Signalling equilibrium, Intergenerational mobility and long-run growth *

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Abstract

This paper provides a signalling model of endogenous growth in which innate talents and education levels of workers drive the basic scientific knowledge and adoptive knowledge accumulation processes. Whether talented individuals get properly educated and are employed in the appropriate technical sectors are determined by the perfectly competitive employers' beliefs about the relationship between talent and education level. Innate talent of a worker is a private knowledge and it is distributed independent of the individual's family backgrounds; education level of workers act as a signalling device for talents as well as it improves their productivities; the family backgrounds and talents of workers determine their optimal education level, which in turn determines the degree of social mobility. The model generates multiple balanced growth paths which differ in the degree of intergenerational social mobility and growth rate. The paper analyzes policies that generate equilibrium paths with higher social mobility, growth in income and Pareto superior allocations.

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> [I]t is not a story that concludes, *Genius will out*-though Ramanujan's in the main, did. Because so nearly did events turn out otherwise that we need no imagination to see how the least bit less persistence, or the least bit less luck, might have consigned him to obscurity. In a way, then, this is also a story about social and educational *systems,* and about how they matter, and how they sometimes nurture talent and sometimes crush it. How many Ramanujans, his life begs us to ask, dwell in India today, unknown and unrecognized? And how many in America and Britain, locked away in racial or economic ghettos, scarcely aware of worlds outside of their own?
>
> Robert Kanigel, *The Man who knew Infinity,* pp.3-4.

1. Introduction

It is a well established fact that an important source of growth in per capita income of modern economies is the increasing stock of tested and useful technological knowledge. Recent literature on endogenous growth is concerned with modeling the process of such knowledge creation. Among others, Lucas [1988] provides a model in which identical workers spend some of their time acquiring skills or vocational training which increase their productivity; the new technologically useful knowledge that the individual workers acquire from their vocational or on the job training adds to the pool of social knowledge. In his model, factors that increase average education level of the work force also create higher long-run growth. In Romer's [1986] model, individual firms spend resources on R&D to create new technological knowledge; the individual firm's knowledge spills over to the rest of the economy creating the pool of social knowledge from which other firms benefit; this process generates increasing returns in production of knowledge and thus leads to long-run growth. Since machines cannot create new knowledge, and only humans working with machines can create new knowledge, in the Romer model human capital of the R&D personnel is important as well.

While the previous endogenous growth models give an important role to the creation of knowledge, these models are deficient in certain respects: These models do not distinguish between different types of knowledge such as basic scientific knowledge which generates process and product innovation, applied industrial technical knowledge which helps to adopt new product or process innovation for industrial production, and the managerial and on the job or vocational training that enhance the efficient organization and implementation of the actual production process of a new technology. A faster growth of an advanced economy requires growth of each type of knowledge.

Another drastic simplification made in the endogenous growth literature is the assumption that individual workers in the work force are all identical in their innate abilities in creating knowledge. In a given population innate abilities or talent levels vary from individual to individual. It is reasonable to presume that very talented workers when properly educated and employed in appropriate sectors will be generating new basic technological knowledge, and workers of somewhat lesser talents with proper education in engineering schools might be suitable for generating applied knowledge suitable to adopting basic scientific knowledge to industrial production purposes, and so on for other talent levels.

The structure of the educational system in a given society affects how individuals with different talent levels and family backgrounds decide about their levels of education. The characteristics of the labor markets determine how these workers of various talents and education levels are placed in different types of jobs. The functioning of these two is also a critical determinant of the growth process. Since there is no well established empirical evidence that innate talents of individuals are genetically inherited[1] we may assume more or less that the innate talent of an individual is independent of his or her family background. However, the family background may affect the level of education attained by an individual. Then it is quite clear that the higher is the rate of mobilization of such valuable talented individuals, the higher will be the intergenerational mobility and growth.

[1] The general wisdom on this is that there is probably a small correlation between the intelligence of parents and children, but the environmental factors are so dominant that it is very difficult to isolate the genetic inheritance, see Levine and Suzuki [1993]

In this paper, we consider a signaling model of overlapping generations. We assume that employers do not observe productive talents but they form subjective beliefs about individuals' productive talents by observing education levels and on that basis they offer wage contracts. Given the wage contract offers, individuals decide on their education levels. Contrary to the traditional growth models with human capital investments, we assume that education not only increases one's cognitive power enhancing productivity, but also acts as a signal of one's productive abilities or talents that are observed only by the individuals themselves. Whether talented individuals get properly educated and are employed in the appropriate technical sector are then determined by the employers' beliefs. The model generates multiple balanced growth paths along which there are varying degrees of intergenerational mobility and mobilization of human resources and growth. I also analyze policies that generate equilibrium paths with higher social mobility and growth rates.

The rest of the paper is organized as follows:

2. The Basic Model

We begin with a simple overlapping generations model. In each period one person is born to each parent. The gender of the individual is not important for this model, and we assume for ease of presentation the male gender. Assume that there is a finite set of discrete talent types $\mathcal{T} = \{1, 2, ..., \hat{\tau}\}$, with a higher number denoting greater talent. Let T be the random variable denoting the talent type of an individual. The probability that a parent has a child of talent $\tau \in \mathcal{T}$ is $h(\tau)$. We assume that the probability mass function $h(\tau)$ does not depend on the parent's talent type.

The talent type of an individual is private information, observed only by him and by no one else. Individuals, however, can choose an education level that is properly adjusted for quality to signal his talent type. Let S be the random variable denoting the education level of an individual. The realization of the random variable T is determined by nature and observed only by the agent. The realization of S is the result of individual choice and it is public information. For simplicity we assume that the set of education levels, $\mathcal{S}$ is discrete and finite and given by $\mathcal{S} = \{1, 2, ..., \hat{s}\}$, with a higher number representing a higher education level.

We can think of each signal in $\mathcal{S}$ as a social class, social status, social rank according to earnings[2] or simply an occupation. Furthermore, the parents' socio-economic status $s \in \mathcal{S}$ summarizes his children's family background. In each period the active members of the society will belong to one of the groups in $\mathcal{S}$. We are interested in modeling the intergenerational mobility of families across the groups in $\mathcal{S}$, without referring to it as social mobility, class mobility or earnings mobility since they all coincide in our model. More specifically, let the probability mass function of the population in period t over the set of signals $\mathcal{S}$ be denoted as $\pi_t = (\pi_t^1, ..., \pi_t^S)$, $t \geq 0$, The economy begins at time $t = 1$ with an adult population whose parents' socio-economic status is distributed as $\pi_0 = (\pi_0^1, ..., \pi_0^S)$. In what follows we provide an economic model of these adults' and their children's and children's children's ... occupation or signal choice problem. That is we provide an economic model to generate equilibrium transition probabilities that characterize the nature of intergenerational mobility and the dynamics of π_t and then study their impact on growth and welfare of the economy.

2.1. Production sector

There are some empirical controversy as to whether years of education is a significant determinant of earnings, see Bowles [1972], and Griliches and Mason [1972]. The general consensus, however, is that both abilities and education level are significant determinants of earnings.

In our model, we assume that education is productive. For all $s \in \mathcal{S}$ and $\tau \in \mathcal{T}$, a unit of labor from an adult of type τ and education level s is equivalent to $e(s, \tau)$ units of labor in efficiency units. Let $e(s, \tau)$ denote the productivity level of an individual of talent type τ and education level s. Let $\mathcal{E} = \{e(s, \tau) | s \in \mathcal{S}, \tau \in \mathcal{T}\}$ denote the set of productivity levels. Let L_t be the total labor in efficiency units used in the production process. To simplify matters and without loss of generality we assume that the aggregate production in period t is represented by the following linear function:

$$F_t(L_t) = A_t L_t \tag{1}$$

where A_t is the shift in the productivity level in period t.

In our model A_t is endogenously determined as follows: As mentioned in the introduction our presumption is that talented workers with higher education can create basic knowledge or new ideas about production processes which will benefit future generations. More specifically, let R_t be

[2] We will see later that earnings are functions of $s \in \mathcal{S}$.

the number of adults in period t with $S_t \geq \bar{s}$ and $T \geq \bar{\tau}$. These are the researchers in period t. A_t evolves over time according to:

$$A_{t+1} = A_t (1 + g(R_t)) \tag{2}$$

where $g(R_t)$ is the growth rate of productivity level, assumed to be a time invariant function of the number of researchers. If $R_t = 0$, $g(R_t) = 0$ and g may be assumed to be an increasing concave function as in $g(R_t) = R_t^\gamma$, $0 < \gamma < 1$.

2.2. Employer's Problem

In the economics of imperfect information there are mainly two ways in which the worker's and employer's problems are formulated: the first approach is the Spence model where the employer announces a wage schedule, then workers make their decisions on education levels; the other approach due to Rothchild-Stiglitz is more applicable in the insurance context, see for other schemes, Kreps [1990]. Spence's approach is more appropriate in our context, and we will follow this approach.

We assume that the production sector is competitive; the producer is risk neutral and he treats A_t as an externality when making his decisions. In each period $t \geq 1$, the producer's role is to announce a wage schedule $w_t(s_t)$ for hiring purposes. He observes the education level $S_t = s$ of an agent but not the realization of his talent type T_t. The employer holds a subjective belief about the conditional probability distribution of the talent type T_t of an worker given his observed education level S_t. We denote it by

$$q_t^{\tau s} = \text{Prob}\{T_t = \tau | S_t = s\}, \tau \in \mathcal{T}, s \in \mathcal{S} \tag{3}$$

Let $w_t(s_t)$ be the wage profile that the producer announces. Perfect competition, and expected profit maximization with respect to all possible probability distributions of S_t imply that

$$\begin{aligned} w_t(s) &= E\left(e(T_t, S_t) | S_t = s\right) \\ &= A_t \textstyle\sum_{\tau \in \mathcal{T}} e(\tau, s) q_t^{\tau s} \\ &= A_t \omega_t(s), \text{ say} \end{aligned} \tag{4}$$

where $\omega_t(s) \equiv \sum_{\tau\in\mathcal{T}} e(\tau,s)q_t^{\tau s}$. Notice that $\omega_t(.)$ depends on the producer's subjective conditional probability distribution, $q_t^{\tau s}$, and dependence of $\omega_t(.)$ on t is through the dependence of $q_t^{\tau s}$ on t.

2.3. Worker's problem

Family background can have great influence on educational attainment in several ways. For instance, suppose that the quality of pre-school investment of parents' time at home affect children's motivation and persistence to continue schooling. Then, of course, more highly educated parents can provide better learning environment for their children at home. similarly, more highly educated parents with their better knowledge base of child care, or simply because of their higher incomes can provide better pre-natal care, and health care for proper cognitive development of their children. We represent these effects of family background in our model by assuming that the cost, $\theta_t(s_t,\tau,s_{t-1})$, of obtaining a certain level of education $s_t \in \mathcal{S}$ for an individual in period t depends on his talent type $\tau \in \mathcal{T}$, and his parent's education level $s_{t-1} \in \mathcal{S}$, for all $s_t, s_{t-1} \in \mathcal{S}$, and $\tau \in \mathcal{T}$.[3] We will assume that $\theta_t(s_t,\tau,s_{t-1})$ is increasing in s_t, and decreasing in τ and s_{t-1}. For further justifications of this assumption of differential cost, see Raut [1985,1991]. The assumption that $\theta(s_t,\tau,s_{t-1})$ varies with τ is necessary for education to act as signals for talent, see Spence [1974] for a justification. We further assume that

$$\theta_t(s_t,\tau,s_{t-1}) = A_t\theta(s_t,\tau,s_{t-1})$$

where A_t is the productivity shift parameter of the aggregate production function defined earlier.

The investment in the level of education of an individual is a complex decision making process. Generally, parents make the initial investments such as pre-school investments and investments up to college or so, until the the individual reaches enough maturity to make his own schooling decisions. The nature of parents' investment may limit the individual's choices later on.

To model the part of the human capital investment that is made by parents we need to introduce intergenerational altruism, which brings complicated intergenerational conflicts and overshadows our main focus. To

[3] There are other ways education of parents can influence the educational achievements of their children, for instance, by providing role models.

make progress, we assume for now that agents have life cycle utility functions which they maximize to decide on their level of education level, and we will come back to the intergenerational altruism issue in a latter section. The effect of parental inputs is assumed to be on the cost of schooling that the individual will face, and is assumed exogenously given.

More specifically, let c_t be the consumption of an adult of period t. All individuals are assumed to have identical utility function $u(c_t)$. Without loss of generality, we can assume $u(c) = c$. An adult of period t with talent type $\tau \in \mathcal{T}$ and from a parent of education level s_{t-1} takes the announced wage function $w_t(s)$ of period t as given and decides his education level $s_t \in \mathcal{S}$ to solve the following problem:

$$\max_{s_t\in\mathcal{S}} \ w_t(s_t) - \theta_t(s_t,\tau,s_{t-1}) \tag{5}$$

Except for degenerate cases, there is a unique optimal solution s_t for each τ and s_{t-1}, which is independent of A_t. We denote the optimal solution of equation (5) by

$$s_t = \sigma_t(\tau,s_{t-1}) \tag{6}$$

2.4. Signaling Equilibrium in the Labor Market

The equilibrium is recursively defined over time. At the beginning of time period t, π_{t-1} and A_t are already known. For given anticipated conditional probabilities $q_t^{\tau s}$ as defined in (3), the producer announces the wage profile $w_t(s)$ as given in (4). Given $w_t(s)$, the worker decides on optimal education level $\sigma_t(\tau,s_{t-1})$ depending on his talent type τ and family back ground s_{t-1} leading to (6). $\sigma_t(\tau,s_{t-1})$ together with π_{t-1} and $g(\tau)$ generate the observed conditional distribution of the talent type T_t given educational level S_t as follows:

Let us introduce a counting variable:

$$\chi(\tau,s_{t-1}|s_t) = \begin{cases} 1 & \text{if } \sigma(\tau,s_{t-1}) = s_t \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Or in other words, $\chi(\tau,s_{t-1}|s_t) = 0$, and

$$\chi(\tau,s_{t-1}|s_t) = 1 \iff w(s_t) - \theta(s_t,\tau,s_{t-1}) > w(s_t') - \theta(s_t',\tau,s_{t-1}) \forall s_t' \neq s_t \tag{8}$$

The empirical conditional distribution $\hat{q}_t^{\tau s}$ of talent type $T_t = \tau$ given $S_t = s$ can be expressed as follows:

$$\begin{aligned} \hat{q}_t^{\tau s} &= \text{observed probability}\{T_t = \tau | S_t = s\}, \tau \in \mathcal{T}, s \in \mathcal{S} \\ &= \frac{\text{Prob}\{S_t = s|\tau)g(\tau)\}}{\sum_{\tau \in \mathcal{T}} \text{Prob}\{S_t = s|\tau)g(\tau)} \\ &= \frac{\sum_{s_{t-1} \in \mathcal{S}} \text{Prob}\{S_t = s|\tau, s_{t-1})\pi_{t-1}^{s_{t-1}} g(\tau)}{\sum_{\tau \in \mathcal{T}} \sum_{s_{t-1} \in \mathcal{S}} \text{Prob}\{S_t = s|\tau, s_{t-1})\pi_{t-1}^{s_{t-1}} g(\tau)} \\ &= \frac{\sum_{s_{t-1} \in \mathcal{S}} \chi(\tau, s_{t-1}|s)\pi_{t-1}^{s_{t-1}} g(\tau)}{\sum_{\tau \in \mathcal{T}} \sum_{s_{t-1} \in \mathcal{S}} \chi(\tau, s_{t-1}|s)\pi_{t-1}^{s_{t-1}} g(\tau)} \end{aligned} \quad (9)$$

The above is the observed distribution that the producer will find after hiring.

A **period t equilibrium** is achieved when the subjective belief about distribution of T_t given S_t coincides with its observed distribution.

Notice that optimal $\sigma(\tau, s_{t-1})$ determines the following transition probabilities

$$\begin{aligned} p_t^{ij} &= \text{Prob}\{S_t = j | S_{t-1} = i\} \ \forall i, j \in \mathcal{S}, t \geq 1 \\ &= \sum_{\tau \in \mathcal{T}} \chi(\tau, i|j) g(\tau) \end{aligned} \quad (10)$$

Let $P_t = \left(p_t^{ij}\right)_{i,j \in \mathcal{S}}$ be the transition matrix in period t. Given π_{t-1}, P_t determines π_t according to the following equation

$$\pi_t = \pi_{t-1} P_t \quad (11)$$

and π_{t-1} and $\sigma(\tau, s_{t-1})$ determine R_t according to equation (2); R_t, in turn determines the growth rate, $g(R_t)$ or equivalently, A_{t+1}. The economy moves to the next period with known π_t and A_{t+1}, and the above process starts all over again.

Definition 1 *:* A **signaling equilibrium** is a sequence of anticipated conditional probability distributions $\{q_t^{\tau s}\}_1^\infty$ defined in (3) such that the induced observed conditional distribution $\{\hat{q}_t^{\tau s}\}_1^\infty$ defined in (9) satisfies the condition that $\hat{q}_t^{\tau s} = q_t^{\tau s} \forall \tau \in \mathcal{T}, s \in \mathcal{S}, t \geq 1$. The equilibrium wage

profile of the producer $w_t(.)$, the education decision rule of the households $\sigma_t(.)$, and the matrix of transition probabilities, P_t are given in equations (4), (6) and (10) respectively.

It is possible to have different types of signaling equilibria. A **pooling equilibrium** is a signaling equilibrium where all types of agents from all economic backgrounds use the same signal, i.e., $\sigma_t(\tau, s_{t-1})$ is independent of τ and s_{t-1} for all $t \geq 1$. A **separating equilibrium** is a signaling equilibrium in which the agents of different talent type and family background use distinct schooling levels, i.e., $\sigma_t(\tau, s_{t-1}) = \sigma_t(\tau', s'_{t-1})$ if and only if $\tau = \tau'$ and $s_{t-1} = s'_{t-1}$ for all $t \geq 1$. These are the kinds of equilibria generally studied in the literature. We define other kinds of equilibria relevant in the present context. A signaling equilibrium is **equal opportunity equilibrium** if $\sigma_t(\tau, s_{t-1}) = \sigma_t(\tau, s'_{t-1}) \equiv \bar{\sigma}_t(\tau)$ say $\forall s_{t-1}, s'_{t-1} \in S$, i.e., all workers of the same talent type get the same education level no matter what their family back-grounds are. A **separating equal opportunity equilibrium** is an equal opportunity equilibrium such that $\bar{\sigma}_t(\tau) \neq \bar{\sigma}_t(\tau')$ if $\tau \neq \tau'$. In our context an equilibrium will lead to maximum growth if $\sigma_t(\tau, s_{t-1}) \geq \bar{s}$ for all $\tau \geq \bar{\tau}$, for all $s_{t-1} \in S$ and $t \geq 1$, we will call it a **growth enhancing separating equilibrium.**

In any given economy there may exist several Pareto ranked multiple equilibria, some of which could be low-level pooling equilibria, some may be separating but not necessarily equal opportunity or growth enhancing. In what follows we will study these possibilities using examples and then we examine how perfect competition can help us to refine equilibria, and the role of public policies to generate and select separating equilibria of desired types such as equal opportunity, or growth enhancing, or Pareto optimal.

For the purpose of studying long-run properties of the system, and the transitional dynamics of it, we restrict ourselves to models that generate stationary transition matrices, i.e., P_t is independent of t. This is needed so that we could apply the known techniques from the theory of Markov chains. A **long-run or steady-state** equilibrium is an equilibrium with $\pi_{t+1} = \pi_t (\equiv \pi)$ say for all $t \geq 1$. This implies that $\pi = \pi P$, i.e., the steady-state distributions of population are the normalized non-negative eigenvectors of the stationary transition matrix P'. Note that not all economies will have stationary transition probabilities, and not all equilibria will converge to a steady-state equilibrium. We will also examine the nature of dynamics for various types of equilibria of our economies.

For the existence of equilibrium, note that the right hand side of equation (9) is function of $q_t^{\tau s}$. Any fixed point of this mapping is an equilibrium. In this paper we will not try to find conditions under which there exists an equilibrium, since we will compute equilibrium for each of the economies that we consider below.

3. First example

Let us consider first an example with $\hat{\tau} = \hat{s} = 2$, and $\bar{\tau} = \bar{s} = 2$. Whether there exists any signaling equilibrium, and if there exists one, whether there exist many equilibria some of which are Pareto superior, some of which are equal opportunity separating, some of which are growth enhancing separating depend on the technology $e(\tau, s)$ and the cost function, $\theta(s_t, \tau, s_{t-1})$. We will illustrate our issues by fixing the following specification of the technology and assuming different forms for the cost function.

$$e(s,\tau) = \begin{cases} 1 & \text{if } s = 1, \forall \tau \in \mathcal{T} \\ 2 & \text{if } s = 2, \tau = 1 \\ 3 & \text{if } s = 2, \tau = 2 \end{cases} \quad (12)$$

An interpretation of the above is that the workers with education level 1 are unskilled workers and the talent of the unskilled workers do not affect their productivity; However, higher educated talented workers have higher productivity than higher educated not-so-talented workers.

Assume that the cost function $\theta(s_t, \tau.s_{t-1})$ satisfies the following:

$$\left.\begin{array}{c} \theta(1,\tau,s_{t-1}) = 0 \ \forall \tau, s_{t-1}, \text{ and} \\ \theta(2,2,2) < \theta(2,1,2) < 1 + p < \theta(2,2,1) < \theta(2,1,2) \end{array}\right] \quad (13)$$

Suppose the employer holds the following subjective probability distribution for the type T_t given S_t:

$$q_t^{\tau s} = \text{Probability}\{T_t = \tau | S_t = s\} = \begin{cases} 1-p & \text{for } \tau = 1 \\ p & \text{for } \tau = 2 \end{cases} \quad \text{for all } s \in \mathcal{S}$$

According to (4), given the above expectations, the employer announces the following wage schedule:

$$w(s_t) = \begin{cases} 1 & \text{if } s_t = 1 \\ 2.(1-p) + 3.p & \text{if } s_t = 2 \end{cases}$$

Given the above wage schedule, one can easily verify that the equilibrium schooling decisions $\sigma(\tau, s_{t-1})$ of an agent of talent type τ from the family background s_{t-1} is as follows:

$$\sigma(\tau, s_{t-1}) = \begin{cases} 1 & \forall \tau \in \mathcal{T} \ if \ s_{t-1} = 1 \\ 2 & \forall \tau \in \mathcal{T} \ if \ s_{t-1} = 2 \end{cases}$$

It can be easily checked that given the above optimum solution, the observed conditional probability distribution of T_t given S_t will coincide with the anticipated one. Note that the the transition matrix associated with $\sigma(.)$ is the following:

$$P_t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \forall t \geq 1$$

Thus in this economy there is no intergenerational mobility. Furthermore, the economy is in steady-state from the beginning. Thus, $R_t = \pi_0^2.p$, and hence the growth rate is given by $g(p\pi_0^2)$ which is strictly less than $g(p)$, the maximum attainable growth rate when the talented individuals from all socio-economic groups obtain education.

This equilibrium is not equal opportunity separating, nor growth enhancing separating type. In this equilibrium, all talent types of the children are pooled within each type of family backgrounds of their parents.

Could there be any other equilibrium for the above economy? For certain economies of the above type, there is one more equilibrium which is growth enhancing separating and is Pareto superior compared to the above equilibrium. To see this, let $v_t \equiv \frac{p}{p\pi_{t-1}^1 + \pi_{t-1}^2}$. Note that $v_t > p \ \forall t \geq 1$. At $t = 1$, v_1 is given. Let us suppose that apart from the assumption (12), the cost function also satisfies the condition, $p < \theta(2,2,1) < v_1 < \theta(2,1,1)$.

Suppose the employer holds the following subjective probability distribution for the type T_t given S_t: $T_t = 1$ with probability 1 when $S_t = 1$, and

$$\text{Probability}\{T_t = \tau | S_t = 1\} = \begin{cases} 1 - v_t & \text{for } \tau = 1 \\ p_t & \text{for } \tau = 2 \end{cases} \quad (14)$$

According to (4), given above expectations, the employer announces the following wage schedule:

$$w(s_t) = \begin{cases} 1 & \text{if } s_t = 1 \\ 2.(1-v_t) + 3.v_t & \text{if } s_t = 2 \end{cases}$$

Given the above wage schedule, the original $\sigma_t(\tau, s_{t-1})$ will be optimal for all (τ, s_{t-1}) except for $\tau = 2$, $s_{t-1} = 1$, who will choose $s_t = 2$. It can be easily checked that for this optimal solution, the observed conditional probability distribution of T_t given S_t will coincide with the anticipated one in equation (14). Note that the transition matrix associated with $\sigma_t(.)$ is as follows:

$$P_t = \begin{pmatrix} 1-p & p \\ 0 & 1 \end{pmatrix}$$

Thus in this economy there is intergenerational mobility. The proportion of the population with higher education will go on increasing and the proportion of the population with lower education will go on decreasing. This process, however, cannot go on for ever, since in that case $v_t \longrightarrow p$, as $t \longrightarrow \infty$, which will mean that there will be some finite $t_0 > 1$ such that $v_{t_0} > \theta(2,2,1)$ for the first time and then on the equilibrium will switch on to the previous one with no mobility. Note, however that the new steady-state equilibrium growth rate will be $g\left(\pi^2_{t_0}.p\right)$ since $\pi^2_{t_0} > \pi^2_0$. Furthermore, the short-run growth rate up to period t_0, is higher in the second equilibrium than in the first type; and the second equilibrium is Pareto superior to the first. Thus, in this economy there may exist multiple equilibria; which one will materialize depends on the expectations of the employer. The question is then, how the employer's expectations are formed? We need a theory of expectations formation of the producers to select an equilibrium, and we do not pursue this theory here.

4. Another Example with more signals

$\mathcal{T} = \{1,2\}$ and $\mathcal{S} = \{1,2,3\}$, $\bar{s} = 3$, $\bar{\tau} = 2$. Let the production technology be of the following type:

$$e(s,\tau) = \begin{cases} 1 & \text{if } s = 1, \forall \tau \in \mathcal{T} \\ 2 & \text{if } s = 2, \tau = 1 \\ 3 & \text{if } s = 2, \tau = 2 \\ 3.5 & \text{if } s = 3, \tau = 1 \\ 4 & \text{if } s = 3, \tau = 2 \end{cases}$$

Let the cost of education be as follows:

$$\theta(s_t, \tau, s_{t-1}) = \begin{cases} \frac{\theta(s_t-1)}{\tau s_{t-1}} - \epsilon & \text{if } s_t = 2, \tau = 2, s_{t-1} = 1 \\ \frac{\theta(s_t-1)}{\tau s_{t-1}} + \epsilon & \text{if } s_t = 2, \tau = 1, s_{t-1} = 2 \\ \frac{\theta(s_t-1)}{\tau s_{t-1}} + 3.5\epsilon & \text{if } s_t = 3, \tau = 1, s_{t-1} = 3 \\ \frac{\theta(s_t-1)}{\tau s_{t-1}} & \text{otherwise} \end{cases}$$

where, $\epsilon > .25$, and $\theta = 3.5$.

Suppose the producer anticipates the following probabilities for talent type T_t types given the education level S_t of the worker:

$$\text{Prob}\{T_t = 1 | S_t = 1\} = 1$$

$$\text{Prob}\{T_t = 1 | S_t = 2\} = \frac{p\pi^1_{t-1}}{p\pi^1_{t-1} + (1-p)\pi^3_{t-1}}$$

$$\text{Prob}\{T_t = 2 | S_t = 2\} = \frac{(1-p)\pi^3_{t-1}}{p\pi^1_{t-1} + (1-p)\pi^3_{t-1}}$$

$$\text{Prob}\{T_t = 2 | S_t = 3\} = 1$$

Given his anticipated conditional probability distributions, he announces the following wage schedule:

$$w(s_t) = \begin{cases} 1 & \text{if } s_t = 1 \\ 2\frac{p\pi^1_{t-1}}{p\pi^1_{t-1}+(1-p)\pi^3_{t-1}} + 3\frac{(1-p)\pi^3_{t-1}}{p\pi^1_{t-1}+(1-p)\pi^3_{t-1}} & \text{if } s_t = 2 \\ 4 & \text{if } s_t = 3 \end{cases}$$

It can be easily checked that the optimal schooling decisions $\sigma(\tau, s_{t-1})$ of an agent of talent type τ and family background s_{t-1} is given by

τ	s_{t-1}	$\sigma(\tau, s_{t-1})$
1	1	1
2	1	2
1	2	1
2	2	3
1	3	2
2	3	3

It can be easily checked that the transition matrix associated with the above optimal decision, $\sigma(\tau, s_{t-1})$ is the following:

$$P_t = \begin{pmatrix} 1-p & p & 0 \\ 1-p & 0 & p \\ 0 & 1-p & p \end{pmatrix}$$

There is now some mobility. The economy converges to a unique stationary distribution $\pi = \left(\frac{(1-p)^2}{1-p+p^2}, \frac{p(1-p)}{1-p+p^2}, \frac{p^2}{1-p+p^2}\right)$ starting from any initial distribution π_0. While the growth rate $g(R_t)$ in period t depends on the initial distribution, π_0, however, the long-run growth rate is independent of initial condition, and is given by $g\left(\frac{p^2}{1-p+p^2}\right) < g(p)$, the maximum attainable long-run growth rate. We can modify the above economy to generate equilibria with higher mobility leading to higher growth. It is possible to have multiple long-run equilibria, and the convergence to a particular equilibrium will depend on the initial distribution π_0 as in the previous example. We have not explored those possibilities here.

We can show that even when the cost of education does not vary with one's family background, if the producer conditions his subjective expectations about a worker's ability on his family background (such as using the last name to find out if one is coming from such and such families with such and such ethnic background), then there are equilibria in which the children of the poorer family backgrounds will not invest in higher education, and thus mobility will be reduced and so will be the economic growth. This has important policy implications.

5. Parental altruism, mobility and growth

As mentioned earlier, pre-school parental investment may be an important determinant of the educational performance of the children.

We can incorporate this aspect by assuming that the cost of producing signal s_t of a child of talent type τ will depend on the parental investment h_{t-1}, and these cost functions are uniform across families, i.e., the cost function is given by

$$\theta(s_t, \tau, h_{t-1})$$

Parents do not observe the talent type of his child when making the pre-school investment decision. Adult children of talent type τ in time period

t from a parent of signal class $s_{t-1} \in \mathcal{S}$, decides

$$\max_{s_t, h_t \in \mathcal{S}} E_{\tau'} \left[u(c_t) + \beta u(c_{t+1})\right]$$

$$subject\ to$$

$$\begin{aligned} c_t &= w_t(s_t) - \theta(s_t, \tau, h_{t-1}) - h_t \\ c_{t+1} &= w_{t+1}\left(s_{t+1}(\tau', h_t) - \theta(s_{t+1}, \tau', h_t)\right) \end{aligned}$$

Definition 2 *:* A **Markov perfect signaling equilibrium** is a sequence of anticipated conditional probability distributions $\{q_t^{\tau s}\}_1^\infty$ defined in (3) and a sequence of reaction functions $\{s_t(\tau, h_{t-1})\}_1^\infty$ such that for all $t \geq 0$, given $s_{t+2}(.,.)$, and given h_{t-1} and τ, $s_{t+1}(\tau, h_t)$ solves the above problem and the induced observed conditional distribution $\{\hat{q}_t^{\tau s}\}_1^\infty$ defined in (9) satisfies the condition that $\hat{q}_t^{\tau s} = q_t^{\tau s} \forall \tau \in \mathcal{T}, s \in \mathcal{S}, t \geq 1$. The equilibrium wage profile of the producer $w_t(.)$, the education decision rule of the households and the matrix of transition probabilities, P_t are given in equations (4) and (10) respectively.

We have simulation results on this type of equilibrium which have similar properties to our earlier models. They are not reported here.

6. Policies and conclusions

The existence of multiple equilibria arising from unprejudiced employer's self-fulfilling expectations raises important empirical questions: how to verify whether an economy is stuck with a low level equilibrium where growth rate, and social mobility are low, and how to design policies that will allow the economy to move from a low level equilibrium to an equilibrium with higher growth and social mobility.

It is clear from our analysis that increasing average education level of the population may not be the most effective way of raising the growth rate. The distribution of education levels according to talents and the assignments of workers to appropriate jobs is important for growth. An important empirical issue in this connection is to examine if observed educational attainment of individuals in a society is according to their innate abilities, or according to their family backgrounds reflecting their cost of education.

If the source of lower social mobility is due to higher cost of education faced by children of poorer family background, subsidizing higher education uniformly to children of all family backgrounds is not necessarily going to be effective in inducing the talented children from poorer family background to opt for higher education. The effective policy is rather to identify the talented individuals from all social backgrounds and give enough subsidies to the ones from the poorer family back ground so that they get higher education and work in the appropriate sectors.

Since educating the talented children of poorer families may require substantial subsidies, it may not be possible to raise that money by taxing the parents of less talented children; however, by borrowing from international markets, and paying it back in the next period, the whole society may have a Pareto superior (even Pareto optimal) allocation of resources and higher growth rates. Furthermore, the short-run cost of subsidies to the poorer families might outweigh the long-run benefits accruing to future generations in terms of higher rate of growth in technological change.

Our analysis has the following implication for the proposed policy: to allow children to borrow for college education. This will be effective only to the students on the margin who have enough pre-school investment so that the rate of return from college is higher than the interest rate for the loans. For those with poorer family back grounds, they would need loans at lower interest rates, or their parents should be given tied subsidies for the purpose of pre-natal and preschool investment to reduce their children's cost of education.

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