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Equilibrium structures in vertical oligopoly

Masahiro Abiru[a,*], Babu Nahata[b], Subhashis Raychaudhuri[a], Michael Waterson[c]

[a] *Department of Economics, Fukuoka University, Nanakuma, Fukuoka 814-0133, Japan*
[b] *Department of Economics, University of Louisville, Louisville, KY 40292, USA*
[c] *Department of Economics, University of Warwick, Coventry, CV4 7AL, UK*

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Abstract

The central purpose of this paper is to examine vertical integration as an equilibrium phenomenon. We model it as integration between Cournot oligopolists in both the upstream and the downstream stages. We consider the issue of private profitability versus collective profitability and show that under several situations the equilibrium outcomes may result in a Prisoner's dilemma. The analysis is extended to consider equilibrium outcomes in a dynamic setting, where we find no integration to be a relatively common outcome. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Consider an industrial sector in which there are two potentially separable levels of production (industries). How is the equilibrium structure determined? Essentially there are two approaches to this question, one focusing on transactions costs issues, and the other on market structure.

Our concern hereafter is with the latter. Though it is clear that forces associated with both factors will have an impact in practice, we feel there are forces in the latter approach which still remain unexplored. When market imperfections exist in both the upstream and

* Corresponding author. E-mail: abiru@jsat.fukuoka-u.ac.jp

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the downstream industries, double marginalization is a clear possibility. This, together with the market structure at each stage, jointly determine the incentive for vertical integration which in turn determines the equilibrium vertical structure. For example, it is well known in the literature that vertically integrated monopoly is an equilibrium structure when successive monopoly would be the alternative. Greenhut and Ohta (1979) show that an integrated duopoly is also the equilibrium structure when successive duopoly would be the alternative.[1] As long as there are equal numbers in both industries, Greenhut and Ohta's conclusions can be extended to successive oligopoly. The intuition behind this is simple. The elimination of double marginalization lowers the cost for the firm that integrates and hence it has a cost advantage which creates a profit incentive for vertical integration. But a firm that remains unintegrated suffers a cost disadvantage and hence it too has an incentive to integrate. Therefore, regardless of what other firms do, each firm has an incentive to integrate (the dominant strategy for each firm is to integrate) and as a result a completely integrated industry emerges as an equilibrium structure in the equal number case.

When numbers at each stage are *unequal*, the elimination of double marginalization offers a cost advantage to an integrating firm. However, the cost disadvantage of unintegrated firms may not be large depending on the difference between numbers at each stage. As a result, the elimination of double marginalization may not provide a sufficiently strong incentive for integration. In the equal number case not only is the degree of competitiveness the same at each stage but also it does not change when any firm integrates. This is not the case with unequal numbers, where the degree of competitiveness also changes. In general, when there are unequal numbers there is no unique dominant strategy, either to integrate or not to integrate for all combinations of numbers of firms at each stage.

Whilst we investigate these various possibilities, our main purpose is to address a somewhat deeper issue. Modelling vertical mergers should, we believe, be predicated on investigating a move from the status quo to an equilibrium, which may either result from cost or demand changes, or more interestingly, a change in behavior. (Of course, this equilibrium might not be reached as a result of antitrust action). Our focus on equilibrium contrasts with much of literature, for example, Salinger (1988) which examines an incremental merger in an industry, *ceteris paribus*. However, we consider that there are often patterns concerning vertical integration which involve most or all of the industry players at or around a particular time. We investigate this issue by examining equilibrium in both static and dynamic terms.[2]

The possible equilibrium structures that can emerge in the presence of double marginalization with an unequal number of firms at each stage is something that has

[1] Greenhut and Ohta show that for a successive duopoly at each stage, producing a homogeneous product, integrated duopoly is an equilibrium structure. Bonanno and Vickers (1988) consider duopoly differentiated products with price setting and conclude that vertical separation is an equilibrium structure.

[2] Examples of industry-wide patterns of change include the cement–concrete industry in the US (Johnson and Parkman, 1995, 1987), soft drink distribution (Muris et al., 1992), and motor car bodies in the UK (Blois et al., 1975). A particularly interesting case is textiles in the early decades of this century, where in the US, vertically integrated firms held sway, whereas attempts at integration in the UK were frustrated (see e.g., Chandler, 1990).

largely remained an open question in the literature. This paper is an attempt to fill that void.[3]

We use a straightforward two-stage Cournot oligopoly model with specific demand and cost functions. In the first stage, firms simultaneously decide whether or not to integrate. The second stage is the market stage in which firms choose output based on the industry structure developed in the first stage. In this model, we characterize the range of equilibrium industry structures and their determinants in order to provide a better understanding of the extent of vertical integration.[4]

Private incentive is the main force that triggers integration. As emphasized by Greenhut and Ohta (1979) p. 140 "... vertical integration requires profit incentives ... However, such mergers do not require greater industry profits." Our analysis confirms this insight, for at the resulting (static) equilibrium industry structure, profits may be higher or lower than the starting point of our model, the no integration industry structure. In an oligopoly the private incentive for a particular firm significantly depends on the reactions of the rival firms and hence a vertical equilibrium structure that is privately profitable may not be collectively profitable. As Lin (1988) pointed out, the existence of such Prisoner's dilemma type situations is not well understood. By characterizing the static equilibrium structures, we provide a better understanding of this private versus collective profitability issue. This then leads us to develop important extensions of the model to a dynamic setting. In particular, we focus on a game in which unintegrated players can choose to integrate in each period, but integration is not easily reversible.

The paper is organized as follows. Section 2 outlines the basic framework. In Section 3 we characterize the static equilibrium structures using the Nash equilibrium concept and then discuss the various implications of the different structures. This gives a policy-relevant backdrop to the question of choices between equilibria. Section 4 discusses the dynamic extensions of our model, whilst Section 5 concludes.

2. Basic framework

The purpose of this section is essentially to set out the framework and derive some key equations for the market stage of our game. Consider successive oligopolies where in an unintegrated state there would be N identical Cournot oligopolists in the upstream stage and m identical Cournot oligopolists in the downstream stage (all notations are defined in

[3] In fact, many recent analyses have modeled vertical integration in specific contexts, see for example, Hart and Tirole (1990); Ordover et al. (1990), Williamson (1990); Grossman and Hart (1986); Bolton and Whinston (1993); Colangelo (1995); Hamilton and Mqasqas (1996). Although all these models have provided a better understanding of vertical integration under different settings, the effect of unequal numbers at the two stages has barely been analyzed. At the late stage in the development of our paper a largely complementary analysis starting from the same basic framework has appeared, namely Gaudet and van Long (1996). Our paper focuses significantly more attention on the equilibrium concept and on the unequal numbers case, whereas they engage in examination of the foreclosure issue.

[4] Clearly a more general framework incorporating additional factors would be superior, but our setting has the advantage of focussing on the problem in hand.

Table 1
Notations

A and B	The two inputs used
m	Number of firms in the downstream industry
N	Number of upstream firms producing input A
μ	Number of integrated firms
$A_n(\mu)$	Quantity of input sold by a non-integrated upstream firm to a non-integrated downstream producer
$X(\mu)$	Total output produced by downstream industry
$X_n(\mu)$	Total output of all non-integrated downstream producers $=(N-\mu)A_n$
$x_i(\mu)$	Output of an integrated firm $= (X-X_n)/\mu$
$x_n(\mu)$	Output of a non-integrated downstream firm $=X_n/(m-\mu)$
c_i	Marginal (=average) cost of an integrated firm
$c_n(\mu)$	Marginal (=average) cost of a non-integrated firm
$P_x(\mu)$	Price of the final product when μ firms are integrated
η	The price elasticity of demand$=-\frac{dX}{dP}\frac{P}{X}=\frac{a-bX}{bX}$
M_a	Marginal cost of input A, assumed to be constant
$P_a(\mu)$	Profit-maximizing price for input A charged by the non-integrated upstream firms to the non-integrated downstream firms
P_b	Price of input B which is supplied by a competitive industry
$\Pi^u(\mu)$	Profits of each non-integrated upstream producer when μ firms are integrated
$\Pi^d(\mu)$	Profits of each non-integrated downstream firm when μ firms are integrated
$\Pi^{int}(\mu)$	Profits of each integrated firm when μ firms are integrated

All variables are written as functions of μ and parameters are independent of μ.

Table 1). If unintegrated, each upstream firm buys the raw material from a competitive market at given unit price M_a, and sells a homogeneous input A at price P_a in an oligopolistic market. Each downstream firm produces a final product using this input A and an input B which is available at given unit price P_b in a competitive market. The downstream firm has no oligopsony power over the input and so treats its price P_a as given. It chooses its final product level x_n so as to maximize profit. We specify linear final demand and assume fixed-proportions technology where one unit of output requires one unit of each of the two inputs.[5]

We obtain expressions for profits for both the upstream and the downstream firms under the assumptions that an arbitrary number of firms ($=\mu$) is integrated and that the integrated firms neither supply the input to non-integrated downstream firms, nor purchase inputs from non-integrated upstream firms.[6] This arbitrary state of course need not represent an equilibrium state.

The final demand function is assumed to be

$$P_x = a - bX, \quad a,b > 0. \tag{1}$$

[5] Our framework is similar to the models used by both Greenhut and Ohta and Salinger. Greenhut and Ohta use more general demand and cost specifications. Differently from us, the main focus of these papers is on welfare and they do not consider the existence of the different equilibrium structures that may result.

[6] Salinger provides an economic rationale for the integrated firms not to supply inputs to non-integrated downstream firms. A formal mathematical analysis leading to the same conclusion is provided by Schrader and Martin (1995). See also Gaudet and van Long, who made some investigation on the question.

2.1. Profit-maximization in downstream industry

The profit of each integrated firm is defined by

$$\Pi^{\mathrm{int}}(\mu) = P_{\mathrm{x}}x_i - (M_{\mathrm{a}} + P_{\mathrm{b}})x_i, \quad i = 1, 2, \ldots, \mu. \tag{2}$$

The profit of each non-integrated firm is defined by

$$\Pi^{\mathrm{d}}(\mu) = P_{\mathrm{x}}x_n - (P_{\mathrm{a}} + P_{\mathrm{b}})x_n, \quad n = \mu + 1, \mu + 2, \ldots, m. \tag{3}$$

The profit maximization condition for each integrated firm is

$$P_{\mathrm{x}} - bx_i = M_{\mathrm{a}} + P_{\mathrm{b}}, \quad i = 1, 2, ..., \mu. \tag{4}$$

The profit maximization condition for each non-integrated firm is

$$P_{\mathrm{x}} - bx_n = P_{\mathrm{a}} + P_{\mathrm{b}}, \quad n = \mu + 1, \mu + 2, \ldots, m. \tag{5}$$

Aggregating in Eqs. (4) and (5) we obtain

$$\mu P_{\mathrm{x}} - \mu bx_i = \mu(M_{\mathrm{a}} + P_{\mathrm{b}}). \tag{6}$$

$$(m - \mu)P_{\mathrm{x}} - (m - \mu)bx_n = (m - \mu)(P_{\mathrm{a}} + P_{\mathrm{b}}). \tag{7}$$

Summing Eqs. (6) and (7) and noting $\mu x_i + (m-\mu)x_n = X$ we get

$$mP_{\mathrm{x}} - bX = (m - \mu)P_{\mathrm{a}} + mP_{\mathrm{b}} + \mu M_{\mathrm{a}}. \tag{8}$$

which holds when partial integration applies, that is, $\mu < \min\{N, m\}$. From Eqs. (1) and (8) we obtain the derived demand for the intermediate good in terms of the parameters and the total output of the final product X:

$$P_{\mathrm{a}} = \frac{m(a - P_{\mathrm{b}}) - \mu M_{\mathrm{a}}}{m - \mu} - \frac{b(m + 1)}{m - \mu}X. \tag{9}$$

We can also derive the expression for derived demand in terms of total output of non-integrated firms, for example, $(m - \mu)x_n$ after obtaining X from Eqs. (7) and (1) and substituting it into Eq. (9) as follows

$$P_{\mathrm{a}} = \frac{a - P_{\mathrm{b}} + \mu M_{\mathrm{a}}}{\mu + 1} - \frac{b(m + 1)}{(\mu + 1)(m - \mu)}X_n. \tag{10}$$

where $X_n = (m - \mu)x_n = (N - \mu)A_{\mathrm{n}}$.

2.2. Profit-maximization in upstream industry

Profit of each non-merging upstream firm is defined by

$$\Pi^{\mathrm{u}}(\mu) = P_{\mathrm{a}}A_n - M_{\mathrm{a}}A_n, \quad n = \mu + 1, \mu + 2, \ldots, N. \tag{11}$$

The profit-maximization condition yields

$$P_{\mathrm{a}}\left(1 - \frac{1}{(N - \mu)E}\right) = M_{\mathrm{a}},\ E = -\frac{\mathrm{d}A}{\mathrm{d}P_{\mathrm{a}}}\frac{P_{\mathrm{a}}}{A}. \tag{12}$$

2.3. Profit functions

Eqs. (10) and (12) give the equilibrium input price P_a as follows

$$P_a = \frac{(a - P_b - M_a)}{(\mu + 1)(N - \mu + 1)} + M_a. \tag{13}$$

Now we can solve for the total output X from Eqs. (9) and (13)

$$X = \frac{(a - P_b - M_a)(mN\mu - m\mu^2 + mN + \mu)}{b(m + 1)(\mu + 1)(N - \mu + 1)}. \tag{14}$$

Using Eqs. (10) and (13) we obtain

$$X_n = \frac{(a - P_b - M_a)(N - \mu)(m - \mu)}{b(m + 1)(N - \mu + 1)}. \tag{15}$$

Final product price is then obtained from Eq. (1) as [7]:

$$P_x = \frac{a(N\mu + N - \mu^2 + m - \mu + 1) + (P_b + M_a)(mN\mu - m\mu^2 + mN + \mu)}{(m + 1)(N - \mu + 1)(\mu + 1)}. \tag{16}$$

Each upstream firm's profit is obtained using Eqs. (11),(13) and (15), and noting that $A_n = X_n/(N - \mu)$

$$\Pi^{u}(\mu) = \frac{(m - \mu)(a - P_b - M_a)^2}{b(\mu + 1)(m + 1)(N - \mu + 1)^2}. \tag{17}$$

Each downstream firm's profit is obtained using Eqs. (1),(3),(8) and (14) and noting that $x_n = X_n(m - \mu)$

$$\Pi^{d}(\mu) = \frac{(a - P_b - M_a)^2 (N - \mu)^2}{b(m + 1)^2 (N - \mu + 1)^2}. \tag{18}$$

Each integrated firm's profit is obtained using Eqs. (1),(2),(4),(8) and (14)

$$\Pi^{int}(\mu) = \frac{1}{b}\left[\frac{(a - P_b - M_a)[(m + N + 1) + \mu(N - \mu - 1)]}{(\mu + 1)(m + 1)(N - \mu + 1)}\right]^2. \tag{19}$$

Eqs. (17)–(19) constitute the core elements in our analysis below. When $\mu = \min\{N, m\}$ the above expression reduces to:

$$\text{For } N < m, \quad \Pi^{int}(N) = \frac{(a - P_b - M_a)^2}{b(N + 1)^2}. \tag{20}$$

$$\text{For } N > m, \quad \Pi^{int}(m) = \frac{(a - P_b - M_a)^2}{b(m + 1)^2}. \tag{21}$$

[7] These expressions are similar to Salinger's but there is an innocuous misprint in his Eq. (4).

3. Equilibrium industry structure

As mentioned earlier, the main objective of this paper is to develop a model in which the extent of vertical merger between firms emerges as an equilibrium strategic decision of the firms. To this end, we can think of a situation where firms play a two-stage game. In the first stage, firms *simultaneously* decide whether or not to integrate. Once such decisions are made by all firms, the game enters in the second stage. In this stage, firms choose their optimal level of output given the industry structure developed in the first stage.[8] In Section 2, we showed the profits or the payoffs of the firms if any arbitrary number of firms, μ, were to be integrated. Here, we first formally describe a normal form game and then characterize the equilibrium industry structures based on firms' decisions of whether or not to integrate. In Section 4 we extend the analysis by considering a more dynamic game.

3.1. Description of the game

A one-shot normal form game G is played between J players. A player of G is defined as an *arbitrarily matched pair* of an upstream and a downstream firm. If there are N upstream firms and m downstream firms, then the number of players in G is given by $J=\min\{N, m\}$. An unmatched firm is *not* a player.

Each player i must choose a *strategy* s_i from its *strategy set* $S_i=\{0, 1\}$ where 0 implies no integration and 1 means integration. In the formal game, players choose their strategies *simultaneously.* Let $s=(s_1, \ldots, s_J)$ be a *strategy profile* and S be the set of all possible strategy profiles. The total number of players, μ_s who choose to integrate is given by: $\mu_s=\sum_{i=1}^{J} s_i$. We denote the resulting *industry structure* when the strategy profile s is observed as μ_s. Several strategy profiles can give rise to the same industry structure. Hence we form a partition[9] ρ of S in such a way that each strategy profile within an element of ρ has the same industry structure. We denote the element of ρ that contains the strategy profile s as $\rho(s)$. Each player i's payoff $v_i : S \to R$ is given by:

$$\begin{aligned} v_i(s) &= \Pi^{\mathrm{int}}(s) && \text{if } s_i=1 \\ v_i(s) &= \Pi^{\mathrm{u}}(s)+\Pi^{\mathrm{d}}(s) && \text{if } s_i=0 \end{aligned}$$

where $\Pi^{\mathrm{int}}(\cdot)$, $\Pi^{\mathrm{u}}(\cdot)$ and $\Pi^{\mathrm{d}}(\cdot)$ are given by Eqs. (17)–(19). Note that an unmatched firm does not play any decision-making role in our model, but their presence may affect the payoffs of the players. This completes the description of G.

Our solution concept is the (Nash) equilibrium of the above game. An *equilibrium* is a strategy profile s^* such that for all i,

$$v_i(s^*) \geq v_i(s_i, s^*_{-i}). \tag{22}$$

[8] The recently published Gaudet and van Long has a similar, though rather less formal structure.

[9] A *partition* of a set X is a collection $\{X_1, \ldots, X_k\}$ of disjoint subsets of X such that $\cup_{z=1}^{k} X_z = X$.

for all $s_i \in S_i$, where the subscript $-i$ is used to denote the strategy profile without the i-th element.

Remark 1. *Note that the above equilibrium concept is defined on strategy profiles, not on the upstream and downstream industry structures (N, m). If the players choose to play an equilibrium strategy profile then an equilibrium industry structure will result. The initial state (that is, N and m) is considered here as the primitive (that is, exogenously given) of the game. It is important to distinguish between the equilibrium (dis-equilibrium) industry structure, which is the result of an equilibrium (dis-equilibrium) strategy profile and the exogenously given primitive.*

Remark 2. It should be noted that our model is in sharp contrast with Salinger's model despite apparent similarities. In Salinger's model a one-shot game is played by a single player and he addresses the issue of what happens to the market price and the profit of the player (that is, the merging firms) when the game ends. In our model all J players play the game simultaneously and we focus on what happens to the resulting equilibrium industry structure. Even though other players (in our sense) are present in Salinger's model, they do not play any decision-making role. Hence his model cannot address the issue of equilibrium industry structure when there are J players.

The following lemma shows the relationship between an equilibrium strategy profile and the resulting industry structure.

Lemma 1. (a) *Suppose that a strategy profile s^* is an equilibrium of G and μ_{s^*} is the resulting industry structure. Then,*

(i) *if $0 < \mu_{s^*} < J$, then μ_{s^*} satisfies the following two conditions.*

$$\Pi^{\mathrm{int}}(\mu_{s^*}) \geq \Pi^{\mathrm{u}}(\mu_{s^*} - 1) + \Pi^{\mathrm{d}}(\mu_{s^*} - 1) \tag{23}$$

$$\Pi^{\mathrm{int}}(\mu_{s^*} + 1) \leq \Pi^{\mathrm{u}}(\mu_{s^*}) + \Pi^{\mathrm{d}}(\mu_{s^*}) \tag{24}$$

(ii) *if $\mu_{s^*} = \beta$, then condition* Eq. (23)*is satisfied. Condition* Eq. (24)*is satisfied when $\mu_{s^*} = 0$.*

(b). *If any μ satisfies the above conditions, then there exists a strategy profile s^* such that each $s \in \rho(s^*)$ is an equilibrium of G with $\mu_{s^*} = \mu_s = \mu$.*

In the above lemma, condition Eq. (23) simply means that at the equilibrium the integrated firms have no incentive to choose any other strategy and condition Eq. (24) implies that at the equilibrium unintegrated firms do not wish to merge.

An industry structure $\mu_{s^*} = J$ refers to the case where firms are completely integrated and $\mu_{s^*} = 0$ refers to the case where firms are completely unintegrated.

We call μ_{s^*} an *equilibrium industry structure* if s^* is an equilibrium of G. In the next subsection we characterize the equilibrium structures of the industry for different values of (N, m).

3.2. Characterization of the static equilibrium

Using the equilibrium concept defined above we state the main results in the following propositions that characterize all possible equilibrium industry structures. Proofs of both propositions are given in the appendix.

Proposition 1. *When the number of firms in the upstream industry (N) is the same or larger than the number of firms in the downstream industry (m), then* $\mu_{s^*} = J$ *(complete integration) is the unique equilibrium industry structure of G.*

Proposition 2. *When the number of firms in the upstream industry (N) is smaller than the number of firms in the downstream industry (m) then*:

(a) $\mu_{s^*} = 0$ *(no integration) is an equilibrium structure, if and only if,* $m > R$

(b) $\mu_{s^*} = J$ *(complete integration) is an equilibrium structure, if and only if,* $m < S$

(c) $0 < \mu_{s^*} < J$ *(partial integration) is an equilibrium structure, if and only if,* $S < m < R$ *where*,

$$R \equiv \frac{2N^3 + N^2 - 1 + 2N\sqrt{N^4 + 4N^3 - 4N^2 - 2N + 2}}{(3N^2 - 2N + 1)}$$

and,

$$S \equiv \frac{(N^3 + 5N - 2) + \sqrt{(N^3 + 5N - 2)^2 - 4(N - 1)^4}}{2(N - 1)^2}$$

Fig. 1 graphically depicts the propositions and we discuss the uniqueness of each equilibrium structure by dividing (*N*, *m*) space into areas.[10] Each region is marked with a number. Line $m = N$ represents the equal number case. Below this line, $N > m$ and vice versa. Line TCFH represents the equation $m = R$ [11] and curve AEFJ represents the equation $m = S$.[12] For $N < m$, we draw a curve BG that represents the combinations of *N* and *m* such that the profit of an integrated pair if the industry were to be fully integrated is equal to the sum of the profits of an upstream firm and a downstream firm when none of the pairs are integrated.[13] To the left of this curve and above TC (regions 1 and 6) full integration is more profitable to each player than no integration and the opposite is the case for the regions to the right of the curve BG and above CH (region 2 and 7). Similarly,

[10] This can be seen as a generalization of Gaudet and van Long which considers outcomes along the axes and the diagonal only.

[11] The equation is derived using $\Pi^{\mathrm{int}}(1) = \Pi^{\mathrm{d}}(0) + \Pi^{\mathrm{u}}(0)$. For complete derivation see proof of Proposition 2a in the appendix.

[12] The equation is derived using $\Pi^{\mathrm{int}}(N) = \Pi^{d}(N - 1) + \Pi^{u}(N - 1)$. For complete derivation see proof of Proposition 2b in the appendix.

[13] The equation for the curve BG is $m = N^2 - 1$ which is obtained by using $\Pi^{\mathrm{int}}(N) = \Pi^{u}(0) + \Pi^{d}(0)$. In Eqs. (17) and (18) by putting $\mu = 0$, and equating the sum to Eq. (20), the stated equation for curve BG follows immediately.

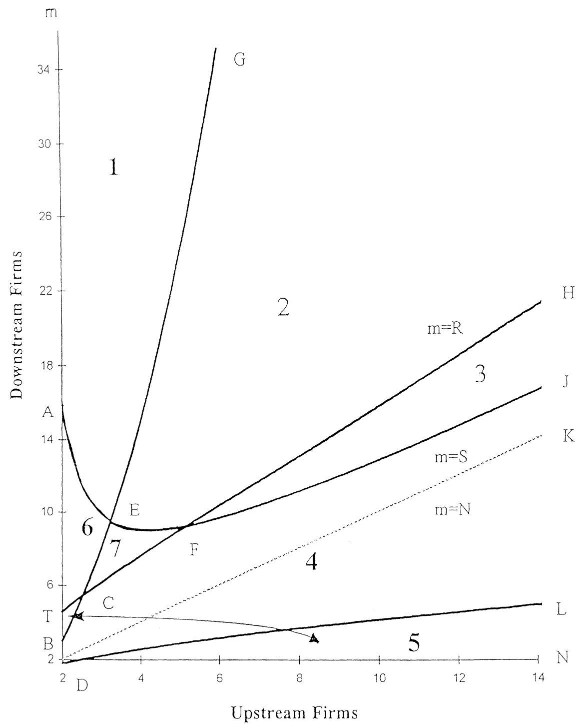

Fig. 1. Equilibrium structures.

for $N > m$, we draw the curve DL that represents those combinations of N and m for which the profit of an integrated pair if the industry were to be fully integrated is equal to the sum of the profits of an upstream firm and a downstream firm when no firms would have integrated.[14] Table 2 describes the static equilibrium which result in the various regions of Fig. 1, together with the propositions which support them.

3.3. Implications for equilibrium structures

The results in the previous subsection suggest that although complete integration can be an equilibrium under all the three cases ($N <$ m, $N =$ m and $N > m$), it is the only equilibrium when the number of upstream firms exceed the number of downstream firms.

This observation may be framed as a prediction about the observability of this outcome. Suppose vertical integration is freely allowed, and our model closely represents reality (clearly the latter point is debatable, but we mean merely to indicate trends). Then we would expect to observe a branch of industry integrated where there were more firms

[14] The equation for the curve DL is $N = (m^2 + m - 1)/2$ which is obtained by using $\Pi^{\text{int}}(m) = \Pi^{u}(0) + \Pi^{d}(0)$. In Eqs. (17)–(19), by putting $\mu = 0$, and equating the sum to Eq. (21), the desired result follows.

Table 2
Description of equilibrium structures

Region	Equilibrium structures	Remarks: integration is	Conditions involved
1	No integration	privately unprofitable but collectively profitable	Proposition 2(a)
2	No integration	privately unprofitable and collectively unprofitable	Proposition 2(a)
3	Partial integration	privately profitable but collectively unprofitable	Proposition 2(c)
4	Complete integration	privately profitable but collectively unprofitable	Propositions 1 and 2(b)
5	Complete integration	privately profitable and collectively profitable	Proposition 2(b)
6	No integration and complete integration	privately unprofitable but collectively profitable	Propositions 2(a) and 2(b)
7	No integration and complete integration	privately unprofitable but collectively profitable	Propositions 2(a) and 2(b)

at the upstream stage than the downstream stage. This is consistent with observation of the oil industry, in which all major refiners have many wholly-owned extraction sites (the number of which is larger than the number of major refiners), and nearly all such sites are in the hands of major refiners. Yet at the same time, not all gas stations are owned by the refiners, a finding consistent with Proposition 2.[15] Our claim is that our model may suggest some general influences, not that it is the only influence.

In a static framework complete integration is the unique equilibrium industry structure both in regions 4 and 5 because elimination of double marginalization together with the industry structure always provides a profit incentive for integration. In spite of private incentives for integration, comparison between these regions shows that while in region 5 complete integration makes all integrated pairs better off compared with no integration, the opposite is the case in region 4. This is what Greenhut and Ohta observed when $N = m = 2$. Our analysis clearly shows that this situation is not limited to the 2×2 case but can exist in a large number of cases. The reason for such a Prisoners' dilemma is fairly straight forward. When there are two or more firms at each level, markups are relatively low compared with the monopoly position. Vertical *separation* raises markups as a result of double marginalization, moving final prices nearer to the monopoly level than they would be under integration and thereby creating higher profits in the industry than under integration. However for each individual player, *integration* reduces the marginal cost and thereby yields a greater share of the final market, by standard Cournot mechanisms.

On the other hand, no integration can only be an equilibrium when $N < m$. Both in regions 1 and 2 no integration is a unique equilibrium structure. In region 1 complete integration is more profitable than no integration and the opposite is true for region 2.

[15] There is one major industry where structure is inconsistent with Proposition 1, namely agriculture. Thousands of small farms exist, unintegrated with food processors. However, this (at least in Europe and to a large extent in the US) is not a free market outcome but rather an industry where prices are externally managed in such a way as to maintain farm incomes.

In all the regions discussed so far, there is only a single static equilibrium in our game. However, because both Propositions 2a and 2b are satisfied in regions 6 and 7, there are two possible static equilibria in these regions, complete integration and no integration, one being Pareto superior to the other. Existence of multiple equilibria poses serious problems in selecting one equilibrium as the most plausible. In the static game-theoretic literature the criteria for selecting a particular equilibrium are not very well understood. If Pareto superiority is the sole criterion of selection then in region 6 complete integration will be observed as an equilibrium structure and no integration in region 7.

Partial integration as an equilibrium structure exists only in region 3 (see Proposition 2c), where at least one pair is not integrated. To understand the difference between the partially integrated industry in our model and in Salinger's model note that in the present context all players move simultaneously and the resulting structure is static equilibrium structure. Hence,the question of an additional merger does not arise. On the other hand Salinger starts out with a partially integrated industry and considers whether or not integration is profitable for a single player, *ceteris paribus*[16]

3.4. Social welfare

For completeness, we now evaluate the effect of vertical integration on the price of the final product. We compare prices between two states where the initial state (primitive) is assumed to involve no integration. Each firm has two options: integrate or not. The equilibrium state is defined to be one where no private profitability incentive exists for any further integration. The effect on the final product price and on welfare is summarized in the following Proposition and Corollary.

Proposition 3. *Compared to the initial state of no integration, the price of the final product is never higher at the static equilibrium.*[17]

Corollary. *The Marshallian measure of welfare is also higher in the static equilibrium*

Proof. See appendix

It is important to note that there is no conflict with the welfare result of earlier models. Here we are examining a model in which all players simultaneously decide whether or not to integrate. In our frame work with $N=6$ and $m=10$ the equilibrium structure of a one-shot game has five players integrated. This contrasts with Salinger who (inter alia) examined the case where there were four integrated firms and found that if an additional merger takes place (since the merger is profitable it should occur) the price of the final product rises. What we compare is the price at this equilibrium state with the initial state when none of these firms were integrated and show that the price at the equilibrium state

[16] Many combinations of *m*, and *N* shown in his Table 1 (p. 354) will not exist as partial integration equilibria in our model. For example, when $m=6$ and $N=2, 3$ in our model the equilibrium industry structure is either no integration or full integration (regions 6, and 7) and hence the industry will not be partially integrated. The same is true for $m=7$ and $N=2, 3, 4$ and $m=8$ and $N=2$,3, 4, 5 and $m=9$ and $N=2, 3, 4, 5, 6$.

[17] This Proposition is essentially a restatement of Greenhut and Ohta's conclusion. Abiru (1988) reaches the same conclusion when the production function is of CES type.

is lower than the initial state of no integration. Since the integration decision is not sequential in our formal model developed in this section, we do not claim that price falls continuously on a sequential merger path.

4. Discussion of dynamic extension

Our game in Section 3 is static in nature, that is, firms simultaneously decide whether or not to integrate. In this static game, we characterized equilibrium industry structures for different values of m and N. Fig. 1 shows that partial integration exists in equilibrium only when (N, m) lies in Region 3. In all other regions either complete or no integration exists. But if we extend our game to a dynamic setting the resulting equilibrium industry structures may change. For example, partial integration may exist in equilibrium in region 4 as well. The purpose of this section is to explore some of the complexities of dynamic extensions and to point out the possible subgame perfect equilibrium outcomes in these extensions.

Several approaches might be considered in a dynamic setting. One is the standard infinitely repeated game approach. Players make their decision of whether or not to integrate repeatedly over time. In other words, choosing integration in one period can readily be revocable in the next period. We conjecture that in such a game, there will be a folk theorem type result – every individually rational outcome can be sustained as a subgame perfect equilibrium outcome. In particular, for region 4, all the three industry structures, namely complete, partial, and Pareto-superior no integration can be sustained as an outcome in a subgame perfect equilibrium.

However, a second, and we consider more convincing, approach would have integration by the players as a process which is irrevocable, or at least involves substantial sunk cost. We use this approach to consider a more interesting dynamic extension Γ of our static game. Consider a dynamic game Γ, where in each period $t = 1, 2, \ldots$, players decide simultaneously whether or not to integrate. Once a player chooses to integrate, it remains integrated in all subsequent periods. The game effectively ends when all players are integrated. That is, once all players choose to integrate, each player enjoys full integration payoff in all successive periods from then on. Note that in this framework the game may continue for ever if some players decide, in all periods, not to integrate.

Suppose that the payoff of each player in Γ is the discounted sum (with a common discount factor) of the payoffs that it receives in each time period. Let us also assume that the players only choose pure strategies in Γ. Note that the folk theorem is no longer applicable, since it is not a standard repeated game. We now investigate which industry structures *fail* to be subgame perfect equilibrium outcomes of Γ.

First, observe that Pareto superior complete integration cannot be sustained as a subgame perfect equilibrium outcome of Γ if (N, m) lies in region 1. There is no private incentive for integration since full integration is not an equilibrium of the first period game of Γ. Since integration is irrevocable, by remaining unintegrated a player can enjoy a higher payoff at the expense of the other players who integrate. In the absence of an effective means of achieving complete integration, the Pareto-superior collusive outcome will not occur in Γ. Similar reasoning establishes that complete integration cannot be sustained as a subgame perfect equilibrium outcome for region 2 either.

Next we show that, in general, partial integration *does not always* exist as a subgame perfect equilibrium outcome of Γ for every (N, m) in region 4. First, note that for such a game Γ there are always two subgame perfect equilibria. In one full integration is the outcome. In this equilibrium, the strategy of each player i is to choose integration at each of i's decision nodes. Since players move simultaneously in each time period, for any subgame, choosing integration is optimal given that all other players do the same in that subgame.

In the other subgame perfect equilibrium, no integration is the outcome provided the discount factor is sufficiently high. In this equilibrium each player i chooses a threat strategy of no integration as long as the other players do not integrate. If in any period player $j \neq i$ deviates by switching to integration then in the next period player i also switches to integration. That is, players may tacitly collude to prevent integration by keeping to a no-integration strategy so long as other players do likewise. This may perhaps have been a factor in the British textile experience alluded to in Footnote 2.

Consider now any three-player *static* game in region 4. That is, $\min\{N, m\} = 3$. If we extend this three-player game to our dynamic game Γ, is it possible to have a subgame perfect equilibrium of this dynamic game Γ in which partial integration is an outcome?

It is clear that there is no subgame perfect equilibrium in which only two integrations take place in the course of the game Γ and the third player remains unintegrated forever. The reason is that once two players integrate in the course of the game, the remaining player finds it beneficial to integrate in the next period if the common discount factor is positive.

We now show that it is also not possible to have a subgame perfect equilibrium in which only one integration takes place in the course of the game Γ and the remaining two players stay unintegrated for ever. The reason is that once a player chooses to integrate in some period T, the remaining two players face the following game in the first period of the subgame that starts at period (T+1):

	0	1
0	$\pi^n(1), \pi^n(1)$	$\pi^n(2), \pi^{int}(2)$
1	$\pi^{int}(2), \pi^n(2)$	$\pi^{int}(3), \pi^{int}(3)$

Here $\Pi^n(\cdot)=\Pi^u(\cdot)+\Pi^d(\cdot)$. We now state the following lemma, the proof of which is given in the appendix.

Lemma 2. *If (N, m) lies in region 4 and min* $\{N, m\} = 3$, *then*

$$\max\{\Pi^n(1), \Pi^n(2)\} < \min\{\Pi^{\text{int}}(2), \Pi^{\text{int}}(3)\}$$

From the above lemma, it is easy to see that (1, 1) is the unique Nash equilibrium of the above two-player game. Moreover, (1, 1) is Pareto undominated and thus integration is the only rational choice for each player in the first period of the subgame starting at period $(T + 1)$. Hence, in our entire dynamic game Γ with three players, it is not possible to have a subgame perfect equilibrium in which only one integration takes place. To summarize

Proposition 4. *Suppose that (N, m) lies in region 4 and min* $\{N, m\} = 3$. *Then the dynamic game* Γ *has only two pure strategy subgame perfect equilibria. In one, complete integration takes place, and in the other, no integration takes place*[18].

It remains an open question to us whether such a Proposition holds true for four or more players. However we have established that the equilibrium industry structure crucially depends on the way one models a dynamic extension and that not every possible degree of integration may be assumed to be an equilibrium. This again relies upon the setting of our dynamic game Γ in which integration is irrevocable and hence disintegration is viewed as having an infinite cost. But we would argue that the process of disintegration is time consuming and in any dynamic extension one would wish to include such a feature.

5. Conclusions

Our model is clearly specific.[19] Yet, the results obtained suggest outcomes that one would expect to remain important under a more general framework. When upstream and downstream numbers are not equal, even with a specific static model, all three equilibrium structures, namely, no integration, partial integration or complete integration are possible for particular number constellations, a conclusion that is significantly different from the equal number case which only admits complete integration as an equilibrium. When we extend the analysis to a dynamic game, we find these are powerful forces suggesting vertical separation as an equilibrium outcome. This is a novel result.

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[18] The reason that Γ has two pure strategy subgame perfect equilibria is because in each period all players move *simultaneously*. Let us for a moment consider a different dynamic extension where in period 1, only player 1 decides whether or not to integrate and other players simply observe the move of player 1. In period 2, only player 2 decides whether or not to integrate, and in period 3, only player 3 decides whether or not to integrate based on the moves of the previous players. Further, if all players receive the payoff only at the end of this three-period game, then under the assumption on (N, m) in Proposition 4 nointegration would be the *unique* subgame perfect equilibrium (a formal proof is available from the authors).

[19] For example, we rule out any form of entry and also do not consider the possibility of any type of trading between integrated and non-integrated firms. Allowing mergers only in one-to-one ratio of upstream to downstream firms may be a restrictive framework. We leave investigation of these important considerations for future research.

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Appendix

Proof of Proposition 1. Proposition 1 is true if for any $m \leq N$, inequality Eq. (23) holds for $1 \leq \mu + 1 \leq m$. Inequality Eq. (23) can be written as (Using Eqs. (17)–(19)):

$$\left(\frac{m+1+(\mu+2)(N-\mu-1)}{(\mu+2)(N-\mu)}\right)^2 > \frac{(\mu+1)(N-\mu)^2+(m-\mu)(m+1)}{(\mu+1)(N-\mu+1)^2}$$

Rewriting the above expression, we get

$$\frac{(m+1)}{(\mu+2)(N-\mu)} + \frac{N-\mu-1}{N-\mu} > \frac{N-\mu}{N-\mu+1}\left[1+\frac{(m-\mu)(m+1)}{(\mu+1)(N-\mu)^2}\right]^{\frac{1}{2}}$$

Using the binomial inequality $(1+y)^{\alpha} \leq 1+\alpha y$ for $0 < \alpha < 1$ and $y \geq 0$ it is sufficient to prove that

$$\frac{(m+1)}{(\mu+2)(N-\mu)} + \frac{N-\mu-1}{N-\mu} \geq \frac{N-\mu}{N-\mu+1}\left[1+\frac{(m-\mu)(m+1)}{2(\mu+1)(N-\mu)^2}\right]$$

or,

$$\frac{(m+1)}{(\mu+2)(N-\mu)} - \frac{(m-\mu)(m+1)}{2(\mu+1)(N-\mu)(N-\mu+1)} \geq \frac{N-\mu}{N-\mu+1} - \frac{N-\mu-1}{N-\mu}$$

simplification yields

$$(m+1)[2(\mu+1)(N-\mu+1)-(m-\mu)(\mu+2)] \geq 2(\mu+1)(\mu+2)$$

Since $N-\mu \geq m-\mu$ it is enough to show that

$$(m+1)[2(\mu+1)(N-\mu+1)-(N-\mu)(\mu+2)] \geq 2(\mu+1)(\mu+2)$$

or,

$$(m+1)[2(\mu+1)(N-\mu)+2(\mu+1)-(N-\mu)(\mu+2)] \geq 2(\mu+1)(\mu+2)$$

or,

$$\mu(N-\mu)(m+1) \geq -2(\mu+1)(m-1-\mu)$$

the LHS > 0 and the RHS ≤ 0 for $1 \leq (\mu+1) \leq m$. □□

Proof of Proposition 2 (a) No integration is Nash equilibrium if $\Pi^{\text{int}}(1) < \Pi^{\text{d}}(0) + \Pi^{\text{u}}(0)$. Using Eqs. (17)–(19), the above condition is equivalent to

$$\left(\frac{2N+m-1}{2N}\right)^2 < \frac{N^2+m^2+m}{(N+1)^2}$$

The above expression may be written as

$$(3N^2 - 2N - 1)m^2 - 2(2N^3 + N^2 - 1)m - (4N^3 - 3N^2 - 2N + 1) > 0$$

For any given value of N there are two roots for m. The positive root represents the value of m that results in no integration as a Nash equilibrium.□□

Proof of Proposition 2 (b) Full integration is an equilibrium if proposition 2a does not hold and at the same time inequality Eq. (23) holds for $1 \leq \mu \leq N$. In other words, $\Pi^{\text{int}}(N) > \Pi^{\text{d}}(N-1) + \Pi^{\text{u}}(N-1)$. By Putting $\mu = N$ in Eqs. (17)–(19), we get

$$(N-1)^2 m^2 - m(N^3 + 5N - 2) + (N-1)^2 < 0$$

For any given numbers of upstream firms, N, the solution to the above inequality gives two roots for m. The positive root represents the value of below which m must be for full integration to be the equilibrium. □□

Proof of Proposition 2 (c) Partial integration is an equilibrium if inequalities Eqs. (23) and (24) hold for some value of μ in the range $1 < \mu < N$. The stated condition implies that Proposition 2a does not hold and the second inequality of the Proposition 2b does not hold for given combinations of m and N. To see that the set of (m, N) values are non-empty, consider for example $N = 6,\ m = 10$. Then Eqs. (23) and (24) are satisfied for $\mu = 5$. Thus, the statement of the Proposition represents partial integration as an equilibrium. □□

Proof of Proposition 3. It is equivalent to prove that $X(\mu = 0) < X(\mu)$ for $\mu > 0$. From Eq. (14) the expressions result in the following inequality.

$$\frac{(a - P_{\text{b}} - M_{\text{a}})mN}{b(m+1)(N+1)} < \frac{(a - P_{\text{b}} - M_{\text{a}})(mN\mu - m\mu^2 + mN + \mu)}{b(m+1)(\mu+1)(N-\mu+1)}$$

$$mN(\mu+1)(N-\mu+1)$$

$$< (N+1)(mN\mu - m\mu^2 + mN + \mu)$$

$$0 < (N+1) + m(N-\mu)$$

which is always true. □□

Proof of Corollary. The firms in the industry may be worse off as a result of integration. However, since we assume fixed proportions in production, there is no producer deadweight loss, whether or not inputs are priced above marginal cost. A lower final price implies greater final output which together with unchanged marginal cost of upstream firm raises the total welfare.

Proof of Lemma 2. Set $(a - P_{\text{b}} - M_{\text{a}})/b = 1$, since it appears in the expressions of all profit functions. First we consider the case when $m = 3$ and $N \geq 3$. In this case, using Eqs. (17)–(19), we obtain the following:

$$\Pi^n(1) = \frac{4 + (N-1)^2}{16N^2}; \Pi^n(2) = \frac{4 + 3(N-2)^2}{48(N-1)^2}; \Pi^{\text{int}}(2) = \frac{(3N-2)^2}{144(N-1)^2}; \Pi^{\text{int}}(3) = \frac{1}{16}$$

It is easy to see that if $N \geq 3$, then both $\Pi^n(1)$ and $\Pi^n(2)$ are less than 1/16 but $\Pi^{int}(2)$ is greater than $1/16$.

Since (m, N) lies in region 4, we now need to consider only the case where $N = 3$ and $3 \leq m \leq 7$. Again, using Eqs. (17)–(19) we obtained the following:

$$\Pi^n(1) = \frac{7+m^2}{18(m+1)^2};\Pi^n(2) = \frac{1-m+m^2}{12(m+1)^2};\Pi^{\text{int}}(2) = \frac{(m+4)^2}{36(m+1)^2};\Pi^{\text{int}}(3) = \frac{1}{16}$$

For $3 \leq m \leq 7$, both $\Pi^n(1)$ and $\Pi^n(2)$ are less than 1/16 but $\Pi^{int}(2)$ is greater than 1/16. This completes the proof of Lemma 2.□□

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