# Child Labor and Fertility

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**Abstract**

This essay analyzes the economic causes and effects of household decisions concerning fertility, education and child labor when children can supplement family income early in life and must support their parents in old age as adults. Parents, who raise and educate children for both financial and altruistic reasons, will typically choose a too little schooling for the economy to grow when all are poor. High child-raising costs or an educational process which is not sufficiently productive are the main reasons for the existence of a poverty trap with a high population growth rate and little or no schooling. Interventions such as taxes and subsidies can lead to sustained long-term economic growth, with fulltime schooling and a low population growth rate, even without outside aid, if the child-raising costs are not too high and the educational process is at least moderately productive.

JEL Classification: O1, I2

# Contents

# 1 Introduction

With more than 250 million children working worldwide, the overwhelming majority of them in poor countries, child labor is a major problem. At the same time, fertility is still well above replacement levels, even though it has started to fall from the very high levels that prevailed for most of the 20th century.

As population growth rates in most developing countries have started to fall within the last 10 years, perhaps as a delayed response to improved health conditions and an increase in life expectancy, it seems clear that families can influence the number of children they have. This is confirmed by statistics on the use of contraception (contraceptive prevalence), which has increased from 18% in 1990 to 32% in 2000 in the least developed countries, and data on the total fertility rate, which has decreased from 5.9 children in 1990 to 5.4 in the year 2000 in the least developed countries[1].

Families have several reasons for raising and educating children – altruistic, social and financial. Some religious groups are known to encourage their adherents to have children, and stigmatize families who do not. Parents in highly developed countries do not raise children for financial reasons, as consumption in all stages of adult life is ensured either by their own income or through savings for retirement and social insurance. With access to capital markets being limited in most developing countries, saving for retirement is not a viable option, so that parents have to ensure consumption in old-age by having enough children to support them. Raising children is costly, however, especially when they are very young and cannot earn, and even prevent one parent from working. When they become old enough to earn or help in running the family entreprise, educating them involves opportunity costs, at the very least. Therefore, financial reasons will play a major role in fertility decisions.

There is a growing literature on child labor, which has been recently surveyed by Basu (1999). When parents decide about their children's education, multiple equilibria can rise, even when fertility is exogenous. Basu and Van (1998) and Swinnerton and Rogers (1999) were the first to analyze such a setting. Ranjan (1999) focusses on the connection between capital markets and child labor: without access to credit,

[1]Source: http://www.childinfo.org/eddb/fertility/index.htm, UNICEF and UN

parents have to send their children to work. If borrowing were possible, parents could finance the (opportunity) costs of their children's education; for the loans could be paid back through the additional income of well-educated children. Baland and Robinson (2000) were the first to analyze child labor and fertility simultaneously. Inefficiency arises, as parents fail to internalize the negative effects of child labor. The setting in Dessy (2002), who also analyzes both fertility and child labor, also exhibits multiple equilibria: a poverty trap with high population growth and little education, and a steady state with low fertility and high productivity. Hazan and Berdugo (2002) focus on the effects of technological progress, which decreases the relative wage of child labor and therefore encourages education. Even if the economy is initially in the low steady state it can escape to sustained growth, with rising education and decreasing fertility.

This paper will analyze the combined problem of child labor, fertility and provision for old age when the accumulation of human capital can lead to sustained growth. Parents educate children for both altruistic reasons and as a means of financing consumption during retirement. The framework draws on Raut and Srinivasan (1994) and Bell and Gersbach (2002). The former analyze the effects of endogenous fertility on economic growth in the absence of altruism. As capital markets are such that saving in the form of physical capital is possible while borrowing is not, parents have children solely as a means of financing consumption during retirement. Various growth paths are possible: convergence to a steady state, chaotic, and divergent. Bell and Gersbach examine the interplay between child labor, education and growth when an intergenerational transmission mechanism plays a vital role in the accumulation of human capital. They do not, however, concern themselves with fertility and provision for old age.

The structure of the paper is as follows: Section 2 presents the basic model, which is an OLG structure with three generations. Section 3 analyzes the different solutions, both interior and at the corner as well as the household decision. Economic growth and possible steady states are examined in section 4. Section 5 explores governmental intervention, with a focus on financial measures such as taxes and subsidies, and it compares the results derived with those of a model with exogenous fertility. In contrast to Bell and Gersbach (2002), this paper will also analyze fertility decisions. Economic growth depends on the educational technology: for a sufficiently high transmission factor the economy will grow steadily. The total size of the population depends on child-raising costs: while low costs lead to exponential population growth, families might become virtually extinct if raising children is very expensive.

# 2 The model framework

In the present setting, a household consist of three generations: children, parents and grandparents. Each generation is endowed with one unit of time. Children divide their time between working and learning, parents work full-time and grandparents do not work at all. The fraction of a childhood assigned to education will be denoted by $\tau \in [0, 1]$.

For simplicity, assume that only parents with identical labor efficiencies form families. It is assumed that parents raise and educate children in part to increase their own current consumption and to finance their old age. They decide how many children to have (denoted by $n$) and how well to educate them. Except for the opportunity costs of the children's labor, education is free. Still, raising children involves costs, and well-educated parents normally spend more on their children out of a sense for what is proper and for altruistic reasons. Let a fixed fraction of the parents' total income be allocated for the consumption of each child. All children are treated identically.

As the grandparents do not work and investment in physical capital is ruled out by assumption, their consumption is financed solely through a grant from the parents. It is assumed that there is a fixed social norm, under which the parents must transfer a fixed fraction $\chi \in (0, 1)$ of their income to their parents. Grandparents make no bequests.

Each generation of parents is therefore linked with the two adjoining generations, but there is no direct link between generations more than one period apart. Grandparents cannot influence their children's decisions concerning fertility and education, and these decisions, in turn, do not affect the grandparents' consumption.

Income is generated through the production of a single, non-storable good. Labor – measured in efficiency units – is the only input in production. Let the efficiency of a child be fixed at $\mu$, and denote the each parents' endowment of labor in efficiency units by $\epsilon$. Hence, the total labor supplied by the household, measured in efficiency units is:

$$L = 2\epsilon + (1-\tau)\mu n. \tag{1}$$

The production function is assumed to exhibit constant returns to scale with respect to labor:

$$y = A\,L = A\,[2\epsilon + (1-\tau)\mu n] \quad A > 0 \tag{2}$$

Turning to preferences, the only active decision makers are the parents, and the decisions they make determine the levels of their consumption in the last two periods of life and the efficiency their offspring will attain in adulthood.

Consider a household in period $t$. The parents' current consumption, $C_{1t}$, is given by their income $(2A\epsilon_t)$, that of the children $(A\mu n_t(1-\tau_t))$, the costs of raising the children $(n_t b\epsilon_t,\ b < 0)$ and the required transfer to the grandparents $(\chi \cdot 2A\epsilon_t)$ as follows:

$$C_{1t} = 2A\epsilon_t(1-\chi) + A\mu n_t(1-\tau_t) + n_t b\epsilon_t \tag{3}$$

Their old-age consumption is given by the number of children and the efficiency each attains in adulthood in period $t+1$:

$$C_{2t} = A\epsilon_{t+1}\chi\, n_t \tag{4}$$

The parents' altruism expresses itself not only through the expenditures on educating and raising children but also in their concern for the children's future income. In contrast to Barro [1974], we do not assume a nested utility function. Instead, parents consider their children's ability to purchase consumption and education for their own children, an ability which is largely determined by the children's efficiency in adulthood. For reasons of simplicity, it is further assumed that the utility function is additively separable and has the following form:

$$U(C_{1t}, C_{2t}, \epsilon_{t+1}) = ln(C_{1t}) + \beta\, ln(C_{2t}) + \beta_1\, ln(\epsilon_{t+1}) \tag{5}$$

To summarize: Parents are the sole decision makers in a household. They determine the number of children and their education, and therefore implicitly the amount of child labor and the future efficiency of the children, and the consumption vector $(C_{1t}, C_{2t})$.

Using equations (3) and (4), the utility function can be rewritten such that it contains only $n_t$, $\tau_t$ and $\epsilon_{t+1}$:

$$\begin{aligned} U(C_{1t}, C_{2t}, \epsilon_{t+1}) &= ln\Big[2A\epsilon_t(1-\chi) + A\mu n_t(1-\tau_t) + n_t b\epsilon_t\Big] \\ &+ \beta ln\Big[A\chi\epsilon_{t+1}n_t\Big] \\ &+ \beta_1 ln\Big[\epsilon_{t+1}\Big] \end{aligned}$$

It is assumed that the efficiency of a grown-up depends on the time she spent at school, the average efficiency of her parents and the productivity of the education process [see Bell and Gersbach (2002)]. If an individual does not spend any time at school, she will attain the minimum level of efficiency $\epsilon = 1$. We choose the simplest form:

$$\epsilon_{t+1} = z\tau_t\epsilon_t + 1 \tag{6}$$

where $z(> 0)$ can be thought of as the strength of the inter-generational transmission mechanism. Consider a highly developed economy without child labor ($\tau = 1$). In this case, the growth rate of the parents' level of efficiency is given by:

$$g_\epsilon \equiv \frac{\epsilon_{t+1}}{\epsilon_t} - 1 = (z-1) + 1/\epsilon_t$$

If $z > 1$, then $\tau_{crit} < 1\,\forall\epsilon$, so that long-term economic growth is possible. With $g_\epsilon > 0$ for sufficiently high levels of efficiency, the growth rate will always be nonzero. If, on the other hand, $z < 1$, no long-term growth is possible. In this case, the maximal efficiency a society can reach is limited, the upper bound for $\epsilon$ being $1/(1-z) > 1$[2]. In the case $z = 1$, long-term unbounded economic growth is possible, as in the case $z > 1$. The two cases differ only as far as the asymptotic rate of growth is concerned: for $z = 1$, $g_\epsilon \to 0$ for very high levels of efficiency.

Using equations (3), (6), and (4), the utility function can be rewritten such that

[2] This is not necessarily the highest efficiency a society can reach, as this upper limit is computed assuming that $\tau = 1$. If the schooling parents choose at the upper limit is $\tau < 1$, then the maximal efficiency will be lower, namely, $1/(1-z\tau) \geq 1$

it contains only the decision variables $n_t$ and $\tau_t$ and the various constants:

$$\begin{aligned} U(C_{1t}, C_{2t}, \epsilon_{t+1}) = U(n_t, \tau_t, \epsilon_t) &= ln\Big[2A\epsilon_t(1-\chi) + A\mu n_t(1-\tau_t) + n_t b\epsilon_t\Big] \\ &+ \beta ln\Big[A\chi(z\tau_t\epsilon_t + 1)n_t\Big] \\ &+ \beta_1 ln\Big[z\tau_t\epsilon_t + 1\Big] \end{aligned} \tag{7}$$

Note that the utility function $u(n_t, \tau_t; \epsilon_t)$ is not necessarily concave everywhere: it is convex in $(n_t, \tau_t)$.

# 3 The Household's Optimum

The solution is a tuple $(n_t(\epsilon_t), \tau_t(\epsilon_t))$ that maximizes (7) for the given level of efficiency $\epsilon_t$. For the solution to be relevant economically, it has to fulfill four conditions, with both $n_t$ and $\tau_t$ being bounded in all periods. The number of children a family can have is bounded above by biological constraints ($n_t \leq n_{max} < \infty$). The total fertility rate of the Hutterites (a religious group in North America) – the highest ever measured historically – was about 9.5 children per woman, suggesting that $n_{max} < 10$. A lower bound might be applicable for social reasons ($0 < n_{min} \leq n_t$)[3]. For the classical family with two parents, $n < 1$ can never be optimal, as $C_{2t} = 0$ in this case. In order to skirt this problem, the paper will deal with an extended family instead: pooling plays a major role in this case, as in Bell, Devarajan and Gersbach (2003), so that $n < 1$ becomes possible. As each child is endowed with one unit of time, schooling is also limited ($0 \leq \tau_t \leq 1$).

## 3.1 The interior solution

The system described above has a unique interior solution. Unfortunately, this does not describe a maximum, but rather a saddle point; for the determinant of the hessian of the function is negative for all values of the parameters:

$$det(H) = -\frac{\beta_1\ (1+\beta)^3\ \mu^2}{4{\epsilon_t}^2\ (\beta_1+\beta)\ (-1+\chi)^2} < 0$$

Therefore the interior solution is not relevant, and corner solutions need to be computed. The following sections will present the three corner solutions and analyze the

[3]One can allow $n_{min}$ to be exactly zero is one introduces the assumption that output is storable or that credit contracts can be entered into by members of adjacent generations. This possibility will be pursued at a later stage of the research.

areas where they are relevant economically, that is, where they fulfill the conditions stated above. Both corner solutions with respect to one variable alone are maxima, with their respective second derivatives being negative.

## 3.2 Interior solutions w.r.t. one variable

Whether an interior solution exists and makes sense economically (i.e. $\tau \in [0, 1]$ and $n \in [n_{min}, n_{max}]$) depends on the child-raising costs and the parents' efficiency. A detailed discussion of the solutions is given in the appendix. Figures 1 and 2 and Tables 1 and 2 outline the results.

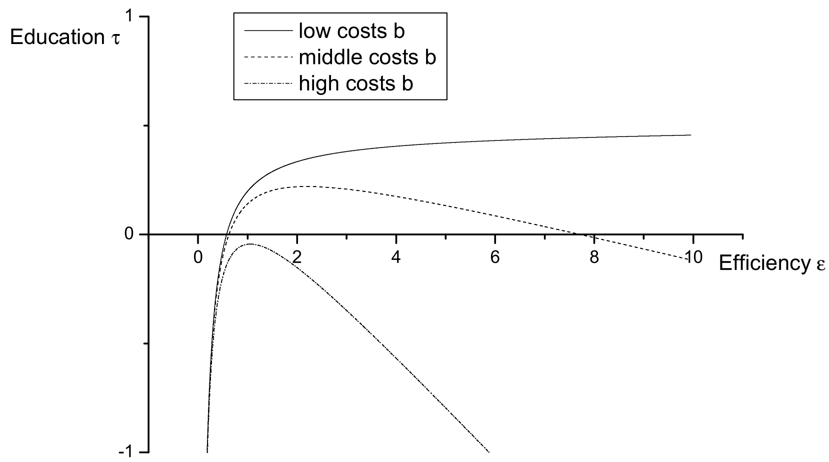


Figure 1: Interior solution with respect to education for different costs.

Table 1: Interior solution w.r.t education

| Child-raising costs | Interior solution w.r.t. education |
|---|---|
| $\lvert b\rvert < \frac{2A(1-\chi)}{\bar{n}}$ | $\tau^*(\cdot)$ relevant for moderate levels of efficiency |
| $\lvert b\rvert > \frac{2A(1-\chi)}{\bar{n}}$ | $\tau^*(\cdot)$ relevant for moderate levels of efficiency $\epsilon \in [\epsilon_a, \epsilon_b]$ |
| $\lvert b\rvert > \frac{2A(1-\chi)}{\bar{n}} + \frac{Az\bar{n}\mu(\beta+\beta_1)}{4\bar{n}}$ | $\tau^*(\cdot)$ is never relevant |

If the child-raising costs are low, the interior solution w.r.t $n$ is either negative or too large ($n > n_{max}$) for all levels of efficiency. Depending on the parameters, the interior solution w.r.t. education could exist for low levels of efficiency and be feasible economically. Therefore, parents will choose $\tau = \tau^*(\bar{n}, \epsilon)$ for low levels of efficiency

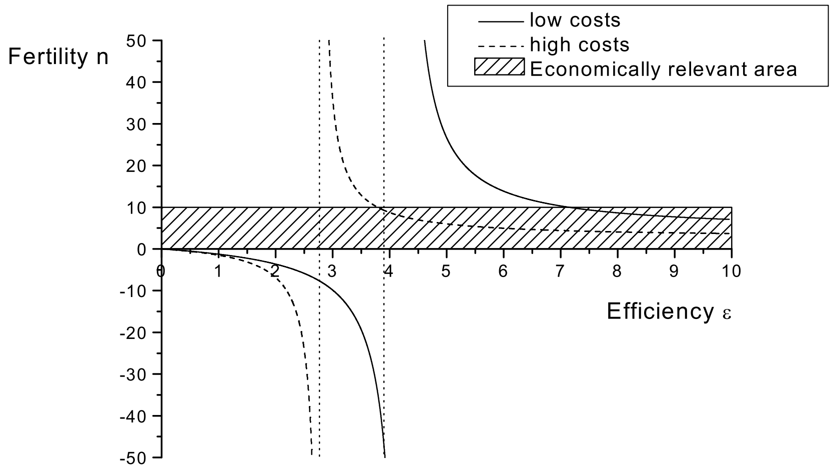


Figure 2: Interior solution with respect to fertility.

Table 2: Interior solution w.r.t fertility for $\bar{\tau} = 0$

| | $\|b\| \in \left[\frac{2A\beta(1-\chi)}{n_{max}(1+\beta)}, \frac{2A\beta(1-\chi)}{n_{min}(1+\beta)}\right]$ | $\|b\| \notin \left[\frac{2A\beta(1-\chi)}{n_{max}(1+\beta)}, \frac{2A\beta(1-\chi)}{n_{min}(1+\beta)}\right]$ |
|---|---|---|
| $\bar{\tau} = 1$ | $n^*(\cdot)$ relevant for all $\epsilon$ | $n^*(\cdot)$ is never relevant |
| $\bar{\tau} = 0$ | $n^*(\cdot)$ is relevant for high levels of efficiency $\epsilon > \frac{A\mu n_{max}(1+\beta)}{2A(1-\chi)\beta - n_{max}b(1+\beta)}$ | $n^*(\cdot)$ is never relevant |

and low costs, where $\bar{n}$ still needs to be determined. For very small and high levels of efficiency, the interior solutions with respect to neither variable will be economically feasible.

For moderate child-raising costs and low levels of efficiency, the interior solution w.r.t. fertility is not feasible. The interior solution $\tau = \tau^*(\bar{n}, \epsilon)$ will be feasible for low and moderate levels of efficiency, but not available for large $\epsilon$, where $n^*(\bar{\tau}, \epsilon)$ could be feasible. The values for $\bar{\tau}$ and $\bar{n}$ as well as the optima in the cases where neither or both of the interior solutions are feasible, still need to be computed.

If the child-raising costs are high, the solution $n^*(\bar{\tau}, \epsilon)$ is available for low levels of efficiency only if $\bar{\tau} = 1$, while the interior solution w.r.t. education yields a negative level for all levels of efficiency. The optimum for small $\epsilon$ might, therefore, be either a corner solution w.r.t both variables or $n^*(\bar{\tau} = 1, \epsilon)$. For high levels of efficiency, consumption in the first period needs to be financed by child labor, as the child-raising costs are relatively high. Therefore, the solution $n^*(\bar{\tau}, \epsilon)$ will be optimal, if biologically feasible.

## 3.3 The corner solutions w.r.t. both variables

As established above, the interior solutions w.r.t. one variable are not feasible for all levels of efficiency. Therefore, full corner solutions need to be analyzed. In order to find the optimum, it is necessary to compare the utilities generated by any combination of $n$ and $\tau$ at the boundary of the feasible set. It suffices to consider the following two cases ($n_{min} \neq 0, n \neq 0$):

$$\begin{aligned} (i)\, \triangle u(n) &:= u(n_{max}, \bar{\tau}) - u(n_{min}, \bar{\tau}) \\ (ii)\, \triangle u(\tau) &:= u(\bar{n}, \tau = 0) - u(\bar{n}, \tau = 1) \end{aligned}$$

Table 3: $\triangle u(n)$ as a function of efficiency and child-raising costs

| | |
|---|---|
| $\lvert b \rvert \leq \frac{2A(1+\chi)(n_{max}^{\beta} - n_{min}^{\beta})}{n_{max}^{1+\beta} - n_{min}^{1+\beta}}$ | $u(n_{max}, \bar{\tau}) > u(n_{min}, \bar{\tau})$ for all levels of efficiency |
| $\lvert b \rvert > \frac{2A(1+\chi)(n_{max}^{\beta} - n_{min}^{\beta})}{n_{max}^{1+\beta} - n_{min}^{1+\beta}}$ | $u(n_{max}, \bar{\tau}) > u(n_{min}, \bar{\tau})$ for sufficiently low levels of efficiency |

It can be shown that if both variables are at the corner and schooling is fixed, well-educated parents will be better off choosing the minimum number of children if the child-raising costs are sufficiently large. Otherwise – that is, for low levels of efficiency or low costs – choosing $n = n_{max}$ will be optimal. If $n_{min}$ is sufficiently close to zero, then choosing $n = n_{max}$ will always be optimal if the interior solution w.r.t $n$ is not economically feasible. The results and critical values are derived in the appendix.

Table 4: $\triangle u(\tau)$ as a function of efficiency and child-raising costs

| | |
|---|---|
| $\lvert b \rvert > \frac{2A(1-\chi)}{\bar{n}}$ | $u(\tau = 0, \bar{n}) > u(\tau = 1, \bar{n})$ for all levels of efficiency |
| $\lvert b \rvert \leq \frac{2A(1-\chi)}{\bar{n}}$ | $u(\tau = 0, \bar{n}) > u(\tau = 1, \bar{n})$ for sufficiently low levels of efficiency |

Analyzing case $(ii)$ for $\bar{n} \neq 0$ yields an inequality involving a transcendental expression. It is not possible to express this condition analytically for the critical efficiency. As was the case for $\triangle u(n)$, the sign of $\triangle u(\tau)$ depends on the child-raising costs and efficiency. If the interior solution w.r.t education is not feasible economically, choosing $\tau = 0$ will be optimal for all levels of efficiency if the child-raising costs are large.

If, on the other hand, $|b|$ is small, well-educated parents will be better off choosing fulltime schooling, while poor parents need child labor and will choose $\tau = 0$. Although it is not possible to compute the critical efficiency analytically, it can be shown that it is single-valued.

## 3.4 The household's decision

With all solutions being, it is now possible to outline how the household's decision depends on the parameters.

If $|b| < \frac{2A(1-\chi)}{n_{max}}$, and the parents' efficiency is very low, none of the interior solutions will be feasible. Parents will therefore choose $\tau = 0$ and $n = n_{max}$, as stated in the previous section. With increasing efficiency, the interior solution w.r.t education becomes feasible, so that parents will choose $\tau = \tau^*(\bar{n}, \epsilon)$ and $\bar{n} = n_{max}$ [4]. For large levels of efficiency and small costs, neither of the interior solutions will be feasible, as both variables exceed their maximal values. Parents will therefore choose $n = n_{max}$ and $\tau = 1$.

If $|b| \in [\frac{2A(1-\chi)}{n_{max}}, \frac{2A(1-\chi)}{n_{min}}]$, and $\epsilon$ is small, the result is the same as above. For moderate levels of efficiency, the interior solution w.r.t. education is feasible, so that parents choose $\tau = \tau^*(\bar{n}, \epsilon)$ and $n = n_{max}$. For large levels of efficiency, $\triangle u(\tau)$ will be negative while the interior solution w.r.t. education is not feasible. As the interior solution w.r.t. fertility might be feasible, the household's optimum for high levels of efficiency will be $n = max[n_{min}, n^*(\bar{\tau}, \epsilon)]$ and $\tau = 1$.

If $|b| > \frac{2A(1-\chi)}{n_{min}}$, choosing full-time schooling always leads to $C_{1t} \leq 0$. As none of the two interior solutions is feasible for low levels of efficiency while $\triangle u(\tau) < 0$ and $\triangle u(n) > 0$, parents will choose $n = n_{max}$ and $\tau = 0$ for low $\epsilon$. With increasing levels of efficiency, as the interior solution w.r.t. fertility becomes feasible, parents will choose $n = max[n_{min}, n^*(\bar{\tau}, \epsilon)]$ and $\tau = 0$[5]. For still higher levels of efficiency, the interior solution w.r.t education is feasible, so that choosing $n = n_{min}$ and $\tau = \tau^*(\bar{n}, \epsilon)$

[4] With the parents' efficiency being small, term $z\epsilon$ is small while the child-raising costs are relatively large. Therefore, fulltime education cannot be optimal, and parents finance old-age consumption by having numerous children.

[5] As can be seen from figure 2, the range of efficiency for which this solution is optimal is very narrow.

will be optimal. For high levels of efficiency, none of the interior solutions will be feasible, while $\triangle u(\tau) < 0$ and $\triangle u(n) < 0$,so that choosing $\tau = 0$ and $n = n_{min}$ is optimal. Note that for very high levels of efficiency, even full-time child labor cannot ensure $C_{1t} > 0$, so that this case is of mathematical interest only.

Extremely high costs will be analyzed in the context of taxes, as societies cannot carry the burden of such costs for long.

# 4 Economic Growth

The next step is to examine the factors influencing economic growth, whereby 'growth' can be measured according to three indices: the parents' efficiency ($\epsilon_t$), the family's income $y_t$ and the lifetime utility of a generation ($U_t$). The second index is also the social product of the household, and is therefore easy to compare with macroeconomic variables like the GNP. Finding the growth rate of income or utility is complicated by the fact that efficiency, the number of children and schooling could all change simultaneously within periods, so that $\triangle y_t$ and $\triangle u_t$ might have either sign. Given the fact that it is not possible to find explicit functions for the schooling and fertility for all $\epsilon$, as critical efficiencies cannot be computed, technical problems arise when trying to compute growth rates for $y_t$ or $u_t$. Hence, we restrict ourselves to efficiency as a measure of economic growth. A big difference in efficiency will always lead to a big difference income and utility, and $lim_{\epsilon \to \infty} u = lim_{\epsilon \to \infty} y = \infty$.[6] Hence, an economic policy aimed at increasing a family's utility or income through an increase in efficiency will be efficacious if increasing $\epsilon$ is at all possible[7].

## 4.1 The critical level of schooling

The efficiency of a lineage will grow with time only if children spend sufficient time at school. The critical level of schooling depends on the parents' efficiency and the educational technology. It is obvious that efficiency cannot fall below its minimum

[6] If $\epsilon \to \infty$ is feasible so that $C_{it} > 0$, $i = 1, 2$.

[7] Sustained growth is only possible if $|b| < \frac{2A(1-\chi)}{n_{min}}$. In this case, parents will choose $n = max[n_{min}, n^*]$ and $\tau = 1$ when $\epsilon$ is large. Computing income and utility, and then differentiating the results w.r.t. efficiency immediately yields that income grows with efficiency, $\partial y/\partial \epsilon = A > 0$ and $\partial u/\partial \epsilon = \frac{1+z\epsilon(1+\beta+\beta_1)}{\epsilon(z\epsilon+1)} > 0$. Therefore, growth of efficiency is equivalent to growth of utility and total income, when $\epsilon$ is large, although the growth rates differ.

$\epsilon = 1$ independently of $z$ and $\tau$. The critical level of schooling can easily be computed from the following condition for an unstable steady-state:

$$\begin{aligned} \epsilon_{t+1} &= \epsilon_t \Leftrightarrow \\ z\tau\epsilon_t + 1 &= \epsilon_t \Leftrightarrow \\ \tau &= \frac{1}{z} - \frac{1}{z\epsilon} \\ \Rightarrow \quad \tau_{crit} &= \frac{1}{z} - \frac{1}{z\epsilon} \end{aligned}$$

Therefore, the values of $\tau$ that induce a steady-state are:

$$\tau_{crit} := \frac{1}{z} - \frac{1}{z\epsilon} \tag{8}$$

The higher the initial efficiency, the higher the level of schooling required to attain it. For very high levels of efficiency, the critical level of schooling will approach $1/z$ asymptotically from below. As parents who already are highly efficient choose $\tau = 1$ automatically if possible,[8] the speed and direction of economic growth depend on $z$ only.

## 4.2 Low and moderate levels of efficiency

If parents are not well educated, they will decide to have as may children as possible ($n = n_{max}$) and to send their children to work. The exact amount of schooling depends on $b$ and $z$, with $\partial\tau/\partial z > 0$ and $\partial\tau/\partial b > 0$, and can be computed using equation (14). For low costs and low levels of efficiency where the critical level of schooling is zero, parents can easily maintain the current level of knowledge. For slightly higher levels of efficiency, the relation $(\tau_{crit} - \tau)$ depends on $b$ as parents choose $\tau = \tau^*(\bar{n}, \epsilon)$. Numerical examples show that parents will usually choose $\tau > \tau_{crit}$ for low costs, although exceptions are possible for extreme parameter values that reduce $\tau$. Such values will be left out of the analysis. For larger costs, $\tau^*(\bar{n}, \epsilon)$ and $\tau_{crit}$ will intersect at most twice. A mathematical analysis of potential points of intersection is given in the appendix.

A point of intersection of the two functions is a steady state with respect to efficiency, provided parents choose the interior solution w.r.t education. It is obvious, therefore, that not all points of intersection computed above will yield a steady state

[8] Choosing $\tau = 1$ is always possible if $|b| < 2A(1 - \chi)/n_{min}$.

if the parents prefer choosing the interior solution w.r.t. fertility rather than the interior solution w.r.t. education for the relevant level of efficiency. As it is not possible to compute the household's choice analytically, the steady states will also have to be computed numerically. Calculations show that there is only a very limited range of parameters in which two steady-states exist. In most cases, there will be either no steady state (for very low costs) or a single steady-state (for moderate costs) if the parents' efficiency is not too large.

Before turning to the analysis of high levels of efficiency, the stability of the potential steady states need to be analyzed. As $lim_{\epsilon \to 0}(\tau - \tau_{crit}) = -\infty$, the level of schooling parents choose will be higher that the critical level of schooling for levels of efficiency lower than the first steady-state and lower for higher levels of efficiency. As both functions are continuous for low and moderate levels of efficiency, it follows that the first point of intersection will yield a stable steady state, while the second steady state, if it exists, will be unstable.

## 4.3 High levels of efficiency

If the parents' efficiency is large, households will always choose full-time education if they can afford it[9]. Whether the economy will grow, stagnate or contract depends only on $z$. The possible outcomes have been presented in section 2. Long-term economic growth is not possible if $z < 1$; in this case, a steady state will exist even if parents always choose $\tau = 1$.

## 4.4 Conclusion: Economic growth and steady states

As stated earlier, the economic prospects of a society depend on the child-raising costs, the productivity of the education function, $z$, and the initial state of the economy, that is, the parents' efficiency in the first period of the analysis.

If $|b| < \frac{2A(1-\chi)}{n_{max}}$, parents usually choose $\tau = 0$ and $n = n_{max}$ when $\epsilon$ is small, and $0 \leq \tau \leq 1$ and $n = n_{max}$ for moderate levels of efficiency. If $z < 1$ there will be a single steady-state for low or moderate levels of efficiency. If $z = 1$, there will be at least a single steady-state, and up to two for moderate costs. If $z > 1$, the

[9]Parent can always afford $\tau = 1$ for high levels of efficiency if $|b| < \frac{2A(1-\chi)}{n_{min}}$.

parents will always choose $\tau > \tau_{crit}$ if $|b|$ is very small. Otherwise, the economy will have a single steady-state. For high levels of efficiency, parents choose $\tau = 1$ and $n_{min} \leq n \leq n_{max}$. The economy will grow if $z \geq 1$, and will contract otherwise.

If $\frac{2A(1-\chi)}{n_{max}} \leq |b| \leq \frac{2A(1-\chi)}{n_{min}}$, the optimal choice will be $\tau = 0$ and $n = n_{max}$ for very low levels of efficiency, and the interior solution w.r.t. education and $n = n_{max}$ for moderate levels of efficiency. A steady state will exist for either low or moderate levels of efficiency. Just as above, there will be at least a single steady state for low and moderate levels of efficiency. For a limited range of parameters, up to three steady states will exist. For high levels of efficiency parents will choose $\tau = 1$ and $n = max[n_{min}, n^*(\bar{\tau}, \epsilon)]$. The economy will always grow if $z \geq 1$.

The case $|b| > \frac{2A(1-\chi)}{n_{min}}$ is only of interest if $n_{min} \neq 0$.

If the parents' efficiency is small, choosing $\tau = 0$ and $n = n_{max}$ will be optimal, as long as the interior solution w.r.t. fertility is not feasible [10]. Again, a steady state could exist for $\epsilon = 1$.

If the efficiency of the parents is large parents will have to reduce schooling as soon as the interior solution w.r.t fertility falls below $n_{min}$. As $C_{1t}$ will be negative for high levels of efficiency for any schooling and any $n \geq n_{min}$, an economy where $|b| > 2A(1-\chi)/n_{min}$ can never reach high levels of efficiency. A steady-state will exist for $\epsilon \geq 1$. For $z > 1$, there will be up to two steady-states, with the second one always being unstable and the first one stable.

An economy will therefore grow forever if $z$ is large while the child-raising costs are very low[11]. In all other cases, stable steady states with low values of $\epsilon$, that is, 'poverty traps', will exist. If the initial level of efficiency is high enough, sustained growth in $\epsilon$ is possible for all but very high levels of child-raising costs if $z \geq 1$. If $z < 1$ unbounded growth is not possible for any $|b|$ and any initial efficiency.

Before turning to the analysis of governmental interventions, however, it is of interest to find parameters so that the economy grows by itself. As this can only be the

[10] The interior solution w.r.t. education always yields a negative $\tau$ for very large costs.

[11] That is, the interior solution w.r.t. education is feasible for $\epsilon = 1$.

case if the child-raising costs are low and $z$ is large, the household's decision can be easily modelled: for low levels of efficiency, parents will choose $\tau = 0$, followed by the interior solution w.r.t. education for higher levels of efficiency. With the child-raising costs being low, parents will always choose $n = n_{max}$. For high levels of efficiency, $\tau = 1$ and $n = min[n_{max}, \frac{2A(1-\chi)\beta}{(1+\beta)|b|}]$ will be optimal. As $z$ is large, it follows immediately that the economy will grow if the initial level of efficiency is large enough. If the level of efficiency in the first period is low, but one can show that the interior solution w.r.t. education always yields $\tau > \tau_{crit}$ it follows that the economy will grow independently of its initial state. Since $\tau > \tau_{crit}$, implies $\epsilon(\tau = 0) < \epsilon(\tau_{crit} = 0) = 1$, it follows immediately that the parents' choice of schooling – be it $\tau = 0$, $\tau = 1$, or $\tau^*(\bar{n}, \epsilon)$ – will always exceed the critical level of schooling. The upper bound for the costs such that the economy always grows has been computed in the appendix. If $b$ does not fulfill condition (21), then $\tau > \tau_{crit}\,\forall\epsilon$, and therefore the economy will always grow for $z \geq 1$. For economies with extremely productive education functions, sustained growth will be possible for a very broad range of costs. On the other hand, if the education function is not very productive, long-term growth will only be possible for very low costs. The paths of education and fertility for different costs, efficiencies and $z$ are depicted in figure 6 at the end of the document.

# 5 Governmental Intervention

Assuming an economy starts with a low initial level of efficiency, long-term growth is not possible unless the child-raising costs are very low and the education function is sufficiently productive ($z > 1$). In all other cases, the economy will be 'stuck' after some periods in a low-level, stable poverty trap, from which it cannot escape without outside intervention. It is assumed that the governments' major goal is to induce sustained economic growth, and that it tries to do so by measures designed to promote higher levels of efficiency. If $z > 1$ and $|b| < 2A(1-\chi)/n_{min}$, there will always exist some level of efficiency, $\epsilon_1$ say, so that for all $\epsilon > \epsilon_1$ parents will choose $\tau > \tau_{crit}$. As soon as the government manages to increase the parents' efficiency to $\epsilon_1$ or above, it will have accomplished its aim. In order to increase efficiency, the government will try to induce households to choose education so that efficiency in the following period will be at least $\epsilon_1$. If this is not possible on account of $z\epsilon + 1 < \epsilon_1$, intervention will have to stretch over more than one period. Without the introduction of a social welfare function encompassing more than one generation, it is not possible to describe the optimal path to permanent growth in detail. A welfare function has problems of its own, however, such as the choice of the temporal discount rate, variations in which can alter the result of the optimization substantially. For reasons of simplicity, therefore, it is assumed that the government will try to induce parents to choose full-time schooling as long as $z\epsilon + 1 < \epsilon_1$ and to choose at least the necessary schooling such that $z\tau\epsilon = \epsilon_1$ in the last period of intervention. This simplification makes the analysis relatively tractable, and the policy program that emerges from it is plausibly a 'good' one, in the sense that the goal is sensible.

In the present setting, the government can implement both fiscal and regulatory measures: the classical policy is to reduce child labor and increase school attendance through the introduction and enforcement of compulsory education. Another regulatory measure would be to limit the number of children a family may have, China being the most prominent example.

There is a broad range of fiscal measures, all of which can be divided into two groups, namely taxes and subsidies. Governmental measures will usually include a combination of instruments from both classes, with taxes being used to finance subsidies.

## 5.1 Regulatory measures

The impact of a prescribed level of either schooling or fertility has already been analyzed in the sections before. If $n$ or $\tau$ is fixed, parents will choose the remaining variable using (14) and (16), respectively. If the resulting variables are outside their natural ranges, the corner solutions w.r.t. both variables, as discussed in section 3.3, will yield the optimum. This class of measures will not be analyzed further in this context: historical evidence shows that compulsory schooling is hard to enforce when the family would experience a heavy loss in income and there are often not enough schools or teachers. Limiting $n$ by decree would also lead to an increase in schooling, but this measure would be very unpopular in most countries, and therefore hard to implement, so that this second policy is mostly of academic interest.

## 5.2 Fiscal Measures

Fiscal measures are designed to induce families voluntarily to choose the schooling the government wishes. It is clear that this class of policies is not without its own problems: taxes are hard to collect, they can lead to social unrest, especially if they impose a heavy burden or if they are perceived to be unfair, and subsidies can fail to reach the needy through corruption and mismanagement. Some of the measures analyzed in the following are not free of these problems, but it will be assumed that these difficulties can be solved. The following three measures will be dealt with in detail:

- Lump-sum transfers $T^f$
- Taxation/subsidization of the expenditures on raising children, $T^b = tn\epsilon$
- Subsidization of school attendance, $S^\tau = sn\tau$

For simplicity, it will be assumed that only parents pay taxes and receive subsidies, while the grandparents' income remains untouched. It is possible to rewrite consumption in the first period of adult life in the following form, which allows the analysis of any combination of the measures enumerated above:

$$C_{1t} = 2A\epsilon(1-\chi) + An\mu(1-\tau) + nb\epsilon - tn\epsilon + sn\tau - T^f \tag{9}$$

Note that none of these taxes alters the obligation to pay $A\epsilon_t\chi$ to the grandparents[12]. The impact of these different fiscal measures on the household's decision will be examined in sections 5.2.1 to 5.2.4. In all these sections, it will be assumed that subsidies are financed through the taxation of other families or through grants from international organizations: a household is either taxed or subsidized, but not both. Therefore the issue of financing the subsidies and the use of the revenues from taxation respectively will be irrelevant for the analysis of the individual household's decision. As the total revenues which can be raised through each measure and the total costs of subsidizing a household play an important role in the setup of a program, the different measures will also be compared with respect to their benefits and costs in the following sections. An integrated analysis of a program comprising both taxes and subsidies will be developed in sections 5.3 and 5.4.

[12] The normal procedure in analyzing such problems is to write down the lifetime budget constraint, and then to appeal to the normalcy of goods in consumption to obtain comparative static results. This is not possible in the present cases, as output is not storable and there are two separate budget constraints which cannot be combined.

### 5.2.1 Lump-sum transfers

Note first that, the adults' level of efficiency being fixed, a lump-sum transfer $T^f$ will be equivalent to taxing the parents' income $(t \cdot (2A\epsilon))$, with $t = T^f/(2A\epsilon)$. Therefore, the following analysis of a lump-sum transfer will also hold for an income tax.

If a family transfers a fixed amount of money $T^f$ to the government, or receives such a transfer, it will in general change its desired level of fertility or the children's education or both. As we have seen, if parents are poor ($\epsilon$ low), they will typically choose $\tau < 1$ and $n = n_{max}$. If the budget set is sufficiently enlarged through a lump-sum subsidy, parents will increase the children's education, as a further increase in $n$ is not possible. The lump-sum subsidy in the first period is partially transferred to the second period through additional education, and lifetime consumption is smoothed.

If parents are rich, they choose $n \leq n_{max}$ and $\tau = 1$. Since a transfer cannot increase education in this case, parents will 'invest' part of it in raising more children. The higher the parents' efficiency, the lower will be the impact of the transfer. If parents chose $n = n_{max}$ and $\tau = 1$ before receiving the transfer, all subsidies will be fully consumed in the first period of adult life.

The impact of a lump-sum tax will be similar: poor parents will rather increase child labor than reduce fertility, as the returns from education are limited if the parents' efficiency is low. Again, this leads to a smoothing of income over the life cycle. If the parents are rich, educating children is highly productive, so that rich parents will leave $\tau(= 1)$ unchanged and reduce fertility, unless raising children is extremely cheap ($b \approx 0$). In that case, parents will maintain fertility and decrease $\tau$ instead. The impact of the transfer decreases as the parents' efficiency increases. If the tax is so large that a further reduction of fertility is not possible, parents will have to reduce education to finance their tax obligations. The maximal tax a rich family can pay depends on $n_{min}$ and $|b|$ as follows.

If $|b| < A\mu/\epsilon$, parents will reduce schooling rather than fertility.

$$T^f_{max} = 2A\epsilon(1 - \chi) + n_{max}(A\mu + b\epsilon) > 2A\epsilon(1 - \chi).$$

As fertility remains unchanged this result is independent of $n_{min}$. For $|b| > A\mu/\epsilon$, parents will always reduce fertility, if possible, before reducing schooling.

If $n_{min} > 0$, large taxes will force families to reduce schooling, as child labor will be used to finance the tax and to ensure $C_{1t} > 0$. The maximal tax they can pay in this case is

$$T^f_{max} = 2A\epsilon(1-\chi) + n_{min}(A\mu + b\epsilon) < 2A\epsilon(1-\chi).$$

If fertility is given exogenously ($n_{ex}$), the total tax such families can pay is

$$T_{ex} = 2A\epsilon(1-\chi) + n_{ex}(A\mu + b\epsilon).$$

As $|b| > A\mu/\epsilon$ for large levels of efficiency, it follows that $T^f_{max} \geq T_{ex}$ if $n_{ex} \geq n_{min}$. Therefore, the total tax revenues in the case where fertility is exogenous will be no higher than the revenues in the case where fertility is endogenous. In the case with endogenous fertility and $n_{min}$ sufficiently close to zero, large taxes will only lead to a strong reduction of fertility, while education remains unchanged, being highly productive: $T^f_{max} = 2A\epsilon(1-\chi)$.

If the parents' efficiency is sufficiently high, they will be able to pay any lump-sum tax. Ignoring the case where $|b| < A\mu/\epsilon$, as it is irrelevant for high levels of efficiency, a society without constraints on $n_{min}$ will be able to afford higher taxes than a society where $n_{min}$ is appreciably different from zero. The 'price' such a society pays is still very high: rich families could virtually die out when confronted with extremely high taxes. Although a community with $n_{min} > 0$ will pay less in taxes, there is still a danger in requiring rich families to pay the maximum they can afford: as they will finance their tax payments by sending their children to work, it is future generations who will bear the burden of the measure. As education can fall to $\tau = 0$, efficiency in the next period can be as low as $\epsilon = 1$, leading the economy in the poverty trap. It is highly probable that families confronted with such taxes will try to avoid paying them independently of $n_{min}$. Therefore, these cases are mainly of mathematical interest.

#### 5.2.2 Taxes and subsidies on the child-raising costs

In the present framework, a tax on the expenditures incurred in raising children will not be shifted. The tax can be interpreted as an increase of the expenditures on raising children $|b|$. Its impact on the household's decision can be derived analyzing (14) for $n = \bar{n}$, (16) for $\tau = \bar{\tau}$ and (17) and (19) for the full corner solutions, while taking into account that switching between solutions is possible. As in the previous case, it is necessary to differentiate between poor and rich families, and low and high taxes/subsidies.

Poor parents – who usually choose $n = n_{max}$ and the interior solution with respect to education – will reduce schooling and leave fertility unchanged, as $\partial\tau^*(\cdot)/\partial b > 0$. Rich parents – who choose the corner solution with respect to education – will reduce fertility ($\partial n^*(\cdot)/\partial b > 0$). If the child-raising costs are larger, the interior solution w.r.t. fertility will become feasible for lower levels of efficiency than in the case where $|b|$ is low, as depicted in figure 2. Therefore, even a small tax on the expenditures on raising children can change the solution dramatically for some $\epsilon$. In this case, parents confronted with low costs will choose $\tau \leq 1$ and $n = n_{max}$ while parents confronted with a tax reduce fertility and select full-time schooling $\tau = 1$ and $n < n_{max}$.

When confronted with a very high tax on the expenditures on raising children, even those poor parents who would have chosen $n = n_{max}$ and $\tau = 0$ will reduce the number of children. In order to maintain consumption in the last period of life, they will also increase schooling to $\tau = 1$. Therefore, a very high tax on the child-raising costs will lead to full-time schooling, while reducing consumption in all periods of life and population growth significantly. As in the above cases, it is very improbable that such a tax can be enforced.

If the tax is not too high and parents do not reduce fertility, the maximal revenue to be gained from such a measure will be $n_{max}t\epsilon$. In this case, the total tax revenue will be identical to the case where a lump-sum tax was raised, as all variables have identical values for both taxes. The critical level of $t$ so that parents do not reduce fertility – that is, $u(\tau = 0, n = n_{max}) > u(\tau = 1, n = n^*)$, which means that the interior solution w.r.t. $n$ becomes feasible – cannot be computed analytically. As soon as $t$ exceeds this level, $n$ will decrease with $\tau = 1$, and the total tax revenue will approach $\frac{\beta\,(2A\epsilon(1-\chi))}{1+\beta} < 2A\epsilon(1-\chi)$ asymptotically with $t \to \infty$ if $n_{min}$

is sufficiently close to zero[13]. If $n_{min} > 0$, it is not possible to impose an unlimited tax on the child-raising costs, so that $t$ is limited. The maximal tax revenue will be $2A\epsilon(1-\chi) + n_{min}(A\mu + b\epsilon)$. It depends on parameters whether the total tax revenue in the case where $n$ is reduced is higher or lower than in the case where $\tau$ is reduced. In any case, if the parents are poor the total tax revenue raised through a lump-sum tax will be at least as high as the revenue from a tax on the child-raising costs.

Rich parents confronted with a tax will reduce fertility first. If $n_{min}$ is very low ($n_{min} \approx 0$), families will respond to any increase in $b$ by reducing $n$ while leaving $\tau = 1$ unchanged. The maximal tax revenue is $\frac{\beta\,(2A\epsilon(1-\chi))}{1+\beta} < 2A\epsilon(1-\chi)$, as in the case where parents are poor. If, on the other hand, $n_{min} > 0$, parents will reduce $\tau$ if the child-raising costs exceed some level. The result for fertility, education and total tax revenue and its consequences for future generations will be the same as in the case where a lump-sum tax was raised; the comparison with the case where fertility is endogenous will also yield similar results.

As soon as parents decide to reduce fertility in order to finance the tax, total government revenues will be reduced. Therefore, this form of taxing is not very efficient as a means of raising public revenue.

If raising children is sufficiently subsidized ($t < 0$), poor parents[14] will choose $\tau = 1$. Therefore, such a subsidy will yield the result desired by the government. The amount of subsidy needed to raise $\tau$ to the level $\tau'$ can easily be computed using (14) for the case where no switching between solutions takes place:

$$|b| = \frac{2A(1-\chi)}{n} - \frac{An\mu\{1 + z\epsilon[\tau' + (\tau'-1)(\beta+\beta_1)]\}}{z\,\bar{n}\,\epsilon^2\,(\beta+\beta_1)}$$

[13]The term is computed as follows:

$$\lim_{t\to\infty} |tn\epsilon| = \lim_{t\to\infty} |t\epsilon \frac{2A\epsilon(1-\chi)}{(b-t)\epsilon} \frac{\beta}{1+\beta}| = \frac{\beta}{1+\beta} 2A\epsilon(1-\chi) \lim_{t\to\infty} |\frac{t}{b-t}| = \frac{\beta}{1+\beta} 2A\epsilon(1-\chi)$$

[14]Poor parents usually choose the interior solution w.r.t education and $n = n_{max}$. If the subsidy does not trigger switching between solutions, and if it is sufficiently large, they will choose $n = n_{max}$ and $\tau = 1$. If the subsidy triggers switching between solutions, parents will choose the interior solution w.r.t education and $\tau = 1$.

### 5.2.3 School-attendance subsidies

The subsidy to promote education takes the form of a fixed cash transfer $s \geq 0$ for each unit of time each child spends at school. Therefore the total subsidy a family receives will be $S^{\tau} = sn\tau$. School fees (i.e. $s < 0$) will be ignored.

As in the previous section, the families receiving this subsidy are not taxed in any way. Therefore, (4) will not change, while (3) can be rewritten:

$$\begin{aligned} C_{1t} &= 2A\epsilon(1-\chi) + An\mu(1-\tau) + nb\epsilon + sn\tau \\ &= 2A\epsilon(1-\chi) + nb\epsilon + An\mu + n\tau(s - A\mu) \end{aligned}$$

If the attendance-subsidy per child exceeds $A\mu$, the opportunity costs of education will be negative, so that families will never choose $\tau < 1$. Trivially, full-time education can easily be achieved if the government, in effect, makes good for the income loss families would otherwise experience. It remains to be seen whether there is a smaller subsidy ($s < A\mu$) such that parents still choose $\tau = 1$, and whether this way of inducing an increase in $\tau$ is more or less costly than other measures.

Analyzing first the interior solution w.r.t. education ($n = \bar{n}$), the new level of schooling depends on the subsidy as follows:

$$\tau(\bar{n}, \epsilon, s) = \frac{z\epsilon(\beta + \beta_1)[2A\epsilon(1-\chi) + \bar{n}A\mu + b\bar{n}\epsilon] + n(s - A\mu)}{\bar{n}z\epsilon(1 + \beta + \beta_1)(A\mu - s)} \tag{10}$$

Obviously, $\tau$ can take any value for $s \leq A\mu$, and $\lim_{s \to A\mu_-} = \infty$ if $|b| < \frac{2A(1-\chi)}{\bar{n}} + \frac{A\mu}{\epsilon}$[15]. As the level of schooling the government wants to achieve is $\tau' \leq 1 (\ll \infty)$, it is obvious that there exists some subsidy $s < A\mu$ that will suffice to induce parents to choose $\tau'$ in this case (see figure 3). With $\partial\tau/\partial s > 0$, moreover, the efficiency for which the interior solution w.r.t. education becomes feasible ($\tau \geq 0$) will be lower for higher subsidies. If $s$ is sufficiently high ($s < A\mu$), any level of schooling can be achieved for any efficiency. Therefore, even if the schooling parents choose for some (low) efficiency is zero, there will exist some subsidy $s < A\mu$ such that parents prefer $\tau = \tau'$. For high levels of efficiency, three scenarios are possible. If $b \leq \frac{2A(1-\chi)}{n_{min}}$ and

[15] This condition for the costs states that consumption in the first period of adult life is nonnegative if children work full-time. If the condition is not fulfilled, parents cannot survive in the first period without outside help even for $\tau = 0$. In reality, therefore, one can assume this condition to be fulfilled for some $n$ for all societies.

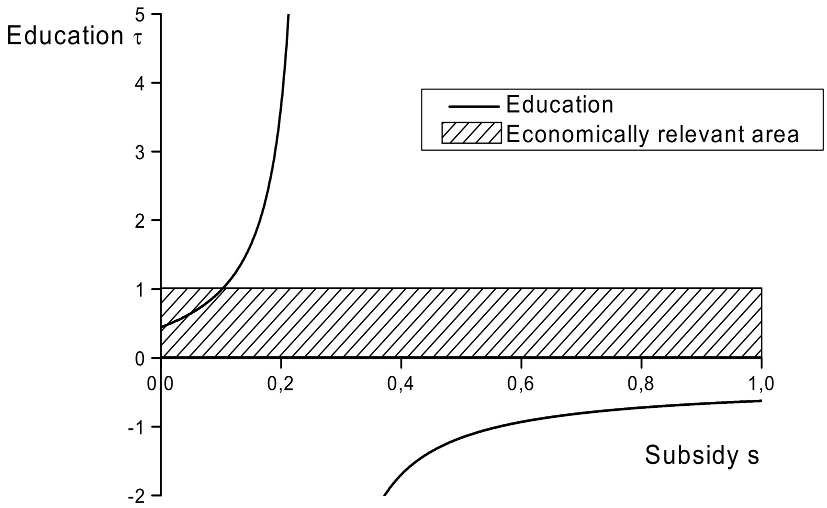


Figure 3: Interior solution with respect to education with subsidies (for $A\mu = 1/4$)

$z \geq 1$, parents will choose $\tau > \tau_{crit}$, and no subsidy is needed. If $b \leq \frac{2A(1-\chi)}{n_{min}}$ and $z < 1$, long-term growth is not possible, so that this case will not be analyzed. If $b > \frac{2A(1-\chi)}{n_{min}}$, very high levels of efficiency are not relevant economically, as parents cannot maintain $C_{1t} > 0$, even if their offspring work full-time. The value of $z$ is irrelevant in this case.

### 5.2.4 Comparing subsidies: the interior solution w.r.t. education

In this section, the total subsidies needed to induce parents to choose some value of $\tau_1 \leq 1$ will be compared. The focus will be on the interior solution w.r.t. education, as this case can be analyzed more easily and as the steady-state of most economies lies in this area. Therefore, regime-switching will be left out of the analysis at this stage, so that fertility is fixed ($n = \bar{n}$). The problems arising when regime-switching is taken into account – that is, where fertility is not fixed – are addressed at the end of the section.

It is easy to show that subsidizing the expenditures on raising children and a flat transfer will cost the same. The total subsidy needed in both cases is[16]:

$$\begin{aligned} S_1 &= \frac{An\mu(1+z\tau_1\epsilon) - z\epsilon(\beta+\beta_1)\{An\mu(1-\tau_1)+\epsilon[2A(1-\chi)+nb]\}}{z\epsilon(\beta+\beta_1)} \\ &= \frac{K_1}{z\epsilon(\beta+\beta_1)} \end{aligned} \tag{11}$$

In the case where the government subsidizes school-attendance directly, the total subsidy required to raise the level of schooling from $\tau$ to $\tau_1$ is:

$$S_2 = \frac{K_2}{z\epsilon(\beta+\beta_1)+z\epsilon+1/\tau_1} \tag{12}$$

The numerators of both equations are identical ($K_1 = K_2 > 0$), but the denominators differ. As $z\epsilon(\beta+\beta_1)+z\epsilon+1/\tau_1 > z\epsilon(\beta+\beta_1)$, $\forall \epsilon, \tau_1 > 0$, it follows immediately that $S_2 < S_1$ for all $\epsilon$ and $\tau_1 > \tau$. That is, the total subsidy required to induce parents to choose some level of schooling $\tau_1 > \tau$ will be lower if education is subsidized directly. This exemplifies the principle of targeting: as a school-attendance subsidy directly attacks the distortion arising from externalities from education it will be the most efficient way to increase the level of schooling. Comparing the influence on consumption in the first period of a school-attendance subsidy $s$ and a subsidy $s_c$ confirms this result: $\partial C_{1t}/\partial s < 0$ and $\partial C_{1t}/\partial s_c > 0$. A subsidy on expenditures on raising children will increase $C_{1t}$. A subsidy on school-attendance will reduce $C_{1t}$.

In the case where corner solutions w.r.t education hold, the analysis is much more difficult, as regime-switching becomes relevant and no analytic results are possible. The exact subsidy needed in each case depends on the parents' initial efficiency, the child-raising costs and the education productivity factor $z$, and can only be computed numerically. Therefore, it is not possible to state in advance which subsidy will be the cheapest. The interventions calculated above in equations (11) and (12) constitute an upper limit for the total subsidy needed in the case where corner solutions w.r.t. education are optimal after the intervention, and the total transfer required can be much lower in some cases[17].

[16]This is computed as follows: find the subsidy $t$ needed to achieve the level of schooling $\tau_1$ for the interior solution. Then compute $tn\epsilon$. In an analogous way, one can directly compute the cash transfer $T^f$ needed to raise the level of schooling from $\tau$ to $\tau_1$. It can then be shown that $K_1 > 0$ for $\tau < \tau_1$, that is, a positive subsidy is needed to raise the level of schooling.

[17]Particularly in the immediate vicinity of the point of discontinuity w.r.t. education and $n$ (see figure 6), where the interior solution w.r.t. fertility becomes optimal, the total subsidy will be very low.

## 5.3 Policy Programs

Subsidizing education for a finite number of periods will always lead to parents eventually choosing $\tau > \tau_{crit}$ forever after, if $z > 1$ and $|b| < \frac{2A(1-\chi)}{n_{min}}$, that is, if sustainable growth is possible. The total resources needed depend on everything in the system but an upper limit thereon can be computed as follows: Assuming that the parents' initial level of efficiency is $\epsilon_0 < \epsilon'$[18], let $P$ periods be needed to reach $\epsilon'$. The program will consist of subsidies, so that parents are induced to choose $\tau = 1$ during the first $P - 1$ periods. In the last period $P$, parents will choose the level of schooling such that $z\tau_P\epsilon_P + 1 = \epsilon'$. Without discounting, the total amount needed, measured in units of output, is $S = S_1 + S_2 + ... + S_P$, where $S_i$ denotes the total subsidy needed in period $i$. As subsidizing school attendance is cheaper as long as the interior solution w.r.t. education is chosen, one can compute $S_i$ using equation (12) and update the parents' efficiency in any period as follows: $\epsilon_p = z\epsilon_{p-1} + 1$ for $p < P$ and $\epsilon_P = z\tau_P\epsilon_{P-1} + 1 = \epsilon'$. Backwards induction then yields the minimum number of periods needed to reach $\epsilon'$. Given that only the interior solution w.r.t. education is analyzed, the number of children born in any period will be constant.

If it is possible to finance the measure from abroad, for example through loans to be paid back no earlier than after $P$ periods, or in some other way which does not involve taxation during the periods in which the subsidy is paid, it is possible for the whole society to escape the poverty trap simultaneously and in finite time. If the measure has to be financed through taxes, whether a successful programme can be set up depends on the system's parameters.

The most simple program, in which no subsidies whatsoever are required, is one where poor families – who would otherwise choose $\tau \leq \tau_{crit}$ and $n = n_{max}$ – have to pay a very high tax on the expenditures incurred in raising children. Confronted with this measure, parents will reduce the number of children and increase schooling to $\tau = 1$. As in the case of a subsidy fully financed from abroad, it is possible to raise the efficiency of the whole society simultaneously to $\epsilon'$ or above. Such a program would not be easy to implement, however, as it would reduce the consumption and utility of at least one generation dramatically. If the taxes raised were used to subsidize the same families, this measure would be equivalent to a change in relative

[18] Note that $\epsilon'$ is chosen so that parents choose $\tau > \tau_{crit}$ for all $\epsilon > \epsilon'$.

prices. For a homogenous society, the governments' balanced-budget condition is $s_t \tau_t n_t = t_t n_t \epsilon_t$. Therefore, for any choice of $s_t$, $t_t$ is fully implied. Families experiencing such a measure might change their decision to reduce fertility and increase education.

In the present setting, there are no other programs such that the whole society can escape the poverty trap simultaneously. If subsidies need to be financed currently, and if this financing is not ensured using measures other than taxes, part of the population will have to pay for them while the rest will enjoy them. Therefore, inequality will rise after the first period in which the measure is introduced. If the process is continued, the efficiency levels of the families so subsidized will exceed $\epsilon'$ after a finite number of periods. This group can now be taxed in some measure, and the revenue obtained can be used to subsidize the poor families. If the tax schedule is chosen such that the efficiency of no succeeding 'rich' generation falls below $\epsilon'$, and if enough revenue is raised to finance a subsidy for the poor which enables the latter to reach $\epsilon'$ after some time, the program will lead to sustainable growth for the entire society after a finite number of periods. The inequality that arises through such a program – due to fertility differences between the groups and due to differences in the level of efficiency – will be discussed in the following section.

## 5.4 Inequality

Consider a society of homogenous adults. Assume that in the first period the entire economy is in the poverty trap, which is the single stable state of the economy. All families' level of efficiency is low, and households typically choose the corner solution w.r.t fertility and $\tau < 1$. Let $z > 1$, so that unbounded growth is possible, and let $\tau > \tau_{crit}$ for all $\epsilon > \epsilon'$. As already discussed in previous sections, taxes and subsidies which lead to switching between solutions are hard to analyze. For simplicity, therefore, we will assume that no switching takes place, that is, fertility will be fixed in the first stage of the program. In this case, lump-sum taxes will yield at least the same revenue as all other taxes discussed so far, and school-attendance subsidies will increase schooling more cheaply and efficiently than any other subsidy. In the first part of the program, therefore, a lump-sum tax should be levied on some part of the population and the education of the children of another part should be subsidized.

Assuming that the subsidy is sufficient to raise the receiving families' level of efficiency to $\epsilon'$ or above, it follows that these will subsequently choose $\tau = 1$. The new situation is therefore one where two groups exist: 'rich' families, who usually choose $\tau = 1$ and $n^r \in [n_{min}, n_{max}]$ and 'poor' families, who usually choose $\tau < 1$ and $n^p = n_{max}$. Given the assumption of assortative mating, the inequality will be persistent, whereby the 'rich' families' level of efficiency will rise and 'poor' families will slide into the poverty trap. Typically, the population growth rates of the two groups will be different, with $n^r < n^p$ so that the relative number of rich families will decline over time.

Whether rich families will ever be able to raise sufficient revenues to subsidize all poor families depends on several factors. First, the efficiencies of the two groups: the more efficient they are, the more taxes rich parents can pay, and the smaller the subsidy needed by poor families to reach $\epsilon'$. Second, the relative size of the two groups: if there are only few 'rich' and many 'poor' families, it will be not possible to raise sufficient revenue for all the 'poor'. Third, the minimum number of children a family can have. As discussed in section 5.2.1, if $n_{min}$ is large, taxable capacity is lower than in the case where $n_{min}$ is very low.

The following simplified example with $n_{min}$ sufficiently close to zero will illustrate the underlying problems. Assume that all poor families are in the poverty trap, that there is only one stable steady-state, that sustainable economic growth is possible for $\epsilon > \epsilon'$ and that the total subsidy 'poor' families need in a given period is $S$. Rich families – whose initial level of efficiency is $\epsilon^r > \epsilon'$ – can pay at most $2A\epsilon^r(1-\chi)$ each, while still choosing $\tau = 1$. In the first period, let there be $r$ rich families, so that total tax revenue will be $r2A\epsilon^r(1-\chi) \equiv T\epsilon^r$. If the total tax revenue in the first period is not sufficient to pay for the subsidies, the inequality $T\epsilon^r < S$ will hold. In the following period, with population growth rates $n^r$ for the rich and $n^p = n_{max}$ for the poor, sufficient revenue can be raised if and only if:

$$\frac{n^r}{n_{max}}(T\epsilon^r z + T) \geq S.$$

Obviously, the relative population growth rates $n^r/n_{max} < 1$ and the productivity of the education function $z$ influence whether 'waiting' will ever lead to the government being able to raise sufficient revenue to finance a subsidy for all the poor. After $t$

periods, the difference between revenues and spending $\delta(t)$ will be:

$$\delta(t) \equiv T\Big(\frac{n^r}{n_{max}}\Big)^t\Big[\sum_{i=1}^{t} z^{i-2} + z^{t-1}\epsilon^r\Big] - S \tag{13}$$

Depending on $z$ and the fertility rates of the two groups, the tax revenues after $P$ periods might suffice to finance the subsidies for the 'poor', that is $\delta(t = P) \geq 0$. It is not possible to compute $P$ analytically, but it is possible to find an upper limit for $P$ as follows:

$$\delta(t) \geq 0 \Leftrightarrow$$

$$T\Big(\frac{n^r}{n_{max}}\Big)^P\Big[\sum_{i=1}^{P} z^{i-2} + z^{P-1}\epsilon^r\Big] \geq S$$

Since $\sum_{i=1}^{P} z^{i-2} > 0$, a sufficient condition of $\delta(t) \geq 0$ is:

$$T\Big(\frac{n^r}{n_{max}}\Big)^t\Big[z^{P-1}\epsilon^r\Big] \geq S \Leftrightarrow$$

$$P \cdot \ln\Big(\frac{zn^r}{n_{max}}\Big) > \ln\Big(\frac{S}{T\epsilon^r z}\Big)$$

Hence, if $zn^r > n_{max}$, there will be some period $P = \ln\Big(\frac{S}{T\epsilon^r z}\Big) \cdot ln\Big(\frac{n_{max}}{zn^r}\Big) > 0$ such that $\delta(t = P) \geq 0$, that is, revenues from taxing 'rich' families after $P$ periods will suffice to finance the subsidies for all poor families.

# 6 Conclusion

In a society where parents decide freely how many children to have and how well to educate them (as opposed to making them work), and also have some measure of altruism towards their children, the child-raising costs, the social norms that govern the provision of support in old age and the productivity of the underlying educational process all have a vital influence. One possibility is that the economy will be trapped in a low-level, stable steady state – or poverty trap – in which adults' labor efficiency and lifetime utility are low, and child labor is the rule. Fertility will usually be at its exogenously given upper limit, so that, while consumption and income per family are constant, the total population grows exponentially. Only if the child-raising costs are sufficiently low and the educational process is highly productive can such a state be avoided.

Escape from this poverty trap is theoretically always possible if the educational process is sufficiently productive ($z > 1$) and the child-raising costs are not extremely high. If the government is sufficiently strongly constrained in raising taxes, however – for example, by the ease with which taxes can be evaded or by the upper limits on taxes imposed by minimal consumption needs – it might not be possible to devise a policy such that the whole society can escape the poverty trap.

Compared to the situation where parents decide only about the extent of schooling, fertility being given exogenously, the range of child-raising costs such that sustainable economic growth is possible is much narrower, and parents have a more limited choice of how to react to taxes. As they can only reduce current consumption, or education, or both, taxes will usually lead to a stronger reduction in overall schooling than if fertility is endogenous. Therefore, the maximal taxes rich families can pay without falling back into the poverty trap is lower. On the other hand, the danger of extinction is not relevant in such a setting. Poor parents could theoretically be induced to choose full-time schooling through high taxes on the expenditures on raising children, a step they could afford by reducing their fertility. If fertility is fixed, however, the measure cannot have this effect. Therefore, in a setting where schooling alone is analyzed, simultaneous escape of the whole society from the poverty trap is not possible without outside help.

# 7 Appedix

## A.1 The corner solution with respect to fertility

If the number of children is fixed ($n_t = \bar{n}$), parents will choose the following level of schooling (the index $t$ has been suppressed):

$$\tau^*(\bar{n}, \epsilon) = \frac{z\epsilon(\beta + \beta_1)[2A\epsilon(1-\chi) + \bar{n}A\mu + \bar{n}b\epsilon] - A\bar{n}\mu}{A\bar{n}\mu z\epsilon(1+\beta+\beta_1)} \tag{14}$$

In order to investigate the properties of $\tau^*(\cdot)$, we begin by disregarding the restriction $\tau^* \in [0,1]$. The shape of the function $\tau^*(\cdot)$ depends on two parameters: the child-raising costs, $b$, and the parents' efficiency, $\epsilon$. Analyzing the function at the borders of its domain reveals that poorly educated parents cannot afford to send their offspring to school, the same being true for the case where raising children is very expensive.

$$\begin{aligned}
\lim_{\epsilon \to 0} \tau^*(\bar{n}, \epsilon) &= -\infty \\
\lim_{b \to -\infty} \tau^*(\bar{n}, \epsilon) &= -\infty \\
\lim_{\epsilon \to \infty} \tau^*(\bar{n}, \epsilon) &= sign[2A(1-\chi) + b\bar{n}]\infty
\end{aligned}$$

Consumption in the first period of adult life can be rewritten so the impact of $\epsilon$ and $b$ becomes clear:

$$C_{1t} = \epsilon_t[2A(1-\chi) + bn] + A\mu(1-\tau)n$$

If the child-raising costs are sufficiently small ($|b| < \frac{2A(1-\chi)}{n}$), the term in the square brackets will be positive and $\epsilon_t[2A(1-\chi) + bn]$ will grow without bound for high levels of efficiency. The term describing income from child labor will then be negligible in comparison: as a consequence, parents will be able to afford to send their offspring to school if the child-raising costs are low. If, on the other hand, $|b|$ is large, the term in squared brackets will be negative, and child labor will be crucial to financing $C_{1t}$. For very high levels of efficiency, parents will not be able to maintain a nonnegative level of consumption in the first period: a negative school-time would be needed, if that were possible.

With $2A(1-\chi)\epsilon_t$ being the residual income of a family after payment of the transfer to the grandparents, the condition $2A(1-\chi) + bn \geq 0$, has a natural interpretation.

If the condition is fulfilled, parents do not have to resort to child labor in order to enjoy positive consumption in the first period of adult life. If, on the other hand, $|b| > \frac{2A(1-\chi)}{\bar{n}}$, some child labor will be optimal.

Apart from the behavior of the function $\tau^*(\cdot)$ for extreme values of efficiency and child-raising costs, the schooling chosen is characterized by its zeroes and maxima, which are of major importance for the economic interpretation and relevance of the closed-form solution in (14). As the numerator is a quadratic function of $\epsilon$, one expects to find up to two zeroes:

$$\epsilon_{1,2} = -\frac{1}{2}\frac{Az\bar{n}\mu(\beta+\beta_1) \pm \sqrt{Az\bar{n}\mu(\beta+\beta_1)[Az\bar{n}\mu(\beta+\beta_1) + 8A(1-\chi) + 4bn]}}{z(\beta+\beta_1)[2A(1-\chi)+b\bar{n}]} \tag{15}$$

For sufficiently small costs ($|b| < \frac{2A(1-\chi)}{\bar{n}}$) only one of these will be positive and the function will have no extrema. For larger costs ($|b| > \frac{2A(1-\chi)}{\bar{n}}$), there will be two zeroes and a maximum. As the factor under the square root falls with increasing costs, and becomes negative when they are sufficiently large ($|b| > \frac{2A(1-\chi)}{\bar{n}} + \frac{Az\bar{n}\mu(\beta+\beta_1)}{4\bar{n}}$), the function $\tau^*(\cdot)$ will be negative for all levels of efficiency if $|b|$ exceeds some limit. With increasing costs, the first zero of the function will move towards larger levels of efficiency: the larger the costs, the less parents can afford to send their children to school, given that the number of children they have is fixed ($\tau(b) > \tau(b')\,\forall\,|b| < |b'|$).

The shape of $\tau^*(\bar{n}, \epsilon)$ has been plotted in figure 1 for different costs. For the purposes of depiction, the intervals for $|b|$ that have been established above will be defined as 'low', 'moderate' and 'high', whereby, in order to depict the shape of the functions fully, it is necessary to stray way outside the meaningful ranges of the variables. If the function has no maximum and grows without bound for high levels of efficiency, it will be economically relevant for some levels of efficiency $\epsilon \in [max(1, \epsilon_a), min(1, \epsilon_b)]$ so that $\tau(\epsilon_a) = 0$ and $\tau(\epsilon_b) = 1$. If the function has a maximum, the ranges can be defined using the zeros of the function, as computed in (15) [19].

[19] As it can be shown that the maximum of the function lies below 1 for all parameters, the case where the function is relevant in two separate sections can be ignored.

$$\tau_{max} = \frac{\beta+\beta_1}{1+\beta+\beta_1}\Big[1 - \frac{4A(1-\chi)+2nb}{\sqrt{z\,(\beta+\beta_1)\,(2A\chi - 2A - bn)\,n\mu\,A}}\Big] < 1$$

The impact of changes in $\bar{n}$ on $\tau^*(\bar{n}, \epsilon)$ remains to be analyzed. With $\partial\tau/\partial\bar{n} < 0\,\forall\epsilon, \bar{n}$, it follows immediately that $\tau(n_{min}) > \tau(n_{max})$: the fewer children a family has, the better it will educate them. As parents do not have to spend so much money just raising children, they can afford to educate them better. On the other hand, additional education is necessary in order to finance and maintain consumption in the last period of life. Therefore, if a family has fewer children for any reason, schooling will increase. A reduced number of children increases $C_{1t}$ if the level of schooling remains unchanged, while $C_{2t}$ will fall. Therefore, a shift in consumption between the two periods is needed, and as education is the only variable in which $C_{1t}$ is decreasing and $C_{2t}$ is increasing, schooling will be increased – if that is possible[20].

## A.2 The corner solution with respect to education

In this case, too, the solution depends on the parents' level of efficiency and the child-raising costs as well as the fixed level of schooling.

$$n^*(\bar{\tau}, \epsilon) = \frac{-2A\beta\epsilon(1-\chi)}{(1+\beta)[A\mu(1-\bar{\tau}) + b\epsilon]} \tag{16}$$

For very small $\epsilon$, the function will be zero or negative for all parameters, while the shape of the function for moderate and high levels of efficiency will depend on $\tau$ and $b$:

$$\lim_{\epsilon \to 0} n^*(\bar{\tau}, \epsilon) \;=\; 0$$

$$\lim_{\epsilon \to \infty} n^*(\bar{\tau}, \epsilon) = \frac{2\beta\, A\,(1+\chi)}{|b|\,(1+\beta)} \;>\; 0$$

If $\tau = 1$, the function $n^*(\cdot)$ will independent of $\epsilon$ ($n = \frac{2A\beta(1-\chi)}{(1+\beta)|b|} > 0$), and decreasing in $|b|$. For the value of $n$ to be economically relevant in this case, the child-raising costs have to fulfill the condition

$$|b| \in \Big[\frac{2A\beta(1-\chi)}{n_{max}(1+\beta)}, \frac{2A\beta(1-\chi)}{n_{min}(1+\beta)}\Big].$$

[20]These considerations are only valid – at this point of the analysis – if the interior solution w.r.t. education is the household's optimum. The intuitive argument will be valid for all solutions.

If $\tau = 0$, $n$ will be negative for low levels of efficiency up to the point of discontinuity and positive thereafter. In general, the critical efficiency (point of discontinuity) depends on the chosen schooling and the child-raising costs: $\epsilon = \frac{A\mu(1-\tau)}{|b|}$. For extremely large $\epsilon$, $n$ approaches a positive value asymptotically from above, since $\partial n/\partial\epsilon \leq 0\,\forall\epsilon$:

$$\lim_{\epsilon\rightarrow\infty} n^*(\bar{\tau},\epsilon) = \frac{2A\beta(1-\chi)}{|b|(1+\beta)} > 0$$

For $n^*(\bar{\tau},\epsilon)$ to be economically feasible for any $\epsilon$, the child-raising costs have to fulfill the same condition as in the case $\tau = 1$. An exogenous increase in schooling will lead to a decrease in the number of children a couple decide to have: $\partial n/\partial\tau < 0\,\forall\,\epsilon, b$. The intuitive argument is the same as in the case where education was interior: An increase in schooling will lead to a reduction in family income in the first period, and hence in $C_{1t}$, if all other variables remain unchanged, while $C_{2t}$ will rise. Therefore, parents will try to shift consumption between periods by reducing fertility.

## A.3 The corner solutions w.r.t. both variables

### A.3.1 $\triangle u(n)$

$$\triangle u(n) := u(n_{max},\bar{\tau}) - u(n_{min},\bar{\tau}) > 0$$

$$\Leftrightarrow$$

$$ln\Big[\frac{2A(1-\chi)\epsilon + A(1-\bar{\tau})\mu n_{max} + b\epsilon n_{max}}{2A(1-\chi)\epsilon + A(1-\bar{\tau})\mu n_{min} + b\epsilon n_{min}}\Big] +$$

$$+\beta ln\Big[\frac{A(z\epsilon\bar{\tau}+1)\chi n_{max}}{A(z\epsilon\bar{\tau}+1)\chi n_{min}}\Big] + \beta_1 ln\Big[\frac{(z\epsilon\bar{\tau}+1)}{(z\epsilon\bar{\tau}+1)}\Big] > 0$$

$$\Leftrightarrow$$

$$\frac{2A(1-\chi)\epsilon + A(1-\bar{\tau})\mu n_{max} + b\epsilon n_{max}}{2A(1-\chi)\epsilon + A(1-\bar{\tau})\mu n_{min} + b\epsilon n_{min}} > \Big(\frac{n_{min}}{n_{max}}\Big)^{\beta}$$

A rearrangement yields: $\triangle u(n) > 0$ if and only if:

$$\epsilon[2A(1-\chi)(n_{max}^{\beta} - n_{min}^{\beta}) + b(n_{max}^{1+\beta} - n_{min}^{1+\beta})] > -A\mu(1-\bar{\tau})(n_{max}^{1+\beta} - n_{min}^{1+\beta}) \quad (17)$$

As before, $b$ and $\epsilon$ determine whether the condition above is fulfilled or not.

If

$$|b| \leq \frac{2A(1-\chi)(n_{max}^{\beta} - n_{min}^{\beta})}{n_{max}^{1+\beta} - n_{min}^{1+\beta}},$$

the left-hand side of (17) will be nonnegative. Since $n_{min} < n_{max}$ and $1+\beta > 1$, the term on the right-hand side of the inequality will be negative or zero. Therefore, the critical efficiency for $\triangle u(n) > 0$ will be negative or zero, so that $u(n_{max}, \bar{\tau}) > u(n_{min}, \bar{\tau})$ for all levels of efficiency. If the child-raising costs are low, choosing the maximal number of children ($n_{max}$) will be optimal for all families when $\tau$ is fixed.

If

$$|b| > \frac{2A(1-\chi)(n_{max}^{\beta} - n_{min}^{\beta})}{n_{max}^{1+\beta} - n_{min}^{1+\beta}},$$

both sides of condition (17) will be negative, and $u(n_{max}, \bar{\tau}) > u(n_{min}, \bar{\tau})$ only if

$$\epsilon < \frac{A\mu(1-\bar{\tau})(n_{max}^{1+\beta} - n_{min}^{1+\beta})}{2A(1-\chi)(n_{max}^{\beta} - n_{min}^{\beta}) + b(n_{max}^{1+\beta} - n_{min}^{1+\beta})}.$$

Choosing the maximal number of children, then, will only be optimal for poor parents, while well-educated households will prefer having few children if the costs of raising them are large. The lower the child-raising costs and the lower the level of schooling, the higher is the critical level of efficiency.

### A.3.2 $\triangle u(\tau)$

$$\triangle u(\tau) > 0 \Leftrightarrow \tag{18}$$

$$ln\Big[\frac{2A(1-\chi)\epsilon + A\bar{n}\mu + \bar{n}b\epsilon}{2A(1-\chi)\epsilon + \bar{n}b\epsilon}\Big] + \beta ln\Big[\frac{A\chi\bar{n}}{A\chi\bar{n}(z\epsilon+1)}\Big] + \beta_1 ln\Big[\frac{1}{z\epsilon+1}\Big] > 0 \Leftrightarrow$$

$$\frac{2A(1-\chi)\epsilon + A\bar{n}\mu + \bar{n}b\epsilon}{2A(1-\chi)\epsilon + \bar{n}b\epsilon} > \Big(\frac{1}{z\epsilon+1}\Big)^{-(\beta+\beta_1)} \tag{19}$$

Analyzing first the behavior of $\triangle u(\tau)$ for very small and very high levels of efficiency, one gets:

$$\lim_{\epsilon\to 0} \triangle u(\tau) = \infty$$
$$\lim_{\epsilon\to\infty} \triangle u(\tau) = -\infty$$

Therefore $\triangle u(\tau)$ has at least one zero or point of discontinuity. For $|b| > \frac{2A(1-\chi)}{\bar{n}}$, the function will have a single point of discontinuity for positive levels of efficiency, and none otherwise. As all 'goods' are necessary in consumption, and as the condition $|b| \geq \frac{2A(1-\chi)}{\bar{n}}$ yields $C_{1t}(\tau = 1) \leq 0$, it follows that $\tau_t = 0$ whenever $|b| \geq \frac{2A(1-\chi)}{\bar{n}}$. That is to say, the children must then be put to work full-time in order to finance their parents' consumption in the first period of adult life.

If, on the other hand, $|b| < \frac{2A(1-\chi)}{\bar{n}}$, parents can afford to educate their children, both $\tau = 0$ and $\tau = 1$ are possible optima, and the function $\triangle u(\tau)$ has no point of discontinuity. The derivative $\partial \triangle u(\tau)/\partial \epsilon$ being always negative for small costs and as $\triangle u(\tau)$ takes all values between $-\infty$ and $\infty$, it follows that the function $\triangle u(\tau)$ will have a single zero in the interval $0 \leq \epsilon < \infty$.

By analyzing the two cases $\beta + \beta_1 > 1$ and $\beta + \beta_1 < 1$ separately, it is possible to approximate the critical levels of efficiency for $\triangle u(\tau) > 0$ and $\triangle u(\tau) < 0$. If $\beta + \beta_1 < 1$ a simplification [21] of the system yields:

$$\triangle u(\tau) > 0 \Leftarrow$$
$$\frac{2A(1-\chi)\epsilon + A\bar{n}\mu + \bar{n}b\epsilon}{2A(1-\chi)\epsilon + \bar{n}b\epsilon} > z\epsilon + 1$$

This inequality can easily be solved for the critical level of efficiency:

$$\triangle u(\tau) > 0 \Leftrightarrow \epsilon \in \left(-\sqrt{\frac{A\bar{n}\mu}{z[2A(1-\chi)+\bar{n}b]}}, \sqrt{\frac{A\bar{n}\mu}{z[2A(1-\chi)+\bar{n}b]}}\right)$$

The result in the case $\beta + \beta_1 > 1$ will be identical. In both cases, the condition for the critical level of efficiency derived above is necessary but not sufficient.

It should be noted that for extremely large child-raising costs, even full-time child labor cannot ensure nonnegative $C_{1t}$, with the critical value for the costs depending on efficiency and the number of children: $|b| < \frac{2A(1-\chi)}{\bar{n}} + \underbrace{\frac{A\mu}{\epsilon} \rightarrow}_{\epsilon\rightarrow\infty} \frac{2A(1-\chi)}{\bar{n}}$.

## A.4 Potential Steady States: the interior solution w.r.t. $\tau$

There are two possible efficiencies such that $\tau^*(\bar{n}, \epsilon) = \tau_{crit}$:

$$\epsilon = -\frac{1}{2}\frac{An\mu[(\beta+\beta_1)(z-1)-1]}{z(\beta+\beta_1)(2A(1-\chi)+nb)} \pm \frac{\sqrt{An\mu\left[\left(An\mu[(\beta+\beta_1)(z-1)-1]^2\right) - 4z(\beta+\beta_1)^2(2A(1-\chi)+nb)\right]}}{2z(\beta+\beta_1)(2A(1-\chi)+nb)} \tag{20}$$

[21]The simplification used is: $\frac{2A(1-\chi)\epsilon + A\bar{n}\mu + \bar{n}b\epsilon}{2A(1-\chi)\epsilon + \bar{n}b\epsilon} > (z\epsilon+1)^1 > (z\epsilon+1)^{(\beta+\beta_1)}$

For the term under the square root to be positive, the costs of child-rearing must fulfill the condition:

$$|b| \geq \frac{2A(1-\chi)}{n} - \frac{A\mu[(\beta+\beta_1)(z-1)-1]^2}{4z(\beta+\beta_1)^2} \tag{21}$$

If (21) is not satisfied, there will be no $\epsilon$ so that $\tau = \tau_{crit}$ for the interior solution w.r.t education[22]. If, on the contrary, (21) is satisfied, the two functions will have up to two points of intersection, depending on the exact size of $b$, $z$ and $(\beta + \beta_1)$. If, further, the total weight of the future arguments of utility is sufficiently smaller than that of $C_{1t}$, that is, $(\beta+\beta_1) < 1$, and if the productivity of education ($z$) is not too large, then the term $[(\beta + \beta_1)(z-1) - 1]$ will be negative. In this case, and if $\frac{2A(1-\chi)}{n} - \frac{A\mu[(\beta+\beta_1)(z-1)-1]^2}{z(\beta+\beta_1)^2} \leq |b| \leq \frac{2A(1-\chi)}{n}$, there will be two points of intersection [23]. If, however, $|b| > \frac{2A(1-\chi)}{n}$, the root in (20) being larger than the term before it, only one of the signs will yield $\epsilon > 0$, so that the two functions will cross only once.

[22]Both values in (20) will be complex numbers.

[23]The term under the square root in (20) will be positive but smaller than the term before it, so that both signs will yield a positive efficiency.

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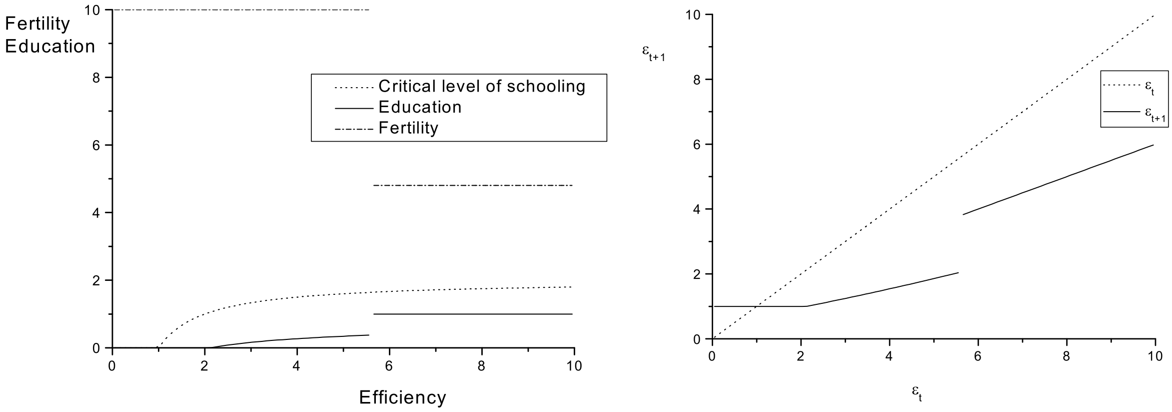


Figure 6a: The Household's Decision and Phase Diagram for $z < 1$ and $|b| < \frac{A(1-\chi)}{n_{max}}$

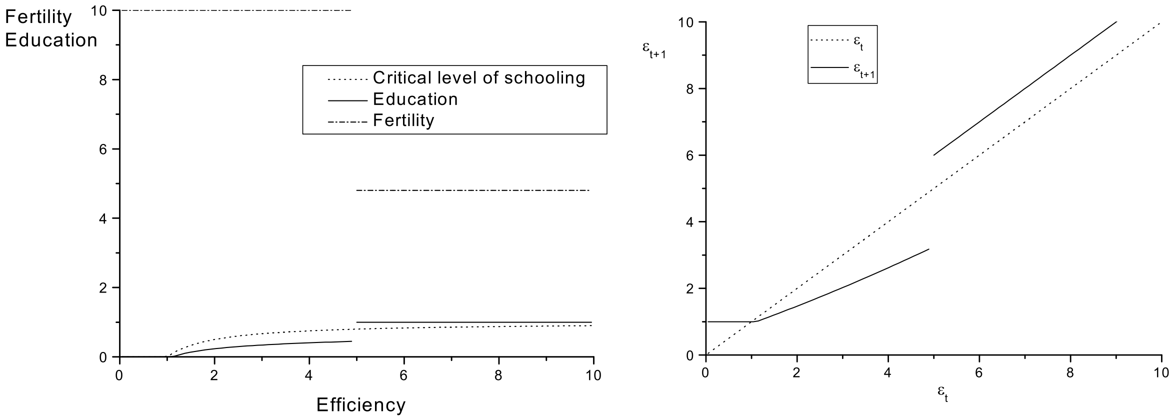


Figure 6b: The Household's Decision and Phase Diagram: $z = 1$ and $|b| < \frac{A(1-\chi)}{n_{max}}$

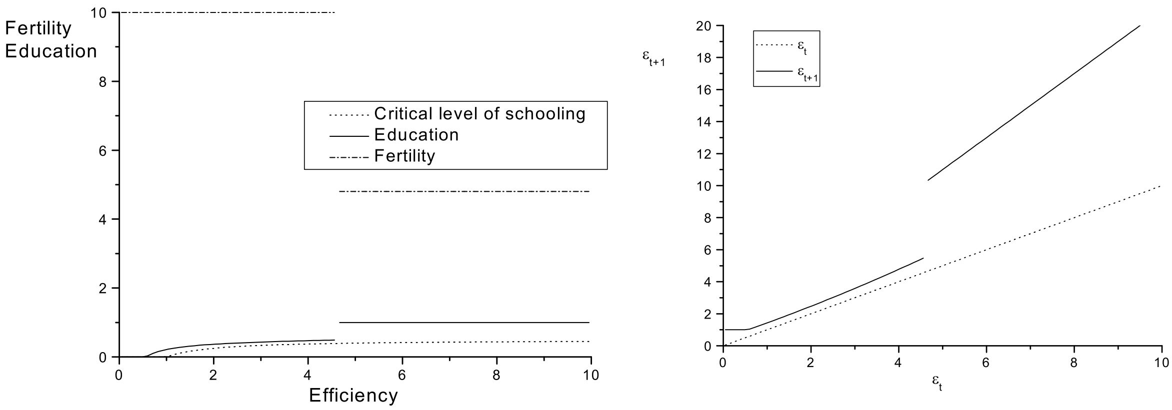


Figure 6c: The Household's Decision and Phase Diagram: $z > 1$ and $|b| < \frac{A(1-\chi)}{n_{max}}$

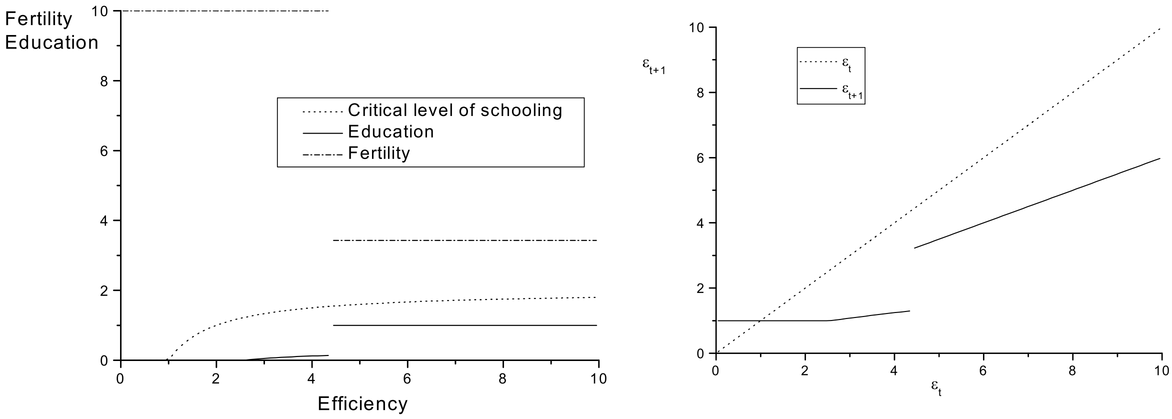


Figure 6d: The Household's Decision and Phase Diagram: $z < 1$ and $|b| \in [\frac{A(1-\chi)}{nmax}, \frac{A(1-\chi)}{nmin}]$

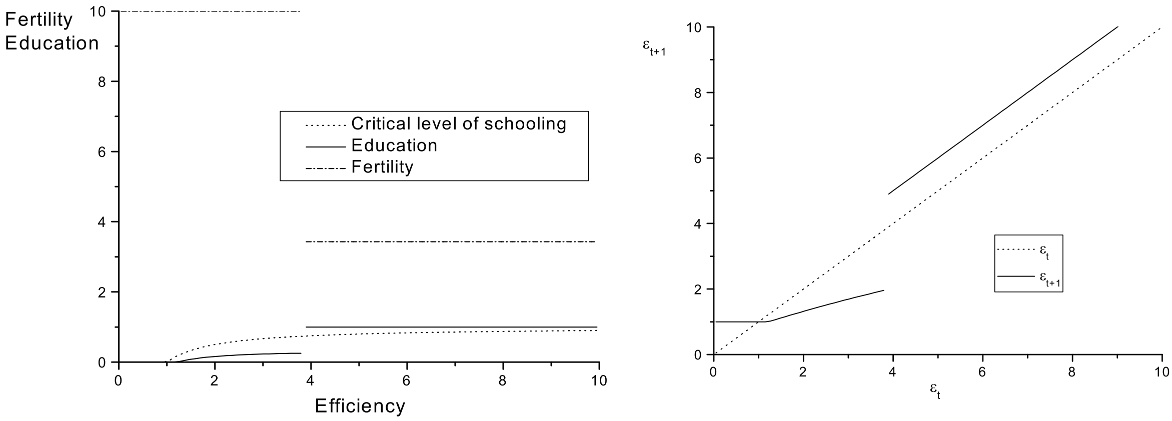


Figure 6e: The Household's Decision and Phase Diagram: $z = 1$ and $|b| \in [\frac{A(1-\chi)}{nmax}, \frac{A(1-\chi)}{nmin}]$

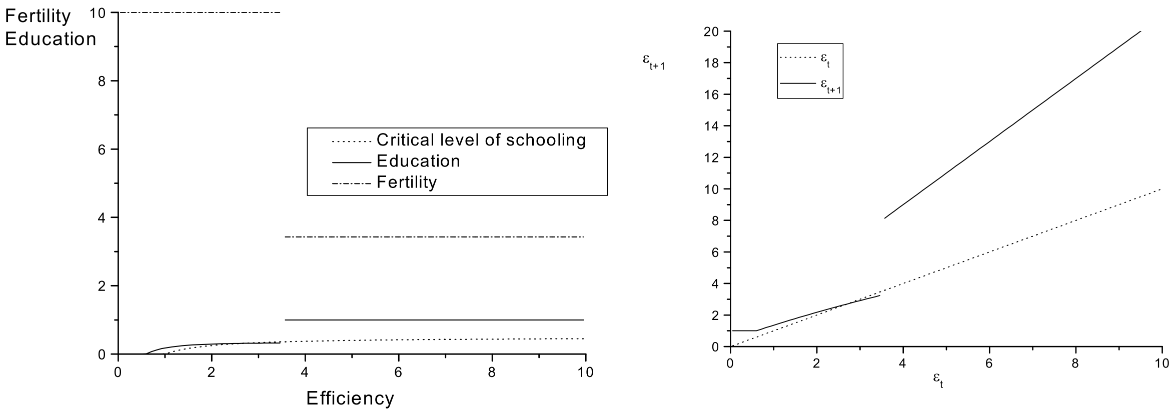


Figure 6f: The Household's Decision and Phase Diagram: $z > 1$ and $|b| \in [\frac{A(1-\chi)}{nmax}, \frac{A(1-\chi)}{nmin}]$

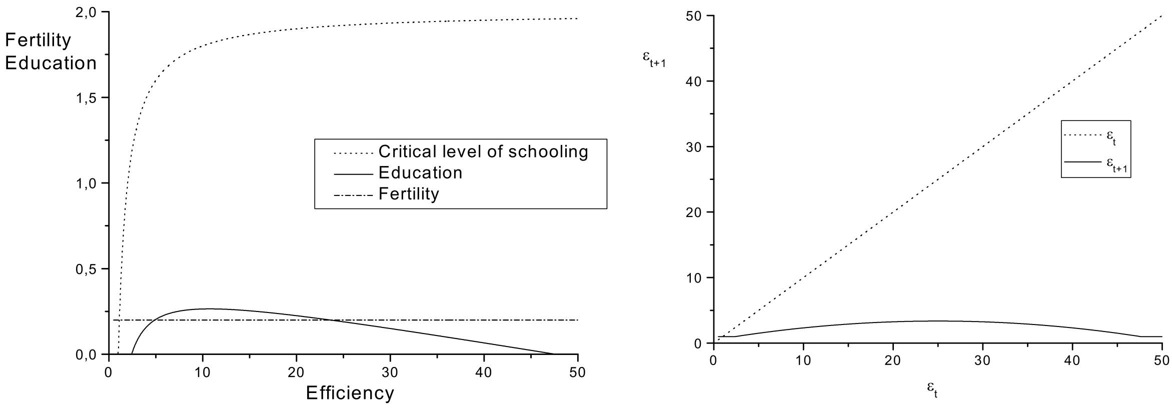


Figure 6g: The Household's Decision and Phase Diagram for $z < 1$ and $|b| > \frac{A(1-\chi)}{nmin}$

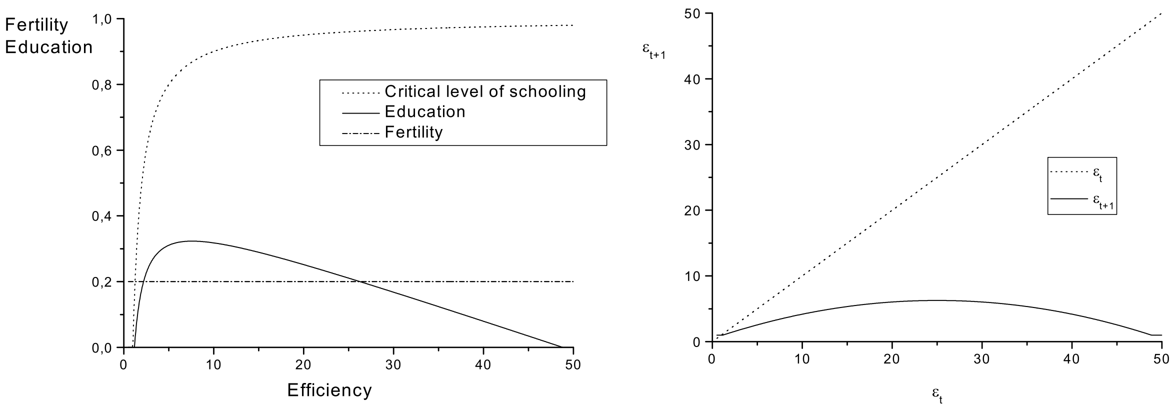


Figure 6h: The Household's Decision and Phase Diagram: $z = 1$ and $|b| > \frac{A(1-\chi)}{nmin}$

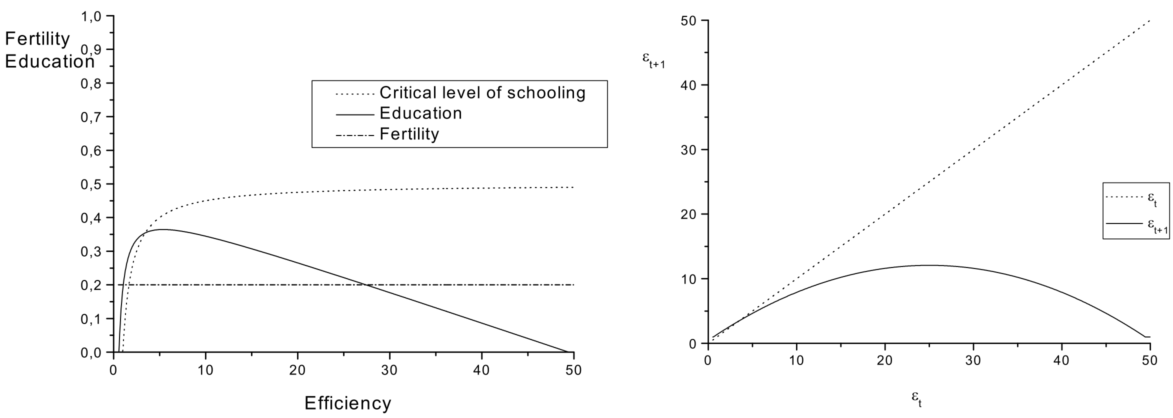


Figure 6i: The Household's Decision and Phase Diagram: $z > 1$ and $|b| > \frac{A(1-\chi)}{nmin}$