Differential impact of the minimum wage on exposed and sheltered sectors

Some sectors of the economy are exposed to and some are sheltered from foreign competition. These exposed and sheltered sectors are likely to have a different composition of their labour force, notably different rates of dependency on the minimum wage. If a national incomes policy does not respect these differences, a country can have both unemployment and a surplus on the trade account.

Thomas Cool, August 5 1996, Report no 96-09

Introduction

This paper originates from the problem that the Second Oil Crisis of 1979 caused for Holland. That second oil price hike aggravated the so-called “Dutch Disease” - under which name the problem has become known in the literature. Briefly put: when a nationally produced but internationally traded resource (Holland is rich in natural gas) has a price hike, then this causes the exchange rate to rise, and then this indirectly causes a reduction of the other exports and their industries and employment. Thus the original increase in national wealth paradoxically combines with an increase in unemployment.

The Dutch Disease may have an addendum on the policy reaction. If policy is not targetted at stabilisation of the exchange rate by monetary means and capital flows, but at tinkering with the labour market, then the situation can grow worse. The Dutch policy reaction to the Second Oil Crisis was a general restraint of wage growth. The presumption was that a relative reduction of production costs could compensate for the rise in the exchange rate, restoring competitivety and employment. However, in a brilliant exposition that has been neglected to the shame of the Dutch economics profession, Marein van Schaaijk (1983) of the Dutch Central Planning Bureau showed that a general wage restraint neglects the differential composition of labour factors, and with important effects in the exposed and sheltered sectors. He noted that the exposed sector is industrial and has the larger share of well educated, highly productive or high value added labour; while the sheltered sector concerns services and has the larger share of lowly educated, lowly productive or low value added labour. A uniform wage restraint is too high for the exposed sector and thus subsidises exports; and the restraint is too low for the sheltered sector and thus generates unemployment. Thus the phenomenon of a strong external position but internal unemployment is prolonged, but now by policy instead of the original energy price hike.

Van Schaaijk’s suggestion of the remedy was standard and sound. It was to let wages develop in line with productivity. Since Dutch policy is oriented to maintaining a more equal distribution of income - which explains part of the policy drive to see a *general* development in wages - Van Schaaijk advised to use tax policy to correct the differential development of gross wages for its effect on net incomes. However, Van Schaaijk's analysis has been neglected to this day, and Holland now suffers from a long period of unemployment *and* a trade surplus *and* a general restraint of wages and net incomes. [1]

Van Schaaijk's remedy was standard. But his analysis as a whole opened my eyes to the importance of tax policy for unemployment, and thereby led to my papers (Cool (1989-1996)) on the solution to the current mass unemployment in the OECD countries in general.

Note that I've always referred to Van Schaaijk's 1983 article whenever it was proper. However, in this paper I finally have occasion to more specifically combine his analysis with my own. This combination has been in my mind for a long time, but there was no time to develop it, as, in fact, this paper suffers from time constraints too.

We shall use a general equilibrium model where the exposed and sheltered sectors have differential factors of labour as in the Van Schaaijk observation. But now we take my analysis on the minimum wage, and let the minimum wage have the differential impact. This is more relevant for the OECD in general. Note, though, that I do not want to imply that all OECD countries have a trade surplus; other conditions are relevant here too, of course.

Due to lack of time we are forced to use a closed model. Thus we cannot reproduce the external imbalance. But we can reproduce the difference in reactions of the two sectors.

I wish to express my gratitude and respect to Asahi Noguchi (1994), who has created the *Mathematica* programs that we shall use in this exposition, see also Cool (1995e). The application program is added as an appendix. It can simply be pasted into a *Mathematica* notebook and be evaluated, provided that you have the packages that make it work.

Before we apply the general equilibrium model, let me briefly restate part of the analysis on the minimum wage.

The minimum wage

Let y be income, $T(y)$ be taxes, X exemption, and τ the statutory marginal rate. Then:

$$T(y) = \tau\ (y - X)$$

There is also subsistence level B. Under current rules of (European) welfare states, people who cannot earn that subsistence are entitled to a benefit of that level. This puts a restriction on taxes, since taxes $T(y)$ cannot be larger than taxable income $y - B$. The minimum wage M is determined by equality of these two terms, when taxes absorb all above B.

$$\tau\ (y - X) \leq y - B$$

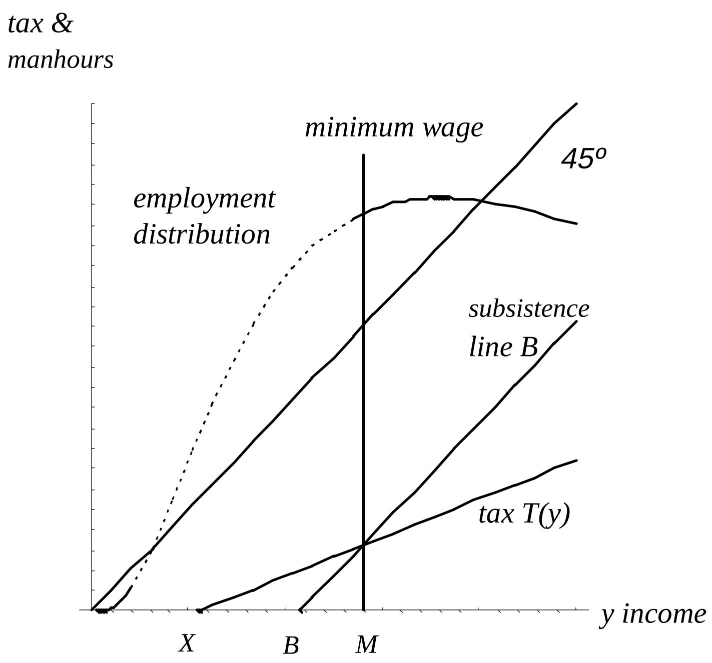
$$M = (B - \tau X) / (1 - \tau)$$

[1] It may be added that Van Schaaijk noted that restraint of incomes also means a restraints of imports, aggravating the situation. Note also the 'consistency' in the delusion with policy makers, that restraint is required to maintain employment by generating a trade surplus.

If $X = B$ then also $M = B$. Unfortunately tax policy makers seldomly choose $X = B$, and they thereby cause a difference between “taxable income $y - X$ according to the tax statute” and real taxable income $y - B$. Then the minimum wage lies above susbsistence, which makes many unemployed, and which both erodes the tax base and causes benefit outlays.

Figure 1 summarises the situation. The minimum wage is determined as the intersection of the tax line and the subsistence line parallel to the 45° line. One may note that the difference between the tax function and the 45° line gives net income, and that the subsistence line secures its part of net income. We also draw a lognormal employment distribution, which gives the number of people (manhours) at the various levels of income (productivity). The employment distribution is dotted below the minimum wage, since it is not observed there since those workers are unemployed. One can see that the tax code below the minimum wage has no real meaning, except that it needlessly drives up the minimum wage.

Figure A: Employment, tax and subsistence



As discussed elsewhere, exemption *X* and subsistence *B* are indexed on a different base. Exemption generally is indexed on inflation (OECD (1986)) while subsistence has a tendency to grow with the general level of welfare (see Aronson (1992) for “keeping up with the Jones’s” behaviour). This differential indexation causes a more than proportional growth of the minimum wage.

We may study the consequences by regarding situations *with* (1970-1990) and *without* (1950-1970) a binding minimum wage. Below, we give a model, tables and graphs.

Model

Regard a general equilibrium model with 20 units of highly productive labour (*h*), 70 units of lowly productive labour (*l*) and 10 units of minimum wage workers and possible benefit recipients (*b*). The economy has exposed and sheltered sectors that produce output x_E and x_S, while a social welfare function (SWF) determines the optimal combination. The SWF will here be a Constant Elasticity of Subsitution (CES) function that neglects the distribution of income:

$$SWF = \left(0.6\, x_S^{-0.667} + 0.4\, x_E^{-.0667}\right)^{-1.5} \quad (CES = .6)$$

Output of the sectors is determined by production functions that depend upon the allocation of the labour factors *b, h & l*. Here we take CES-functions, that have a constant elasticity of substitution between the labour factors:

$$x_E = \left(0.44\, h_E^{-1.5} + 0.56\ l_E^{-1.5} + .01 b_E^{-1.5}\right)^{-.667} \quad (CES = .4)$$

$$x_S = \left(0.34\, h_S^{.334} + 0.56\ l_S^{.334} + .10 b_S^{.334}\right)^{3} \quad (CES = 1.5)$$

The exposed sector is industrial, the labour factors are hardly substitutable, and due to the high standards of production, the personal productivity coefficient of the subsistence workers is low. The sheltered sector contains more services and the labour factors are very substitutable.

When wages are free, then the equilibrium and the optimum are found at an output of the exposed sector of 16.8 and an output of the sheltered sector of 23.2 (Table 1, first row). The associated allocation of the factors of production can be found in Table 2.

When there is a binding minimum wage, then the equilibrium and the optimum are found at an output of the exposed sector of 16.7 and an output of the sheltered sector of 19.9 (Table 1, second row). The associated allocation of the factors of production can be found in Table 3. (Though see the appendix on the use of the CES function.)

For prices and wages, the numeraire is the output price of the sheltered sector. For example, in the economy with free wages, the highly productive enjoy a wage of .80, the lowly productive have a wage of .34, and the subsistence workers get only .26 (see Table 2). [2]

One may note, then, that the coefficients have been chosen so that they resemble a real economy.

[2] The highly productive workers have a lower direct per person index of productivity (i.e. coefficients .34 and .44 versus twice .56) but they are relatively scarce, and thus command a higher price.

Tables

These tables give the outcomes of two scenarios, one *Without* a binding minimum wage (*M*), and one *With* such binding *M*. With a binding *M*, the subsistence workers *b* are unemployed and dependent on a benefit. Since they do not work, national income and social welfare is lower.

The coefficients in the production functions show that subsistence workers are more productive in the sheltered sector. When they drop out, real output of the sheltered sector drops markedly. Real output of the exposed sector is hardly affected. However, the output price of the exposed sector drops markedly too. Due to the relative scarcity of the product of the sheltered sector, it can demand a higher output price (Table 1). One can also check that the wage gap between the *h* and *l* workers increases (due to the relative scarcity of the first factor).

Table A: Utility, production and national income for two scenarios

	Utility level	*National income*	*Product prices Sheltered & exposed*		*Production Sheltered & exposed*	
Without *M*	20.2708	42.5931	1	1.14783	23.2939	16.8137
With *M*	18.588	34.9718	1	0.893309	19.9901	16.771

Note: all prices are scaled to the product price of the sheltered sector = 1

Table B: Allocation without a binding Minimum Wage

Allocation without M	*High*	*Low*	*Subsistence*
Labour years Exposed	13.5707	21.0348	4.64197
Labour years Sheltered	6.42933	48.9652	5.35803
Labour years Total	20	70	10
Wage	0.802042	0.341265	0.266374
National Income Share	0.376606	0.560854	0.0625392

Table C: Allocation with a binding Minimum Wage

Allocation with M	*High*	*Low*	*Subsistence*
Labour years Exposed	13.0314	19.8917	-
Labour years Sheltered	6.96861	50.1083	-
Labour years Total	20	70	-
Wage	0.686422	0.303476	-
National Income Share	0.392558	0.607442	-

Graphs

Figure 2 plots the production possibility curves and the SWF indifference maps of the two situations. One can see that mainly the sheltered sector is affected, with a reduction of its output.

Figure B: Production Possibility Curves & Indifference Maps

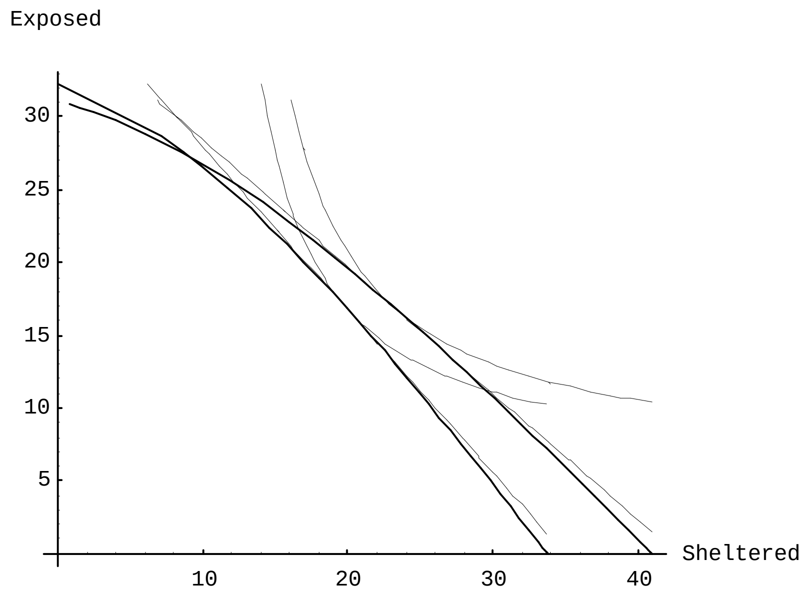
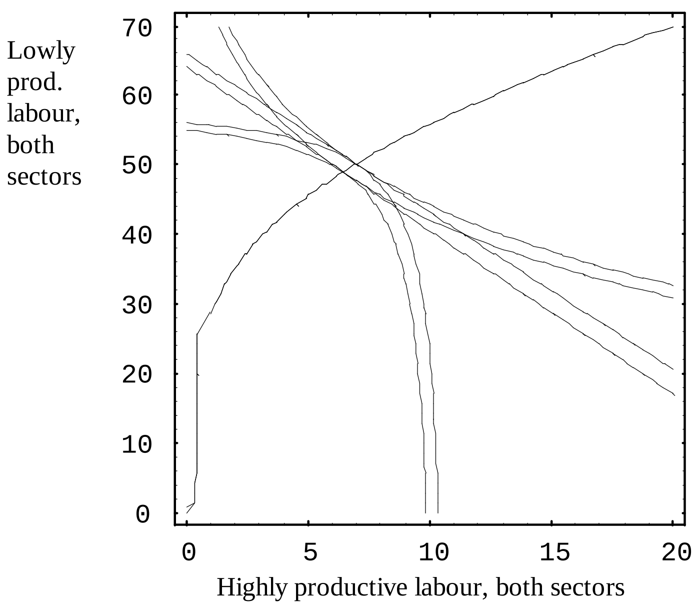


Figure 3 plots the Edgeworth-Bowley diagram for factors *h* and *l*. The movement is along the contract curve, and not a movement of the contractcurve.

Figure C: Edgeworth-Bowley Diagram



Conclusion

By proper choice of functions and parameters we have succeeded both in approximating a real economy and in reproducing the Van Schaaijk observation & analysis of the differential reaction of the exposed and sheltered sectors on incomes policy. Instead of having the mismatch caused by a general reduction of wage growth - as is typical for Holland - we have it caused by a minimum wage - as is more applicable for the OECD.

As Van Schaaijk did, we find that the sheltered sector loses most, and that it would be optimal to have wages reflect productivity. And similarly, this can be supported by tax policy.

Thomas Cool

PO Box 84399
NL 2508 AJ Scheveningen
Holland

Cool@CAN.NL

Http://www.can.nl/~cool

References

Aronson, E. (1992), "The social animal", Freeman

Asahi Noguchi (1993), "General Equilibrium models", in Varian (Ed), "Economic and financial modeling with Mathematica", Springer Telos 1993 pp 104-123

Cool, T. (1992a), "Definition and Reality in the general theory of political economy; Some background papers 1989-1992", Magnana Mu Publishing & Research (English & Dutch)

Cool, T. (1992b, 1995c), "On the political economy of employment in the welfare state", revised paper taken from (1992a) (ewp-mac/9509001) contains theorem & proof

Cool, T. (1994a, 1995a), "Tax structure, inflation and unemployment", Magnana Mu Publishing & Research (ewp-mac/9508002)

Cool, T. (1994b), "Trias Politica & Centraal Planbureau", Samuel van Houten Genootschap (Dutch)

Cool, T. (1995b), "How a dead wrong OECD tax policy causes mass unemployment: An explanation using data for Holland 1950-1995" (ewp-mac/9508003)

Cool, T. (1995d), "Belastingstructuur, inflatie en werkloosheid", in NAD/CBS (1996) "Nederlandse Arbeidsmarktdag 1995, congres rapport", pp 173-188 (Dutch)

Cool, T. (1995e), "Economics programs written in *Mathematica*" (ewp-prog/9508001)

Cool, T. (1996a), "Unemployment solved ! A breakthrough in economic analysis", ewp-get/9604002

OECD (1986), "An empirical analysis of changes in personal income taxes", Parijs

Schaaijk, M. van (1983), "Loondifferentiatie en werkloosheid", ("Wage differentiation and unemployment"), Economisch Statistische Berichten September 21

Appendix: A note on the use of CES production functions

There is a complication in using CES production functions. While factors are substitutable they still can be vital for production when CES ≤ 1. For example setting $b_E = 0$ in x_E actually causes production of the exposed sector to drop to zero too. (The negative power causes in infinite value that is inverted again by the outer negative value.)

To confess: for the situation with a binding minimum wage I have used a slightly different production function for the exposed sector, namely one with b_E fully dropped from the function. In a sense, I thus cheated. But did I really cheat ?

One can see the error by using the unchanged functions and then making a path where *b* gradually drops from the economy. Figures 4 and 5 give the results of this. Compare:

- balanced growth - all factors grow unconstrained at a rate of 2% for 20 years
- stunted growh - factors *h* and *l* grow at 2%, but the minimum wage has a gradual rise so that the number *b* employable at the current rate drops 5% per year.

Figure 4 shows equilibrium output (*XEq*) over time. The exposed sector starts form 16.8 and for the sheltered sector from 23.2. Balanced growth in the inputs generates balanced growth in the outputs. Stunted growth however reduces the growh paths, and markedly in the exposed sector.

Figure D: Stunted growth due to minimum wage

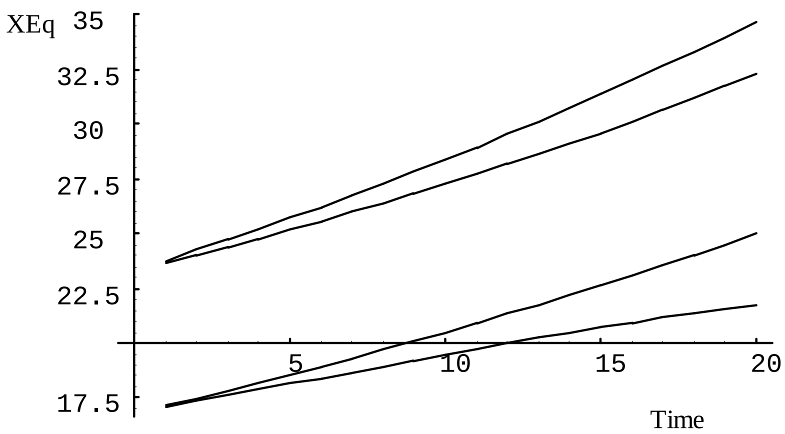


Figure E: Sloped growth due to minimum wage

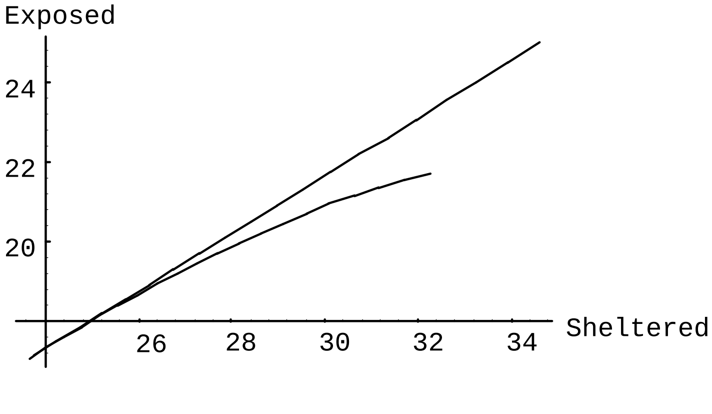


Figure 5 plots the result in the two-sector output-space. Balanced growth does not affect the ratio of the two outputs, and thus gives a ray through the origin. Stunted growth however affects total growth and the ratio between both sectors, with the exposed sector lagging behind.

The important observation is that these latter plots generate a different conclusion than we reached earlier. Namely, earlier it was the sheltered sector that was most affected.

These plots clarify the nature of the deceit in the main body of the text. These plots are caused by the assumption that we maintain a CES = .4 for the exposed sector, i.e. also including b_E so that minimum wage labour would be vital for the exposed sector. It is not likely however that minimum wage labour is vital for exports. Figure 4 and 5 thus misrepresent the actual argument.

I have run the program again for CES = 1.1 for the exposed sector. Indeed we then find that the sheltered sector is most affected, as is consistent with the actual argument.

However, the plots for CES = 1.1 are less clear from a didactic point of view. Therefor I maintain the CES = .4 presentation in the body of this text, and have cheated a bit by using an adapted function when minimum wage labour drops out.

Actually, my impression is now that we might use a CES in levels (nested CES), with the CES between *h* and l indeed .4, and the CES between this compound and minimum wage labour greater than 1. However, I lack the time to investigate this and rework this paper.

Appendix: The application program in *Mathematica*

The following program produces the figures and plots used in this article. It requires about 8 minutes on a 486 25 MH machine with 12 MB.

```
Needs["Cool`Common`"]
ResetAll
(*Windows note: The MyFindRoot.m package will have to be known
by the name Myfindro.m rather than Mfroot.m.*)
Contents["Cool`AGE`"]
SetOptions[Aggregator, Aggregator -> CES] (*default*)
(*
Sheltered & Exposed with 3 factors
*)
SetModel[NumberOfSectors -> 2, NumberOfFactors -> 3, IntermediatesQ -> False]
parsgrow[t_] := {Resources -> {20 * 1.02^t, 70 * 1.02^t , 10 * 1.02^t}}
parsdown[t_] := {Resources -> {20 * 1.02^t, 70 * 1.02^t , 10 * .95^t}}
pars =  {Utility ->  {Scale[Utility] -> 1, RTS[Utility] -> 1, S[Utility] -> .6,
                    FactorE[1] -> 0.6, FactorE[2] -> 0.4 },
        Production ->   {Sector[1] ->  {Scale[1] -> 1., RTS[1] -> 1, S[1] -> 1.5,
                                  FactorC[1, 1] -> 0.34, FactorC[1, 2] -> .56,
                                  FactorC[1, 3] ->  .1},
                         Sector[2] ->  {Scale[2] -> 1.05, RTS[2] -> 1, S[2] -> .4,
                                  FactorC[2, 1] -> 0.44, FactorC[2, 2] -> .56,
                                  FactorC[2, 3] ->  .01}} };
pars23 = pars ~Join~ parsgrow[0];
Assign[pars23]
al23 = Allocation[pars23, StartValues ->
```

```
            {FactorX[1, 1] -> 13,  FactorX[1, 2] -> 17,  FactorX[1, 3] -> 19,
             FactorX[2, 1] -> 17,   FactorX[2, 2] -> 20,  FactorX[2, 3] -> 19}]
eq = Equilibrium[pars23]
shares = (FactorPrices /. eq) * (Resources /. pars23) / (YEq /. eq)
cpc23 = CPCDiagram[pars23, AxesLabel -> {"Sheltered", "Exposed"},
                          AspectRatio -> Automatic]
ploteq1 = EdgeworthBowley[pars23, Factor -> {1, 2}, PlotPoints -> 50]
ploteq2 = EdgeworthBowley[pars23, Factor -> {1, 3}, PlotPoints -> 50]
ploteq3 = EdgeworthBowley[pars23, Factor -> {2, 3}, PlotPoints -> 50]
Path[ {XEq[1], XEq[2]}, Table[pars ~Join~ parsgrow[i], {i,1, 20, 1}]]
pathgrow = Results[Path];
Path[ {XEq[1], XEq[2]}, Table[pars ~Join~ parsdown[i], {i,1, 20, 1}]]
Show[Out[-1], Out[-3]]
pathdown = Results[Path];
paths = ( {XEq[1], XEq[2]} /. pathgrow) ~Join~ ( {XEq[1], XEq[2]} /. pathdown);
aid = ListPlot[#, PlotJoined -> True, DisplayFunction -> Identity]& /@ paths;
Show[aid, DisplayFunction -> $DisplayFunction]
(*
Sheltered & Exposed with 2 remaining factors
*)
SetModel[NumberOfSectors -> 2, NumberOfFactors -> 2,
IntermediatesQ -> False]
pars22 =  {Utility ->  {Scale[Utility] -> 1, RTS[Utility] -> 1, S[Utility] -> .6,
                 FactorE[1] -> 0.6, FactorE[2] -> 0.4 },
             Production ->  {Sector[1] -> {Scale[1] -> 1., RTS[1] -> 1, S[1] -> 1.5,
                                    FactorC[1, 1] -> 0.34, FactorC[1, 2] -> 0.56},
                          Sector[2] ->  {Scale[2] -> 1.05, RTS[2] -> 1, S[2] -> .4,
                                    FactorC[2, 1] -> 0.44, FactorC[2, 2] -> 0.56}},
            Resources -> {20, 70} };
Assign[pars22]
al22 = Allocation[pars22, StartValues ->
            {FactorX[1, 1] -> 13,  FactorX[1, 2] -> 17,
            FactorX[2, 1] -> 17,   FactorX[2, 2] -> 20}]
eq22 = Equilibrium[pars22]
shares22 = (FactorPrices /. eq22) * (Resources /. pars22) / (YEq /. eq22)
cpc22 = CPCDiagram[pars22, AxesLabel -> {"Sheltered", "Exposed"},
                          AspectRatio -> Automatic]
ploteq4 = EdgeworthBowley[pars22, Factor -> {1, 2}, PlotPoints -> 50]
Show[cpc23, cpc22]
Show[ploteq1, ploteq4]
```