Accumulation and Distribution of Human Capital: The Interaction Between Individual and Aggregate Variables

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Abstract

The paper analyzes the joint evolution of accumulation and distribution of human capital in an OLG framework. Dynamics arises as a result of a continuous interplay between features of the human capital distribution and individual variables - inherited human capital and inborn ability. Such interaction drives individual investment in human capital and accumulation in the economy. According to the initial distribution the model provides different dynamical behaviours linking growth and inequality; in general more equal economies grow faster, and with less social conflicts, than the ones characterized by high inequality in the initial distribution, but several other cases are possible. Moreover, since the model provides an endogenous threshold for investing in human capital, the final distribution is characterized by multimodality.

Keywords: Human Capital, Accumulation, Bargaining, Inequality, Poverty trap.

J.E.L. D31, D82, I20, J41, O41

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1 Introduction.[1]

The recent years have been characterized by a large number of contributions in the field of personal distribution and economic growth whose aim is to shed a light on the interplay between these two important economic variables; such a line of research relies mainly on the human capital approach to economic growth. A first strand of this research concluded that a trade-off exists between growth and inequality, in sense that a higher inequality, calling for considerable redistributive schemes, depresses the accumulation; following refinements of the research have in some cases reversed such a result, concluding that more equal economies grow faster thanks to low social conflicts. More recently, several authors have stressed the importance of social, local and parental variables on the accumulation of human capital, as a source of stratification and segregation among individuals belonging to a given community; in general the final effect of such social exclusion is a slowing down in the growth process. These works focuses on the presence of spillovers among individuals, or communities, which affect the individual investment in human capital; these "neighborhood" effects, as defined by Benabou[4] and Durlauf[8], introduce some complementarities in the individual choices, thanks to which the human capital in the economy affects the individual behaviour but, at the same time, is itself affected. From this continuous interplay between "micro" and "macro" variables emerges the common dynamics of growth and distribution, which is very often related to the initial conditions. Recent works - Galor and Tsiddon [11], Benabou[4], Durlauf[8], Gradstein and Justman[14] - show that the interplay between "local" home environment variables, as the parental level of human capital, and the global technological externality *governs the evolutionary patterns of the distribution of human capital, the distribution of income and economic growth*[2]. In Gradstein and Justman the average human capital affects the individual earning capacity with the result that *We find little conflict between democracies and growth*[3], while in Durlauf *Parents affect the conditional probability distribution of their children's income through the choice of a neighborhood in which to live. Neighborhood location affects children both through local pub-*

[1] I started to work on this paper during my visiting spent at Nuffield College, University of Oxford; I am indebted to Anthony Atkinson for helpful comments and suggestions. Usual disclaimers apply.

[2] Galor and Tsiddon, 1997, page 93.

[3] Gradstein and Justman, 1997, page 169.

lic finance of education as well as through sociological effects[4]. Finally, in Benabou, where complementarities exist both in the individual technology for human capital and in the demand curve for the intermediate inputs, the final effect is that *Integration tends to slow down growth in the short run yet raise it in the long run*[5].

The present work should be placed among these recent works, since it focuses on the interplay between the distribution of human capital in the economy and the individual behaviour. We perform this in an imperfect labour market characterized by bargaining; the latter is necessary because of the presence of asymmetric information between firms and workers. Bargaining over unskilled workers provides a reference point for a second bargaining involving skilled agents.

In the model we are going to assume that the individual investment in human capital comes from a "cost-benefit" analysis, given the inherited human capital, schooling cost, genetic ability, unskilled wage, and the employment rate of the previous generation, which depends on the human capital distribution. In this way both micro and macro variables affect the individual choice. The private agent chooses the investment in human capital if the discounted flow of future income is higher than the wage obtained working as "unskilled". Individual choices update the stock of human capital in the economy, modifying the distribution which, in its turn, affects the accumulation of human capital for the next generation and so on. This path-dependence gives birth to a rather complex stochastic process (i.e. a non-linear Markov Chain) describing the evolution of human capital, which will be investigated through a qualitative analysis[6].

The interplay between accumulation and distribution of human capital gives birth to an endogenous poverty trap whose measure depends on the characteristics of the distribution itself; according to the initial conditions, the economy can exhibit low accumulation with high inequality and viceversa, and the transition to the poverty trap follows monotonic or oscillatory trajectories. The model shows a strong dependence of results on the characteristics

[4] Durlauf, 1996, page 75.

[5] Benabou, 1996, page 584.

[6] Linear Markov chains can be analyzed dynamically by means of an approximation in continuos time, i.e. via Fokker Planck equation, which is a parabolic partial differential equation. An application to the analysis of human capital distribution is provided in Giannini[13]. Nevertheless, the resulting Fokker Planck equation for a non linear chain does not allow a closed form solution. Details are in appendix.

of the human capital distribution and parameters; in general more unequal initial distributions tend to enlarge the poverty trap, slowing down accumulation and increasing inequality. Nevertheless further cases are possible and inequality in the initial conditions not always imply a lower accumulation. In other words, the main conclusion offered by the model relies on the fact that the relationship between growth and distribution can be extremely complicated and attempts in providing a well-established and predictable linkage could be frustrated. This makes difficult to compare different economies as well as different initial conditions; the paper tries to embed the dynamics in a series of propositions, but the paper itself shows as the final result is particularly sensitive to parameters and initial distribution.

2 The Model.

The economy described by the model involves non-altruistic, heterogeneous individuals living for two periods in overlapping generations. Sources of heterogeneity are: endowment of human capital and innate ability; the latter does not affect the individual productivity on job, which is solely related to human capital, but influences the educational effort. The human capital is distributed among the N agents in the economy according to the distribution $G(h)$, c.d.f. hereafter, with density $g(h)$; hence $\int_0^H dG(h)$ represents the percentage of population whose human capital is equal or less to H. The birth of each individual coincides with the end of compulsory school; for such a reason, in the first period of life, agents must decide whether to invest in higher education or to work as "unskilled". Hence the investment in human capital follows a binary choice, and for "unskilled worker" is meant an individual who does not invest in higher education at the end of compulsory school, giving up to signal herself as skilled agent. The latter instead invests in higher education so as to increase her human capital endowment by a fixed quantity; we are in fact measuring human capital as a discrete variable - educational qualification - rather as a continuous variable - length of education. As far as firms is concerned, they are characterized by a technology whose only argument is the worker's human capital - $y^i = (h^i)^\alpha$, $\alpha < 1$, for the $i-th$ firm - but they are different with respect to the type of hired worker (skilled or unskilled); we are going to come back on this point after describing the informative question entailed in the model. Finally, since at each instant t the labour supply changes jointly to the distribution of the human capital in the economy, I assume a unitary horizon of production.

Although the model does not involve an intentional bequest of human capital from parents to offspring, a key assumption of the model is that such intergenerational transfer does exist thanks to parental spillovers; this involve a strong form of heterogeneity among workers, even among "unskilled workers". In other words, I assume that unskilled workers are characterized by a different human capital, even if they do not update the latter at the end of compulsory school. Unskilled workers know their human capital endowment and innate ability, since the time spent in compulsory school is sufficient to reveal the latter to themselves. Nevertheless this information is private and firms are not able to identify the human capital of an unskilled worker by means of education, since this kind of workers are characterized by a common educational qualification; this means that such asymmetry of information can not be ruled out by means of education, as in the Cho and Kreps'[6], solution to the Spence's model. As it will be clearer later, the model involves a budget constraint due to the cost of investing in higher education, which causes less endowed individuals not to invest in human capital. The presence of such a constraint in the signal set produces a break down of the separating equilibrium provided by Cho and Kreps, with the result that pooling equilibria are also possible in such a context (see Cho and Sobel [7], for a general analysis of the problem and Giannini [12], for the particular case of the Spence's model).

As previously stated, the model assumes two types of firms according to the characteristic of the hired worker; a key assumption is given by the impossibility for a firm to change its type. Stated differently, technology is characterized by different "slots", ranked by human capital, and each slot represents a firm. Slots can be ordered, being in a one-to-one relation with human capital, and partitioned in two subsets, say $\mathcal{A}$ and $\mathcal{B}$; each slot belonging to $\mathcal{A}$ is filled by "unskilled workers" and viceversa. At the same way, human capital can be divided into two separated subsets, depending on the individual choice about human capital investment. The threshold separating unskilled from skilled agents, h_c, is the main object of the paper.

The set of assumptions stated so far requires two different strategies, for firms, in hiring workers; as said in fact, the informative problem affects only firms hiring unskilled workers which can not infer the real worker productivity by means of the educational choice. Such imperfection opens to the possibility of a mismatch between workers and "slots", or firms. The signalling approach before mentioned, suggests that in a situation characterized by a common educational choice for workers - pooling equilibrium - the firm "best reply" - wage - acts on the base of an average productivity; in our situation such behaviour is not viable, since the possibility of a mismatch is not ruled out. Moreover, because of the length of the productive process, workers' human capital can not be revealed "on job".

Under an assumption of risk aversion for firms, the uncertainty caused by the mismatch can be faced adopting a cooperative behaviour, in sense that firms have an advantage in constituting a trust acting as a representative firm on the base of the average productivity of the unskilled workers, and redistributing equally the total profit among firms which hire this type of workers, providing a form of risk sharing. The production function taken into account from the firms' union is the $F(\bar{y}_t L^u(\bar{y}_t))$, where $\bar{y}_t = \frac{\left(\int_0^{h_{c_t}} h_t^\alpha dG_t(h_t)\right)}{\left(\int_0^{h_{c_t}} dG_t(h_t)\right)}$, is the average productivity and L_t^u the employment among unskilled workers. The workers' reply to such a cooperative behaviour is the creation of a workers' union bargaining for wages[7]; the reference point for wage is the reservation one, which is assumed to be the same for every unskilled worker. At the same time, firms receive from bargaining an equal profit with certainty instead of a higher profit with uncertainty. For these reasons, I assume that, for unskilled workers, the union tries to maximize the distance between the wage and the reference point, i.e. the reservation wage, independently from the workers' endowment, and at the same time, firms union maximize the distance between current profit and the reference point, i.e. the zero profit.

2.1 Bargaining over unskilled wage.

In the following we are going to assume that unskilled workers are organized in a trust from which they are randomly drawn; the number of unskilled workers is $M_t = N \int_0^{h_{ct}} dG(h_t)$, where h_{c_t} is the level of human capital separating unskilled from skilled individuals and N the population - stationary by assumption. All union members are treated uniformly and the union utility function is simply the sum of the individual ones. As stated, the wage for unskilled workers comes from an asymmetric Nash bargaining function where the workers' union sets the wage and firms the employment level L_t^u as a consequence; in the following we are going to assume $L_t^u < M_t$. The Nash

[7] Since bargaining involves unemployment, we are implicitly assuming that benefits of forming a union are at least equals to the costs in term of unemployment.

asymmetric bargaining function is then:

$$\mathcal{N}_t = L_t^u \left[U(w_t^u) - U(R)\right]^{\beta} \left[F(\bar{y}_t L_t^u) - w_t^u L_t^u\right]^{1-\beta} \tag{1}$$

where w_t^u is the bargained wage. Both utility and production function involved in the game are characterized by a monotonic, increasing, concave graph; we are hence assuming diminishing returns to scale in the labour per efficiency unit for the representative firm. By maximizing the (1) w.r.t. w_t^u and L^u, we obtain a point on the contract curve:

$$w_t^u = \frac{1}{2-\beta}\left[\frac{F(\bar{y}_t L^u(\bar{y}_t))}{L^u(\bar{y}_t)} + (1-\beta)\, F'(\bar{y}_t L^u(\bar{y}_t))\right] \tag{2}$$

$$L^u(\bar{y}_t) = \frac{\beta U' w_t^u F(\bar{y}_t, L^u(\bar{y}_t))}{\beta U' w_t^u + (1-\beta))\left[U(w_t^u) - U(R)\right]} \tag{3}$$

with $L^{u'}(\bar{y}_t) > 0$. Eq. (2) shows the wage as a weighted average between the mean and marginal productivity. In this way a CES production function provides a wage rigidity until there are available workers.

According to (2) and (3), the rent sharing can be expressed in:

$$\frac{L^u(\bar{y}_t)\,(w_t^u - R)}{\pi\,(w_t^u, L^u(\bar{y}_t))} = \frac{\beta}{(1-\beta)}\eta\,(w_t^u)$$

where $\eta(\cdot)=\frac{U'(w)(w-R)}{U(w)-U(R)}$ is the union's incremental elasticity of substitution.

By assuming $F(\bar{y}_t L^u(\bar{y}_t)) = (\bar{y}_t L^u(\bar{y}_t))^{\alpha}$, $U(w_t^u) = \frac{(w_t^u)^{\sigma}}{\sigma}, \sigma < 1$, and $U(R) = R$, we have:

$$\begin{aligned} w_t^u &= w^u > R \qquad \forall t \\ L_t^u &= \left[\frac{\beta(w^u)^{\sigma-1}}{\beta(w^u)^{\sigma} + (1-\beta)\left(\frac{(w^u)^{\sigma}}{\sigma} - R\right)}\right]^{\frac{1}{1-\alpha}} \bar{y}_t^{\frac{\alpha}{1-\alpha}} = \phi \bar{y}_t^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{4}$$

In this way the profit for each firm producing with unskilled labour is:

$$\pi_t^u = \frac{(\bar{y}_t L(\bar{y}_t))^{\alpha}}{L(\bar{y}_t)} - w^u = \phi^{-(1-\alpha)} - w^u = \frac{\beta(w^u)^{\sigma} + (1-\beta)\left(\frac{(w^u)^{\sigma}}{\sigma} - R\right)}{\beta(w^u)^{\sigma-1}} - w^u$$

2.2 Bargaining over skilled wage.

The presence of skilled individuals in the economy opens to a one-to-one bargaining between firms hiring skilled workers and the latter; the key assumption in this case is represented by the perfect signalling provided by the investment in higher education; this implies absence of mismatch and unemployment for skilled individuals. Firms can increase their per-worker profit, at the beginning of time $t+1$, beyond the level of π_t^u, by hiring individuals with human capital $h > h_{ct}$; at the same time, individuals invest in higher education if they receive a wage $w^s > \bar{w}^u = pw^u + (1-p)R$, where p is the probability to be employed as unskilled worker. In this way, players maximize the following Nash function:

$$\mathcal{N}_{t+1}^s = \left(\pi_{t+1}^s - \pi_{t+1}^u\right)^{1-\gamma} \left(w_{t+1}^s - \bar{w}_{t+1}^u\right)^{\gamma}$$

where $\pi_{t+1}^s = h_{t+1}^{\alpha} - w_{t+1}^s, h_{t+1} = h_t^s + \delta, h_t^s > h_{c_t}$. By so doing we obtain the skilled wage for an individual characterized by a human capital h^s :

$$w_{t+1}^s = \gamma \left[(h_t^s + \delta)^{\alpha} - \frac{1}{\phi} \right] + \bar{w}_{t+1}^u$$

which is a mark-up on the unskilled wage.

2.3 The individual behaviour

As said, economy is populated by a continuum of heterogenous agents, of mass N, living for two periods in overlapping generations; individuals are born at each instant t and they are old in $t+1$. I assume that each individual has one child, so population is stationary. At the birth each agent is endowed with human capital inherited from the parents: I assume that the parental human capital accumulated during the life accrues completely to the offspring through family spillovers rather than an intentional bequest. Moreover, at the birth, Nature sets the individual's inborn ability, ϵ^i, according to a $n.i.i.d.(0, \sigma_\epsilon^2)$ law. Each individual is characterized, *ex-post*, by a given ϵ coefficient. Birth coincides with the end of compulsory school. In this way, at the birth t, each individual i knows:

1. Her inherited human capital, h_t^i;

2. Her inborn ability, i.e. the ϵ^i ex-post realization;

3. The critical human capital in the previous generation, $h_{c_{t-1}}$, hence employment in the previous generation.

By using this information set, the individual decision making acts as follows: in the first period of life the agent decides whether to invest in higher education, increasing her stock of human capital and employing herself in the second period of life, or to work for both periods. The agent performs such a decision comparing the discounted income stream in both cases. In the first case - schooling - the individual goes to school during her youth; such a process involves a cost, both in pecuniary and effort term and at this end I assume that it consists of a fixed component χ, representing the pecuniary term, and a variable one, represented by the individual ability coefficient ϵ^i. In this way the scholastic cost is $c^i = \chi + \epsilon^i$; individuals characterized by $\epsilon^i < 0$ are highly skilled and their scholastic cost is lower than the ones having $\epsilon^i > 0$. Individuals investing in human capital can borrow from an international financial market at the constant world rate r, in order to cover the pecuniary cost χ, since their inherited human capital can not be traded or resold. Note that the ability parameter, which is known ex-post to the individual, can operate either in a positive or in a negative way when related to the inherited human capital, in sense that individuals characterized by a high inherited human capital but low aptitudes - ϵ high - are not different from individuals in the opposite situation. Conversely, individuals characterized both by a high inherited human capital and a negative skill coefficient are particularly endowed. In other words, the dynamics of the individual human capital is driven by the convolution of two distributive laws: the one related to the inherited human capital and the one describing the inborn ability.

Viceversa if the $i-th$ individual employes herself from the first period of life, she simply receives the central wage w^u in both periods. Nevertheless, due to the unemployment among unskilled workers, the wage w^u is obtained according to a given probability; I assume that such probability is given by $p_t = \frac{L_{t-1}}{M_{t-1}}$, i.e. by the employment rate at the end of the previous period, which enters in the individual information set at the beginning of the decisional horizon ; for the same reason the reservation wage is obtained with probability $(1 - p_t)$.

In this way the individual decision about human capital investment is performed comparing the following valuation indexes W - life income stream - at the beginning of the first period of life:

$$
\begin{aligned}
W_{st}^i &= \lambda \left[w_{t+1}^s \left(h_{t+1}^i\right) - (1+r)\chi\right] - (\chi + \epsilon_t^i) \quad \text{with } h_{t+1}^i = h_t^i + \delta \quad (5)\\
W_{ut}^i &= \bar{w}_t^u + \lambda \bar{w}_{t+1}^u \qquad \text{with } \bar{w}^u = pw^u + (1-p)R
\end{aligned}
$$

where λ is the discount factor. The first equation shows the income stream when the agent decides to stay at school during youth; the first term on the R.H.S. (Right Hand Side) is the discounted - expected - wage, net of refunding, which accrues to the individual during the second period of life while the second one is the scholastic cost which must be paid during the first period. The second equation represents the income stream in case the individual decides to work in both periods.

By so doing we obtain a simple investment decision rule: if $W_s^i > W_u^i$ then the $i-th$ agent prefers schooling to working during youth. By defining the variable $z^i = W_s^i - W_u^i$, our decision rule simply becomes:

$$
z_t^i = \lambda\gamma \left(h_{t+1}^i\right)^\alpha - \left[\frac{\lambda\gamma}{\phi} + \lambda(1+r)\chi + (\chi + \epsilon_t) + p_t w^u + (1-p_t)R\right] \qquad (6)
$$

and the $i-th$ agent invests in human capital only if $z^i \geq 0$.

Individuals not investing in human capital do not update their inherited human capital [8]. With such a remark the individual human capital evolves according to the following rule:

$$
\begin{aligned}
h_{t+1}^i &= h_t^i + \delta \quad if \quad z_t^i(h_t^i, p_t, \epsilon_t^i) \geq 0\\
h_{t+1}^i &= h_t^i \qquad\quad if \quad z_t^i(h_t^i, p_t, \epsilon_t^i) < 0
\end{aligned}
$$

In this way each individual decides the level of human capital during youth; such a level is preserved during the working life and totally transmitted to the offspring, although there is not altruistic behaviour in the model.

2.4 The Marginal Agent.

In this section we are going to analyze the behaviour of the marginal agent, for whom $z = 0$, which implies $h_{t+1}^i = h_{c_t} + \delta$; at the moment we investigate the deterministic case, viz. for $\epsilon^i = 0$. From Eq. (6) we have:

[8] I do not assume *learning by doing* effects on job.

$$h_{c_t} = \left[\frac{1}{\lambda\gamma} \left(\frac{\lambda\gamma}{\phi} + \lambda(1+r)\chi + \chi + \frac{\phi^{\frac{1}{1-\alpha}} \left(\int_0^{h_{c_{t-1}}} h_{t-1}^{\alpha} dG_{t-1}(h_{t-1}) \right)^{\frac{\alpha}{1-\alpha}}}{N \left(\int_0^{h_{c_{t-1}}} dG_{t-1}(h_{t-1}) \right)^2} (w^u - R) + R \right) \right]^{\frac{1}{\alpha}} - \delta \tag{7}$$

Equation (7) is a first-order difference equation in h_{c_t} on $h_{c_{t-1}}$ and G_{t-1}. The problem that we have to face in this equation is the fact that we do not know the evolution of G over time; the latter is affected recursively by h_c which in turn is affected by G itself. The peculiarity of equation (7) is to be a Markov chain whose transition probabilities depend on the distribution itself and for such a reason it is not possible apply the conventional dynamical operators, as for instance the Fokker-Planck equation for approximation in continuous time; technical details are provided in appendix; in the following we are going to provide a qualitative analysis.

We start at $t = 0$ with a given distribution of human capital in the economy; at this time Nature draws randomly the threshold h_{c0} so as individuals whose initial human capital lies below h_{c0} do not invest in human capital and vice versa; such an investment process modifies the human capital distribution and (7) provides a new value for h_{c1} and so on. The mechanics involves a complex relationship between the characteristics of the human capital distribution and the individual behaviour. To describe it we start from the effect of h_c on the human capital c.d.f.. We said that individuals characterized by a human capital higher than the critical level do invest in human capital, increasing their distance from individuals trapped below h_c. The effect on the human capital distribution is to shift forward individuals above the critical level, increasing dispersion and creating a bimodality in the curve; Figure 1 shows the result in term of the human capital c.d.f.. Such a bimodality is involved in eq. (7) since the c.d.f. $G_t(h_t)$ is present in the r.h.s. of the equation; it is worth stressing that the difference between G_t and G_{t+1} lies only in the distribution of individuals above h_c. Next proposition gives a qualitative analysis of equation (7):

Proposition 1 *According to the α coefficient, equation (7) is steadily decreasing or increasing in the $\left(h_{c_t}, h_{c_{t-1}}\right) \subseteq \Re^+ \nwarrow \Re^+$ plane and it crosses once the $\left(h_{c_t} = h_{c_{t-1}}\right)$ line. Nevertheless the position of (7) in the plane is not constant: the continuous modification of $G(h)$ over time, induced by*

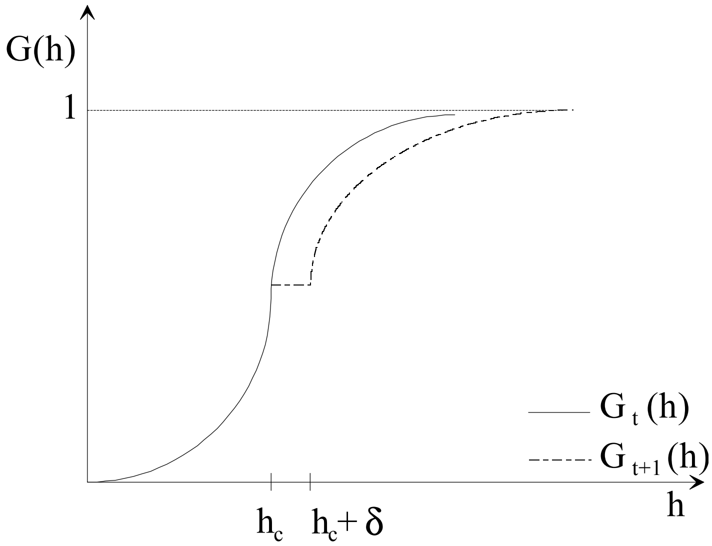


Figure 1: C.d.f. between t and $t+1$

changes in h_c, shifts (7) in the space according to α. Such a shifting affects the transition to the steady state h_c^*.

Proof. We begin with the first part of the proposition. Looking at equation (7) the only variable term is represented by (we omit the subscript $t-1$):

$$V = \frac{\left(\int_0^{h_c} h^\alpha dG(h)\right)^{\frac{\alpha}{1-\alpha}}}{\left(\int_0^{h_c} dG(h)\right)^2} \tag{8}$$

and it is straightforward to see that $\lim\limits_{h_c \to \infty} V = (m_\alpha)^{\frac{\alpha}{1-\alpha}}$ where m_α is the $\alpha - th$ moment of $G(h)$. Instead when h_c approaches zero, V can be either limited or superior unlimited according to the value of α. In fact we can write:

$$\log(V) = \frac{\alpha}{1-\alpha} \log\left(\int_0^{h_c} h^\alpha dG(h)\right) - 2\log\left(\int_0^{h_c} dG(h)\right)$$

The second term on the r.h.s. is independent from α which affects only the first one; the latter tends to a negative unbounded value when h_c approaches zero from the right and vice versa for the former, given the sign in front of

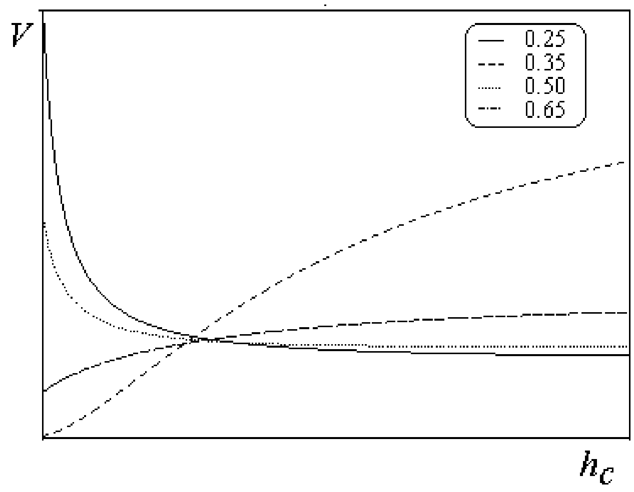


Figure 2: Modification of V induced by α

it. Nevertheless, being $\frac{\alpha}{1-\alpha} \in [0, \infty)$, the velocity of divergence of the first term is greatly affected, in a positive way, by the value of α ; in other words, according to α, the first term can be a superior or inferior infinity w.r.t. the second term. In general low values of α makes $\frac{\alpha}{1-\alpha} \log \left(\int_0^{h_c} h^\alpha dG(h) \right)$ an infinity of an inferior order[9] w.r.t. $2 \log \left(\int_0^{h_c} dG(h) \right)$, causing eq. (7) to show a decreasing pattern whose slope depends negatively on α. As far as α increases, the order of divergence of the two terms changes, inducing in (7) an upward slope which is as steep as α is high and an intercept which tends to zero. It is worth stressing that the slope of (7) is also affected by the shape of $G(h)$. Figure 2 shows (8) for a log-normal distribution in four cases ranked by α.

As far as the second part of the proposition is concerned, it comes from the effect of h_c on the human capital c.d.f., as shown in Figure 1, which increases dispersion among agents, pushing forward individuals above h_c; we shall be afterwards returning to this point in a greater details. ■

As we said, the characteristics of the human capital distribution function do affect (7) and we are especially interested in the variance of $G(h)$ which

[9] Alternatively, $\left(\int_0^{h_c} dG(h) \right)^2$ is an infinitesimal of a higher order w.r.t. $\left(\int_0^{h_c} h^\alpha dG(h) \right)^{\frac{\alpha}{1-\alpha}}$.

provides a broad measure of inequality. It is known that an increase in the variance for a given distribution makes the new c.d.f. flatter than the previous one, since the probability - or frequency - to find individuals lying in the tails is higher. For the same reason, a distribution characterized by a higher density in the low tail shows a flatter c.d.f. for medium-low classes w.r.t. to a more equal - concentrated - distribution. The effect of this higher c.d.f. in the left tail affects mainly the denominator of Eq. (8) with the result to increase it. From here we have:

Remark 1 *For α given, Eq. (7) is flatter for positively skewed distributions. Given a distributive law, then the same holds for an increase in the variance of the distribution.*

3 Dynamics.

In this section we are going to analyze the evolution of h_c over time by means of Eq. (7). As we said, because of the non-linearity of the Markov Chain describing the evolution of h_c, it is not possible to analyze the dynamics in a formal way but we can fruitfully perform this by means of a phase diagram. We start with the decreasing case for (7). Let G_0 be the initial human capital distribution function and h_{c0} the initial threshold drawn from a given distribution H_c ; individuals above h_{c0} increase their inherited human capital by δ and so does the marginal agent whose human capital increase to $h_1 = h_{c0} + \delta$. The effect is described in Figure 1 for the initial c.d.f.; the bimodality is embodied in (7) which is shown in Figure 3.

The solid line shows the final trajectory and the arrows the transition to h_c^*; the thin ones the modification of (7) over time, induced by changes in the human capital c.d.f. due to the evolution of the critical threshold; this is the sense of the last part of Proposition 1. The transition to the equilibrium value h_c^* shows an oscillatory dynamics, which is complicated by the shifting of (7); it must be remarked, however, that h_c^* is achieved in a finite time which depends on the slope of (7), i.e. on the characteristics of the human capital c.d.f. From here we have the following:

Proposition 2 *The transition to h_c^* is as fast as the human capital c.d.f. is flat. Comparing a given human capital distribution with two different variances, for the same initial condition h_{c0}, the equilibrium value h_c^* is as low as the variance is high, i.e. as (7) is flat. The same conclusion does not hold*

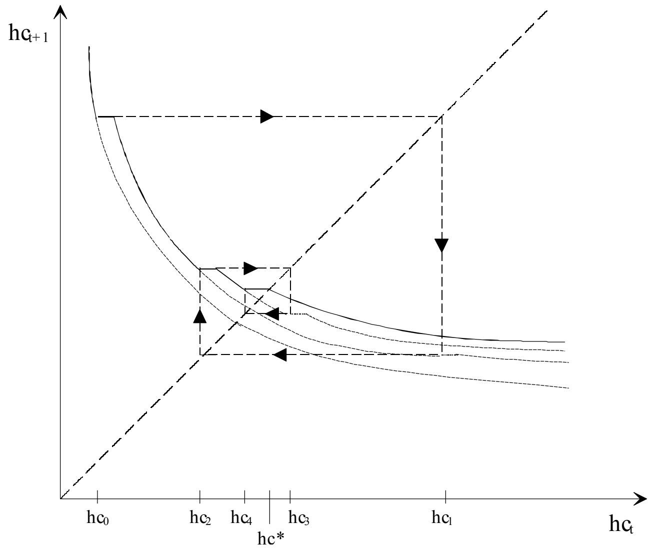


Figure 3: Oscillatory Transition

comparing different distributive laws with different dispersion, since in this case, the position of (7) changes in the phase diagram. In general, Eq. (7) is flatter for an increasing dispersion and steeper in the neighboring of the origin for an increase in the average value.

Proof. It comes directly from Remark 1; the last part refers to Eq.(8) since an increase in the average value reduces the dominator, for $h_c \to 0$, causing the latter to converge faster to zero than the numerator. ■

As soon as h_c^* is achieved, individuals below it are trapped for ever since the conditions in the labour market for unskilled workers change no longer over time; the result is a steady increase in the human capital for individuals above h_c^*, which amplify inequality over time. A way for trapped individuals to escape is provided by the genetic shock, as we shall see.

The medium class is particularly involved in the transition path; the oscillatory behaviour creates a sort of "stop and go" in human capital investment for such individuals. The final effect is a flattening and compression in the medium class in an alternative way until the achievement of h_c^*. Such a fact involves multi-modality in the human capital distribution.

The transitional dynamics is related to the value of α that affects the slope of (7), as stated in Proposition 1. When α is such to induce a positive

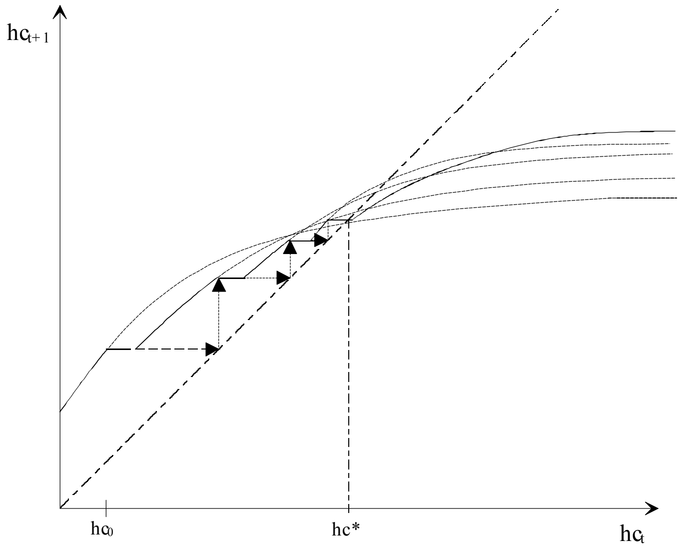


Figure 4: Monotone Transition

slope in (7), then the transition to h_c^* is monotonic and shown in Figure 4.

As said, in this case we have a monotone convergence in a finite time to h_c^*, although the displacement of (7) makes longer or shorter the transition depending on the slope coefficient, which is in turn related to α and inequality - variance - in the human capital c.d.f., as previously stated.

4 Accumulation.

The previous section analyzed the evolution of the human capital distribution over time; now we are interested in examining the evolution of the average value of such distribution which can be considered the accumulation path of human capital in the economy. It is obvious, in fact, that the higher the number of individuals investing in human capital, the higher the level of average human capital; such a simple fact generates a link between the evolution of h_c and the growth of the average human capital. Clearly, the average value is as high as h_c is low; nevertheless the former depends on several other variables, as we are going to investigate.

For the sake of simplicity, in the following we shall assume a maximum value for human capital, say h_M; this upper bound grows by δ at each time if the highest endowed individual invests in human capital; we can easily extend our results to the limit case $h_M = \infty$. We start from the observation that the statistical average between t and $t+1$ is given by:

$$\begin{aligned}\bar{h}_t &= \int_0^{h_{ct}} h_t g_t(h)dh + \int_{h_{ct}}^{h_M} h_t g_t(h)dh \\ \bar{h}_{t+1} &= \int_0^{h_{ct+1}} h_{t+1} g_{t+1}(h)dh + \int_{h_{ct+1}}^{h_M+\delta} h_{t+1} g_{t+1}(h)dh\end{aligned}$$

As previously remarked, individuals below h_{ct+1} do not increase their inherited human capital, staying on the same level; for such a reason the part of the c.d.f. under the threshold does not change between t and $t+1$; in other words $\int_0^{h_{ct+1}} h_t g_t(h)dh = \int_0^{h_{ct+1}} h_{t+1} g_{t+1}(h)dh \; \forall h < h_{ct+1}$; for such a reason we can write: $\bar{h}_t = \int_0^{h_{ct+1}} h_t g_t(h)dh + \int_{h_{ct+1}}^{h_M} h_t g_t(h)dh$. Hence, taking the difference in the statistical mean between the two temporal instants, we obtain:

$$\Delta\bar{h} = \bar{h}_{t+1} - \bar{h}_t = \int_{h_{ct+1}}^{h_M+\delta} h g_{t+1}(h)dh - \int_{h_{ct+1}}^{h_M} h g_t(h)dh \tag{9}$$

From here the following proposition holds:

Proposition 3 *$\Delta\bar{h}$ is positive. Moreover it depends positively on δ and $G(h)$, and negatively on h_c.*

Proof. In order to prove the proposition, we rewrite Eq.(9) as (ignoring the subscript $t+1$ for h_c since there is not ambiguity in the notation):

$$\bar{h}_{t+1} - \bar{h}_t = \int_{h_M}^{h_M+\delta} h dG_{t+1}(h) + \int_{h_c}^{h_M} h dG_{t+1}(h) - \int_{h_c}^{h_M} h dG_t(h)$$

Through the mean value theorem on integration, recalling that $G_{t+1}(h_M + \delta) = G_t(h_M) = 1$,we can finally write:

$$\bar{h}_{t+1} - \bar{h}_t = (h_1 - h_2)\left(1 - G_{t+1}(h_M)\right) \tag{10}$$

where $h_1 \in (h_M, h_M + \delta]$ and $h_2 \in [h_c, h_M]$. Hence $\bar{h}_{t+1} - \bar{h}_t$ is positive and depends positively on the area of the tail of the human capital density function above h_M; moreover it is positively related to δ via h_1 and negatively to h_c via h_2. ■

By crossing Propositions 1 to 3, we obtain a more complete dynamical scenario with the following Proposition:

Proposition 4 *In general initial distributions characterized by a high average value jointly to a large dispersion around it, induce a higher value of* h_c^* *and a faster convergence; this in turn implies a lower accumulation with an increasing dispersion. The converse also is true. This brings to the idea that inequality is harmful for growth. Nevertheless the final result is strongly dependent on the parameter set, particularly* α, *and on the characteristics of the initial distribution.*

The above proposition provides a general description of the dynamics but several particular cases are also possible; a comprehensive analysis is not possible because of the strong dependence of Eq.(7) on the moments of the initial distribution, other than on the parameters. As an example, a normal distribution with high average value and low dispersion can provide a higher h_c than a log-normal with the opposite characteristics. This is due to the way in which Eq.(8) is affected, as we have pointed out in Proposition 2. From an economic point of view, the modification of (8) affects directly the individual decision about human capital investment, via Eq.(6); for h^i given, an increase in (8) induces individuals to give up to the higher education, shifting forward the level of human capital necessary to invest in human capital.

In other words, comparisons among different initial conditions are rather difficult, as well as the identification of an unambiguous, well-founded relationship between accumulation and distribution of human capital; each case should be carefully analyzed individually, since it greatly depends on the features of the economy under investigation, features which are represented by parameters and initial conditions.

5 The Stochastic Case.

In this section we are going to analyze how the stochastic component affects the results obtained so far. In this case each individual is characterized by a shock ϵ representing the "genetic" endowment; individuals with $\epsilon < 0$ are more skilled than the average ($\epsilon = 0$) and viceversa. As previously remarked, ϵ affects only the educational effort and not the productivity on job; for such a reason it affects the educational cost but not the wage. In the following we assume that ϵ^i is $n.i.i.d.$ $(0,\sigma_\epsilon^2)$ across individuals and over time, $i = 1, 2, ...N$.

The effect of the genetic shock is to shift Eq. (6) upward and downward around the mean value provided by $\epsilon = 0$; the magnitude of the shift depends obviously on σ_ϵ^2. As previously noted, the genetic shock represents the only way for escaping from the unskilled area; under this point of view, the stochastic model provides a more general result than the deterministic one, and this agrees with authors (see Benabou,[4], for example) who stresses the importance of the stochastic component in this type of models.

The stochastic component induces a continuous two-way jump across the critical threshold provided by the marginal agent; the final effect depends on the total number of agents migrating from unskilled to skilled and viceversa. Since symmetrical agents have the same probability to jump up or down the critical threshold, the final balance depends on the number of individuals entailed in the process, which obviously depends on σ_ϵ^2; in other words, the final result depends on the distribution of individuals around the critical threshold. Let $\left[\underline{h}, \bar{h}\right]$ be the interval of confidence, centered on h_c, for a given statistical level - say 95%; then the following proposition holds:

Proposition 5 *If the percentage of individuals lying in the* $\left[\underline{h}, h_c\right]$ *region is greater than the ones in* $\left(h_c, \bar{h}\right]$, *then in the stochastic case there is a larger share of population investing in human capital than the deterministic one, reducing inequality and fostering accumulation. The converse also holds.*

Proof. The proof comes straightforwardly from a urn model. Individuals lying in $\left[\underline{h}, h_c\right]$ have a positive probability to jump the threshold h_c thanks to the genetic shock: this probability is as high as the individual human capital, h^i, is close to the threshold h_c. To clear the point let us choose an individual in the $\left[\underline{h}, h_c\right]$ region; for such an individual, in the deterministic case, the variable z is negative and there is not chance to jump the threshold h_c. In the stochastic case instead, if the genetic shock ϵ^i is sufficiently low, the variable z will be positive, allowing the individual to invest in human capital. The value of ϵ making $z = 0$ is the following:

$$\epsilon^{i*} = (h^i - h_c) < 0$$

and being $\epsilon^i \sim N(0, \sigma^2)$ we obtain:

$$P(\epsilon^i \leq \epsilon^{i*}) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{\epsilon^{i*}} e^{-\frac{\epsilon^2}{2\sigma^2}} d\epsilon^i$$

moreover $\partial P(\epsilon^i \leq \epsilon^{i*})/\partial h^i > 0$. In other words each individual in the $\left[\underline{h}, h_c\right]$ region has a probability $p = P(\epsilon^i \leq \epsilon^{i*})$ to jump the threshold h_c and a probability $q = 1 - p$ to in $\left[\underline{h}, h_c\right]$. At the same way, because of the symmetrical shock, an individual lying in the $\left(h_c, \bar{h}\right]$ region, at the same distance $|\epsilon^*|$, has the same probability $p = P(\epsilon^i \geq \epsilon^{i*} = (h^i - h_c) > 0)$ to jump the threshold h_c, getting "poor".

Now we are finally in position to demonstrate the proposition approximating the probability of transition from $\left[\underline{h}, h_c\right]$ with the probability of the lowest individual in this domain, i.e. for $h^i = \underline{h}$ which implies $\epsilon^* = (\underline{h} - h_c) < 0$; at the same way, individuals in $\left[h_c, \bar{h}\right]$ are approximated by the maximal individual $\bar{h}$, where now $\epsilon^* = (\bar{h} - h_c) > 0$. We mean by $p = P(\epsilon \leq \epsilon^*)$ the probability that an individual randomly drawn from $\left[\underline{h}, h_c\right]$ jumps the level h_c getting "rich". At the same way p is the probability for an individual lying in $\left[h_c, \bar{h}\right]$ to become poor, $p = P(\epsilon \geq \epsilon^* = (\bar{h} - h_c) > 0)$. We mean by N_1 the number of individuals in the region $\left[\underline{h}, h_c\right]$, and N_2 the ones in $\left(h_c, \bar{h}\right]$. Given that for each individual there exists a probability p to jump the critical level h_c and a probability $q = 1 - p$ to remain in the same state, the problem can be seen as a classical urn model. It is known that in such a case the probability that n agents out of N_i cross (are drawn) the threshold h_c is given by the binomial distribution:

$$P(n, N_i) = C_{N_i}^n p^n q^{N_i - n}$$

where $C_{N_i}^n$ are the binomial coefficients. If $N_1 > N_2$ we have $P(N_2, N_1) > P(N_2, N_2)$, i.e. the probability that N_2 agents, out of N_1, jump from $\left[\underline{h}, h_c\right]$ to $\left[h_c, \bar{h}\right]$ is greater than the opposite event. The converse holds. ■

By so doing we have demonstrated a rather intuitive result: in the stochastic economy less endowed individuals have a chance, missing in the deterministic case, to jump the critical level h_c. There are at least two relevant factors affecting the magnitude of the probability. The first one concerns the variance of the genetic shock, since the probability grows according to the latter; two normal distributions centered on zero but with even moments -

variance and kurtosis - different in magnitude provide two different values of the jump probability. The second point concerns the possibility of asymmetric, rather than symmetric, genetic shocks; if negative shocks have a higher probability to occur than positive ones, then poor and rich individuals would have a different jump probability irrespectively of the skewness of the human capital distribution. This opens to the possibility to make endogenous the distribution of the innate ability, as in Durlauf, [8]; for instance, one can assume that the probability to jump from unskilled to skilled grows with the proceeding of the accumulation process, which represents an improvement in the standard of life. This model extension would provide an interesting feedback between distribution and development.

6 A numerical exercise.

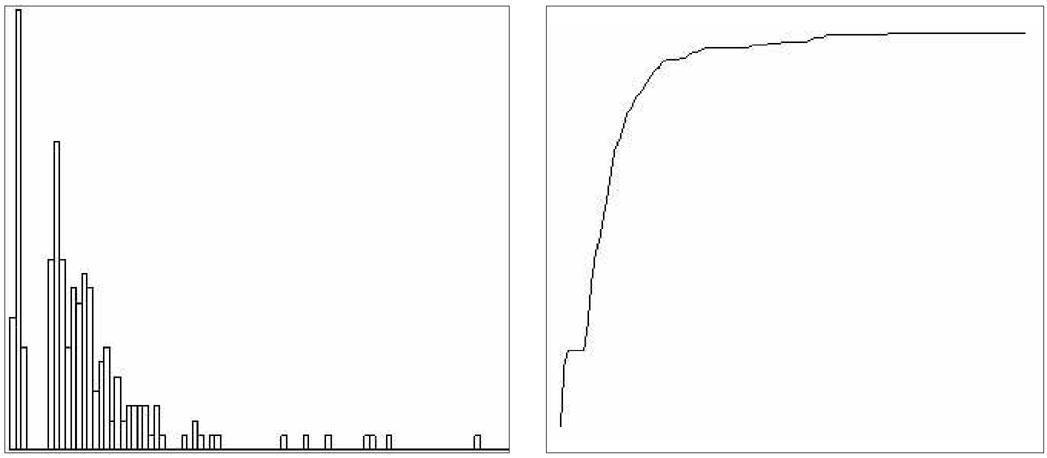
Because of the difficulty in providing an explicit analysis of the dynamics, the model was submitted to several numerical exercises, aiming to verify the dynamical behaviour in different situations, with a particular interest in the initial conditions. It is worth stressing that the results obtained agree fully with the qualitative analysis previously described.

Due to the impossibility of reproducing totally the performed numerical exercises, in the following will be shown two situations which refer to two different initial distributions. The numerical simulation uses a discrete version of the model and refers only to the deterministic case; the human capital is distributed among 35 classes of equal measure and there are 200 individuals in the economy.

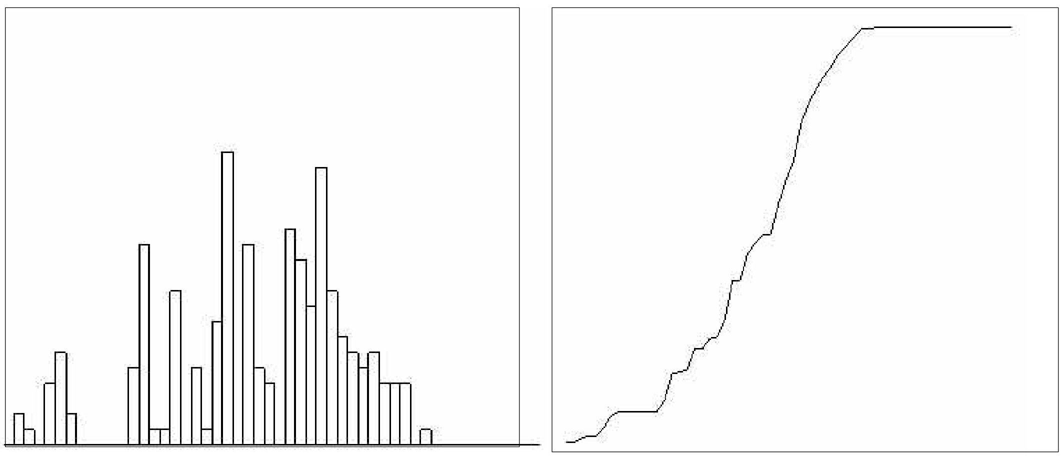
From a dynamical point of view, the most interesting pattern is provided by the oscillatory path and for such a reason the following pictures refers to $\alpha = 0.4$, which is sufficiently low to cause a downward slope in Eq.(7), as stated in Proposition 1.

Figure 5 refers to two exercises built with the same parameter set but with different initial distributions, viz. a log-normal[10] $[1, 1]$ (at the top) and a normal $[3, 1]$ (at the bottom). The figure shows the density function and the corresponding c.d.f. for each simulation after the achievement of h_c^*. The convergence is much faster for the log-normal than the normal distribution, as shown by the c.d.f. in both cases; while for the log-normal

[10] The cited log-normal is $\frac{1}{x\sqrt{2\pi s}}\exp(-(\log(x)-\mu)^2/2s^2)$; the parameters $[1, 1]$ refers to μ and s. This brings to an average value of 4.5 and a variance of 35.



Log-Normal



Normal

Figure 5: Numerical Simulations

the convergence is so fast that the oscillatory dynamics rests hidden, the c.d.f. for the normal law shows a more complex transitional dynamics which affects mainly the medium class. This different dynamical behaviour is due to the larger variance of the log-normal distribution; as stated in Proposition 2, this causes a flattening in Eq.(7), with a faster convergence process.

Simulating over initial distributions characterized by a high average value and low variance, it is possible to obtain a longer transition phase, jointly to a large oscillatory behaviour. As said, the transitional dynamics is greatly affected by the initial distributions, and in general it is hard to compare different initial conditions.

7 Conclusions.

The results of the paper show how the dynamics of human capital accumulation and distribution depends strongly on the interaction between the characteristics of the distribution itself - the "macro" variable - and the "micro" variables - human capital endowment, genetic ability - via the individual choice about human capital investment. When individual decisions are affected by "environmental" variables the economic evolution becomes more complex and several dynamical behaviours are possible according to parameters and initial conditions.

About policy implications, it is obvious that any reduction of the endogenous poverty trap fosters growth. This opens to a redistributive policy which can be implemented in two different forms: either reducing the educational cost for trapped individuals or subsidizing directly the latter. Both cases entail a taxation on skilled individuals. In a democratic context, the application of the median voter principle is made more complex by the presence of the critical threshold h_c; even when the median agent is included among trapped individuals, the optimal tax rate of the latter could not be sufficient to ensure the necessary tax-revenue which would allow to all agents in the poverty area to escape from it. Given the economy described by the model, the optimal tax rate for the median voter is the one which allows him/her to jump the threshold h_c but, as said, such a value could not "polarize" the preferences of the majority, if the tax-revenue is unsatisfactory. In other words, in this scheme, the median voter looses the leading role played in the traditional models of endogenous fiscal approach. Under this point of view, the policy action should be implemented by a central authority on the base of some measure of collective welfare; the more direct way is to make the educational cost as low as possible, financing it by a taxation scheme whose characteristics should be analyzed through the impact on growth and inequality.

Finally a word on further refinements. Given the dependence of results on parameters, it is possible to add further non-linearities to the model making endogenous some crucial parameters, enriching the range of possible dynamical behaviours. I refer especially to the coefficients measuring the bargaining power, viz. β and γ, which could be depending on the unemployment rate in the economy; these refinements should increase the possibility of non-monotonic paths for accumulation and inequality. As remarked by Atkinson,[2], in fact, the empirical observation is characterized by relatively sudden changes in the inequality indexes; embodying such a feature in a theoretical framework, although simple, allows us to take into account a large number of dynamical behaviours relating accumulation and distribution of human capital; I hope this paper partially contributes to the goal.

Appendix

In this appendix, we provide some technical details about the particular Markov chain describing the accumulation of human capital. As we have seen, the transition probability, for human capital, between two neighboring instants is represented by the valuation index z_t; more explicitly the transition probability depends on the state occupied at the present time h_t^i, distribution law of the noise term ϵ and human capital distribution $g(h)$ through the probability p. In the following we ignore the ϵ component, since the latter is not the source of the non-linearity for the Markov chain.

The non-linearity is induced by the dependence of the transition rates on the human capital distribution. In fact, if $P(h+\delta, t+1)$ means the probability of finding the process in the $h+\delta$ state at time $t+1$ being in state h at time t, then we can write:

$$P(h+\delta, t+1) = p(h+\delta, h)P(h,t)$$

where $p(h+\delta, h)$ is the transition probability from state h to state $h+\delta$ in the unit of time. This is the traditional way one can write down the probabilistic law for a Markov Chain; usually in such a process, the transition probability does not depend on the distribution of h and for such a reason we define it as a linear Markov Chain. In our case this property holds no longer since both terms on the R.H.S. depend on the distribution of h and from here the non-linearity of the process. In the following we are going to identify the partial differential equation describing the evolution of such a process, although we have no hope to provide a solution in a closed form.

For the sake of simplicity I assume that when agents do not invest in human capital they incur in an obsolescence equal to the δ coefficient; this assumption makes simpler the analysis inducing symmetry in the elementary steps, without affecting results. In this way the chain can hop at one elementary step only over a fixed distance δ, i.e. $h_{n+1} = h_n \pm \delta$ where n is the number of steps. Normalizing $h_{t0} = 0$, then the space state is the set $\{0, \pm\delta, \pm 2\delta, \pm 3\delta ....\}$. By $P(h, n+1)$ we mean the probability of finding the $i-th$ agent at the position h after $n+1$ steps; our goal is to derive an equation describing the temporal evolution of this probability. We start from the fact that the process can hop only over a distance δ which means that the position h can be achieved only from $h-\delta$ or $h+\delta$; these states are occupied respectively with probability $P(h-\delta, n)$ and $P(h+\delta, n)$. Thus $P(h, n+1)$

consists of two parts stemming from two possibilities for the process to jump. By $p(h, h')$, $h' = h \pm \delta$, we mean the transition probability to move from h' to h. By so doing, we can write down the probability of finding the variable at the position h after $n+1$ steps:

$$P(h, n+1) = p(h, h-\delta)P(h-\delta, n) + p(h, h+\delta)P(h+\delta, n) \tag{11}$$

where $p(h, h') = 0$ unless $h' = h \pm \delta$. Now we can approximate such a process in continuous time and at this end we "rescale" the integer n in $n = t/\tau$, where τ is the transition time per elementary step. In this way $\tilde{p}(h, h') = p(h, h')/\tau$ is the transition probability per unit time. By subtracting $P(h, n)$ from both sides of (11) and dividing both sides by τ we obtain:

$$\frac{P(h, n+1) - P(h, n)}{\tau} = \tilde{p}(h, h-\delta)P(h-\delta, n) + \tilde{p}(h, h+\delta)P(h+\delta, n) - P(h, n) \tag{12}$$

By indicating $\tilde{P}(h, t) = P(h, n/\tau)$ we mean a new probability measure on the rescaled variable $t = n\tau$. Moreover, letting $\tau \to 0$,we rewrite the above equation as the following:

$$\frac{d\tilde{P}(h, t)}{dt} = \tilde{p}(h, h-\delta)\tilde{P}(h-\delta, t) + \tilde{p}(h, h+\delta)\tilde{P}(h+\delta, t) - \tilde{P}(h, t)$$

This linear differential equation is known in literature as the Master Equation. Despite its seemingly simplicity, such equations have not closed solutions and numerical simulations are often implemented. In our case however the situation is even more complex because of the non-linearity induced by the dependence of the transition probabilities on $\tilde{P}(h, t)$ itself.

The master equation represents the evolution of the probabilistic law of a stochastic process assuming values on a discrete set in continuous time. To avoid such coexistence of discrete and continuous variables the master equation is often approximated by a continuous-state, continuous-time process by means of a rescaling analogous to the one used for the time variable. In this way we bring the master equation to a partial differential equation which is usually more tractable, although our case does not allow such a facility.

By explicating the probability transitions in our case, the Master equations assume the following form:

$$\frac{d\tilde{P}(h,t)}{dt} = \left(\int_{\Re^{++}} Z(z_t)dz_t\right)\tilde{P}(h-\delta,t)+\left(1-\int_{\Re^{++}} Z(z_t)dz_t\right)\tilde{P}(h+\delta,t)-\tilde{P}(h,t) \tag{13}$$

where $Z(z_t)$ is the density function of z_t. which depends on h and $g(h_t) = \tilde{P}(h,t)$.

Approximating the (13) by a continuous space state variable means that $\tilde{P}(h,t)$ must be viewed as a density function, where now h acts as a continuous variable[11]; in order to avoid mistakes, from now and on we replace $\tilde{P}(h,t)$ with $g(h,t)$ where $g(\cdot,t)$ is the density function for h. By expanding the R.H.S. of (13) into a Taylor series up to second order in $\delta = 0$, and normalizing δ to one, we obtain the following partial differential equation:

$$\frac{dg(h,t)}{dt} = \frac{\partial g(h,t)}{\partial h}\left(1-2\int_{\Re^{++}} Taylor\left[Z(z_t),2,\delta=0\right]dz_t\right)+\frac{\partial^2 g(h,t)}{\partial h^2} \tag{14}$$

The solution of (14), jointly to the boundary conditions $g(h,0) = g(h)$ and $g(0,t) = 0$, provides the unknown density function for the human capital.

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