The Solution to Arrow's Difficulty in Social Welfare

Thomas Cool, September 4 1997

Rotterdamsestraat 69, 2586 GH Scheveningen, The Netherlands
http://www.can.nl/~cool cool@can.nl

Summary

Arrow's Theorem holds that no constitution can satisfy certain properties. In annex to that theorem, Arrow claims that those properties are reasonable and morally desirable. In Arrow's view there thus is the difficulty that people desire a constitution that cannot exist. While the Theorem stands as a mathematical result, the additional claims concern other domains, i.e. the domains of reasonableness and morality. It are these claims that have caused much confusion in the literature. It is shown here that the claims are unwarranted, since inconsistent properties are neither reasonable nor morally desirable. It is shown too that Arrow's axiom of the Independence of Irrelevant Alternatives is not realistic, and thus unattractive. We show the existence of some constitutions that are consistent and might be optimal to many. The major error made by Arrow and his students is to mix up the context of scientific discovery and learning with the context of application to the real world by educated people.

Keywords

Social Choice, social welfare, welfare economics, economic policy, decision theory, information, learning, scientific discovery, politics, ethics, philosophy

Introduction

Arrow (1951, 1963) showed that if certain properties are postulated for a constitution, then such a constitution would not exist. This result stands as a mathematical theorem, has been checked by numerous scholars, and is accepted by this author. In fact, we will give a short proof below.

Arrow also claimed, annex to the theorem, and *this* will be at issue here, that those properties would be reasonable and morally desirable. He recently repeated that claim in the Palgrave (1988 p125). He writes:

> "(...) conditions to be imposed on constitutions (...)"
>
> "(...) there is no social choice mechanism which satisfies a number of reasonable conditions".

For clarity it is useful to introduce the following abbreviations for the theorem and its companion claims, and their conjunction:

> *AT* = the Arrow Theorem
>
> *ARC* = the Arrow Reasonableness Claim = that the properties are *reasonable*
>
> *AMC* = the Arrow Moral Claim = that they are *to be imposed.*
>
> *AGV* = the Arrow General View = *AT & ARC & AMC*

Note that Arrow's phrasing on *ARC* and *AMC* is a bit ambiguous. The "to be imposed" might not be moral but merely logical, in a sense that one needs at least some conditions to make a constitution. However, the topic of collective choice is distinctly a moral one. Secondly, Arrow emphasises what is to be imposed and what is reasonable, but he may not be in a position to impose his views and morals on us. The best interpretation of the situation likely is as follows. Presume that Arrow sees the Founding Fathers at work. He then retreats to his office, and conjectures: 'If I interprete correctly what they want, then it are these properties.' Thus the *ARC* and *AMC* are not quite Arrow's personal ideas. Above quotes can best be interpreted as factual statements on what people apparently want and consider reasonable.

Arrow's general view has been accepted in many places in the literature and textbooks, see Luce & Raiffa (1957), Johansen (1969), Sen (1986) or various other entries in that same Palgrave. For example, Tobin (1990):

> "We know there is no way to aggregate individual preferences into social rankings (...). As if this were not obvious, Kenneth Arrow proved it rigorously years ago. The impossibility applies to aggregations across contemporaneous cohorts, a fortiori across generations living and unborn."

In a much used book on cost benefit analysis, Dasgupta & Pearce (1980):

> "(...) no escape route (...) seems yet to be available."

Jorgenson (1990) concludes 'more positively' to dictatorship:

> "The classic result of social choice theory is Arrow's (...) impossibility theorem, which states that ordinal noncomparability of individual welfare orderings implies that a consistent social ordering must be dictatorial, corresponding to the preferences of a single individual."

Not everybody falls for dictatorship. The impact of the *AGV* generally comes from the fact that people find themselves, either from moral obligation or from reasonableness, wanting the impossible.

Note the subtlety in that fixture. The impossibility is logical and not just empirical. An example may help. Let me confide that I want to found a new university on the island of Crete. However, I am not that rich, so I want something impossible. This however does not put me into a fixture, since I am used to the fact that I cannot afford some things that I want. However, the Arrow general view concerns a logical impossibility, which is something quite different.

We can usefully recognise:

reasonable = rational & realistic

Reasonableness is the intersection of rationality and empirical realism. Nonexistence may derive from empirical circumstances or from logical impossibility. Irrationality however is always unrealistic. Inconsistency cannot *exist*, in the true empirical sense. For example a round square cannot exist. The nonexistence of the Arrowian constitution similarly derives not from empirical reality but from logical necessity.

Given the *AGV*, the question arises what the reasonableness and moral presumptions of Arrow's claims actually are. Are these claims as strong as conjectured ?

The position of this paper is as follows:

1. As has been said on 'round tables', it is not rational to postulate inconsistent properties. People involved in a learning process may indeed make inconsistent assumptions. However, once the inconsistency is discovered, it is no longer considered to be rational to adopt those assumptions. People may enjoy 'roundness' and 'squareness', but both simultaneously is seen to be inconsistent, even inconceivable, and hence unreasonable. The Arrowian properties are unreasonable in the exactly same manner. Arrow's pitfall is to confuse the learning process, his context of discovery, with real world applications by educated people.
2. Similarly, one cannot be morally obligated to a logical impossibility. Hence Arrow's properties are morally undesirable.

These points will be clarified below.

Note that people have in practice rejected some of Arrow's properties. Even those scholars that seem to accept the general claim *AGV*, accept, a fortiori, the implied inconsistency, and thus in practice drop some assumptions to cope with the real world. Unfortunately, however, the literature has not converged to some agreement on which properties are best to drop. The position of this paper will be to forward the proposition that the Arrow axiom of the Independence of Irrelevant Alternatives is the culprit to kill. It is a bad axiom for rational collective decision making, since it appears to be incongruent with that very notion itself.

In the following we develop the concepts, give a short proof and discussion of Arrow's Theorem, construct the argument against the claims, reappraise the literature, and conclude.

Basic concepts

Let X be the commodity domain. An agent is a compound of various properties such as utility, wealth etcetera. Let S be the set of possible compounds on X. With n agents, our interest concerns the function $c: S^n \rightarrow S$. which maps the society into an aggregate compound. This is generally called the 'Arrow type of social welfare function' or simply a constitution.

A constitution differs from the 'Bergson type of social welfare function' (SWF) which is defined directly over X, as a utility index $SWF: X \rightarrow [0, \infty)$.

It suffices to restrict S to preference orderings. These orderings satisfy reflexivity, transitivity and completeness. It is important to add that there is no cheating. Let R denote normal preference, P strict preference, and I indifference. A suffix denotes an individual preference, otherwise it is the aggregate. An element in S^n is called a *profile*, and $R = c(R_1, ...R_n)$.

There are the following Arrowian properties (axioms, conditions):

AWP the weak Pareto principle

AU universal domain (wide ranging preferences)

AD no dictator

AIIA independence of irrelevant alternatives

a *AWP & AU & AD & AIIA.*

It is useful to discuss the notion of inconsistency a bit deeper. There are two approaches:

- The conjectural approach is like the following. Properties like 'squareness' and 'roundness' may apply to different objects, and they thus can be used consistently next to each other. The properties do not apply simultaneously, since there are no objects that are round and square simultaneously. As long as one does not assert the existence of such an object, there is no inconsistency. In the same way above Arrowian properties need not result into an inconsistency when they only result into the conclusion that no constitution can satisfy them all.
- The realistic approach is that collective decision making is happening all the time around us, or, in other words, there is a constitution. We want to know what that constitution is, or, perhaps change it into something more to our liking. In this case, we may run into a real inconsistency.

In the following we will presume existence, so that e.g. 'squareness' and 'roundness' are inconsistent.

The Arrow Theorem then can be expressed in various equivalent logical forms: [1]

AT $a \Rightarrow falsum$

AT' $a \Rightarrow \sim a$

AT" $\sim a$

with *falsum* a contradiction or falsehood and ~ the negation sign. If something leads to a contradiction, then we conclude to the falsehood of the assumptions themselves.

There is a Kantian distinction between technical, pragmatic and moral imperatives. Utility, as commonly regarded by economists, likely is of the pragmatic kind. Interestingly, theorists on morality have developed something called 'deontic logic', which appears to give many similar results as economic theory. Deontic logic however applies to *propositions* and not to *commodity domains*. It is possible, though, to integrate all these kinds of preferences into an integral utility index, when we replace a point x in the commodity domain by a statement "The state of the world is x". The resulting integral utility likely would be lexicographic, in that some moral and constitutional issues might dominate pragmatic results in the commodity domain.

[1] We thus do not discuss the format *AWP & AU & AIIA* $\Rightarrow$ *AD.* These are just logical transformations.

Whatever that might be, though, it remains that we can usefully introduce and apply some terms from deontic logic. Define:

$Ap \Leftrightarrow pR(\sim p)$ means that p is allowed (at least as good as $\sim p$)

$Op \Leftrightarrow pP(\sim p)$ means that p is a moral obligation (one ought to p)

An exemplaric deontic result is:

$Op \Leftrightarrow \sim(A(\sim p))$

The gap between Ought and Reality (*Sein und Sollen*) comes from the rejection of $Op \Rightarrow p$ and $p \Rightarrow Op$.

Moral consistency is reflected in the Deontic Axiom:

DA $(Op \ \& \ (p \Rightarrow q)) \Rightarrow Oq$

There is some discussion between moral theorists whether *DA* really holds. It may be felt that the logic is not very compelling for empirical relations of dubious causality. However, if $p \Rightarrow q$ reflects a logial truth, then *DA* is commonly accepted.

Deontic logic allow us to translate:

$AMC = Oa$

The use of deontic logic allows a forceful restatement of Arrow's difficulty in social choice:

$Oa \ \& \sim a$

On reasonableness, it seems a bit better to attach the properties to the agents rather than to the propositions or commodities. Useful axioms then are:

AF feasibility, *X* is the budget set

ARe agents are realistic (they only consider feasible options)

I thus agree with Arrow's 1950 statement: "My own feeling is that tastes for unattainable alternatives should have nothing to do with the decision among the attainable ones; desires in conflict with reality are not entitled to consideration." Thus, also, when one point is (socially) most preferred, it is the one consumed.

The most complex property seems to be good old *rationality*:

ARa agents are rational (they accept logic, [2] have a preference ordering, are morally consistent (*DA*), and are educated on the Arrow's Theorem (*K(AT)*))

The latter is a novel aspect, that, however, should not come as a surprise, given what we said in the introduction. The information set or knowledge base *K(.)* must contain the Arrow Theorem. There is a difference between a learning process and a result. In a common classroom or used-car-salesman strategy, people are goaded into buying some axioms as reasonable and attractive, and then burn themselves, which teaches them. This may be called rational from the viewpoint of learning. This paper however concentrates on the after-learning-rationality, the kind of rationality that makes learning so worthwhile.

Hence:

[2] This is not without problem, since there are many logics, such as standard, threevalued, fuzzy, intuitionistic logic, and my own scheme of 'the logic of exceptions'' (that I use to solve the liar paradox, and Russells and Gödels problems). However, here it suffices to presume standard logic.

ARC = ARe & ARa

How does Arrow's original approach relate to the *K(AT)* assumption ? Arrow (1951, 1963) has no incorporation of learning, so it might be that he assumes standard economic rationality. If that would be perfect foresight, then *K(AT)* is implied. However, it is better to hold that Arrow in that period discussed constitutional choice *for* agents and not *by* agents. His axioms do not describe educated people involved in constitutional choice. [3]

Alternatively put, the new result in this paper comes from widening the scopes of utility and rationality to the inclusion of the constitutional process itself.

Restatement of Arrow's Theorem & Discussion

It appears very useful to discuss the example given by the Marquis de Condorcet 1785. This example is reproduced in table 1. There are three parties and three topics on ballot, and the numbers of seats and the preferences are such that, with pairwise voting and a majority rule, a cycle results: A > B > C > A.

Table A: Condorcet 1785

Party	Seats	Topics ordered by preference		
		High	Mid	Low
Red	25	*A*	*B*	*C*
Green	35	*C*	*A*	*B*
Blue	40	*B*	*C*	*A*
Total	*100*			

Pairwise vote							
A	*B*		*A*	*C*		*C*	*B*
25			25				25
35				35		35	
	40			40			40
60	*40*		*25*	*75*		*35*	*65*
	A			*C*			*B*

It is, in all clarity, not that easy to aggregate votes on more than two topics. For two topics one can indeed ask for pro and contra, and find a majority (and occasional ties). For more topics, votes will scatter across the topics, and there will often be no clear majority. Therefor, pairwise voting is a good strategy to get the required information on the preferences. However, pairwise voting apparently also causes problems. So, basically, the search is for a strategy without such problems. And that is, basically, also the suggested value of Arrow's Theorem: that it states that there would be no such 'optimal' strategy.

However, in this Condorcet example, we may clearly conclude that the cycle primarily means that there is a tie. The situation is in a deadlock, and the group, as a collectivity, is *indifferent*.

[3] In that sense the axioms can be called incomplete. Alternatively, if the idea is that these axioms concern educated people, then there is a hidden inconsistency, in that reasonable agents are assumed to regard inconsistent axioms as reasonable. (Arrow likely prefers incompleteness to inconsistency.)

That there are indifferences or ties, is nothing special. Standard economic analysis allows agents to be indifferent (we even draw indifference loci), so groups should be allowed to be indifferent too. In Condorcet's example, indifference is even a logical choice, since when we assume something else, then we quickly run into difficulties.

There is the famous case of Buridan's Ass (AD 1358). An ass stands between two equal stacks of hay, at equal distances. He cannot decide what stack to take, and dies of starvation. The upshot of this parable is that rational beings can devise a decision. Constitutions generally state what happens when there are ties. Commonly the Status Quo persists. (This may happen even if it was one of the topics under ballot, and apparently was rejected at that stage.) Alternatives are that the chairman decides, or points are negotiated, and one can use dice. It is important to see the difference between *voting* and *deciding*. Table 1 essentially gives a voting field, and no decision yet. There are additional rules that translate the field into a unique decision.

We can use Condorcet's example to give a short proof of Arrow's Theorem. The group decision in the Condorcet case is indifference, so that *AIB*. Under the axioms of universality we can look at various preference *profiles*, of which Condorcet's example is only one. Now regard another profile (call it "Condorcet-Adjusted") in which the preferences on *A* and *B* remain the same, but the preference on *C* change, e.g. *C* drops to the end of the line. The Condorcet-Adjusted profile thus is *{{A, B, C}, {A, B, C}, {B, A, C}}*. Majority voting now results into *APB*. It thus differs from *AIB*. But this contradicts Arrow's *AIIA*. Since the preferences on *A* and *B* have not changed, the outcome on *A* and *B* should be the same, but here it isn't. Thus there is a counterexample to the axioms. So the axioms are inconsistent. Q.E.D.

The merit of this short proof is that it clearly shows the awkwardness of the *AIIA*. In the case of Condorcet's example the conclusion *AIB* is a sound decision, and in the case of the Condorcet-Adjusted example the conclusion *APB* is sound too. That preferences outside of the pair *A* and *B* have changed is *vital* to the group decision, since the shift helps a change from clear indifference to clear preference. The preferences on other topics are quite relevant, and not 'irrelevant'. Arrow's axiom excludes vital information about the preferences, and it should come as no surprise that paradoxes and inconsistencies arise. The *AIIA* is incongruent with the notion of group decision making; perhaps an individual can exclude information about other topics, but a group cannot. It is a surprise that *AIIA* has not been killed right in 1951.

The following sections use formal logic.

A lemma

Lemma: *AF* implies that a constitution *p* satisfies the property $Op \Rightarrow p$.

First proof: The definition of *AF* is that desires (*Op*) in conflict with reality (*~p*) are not entitled to consideration. But $\sim(Op \,\&\, (\sim p))$ is equivalent to $Op \Rightarrow p$. Q.E.D.

Second proof: We already concluded that the most preferred point (*Op*) would also be the chosen point (*p)*. Thus $Op \Rightarrow p$. Q.E.D.

Discussion: We have enlarged the domain with constitutions, and hence the axiom of feasibility becomes stronger. The extension itself is rather weak, since we only extend on consistency (and not empirical validity). Our criterion might be rephrased as that a reasonable society would stick to its rules. The gap between Ought and Reality still exists in principle, but can in practice be bridged by the human effort to attain one's ends.

Note: $q \Rightarrow p$ is equivalent to $\sim p \Rightarrow \sim q$, and we may take $q = Op$.

Rejection of the Arrow Moral Claim (AMC)

Theorem I: For a reasonable society, the *AMC* is invalid.

First proof by rationality & moral consistency (*DA*): Assume *Oa.* But *a => ~a,* and with *DA* we get *O~a.* But this gives a preference inconsistency *Oa & O~a*. Hence *~Oa.*Q.E.D.

Second proof by rationality & moral consistency (*DA*): Assume *Oa.* Since $a \Rightarrow falsum$ we find *Ofalsum.* Thus for some *p* we have *O(p & ~p).* But this means *Op & O~p,* and that is a preference inconsistency. Hence *~Oa.* Q.E.D.

First proof by realism (*AF)*: Assume *Oa.* By the lemma $Op \Rightarrow p$ we find *a*. But then we have *~a & a,* which is an inconsistency. Hence *~Oa.* Q.E.D.

Second proof by realism (*AF*): Since *~a* and above lemma $\sim a \Rightarrow \sim Oa$, hence *~Oa.* Thus the axioms are not morally desirable either. Q.E.D.

Rejection of the Arrow Reasonableness Claim (ARC)

Theorem II: For a reasonable society, the *ARC* is invalid.

Proof: Given *AF,* infeasible choices are not considered. Since *~a,* apparently *a* is not feasible, and the Arrow constitution is not reasonable. So it is invalid that the axioms would be reasonable. Q.E.D.

Discussion: No reasonable society in its right mind would want to accept Arrow's axioms as its constitution. Supposedly at a chaotic Boston Tea Party a constitution *c = a* might be tried, but pretty soon rational people would see that they should make another constitution, for otherwise the situation will remain chaotic, and the Tea Party will not go down into history as a notable event.

Note that Arrow adopts feasibility, but apparently wants to impose infeasible conditions.

Selection of the culprit axiom.

The selection of the culprit axiom is straightforward. We order the axioms by preference, for example *AD > AWP > AU > AIIA.* From *~a,* we conclude that we have to drop one of the axioms. We drop the least preferred one. My discussion on Condorcet's example should generate support for the rejection of *AIIA.* Basically though, scientists can only advise on preferences, and the proper decision is up to the body politic.

Note that ordering the axioms means that the deontic predicate *O* is not homogeneous. This means that deontic logic may be more related to preference theory than deontic theorists think.

Example consistent constitutions

Consistent constitutions violate one of the axioms of Arrow's Theorem. Violating one of these axioms is to be considered useful for reasonableness and morality, rather than the reverse. (That is what we proved above.)

One general feature is a Status Quo that persists when there are ties.

One example already has been mentioned in the discussion of the Condorcet problem. With majority voting, a cycle means indifference, and there are various ways to solve ties. One possible solution is the persistence of the Status Quo.

Another example constitution is the "Pareto-Majority" rule. One first selects all Paretian improvements from the Status Quo. That is, those points where some advance while nobody loses. There may be more Paretian points, such as $B > A$ and $C > A$, with the Status Quo as A. When there is no Paretian order between B and C, then it suffices to decide on these points by simple majority. Of course, with more than two points, majority voting can result into cycling, but that has been treated above.

See my home page http://www.can.nl/~cool for implementation of these rules in the program *Mathematica*. Little helps so much as trying it out for yourself.

Note that the Tobin quote above was misleading. The problem with 'unborn generations' should not be mixed up with the Arrow difficulty. Secondly, that problem of the 'unborn generations' actually can have a rather simple solution. It are the preferences of the currently living that matter, and what they prefer for the unborn ! The preferences of the unborn cannot logically be included.

Domain completeness and hysteresis

The issue may be enlightened a bit more by regarding domain completeness and hysteresis.

Above 'Example Constitutions' implicitly rely on the thought that the list of items on ballot is complete. This may be stated as a separate axiom:

DC domain completeness: the list of items on ballot is complete at the moment of decision

This property of completeness applies only to the moment when the votes are cast and the decision is made. The collective decision only concerns the items on ballot.

For example, regard the Condorcet situation again, above. Suppose that only items A and B are available, that B is the Status Quo. A majority decision will be that $A > B$, and then to select A. This decision can only be taken under the assumption of *DC*. There are at least two aspects on this:

1. To warrant completeness, good constitutions will allow all participants to put items on ballot. In this case, the Green party will favor C, and will put C on ballot. When all parties have put their favorite items on ballot, there is some guarantee that the ballot list is complete. (Research should highlight theorems on this issue.) With the list now complete, the choice of A is averted, and the Status Quo B will prevail.
2. Suppose that C is not put on ballot, perhaps because the Green Party did not think of it, or because it was too costly (the group adopts the axiom of feasibility). Suppose that A is selected and becomes the new Status Quo. Then, let C be discovered, or income rises and C becomes feasible. Casting votes again, it now appears that the group is indifferent about A, B and C. What to make of the situation ? Major points are:
    a) The axiom of completeness *DC* applies to each decision moment. So there is no inconsistency between the two results even though they differ.

b) A common result is that *A* remains chosen, since it is the Status Quo (i.e. the new Status Quo given the former decision).

c) With hindsight, it appears that the old Status Quo *B* could have prevailed, if *C* had been put on ballot sooner. This means that collective decisions can be subject to *hysteresis.*

Hysteresis means 'path-dependency'. Macro-economists have learned to reckon with hysteresis in the development of inflation, since it appears that economies that are subject to the same impulses may generate different results, depending upon the position they start from. In a similar way, collective decisions can be subject to path dependency too.

Note that this hysteresis is only caused by human practical limitation in real world situations. If the whole universe of all possible items were known and rated, then a complete collective index could be made. However, there are constraints of time and other resources, and hence the ballot list will be limited, causing the possibility of hysteresis. The 'paradoxes of voting' then also appear to be true paradoxes, i.e. seeming contradictions and not true contradictions, since the different decision outcomes are separated in time and place, and are conditional to practical circumstances. The objective for research now becomes to deal with time and resource constraints as effectively as possible, reducing hysteresis as much as possible.

Note too that the Arrow axiom *AIIA* tries to do two things at the same time: to solve the issue of completeness and to aggregate individual ratings. We separate these aspects: (a) an axiom of (local) domain completeness restricts attention to the items on ballot, and makes the collective choice independent of items not considered, (b) a pairwise aggregate result remains dependent on all items on ballot and not just the pair.

A reappraisal of the literature

Our discussion arrives at a conclusion that differs from the literature, and thus warrants a reappraisal of that literature. This reappraisal is not the topic of this paper, but some examples are useful.

1) Arrow 1951 also stated:

> "If consumers' values can be represented by a wide range of individual orderings, the doctrine of voters' sovereignty is incompatible with that of collective rationality."

This is clearly inaccurate. The statement suggests that we have to adopt Arrow's axioms, while the sensible thing is to reject these axioms and to adopt both voters' sovereignty and collective rationality.

2) One of the more interesting points made here is the distinction between the learning process and the end result. How should Arrow's result be presented in the future ? Is it possible to maintain the teaching strategy to call the axioms 'reasonable', then have the students get into a fixture, and them let them find a way out ? It is good teaching practice ! However, in a Palgrave meant for a wider audience (or a general encyclopedia that even might be read by dictators), it might be improper to call Arrow's axioms 'reasonable'. It should be 'seemingly reasonable' at the least.

Note that the phrase then becomes less enchanting:

> 'there is no social choice mechanism which satisfies a number of seemingly reasonable conditions'.

3) Interestingly, Goodman & Markowitz (1952) express the same intuition that my analysis started from. They say:

> "In this paper we argue that the Arrow postulates are not as plausible as they at first appear. (...) Suppose you intended to serve refreshments to two friends. You could serve them either coffee or tea but not both; *A* preferred coffee, *B* preferred tea. It seems clear that a symmetric ("democratic") welfare function would rank coffee and tea equally. (...)"

Sen (1979) discusses this paper, but concentrates on the solution approach and not the intuition.

4) I am a bit shocked by Mueller's (1989, p406-407) discussion of Arrow's general view. One would expect a more critical attitude, but finds instead:

> "The Arrow and Sen theorems (...) raise fundamental questions about the possibility of establishing collective choice procedures satisfying minimally appealing normative properties (...) But the negative side should not be overemphasized. We have suggested that both sorts of paradoxes might be avoided with the use of cardinal, interpersonally comparable utility information. Arrow explicitly eschewed the use of such information, and the independence of irrelevant alternatives axiom was imposed to rule out voting procedures that might make use of such information (... But it) is possible that the citizens may be trusted to make these comparisons in an ethically acceptable way."

Well, interpersonal comparison of course occurs, minimally, when we assign votes to people, assign rights to put topics on ballot, and the like. So interpersonal comparison is not as bad as many economists seem to think. But my solution to Arrow's difficulty does not rely on cardinality and cardinal comparison. So, disappointingly, Mueller both accepts the idea that Arrow would cause 'questions' about the possibility of social choice, and he comes with a wildly wrong conclusion. This is supposed to be a modern textbook !

5) What is important, is that the development of economic theory and the development of real economies have been hindered by the confusion generated by the standard explanation. Where decision makers were divided, some interested in social welfare and others not, the latter group was provided with decisive gunpowder - and beware of people who have an ideology and even wield a mathematical theorem to prove their lunacy. Generations of students have been taught by Nobel Prize laureats that research into social welfare would be subject to impossibilities. Creative energy has been directed to enlarging the impossibilities rather than to devising structures that might improve practical situations. Practical research into social choice functions and parameters has been aborted, all with reference to a misunderstood theorem.

Recently, I showed that economic research leads to a suggestion of a constitutional amendment, see Cool (1996). I hope that this present paper helps to clarify that this kind of research is a useful type of economics.

6) It is relevant to note that I gave this analysis earlier, in Cool (1990) and Cool (1992a). I have had no success so far in getting a publication in the main channels. This present paper is a rephrasing of the main principles. Thus, the quality of academic discussion was not just a problem in the 1950s but is in the 1990s too. Let us draw some lessons here.

Conclusion

Arrow's Theorem has given some problems in the literature, see the quotes above. We have achieved the following solution:

- There is more clarity now, by the distinction between the theorem proper ($a \Rightarrow falsum$), the moral claim (*Oa*) and the claim on reasonableness (*AF* and *K(~a)*).
- From a mathematical point of view, the Arrow axioms are incomplete for decision making in a reasonable society (i.e. issue incompleteness).
- It has been shown that the *AIIA* is undesirable. Dropping *AIIA* is not a sad state of affairs, as is sometimes suggested in the literature, but a sign of understanding group decision making.
- Hence, the Arrow axiomatisation does not capture the truely desirable properties required for a constitution, both by issue incompleteness and *AIIA.*
- We have clarified the relation between domain completeness and the phenomenon of hysteresis in practical decision making. We have clarified that *AIIA* tries to do too much at the same time.
- There are detail results, such as the distinction between voting and deciding, the integration of preference theory and deontic logic, and a proof of Arrow's Theorem that shows clearly the abuse by *AIIA.*
- We have given examples of consistent constitutions that might be seen as optimal by many.

Literature

Arrow (1950), “A difficulty in the concept of social welfare”, J. of political economy, pp328-346, reprinted in Arrow & Scitovsky (1969) pp147-168

Arrow (1951, 1963), “Social choice and individual values”, J. Wiley

Arrow & Scitovsky eds. (1969), “Readings in welfare economics”, Allen & Unwin

Bochenski (1956, 1970), “A history of formal logic”, 2nd edition, Chelsea

Cool (1990), “Why a social welfare (meta) function *does* exit: The Arrow Impossibility Theorem for Social Choice resolved, A better analysis suggested,” internal note Central Planning Bureau, The Hague, reprinted in Cool (1992b) pp57-104

Cool (1992a), “On the reasonableness and the moral desirability of the conditions of Arrow’s Theorem”, published in Cool (1992b) pp273-285

Cool (1992b), “Definition and Reality in the general theory of political economy, Some background papers 1989-1992”, Magnana Mu Publishing & Research, Rotterdam

Cool (1996), “A constitutional amendment for an Economic Supreme Court”, ewp-get/9604003

Dasgupta & Pearce (1980), “Cost-benefit analysis”, Macmillan

Goodman & Markowitz (1952), “Social welfare functions based on individual rankings”, American Journal of Sociology Vol LVIII, No 3, November, reprinted as Cowles Commission Papers New Series, No 67, Chicago

Johansen (1969), “An examination of the relevance of Kenneth Arrow’s general possibility theorem for economic planning”, Economics of planning 9, pp5-41

Jorgenson (1990), “Aggregate consumer behavior and the measurement of social welfare”, Econometrica 58 no 5 pp1007-1040

Luce & Raiffa (1957), “Games & decisions”, J. Wiley

Mueller (1989), “Public Choice II”, Cambridge

The New Palgrave, Macmillan 1988

Sen (1979), “Collective choice and social welfare”, North-Holland

Sen (1986), “Social choice theory,” p1073-1181 in Arrow & Intrilligator eds. (1986), “Handbook of mathematical economics, Volume III”, North Holland

Tobin (1990), “On the theory of macroeconomic policy”, Tinbergen Lecture, De economist 138 pp1-14