ASYMMETRY, PERSISTENCE AND NON-LINEARITY OF SPANISH UNEMPLOYMENT RATES

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Abstract: The asymmetry or counter-cyclical nature and its influence on the persistence of the number of registered unemployed is one of the classic subjects of analysis in economic theory, which has not been tackled in the studies carried out on Spanish unemployment, focusing on demonstrating its long memory and generating macro-economic models with autoregressive vectors in which the unemployment variable is presumed to be non-stationary and co-integrated. Its consideration in this article, with smooth transition autoregressive (STAR) models, which detect the different existing regimes and the velocity of change, not only allows us to improve the short-term specification and forecast of the variable, but also it leads us to suppose, the same as Skalin and Teräsvirta (2002), a behaviour for unemployment which is locally non-stationary in a globally stable model.

Key Words: Persistence, asymmetry, non-linearity, smooth transition autoregressive models, time series, unemployment.

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1. INTRODUCTION

The most important characteristics of unemployment when analysing its evolution are, on one hand, its persistence in time, and on the other hand, its counter-cyclical nature, demonstrating accelerated growth in periods of recession and gradual reduction in periods of expansion.

The persistence of employment at the level attained is traditionally known as hysteresis (Lindbeck and Snower, 1985; Blanchard and Summers, 1987). In econometric terms, hysteresis in unemployment seems to assume a long memory, in the sense of unitary roots, and understands as a measurement of persistence the sum of the coefficients in an autoregressive process. One popular line of research has been to demonstrate the phenomenon of hysteresis or long-term persistence in unemployment rates checking the null hypothesis of the unit root, and measuring the dimension of the memory through the coefficients in an ARMA process (see Sachs, 1986 and Blanchard and Summers, 1990).

However, these studies do not take into account the characteristics associated with unemployment in the construction of the models, which suggest an estimate by non-linear procedures and which can significantly affect the measurement of the persistence of localized shocks. Bianchi and Zoega (1998) and Skalin and Teräsvirta (2002) even assume that if these peculiarities are taken into account, it can be affirmed that the unemployment rate is globally stationary but possibly non-linear and locally non-stationary.

Among the reasons that justify the non-linearity is the clear counter-cyclical nature of unemployment, increasing faster in periods of recession than it decreases in times of economic expansion. This result is consistent with the theory of asymmetric costs of hiring and firing of Bentolila and Bertola (1990) as well as the insider-outsider of Lindbeck and Snower (1988) who argue that in periods of economic expansion the hard core of workers will apply pressure to transfer in the form of increased salaries the increase in aggregate demand, instead of increasing the level of employment. This ability of the insiders to transfer the increase in demand to salaries is clearly connected to the concept of persistence and hysteresis, given that the pronounced aggregate shocks which displace demand for work can have persistent effects on salaries and employment, this persistence effect being more pronounced in countries with high rotation costs and strong union movements.

Persistence and asymmetry, hysteresis and *insiders-outsiders* are related concepts and theories which try to explain the causes of high rates of unemployment that exist in Europe and which have apparently become endemic in Spain in particular, with neither salary moderation nor expansive policies reducing them in any significant way.

It is these characteristics which make unemployment a variable which must be studied following models of non-linear temporal series. There are many works which have developed this line, notably the pioneering work of Neftçi (1984) in which he uses the series of unemployment as an indicator of the economic cycle analysing the asymmetries through a two-stage Markov chain. Parker and Rothman (1997), Rohtman (1998), Montgomery et al (1998), and van Dijk, Teräsvirta and Franses (2002) analyse unemployment in the United States with different non-linear models. Other authors such as Hansen (1997) have developed the statistical inference for the threshold parameter in a threshold autoregressive models (TAR). Bianchi and Zoega (1998) take up the study of multiple equilibriums and analyse their existence for the fifteen member states of the OECD unemployment series using a variant of the Markov switching models introduced by Lindgren (1978). Koop and Potter (1999) develop the logistic transformation of unemployment by applying Bayesian techniques to TAR models. Bränäs and Ohlsson (1999) generate formulas of temporary aggregation for average asymmetric autoregressive models (ARasMA) applying it to monthly and quarterly unemployment in Sweden. Finally, mention must be made of the work of Caner and Hansen (2001) and the modelling of the asymmetries of the unemployment rate for some OECD countries carried out by Skalin and Teräsvirta (2002).

In spite of the fact that many economic theories recommend taking into account the asymmetric nature of unemployment, these recommendations have not been heeded in the different studies carried out on the Spanish labour market. The macro-economic models traditionally employed are linear vector models in which the variable of unemployment is assumed to be non-stationary and co integrated with the other macro-economic variables. In this article, we attempt to demonstrate that the Spanish unemployment rate should be modelled taking into account its asymmetries following a smooth transition autoregressive model (STAR) which supposes that unemployment is stationary but possibly non-linear and with which we will obtain not only better adjustments but also enhanced forecast quality than with a traditional linear model.

The remainder of this paper is structured as follows: in section 2, we present a justification of the non-linear modelling of time series; in section 3 the modelling cycle is concentrated on non-linear models, presenting the principal results; in section 4 the methodology described is applied to time series data on unemployment in Spain. Finally, in section 5, demonstrating the principal conclusions reached by the use of applied economics.

2. JUSTIFICATION OF NON-LINEAR MODELLING OF TIME SERIES

The main reason for treating time series as non-linear has its origin in the different regimes or states which can occur in the dynamic behaviour of the economic variables over time. For each dynamic state of the time series we will see a differentiated behaviour from the average, variance or autocorrelation.

This behaviour is often reflected in economic theory. Some examples of non-linearity take in the economic processes with thresholds, capacity constraints restricting production, persistent disequilibria due to rationing, institutional restrictions such as tax brackets, multiple equilibriums and asymmetries of different types, such as the cyclical fluctuation of employment or unemployment due to asymmetries hiring and firing costs.

If the non-linearity which economic theory assumes has been verified through the study of real data, it is to be expected that the estimated equations are also nonlinear. However, the vast majority of econometric models are linear, in many cases because the core equations have been replaced by a linear approximation which is considered sufficient in practice. Another reason for using linear equations is the desire to avoid "incredible theories" -Sims (1980) – and to carry out the modelling with the least number of theoretical assumptions possible. Finally, the great development of statistical theory for linear models has allowed a steady advance of modelling strategies based on linearity.

On the other hand, linear models have not completely escaped criticism. The usual rejection of the constancy of the parameters, indicative of the so-called structural breakdown has been corrected by rejecting the model or amending it with dummy variables. But the behaviour of an economy, a market, a business or a family is too complicated to be adequately described by a series of linear equations, therefore a breakdown of the same is to be expected over time. The frequent breakdowns that occur in some series and the necessity to apply various groups of variables of change cast doubt on the credibility of the model, being a more satisfactory response the application of specifications which seek modelling of the different states or regimes which the series produces throughout its dynamic behaviour.

There is no doubt that the problems encountered in the linear models as well as the spectacular growth in available computing power have been instrumental in the rise in popularity of non-linear econometrics, but it has been accompanied by an explosion in the number of studies in the last twenty years, which, from a non-linear perspective have dealt with the principal topics and difficulties of classic econometrics.

There is a wide variety of models of variable regime. A detailed revision of the existing through the published bibliography available to date can be found in Casado (2003)[1].

In spite of the large number of existing models, they can be divided into two groups. The first is formed by those that assume that the different behaviours or regimes can be determined by an observable variable and as such can be fixed with absolute certainty. The second group of models contains those whose initial hypothesis is that the different states are not currently observable, but they are determined by an unobservable stochastic process.

In the following sections we will concentrate on the interpretation, specification, linearity test, estimation, evaluation and prediction of those models whose variable of breakdown is observable, concentrating our analysis on those which suppose a smooth transition between the different regimes (STAR) and complete the modelling cycle for non-linear time series, recommended by Teräsvirta (1994).

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[1] Although not intended to be exhaustive, we cite the following as among the models considered in the literature: *Threshold Autoregressive* (TAR), see Tong (1978, 1990); *Self Exciting TAR* (SETAR), see Tong and Lim (1980*); Smooth Transition Autoregresive* (STAR), see Chang and Tong (1986), *Logistic STAR* (LSTAR), see Teräsvisrta and Anderson (1992), *Exponential STAR* (ESTAR), seeTeräsvisrta (1994), *Multiple Regime STAR* (MRSTAR), see van Dijk and Franses (1999), *Time-Varying STAR* (TVSTAR), see Granger and Teräsvirta (1999); *Smooth Transition Error-Correction Model* (STECM), see Granger and Swanson (1996); *Fractionally Integrated STAR* (FI-STAR), see Van Dijk, Franses and Paap (2002).

3. THE MODELLING CYCLE IN THE ANALYSIS OF NON-LINEAR TIME SERIES

In the same way the procedure by stages suggested by Box and Jenkins (1970) for the modelling of linear time series, Granger (1993) recommends using the procedure "from specific to general" when we consider the use of models of non-linear time series, to describe the characteristics of a specific variable. This means starting with a simple and restrictive model, and proceeding with the elaboration of a more complex one only if the tests and diagnoses indicate a lack of suitability of the approach. The fundamentals of the modelling cycle were initially proposed by Teräsvirta (1994), consisting of, see figure 1, the following stages.

1. Specification of the autoregressive model of the order p for the time series object of the study, using appropriate criteria in the selection of models.
2. Elaboration of linearity tests.
3. If linearity is rejected, selection of the transition variable s_t and of the functional form of the transition function $G(s_t;\gamma,c)$.
4. Estimation of the parameters of the chosen model.
5. Evaluation of the model, using the mechanisms of specific validation for non-linear models.
6. Modification of the model as necessary.
7. Exploitation of the model to descriptive ends. Prediction and impulse-response analysis.

(Insert Figure 1)

Stages 2 to 7 will be discussed in detail in the following sections, but first we will present a brief introduction to the characteristics and specification of the smooth transition autoregressive models most frequently used.

3.1. Interpretation and specification of STAR models

The smooth transition autoregressive model (STAR) for a univariant time series y_t, whick is observed at $t = 1-p, 1-(p-1), \ldots, -1, 0, 1, \ldots\ldots, T-1, T$, is defined as:

$$y_t = (\phi_{1,0} + \phi_{1,1} y_{t-1} + \ldots + \phi_{1,p} y_{t-p})(1 - G(s_t;\gamma,c)) + \\ + (\phi_{2,0} + \phi_{2,1} y_{t-1} + \ldots + \phi_{2,p} y_{t-p}) G(s_t;\gamma,c) + \varepsilon_t, \quad t = 1,2,\ldots,T \quad (1)$$

which we express, in reduced form, as:

$$y_t = \phi_1' x_t (1 - G(s_t;\gamma,c)) + \phi_2' x_t G(s_t;\gamma,c) + \varepsilon_t \qquad (2)$$

where $x_t = (1, \tilde{x}_1')'$ with $\tilde{x}_t = (y_{t-1}, ..., y_{t-p})'$ and $\phi_i = (\phi_{i,0}, \phi_{i,1}, ..., \phi_{i,p})'$, where $i = 1,2$ for the cases of the existence of two regimes. Where the random disturbances ε_t are independent of the history of the series up to the period $t-1$, which is expressed as $E(\varepsilon_t / \Omega_{t-1}) = 0$, where $\Omega_{t-1} = \{y_{t-1}, y_{t-2}, ..., y_{1-(p-1)}, y_{1-p}\}$ and we assume that the conditional variance of ε_t is constant, $E(\varepsilon_t^2 / \Omega_{t-1}) = \sigma^2$.

The transition function $G(s_t; \gamma, c)$, is a continuous function, which takes values in the interval (0,1). It can adopt different functional forms, and the transition variable, s_t is usually the lagged endogenous variable, $s_t = y_{t-d}$ for certain integer $d > 0$. However, the transition variable can also be an exogenous variable $(s_t = z_t)$, see van Dijk and Franses (2000), or a linear time trend, $s_t = t$ (see Lin and Teräsvirta, 1994).

The interpretation of the STAR models can be analysed from two standpoints. On one hand, as models of two variable regimes, each one associated with an extreme value of the transition function $G(s_t; \gamma, c) = 0$ y $G(s_t; \gamma, c) = 1$, where the transfer of one model to the other is smooth, or on the other hand, as a "continuous" regime with different values of $G(s_t; \gamma, c)$ which move in the interval (0,1).

The functional forms which the transition function can adopt are different, but we will centre our study on two. One popular selection is the first-order logistic function:

$$G(s_t; \gamma, c) = \left(1 + exp\{-\gamma(s_t - c)\}\right)^{-1}, \quad \gamma > 0 \tag{3}$$

In this case the STAR model becomes known as LSTAR (*Logistic STAR*). The parameter c in (3) must be interpreted as the threshold points between the two regimes given that the function $G(s_t; \gamma, c)$ equals 0.5 when $s_t = c$. The parameter γ measures the smoothness in the speed of change from one regime to another. When γ takes high values the transition velocity is high, and the logistic function $G(s_t; \gamma, c)$ approaches the indicator function $I[s_t > c]$, defined as $I[A] = 1$ if A is true and $I[A] = 0$ in the rest of the cases and so the change of $G(s_t; \gamma, c)$ from 0 to 1 is instantaneous in which case the LSTAR model becomes an autoregressive double regime model (TAR) whenever $s_t = c$ and in a *self-exciting TAR* (SETAR) model when $s_t = y_{t-d}$. In the opposite case, that is to say, when γ tends to zero the logistic function makes the model change regime smoothly, the limit being that in which $\gamma = 0$ (the function $G(s_t; \gamma, c) = 0.5$) and the LSTAR model is reduced to an AR with parameters $\phi_j = \frac{(\phi_{1,j} + \phi_{2,j})}{2}$, $j = 0,1, ..., p$.

In some cases, it is more appropriate to specify the transition function, such that the regimes are associated with small and large absolute values of s_t. This can be achieved by using exponential function (ESTAR, exponential smooth transition autoregressive model), which is defined as,

$$G(s_t;\gamma,c)=1-exp\left\{-\gamma(s_t-c)^2\right\} \quad \gamma>0 \qquad (4)$$

3.2. Tests of linearity in STAR models

Before constructing a non-linear model it is advisable to check that if a linear model is really appropriate to characterize economic relationships that are the subject of analysis. But as we will demonstrate later, the tests of linearity are also useful when it comes to specifying the type of non-linear model, as well as in the validation phase, as it is recommendable to subject the residuals of the specified model to the said test, to verify that the non-linear characteristics of the original series have been captured adequately.

The null hypothesis of linearity can be expressed as the equality of the autoregressive parameters of both regimes of the STAR model (2), that is:

$$\begin{aligned} &H_0:\phi_1=\phi_2 \\ &H_1:\phi_1\neq\phi_2 \end{aligned} \qquad (5)$$

The carrying out of this test is complicated by the presence of unidentified nuisance parameters under the null hypothesis. The void hypothesis does not limit the values of the parameters of the transition function, γ y c, when the null hypothesis is true, and the likelihood is affected by the presence of the values of both measurements. This problem of non-identification in the linearity tests can be reformulated more clearly from the following contrast.

$$\begin{aligned} &H_0':\gamma=0 \\ &H_1':\gamma\neq 0 \end{aligned} \qquad (6)$$

which gives rise to a linear model.

This inconvenience was dealt with by Luukkonen et al (1988a), whose proposal for the solution was to substitute the transition function $G(s_t;\gamma,c)$ for a Taylor series approximation. In the reparametrized equation, the problem of identification is not present and the linearity can be checked with Lagrange multiplier statistic which follows asymptotically a distribution χ^2 under the null hypothesis.

Starting from the LSTAR model:

$$y_t=\phi_1'x_t+(\phi_2-\phi_1)'x_tG(s_t;\gamma,c)+\varepsilon_t \qquad (7)$$

and assuming that $\varepsilon_t \sim N.I.D(0,\sigma^2)$, Luukkonen et al. (1988a) advise approximating the logistic function $G(s_t;\gamma,c)=\left(1+exp\left\{-\gamma(s_t-c)\right\}\right)^{-1}, \gamma>0,$ with a first-order Taylor approximation around $\gamma=0$, giving as a result the auxiliary regression:

$$y_t=\beta_0'x_t+\beta_1'x_ts_t+e_t \qquad (8)$$

where $\beta_i = (\beta_{i,0}, \beta_{i,1}, ..., \beta_{i,p})'$, i = 0,1; $e_t = \varepsilon_t + (\phi_2 - \phi_1)' x_t R_1(s_t; \gamma, c)$, being $R_1(s_t; \gamma, c)$ the remainder term of the Taylor expansion. Under the null hypothesis $R_1(s_t; \gamma, c) \equiv 0$ and $e_t = \varepsilon_t$ and the remainder does not affect the properties of the random disturbances. The parameters $\beta_i,\ i = 0,1$ in the regression (8) carry out the function of parameters of the model (7), so that $\gamma = 0$ implies $\beta_{0,j} \neq 0$ and $\beta_{1,j} \neq 0$ for $j = 0,....,p$. Hence, testing the null hypothesis $H_0' : \gamma = 0$ in (7) is equivalent ot testing the null hypothesis[2]

$$\begin{aligned} H_0'' &: \beta_1 = 0 \\ H_1'' &: \beta_1 \neq 0 \end{aligned} \tag{9}$$

in (8). The statistical test to contrast this restriction, designated LM_1, follows an asymptotic distribution χ^2 with $p+1$ degrees of freedom under the null hypothesis of linearity.

Luukkonen et al. (1988b) shows that the test LM_1 does not have sufficient power in the cases in which only the intercept differs across regimes of the STAR model, that is to say:

$$\begin{aligned} &\phi_{1,0} \neq \phi_{2,0} \quad \text{and} \\ &\phi_{1,j} = \phi_{2,j} \qquad j = 1,...,p \end{aligned} \tag{10}$$

developing for its solution the test LM_3 with an approximation of $G(s_t; \gamma, c)$ generated with a third-order Taylor expansion whose auxiliary regression remains specified as:

$$y_t = \beta_0' x_t + \beta_1' x_t s_t + \beta_2' x_t s_t^2 + \beta_3' x_t s_t^3 + e_t \tag{11}$$

where $e_t = \varepsilon_t + (\phi_2 - \phi_1)' x_t R_3(s_t; \gamma, c)$ and again $\beta_{0,0}$ y $\beta_i, i = 1,2,3$ act as the parameters $\phi_{1,\phi}, \phi_{2,\phi}, \gamma$ and c now allowing the expression of the la hypothesis $H_0' : \gamma = 0$ as:

$$\begin{aligned} &H_0'' : \beta_1 = \beta_2 = \beta_3 = 0 \\ &H_1'' : \text{At least one } \beta_i = 0 \end{aligned} \tag{12}$$

Under the null hypothesis, the LM test follows an asymptotic distribution χ^2 with $3(p+1)$ degrees of freedom.

The expression of $\beta_i, i = 1,2,3$, in (11) in terms of ϕ_1, ϕ_2, γ and c is such that only the parameters $\beta_{1,0}, \beta_{2,0}$ y $\beta_{3,0}$ represent the intercepts $\phi_{1,0}$ y $\phi_{2,0}$. A more parsimonious version of the estimator LM_3 can be obtained by increasing the auxiliary estimation (8) with regressors such as s_t^2 y s_t^3, so that:

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[2] The order *p*, number of lagged in the LM models of linearity contrast, are the result of the specification, estimation and validation of a linear autorregressive model(AR) previously using appropriate criteria of selection of the models.

$$y_t = \beta_0^{'} x_t + \beta_1^{'} x_t s_t + \beta_{2,0} s_t^2 + \beta_{3,0} s_t^3 + e_t \qquad (13)$$

the test is carried out:

$$\begin{aligned} H_0'' &: \beta_1 = 0 \\ & \beta_{2,0} = \beta_{3,0} = 0 \\ H_1'' &: \text{At least one } \beta_i \neq 0 \end{aligned} \qquad (14)$$

The resulting statistical test is called LM_3^e which follows asymptotically a distribution χ^2 with $p+3$ degrees of freedom under the null hypothesis of linearity.

The tests of linearity for ESTAR models were initially expounded by Saikkonen and Luukkonen (1988) suggesting the checking of the regression

$$y_t = \beta_0' x_t + \beta_1' x_t s_t + \beta_2' x_t s_t^2 + e_t \qquad (15)$$

where $e_t = \varepsilon_t + (\phi_2 - \phi_1)' x_t R_2(s_t; \gamma, c)$.

This equation is obtained from the first order Taylor expansion for the expression (2) using as an expression of the transition variable (4), where the restriction $\gamma = 0$, is analysed with the hypothesis,

$$\begin{aligned} H_0'' &: \beta_1 = \beta_2 = 0 \\ H_1'' &: \text{al menos un } \beta_i \neq 0 \end{aligned} \qquad (16)$$

The statistical LM_2, follows asymptotically, under the null hypothesis, a distribution χ^2 with $2(p+1)$ degrees of freedom.

Finally, Escribano and Jordá (1999) show that the first-order Taylor approximation for the exponential function is not sufficient to capture both points of inflexion of this function and they suggest a second-order Taylor expansion,

$$y_t = \beta_0' x_t + \beta_1' x_t s_t + \beta_2' x_t s_t^2 + \beta_3' x_t s_t^3 + \beta_4' x_t s_t^4 + e_t \qquad (17)$$

As in previous cases, here the hypothesis is contrasted

$$\begin{aligned} H_0'' &: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0 \\ H_1'' &: \text{al menos un } \beta_i \neq 0 \end{aligned} \qquad (18)$$

In this final case, the LM_4 test follows an asymptotic distribution χ^2 with $4(p+1)$ degrees of freedom under the null hypothesis.

Before finalizing this brief revision of the linearity tests in STAR models we must refer to the fact that these tests, in the cases where there are problems of heteroscedasticity or the presence of outliers is large, they are not efficient. In these circumstances, therefore, it is necessary to adopt a robust specification of the LM tests.

Wooldridge (1990) has designed the specification of the tests which are to be used in the presence of heteroscedasticity, without the need to specify the functional form that the said heteroscedasticity follows. An example for the test LM_3 would be to develop the following steps:

- Regress y_t in x_t and obtain the residuals $\hat{\varepsilon}_t$.
- Regress the auxiliary regressors $x_t s_t^i, i = 1,2,3,$ over x_t and calculate the residuals $\hat{r}_t$.
- Regress 1 on $\hat{\varepsilon}_t \hat{r}_t$. The explained sum of squares from this regression is the $LM-type$ statistic.

In the case of the presence of outliers, as in the case for heteroscedasticity we present, by way of an example, the procedure of elaboration of the robust estimator for the statistical LM_3, from the result of nR^2 where the coefficient of determination R^2 is the derivative of the regression of the weighted residuals $\hat{w}_r(\hat{r}_t)\hat{r}_t$ over the regressors $\hat{w}_x(x_t) \otimes (x_t', x_t' s_t, x_t' s_t^2, x_t' s_t^3)'$, where $\otimes$ denotes element-by-element multiplication. The weighted of the residuals $\hat{w}_r(\hat{r}_t)$ and of the regressors $\hat{w}_x(x_t)$, is obtained from a model $AR(p)$ under the null hypothesis. The resulting statistical LM asymptotically follows a distribution χ^2 with $3(p+1)$ degrees of freedom.

3.3. Estimation

Once the linearity has been rejected, it is necessary as a step previous to the process of estimation, to select the transition s_t, as well as the most appropriate functional form of the transition function $G(s_t;\gamma,c)$ (in our case LSTAR vs. ESTAR).

A traditional way of selecting the transition variable has been from the p-value of the statistical LM_3, as the ESTAR specification of (15) in (11) is integrated, the variable s_t can be selected independently of the functional form of $G(s_t;\gamma,c)$. This procedure suggests that the appropriate transition variable of the STAR model can be determined by computing the statistical LM_3 for the transition variables candidates selecting that in which the p-value is the smallest. The rationality of this procedure, confirmed in Teräsvirta (1994), is based on the fact that the tests should be more powerful in the cases in which the alternative model is correctly specified, and this is, when we have used the appropriate transition variable.

Once linearity has been rejected and the transition variable has been chosen, but before proceeding with the estimation, we must select the functional form of the transition function $G(s_t;\gamma,c)$. Traditionally, two tests are employed. On one hand that proposed by Teräsvirta (1994), based on the contrast of a sequence of nested void hypotheses, $H_{03}: \beta_3 = 0$, $H_{02}: \beta_2 = 0 / \beta_3 = 0$ and $H_{01}: \beta_1 = 0 / \beta_3 = \beta_2 = 0$ over the regression (11) selecting a ESTAR model if the *p-value* of H_{02} is lower and an LSTAR model in the rest of cases. On the other hand, Escribano and Jordá (1999) proposed a test based on the checking of two separate hypotheses carried out from the expression (17):

$$H_{0E} : \beta_2 = \beta_4 = 0$$
$$H_{0L} : \beta_1 = \beta_3 = 0 \qquad (19)$$

and comparing the *p-values* of both estimators. The LSTAR will be chosen if the minimum *p-value* is obtained for H_{0L}.

Once the variable and the transition function have been selected, the next phase in the modelling cycle described in figure 1 is the estimation of the parameters of the STAR model through a procedure of non-linear least squares (NLS)[3], where the aim is to obtain the parameters $\theta = (\phi_1', \phi_2', \gamma, c)'$ through the minimization of,

$$\hat{\theta} = \underset{\theta}{arg\,min}\, Q_T(\theta) = \underset{\theta}{arg\,min} \sum_{t=1}^{T} (y_t - F(x_t;\theta))^2 \qquad (20)$$

where $F(x_t;\theta)$ is the skeleton of the non-linear model, that is to say,

$$F(x_t;\theta) = \phi_1' x_t (1 - G(s_t;\gamma,c)) + \phi_2' x_t G(s_t;\gamma,c). \qquad (21)$$

Wooldridge (1994) and Pötscher and Prucha (1997) showed that non-linear least squares generate consistent and asymptotically normal estimators,

$$\sqrt{T}(\hat{\theta} - \theta_0) \to N(0,C) \qquad (22)$$

where θ_0 is the true value of the parameter.

The asymptotic covariance matrix C of $\hat{\theta}$ can be estimated consistently by the product of $\hat{A}_t^{-1}\hat{B}_T\hat{A}_t^{-1}$, where $\hat{A}_T$ is the Hessiano of $\hat{\theta}$, that is:

$$\hat{A}_T = -\frac{1}{T}\sum_{t=1}^{T} \nabla^2 q_t(\hat{\theta}) = \frac{1}{T}\sum_{t=1}^{T} (\nabla F(x_t;\hat{\theta}) \nabla F(x_t;\hat{\theta})' - \nabla^2 F(x_t;\hat{\theta})\hat{\varepsilon}) \qquad (23)$$

with $q_t(\hat{\theta}) = (y_t - F(x_t;\hat{\theta}))^2$ y $\hat{B}_T$:

$$\hat{B}_T = \frac{1}{T}\sum_{t=1}^{T} \nabla q_t(\hat{\theta}) \nabla q_t(\hat{\theta})' = \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t^2 \nabla F(x_t;\hat{\theta}) \nabla F(x_t;\hat{\theta})' \qquad (24)$$

The estimation of the parameters can be carried out by different types of logarithms of convergence for nonlinear optimisation procedures, for example those developed in Hamilton (1994) or Hendry (1995).

3.4. EVALUATION

Before the STAR model is admitted as valid, it must undergo an exhaustive evaluation, which must include, as well as the classic steps of linear models, other specific tests for non-linear models such as those of non-autocorrelation, remaining non-linearity or constancy of the parameters.

[3] Under the hypothesis that the random disturbances are distributed normally, non-linear least squares (NLS) is equivalent to maximum liklihood and so NLS can be interpreted as a quasi-maximum likelihood estimation.

(a) Residual autocorrelation Tests.

From the STAR model considered in (2) and from the functional form of the skeleton of the model (21) a qth -order LM test for the serial dependence of ε_t can be obtained as nR^2, where R^2 is the coefficient of determination of the regression of $\hat{\varepsilon}_t$ in

$$\nabla F(x_t;\hat{\theta}) = \partial F(x_t;\hat{\theta})/\partial\theta \tag{25}$$

Being $\theta = (\phi_1', \phi_2', \gamma, c)'$ and q the lagged of the residuals $\hat{\varepsilon}_{t-1},....,\hat{\varepsilon}_{t-q}$, where the circumflex symbols indicate that they have been estimated under the null hypothesis of serial independence of ε_t. The result of the statistic, is distributed asymptotically as a χ^2 with q degrees of freedom.

(b) Remaining non-linearity test

Up to now we have considered that non-linearity assumes la existence of two regimes. On the other hand, the general configuration of the STAR models allows the existence of various multiple regime models which contain sub-models with different speeds of adjustment and breakdown points.

The STAR models which incorporate more than one regime must be classified in two groups, in one those in which each state is characterized by a single transition variable s_t, and in the other group those in which a combination of various variables $s_{1t},...,s_{mt}$ determine the regime in which we find ourselves. In the first case we must rewrite (2) in order to obtain a three-regime model as follows:

$$y_t = \phi_1' x_t + (\phi_2 - \phi_1)' x_t G_1(s_t;\gamma_1,c_1) + (\phi_3 - \phi_2)' x_t G_2(s_t;\gamma_2,c_2) + \varepsilon_t \tag{26}$$

If we assume that $c_1 < c_2$, the autoregressive parameters in this model change slowly from ϕ_1 through ϕ_2 to ϕ_3 before increment in the value of s_t, when the first of the functions G_1 changes from 0 to 1, followed by similar changes of G_2. In a more general way the definition is possible of a group of $m-1$ parameters of smooth transition autoregressive model, $\gamma_1,...,\gamma_{m-1}$ and a group of $m-1$ parameters of localization $c_1,...,c_{m-1}$ to achieve a STAR model with m regime of change,

$$y_t = \phi_1' x_t + (\phi_2 - \phi_1)' x_t G_1(s_t) + ... + (\phi_m - \phi_{m-1})' x_t G_{m-1}(s_t) + \varepsilon_t \tag{27}$$

where $G_j(s_t) \equiv G_j(s_t;\gamma_j,c_j), j = 1,...,m-1$ can adopt the form of the logistic function (3) becoming (6) in a SETAR multiple regime model when the parameter of transition velocity γ, takes high values.

The formalization of the STAR model for the case in which the regimes are determined by a combination of different variables can be carried out by "encapsulating" two double regime STAR models in the following way:

$$y_t = \left\{ \left[\phi_1' x_t (1 - G_1(s_{1t};\gamma_1, c_1)) \right] + \phi_2' x_t (G_1(s_{1t};\gamma_1, c_1)) \right\} \left[1 - G_2(s_{2t};\gamma_2, c_2) \right] +$$
$$+ \left\{ \left[\phi_{31}' x_t (1 - G_1(s_{1t};\gamma_1, c_1)) \right] + \phi_4' x_t (G_1(s_{1t};\gamma_1, c_1)) \right\} \left[G_2(s_{2t};\gamma_2, c_2) \right] + \varepsilon_t \quad (28)$$

where the relation between y_t and its lagged is given by the linear combination of four AR models, each one associated with a particular combination of $G_1(s_{1t})$ and $G_2(s_{2t})$ which take values between 0 and 1.

The MRSTAR, see van Dijk and Franses (1999) takes into account a maximum of one model with four different regimes, but by repeatedly applying the principle of encapsulating, the model can be extended to 2^m regimes.

Eitrheim and Teräsvirta (1996) develop a statistical LM to check the existence of a double regime LSTAR against that specified in (26), where the null hypothesis of the existence of a double regime model equals

$$H_0' : \gamma_2 = 0 \ \text{ ó } \ H_0' : \phi_3 = \phi_2 \quad (29)$$

As in the linearity tests considered, these tests present problems unidentified parameters, a question which is again resolved, by specifying a Taylor expansion following a third order approximation to replace the transition function $G_2(s_t;\gamma_2, c_2)$ with its approximation, to be expressed as:

$$y_t = \beta_0' x_t + (\phi_2 - \phi_1)' x_t G_1(s_t;\gamma_1, c_1) + \beta_1' x_t s_t + \beta_1' x_t s_t^2 + \beta_1' x_t s_t^3 + e_t \quad (30)$$

where $\beta_i, i = 0,1,2,3,$ are functions of the parameters $\phi_1, \phi_2, \phi_3, \gamma_2$ y c. In such a way that the hypothesis (29) in (26) can be identified, considering la the expression (30), with:

$$\begin{array}{l} H_0'' : \beta_1 = \beta_2 = \beta_3 = 0 \\ \text{At least one } \beta_i \neq 0 \end{array} \quad (31)$$

The resulting statistical test traditionally defined as $LM_{AMR,3}$ follows an asymptotic distribution χ^2 with $3(p+1)$ degrees of freedom.

(C) Parameter Constancy Test

Non-linearity is one of the many characteristics of time series data. However, another of the most common peculiarities is the existence of structural changes. Over time both have been seen as different ways of analysing the same reality, however, recent years have seen the design of a new typology of non-linear models, called time-varying STAR models (TVSTAR) which include both characteristics.

TVSTAR models, assume that y_t follow a STAR model all the time, but with a smooth change in the autoregressive parameters in both regimes, that is to say from $\phi_1 \leftrightarrow \phi_2$, to $\phi_3 \leftrightarrow \phi_4$, for $G\ (s_{1t};\gamma_1, c_1) = 0$ and $G\ (s_{1t};\gamma_1, c_1) = 1$, respectively.

Its general expression is as follows:

$$y_t = \phi_1(t)' x_t (1 - G_1(s_{1t}; \gamma_1, c_1)) + \phi_2(t)' x_t (G_1(s_{1t}; \gamma_1, c_1)) + \varepsilon_t \qquad (32)$$

being: $\phi_1(t) = \phi_1 [1 - G_2(t; \gamma_2, c_2)] + \phi_3 G_2(t; \gamma_2, c_2)$

$\phi_2(t) = \phi_2 [1 - G_2(t; \gamma_2, c_2)] + \phi_4 G_2(t; \gamma_2, c_2)$

Therefore, contrasting $\gamma_2 = 0$ in (32) we contrast the constancy of the parameters in a double regime STAR model. The corresponding LM test, obtained from a third-order Taylor approximation of $G_2(t; \gamma_2, c_2)$, called $LM_{C,3}$ and follows an asymptotic distribution χ^2 with $6(p+1)$ degrees of freedom.

3.5. Forecast

Non-linear models can be used for various purposes. Sometimes the principal objective is simply to obtain an adequate description of the dynamic behaviour that a certain variable shows. But frequently, an additional objective is to use the model to forecast future values of the time series, and this forecast is used to evaluate the quality of the STAR model, comparing it with a traditional linear model.

We will consider the case where y_t is explained by a STAR model with $s_t = y_{t-1}$:

$$y_t = F(x_t; \theta) + \varepsilon_t \qquad (33)$$

where the skeleton of the model is formed by:

$$F(x_t; \theta) = \phi_1' x_t (1 - G(y_{t-1}; \gamma, c)) + \phi_2' x_t G(y_{t-1}; \gamma, c) \qquad (34)$$

being $x_t = (1, y_{t-1}, ..., y_{t-p})'$

The point forecast of y_{t+h} for the period t is given by its conditional average, and must be expressed for a forecast horizon h as:

$$\hat{y}_{t+h/t} = E[y_{t+h} / \Omega_t] \qquad (35)$$

where Ω_t represents the history of the time series up to the present moment, including the observation of the period t.

From (35), and taking into account that $E[e_{t+1} / \Omega_t] = 0$, the forecast with a horizon equal to 1, can be obtained as:

$$\hat{y}_{t+1/t} = E[y_{t+1} / \Omega_t] = F(y_t; \theta) \qquad (36)$$

But when the forecast horizon is greater than 1, the point forecast becomes complicated. Let us suppose the case in which $h = 2$:

$$\hat{y}_{t+2/t} = E[y_{t+2} / \Omega_t] = E[F(y_{t+1}; \theta) / \Omega_t] \qquad (37)$$

In this case, the expect value cannot be interchanged with the non-linear operator $F(\bullet)$, that is to say:

$$E[F(\bullet)] \neq F(E[\bullet]) \qquad (38)$$

So, the expected value of the non-linear function is not equal to the calculated function of the expected value, that is: $E[F(y_{t+1};\theta)/\Omega_t] \neq F(E[y_{t+1}/\Omega_t];\theta)$.

When the forecast horizon is greater than a period, the expression of $\hat{y}_{t+h/t}$ is not possible, given that the expression $E[y_{t+h}/\Omega_t]$ requires a dimension $h-1$. Although at first numerical integration techniques were proposed to solve this problem, in recent years the methods of Monte Carlo and the iterative bootstrap have become standard in approximating the expected value of $E[y_{t+h}/\Omega_t]$, calculating the Monte Carlo prediction for a forecast horizon of order 2 as,

$$\hat{y}_{t+2/t}^{mc} = \frac{1}{k}\sum_{i=1}^{k} F\left(\hat{x}_{t+2/t}^{(i)};\theta\right) \tag{39}$$

where k is a high value and the values of ε_{t+1} in $\left(\hat{x}_{t+2/t}^{(i)}\right)$ are obtained from the supposed distribution of ε_t.

The forecast obtained by the iterative method is very similar and the only difference is that ε_{t+1} is obtained by the substitution of the residuals of the estimated model $\hat{\varepsilon}_t, t = 1,....,T.$ having the advantage that it is not necessary to assume any type of distribution for ε_t. Another advantage of the Monte Carlo and iterative methods is that $F\left(\hat{x}_{t+2/t}^{(i)};\theta\right)$ constitutes the form of the forecast density function, allowing its use in the construction of interval forecasts.

Up to now we have assumed the parameters of the STAR model are known, but in practice they are estimated and this adds an additional degree of uncertainty to the estimation which can be absorbed by adding to the Monte Carlo or Iterative methods a mean component on the different values of the parameters. That is, obtaining:

$$\hat{y}_{t+2/t}^{mc} = \frac{1}{kr}\sum_{i=1}^{k}\sum_{j=1}^{r} F\left(\hat{x}_{t+2/t}^{(i)};\theta^{(j)}\right) \tag{40}$$

where $\theta^{(j)}$ is obtained from the distribution of the estimated parameters $\hat{\theta}$.

3.6. Impulse Response Functions

The impulse response functions used in the linear models, are defined as the difference between the realizations of y_{t+h} defined for an identical history of the time series, $t-1$, which we denote as, w_{t-1}. In the first, the process is subjected a shock of dimension δ at the moment t, while the second is not "shocked" by any impulse. All the shocks in the intermediate periods between t and $t+h$, are equal to zero in both realizations, therefore we can represent the traditional impulse response functions (*TI*) as:

$$TI_y(h,\delta,\omega_{t-1}) = E\left[y_{t+h} \middle| \varepsilon_t = \delta, \varepsilon_{t+1} = ... = \varepsilon_{t+h} = 0, \omega_{t-1}\right] - \\ - E\left[y_{t+h} \middle| \varepsilon_t = 0, \varepsilon_{t+1} = ... = \varepsilon_{t+h} = 0, \omega_{t-1}\right] \tag{41}$$

for $h = 1,2,3,...$

Symmetry and historical independence are the properties of the functions *TI* in linear time series. The first supposes that a shock of dimension, $-\delta$ has exactly the opposite effect to a shock of dimension $+\delta$, and the response is proportional to the dimension of the shock. The second eliminates the dependence of the impact in relation to the particular history ω_{t-1}. Both properties are easily observable in the *TI* of a process AR (1).

$$TI_y(h,\delta,\varpi_{t-1}) = \phi_1^h\delta, \quad h = 0,1,2,... \tag{42}$$

On the other hand, if we make the *TI* extendable to non-linear models, we observe that some of these properties are not maintained. Thus, assuming a simple non-linear model (SETAR):

$$y_t = \begin{cases} \phi_{1,1}y_{t-1} + \varepsilon_t, & si \ \ y_{t-1} \le 0 \\ \phi_{1,2}y_{t-1} + \varepsilon_t, & si \ \ y_{t-1} > 0 \end{cases} \tag{43}$$

obtaining:

$$TI_y(1,\delta,\omega_{t-1}) = \begin{cases} \phi_{1,1}\delta & si \ \ y_{t-1}+\delta \le 0, \quad y_{t-1} \le 0 \\ \phi_{1,1}\delta + \delta_{1,2}(\phi_{1,1}-\phi_{1,2})y_{t-1} & si \ \ y_{t-1}+\delta \le 0, \quad y_{t-1} > 0 \\ \phi_{1,2}\delta + \delta_{1,1}(\phi_{1,2}-\phi_{1,1})y_{t-1} & si \ \ y_{t-1}+\delta > 0, \quad y_{t-1} \le 0 \\ \phi_{1,2}\delta & si \ \ y_{t-1}+\delta > 0, \quad y_{t-1} > 0 \end{cases} \tag{44}$$

From (44) it can be deduced clearly that in non-linear models, the response to a shock depends on the sign and dimension of the shock, and on the history of the process. So neither the symmetry nor the historic dependence will be properties of the non-linear models, as they have different values for $y_{t-1}+\delta > 0$ y $y_{t-1}-\delta \le 0$.

As the impulse response functions are a good tool to consider the dynamic behaviour of the estimated non-linear models and given the peculiarities of the same, Koop et al (1996) designed a useful generalization of the concept of impulse response functions for non-linear models defined as *GI*, which is defined, for a specific shock $\varepsilon_t = \delta$ and a history ω_{t-1}, as:

$$GI_y(h,\delta,\omega_{t-1}) = E[y_{t+h}|\varepsilon_t = \delta,\omega_{t-1}] - E[y_{t+h}|\omega_{t-1}] \tag{45}$$

for $h = 0,1,2,\ldots$

In the *GI*, the expectancy of y_{t+h} given for a shock δ occurring in t is conditioned by the history and the shock, being a function of δ and ω_{t-1} that can be understood as different results of the random variable ε_t and Ω_{t-1}. Koop et al (1996) point out that $GI_y(h,\delta,\omega_{t-1})$ is in itself a succession of random variables, defined as:

$$GI_y(h,\delta,\Omega_{t-1}) = E[y_{t+h}|\varepsilon_t,\Omega_{t-1}] - E[y_{t+h}|\Omega_{t-1}] \tag{46}$$

From this general interpretation of GI as a random variable, various conditioned versions can be considered to be of interest. Only one particular history can be taken into account ω_{t-1} considering the *GI* as a random variable in terms of ε_t:

$$GI_y(h,\varepsilon_t,\omega_{t-1}) = E[y_{t+h}|\varepsilon_t,\omega_{t-1}] - E[y_{t+h}|\omega_{t-1}] \quad (47)$$

or, on the other hand, determine for the different histories the *GI* for a determined shock a priori $\varepsilon_t = \delta$, considering it as a random variable in terms of the history Ω_{t-1}.

In practice, it is usual to calculate the GI conditioned to a particular combination of (A and B) shocks and histories respectively $GI_y(h,A,B)$. For example for all the histories in a particular regime and with negative shocks.

Finally Koop et al (1996) show us, that in the case of the STAR models, the analytical expression for the conditional expectancy of GI when $h > 1$, is not available, and so we must use stochastic simulation in order to obtain the value of the different responses as we will see in the empirical development.

It is perhaps the measurement of the persistence, that is the greatest of the capacities which explain the impulse response functions. We must remember that a shock $\varepsilon_t = \delta$ is transitive for a history ω_{t-1} if $GI_y(h,\delta,\omega_{t-1})$ tends to 0 when h tends to infinity, in the opposite, we say that the shock is persistent. Potter (1995) and Koop et al (1996) suggest that the dispersion of the distribution $GI_y(h,\varepsilon_t,\Omega_{t-1})$ in a finite horizon can be interpreted as a measurement of the persistence of the shocks. This allows us to compare and measure densities (persistence) of GIs conditioned to types of shocks (negative-positive) taking into account at the same time the differences generated by the regimes of the specified STAR model.

4. MODELLING SPANISH UNEMPLOYMENT. A NON-LINEAR APPROACH

The study of Spanish unemployment must take into account three characteristics of the series, which can be clearly seen in figure 2, which represents the unemployment rate registered by INEM (National Employment Institute) in the period January 1982 to December 2002.

(Insert figure 2)

- The asymmetric nature with respect to the economic cycle. Accelerated growth in periods of recession and slow and moderate decrease in periods of expansion, characteristic which should be captured with models which contemplate differentiated regimes and counter-cyclical behaviours.
- Persistence. Although traditionally related, following the concept of hysteresis, with the non-stationary condition, and checked with unitary root tests, Skalin and Teräsvirta (2002) check by applying the Dickey-Fuller test to different simulated non-linear series which the models of regime change do not provide additional evidence against the

hypothesis of unitary root compared with the corresponding linear model. So the search for unitary roots on the one hand and the non-linearity (asymmetry) on the other, using the same data, should not necessarily be seen as a contradiction. Therefore in this analysis we follow the focus of Bianchi and Zoega (1998) and Skalin and Teräsvirta (2002) supposing that the rate of unemployment is globally stationary but possibly non-linear and locally non-stationary.

- However, the specification STAR models in (1) does not contemplate another of the observable characteristics in figure 2, the mentioned seasonal behaviour, (observable in the graphs in figures 3 and 4) showing greater values in autumn and winter (November-February) than at the end of the summer months (July - October). Although some works assume that the seasonal component changes over time, the study of the seasonal characteristic of the series in our case, shows us that it is not subject to change (figure 4) during the periods of expansion/recession and so must be reflected in the non-linear model using monthly dummy variables, $D_{s,t}, s=1,...,11,$ where $D_{s,t}=1$ if the observation t corresponds to the month s and $D_{s,t}=0$ for the remaining cases, which are not influenced by the cycles of recession/expansion.[4]

(Insert figures 3 and 4)

Using the sample from January 1985 to analyse the series described, and reserving the last three years for forecast, we begin the modelling cycle, described in the previous section, specifying an autoregressive linear model. Carrying out behaviour from specific to general, we observe that both the AIC and the BIC indicated that a model AR (4) was the most appropriate. On the other hand its parsimonious behaviour and the existence of autocorrelation in the residuals has led us to increase the lagged of autoregressive model to 13, eliminating those variables whose parameter were not significant. Thus, we have obtained:

$$\begin{aligned}\Delta y_t = & -\underset{(0.061)}{0.122} - \underset{(0.003)}{0.002}\, y_{t-1} + \underset{(0.071)}{0.337}\,\Delta y_{t-1} + \underset{(0.069)}{0.213}\,\Delta y_{t-2} + \underset{(0.068)}{0.168}\,\Delta y_{t-3} - \underset{(0.059)}{0.163}\,\Delta y_{t-5} + \\ & + \underset{(0.060)}{0.153}\,\Delta y_{t-8} + \underset{(0.065)}{0.237}\,\Delta y_{t-12} - \underset{(0.067)}{0.093}\,\Delta y_{t-13} + \underset{(0.050)}{0.284}\, D_{1,t} + \underset{(0.049)}{0.168}\, D_{2,t} + \\ & + \underset{(0.056)}{0.144}\, D_{3,t} - \underset{(0.044)}{0.047}\, D_{4,t} - \underset{(0.044)}{0.048}\, D_{5,t} + \underset{(0.048)}{0.167}\, D_{6,t} + \underset{(0.049)}{0.292}\, D_{8,t} + \\ & + \underset{(0.054)}{0.401}\, D_{9,t} + \underset{(0.056)}{0.321}\, D_{10,t} + \underset{(0.044)}{0.191}\, D_{11,t} + \hat{\varepsilon}_t\end{aligned}$$

$\hat{\sigma}_e = 0.1091, SK = 0.1241(0.2478)_p, EK = 3.5300(0.0727)_p, JB = 2.5830(0.2745)_p,$

$LM_P(1) = 0.9845, LM_P(4) = 0.6616, LM_P(8) = 0.7036, LM_P(12) = 0.2736,$

$ARCH_P(1) = 0.2484, ARCH_P(2) = 0.4730, AIC = -4.220, BIC = -3.884$

p = Probability Value

[4] It has been checked by applying the contrast by Hylleberg et al (1990) the determinist nature of the seasonal component of Spanish unemployment, being demonstrated the convenience of dummy variables as instruments for expressing the seasonal component. (Franses, 1996).

The values in parentheses are the residual standard deviations obtained by ordinary minimum least squares of the estimated parameters, and $\hat{\varepsilon}_t$ the residuals of the regression at the moment t, $\hat{\sigma}_{\varepsilon}$ the typical deviations of the residuals, SK the asymmetry, EK the level of kurtosis, JB the Lomnichi-Jarque-Bera residual normality test, $ARCH(q)$ the LM test of the non-existence of conditional autoregressive heteroscedasticity of the order q and $LM(p)$ the Breusch-Godfrey test of the non-existence of autocorrelation of the order p.

The linear model shows adequate behaviour of the residuals, given that they are not correlated, and there is no conditional autoregressive heteroscedasticity in them. There is no significant asymmetry, but there is a slight kurtosis which does not become relevant when it comes to accepting the normality with test JB.

The following stage is to check the linearity against the STAR using the statistical LM tests explained in section 3.2 (see table 1). As our interest is centred on the behaviour of unemployment with respect to the economic cycle, we must use a transition variable of the STAR model, which reflects the properties of the periods of expansion and recession. Therefore the monthly change in the unemployment rate is an inappropriate variable as a proxy of the economic cycle given that it reflects the seasonal fluctuations in the unemployment rate. Following Skalin and Teräsvirta (2002), we will use the twelfth difference as a variable $s_t = \Delta_{12} y_{t-d} \equiv y_{t-d} - y_{t-d-12}$ selecting six as the maximum value for d.

(Insert Table 1)

Firstly, focusing on the standard tests, we observe that for the contrast LM_1 and LM_3^e, any of the levels can be selected as a transition variable at 5% but that is not the case of the tests LM_3 and LM_4, for which it would seem more appropriate to use any of the first three. On the other hand, the general result of the tests seems to indicate a clear non-linear behaviour of the series, and this decision is not influenced by the presence of outliers, because the robust tests of non-linearity would lead us to adopt the same conclusion.

In a third stage, and once linearity has been rejected, we select the transition variable s_t and the functional form of transition $G(s_t;\gamma,c)$. It is the first seasonal difference, $\Delta_{12} y_{t-1}$, which demonstrates a lower value of the probability in the statistical LM_3. With respect to the optimum specification and basing ourselves on the Teräsvirta (1994) rule of decision, the three variants of the test suggest an LSTAR model as the most suitable considering any of the transition variables, and the Escribano-Jordá statistics confirm the decision taken.

(Insert Table 2)

Therefore until now and with the combination of the tables 1 and 2, the variable $\Delta_{12} y_{t-1}$ has been selected as the transition variable of a LSTAR model. On the other hand, and with the aim of selecting the most appropriate, we have estimated different LSTAR models for $s_t = \Delta_{12} y_{t-d}$, being d = 1, 2, 3, obtaining in the different stages of evaluation and forecast the best results for $d = 1$ and therefore confirming the variable selected by the traditional procedures.

Starting from a thirteenth order autoregressive model for each regime, we carry out the process of the estimation of the LSTAR, obtaining:

$$\begin{aligned}
\Delta y_t =& \underset{(0.056)}{-0.006} + \underset{(0.038)}{0.202} D_{1,t} - \underset{(0.039)}{0.050} D_{3,t} - \underset{(0.039)}{0.231} D_{4,t} - \underset{(0.042)}{0.234} D_{5,t} - \underset{(0.043)}{0.157} D_{7,t} + \\
&+ \underset{(0.045)}{0.160} D_{8,t} + \underset{(0.050)}{0.381} D_{9,t} + \underset{(0.050)}{0.288} D_{10,t} + \underset{(0.048)}{0.190} D_{11,t} + \Big[- \underset{(0.004)}{0.007} y_{t-1} + \underset{(0.098)}{0.238} \Delta y_{t-2} + \\
&+ \underset{(0.099)}{0.110} \Delta y_{t-4} + \underset{(0.096)}{0.149} \Delta y_{t-8} - \underset{(0.084)}{0.167} \Delta y_{t-9} - \underset{(0.093)}{0.101} \Delta y_{t-10} + \underset{(0.077)}{0.165} \Delta y_{t-12} \Big] \cdot \\
&\cdot \left[1 - G(\Delta_{12} y_{t-1}; \hat{\gamma}, \hat{c})\right] + \Big[\underset{(0.003)}{0.002} y_{t-1} + \underset{(0.073)}{0.372} \Delta y_{t-1} + \underset{(0.078)}{0.132} \Delta y_{t-2} + \underset{(0.082)}{0.163} \Delta y_{t-3} - \\
&- \underset{(0.071)}{0.105} \Delta y_{t-8} - \underset{(0.070)}{0.089} \Delta y_{t-12} - \underset{(0.073)}{0.098} \Delta y_{t-12} \Big] G(\Delta_{12} y_{t-1}; \hat{\gamma}, \hat{c}) + \hat{\varepsilon}_t
\end{aligned}$$

$$G(\Delta_{12} y_{t-1}; \hat{\gamma}, \hat{c}) = \left(1 + exp\left\{4.743(\Delta_{12} y_{t-1} - 0.668) / \hat{\sigma}_{\Delta_{12} y_{t-1}}\right\}\right)^{-1}$$

$$\begin{aligned}
&\hat{\sigma}_e = 0.1034, \hat{\sigma}(LSTAR/AR) = 0.94, SK = 0.049(0.3929)_p, EK = 3.393(0.1404)_p, \\
&JB = 1.236(0.5390)_p, LM_P(1) = 0.8376, LM_P(4) = 0.4041, LM_P(8) = 0.4255, \\
&LM_P(12) = 0.2144, ARCH_P(1) = 0.3542, ARCH_P(2) = 0.3435, AIC = -4.251, \\
&BIC = -3.79
\end{aligned}$$

$p =$ Valor de la probabilidad

Where $\hat{\sigma}(LSTAR/AR)$ represents the ratio of the typical deviation of the residuals of the LSTAR model against the autoregressive one, reducing 6% the deviation of the LSTAR with respect to the AR. Also this reduction is sufficient to compensate the increase in the number of parameters (from 20 to 26) because the LSTAR is preferred over the AR with respect to the AIC, although we cannot say the same with respect to the BIC. The decrease of the asymmetry and the excess of kurtosis, leads us to accept with greater intensity the normality of the residuals. The LM and ARCH tests do not reject the null hypothesis for any of the lagged analysed.

Figures 5 and 6 represent, respectively, the residuals of the LSTAR model corresponding to the series, which are the object of our study and the Kernel distribution function of the residuals of the estimated LSTAR model.

(Insert Figures 5 and 6)

We conclude the LSTAR evaluation stage with the tests of autocorrelation, remaining non-linearity constancy of the parameters (see table 3), validating the selected model.

(Insert Table 3)

The transition function $G(\Delta_{12}y_{t-1};\hat{\gamma},\hat{c})$ estimated against the transition variable $\Delta_{12}y_{t-1}$ is shown in figure 7. The estimation of $\hat{\gamma}$ and $\hat{c}$, shows that the transition of the logistic function from 0 to1 takes place for values of $\Delta_{12}y_{t-1}$ between -2 y 0.5, with moderate transition velocity and equal to 4.74 and the point of inflexion of the transition is $\Delta_{12}y_{t-1} = -0.668$.

(Insert Figure 7)

The last three years of the time series, from January 2000 to December 2002, were reserved to evaluate the forecast quality of the estimated AR and LSTAR model. For each moment of time from January 2000 and until January 2002 we computed the forecast from 1 to 12 horizons both for the AR model and for the LSTAR. The latter have been calculated using the iterative bootstrap method specified in section 3.5.

The forecast has been carried out statically, without updating the estimated parameters when a new observation was available. Table 4 contains the evaluation of the forecast making use of some of the most generalized criteria. From the MPE criteria, mean prediction error, the LSTAR model, in spite of being pessimistic with respect to the reduction of unemployment, because its forecast value is higher than the actual value, presents a lesser deviation than the autoregressive one for a short time horizon (1 and 2 periods). On the other hand the AR model is more constant in the quality of the forecast and less influenced by the time dimension in which it is measured. Comparing the MSPE, (mean squared prediction error), of both models we reach a very similar conclusion, given that although the quality of the LSTAR model is higher in the short term it is more recommendable to use the linear for long term horizons.

(Insert Table 4)

To know the dynamic properties of the estimated STAR model, we analyse the behaviour of the model with different shocks, through the creation of generalized impulse response functions specified in 3.6. Calculate the GIs, as defined in (45), taking into account the sample size of the estimated model for different normalized shocks $\delta/\hat{\sigma}_{\varepsilon} = \pm 3, \pm 2.9, \ldots, \pm 0.2, \pm 0.1,$ where $\hat{\sigma}_{\varepsilon}$ represents the typical mean deviation of the estimated residuals in the specified LSTAR model. For each combination of history (180) and shocks (60) calculate a $GI_{\Delta y}(h,\delta,\omega_{t-1})$ with a horizon of $h = 0,1,\ldots,60$. The conditional expectancy of (45) is estimated as the mean of 1000 operations of Δy_{t+h}, obtained through iterations of the specified LSTAR model, with and without the use of the selected shock and using randomly samples of estimated residuals in the specified non-linear model. The impulse responses for the unemployment variable at levels is obtained by the aggregation of the impulse responses in first differences, that is:

$$GI_y(h,\delta,\omega_{t-1}) = \sum_{i=0}^{h} GI_{\Delta y}(i,\delta,\omega_{t-1}).$$

The different GIs are calculated for A shocks and B histories, $GI_y(h, A, B)$ respectively, where A represents the total of all the positive and negative shocks, and B the histories for which the estimated transition function $G(\Delta_{12}y_{t-1};\gamma,c)$ is greater (recession) or less (expansion) than 0.5.

Finally the densities are calculated with a Kernel Nadaraya-Watson estimator, using $\phi(\delta / \hat{\sigma}_{\varepsilon})$ to weight $GI_y(h,\delta,\omega_{t-1})$ where $\phi(z)$ represents the typical distribution of normal probability, and with the HDR (Highest Density Regions) using the methodology typified in Hansen (1996).

(Insert Figure 9)

Figure 9 shows the HDRs for the distributions of $GI_y(h, A, B)$ with horizons h=0,3,6,…,60, showing how the shocks which occur during the recessions tend to reach their peak response in the sixth month. On the other hand in the periods of expansion the shocks do not reach their peak values until a year has passed. In general, the effect of the negative shocks is less than that of the positive ones, which would lead us to demonstrate the limited power of employment creation policies and the rigidity of the labour market, independently of the moment of the economic cycle in which we find ourselves. Also we must highlight the lesser asymmetry of the impacts in periods of recession compared to those occurring in periods of expansion.

Finally, one of the fundamental reasons for this work can be seen by comparing the graphs, and that is that employment creation policies if implemented in times of recession, have a far lesser effect than if in periods of expansion a shock occurs which affects its evolution, thus demonstrating the counter-cyclical nature of unemployment.

5. CONCLUSIONS

The Spanish unemployment rate and in general the labour market is, as we said in the introduction, a variable which must be analysed taking into account its peculiarities. Its asymmetrical behaviour, increasing more rapidly in periods of recession than it decreases in periods of expansion, as has been demonstrated in this work, two totally differentiated behaviours of the series, recommending the use of non-linear models for its specification

The demonstration in this work that Spanish unemployment has a completely different behaviour in expansion and recession periods requires that the decision-making bodies, active and passive unemployment policies differentiated in function of the phase of the economic cycle in which we find ourselves. The design of long-term employment policies and independent of the current situation of the economy, are not only inefficient but they prejudice the expected behaviour of the labour market. We must bear in mind that we are not faced with the same pattern in which only the positive/negative evolution varies but as can be observed from the estimated model, there are two totally differentiated autoregressive models.

The correct non-linear specification of the series, has not only been confirmed, through linearity tests, but also in both the evaluation of the estimated model and in the short-term forecast. A substantial improvement has been demonstrated in all the contrasts carried out with respect to the linear model, highlighting the suitability of selecting two regimes. Structural permanence (rendering unnecessary estimation by TVSTAR procedures), the non-existence of autocorrelation or the absence of conditional autoregressive heteroscedasticity, are some of the optimum qualities that the non-linear model offers.

Analysing the dynamic behaviour of the estimated STAR model with the impulse-response functions, the counter-cyclical and asymmetric nature of Spanish unemployment is confirmed, and the limited capacity of employment creation policies due to the rigidity of the Spanish labour market is also demonstrated.

Finally we should highlight the fact that the results obtained for Spanish unemployment do not differ from those found in other countries, although it must be pointed out that, following the article by van Dijk, Teräsvirta and Franses (2002) the greater effect of the responses that employment policies have in the United States, demonstrating the relative rigidity of the Spanish labour market and the smaller margin for manoeuvre of the employment policies in Spain.

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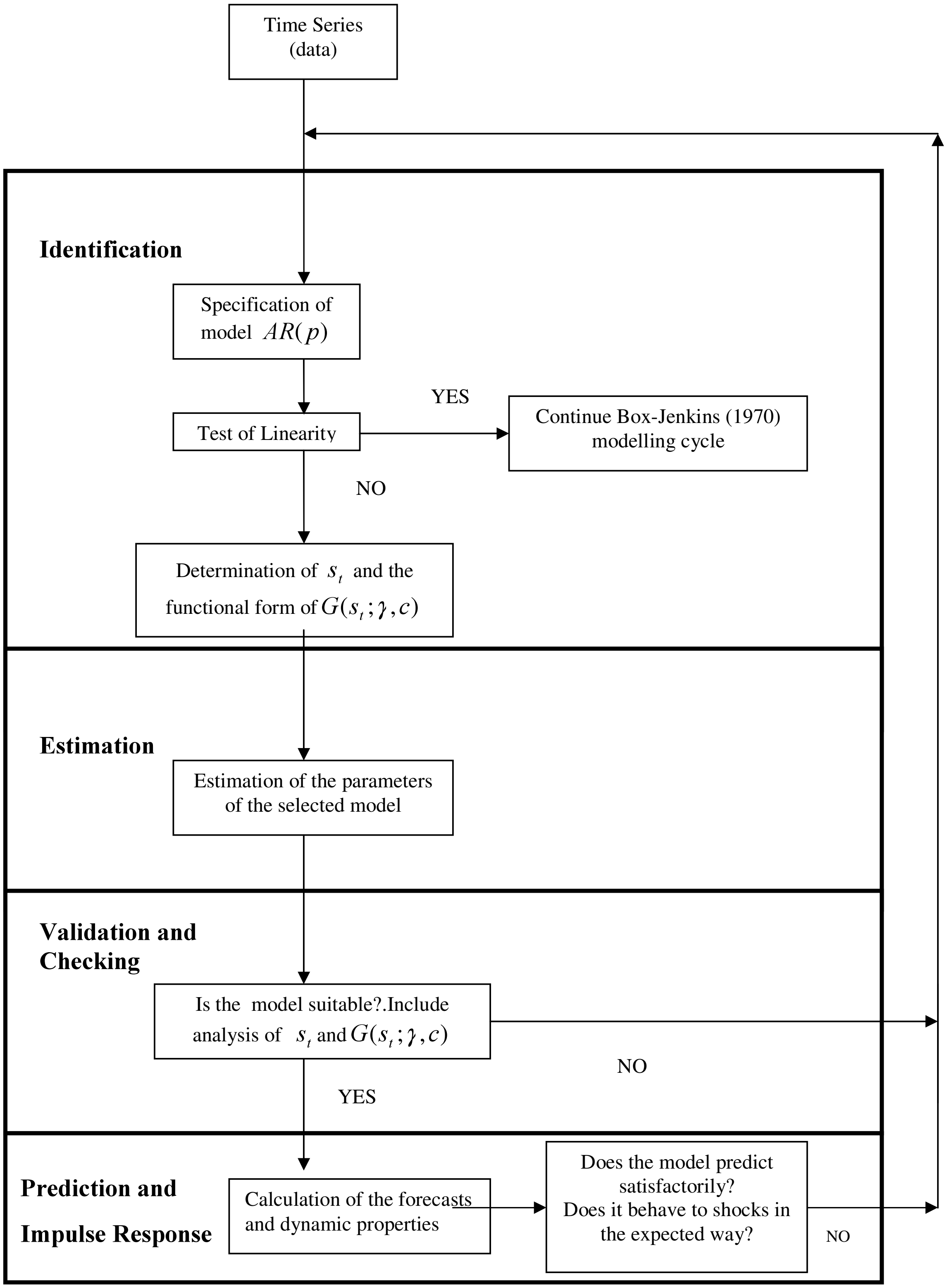


Figure 1: The modelling cycle in non-linear time series

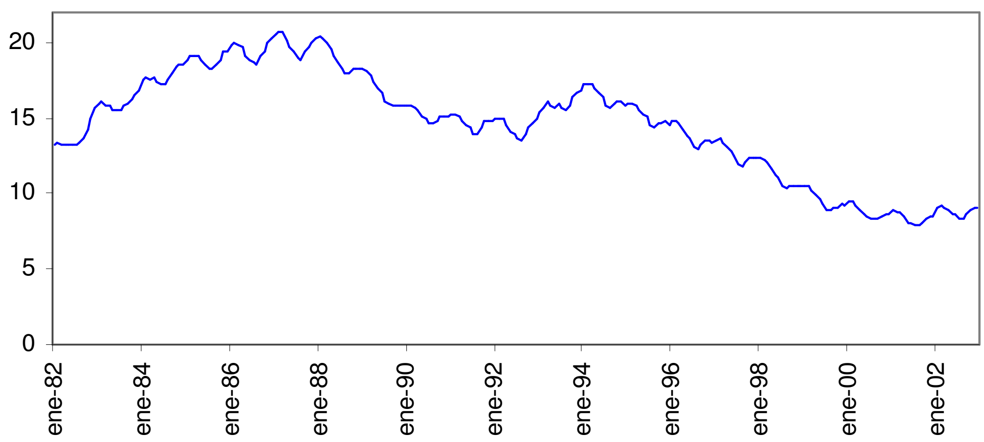


Figure 2: Evolution of the monthly unemployment rate in Spain. Period: January 1982 to December 2002.

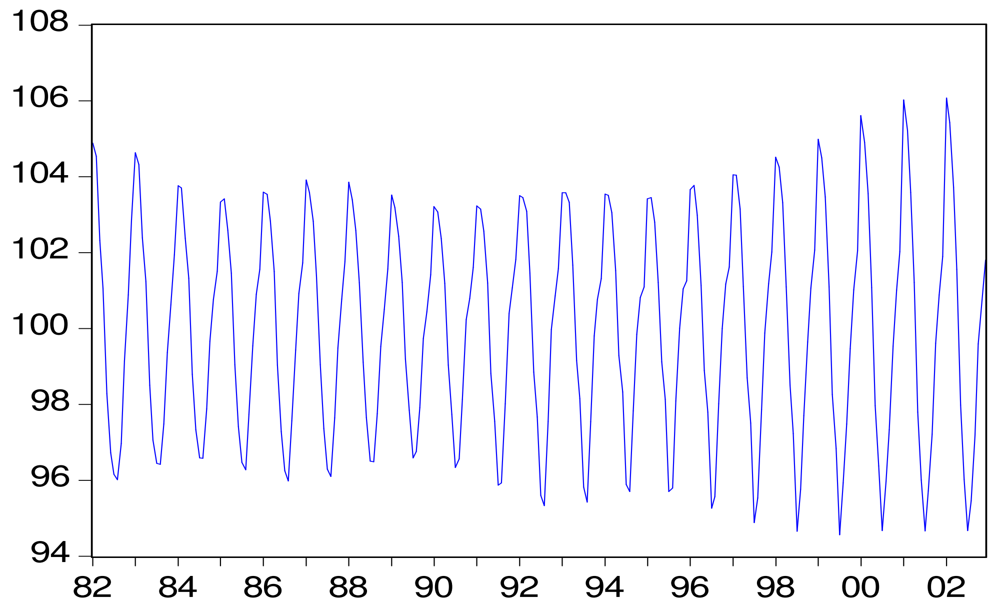


Figure 3: Seasonal component of the unemployment rate obtained through the application of TRAMO/SEATS

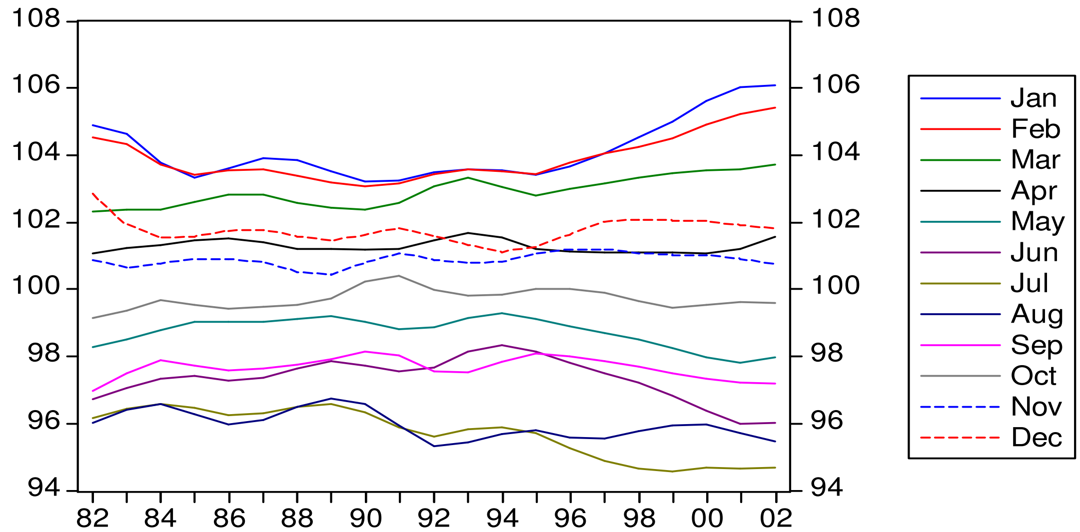


Figure 4: Seasonal Component of the unemployment rate by period.

Table 1: LM-type test for STAR non-linearity

	Standard Test				Robust Outliers Test			
Variable Transition (s_t)	LM_1	LM_3	LM_3^e	LM_4	LM_1	LM_3	LM_3^e	LM_4
$\Delta_{12}y_{t-1}$	0.000	0.001	0.000	0.023	0.007	0.007	0.004	0.089
$\Delta_{12}y_{t-2}$	0.000	0.006	0.000	0.019	0.009	0.013	0.009	0.050
$\Delta_{12}y_{t-3}$	0.001	0.028	0.001	0.022	0.011	0.039	0.013	0.152
$\Delta_{12}y_{t-4}$	0.004	0.312	0.006	0.128	0.020	0.253	0.026	0.341
$\Delta_{12}y_{t-5}$	0.007	0.378	0.015	0.082	0.036	0.393	0.060	0.114
$\Delta_{12}y_{t-6}$	0.012	0.285	0.020	0.364	0.043	0.189	0.059	0.333

Note: *p-values* of the LM type tests for non-linearity of the Spanish unemployment rate, January 1985-December 1999. The tests are applied to an AR(13) model for the first difference, including the variable at **en niveles retardada y las ficticias estacionales**.

Table 2: Selection of STAR model

	Teräsvirta			Escribano-Jordá	
Variable Transition (s_t)	H_{03}	H_{02}	H_{01}	H_{0L}	H_{0E}
$\Delta_{12} y_{t-1}$	0.024	0.459	0.000	0.0464	0.229
$\Delta_{12} y_{t-2}$	0.038	0.744	0.000	0.0606	0.198
$\Delta_{12} y_{t-3}$	0.0826	0.869	0.001	0.0264	0.106

Note: The tests are applied to an AR(13) model for the first difference, including the variable at **en niveles retardada y las ficticias estacionales**. In the la specification of Teräsvirta (1994), The hypotheses checked are $H_{03} : \beta_3 = 0, H_{02} : \beta_2 = 0 / \beta_3 = 0, H_{01} : \beta_1 = 0 / \beta_3 = \beta_2 = 0$ over (11). If the p-value of H_{02} is less, an ESTAR model will be selected, with an LSTAR for the rest of the cases. The procedure of Escribano-Jordá consists of checking the hypotheses $H_{0E} : \beta_2 = \beta_4 = 0$ and $H_{0L} : \beta_1 = \beta_3 = 0$ over (17), selecting an LSTAR (ESTAR) model if the minimum p-value is obtained for $H_{0L} (H_{0E})$.

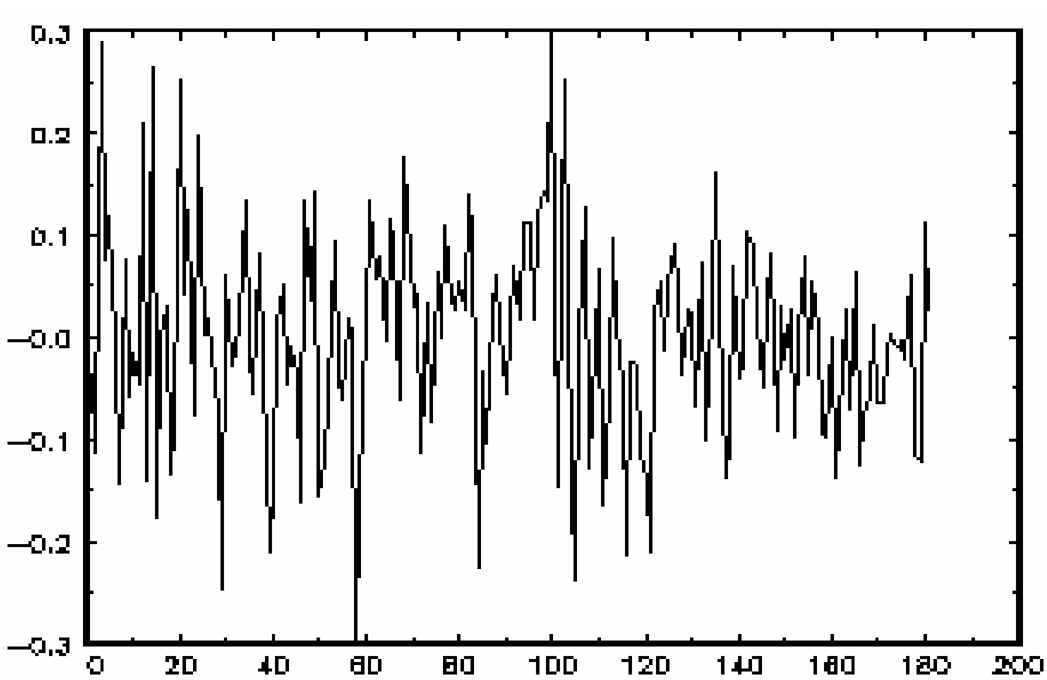


Figure 5: Residuals of the LSTAR model corresponding to the monthly unemployment rate in Spain

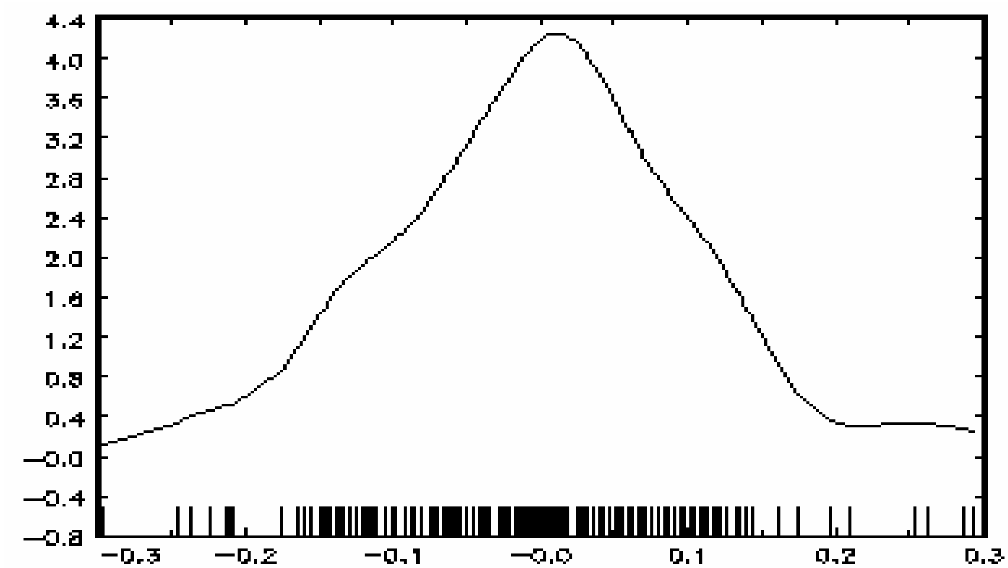


Figure 6: Kernel distribution function of the residuals of the estimated LSTAR model

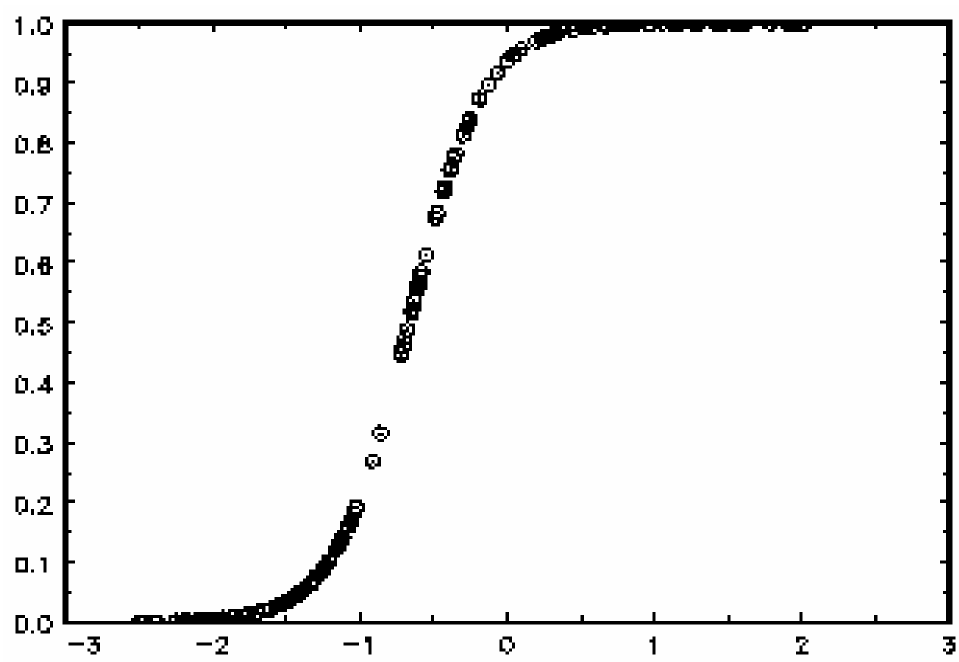


Figura 7: Transition function of the LSTAR model corresponding to the monthly unemployment rate against the transition variable $\Delta_{12} y_{t-1}$

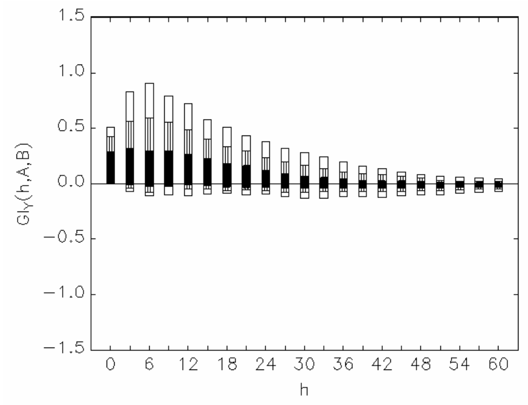
Table 3: Diagnostic Testing of the estimated LSTAR Model

	Test of serial Correlation of the order *q*					
q	2	4	6	8	10	12
p-value	0.965	0.488	0.791	0.686	0.750	0.652
	Test of Constancy of the parameters					
	Constants and Dummies			Lagged dependent variables		
	$LM_{C,1}$	$LM_{C,2}$	$LM_{C,3}$	$LM_{C,1}$	$LM_{C,2}$	$LM_{C,3}$
p-value	0.139	0.089	0.122	0.149	0.138	0.359
	Test of remaining non-linearity					
	Constants and Dummies			Lagged dependent variables		
	$LM_{EMR,1}$	$LM_{EMR,2}$	$LM_{EMR,3}$	$LM_{EMR,1}$	$LM_{EMR,2}$	$LM_{EMR,3}$
$\Delta_{12} y_{t-1}$	0.218	0.689	0.710	0.609	0.579	0.627
$\Delta_{12} y_{t-2}$	0.314	0.638	0.630	0.669	0.575	0.723
$\Delta_{12} y_{t-3}$	0.388	0.661	0.603	0.662	0.693	0.641
$\Delta_{12} y_{t-4}$	0.478	0.726	0.619	0.640	0.668	0.685
$\Delta_{12} y_{t-5}$	0.629	0.777	0.466	0.425	0.563	0.663
$\Delta_{12} y_{t-6}$	0.560	0.734	0.674	0.492	0.711	0.569

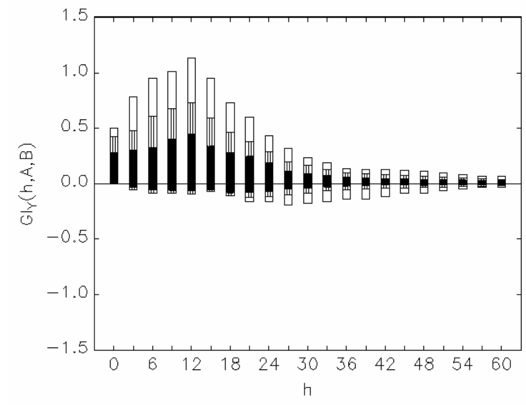
Table 4: Evaluation of the forecast of AR and LSTAR models

	MPE		**MSPE**	
h	AR	LSTAR	AR	LSTAR
1	-0.013	0.003	0.018	0.016
2	-0.005	0.004	0.018	0.047
3	-0.005	0.019	0.019	0.079
6	-0.008	0.038	0.018	0.132
9	-0.011	0.052	0.018	0.205
12	-0.009	0.089	0.020	0.166

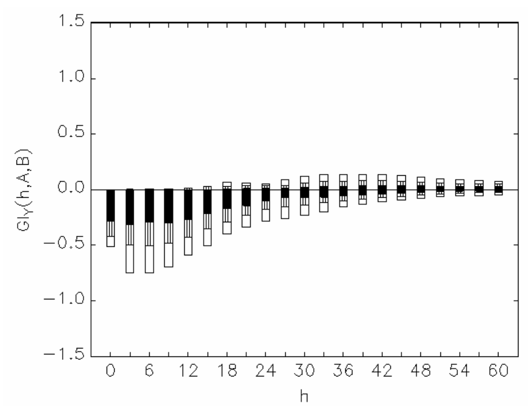
Note: The forecast period is from January 2000 to December 2002



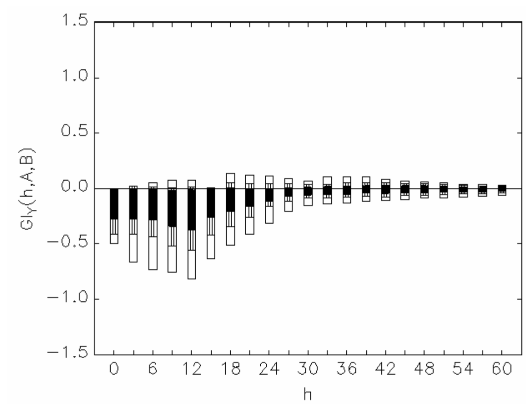
(a) Positive shocks in recession



(b) Positive Shocks in expansion



(c) Negative Shocks in recession



(d) Negative Shocks in expansion

Figure 8: HDRs of the impulse response functions of the LSTAR model for monthly unemployment in Spain.

Note: The HDRs are calculated at 50, 75 and 90% (black, grey and white, respectively, in the graph). Recession and expansion are considered with different histories, which take into account the values of the transition function depending on the fact that $G(\Delta_{12}y_{t-1};\gamma,c)$ is less or greater than 0.5 respectively.