

# Informational Structure and Efficiency in Monopoly

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**Abstract:** The paper focuses on efficiency under monopoly. Contrary to common wisdom, nine examples given in the paper show that a Pareto-efficient output in monopoly is possible under both linear and nonlinear pricing. Pareto efficiency can be achieved when consumers are homogeneous as well as heterogeneous. Since Pareto-efficiency is possible under different demand and cost conditions; different pricing strategies; and different degree of consumer heterogeneity, in general, monopoly per se is not the cause for inefficiency.

JEL Codes: D42, L10, L40

# Informational Structure and Efficiency in Monopoly

## 1. Introduction

Common economic wisdom is that monopoly pricing results in an inefficient allocation of resources because of some deadweight loss. However, in the literature there exist some counter examples contradicting this general view. By presenting different examples, this paper attempts to review plausible conditions that can lead to Pareto-efficient outcomes. In addition, we compare different situations when Pareto efficiency is not attainable. This comparison may be useful for efficiency-motivated public policy.

Besides theoretical interest in studying efficiency, several comments from Bill Gates (1999) related to acquisition of information using the Internet have also motivated us. “...Some Web merchants will adopt flexible pricing. Flexible prices are already a fixture of the ordinary marketplace. ... Direct-mail marketers often publish different prices in different catalogs targeted at different market segments. ...merchants are setting prices according to an individual’s willingness to pay. ...Sellers will identify repeat visitors to their online stores and give them personalized information and services. If a store’s Web site comes to know what kinds of prices a customer has or hasn’t been willing to pay in the past, it may reduce a price to spur that customer to buy. Many Web sites ask users for registration information, including name, address, demographic data and credit information. While this data enables businesses to offer better services and support for customers and do more targeted marketing, consumers should be able to approve in advance the use of any personal data and whether that data can be passed on to other entities” (p. 76-77). These emerging capabilities beg an important question: Will the

society be better off when a monopolist motivated by acquisition of some information or forced by legal statutes switches from one type of pricing to another? Although an unequivocal answer to such a broad question is hardly possible, our modest goal is to shed some light on the implication of such practices based on the economic theory of pricing and simple intuitions.

We approach this question by classifying different market situations (to some extent following Armstrong and Vickers (2001)) that are relevant for pricing strategies typically chosen by a single-product monopolist. The first factor to consider is the structure of demand or the degree of its *heterogeneity*. Indeed, the aggregate demand can be comprised of many similar consumers (homogeneous demand), or in a limiting case, the same aggregate demand is obtained from heterogeneous consumers, i.e., each consumer has a different demand curve. More typical are the intermediate cases representing several ‘more-or-less-homogeneous’ groups. For these cases, *two kinds of information* about the consumer groups are important: (1) Does the monopolist know the demand structures of the homogeneous groups? (2) Is the market ‘segmentable’?

An affirmative answer to the first question enables, at least, the use of non-linear pricing schemes (second-degree price discrimination), in particular, ‘package-pricing.’<sup>1</sup> An affirmative answer to the second question enables the monopolist to practice third-degree price discrimination based on market segmentation, or in the extreme case, perfect discrimination.<sup>2</sup> Of course, the possibility of an affirmative answer in itself does not

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<sup>1</sup>Package pricing is a form of nonlinear pricing. It can be used when a monopolist can only observe purchases by different groups of homogeneous consumers. It does not require identification of consumers and hence consumer types may be hidden. Because consumers are hidden, the monopolist offers different quantity-tariff bundles (packages) on ‘take-it-or-leave-it’ basis and consumers self-select.

<sup>2</sup>The necessary prerequisite for the practice of third-degree price discrimination is that the aggregate

guarantee that these pricing schemes will really be practiced. The strategy that will actually be implemented depends on the relative profitability of each scheme and also on the possibility of arbitrage. The likely choices of pricing strategies depending on different informational situations are stated in Table 1.<sup>3</sup>

TABLE 1- INFORMATION AND PRICING STRATEGIES UNDER NO ARBITRAGE

	<i>Information about consumer groups</i>		
<i>Identification of consumer types</i>	<i>No information</i>	<i>Incomplete information</i> ( <i>heterogeneous groups</i> )	<i>Full information</i> ( <i>homogeneous groups</i> )
<i>Non-observable</i>	1. Linear pricing	?	2. Package pricing
<i>Observable</i>	–	3. Pricing using segmentation (e.g., 3rd-degree discrimination)	4. Personal pricing (perfect discrimination)

The classification in Table 1 has an intuitive appeal about relative profitability. Generally, price discrimination of any degree, when it can be practiced *without* any arbitrage or other impediments, is more profitable than uniform pricing. More precisely, price discrimination gives at least no less profit than uniform price because the uniform price is one of the choices available when choosing the most profitable discrimination. Thus,

(1) Generally, simple uniform pricing should be practiced when, except for the total market demand, no other information about heterogeneous consumers is known, or because of arbitrage any other pricing scheme is not implementable. (2) Nonlinear pricing,

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demand must be segmentable into submarkets based on some identifiable characteristics of consumers. Because the pricing used in third-degree price discrimination is linear, consumer homogeneity, although preferable, is not necessary as is the case with package pricing.

<sup>3</sup>Table 1 does not give a complete taxonomy of optimal choices of pricing strategies for a broad spectrum of market situations. It would be too complicated to incorporate different arbitrage possibilities and different degrees of heterogeneity and observability of consumers. The picture would become even more complex if additional factors such as transaction costs, legal constraints, etc., which we *ignore* here, are taken into account. For simplicity, we consider pricing schemes *without* any arbitrage.

and in particular, package pricing, can be used when a monopolist *knows* the demands of different groups of consumers, each group consisting of homogeneous consumers, but cannot *observe* a consumer's affiliation to a particular group (non-segmentable market).<sup>4</sup> In addition, inter-group arbitrage is impossible (3) Third-degree price discrimination can be used when in addition to no arbitrage, observability enables sorting of many heterogeneous consumers into segments or sub-markets.<sup>5</sup> Is linear pricing the only option under third-degree price discrimination? It is often assumed to be the case in the literature, and we adhere to the tradition. (4) Perfect price discrimination is the limiting case of perfect knowledge *and* observability without arbitrage. In particular, when *all* consumers are observable, then each consumer can be treated as a separate homogeneous market, and we arrive at cell 4 (see example 7 below). In this situation, several pricing strategies can be used to capture the consumer surplus, and personal pricing includes all such pricing strategies.

Our focus is on social (Pareto) efficiency under the optimal solutions for the four choices of pricing strategies mentioned above. Giving a complete picture of efficiency for all combinations of assumptions and the resulting solutions is hardly possible. Generally, the effects are ambiguous. The important exception is the well-known Robinson-Schmalensee result.<sup>6</sup> In Section 2, we give examples that show the possibility of Pareto-efficient out-

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<sup>4</sup>For a non-segmentable market, although price discrimination based on direct segmentation is not possible, a nonlinear pricing based on self-selection by consumers is implementable. In cell 2, we mention only package pricing because among different nonlinear pricing schemes, package pricing can be shown to be most profitable. Other pricing schemes may be applicable in the cell marked by ?, but exact conditions remain unclear.

<sup>5</sup>Specifically, by 'observable' or 'segmentable' market structure, we mean that the monopolist not only can distinguish consumers based on some identifiable characteristics (e.g., age, location, gender etc.), but is also able to use these characteristics legally in the design and implementation of pricing strategies of her choice.

<sup>6</sup>For some definite conclusions about efficiency in some cases, see Schmalensee (1981), Varian (1985)

comes under *all* types of pricing strategies. When the first-best solution is not possible, then social efficiency often increases downwards in our classification 1– 4.<sup>7</sup> A complete analysis of specific situations showing some exceptions to this general tendency is presented in Section 3. We compare efficiency between uniform pricing, package pricing, and third-degree price discrimination for two linear demands. The results have some public policy implications. Section 4 concludes with some suggestions for possible extensions of the topic. The appendix includes formal derivations and proofs.

## 2. Pareto-efficiency examples

Our primary goal in this Section is to present a sufficiently broad collection of examples of efficient outcomes under monopoly. These examples shed some light on the controversy between profitable monopoly pricing schemes and social efficiency under some combinations of three main market characteristics, i.e., heterogeneity, knowledge of consumer groups and observability. Though some of our examples may seem too specific, they are contrary to the common wisdom of ‘always-inefficient-monopoly.’

### A. *Uniform linear pricing under arbitrage or unknown groups*

For this case, we can mention at least three counter-examples to the generally good assertion of ‘inefficient monopoly.’ For differentiable functions, the profit-maximizing monopoly output can coincide with the Pareto-efficient, if and only if,  $x\dot{p}(x) = 0$ , where  $x$  denotes quantity,  $p(x)$  is the inverse demand function,  $c(x)$  is the cost function, and dot denotes the derivative.

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and Schwartz (1990).

<sup>7</sup>See also Armstrong and Vickers (2001) for welfare effects of price discrimination in oligopoly using ‘competition-in-utility-space’ approach.

*Example 1:* One such solution with *zero derivative* can be constructed using the functions  $p(x) = 1 + (1 - x)^3$  and  $c(x) = \frac{1}{2}x^2$ , which are shown in Fig. 1(a).

*Examples 2 and 3:* These two examples give another solution of efficient uniform monopoly pricing for non-differential functions. Figure 1(b) relates to a locally perfectly *elastic demand*.<sup>8</sup> Figure 1(c) shows another example demonstrating a similar idea but for a locally perfectly *inelastic supply*. Note that all three examples are based on specific local behavior of demand and supply curves, and the standard proof of inefficiency discourages further attempts to construct more such examples. It is interesting to note here that in these examples, welfare is *divided* between the consumers and the producer so that the consumer surplus is non-zero.

B. *All types of consumers observable (perfectly segmentable market)*

Next, we consider several examples of perfect discrimination when all consumer types are observable (case 4).

*Example 4:* Consider first a very particular case of bilateral monopoly (a single buyer and a single seller). In reality this may be the case when a single producer of tanks, fighter planes, etc., sells these goods to a *single buyer*, namely the government. In this case, there is room for bargaining between the seller and the buyer over the division of welfare. However, in any case, it is reasonable to suppose that the two will arrive at a Pareto-efficient agreement so that the quantity remains optimal.

*Example 5:* Very similar to the single-consumer case is the situation with several

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<sup>8</sup>To understand the figures note that the goal of monopolist is to maximize profit which is the area between the dashed line and the marginal cost curve. In choosing optimal quantity  $\hat{x}$  the monopolist must compare any possible increase in price with decrease in quantity. For this trade off, the slope of the demand curve above the solution point matters for the example to be valid. Thus, among locally elastic demands only the special ones relate to an efficient monopoly.

*cooperating consumers.* For instance, suppose the residents of a village are willing to buy a bridge from a monopolistic builder. When the residents are wise enough to cooperate efficiently (say, by Groves-Clark procedure) then the resulting coalition is, in essence, equivalent to a single consumer, and we arrive at the same efficient outcome as above.

*Example 6:* Something similar may also happen in the case of many *homogeneous consumers.* For this situation, the monopolist again is in a very good informational position to practice perfect price discrimination. It may differ from the above cases because it is the monopolist who now has the bargaining power. To extract the entire welfare, the obvious choice for the monopolist is to offer a single package containing the Pareto-efficient quantity. It is interesting to note that in this case, packages are possible despite the possibility of arbitrage! But because all consumers are the same, no one can be better off by reselling her package.

Can other pricing schemes be used for perfect price discrimination? The answer depends upon the possibility of arbitrage. When arbitrage is possible, a two-part tariff should result in reselling and some loss of profit. However, when arbitrage is prevented, using the same logic as above, a two-part tariff or other tariff functions like  $A + f(x)$  can also give the same result - - maximum profit *and* Pareto efficiency.

*Example 7:* Example 6 can also be generalized to a situation where there are many types of consumers, but the market is *completely segmentable.* Each group consists of homogeneous consumers, and a consumer belonging to a particular group is also observable. Then the previous idea can be implemented for each market segment to practice perfect price discrimination.

### C. Non-segmentable market, no arbitrage and known demand structures

We now turn to a more puzzling situation when the monopolist knows the demands of two homogeneous groups (e.g., a high-demand group and a low-demand group) and arbitrage is prevented, but the market is non-segmentable (case 2). For this case, nonlinear pricing can be used. When consumer groups are homogeneous, it is most practical to use package pricing based on a step-wise outlay function. It is again the common wisdom that Pareto efficiency is *unattainable*. However, this common wisdom is true *only* for the most explored case of several different consumer groups with “ordered” demands (“ordered” means that the demand curves of different consumer types do not cross). For this case, the textbook proofs show that the monopolist offers too small a package (a quantity less than Pareto efficient) for the low-demand consumers to prevent high-demand consumers from taking a smaller package.<sup>9</sup> Example 8 below shows that not only can Pareto-efficiency occur, but such an outcome is *non-degenerate* for the “non-ordered” demands (“non-ordered” means that the demand curves of different consumer types do cross).<sup>10</sup>

*Example 8:* To see why Pareto efficiency is a non-degenerate outcome, consider Figure 2 with two linear demands denoted by  $\dot{v}_i(x_i)$ , and derived from valuations functions  $v_i(x_i)$ . Costs are assumed to be zero. One can check that the monopolist will offer two different incentive-compatible packages  $(\hat{x}_1, t_1)$ ,  $(\hat{x}_2, t_2)$  as shown. The peak of each valuation function is represented by the most preferred quantity  $\hat{x}_i$  ( $i = 1, 2$ ). Incentive-compatibility constraints *are* satisfied when the first peak at the Pareto-efficient quantity for consumer

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<sup>9</sup>Recall that consumers self-select and incentive-compatibility constraints must be satisfied, i.e., no consumer should have an incentive to switch to the package designed for the someone else.

<sup>10</sup>For a complete analysis of package pricing under both ordered and non-ordered demands, see Nahata, Kokovin and Zelobodko (2002).

type 1 is *above* the second curve and the second peak at the Pareto-efficient quantity for the consumer type 2 is *above* the first curve. Nothing else is required. These arguments show that Pareto-efficient package pricing is feasible not only when two demand triangles are equal, but when they are ‘more-or-less-similar.’ This similarity of demand triangles for linear demands amounts to the condition  $\beta/(2\beta - 1) \geq \alpha \geq 2 - \beta$ , where  $\alpha$  is the height of one demand triangle,  $\beta$  is its length and the other triangle is the standard (1,1) simplex.<sup>11</sup>

It should be noted that Pareto-efficiency under package pricing is not only a *non-degenerate* outcome, but it is rather probable.<sup>12</sup> Further, the figure also demonstrates that, in general, Pareto-efficient package pricing is probable *not only* for linear demands, zero costs, and uniform distribution of parameters, but also for nonlinear demands and positive costs.

*Example 9:* Pareto efficiency can also be realized under the limiting case when distinction between ordered and non-ordered demands disappears. This happens when  $\beta = 1$  and  $\alpha$  is sufficiently high. It is easy to see that for this special situation a *single* Pareto-efficient package of size 1 for both types of consumers can be offered with a uniform tariff  $\alpha/2$ . Unlike the case of two consumers case above, Pareto efficiency is realized with a non-zero consumer surplus, which equals  $(1 - \alpha)/2$  for the high-demand consumers. Of course, the question of any arbitrage does not arise in this case.

Some important inferences from Examples 8 and 9 are noteworthy. First, Pareto-

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<sup>11</sup>For the derivation of this condition see the appendix.

<sup>12</sup>To evaluate the probability of such efficient outcomes, assume a uniform distribution of  $\alpha \in (0, 1)$  and another independent uniform distribution of  $\beta \in (1, b)$ , for some  $b > 1$ , describing the maximum possible length of the long triangle. For  $b > 2$ , this probability is more than 0.137, and it increases as  $b \rightarrow \infty$  to 1/2.

efficiency can be realized *without* full information about the unobservable homogeneous consumers. Second, unlike linear pricing, for packages Pareto-efficiency can be realized with zero consumer surplus (no informational rent) for all types of consumers. Third, due to increasing information as we move downwards in our classification 1-4, social efficiency becomes a non-degenerate case.

### **3. Comparison of efficiency between linear and nonlinear pricing**

In Section 2, our focus was on Pareto-efficient allocations under monopoly. However, more typical is the case when most pricing schemes do not result in the first-best or the Pareto-efficient outcomes. For these situations, there is one important question both theoretically and in the context of public policy: Which pricing strategy is socially more efficient?

We present some new results for linear demands by comparing package pricing with both uniform pricing and third-degree price discrimination (comparing cells 1, 2 and 3 in Table 1 with each other). The comparison between uniform pricing (case 1) and second or third-degree price discrimination is easily interpretable. Should legislations and/or innovations enhancing the ability of the monopolist to discriminate be encouraged?

It is not so easy to interpret the comparison between case 2 and case 3 because second and third-degree price discriminations are practiced under different circumstances. Third-degree price discrimination can be practiced when the aggregate demand can be segmented into sub-markets based on some identifiable characteristics of consumers. Such segmentation, in general, may not represent homogeneous groups because willingness to pay may not be the same for all consumers in a segmented sub-market based identifiable

characteristics. This may be the reason why the pricing employed is linear in third-degree price discrimination. On the other hand, for package pricing only group homogeneity is required and consumers need not be observable based on identifiable characteristics. Because of the differences in conditions under which the two pricing strategies are practiced, the question of choosing one over the other should not arise. For example, suppose that the monopolist cannot identify the two hidden types of consumers and initially uses package pricing. Now, because of some new informational innovation (e.g., Internet), suppose the monopolist can identify consumers in each of the two groups based on some observable attributes. As a result, the monopolist should now switch from second-degree to perfect discrimination, which has the highest profitability, rather than to the less profitable, third-degree price discrimination. Similarly, suppose consumers are observable but because their homogeneity is not known, third-degree price discrimination is practiced. Now suppose in addition to identifying consumers their homogeneity is also known then again switching from third-degree to perfect discrimination is more profitable than switching to second-degree price discrimination because switching to the latter would mean wasting useful market information. However, the comparison does become meaningful when perfect discrimination faces some legal hurdles, public outcry, or as noted by Willig (1978), when an overt collection of detailed accurate information about each economic agent can distort the agent's economic behavior. The comparison also becomes relevant in figuring out the relative importance of group homogeneity and the observability of consumers in assessing efficiency loss/gain.

Our comparisons below are based on two homogeneous groups of consumers of equal

number with ordered linear demands.<sup>13</sup> The aggregate linear demand functions are:  $D_1(p) = 1 - p$  and  $D_2(p) = \beta - \frac{\beta}{\alpha}p$ , where  $\alpha < 1$  and  $\beta < 1$ . For simplicity, the costs are assumed to be zero:  $c(x) \equiv 0$ . It should be noted, however, that our setting is rather general for linear demands and linear costs because all situations can be converted to similar demand functions as above by subtracting costs from the inverse demand functions and normalizing. The advantage is that the normalized demand functions enable us to describe *all* possible market situations in terms of only two parameters,  $\alpha$  and  $\beta$ , and also allows us to construct an ‘efficiency-map’ that shows *relative frequency* of various outcomes.

The results are stated below (proofs are in the Appendix) and the efficiency-map is depicted in Figure 3. All regions are marked with the letters U (uniform pricing), P (package pricing), and T (third-degree price discrimination). The order of the letters represent the efficiency from the highest to the lowest. For example,  $U > P > T$  means uniform pricing is more efficient than package pricing, which in turn is more efficient than third-degree price discrimination. Although the map does not depict the relative profitability of different pricing schemes, it is true that in *all regions*, package pricing is more profitable than third-degree discrimination which is more profitable than uniform pricing.

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<sup>13</sup>For third-degree we assume that the monopolist knows demands and can observe the consumers in each market segments but is unable to use perfect discrimination, perhaps being unsure of the homogeneity of consumers. More challenging would be the situation when the monopolist observes only some characteristics of the consumers in two demand groups. For example, the observable groups are younger and older consumers, while the demand groups consist of high and low-demand consumers, the latter being 70 percent of young consumers. Then the demand curves of observable groups do not coincide with those of demand groups and should be re-estimated. Besides, it is not so obvious why instead of third-degree, the monopolist should not use more profitable package pricing.

**Result 1.** (a) *Both consumer types are served under uniform pricing, if and only if,  $\alpha > 1/(2 + \beta)$ ; (b) Both types are served under package pricing, if and only if,  $\alpha > 0.5$ .*<sup>14</sup>

In Figure 3,  $1/(2 + \beta)$  represents curve (1), and  $0.5 = \alpha$  is the dashed line.

**Result 2.** *When both types of consumers are served under both uniform and package pricing then the necessary and sufficient condition for efficiency to be strictly higher under*

*uniform pricing is  $0.5 < \alpha < \frac{\beta(2+2\beta^2+\sqrt{(7\beta^2+10\beta+7)(1-\beta)})}{2(1+\beta)^2}$ .*

In Figure 3, this condition represents the region between  $0.5 = \alpha$  and curve (2).

**Result 3.** *When both types of consumers are served under uniform pricing but only the high-demand consumers are served under package pricing, then the necessary and sufficient condition for efficiency to be strictly higher under uniform pricing is  $\max\{1/(2 + \beta), \frac{1+2\beta-3\beta^2}{4\beta}\} < \alpha < 0.5$ .*

The region represented by the above inequality is the region between  $\alpha = 0.5$  and curves (3) and (1).

It is known that for linear demands when all consumers are served, third-degree discrimination is also less efficient than uniform pricing (Robinson-Schmalensee result, which holds above curve (1) in Fig. 3).<sup>15</sup> In contrast, Results 2 and 3 show that package pricing is, *in most cases*, more efficient than uniform pricing, regardless of how many consumer types are served (see Fig. 3 to estimate the relative probability of efficient outcomes).

Further, Result 3 shows that even when the low-demand consumers are ignored under

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<sup>14</sup>The borders of the regions  $0.5 < \alpha$  and  $\alpha > 1/(2 + \beta)$  are included in the regions where the low-demand consumers are ignored. Although both markets are *always* served under third-degree price discrimination, this need not be the case with package pricing. By ignoring low-demand consumers, the profits can be higher under package pricing.

<sup>15</sup>The opposite (U<T) is the case when uniform pricing excludes one market in the region below the curve (1). One can visually compare the likelihood of each outcome.

package pricing, efficiency, in most cases, is higher under package pricing. The reason is that the high-demand consumers buy the package containing the Pareto-efficient quantity, and the deadweight loss is only from the low-demand triangle. It is mostly smaller than the deadweight loss under uniform-pricing, which amounts approximately 1/4 of the area of the demand rectangle formed by two overlapping triangles.

**Result 4.** *When both types of consumers are served under package pricing, both the efficiency and profits are always higher in package pricing than under third-degree price discrimination.*

**Result 5.** *Profits are higher under package pricing than third-degree price discrimination even when only the high-demand consumers are served under package pricing. The necessary and sufficient condition for efficiency to be strictly higher under third-degree price discrimination is  $\frac{1}{2} \geq \alpha > \frac{1}{3\beta}$ .*

In Figure 3, this region is between  $\alpha = 0.5$  and to the right of curve 4 (the mouth of ‘frog’ shaded in the plot).

There is a simple economic intuition behind the efficiency conclusion in Result 4. Package pricing results in the Pareto-efficient quantity for the high-demand consumers. Although there is a deadweight loss from the low-demand consumers in both third-degree discrimination and package pricing, the elimination of deadweight loss from the high-demand consumers in package pricing is *always* large enough to result in a net increase in efficiency.

The ‘efficiency-map’ in Figure 3 clearly shows that package pricing results in highest efficiency *except* in the region bounded by curves 2, 3 and 1. This is the only region in

which uniform pricing results in higher efficiency. The area of this region ( $U > P$ ) is approximately 0.0586, or about 6 percent of the unit square describing all parameters of ordered demands. Similarly, only in the sub-region ( $U > T > P$ ), with an area of approximately 0.0315, is third-degree price discrimination more efficient than package pricing.

There are some policy implications from our results. First, from an efficiency consideration alone, third-degree price discrimination is least desirable under linear demands when it does not open up new markets. Second, suppose a monopolist has the relevant information about different consumers to practice third-degree price discrimination. Certainly there is a strong profit incentive to switch to perfect discrimination. But when perfect discrimination is not feasible for the reasons mentioned earlier, the more profitable, and in most situations more efficient, second-degree price discrimination becomes an obvious choice. Although third-degree price discrimination can be justified based on efficiency considerations (the Robinson-Patman Act), such considerations becomes redundant here because the more profitable second-degree price discrimination *should and can* be used instead.

#### 4. Conclusions and extensions

The nine examples above show that, at least theoretically, neither monopoly nor the degree of monopoly power is necessarily the cause of Pareto inefficiency. Although our Examples 1-3 are pathological, from purely theoretical stand point even the choice of pricing strategy (linear versus nonlinear) is not the sole determinant of inefficiency. The most general conclusion that can be drawn from these examples is that, although not all, in most cases, the *informational structure* of the market (e.g., observability and homo-

geneity) is a significant determinant of Pareto efficiency. For unobservable heterogeneous situations, generally there are some efficiency losses, except for some very special demand curves shown in Figure 1. Between identifiable but heterogeneous groups (third-degree) and unidentifiable but homogeneous groups (package pricing), efficiency is always higher under the latter case when both types of consumers are served. Even when only one type is served under package pricing efficiency is mostly higher except in the situation depicted in Figure 3 (see Result 5). Thus, if consumer homogeneity is known through sufficient information about consumers, a monopolist can implement a suitable nonlinear pricing strategy that can result in higher efficiency or even Pareto-efficient outcome (examples 8-9). Of course, in cases when both homogeneity and observability are feasible at the same time, a Pareto-efficient outcome can result and it is not a pathological case.

If efficiency is the main consideration then the relevant public policy question is: Should the regulators of monopolies actively engage in legislating monopolistic pricing practices or encourage innovations or practices that enhance abilities to acquire more information about the consumers? For a monopolist profit incentive is a strong motivating factor to acquire relevant information about consumers. Therefore, attempts to acquire information using, for example, the Internet can potentially minimize or in some cases even eliminate the distortions traditionally associated with monopoly. If income redistribution is not taken into account, a policy encouraging an improved informational structure of market due to informational innovations is likely to promote efficiency.

We suggest two extensions of the topic. First, a comprehensive analysis of the most profitable pricing schemes for all informational situations of the market, especially for the

second column in our Table 1, will enhance the understanding of the relationship between information and efficiency. Another useful extension would be to incorporate transaction costs and the cost of information in analyzing the choice of different pricing schemes and efficiency.

## Appendix

### *Condition for Pareto efficiency for non-ordered demands (Example 8)*

Recall that there are two types of consumers with normalized inverse demand functions  $p(x_1) = 1 - x_1$  and  $p(x_2) = \alpha - \alpha x_2/\beta$ . For the non-ordered case  $\alpha < 1$  and  $\beta > 1$  and hence the two demands cross with each other. For perfect discrimination the monopolist would choose quantity-tariff bundle  $(x_i, t_i)$  so that  $\hat{x}_1 = 1$ ,  $\hat{x}_2 = \beta$ ,  $\hat{t}_1 = 1/2$ ,  $\hat{t}_2 = \alpha\beta/2$  (this can be obtained by differentiating the area of each demand triangles and setting it equal to zero). Both participation constraints  $v_1(x_1) - t_1 = x_1 - x_1^2/2 - 1/2 \geq 0$  and  $v_2(x_2) - t_2 = \alpha x_2 - \alpha x_2^2/2\beta - \alpha\beta/2 \geq 0$  are satisfied. The same  $(x_i, t_i)$  constitutes the solution for the first-best package pricing when the self-selection constraints, namely  $v_1(x_1) - t_1 \geq v_1(x_2) - t_2$ , and  $v_2(x_2) - t_2 \geq v_2(x_1) - t_1$ , will be satisfied. By comparing these relations with  $(\hat{x}_i, \hat{t}_i)$  we get the region of parameters  $\beta/(2\beta - 1) \geq \alpha \geq 2 - \beta$  for the Pareto-efficient packages.

### *Formulae for different pricing schemes for ordered demands*

#### *Uniform Pricing*

Under simple monopoly the aggregate demand is

$$D(p) = x_1 + x_2 = x = (1 + \beta) - \left(1 + \frac{\beta}{\alpha}\right)p$$

Case 1: When both markets are served then  $x_1 > 0$  and  $x_2 > 0$ . For the linear

demand, the expressions for the optimal price, optimal quantity and profit are standard.

The optimal price is  $p_{U2}^* = \frac{\alpha(1+\beta)}{2(\alpha+\beta)}$ , the optimal quantities are

$$x_1^* = \frac{\alpha+2\beta-\alpha\beta}{2(\alpha+\beta)}, x_2^* = \frac{\beta(\beta+2\alpha-1)}{2(\alpha+\beta)} \text{ and } x^* = \frac{\beta+1}{2}. \text{ The optimal profit } \pi_{U2}^* = \frac{\alpha(1+\beta)^2}{4(\alpha+\beta)}.$$

Consumer surpluses at the optimum are the areas of triangles and can be written as

$$CS_1 = \frac{(-\alpha-2\beta+\alpha\beta)^2}{8(\alpha+\beta)^2}, CS_2 = \frac{\alpha\beta(-1+\beta+2\alpha)^2}{8(\alpha+\beta)^2} \text{ and } CS_{U2} = \frac{\alpha\beta^2-6\alpha\beta+4\alpha^2\beta+4\beta+\alpha}{8(\alpha+\beta)}.$$

$$\text{The total resulting welfare } W_{U2} = CS_{U2} + \pi_{U2}^* = \frac{3\alpha\beta^2-2\alpha\beta+4\alpha^2\beta+4\beta+3\alpha}{8(\alpha+\beta)}$$

Case 2: When only one market is served under uniform pricing then the corresponding expressions are:

$$p_{U1}^* = \frac{1}{2}, x_1^* = \frac{1}{2}, x_2^* = 0, \text{ and } \pi_{U1}^* = \frac{1}{4}, CS_1 = \frac{1}{8}, CS_2 = 0, CS_{U1} = \frac{1}{8} \text{ the resulting welfare } W_{U1} = \frac{3}{8}.$$

### *Third-degree Price Discrimination*

The expressions under third-degree price discrimination are straight-forward.  $p_1 = \frac{1}{2}$ ,  $x_1 = \frac{1}{2}$ ,  $p_2 = \frac{\alpha}{2}$ ,  $x_2 = \frac{\beta}{2}$ ,  $CS_1 = \frac{1}{8}$ ;  $CS_2 = \frac{\alpha\beta}{8}$ ;  $CS_T = \frac{1+\alpha\beta}{8}$ ;  $\pi_T = \frac{1+\alpha\beta}{4}$ . The resulting welfare  $W_T = \frac{3(1+\alpha\beta)}{8}$ .

### *Package Pricing*

For ordered-demands ( $\beta \leq 1$ ) the expressions for package pricing can be easily obtained (see Varian (1992)).

Case 1: When both consumer types are served then the optimal quantity and tariff ( $x_i, t_i$ ) for the high-demand and the low-demand consumers are

$$(x_1, t_1) = \left(1, \frac{\alpha(4\beta\alpha^2-8\alpha\beta+4\alpha-\beta+2\beta^2)}{2(2\alpha-\beta)^2}\right), \text{ and } (x_2, t_2) = \left(\frac{(2\alpha-1)\beta}{2\alpha-\beta}, \frac{\beta\alpha(2\alpha-1)(2\alpha-2\beta+1)}{2(2\alpha-\beta)^2}\right).$$

The profit  $\pi_{P2} = \frac{\alpha(2\alpha\beta-2\beta+1)}{2\alpha-\beta}$ . The consumer surplus is  $CS_1 = \frac{\beta(-3\alpha+\beta-4\alpha^3+8\alpha^2-2\alpha\beta)}{2(2\alpha-\beta)^2}$ ,  $CS_2 = 0$  and  $CS_{P2} = \frac{\beta(-3\alpha+\beta-4\alpha^3+8\alpha^2-2\alpha\beta)}{2(2\alpha-\beta)^2}$ . The resulting welfare  $W_{P2} = \frac{4\beta\alpha^3+4\alpha^2-5\alpha\beta+2\alpha\beta^2-4\alpha^2\beta^2+\beta^2}{2(2\alpha-\beta)^2}$ .

Case 2: Under the package pricing when it is optimal to serve only one consumer type (ignoring solution) then obviously  $x_1 = 1$ ,  $t_1 = \frac{1}{2}$ ,  $x_2 = 0$ ,  $t_2 = 0$ , profit  $\pi_{P1} = \frac{1}{2}$ . The consumer surplus  $CS_{P1} = CS_1 = CS_2 = 0$ , the resulting welfare is  $W_{P1} = \frac{1}{2}$ .

*Proof: Result 1*

The border of the regions when one or two markets will be served under uniform price can be determined in terms of parameters  $\alpha$  and  $\beta$ . When these two strategies compete (price  $p_{U1}^*$  is above the kink in the demand curve, and price  $p_{U2}^*$  is below), then profits  $\pi_{U2}^* = \frac{\alpha(1+\beta)^2}{4(\alpha+\beta)} > \frac{1}{4} = \pi_{U1}^*$ . This inequality holds, if and only if,  $\alpha > \frac{1}{2+\beta}$ . The combinations of  $(\alpha, \beta)$  when both prices (calculated from different triangles)  $p_{U2}^*, p_{U1}^*$  are on the same side of the kink is given by  $\alpha = \frac{1}{2+\beta}$ , which constitutes the border.

The border under package pricing can be obtained by using the standard textbook arguments. Note that when there are two types of consumers in equal numbers, then at the optimal solution resulting two packages the willingness-to-pay for the marginal unit of good of the low-demand consumer should be one-half of the high-demand consumer's willingness-to-pay for this unit, or  $P_2(x_2) = 0.5P_1(x_2)$ . This condition is satisfied, if and only if,  $\alpha = 0.5$ , and both types of consumers will be served when  $\alpha > 0.5$ .

*Proofs: Results 2-5*

By comparing the applicable welfare formulas for each case, the results follow.

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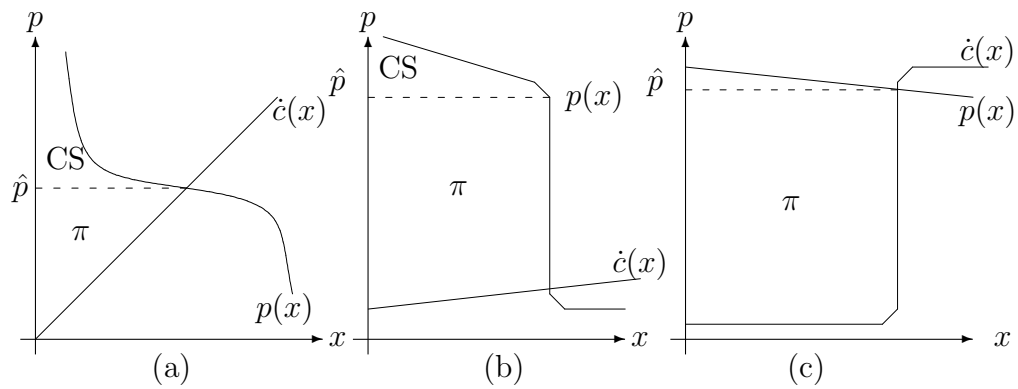


Figure 1: EFFICIENCY OF MONOPOLY UNDER UNUSUAL ELASTICITIES

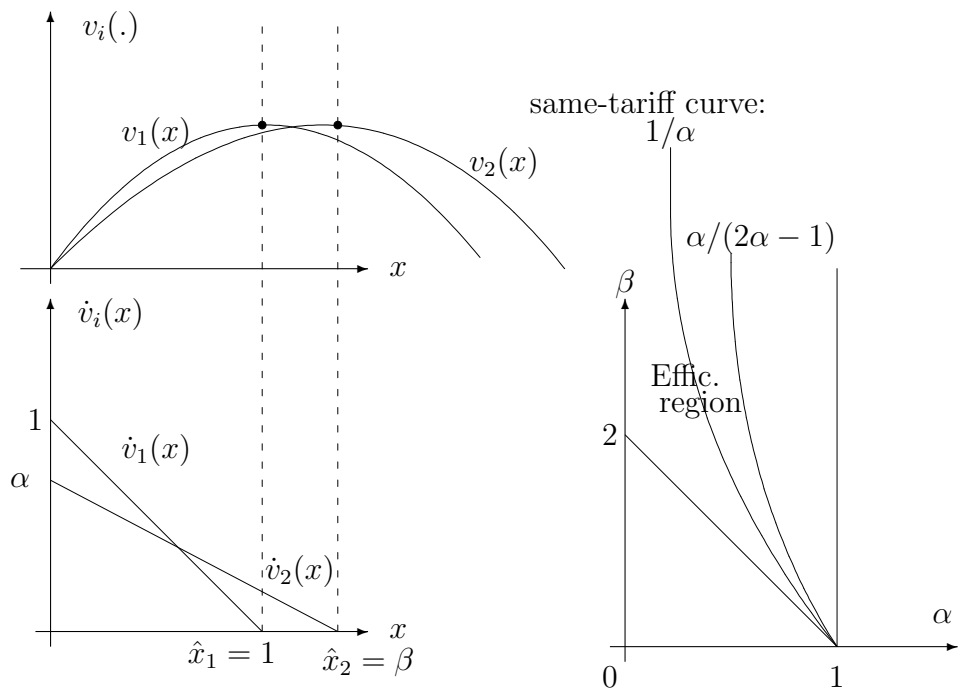


Figure 2: PACKAGES: EFFICIENCY FOR TWO LINEAR DEMANDS

