b] assume the existence of an external authority to enforce loans made by the borrower but not loans made by the lender, as labelled in their article.

<sup>10</sup>Models that adopt a consumption-smoothing motive for international financial flows include Eaton and Gersovitz [1981], Atkeson [1991], Craig [1991], Grossman and van Huyck [1988], Kehoe and Levine [1993], Kletzer [1989 and 1994] and Wörrall [1990]. Cole and English [1992] study expropriation of equity investment in a consumption-smoothing model, and Kletzer, Newbery and Wright [1992] study loan, futures and options contracts for international smoothing in the presence of sovereign risk.

<sup>11</sup>Our approach to modelling credible punishment of sovereigns differs from the analysis of sanctions of Eaton and Engers [1992] in two essential ways. The first is that they model a bilateral relationship and so are not concerned with our main issue — the problem that new entrants might benefit by not cooperating in a punishment. The second is that they study Markov perfect equilibria of a game in which the power to sanction is exogenous to borrowing and lending, as in Bulow and Rogoff [1989a], rather than subgame perfect equilibria of a game in which the incentives to cooperate derive from the surplus internal to the

intertemporal smoothing relationship.

<sup>12</sup>The borrower's endowment follows a Markov chain in the working paper version, Kletzer and Wright [1995].

<sup>13</sup>Important definitions and results for infinitely-repeated games are made by Farrell [1984], Bernheim and Ray [1989], Farrell and Maskin [1989], Abreu, Pearce and Stacchetti [1993] and Ray [1994]. Pearce [1992] surveys and compares concepts of renegotiation-proofness.

<sup>14</sup>Abreu [1988] proves that all perfect equilibria can be found using minmax punishments as done here.

<sup>15</sup>In the sense that one party's current payment cannot be conditioned on the other's current payment.

<sup>16</sup>Perfect equilibria for repeated games satisfy the principles of dynamic programming as shown most generally by Abreu, Pearce and Stacchetti [1990].

<sup>17</sup>Thomas and Worrall study smoothing of a risk-averse worker's consumption when her opportunity spot wage is stochastic and the employer cannot be bound to wage contracts. They assume trigger strategy punishments are used by the employer and worker to enforce the contract. Kocherlakota [1996] solves for a similar equilibrium, supported by trigger strategies, for two risk-averse agents. His equilibrium could equally well have been used here by assuming a risk-averse lender.

<sup>18</sup>The multipliers for the implied Lagrangean are  $\beta^n \bar{c}_{t+1}^n$  and  $\beta^n \bar{A}_{t+1}^n$  where  $\beta^n$  is the probability of state  $n$ .

<sup>19</sup>If  $\beta < \beta^n$ , then  $\bar{c}^n = \bar{c}^n = y^n$  for every  $n$  since consumption smoothing is not possible.

<sup>20</sup>Note that we can find an initial allocation of surplus such that consumption is smoothed for all periods. Pareto optimal allocations are achievable for large enough discount factors, but not every efficient perfect equilibrium path is Pareto optimal.

<sup>21</sup>Grossman and van Huyck [1988] propose this interpretation of sovereign debt renegotiation. They do not impose the constraint that lenders will not take negative repayments.

<sup>22</sup>Fernandez and Rosenthal [1990] model the negotiation of a repayment in an extensive-form game with an exogenous default penalty. They model the one-time termination of a debt relationship, rather than renegotiation in a long-term relationship with endogenous incentives. Gale and Hellwig [1989] present a similar model with incomplete information about the borrower's type.

<sup>23</sup>Grossman and van Huyck [1989] assert if the borrower's income is i.i.d. so is her consumption, inconsistent with the solution under non-commitment. Worrall [1990] shows that consumption is not iid outside the steady state when the risk-neutral agent can commit.

<sup>24</sup>This proposition demonstrates existence, not uniqueness. For example, Figures 2a and 2b show states of nature such that the lender's surplus is zero in the steady state. When these are reached, a new lender could take over smoothing the borrower's consumption, and similar punishments to those described in the text enforce the equilibrium.

<sup>25</sup>In their paper, Bulow and Rogoff note that they make this assumption but write that it is unnecessary for their claim that reputational equilibria alone will not work. See Bulow and Rogoff [1989b], page 45, lines 12-16. Cohen [1991, page 94] makes a similar claim in a consumption-smoothing model. He imposes the constraint that the borrower will just be indifferent in period  $t + 1$  between autarky and repayment if she repays in period  $t$ . Therefore, repaying in period  $t$  can only make her worse off. There is a problem: the continuation values are fixed rather than derived from equilibria for the subgames reached, so that his argument does not address whether lending and repayment can be self-enforcing.

<sup>26</sup>Worrall [1990] solves for the efficient smoothing path in a two-party model under the one-sided commitment assumption made by Bulow and Rogoff [1989b]. Kletzer, Newbery and Wright [1992] show how option contracts can be used in combination with one-period loans to approximate Worrall's efficient solution.

<sup>27</sup>Worrall [1990] and Atkeson [1991] explicitly state this assumption. The assumption is implicit in Grossman and van Huyck [1988]. In the Atkeson and Worrall models, the lender's surplus is negative in some events in equilibrium for the general case.

<sup>28</sup>Cole and Kehoe [1992] pursue the possibility, adumbrated in Bulow and Rogoff [1989b], of a reputational equilibrium in which the borrower is concerned about the implications for her other market relationships of the reputation (the conditional probability that she is "honest") that she establishes in the loan market. Other models of incomplete information include Eaton [1990], Cole, Dow and English [1991] and Thomas [1992].

<sup>29</sup>To apply the results of Abreu, Pearce and Stacchetti [1990], the payoffs are first renormalized by multiplying by  $\beta$ . Define the payoff  $\beta^{-1}V^i(a, \omega)$  as  $\beta^{-1}(u_i(a, y) + E\omega^i)$ . Let  $W$  denote the  $N$ -tuple of the sets  $W^1, \dots, W^N$ , and  $\omega$  denote the  $N$ -tuple of the continuation payoffs,  $\omega^1, \dots, \omega^N$ . The pair  $(a, \omega)$  is admissible with respect to  $W$  if  $\omega^i \in W^i$  and  $V^i(a, \omega)$  is maximal with respect to the action  $a_i \in A_i(y^i)$ , for each agent  $i = 0, 1$ . The set  $B(W)$  defined by Abreu, Pearce and Stacchetti is given by  $\beta^{-1}V(a, \omega)$  is admissible w.r.t.  $W$ .

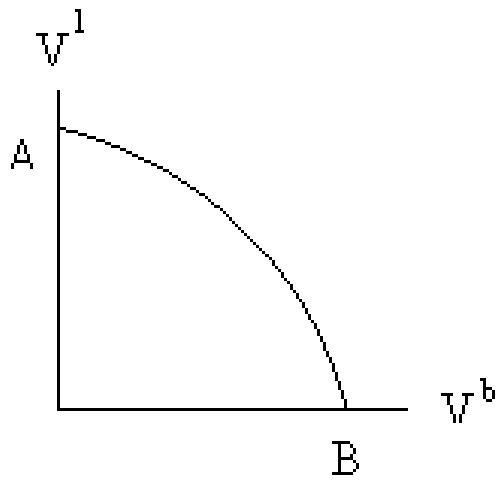


Figure 1

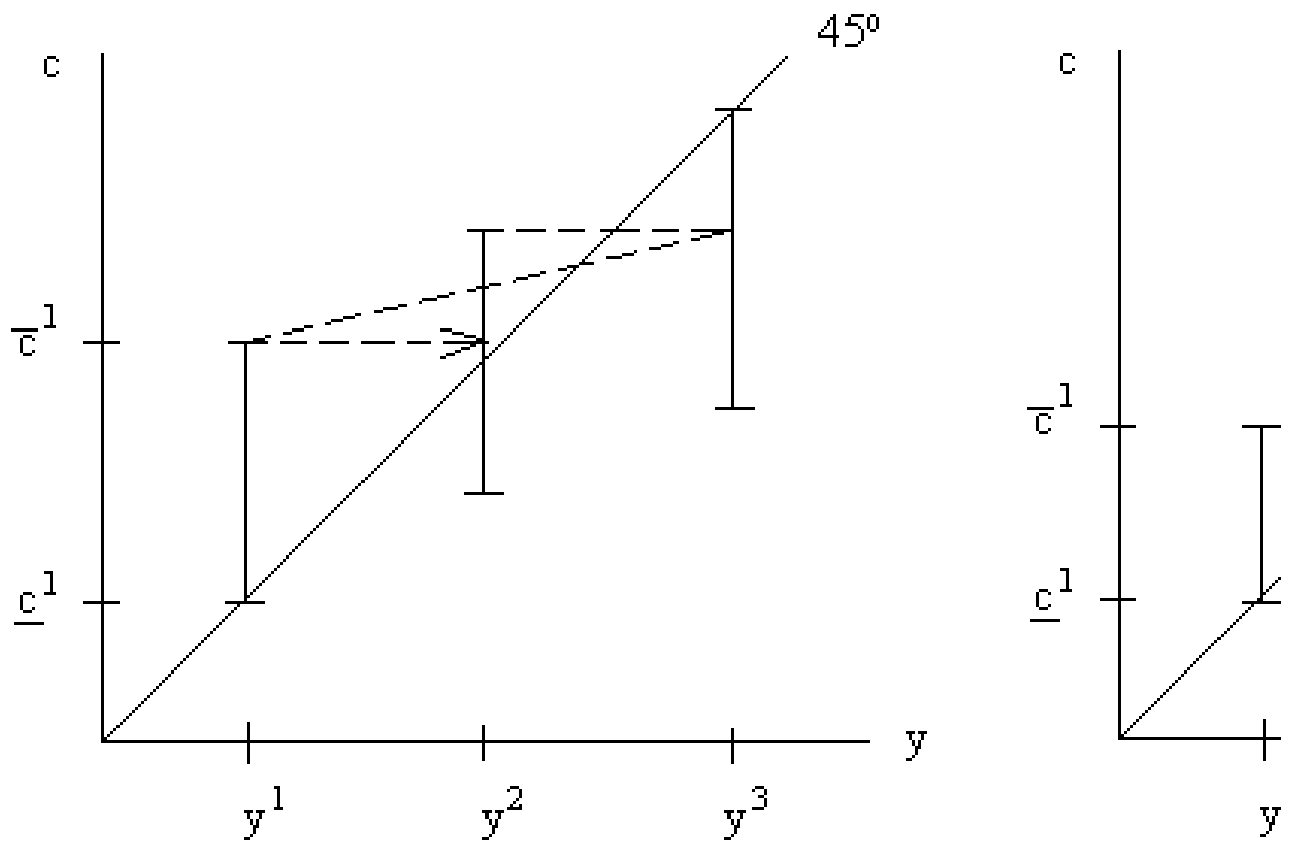


Figure 2a

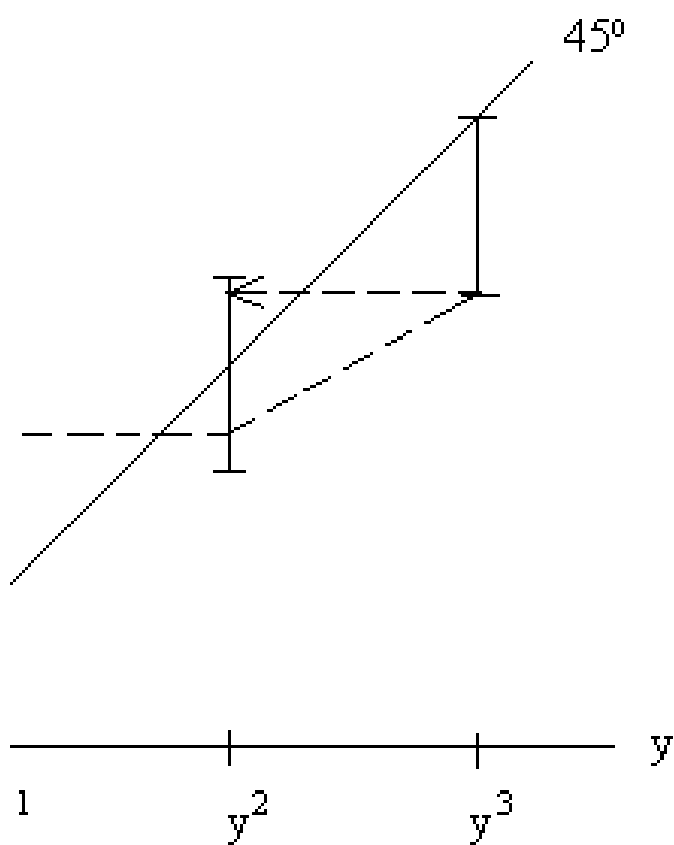


Figure 2b

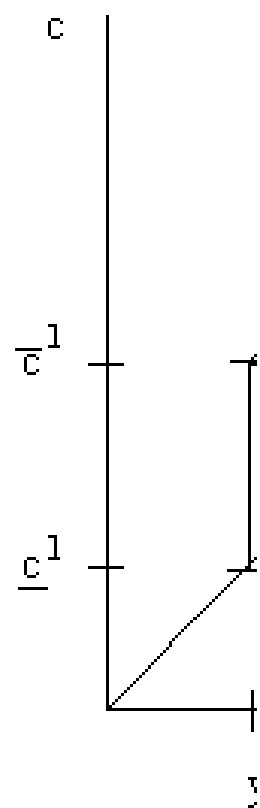
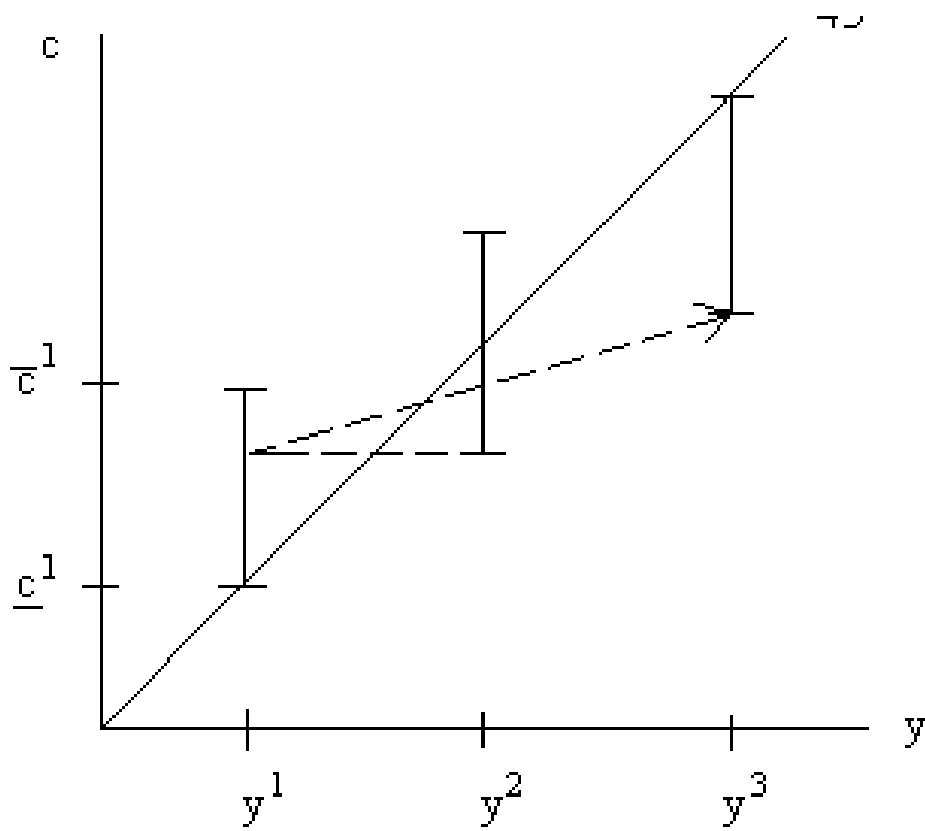


Figure 3a

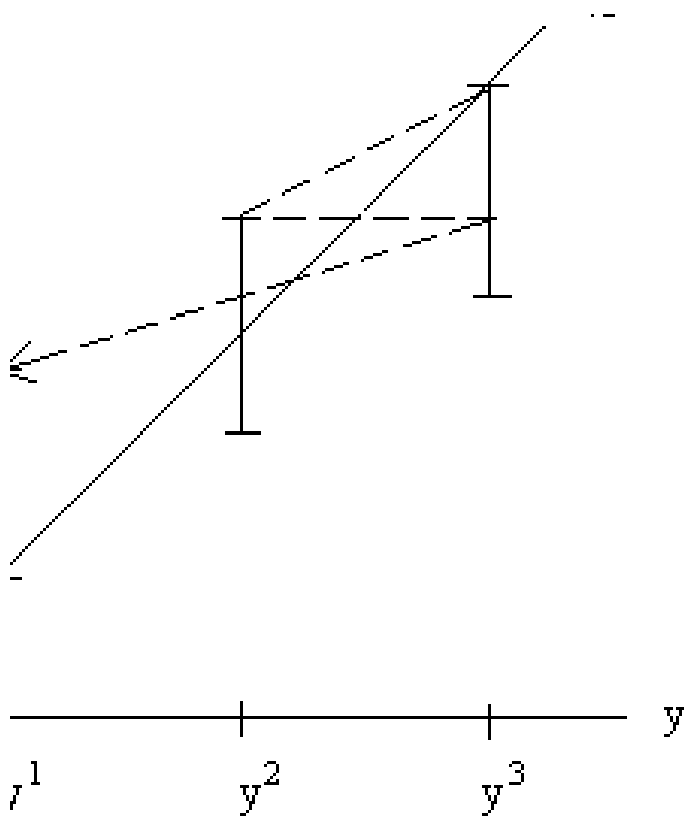


Figure 3b