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THE SOURCES OF GROWTH

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Abstract

In this paper, we survey the principal theoretical models of endogenous growth and explore their common components through analyzing a series of planning problems. Our aim is to provide the reader with a consistent approach to the emerging literature on growth as an introduction to the field.

Introduction

Our purpose in this paper is to provide a concise summary of the different methods that have developed in the recent literature on endogenizing the rate of growth in economic models (see also Sala-i-Martin (1990), Grossman and Helpman (1991c) and Helpman (1992) in this regard). Historically, the method for dealing with the phenomenon of perpetual growth in dynamic economic models comes from Solow's seminal work on the topic (1956) which is based on the assumption of exogenous technical change. The difficulty with this approach lies in its weaknesses in two distinct areas. First, it is impossible using this model to explain the observed long run differences in performance exhibited by different countries. Examples of this are well documented. They include differences in average rates of growth among countries of up to 8% (from -4%, Chad, to +4%, Japan) over periods of up to 30 years. Second, it seems reasonable that the productivity changes that are assumed exogenous in the Solow model are, in fact, the result of conscious decisions on the part of economic agents. If this is the case, it is then important to explore both the mechanism for these productivity changes as well as the factors that can give rise to the observed long run differences if we are to understand these phenomena.

Our emphasis in this paper is on the models that have been used in the literature to help understand these facts. For this reason, we will concentrate on the theories used by different authors to endogenize productivity change. Our main goal in discussing these models is to try and uncover their common properties and in this manner see what are the important features that underlie all models of the process of growth. As we shall see, the essence of the problem in an intertemporal setting with optimizing consumers is guaranteeing that the combination of the aggregate technology and assumed market structure imply that the market rate of interest does not decline to zero as the economy grows without bound. This will be emphasized throughout the presentation.

Our emphasis on basic structural properties of preferences and technologies precludes the possibility of discussing a number of other interesting problems related to the long run performance of the economy. These include (but are not limited to) the effects of tax policy on growth and the form of optimal taxation, the effects of government spending on growth (especially when it has a productive role) and its optimal form, and the role of the collective decision making process on growth. These topics are covered elsewhere in this volume.

The theories that we survey can be roughly classified into two groups. The first uses convex models of growth that satisfy the standard welfare theorems in their simplest form and rely on differences in country specific economic policy variables to understand the differences in country performance. These will be discussed in sections 2 and 3, below. The second group relies on models with either non-convexities on the technological side or externalities (or both). These will be discussed in sections 4 and 5.

At this point, the conflict between these two approaches has not been resolved. Although most researchers agree that there both non-convexities at the individual firm level and some form of externality at the local level, neither of these implies (necessarily) that the aggreate technology will exhibit similar behavior. Examples of this are abundant in the economics literature. It is straightforward to show that even with a fixed cost and constant marginal costs, the Nash equilibrium of a partial equilibrium quantity setting game with free entry is approximately the same as the Walrasian equilibrium when the market is large. Analagously, the Tiebout argument gives a similar result when externalities are local in nature. Thus, this important issue is yet to be decided.

The remainder of the paper is organized as follows. In section 1, we give a brief review of the Solow model and its implications for growth. In sections 2 and 3, we outline two simple convex models that endogenize the rate of growth of the economy. In sections 4 and 5, we discuss two of the more popular models with externalities and aggregate non-convexities that have been used. Finally, in section 6, we consider the effect of including heterogeneity of the overlapping generations sort.

1. Growth and the Solow Model

In the simplest time invariant version of the Solow model, it can be shown that the per capita stock of capital converges to a unique value independent of initial conditions. It is then necessary to assume some exogenous source of productivity growth in order to account for long run growth. In Solow (1956), it is assumed that labor productivity grows continually and exogenously. In response, the capital stock (assumed homogeneous over time) is continually increased allowing for a continual expansion in the level of output and consumption. The literature on endogenous growth has concentrated on replacing this assumed exogenous productivity growth by an endogenous process. If this change in productivity of labor is thought to arise from the invention of techniques consciously developed, the literature on endogenous growth can then be thought of as explicitly modelling the decisions to create this technological improvement (see Shell (1967) and (1973)). For this to go beyond a reinterpretation of the Solow treatment, it must be that the technology for discovering and developing these new technologies does not have itself a source of exogenous technological change. Because of this, these models all feature technologies that are time stationary.

To understand the common elements of all of these models let us first examine the simple intertemporal optimization problem of a representative agent:

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t. (i)} \quad \sum_{t=0}^{\infty} p_t(c_t + x_t) = W_0 + \sum_{t=0}^{\infty} p_t r_t k_t$$

$$\text{(ii)} \quad k_{t+1} = (1 - \delta)k_t + x_t$$

where c_t is the level of consumption, x_t is investment, k_t is the capital stock, p_t is the price of consumption (relative to time 0), and r_t is the rental price of capital, all in period t, and W_0 is initial wealth.

It is easy to derive the first-order conditions from this problem. In particular, it can be shown that if the solution is interior,

$$u'(c_t) = \beta u'(c_{t+1})\{1 - \delta + r_{t+1}\}.$$

Since perpetual growth requires that $c_{t+1} > c_t$ for all t, and u' is decreasing, we can see that for perpetual growth of consumption to be an optimal intertemporal plan for the consumer, it is necessary that

$$\beta\{1 - \delta + r_{t+1}\} > 1 \text{ for all } t.$$

This property is very general, it must hold for any model with maximizing consumers that generates positive long run growth.

One can actually say more than this. In order to have both constant interest rates (roughly a fact) and constant consumption growth rates (roughly a fact) it is necessary that preferences be of the constant elasticity form,

$$u(c) = c^{1-\sigma}/(1 - \sigma), \quad \sigma \geq 0.$$

(In fact, any two of these implies that third--see Rebelo (1991).)

In this case, we see that

$$\gamma^{\sigma} = \beta\{1 - \delta + r\},$$

where γ is the growth rate of consumption. Thus, for $\gamma > 1$, we need that r is large enough so that $\beta\{1 - \delta + r\} > 1$.

It is instructive to see how this is accomplished in the Solow framework. Since this approach is a standard neoclassical treatment, the interest rate is determined as the marginal product of capital. In this formulation, output is given by $y_t = F(k_t, B_t n_t)$, where n_t is the supply of labor and F is constant returns to scale. One can think of n_t as being given exogenously at 1 in the consumer problem above, an assumption we will impose hereafter. Here, B_t determines the productivity of labor and is assumed to follow the dynamic law: $B_{t+1} = (1 + g)B_t$, where g is given exogenously.

Assume that F is CRS in its two arguments and that $u(c) = c^{1-\sigma}/(1 - \sigma)$. Then, letting $F_k(t)$ denote the partial derivative of F with respect to k evaluated at the time t equilibrium value and imposing the equilibrium condition $r_t = F_k(t)$, the first order condition for the comsumer's problem is,

(1.1) $$c_{t+1}/c_t = [\beta(1 - \delta + F_k(t + 1))]^{1/\sigma}.$$

If the solution is going to display balanced growth (or constant growth rates), it is <u>necessary</u> that F_k be constant. However, since $F_k(k_t, B_t)$ is homogeneous if degree zero it follows that k_t/B_t is also constant. Therefore, $k_{t+1} = (1 + g)k_t$.

From the feasibility constraint it follows that $c_{t+1} = (1 + g)c_t$ and the first order condition (1.1) can be used to determine the balanced growth level of capital per effective worker, $\kappa = k_t/B_t$, as the solution to:

$$(1 + g) = [\beta(1 - \delta + F_k(\kappa, 1))]^{1/\sigma}.$$

Thus, in the Solow model, steady state growth in per capita consumption is fueled by an exogenous source of labor productivity growth. This is accompanied by an enabling growth response on the part of the capital stock. In this case, the capital stock, consumption and output all grow at the common rate 1 + g.

Because of the assumed exogeneity of the process of labor productivity, this model is one of exogenous growth. Put another way, <u>if</u> g = 0, it follows that the growth rate of output is also zero in the long run. Thus, in this model, exogenous change in the productivity of labor is the engine of growth. Moreover, it is easy to see that if g = 0, perpetual growth is not feasible if $\delta > 0$. This will hold for any production function as long as $\lim_{k\to\infty} F_k(k,1) < \delta$ since in this case, it follows that for large k, $F(k,1) < \delta k$, effectively limiting the capital stock.

In what follows, we will analyze a series of examples building on this one in which labor productivity does not grow exogenously, and yet output still does grow (even though F is time stationary). As we shall see, how fast output grows in these models depends on a variety of factors (e.g., parameters of preferences). Because of this, these models have the property that the rate of growth is determined by the agents in the model. They are, therefore, known as models of endogenous growth.

Throughout, there will be one common theme. This mirrors the point emphasized above, that for growth to occur, the interest rate (either implicit in a planning problem or explicit in an equilibrium condition) must be kept from being driven too low. This follows immediately from the discussion above. As noted above, this occurs in the standard Solow model because of the assumed growth in B_t along with the enabling growth in k_t. Thus, an alternative interpretation of the goal of endogenous growth theory is to generate and analyze plausible models in which, without assumed exogenous productivity growth, the implied interest rate remains bounded below (so that $\beta\{1 - \delta + r\} > 1$) even as the capital stock grows without bound.

In terms of key features of the environment that are necessary to obtain endogenous growth there is one that stands out: it is necessary that the marginal product of some augmentable input be bounded strictly away from zero in the production of some augmentable input which can be used to produce consumption.

Note that this rules out (as it should) the version of the Solow (or Cass (1965) Koopmans (1965)) growth model with g = 0. To see why, note taht the production function F is used to produce both the consumption good, c, and the investment good, x, and that the only augmentable input is capital. Thus, for this model to violate the principle just stated, it is sufficient to show that the marginal product of capital is not bounded strictly away from zero. In the standard neoclassical model, it is assumed that $\lim_{k\to\infty} F_k(k,1) = 0$. This, however, is stronger than necessary to obtain zero asymptotic growth.

More precisely, if $\lim_{k\to\infty} 1 - \delta + F_k(k,1) < 1$, it is physically impossible for this economy to grow without bound. The argument is as follows: The highest feasible investment policy is the one that allocates all the output to investment. Under this policy, capital evolves according to

$$k_{t+1} = F(k_t,1) + (1-\delta)k_t.$$

However, the condition $\lim_{k\to\infty} F_k(k,1) + 1-\delta < 1$ implies that there exists some $\bar{k}$ such that for all capital stocks, k, greater than or equal to $\bar{k}$, the function on the right hand side has slope less than one. Thus, if it is above the 45° line, it cannot stay there forever. That is, there is a maximum sustainable capital stock, k_m, that solves

$$k_m = F(k_m,1) + (1 - \delta)k_m.$$

In this case the sector providing the augmentable input (capital) is not productive enough to guarantee unbounded output.

In what follows, we will describe a number of economic environments (preferences and technologies) that generate sustained growth and highlight how the technological specification guarantees that the the marginal product of some augmentable input is bounded below. To keep the presentation as simple as possible, we concentrate on the solution of planner's problems in the models that we will examine. Although for some of the examples the well known connection between the solution to planner's problems and equilibrium will hold, this will not be the case for all of the models we will examine. Although this will make some difference in both the quantitative answers to questions that one would like to ask and the details of the algebra, there is no qualitative loss of generality in this restriction when it comes to analyzing the sources of growth. In those cases where there is a difference between the planner's solution and the equilibrium, the reader is invited to consult the original sources (or work out the details!).

2. Simple Convex One-Sector Models of Growth

In this section we outline the details and properties of the simplest class of models of endogenous growth. These are based on straightforward generalizations of the Solow model discussed above. (See Jones and Manuelli (1990) and Rebelo (1991).) They have been used in some form by a variety of authors in studying questions related to growth. A partial list includes Eaton (1981), Easterly (1989), Barro (1990), Greenwood and Jovanovic (1990), King and Rebelo (1990), Alesina and Rodrik (1991), Devereaux and Mansorian (1992), Lee (1992), Obstfeld (1992), Roubini and Sala-i-Martin (1992), Chang (1993), Easterly (1993), Jones, Manuelli and Rossi (1993), Rebelo and Stokey (1993), and Glomm and Ravikumar (1994). Although the specifics of these models differ in the number of capital goods considered and the market structure that is used (e.g., whether or not some of the goods are provided by the government), they all share the feature that the basic aggregate technology is a simple convex generalization of the basic Solow approach.

For simplicity we will assume that labor supply is exogenous. Consider the general planning formulation of a Cass (1965) Koopmans (1965) growth model

$$\text{Max} \sum \beta^t u(c_t)$$

s.t.

$$c_t + x_t \le y_t \equiv F(k_t, n_t)$$

$$k_{t+1} = (1 - \delta)k_t + x_t,$$

where c_t is consumption, x_t is investment, k_t is the capital stock, and n_t is labor supply at time t.

If n_t is set equal to one, the resource constraint becomes

$$c_t + x_t \le F(k_t, 1) \equiv f(k_t),$$

where f is concave and increasing.

From above, we can see that for growth to be possible, $\lim_{k\to\infty} f'(k) > \delta$. (Or else $\delta k > f(k)$ for large k.) Moreover, since in the case usually studied in the literature, $f(k) = k^\alpha$, $f'(\infty) = 0$, and thus growth is not feasible without exogenous labor productivity growth. This (i.e., $f'(\infty) = 0$) is not a property shared by all of the models of this type, however. That is, there are many forms for F for which $f'(\infty) > 0$. The simplest of these is what has become known as the Ak model. Here, $f(k) = Ak$. Other asymptotically equivalent versions include $f(k) = Ak + g(k)$, where g is (for example) Cobb-Douglas.

This also holds for the CES form. $F(k,n) = b\{\alpha k^{-\rho} + (1 - \alpha)n^{-\rho}\}^{-1/\rho}$ for $-1 \leq \rho < 0$. (In this case, the two inputs are substitutes in the sense that the elasticity of substitution between capital and labor $(1/(1+\rho))$ is greater than one. In this case, $\lim_{k\to\infty} f'(k) = b\ \alpha^{-1/\rho} > 0$. Thus, even this case is capable of generating growth endogenously for certain parameter values (i.e., $b\alpha^{-1/\rho} > \delta$).

This gives a large list of simple models that have the potential for generating growth endogenously. To see when growth actually will occur in these models, we must analyze the solution to the planning problem presented above.

The first order condition (for an interior solution) to this problem is given by

$$u'(c_t) = u'(c_{t+1})\beta\{1 - \delta + f'(k_{t+1})\}.$$

If $\lim_{k\to\infty} f'(k) \equiv A > 1/\beta - (1 - \delta)$ (i.e., $\beta(1 - \delta + A) > 1$) it follows (from the concavity of f) that $f'(k) > 1/\beta - (1 - \delta)$ for all k and hence, $u'(c_t) > u'(c_{t+1})$ for all t. Since u is strictly concave, it follows that $c_{t+1} > c_t$ for all t if $\beta(1 - \delta + A) > 1$. It is because of this that conditions like these are typically referred to as 'growth conditions.' This property is also shared by the Solow model. The important difference is that in the Solow model the sequence $\{c_t\}$ converges to the steady state value c^*, while in the model just described $\lim_{t\to\infty} c_t = \infty$. To see this recall that a monotone increasing sequence either converges or grows without bound. To prove that c_t

cannot converge suppose to the contrary that $c_t \to c^*$ for some finite c^*. Since $u'(c)$ and $f'(k)$ are continuous the previous equation is

$$u'(c^*) = u'(c^*)\beta(1 - \delta + f'^*),$$

where f'^* is the limiting value of f'. This implies

$$1 = \beta(1 - \delta + f'^*) \geq \beta(1 - \delta + A) > 1,$$

a contradiction. Note that the last two inequalities follow from the assumption that f is concave and the growth condition.

That is, in equilibrium, consumption will grow. It can also be shown that k_t grows monotonically in time as well in this model. Because of this, it follows that f' is decreasing over time.

If we further specialize to preferences of the form $u(c) = c^{1-\sigma}/(1 - \sigma)$, it follows that for all models in this class,

$$\gamma_t \equiv \frac{c_{t+1}}{c_t} = \left(\frac{u'(c_t)}{u'(c_{t+1})} \right)^{1/\sigma}$$

is monotonically decreasing and has a limiting value, $\gamma = \{\beta[1 - \delta + A]\}^{1/\sigma}$. As we can see the asymptotic rate of growth in these models is determined by the parameters of taste and technology.

Note that although with the preferences given, the model generates asymptotic constant growth, this does not necessarily occur. As an example, consider the case where $u(c) = -e^{-\lambda c}$, $\lambda > 0$. Here, $c_{t+1} - c_t = \ln\{\beta[1 - \delta + f'(k_{t+1})]\}/\lambda \to \ln\{\beta[1 - \delta + A]\}/\lambda$ and $\gamma_t \to 0$. Thus, although consumption (<u>and</u> investment and the capital stock) grows without bound if $\beta[1 - \delta + A] > 1$, it need not be true the asymptotic growth <u>rate</u> is positive. Thus, within this simple class of models, whether the resulting consumption series is 'trend stationary' (i.e., constant growth rates) or 'difference stationary' (i.e., the difference $c_t - c_{t-1}$ is stationary) is determined by the properties of the utility function.

To complete the discussion of the asymptotic growth rate of this class of models, note that if $\beta(1 - \delta + A) < 1$, there are two possibilities. First, if $\beta(1 - \delta + f'(0)) < 1$, the solution has consumption shrink monotonically to zero. On the other hand, if $\beta(1 - \delta + f'(0)) > 1$ the solution averages to a positive steady state. This is the usual Cobb-Douglas case where $f'(0) = \infty$ and $f'(\infty) = 0$.

One striking implication of this class of simple endogenous growth models is that labor's share of income becomes asymptotically zero. To see this consider any function F that is concave, continuous and homogeneous of degree one. In a competitive environment labor's share of income is given by

$$\frac{F_n(k_t, n_t)}{k_t} \frac{n_t}{[F_k(k_t, n_t) \cdot F_n(k_t, n_t) \frac{n_t}{k_t}]}.$$

It can be shown that our assumptions on F imply that for all n, $\lim_{k \to \infty} F_n(k,n)/k = 0$. Since F_k is, by assumption, bounded away from zero and n_t is bounded above, the above expression for labor's share in income shrinks to zero as k goes to infinity.

One interpretation of this result is that capital's share is one asymptotically. This interpretation depends on the assumption that reproducible stocks reflect physical capital. If one of the capital stocks (that is used in positive quantities in an optimal path) is interpreted as human capital, knowledge or any other form of augmenting the efficiency of labor, this class of models puts no restrictions on "measured" labor's share (see below).

In summary, we see that the class of standard neoclassical growth models suitably modified to incorporate a lower bound on the marginal product of capital is quite capable of generating growth endogenously. Growth here is endogenous in three senses. First, that no growth solutions are feasible in the model, but they are not chosen. Second, that different growth rates are possible and the one selected depends on tastes (i.e., β and σ). Finally, growth is not exogenous because the production function is time stationary.

This class of models does bring some problems with it however. In particular, the property noted above that labor's share in income converges to zero is particularly troublesome. In the comments below and in the succeeding sections, simple alterations of the model will be presented in which this is no longer true.

Extensions and Reinterpretations of the Model

A. Human Capital This model can be extended along a variety of dimensions. In particular, although the usual interpretation of k corresponds to physical capital (i.e., machines, structures, etc.), this is not essential. All that is necessary is that there be one fully reproducible productive input whose marginal product (net of depreciation) is bounded away from zero as its level is increased without bound. Although physical capital is one natural interpretation of this factor, others are available.

As a first alternative, drop n from the model altogether and interpret k_t as the individual's level of knowledge or stock of human capital at time t. Then by assuming that output is produced using "effective labor" as in Lucas (1988) and assuming that

(i) effective labor is given by $n_t k_t$ where n_t is the fraction of time spent working and

(ii) labor is supplied inelastically--$n_t \equiv 1$

gives an alternative interpretation in which "knowledge" or investment in knowledge is the engine of the growth machine.

Alternatively, one can think of k_t as being an aggregate of human and physical capital where the ratio of investments in these two assets are constant over time. See Jones and Manuelli (1990 working paper version) for an example of an extreme version of this interpretation.

B. Quality Improvements Some people object to the model of growth presented above (and to growth models in general) because they imply that individuals consume more and more of a given good over time whereas in reality most of the growth that occurs is in the quality rather than the quantity dimension.

A simple reinterpretation of the model is sufficient to handle this problem. Let k_t be the quality of the capital good used in production at time t and assume that only one unit of capital goods can be used at any given time. Assume that c_t and x_t measure, respectively, the quality of consumption and investment goods also available only in indivisible amounts. (The assumption of indivisibilities can be easily relaxed.) Assume that the law of motion for the production of the capital good depends on the quality of the old capital good and the quality of the investment good according to $k_{t+1} \leq (1 - \delta)k_t + x_t$ (perhaps $\delta = 0$) and that the quality of consumption and investment goods possible given the current quality of capital (or knowledge in keeping with (A) above) is given by $c_t + x_t \leq f(k_t)$. Then, the consumer faces the same problem as that outlined above.

Thus, this simple version of the model is sufficient to capture notions of quality improvement (cf. Stokey (1991) and Grossman and Helpman (1991a)). (See also Jones and Manuelli (1993b) for a description of this model without indivisibilities.)

C. Multiple Capital Goods. The results of this section can be extended to models of multiple capital goods. In particular, it is of interest to see if an extension of the growth condition, $\beta[1 - \delta + A] > 1$, beyond the one capital good model is possible. One such extension was alluded to in (A) above. Here, a general treatment is outlined. See Jones and Manuelli (1990) for details.

Suppose that there are n capital goods and that the planner's problem is given by

$$\max \sum \beta^t u(c_t)$$

s.t.

$$c_t + \sum_i x_{it} \leq f(k_t) \equiv y_t, \quad k_t \in \mathbb{R}^n_+$$

$$k_{it+1} = (1 - \delta_i)k_{it} + x_{it}.$$

Suppose that:

<u>Assumption G</u>: There is a function h: $\mathbb{R}^n \to \mathbb{R}$ which is homogeneous of degree one with $f(k) \geq h(k)$ for all k and for some $\hat{k} > 0$, and some $a > 0$

$$\beta[h_i(\hat{k}) + 1 - \delta_i] \geq 1 + a$$

holds.

Then, it can be shown that the solution to the planner's problem exhibits growth. Of course, Assumption G is the generalization of the growth condition that we were looking for. See Jones and Manuelli (1990) for details.

It is straightforward to check that all of the examples discussed to this point fit into this category. In particular, if $n = 1$, using $h(k) = Ak$ where $A = \lim_{k\to\infty} f'(k)$ suffices for condition G in this case.

Other complications of the basic structure are essentially special cases of this extension. A simple example related to what we will discuss below is non-externality versions of learning by doing models. It is straightforward to construct fully convex models with all of these qualitative features by going to a multiple capital good framework and having the law of motion for each individual's human capital depend on his own work effort in the market (see also the discussion in section 4 concerning models of human capital formation).

D. Government Production. Many of the models that have been developed in which government production plays a productive role in the economy are, structurally speaking, special cases of the multiple capital good formulation in C. above. The most well known example of this is Barro (1990). (See also Easterly (1989), Alesina and Rodrik (1991), Devereaux and Mansorian (1992), Lee (1992), Glomm and Ravikumar (1992), Chang (1993), and Jones, Manuelli and Rossi (1993).) Here, the equilibrium theory is different, however, in that the level of the output of the public good is taken as given by all of the private agents in the economy. To see how these models are structured, consider the following special case of the formulation above:

$$\max \sum \beta^t u(c_t)$$

s.t.

$$c_t + x_t + x_{gt} \leq f(k_t, g_t) \equiv y_t,\ k_t,\ g_t \geq 0$$

$$k_{t+1} = (1 - \delta)k_t + x_t$$

$$g_{t+1} = (1 - \delta_g)g_t + x_{gt}.$$

This is clearly a special case of the model above with two capital goods and Assumption G is sufficient to generate long-run growth along an optimal path. If in addition, we assume that $\delta_g = 1$, we have the model of Barro (1990) if f is constant returns to scale.

E. Heterogeneity. Most of the work on growth has concentrated on models with a representative agent. There has been some work with heterogeneity, however. In addition to the research on growth in environments with overlapping generations, some work on has been done on infinitely lived settings with differences in preferences (see Jones and Manuelli (1990)). It can be shown that much of the intuition developed above still holds if discount factors or intertemporal marginal rates of substitution are diverse among the population. In particular, although different individuals will in general have differing growth rates of consumption, the basic necessities for a model to exhibit growth are unchanged.

F. Policies and Differential Growth. The class of models discussed in this section has been used extensively to understand the role played by government policies in the differences in development paths followed by different countries. In particular, from the expression given above, $\gamma^{\sigma} = \beta (1 + r_{t+1})$ it follows that any government policy which lowers the equilibrium rate of return received by investors will adversely affect the rate of growth of the economy in these models. Moreover, as a natural extension, note that policies can even be responsible for shutting off the growth process entirely. This will happen whenever prolicies are such that $1+r_t$ (suitably interpreted as the after tax rate of return on savings) falls below $1/\beta$ as k grows. In such a case, the analysis in the introduction can be used to show that the economy converges to a point where capital per worker, k*, is such that $\beta(1+r(k^*))=1$, where $r(k^*)$ is the after tax rate of return on savings that obtains when the capital stock equals k*.

A variety of examples of this sort have appeared in the literature to this point. In particular, taxes on income (see Jones and Manuelli (1990), Lucas (1990), Yuen (1990), Rebelo (1991), Kim (1992), Zhu (1992), Jones, Manuelli and Rossi (1993), and Rebelo and Stokey (1993)), distorting taxes on capital composition (see Easterly (1991) and Jones, Manuelli and Rossi (1993)) and taxes on investment imports (Jones and Manuelli (1990)) all have these effects. Moreover, in extensions of the model using the Lucas labor supply, distorting taxes on consumption and labor income have similar effects (see Jones, Manuelli and Rossi (1993)) as does inflation (see Jones and Manuelli (1993a)).

Since these issues are discussed in detail elsewhere in this volume, we will not comment on them further here.

3. Two Sector Models of Growth

In this section we outline a simple two sector model of endogenous growth aimed at two potential objections to the models of the previous section while staying within the class of convex models. The class of models with a similar structure in the literature include Rebelo (1991), Fisher (1992a) and (1992b), Jones and Manuelli (1992) and Bond, Wang and Yip (1993). Our treatment follows the development of Rebelo (1991) quite closely.

The two properties of the simple 1-sector models of growth outlined in the previous section that are potentially problematic are first, that labor's share of income converges to zero and that the same must hold for <u>any</u> non-reproducible factor. Natural resources and land provide other examples.

Roughly speaking, what the simplest models outlined in Section 2 requires for growth is that the asymptotic factor shares (under competitive pricing) of those factors which are reproducible sum to unity. The alteration we will make in this section is to relax this requirement to one where the asymptotic factor share of the reproducible factors in the sector in which the reproducible factors are produced must sum to unity. This allows (in the standard interpretation) for any mix in the consumption sector as long as the capital production sector is "all capital" in its inputs. (See also the notes at the end of section 2 and the discussion in section 4 below for other alterations of the one sector model that produce the same result.)

Consider a planner's problem of the form

$$\max \sum \beta^t u(c_t, \ell_t)$$

s.t.

$$c_t \leq F(k_{1t}, n_{1t})$$

$$k_{t+1} = (1 - \delta)k_t + G(k_{2t}, n_{2t})$$

$$k_t = k_{1t} + k_{2t}$$

and

$$n_{1t} + n_{2t} + \ell_t = 1.$$

The interpretation of this is as follows. Some of the capital stock, k_1, and the labor allocated to the market sector, n_1, are used to produce consumption goods with the technology summarized by F. The remainder of the capital stock and labor services, k_2 and n_2, are used to produce new capital goods according to the production function, $G(k_2,n_2)$.

In this case, one of the first-order conditions for this problem is given by

$$u_c(t)\frac{F_k(t)}{G_k(t)} = \beta u_c(t+1)\frac{F_k(t+1)}{G_k(t+1)}[1-\delta+G_k(t+1)].$$

If the system converges to a balanced growth path, (this is a non-trivial assumption since, in general, two sector models can exhibit very complex dynamics- see Bond, Wang and Yip (1993) and Santos (1993) for discussion of this point)--i.e., $\ell_t \to \ell$, $G_k(t) \to G_k$ and $F_k(t+1)/F_k(t) \to \hat{F}_k$--and utility is of the form $u(c,\ell) = c^{1-\sigma}/(1-\sigma)\ v(\ell)$, this condition can be written as

$$\gamma_c^{\sigma} = \beta[1 - \delta + G_k]\hat{F}_k$$

where γ_c is the asymptotic growth rate of production in the consumption sector.

For γ_c to be bounded away from 1, it follows that G_k must be bounded below. To see this, simply note that if $G_k < \delta$ for k sufficiently large, it must be the case that $(1-\delta)k_t + G(k_t,1)$ crosses the 45° line. That is, there is a maximum sustainable capital stock as in the Solow model. However, note that there is no 'natural' lower bound on the growth rate of the marginal product of capital in the consumption sector which is required for the economy to display positive growth. In contrast with the one sector model, a low marginal product of capital in the production of the consumption good (i.e., low F_k) can be 'compensated' for with a high marginal product of capital in the investment sector (i.e., high G_k). This, however, must be interpreted with care as the equilibrium value of these two marginal products are not independent.

To see this better, consider the following example:

$$F(k_1,n_1) = Ak_1^{\alpha}n_1^{1-\alpha},\ G(k_2,n_2) = bk_2,\ v(\ell) = \ell^{\eta(1-\sigma)}.$$

Then, the first order conditions for the planner's problem imply that

$u_\ell/u_c = F_n$ which in this case corresponds to
$\eta c/(1 - n_1) = (1 - \alpha)A(k_1)^{\alpha}(n_1)^{-\alpha} = (1 - \alpha)\ F/n_1 = (1 - \alpha)c/n_1.$

Thus, $n_1/(1 - n_1) = (1 - \alpha)/\eta$. So that $n_1 = (1 - \alpha)/(\eta + 1 - \alpha)$.

From the condition, $c_t = F(k_{1t},n_{1t})$, it follows that (since n_{1t} is constant) that $c_{t+1}/c_t = (k_{1t+1}/k_{1t})^{\alpha}$, or $\gamma_c = \gamma_k^{\alpha}$. Similarly, $\hat{F}_k = \dfrac{A\alpha n_{1t+1}k_{1t+1}^{\alpha-1}}{A\alpha n_{1t}k_{1t}^{\alpha-1}} = \gamma_k^{\alpha-1}$.

Thus, the Euler equation is given by $\gamma_c^{\sigma} = \beta[1 - \delta + b]\gamma_k^{\alpha-1}$.

From this, using the fact that $\gamma_c = \gamma_k^{\alpha}$, it follows that

$$\gamma_k = (\beta(1 - \delta + b))^{\frac{1}{\alpha\sigma+1-\alpha}}$$

and

$$\gamma_c = (\beta(1 - \delta + b))^{\frac{\alpha}{\alpha\sigma+1-\alpha}}.$$

Finally, from the capital accumulation equation it follows that

$$k_{t+1}/k_t = 1 - \delta + b(k_{2t}/k_t),$$

or, $\gamma_k = 1 - \delta + b\phi$, where $\phi = k_{2t}/k_t$ is the share of capital allocated to the investment sector.

Thus, for growth in both consumption and the capital stock to be positive in the limit, we need that $\beta(1 - \delta + b) > 1$. This is, of course, the same condition that we obtained in the one sector model in the previous section since here, $\beta(1 + r) = \{\beta(1 - \delta + b)\}^{\alpha\sigma/(\alpha\sigma+1-\alpha)}$. The difference here, however, is that this need only be satisfied for the capital production sector (i.e., that the technology is "sufficiently productive").

Note that in contrast to the results obtained in the previous section consumption and the capital stock need not grow at the same rate. In fact, this occurs only in the case when $\alpha = 1$ where an aggregation result holds, giving a model equivalent to that in Section 2.

We can see that σ, β, b and δ all affect the growth rate. Additionally, in this example, the "productivity" of capital in consumption production, α, also affects the growth rate. Moreover, α is the only parameter of the consumption production function that does affect the growth rate for this example. In particular, both γ_c and γ_k are independent of A, a measure of overall productivity in the consumption sector.

Note that the long run rate of growth in this example is not affected by η. Here, η determines the elasticity of labor supply and, consequently, the steady state level of employment. In this model, changes in the level of employment in the consumption sector (recall that $n_2 = 0$) do not affect the growth rate of consumption. They do affect the level of consumption, however. More precisely, consider two economies that are identical in every respect (including the inital capital stock) except for the value of η. If the values for η are η_1 and η_2 respectively with $\eta_1 < \eta_2$, then the equilibrium level of employment and output of consumption will be higher in every period in economy 1. However, both economies will grow at the same rate. Even this simple example can be used to illustrate the dangers of trying to find simple explanations of the growth rate. Here, it is possible to show that the price of new capital goods is <u>higher</u> in economy one. Of course, this higher price of capital is matched with higher rental payments to capital so that the rate of return to saving is the same in the two economies.

As can be seen, increasing the productivity of the capital stock in the production of consumption unambiguously increases the growth rate of consumption, the effect on γ_k depends on σ. If $\sigma > 1$, an increase in α decreases the growth rate of k while if $\sigma < 1$ and increase in α increases the chosen rate of growth. In essence, if $\sigma > 1$, growth is less important to the consumer and an increase in α is met with a reduction in the savings rate and the share of capital devoted to capital production. (Recall that in this case, the productivity of capital in the consumption sector increases with α.) In contrast, if $\sigma < 1$, ϕ increases with α causing γ_k to increase.

Note that in both cases, n falls with an increase in α.

Recall that one motivation for examining the two sector model of growth presented here was that in the simplest model in the one sector case, labor's share converges to zero. To see that this does not occur in this case, note that for the special model analyzed here, $GNP_t = p_{kt}bk_{2t} + c_t$ where we have normalized $p_{ct} = 1$ for all t. Since capital must be paid the same in both sectors, it follows that $p_{kt}b = F_k(t) = \alpha A(n_{1t})^{1-\alpha}k_{1t}^{\alpha-1}$. Since $w_t n_{1t} = (1 - \alpha)c_t$ and $p_{kt}bk_{2t} = p_{kt}b\phi k_t = \phi\alpha A k_t(n_{1t})^{1-\alpha}(k_{1t})^{\alpha-1} = \phi k_t \alpha A(n_{1t})^{1-\alpha}(1-\phi)^{\alpha-1}(k_t)^{\alpha-1}$, $GNP_t = \phi\alpha A(n_{1t})^{1-\alpha}k_{1t}^{\alpha}(1 - \phi)^{\alpha} + A(n_{1t})^{1-\alpha}(1 - \phi)^{\alpha}k_t^{\alpha} = A(n_{1t})^{1-\alpha}(1-\phi)^{\alpha}(k_t)^{\alpha} + \phi(1-\phi)^{\alpha-1}\alpha A(n_{1t})^{1-\alpha}(k_t)^{\alpha}$. Hence, $w_t n_{1t}/GNP_t = (1 - \alpha)/(1 + \alpha\phi)$. Thus, labor's share is bounded away from zero. (Note that the price of capital relative to consumption converges to zero here!)

Extensions

A. The Production Function in the Consumption Sector. The reader may wonder if the Cobb-Douglas production function in the consumption sector is necessary for growth in this model. In particular, is it necessary that the marginal product of capital converge to zero? The answer to this is no, in particular, if $F(k_{1t},n_{1t}) = Bk_{1t} + A(k_{1t})^{\alpha}(n_{1t})^{1-\alpha}$, it can be shown that the solution to the planner's problem exhibits growth as long as the capital sector is sufficiently productive. However, in this case there is no exact balanced growth path and n_t converges to zero asymptotically, driving labor's share of income to zero as in Section 2. Alternatively, if $F(k_1,n_1) = Bn_1 + Ak_1^{\alpha}(n_1)^{1-\alpha}$, the asymptotic labor supply is not zero, but there is no balanced growth path.

B. Natural Resources and Growth. One common criticism of the class of endogenous growth models is that they fail to consider the existence of essential factors that are used up in the productive process and are such that their total supply is fixed (i.e., non-renewable natural resources). It is possible to extend the two sector model to incorporate this feature and still preserve the growth result (see Rebelo (1991)). If the production function exhibits sufficient substitutability between augmentable and exhaustive resources and the productivity of capital is sufficiently high, it is possible that the optimal policy for investment is high enough so that growth will occur. In essence, the high rate of investment in capital 'makes up' for the depletion for the resource, allowing for growth of output.

To illustrate this point, we extend the simple example of this section to include the flow of a non-renewable resource, m_t, as an 'essential' input in the production of consumption. The technological side of the model becomes

$$c_t \cdot Ak_{1t}^{\alpha_1} n_t^{\alpha_2} m_t^{1-\alpha_1-a_2}$$

$$k_{t+1} = (1 - \delta)k_t + bk_{2t}$$

and

$$\Sigma_0^{\infty} m_t \leq z.$$

The last constraint reflects the fact that the total stock of the resource, z, is finite and cannot be augmented. Of course, in any feasible allocation, $m_t \to 0$ necessarily. Here it can be shown that (with the preferences used above), $m_{t+1}/m_t = \gamma_m < 1$ and there is growth as long as $1 - \delta + b$ is sufficiently large. Moreover, although $m_t \to 0$, its share in income is constant as its price increases over time to reflect its increasing scarcity. In this model, a high productivity capital sector offsets the negative impact on consumption induced by the essential, exhaustible resource.

C. Taxes and Growth in the Two-Sector Model. As in the one sector model, it can be shown that in this case, taxation does affect growth (see Rebelo (1991)). As discussed in that section, taxes on income overall or on income from capital lower the rate of saving and growth. However, taxation on the income from labor or on consumption directly do not. This is because these taxes are roughly equivalent to reductions in the parameter A (or alternatively a change in η). As discussed above, this change has level, but not growth effects.

4. Models with Externalities

In the models studied so far, it was shown that convexity of the technology is not incompatible with the necessary condition for growth, the lower bound on the marginal product of the augmentable factors. There are, however, other specifications that also satisfy this necessary condition. One popular version emphasizes the role played by externalities. In these models, in contrast to those discussed above, the aggregate production set is non-convex. Various models with the same aggregate behavior of these models have been used extensively in the literature. Examples include Romer (1990), Bencivenga and Smith (1991), Matsuyama (1991a) and (1991b), Murphy, Shleifer and Vishny (1991), Young (1991), Aghion and Howitt (1992a) and (1992b), Baldwin (1992), Gilles (1992), Glomm and Ravikumar (1992) and (1993), Japelli and Pagano (1992), Uhlig and Yanagawa (1992), Yanagawa and Grossman (1993) and Asilis and Ghosh (undated).

In fact, the modern rebirth of growth theory began with models of this type. The first paper of this type was that of Romer (1986) which assumes an externality from physical capital. This was later extended and reinterpreted to human capital by Lucas (1988). This approach is related to some of the work on learning-by-doing (see Arrow (1962), Stokey (1991), and Young (1993)).

To begin, consider the simple example with exogenous labor supply;

consumer preferences are given by $\sum \beta^t c_t^{1-\sigma}/(1 - \sigma)$.

There are a continuum of firms, indexed by $i \in [0,1]$, each with production function

$$y_i \leq \phi(\bar{k})F(n_i, k_i)$$

where $\bar{k}_t = \int_0^1 k_{it}di$ is the average stock of capital, and k_i and n_i are the amount of capital and labor used by firm i. The overall supply of labor is bounded by one, or, $0 \leq \int_0^1 n_i di \leq 1$. The law of motion of firm i's capital stock is $k_{it+1} = (1 - \delta)k_{it} + x_{it}$. Since we will assume that all firms are symmetric, we will assume that $n_i = 1$ for all i.

The Euler equation for the planner's problem for the economy (assuming a symmetric solution) is:

$$(4.1) \qquad \left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta[1 - \delta + g'(\bar{k}_{t+1})]$$

where $g(k) = \phi(k)F(k,1)$.

As we have seen above, this will imply asymptotic growth in consumption as long as $\lim_{k\to\infty} g'(k) \equiv b > 0$ is sufficiently high (i.e., if $\beta[1 - \delta + b] > 1$).

Specializing to the case $\phi(k) = k^{\eta}$, $F(k,n) = Ak^{\alpha}n^{1-\alpha}$, we see that $g(k) = \phi(k)F(k,1) = An^{1-\alpha}k^{\eta+\alpha}$.

In this case, if $\eta + \alpha < 1$, we are back to the analogue of the neoclassical growth environment where $\lim_{k\to\infty} g'(k) = 0$ and no growth is possible. If $\eta + \alpha = 1$, we are (in terms of the characteristics of the planning problem) in the Ak world as long as n is taken as exogenous.

Finally, if $\eta + \alpha > 1$, we see that, from (4.1), the growth rate of consumption is increasing over time since $g'(\bar{k}_{t+1}) > g'(\bar{k}_t)$ if $\bar{k}_{t+1} > \bar{k}_t$. In this case, the <u>growth rate</u> itself is explosive, converging to infinity, since

(4.2) $$\gamma_t = \frac{c_{t+1}}{c_t} = \left\{\beta\left[1 - \delta + (\alpha + \eta)A\bar{k}_{t+1}^{\alpha+\eta-1}\right]\right\}^{\frac{1}{\sigma}}.$$

and $\bar{k}_{t+1}^{\alpha+\eta-1} \to \infty$.

Since this model features an external effect, and competitive equilibria (under the normal definition) no longer coincides with the solution the planner's problem, it is useful to contrast this allocation with some form of market equilibrium. To model market behavior, we consider the case in which consumers and firms behave competitively. In the case of firms, they take the sequence of aggregate capital stocks, $\{\bar{K}_t\}$, as given. Thus, from the perspective of an individual firm, its own technology is time varying with a productivity factor, $\phi(\bar{K}_t)$, that grows as the economy grows. The difference between the planner's solution and the competitive equilibrium lies in the assumption that firms ignore the impact of their own investment decisions on the productivity of other firms in the economy through their effect on $\bar{K}_t$. In this case, in the market solution, the first order condition satisfied by the representative consumer in a symmetric competitive equilibrium is:

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta[1 - \delta + \phi(\bar{k}_{t+1})F_k(\bar{k}_{t+1},1)].$$

In the special case described above, this simplifies to

(4.3) $$\gamma_t^{\sigma} = \beta[1 - \delta + \alpha A\bar{k}_{t+1}^{\alpha+\eta-1}].$$

From (4.2) and (4.3), it follows that if $\eta > 0$, $\alpha + \eta > \alpha$, the planner would choose to grow faster than occurs in the equilibrium. Thus, one manifestation of the inefficiency of the market outcome in this particular case is a downward bias in the growth rate of the economy. This is due to the downward bias of the effective interest rate in the market version of the model to $1 - \delta + \alpha A\bar{k}_{t+1}^{\alpha+\eta-1}$ from the true social marginal product of capital and hence the implicit interest rate in the planner's problem, $1 - \delta + (\alpha + \eta) A\bar{k}_{t+1}^{\alpha+\eta-1}$.

This effect can be quite dramatic. If $\alpha + \eta \geq 1$, there are values of k_o such that the economy should grow and would if it followed the solution to the planner's problem. However, if $\alpha < 1$ the economy does not grow at all in the market equilibrium if k_o is in this range.

To see this, let us consider the competitive case first. The marginal rate of substitution between present and future consumption must equal the interest rate. Thus, in this example, $\beta^{-1}(c_1/c_o)^{\sigma} = 1+r$ at time 0. In equilibrium, it 1+r exceeds the marginal product of capital, there will be no investment and the economy shrinks over time. To verify the conditions under which this will happen, consider the equilibrium rate of interest under the no investment policy. This is easily calculated given that c_o and c_1 are just

$$c_o = A(k_o)^{\alpha+\eta}, \quad c_1 = A[(1-\delta)k_o]^{\alpha+\eta},$$

and hence, $1+r = (1-\delta)^{\sigma(\alpha+\eta)}/\beta$.

The marginal product of capital at time 1 under the no investment policy is just

$$1 - \delta + \alpha A[(1-\delta)k_o]^{\alpha+\eta-1}.$$

Thus, for no investment to be the equilibrium policy, it must be the case that

$$\text{(a)} \qquad (1+r) = (1-\delta)^{\sigma(\alpha+\eta)}/\beta \geq \{1 - \delta + \alpha A[(1-\delta)k_o]^{\alpha+\eta-1}\}.$$

There are two things to note about (a). First, if zero investment is optimal at time zero, it will also be optimal at any time t. The reason for this is simple: The interest rate remains constant, but the marginal product of capital at time t is $1 - \delta + \alpha A[(1-\delta)^t k_o]^{\alpha+\eta-1}$ which is decreasing in t. Second, there is a capital stock $\hat{k}_c$, such that if $k_o = \hat{k}_c$, the economy stays there. This is simply the level of k such that $1 - \delta + \alpha A\ k^{\alpha+\eta-1}$ the right hand side of (a) equals β^{-1}. However, it can be shown that if $\alpha+\eta > 1$, this steady state level of capital is unstable in the sense that if $k_o < \hat{k}_c$, the economy permanently disinvests, while if $k_o > \bar{k}_c$, the growth rate is explosive.

We next study the conditions under which the planner's solution calls for positive investment. This will be the case whenever the marginal rate of substitution is less than the marginal product of capital under the 'no investment' rule. If the economy does not invest today, $c_o = A(k_o)^{\alpha+\eta}$. In this case, c_1 is bounded above by $A[(1-\delta)k_o]^{\alpha+\eta}$. Thus, the marginal rate of substitution satisfies

$$\beta^{-1}[c_1/(A(k_o)^{\alpha+\eta})]^{\sigma} \leq \beta^{-1}\{[A((1-\delta)k_o)^{\alpha+\eta}]/[A(k_o)^{\alpha+\eta}]\}^{\sigma} = \beta^{-1}(1-\delta)^{\sigma(\alpha+\eta)}.$$

Thus, for the planner to invest a positive amount at time zero, it is necessary that

$$\text{(b)} \quad \beta^{-1}(1-\delta)^{\sigma(\alpha+\eta)} < 1-\delta + (\alpha+\eta)A[(1-\delta)k_o]^{\alpha+\eta-1}$$

where the right hand side is simply the social marginal product of capital.

To find conditions under which there is disinvestment in equilibrium but the planner's solution would have positive investment, it is sufficient to pick a level of k_o such that both (a) and (b) are satisfied. This can always be done if $\alpha+\eta > 1$ and $\eta > 0$.

In fact, a complete categorization of the nature of the two equilibrium concepts (i.e., planner's solution and market equilibrium) as a function of k_o is possible (given the values of the parameters of the model). Let $\bar{k}_p$ denote the value of k where the <u>social</u> marginal product of capital is equal to $1/\beta$. It is easy to check that $\bar{k}_p < \bar{k}_c$ if $\eta > 0$. If $\alpha + \eta > 1$, the behavior of the system is:

	Competitive Equilibrium	Planner's Solution
$k_o < \bar{k}_p$	No investment (Negative Growth)	Negative Growth
$k_o = \bar{k}_p$	No investment (Negative Growth)	Steady State
$\bar{k}_p < k_o < \bar{k}_c$	No investment (Negative Growth)	Positive Growth
$k_o = \bar{k}_c$	Steady State	Positive Growth
$k_o > \bar{k}_c$	Positive Growth	Positive Growth

Note two things about this model. First, in contrast to the simplest one-sector model above, there is no problem about labor's share vanishing in the competitive equilibrium. In particular, in equilibrium, $w_t n_t/y_t = 1 - \alpha$ for all t for the special case considered. Thus, it is not necessary to append a second sector or go to the Lucas labor supply technology for this.

Second, notice that nothing about this exercise undoes what was said about the growth effects of taxes under the competitive system. In particular, taxes on income in general or capital income in particular result in lower rates of growth. Moreover, because of the form of the technology, it can be checked that if labor supply is endogenized, taxes on labor income and consumption also have growth diminishing effects.

Human Capital

Some authors argue that rather than an externality in physical capital, one in the knowledge of co-workers, etc., through human capital is more realistic. This is the approach that is adopted in Lucas (1988). An extensive literature building on Lucas' work has developed including Becker, Murphy and Tamura (1990), Goodfriend and McDermott (1991), Persson and Tabellini (1991), Rebelo (1991), Tamura (1991), Boldrin (1992), Buiter and Kletzer (1992), Kim (1992), Glomm and Ravikumar (1992) and (1993), Jones, Manuelli and Rossi (1993), Lucas (1993), Rebelo and Stokey (1993), and Wang and Yip (1993).

The simplest way to accomplish this would be to reinterpret k as h in the preceding section. One could then endogenize the labor supply by utilizing the Lucas' effective labor technology as outlined in Section 2.

This reinterpretation requires the acceptance of the 'standard' law of motion for physical capital for that of human capital as well. This raises the question of the nature of the essential inputs in the production of human capital and highlights the fact that one's own time is one of these inputs. Beyond this, it is of interest to extend the model in order to study the forces for migration in the context of a model of human capital formation.

As emphasized by Lucas (1988), it seems that the international differences in the returns to human capital vastly exceed the differences in returns to physical capital. To capture this behavior, it is necessary to alter the basic model to add a second capital stock with a different law of motion that is supplied jointly with labor hours, human capital.

Assume that consumers' preferences are of the form

$$\sum \beta^t c_t^{1-\sigma}/(1 - \sigma)$$

and that effective labor is given by

$$h^e = \int_0^\infty hn(h)dg(h),$$

where g(h) is the number of individuals (i.e., density) with human capital h, and n(h) is the amount of time spent working by h.

Let $\bar{h} = \int hdg(h)/\int dg(h)$ denote the average level of human capital in the population and assume that output is given by $y_t = F(k_t,h_t^e)\phi(\bar{h}_t)$, where F is concave and homogeneous of degree 1 in its arguments.

Moreover, we assume that

$$k_{t+1} = (1 - \delta)k_t + x_t \text{ and}$$
$$h_{t+1} = h_t G(1 - n(t)),$$

where 1 - n(t) is the amount of time spent by an individual on human capital augmentation. Finally, assume the standard feasibility constraint:

$$c_t + x_t \le y_t = F(k_t,h_t^e)\phi(\bar{h}_t).$$

Of course, in a symmetric equilibrium, $h_t = \bar{h}_t$ for all agents.

From this description of the technology, it follows that both effective inputs into the production of final output are accumulated through technologies that are linear in the reproducible factors. Thus, if ϕ is increasing, it follows that the basic necessity for growth, that y_t be (at least) linear in the reproducible factors holds in this case.

It is instructive to first consider a special case of the model that abstracts away from any external effects due to human capital accumulation. Specifically, assume $\phi(\bar{h}_t)$ is a constant. Moreover, assume that G(1-n) = $\bar{G}$(1-n), where $\bar{G}$ is a constant.

With this specification, if all individuals are identical, effective labor supply is

$$h_t^e = n_t \; h_t \equiv h_{1t},$$

while next period's human capital is simply

$$h_{t+1} = (1-n_t) \; \bar{G} \; h_t \equiv \bar{G} \; h_{2t},$$

where $h_{1t} + h_{2t} = h_t$.

Thus, this version of the model can be fully convexified. In so doing, it becomes clear that this structure is similar to the two sector model discussed above. In this case, h_{1t} is human capital allocated to the consumption sector, while h_{2t} is human capital allocated to the investment (in human capital) sector. The reader can verify that this 'change of notation' approach to convexifying the problem will not work if the function G(1-n) is not linear or if leisure is an argument of the utility function. It is straightforward to adopt this formulation to include learning by doing in a simple form. If we alter the law of motion for h_t to include work in the market, (i.e., $h_{t+1} = G(h_{1t},h_{2t})$), the idea that time on the job has a direct effect on human capital (i.e., learning by doing) can be captured. Here, learning by doing is modelled as the production of a complementary (to general output) capital good that is privately owned by the worker. Thus, even though this special case is delicate, it is a useful simplification in which to explore the implications for growth.

If we assume that F( ) is Cobb-Douglas, the planner's problem is given by:

$$\text{Max} \; \Sigma \; \beta^t \; c_t^{1-\sigma} \; / \; (1-\sigma)$$

$$\text{Subject to} \quad c_t + x_t \leq A \; k_t^{\alpha} \; (h_{1t})^{1-\alpha}$$

$$k_{t+1} \leq (1-\delta) \; k_t + x_t$$

$$h_{t+1} \leq \bar{G}\, h_{2t}$$

$$h_{1t} + h_{2t} \leq h_t .$$

The first order conditions for this problem are given by:

$$\left(\frac{c_{t+1}}{c_t} \right)^{\sigma} = \beta \left(1 - \delta + \alpha A k_{t+1}^{\alpha-1} h_{1t+1}^{1-\alpha} \right)$$

$$\left(\frac{c_{t+1}}{c_t} \right)^{\sigma} = \beta\, \bar{G} \left(\frac{k_{t+1}}{h_{1t+1}} \right)^{\alpha} \Big/ \left(\frac{k_t}{h_{1t}} \right)^{\alpha},$$

along with the constraints.

Simple, but tedious calculations show that in a balanced growth path, the growth rates of consumption, capital and human capital as well as the allocation of the latter between the two sectors is constant.

If we let $\nu \equiv h_{1t}/h_t$, then the balanced growth path can be summarized by

$$\gamma^{\sigma} = \beta\ (1 - \delta + A\ \nu^{1-\alpha}\ (h/k)^{1-\alpha})$$

$$\gamma^{\sigma} = \beta\ \bar{G}$$

$$\gamma = \bar{G}\ (1 - \nu).$$

These equations can be used to solve for the three endogenous variables, ν, γ and h/k. Of course, for this balanced growth path to be the solution of the planner's problem, it must be the case that the value (h/k) that solves these equations equals (h_o/k_o).

There are several things to note about this solution. First, as before, the productivity of an augmentable resource (human capital) in the production of an augmentable resource (human capital) (given by $\bar{G}$ here) determines the growth rate of the economy. Second, if we ignore the output of the human capital sector, factor shares are α for physical capital and $1-\alpha$ for human capital. Thus, this model is an alternative to standard two sector models as a way to match observed factor shares. Third, it is possible to compute the market price of human capital following the same procedure we used in the two sector case. It can be verified that - unlike the two sector model- the price of capital (here human capital) relative to consumption is constant along a balanced growth path and equal to $(1-\alpha)\ A\ (h/k)^{-\alpha}/\bar{G}$. Finally, the model implies that wage rates per level of human capital are independent of the level of output and development of a country. Specifically, if a country has human capital equal to $\bar{h}$ and physical capital equal to $\bar{k}$, an individual who supplies h_{1i} units of effective labor earns income equal to $w\ h_{1i}$ where,

$$w = (1-\alpha)\ A\ (\bar{h}/\bar{k})^{-\alpha}.$$

Since along a balanced growth path the ratio h/k is constant, the level of development (as measured by the <u>level</u> of k and h) does not affect the wage rate, although individual labor income for the average person is affected by this.

That is, if as the model in this form predicts, wages for a given level of human capital are the same in different countries, why would workers in one country wish to migrate? It is clear that to reconcile this, some country specific factor must come into play. One choice would be to have differential taxation (or other policies) in different countries. A different alternative, and the one that is adopted in Lucas, is to have country specific externalities.

To see how the introduction of externalities can result in a path that displays independence of the rate of return on capital from the level of development and yet dependence of the price of human capital on this level, we consider another special case of the function $\phi(\bar{h})$ given by $\phi(\bar{h}) = \bar{h}^{\eta}$.

Thus, a planner's problem for a specific version of this economy is given by:

$$\text{Max} \quad \sum \beta^t c_t^{1-\sigma} / (1-\sigma)$$

$$\begin{aligned} \text{Subject to} \quad & c_t + x_t \le A\ k_t^{\alpha}\ (n_t\ h_t)^{1-\alpha}\ \bar{h}_t^{\eta} \\ & k_{t+1} \le (1-\delta)k_t + x_t \\ & h_{t+1} \le h_t\ G(1-n_t) \\ & h_t = \bar{h}_t, \end{aligned}$$

where the last constraint implies that the planner recognizes that the average ($\bar{h}$) and the representative level of human capital (h) are the same. This recognition that $h_t = \bar{h}_t$ is, in fact, what distinguishes the planner's solution from a competitive allocation. A competitive allocation in which individual agents ignore the impact of their human capital investment decisions on the productivity enhancing aggregate level, $\bar{h}$, can be obtained as the solution to a pseudo-planner's problem. In this case, it is the planner's problem described above except that the last constraint is ignored; that is, the pseudo-planner also ignores the effect of changing h_t upon $\bar{h}$ and takes the aggregate level of human capital as given. Of course, the actual equilibrium allocation will be the one for which $\bar{h}_t = h_t$.

The solution to the planner's problem satisfies:

$$\text{(a)} \qquad \left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta\left[1 - \delta + \alpha A\, k_{t+1}^{\alpha-1}\, n_{t+1}^{1-\alpha}\, h_{t+1}^{\eta+1-\alpha}\right]$$

$$\text{(b)} \qquad \left(\frac{c_{t+1}}{c_t}\right)^{\sigma} \frac{k_t^{\alpha} n_t^{-\alpha} h_t^{\eta-\alpha}}{G'(1-n_t)} = \beta\, \frac{k_{t+1}^{\alpha} n_{t+1}^{-\alpha} h_{t+1}^{\eta-\alpha}}{G'(1-n_{t+1})} \left[G(1-n_{t+1}) + \frac{\eta+1-\alpha}{1-\alpha} G'(1-n_{t+1})\, n_{t+1} \right]$$

and the feasibility constraints as well as the law of motion for the two capital stocks (physical and human). The reader can verify that the competitive allocation satisfies the same equations except that the term $(\eta+1-\alpha)/(1-\alpha)$ in (b) is substituted by $(1-\alpha)/(1-\alpha) = 1$. The reason for this is that competitive decision makers do not take into account the impact of their own capital accumulation decisions on the level of overall output. This impact is measured by η.

Along a balanced growth path, $c_{t+1}/c_t = \gamma_c$ and $n_t = n$. Thus, equation (a) implies that the term $(k_{t+1})^{\alpha-1}\ (h_{t+1})^{\eta+1-\alpha}$ is constant. This, in turn requires that $(\gamma_k)^{1-\alpha} = (\gamma_h)^{\eta+1-\alpha}$. Note that in the convex case $(\eta=0)$, $\gamma_k = \gamma_h$ as we derived before. Inspection of (a) (that holds in a competitive equilibrium) shows that the right hand side is the discounted marginal product of capital. Thus, the restriction across γ_k and γ_h is necessary to keep the interest rate constant. Hence, the balanced growth path is such that the interest rate equals γ_c^{σ}/β independent of the <u>level</u> of output.

Manipulation of (a) and (b) and the feasibility constraints show that the balanced growth path is given by

$$\gamma_c = \gamma_k = (\gamma_h)^{(\eta+1-\alpha)/(1-\alpha)}$$

$$\gamma_h = G(1-n)$$

$$\gamma_h^{(\sigma(\eta+1-\alpha)-\eta)/(1-\alpha)} = \beta\left[G(1-n) + \frac{\eta+1-\alpha}{1-\alpha}\, G'(1-n)\, n\right].$$

This system can be used to solve for γ_c, γ_k, γ_h and n. As mentioned before, for the solution to this system to characterize a competitive allocation, $(\eta+1-\alpha)/(1-\alpha)$ must be equal to one. There are some restrictions that have to be satisfied to guarantee the existence of a solution if $\sigma < 1$. A necessary condition is that $\sigma(\eta + 1 - \alpha) - \eta > 0$. This guarantees that utility is bounded.

As pointed out, the rate of return to capital is constant along a balanced growth path. However, the marginal product of labor is not.

$$w(k,h) = (1-\alpha)\ A\ k^{\alpha}\ n^{-\alpha}\ h^{\eta-\alpha} = (1-\alpha)\ A\ n^{-\alpha}\ k^{\alpha-1}\ h^{\eta+1-\alpha}\ k/h.$$

As we showed before, $k^{\alpha-1}\ h^{\eta+1-\alpha}$ is constant along a balanced growth path. Thus,

$$w(k_t,h_t) = [(1-\alpha)\ A\ n^{-\alpha}\ k^{\alpha-1}\ h^{\eta+1-\alpha}]\ k_t/h_t.$$

Hence, the return to a unit of human capital depends on the ratio, k/h: The higher this ratio is, the more productive human capital is. However, we have shown that $\gamma_k > \gamma_h$ if $\eta > 0$. Thus, along a balanced growth path, k grows faster than h and wages increase. In this case, the return on human capital in a developed economy (high k, h, <u>and</u> k/h) exceed that in a less developed economy and free capital flows do not eliminate this discrepancy. The reader can verify that in a competitive allocation, although the growth rate is lower, it is still the case that $\gamma_k > \gamma_h$ and thus this result holds at competitive allocations as well.

This failure of the factor price equalization theorem of international trade in the presence of externalities has been interpreted by Lucas and others as an essential feature of the development process since it is consistent with the desire of educated individuals to migrate from less developed to more developed countries.

To see the intuition underlying this result, it is useful to understand the role played by the human capital externality along a balanced growth path. From the equation determining the wage rate, it follows that wages increase with the physical capital/ human capital ratio. Thus, to provide the desired result (same interest rates and higher wage rates in richer countries), it is necessary to get

(a) A balanced growth path. As discussed before, this requires a constant, and hence, level independent, interest rate, and

(b) A wage rate increasing in k/h.

In this economy (along a balanced growth path), the interest rate (in the competitive equilibrium) is given by

$$[1-\delta + A\alpha n^{1-\alpha} (k_t)^{\alpha-1} (h_t)^{\eta+1-\alpha}].$$

Thus, constancy of the interest rates implies an increasing k/h ratio. It is interesting to note that a capital based externality would have produced the same result. To see this, consider a model in which the externality is induced by $\bar{K}^{\eta}$ rather than $\bar{H}^{\eta}$. In such a model, the interest rate (in a competitive allocation) is

$$[1-\delta + A\alpha n^{1-\alpha} (k_t)^{\alpha+\eta-1} (h_t)^{1-\alpha}].$$

Assuming that $0 < \alpha+\eta < 1$ (a necessary condition for the existence of a balanced growth path), it follows that k_t/h_t is increasing along any balanced growth path. In this case, it can also be verified that wages are increasing in k/h and, hence, that the same essential properties hold whether the externality is in physical or human capital.

This alternative specification of the nature of the externality shows that it is not essential for a human capital externality to be present to capture the stylized facts about the returns on physical and human capital and the level of development. It also shows that the intuition that this model explains higher wages of educated people in developed regions as arising from their interaction with other highly educated workers (the human capital externality) is somewhat misleading since a physical capital externality can give rise to the same wage differential.

Although these externality based models can explain differences in the price of a non-tradeable factor (human capital) across regions, there are possibilities as well. Among them are differences in policies (especially tax policies), and preferences and technology parameters.

Finally, it must be pointed out that models that rely on aggregate externalities typically have as one of their implications that larger countries (or whatever the region is that the externality extends over) should grow at higher rates than small countries.

5. Growth and the Introduction of New Goods

A common feature of advanced economies is that the process of growth encompasses not only a larger quantity of existing goods, but also both a greater variety of goods and higher qualities of given goods.

A significant fraction of the recent work on growth has dealt with the introduction of new goods either at the final, consumption stage, or the intermediate one. This literature has followed two approaches, one based on the symmetric approach of Dixit and Stiglitz (1977) and Ethier (1982) (see Romer (1987), Grossman and Helpman (1991b)), the other on asymmetric models of quality improvement (see Grossman and Helpman (1991a) and Stokey (1991)). (The paper of Grossman and Helpman (1991a) is a combination of these two approaches.) Papers following the first approach include Romer (1989), Grossman and Helpman (1990), Romer (1990a), Bertola (1991), Chou and Shy (1991), Goodfriend and McDermott (1991), Rivera-Batiz and Romer (1991), Chou and Talmain (1992), Ciccone and Matsuyama (1992), Helpman (1992), Bencivenga, Smith and Starr (1993), and Young (1993). Those following the second include Grossman and Helpman (1991c), Helpman (1992) and Stokey (1992).

Formally, one can think of these models as developing "micro-foundations" for the evolution of a stock thought of as the best current vintage or good.

Since the Dixit-Stiglitz-Ethier approach is somewhat easier to understand, we will present a simple version of it here.

The basic setting is one in which there is one final good - consumption

- and a large number (potentially as many as real numbers) of intermediate goods that, when combined with raw labor, are used to produce consumption. Known intermediate goods are produced with labor using a linear technology. If it is not known how to produce a good, consumption can be used to 'discover' the process (a once and for all expenditure) and thereafter it uses the common technology for intermediate goods.

Let z_t denote the highest level of intermediate good that is known how to produce at time t and for $i \in [0, z_t]$, let $n_{2t}(i)$ denote the amount of labor used in the production of good i, $x_t(i)$ its level of output.

Then assume that $x_t(i) = an_{2t}(i)$ and that final output is produced according to

$$(5.1) \qquad y_t = M\, n_{1t}^{1-\alpha}\left[\int_0^{z_t} [x_t(i)]^{\mu}\, di\right]^{\alpha/\mu}$$

where n_{1t} is the amount of labor used in the production of the final good.

Finally, assume that z_t evolves according to $z_{t+1} = z_t + bx_t$, where we have assumed that there is no depreciation of the "stock" of goods available for production. This specification implies that one unit of consumption is necessary to 'discover' b units of new intermediate goods.

Notice that for a given level of z, the technology described by (5.1) exhibits constant returns to scale in the factors n_1 and $x_t(\bullet)$ and that labor need not be 'trained' in the production of the new goods.

Then, assuming that labor supply is exogenous and fixed at 1, the planner's problem for this economy is given by

$$\max \Sigma\beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$\text{s.t.} \quad c_t + x_t \le y_t = M\, n_{1t}^{1-\alpha}\left(\int_0^{z_t} (x_t(i))^{\mu}\, di\right)^{\alpha/\mu}$$

$$0 \leq x_t(\bullet) \leq an_{2t}(i), \quad 0 \leq i \leq z_t$$

$$n_{1t} + \int_0^{z_t} n_{2t}(i)di \leq 1$$

$$z_{t+1} = z_t + bx_t.$$

This problem can be considerably simplified when $0 \leq \mu \leq 1$ (as we assume) by noting that, given z_t and n_{1t}, the sub-problem of choosing the $n_{2t}(i)$ and $x_t(i)$ is a simple, concave, symmetric problem with the solution

$$n_{2t}(i) = (1 - n_{1t})/z_t \text{ for all } i.$$

Because of this, output can be written as

$$y_t = M\, n_{1t}^{1-\alpha} a^{\alpha} \left[z_t \left(\frac{1 - n_{1t}}{z_t} \right)^{\mu} \right]^{\alpha/\mu}$$

$$= M\, a^{\alpha} n_{1t}^{1-\alpha} z_t^{\alpha(1-\mu)/\mu} (1 - n_{1t})^{\alpha}.$$

Hence, the planner's problem reduces to

$$\max \Sigma \beta^t \frac{c_t^{1-\sigma}}{1 - \sigma}$$

$$\text{s.t.} \quad c_t + x_t \leq M\, a^{\alpha} n_{1t}^{1-\alpha} (1 - n_{1t})^{\alpha} z_t^{\alpha(1-\mu)/\mu}$$

$$z_{t+1} = z_t + bx_t.$$

Notice that because of the form of the production function here, this problem features an element of increasing returns to scale (cf. the discussion in the previous section). Further, this problem can be simplified even more by noting that $n_{1t}^{1-\alpha}(1 - n_{1t})^{\alpha}$ is maximized at $n_t = 1 - \alpha$ for all t. This gives a final version of the planner's problem as

$$\max \Sigma \beta^t \frac{c_t^{1-\sigma}}{1 - \sigma}$$

$$\text{s.t.} \quad c_t + x_t \leq A z_t^{\eta}$$

$$z_{t+1} = z_t + bx_t$$

where $\eta = \alpha(1 - \mu)/\mu$ and $A = Ma^{\alpha}\alpha^{\alpha}(1 - \alpha)^{1-\alpha}$.

This reduces the problem to one similar to those studied above. Because of this, we can cay a lot about the solution to the problem.

In particular, we know that if $\alpha < \mu/(1 - \mu)$ $(\eta < 1)$, the solution to this problem is such that growth will not occur. That is, $c_t \rightarrow c^* < \infty$. In this case, the model can be interpreted as a standard neoclassical growth model a la Cass-Koopmans in which z_t plays the role of a non-depreciating capital stock. If $\alpha = \mu/(1 - \mu)$ $(\eta = 1)$ balanced growth will result (depending on the other parameters) as this is a special case of the linear or Ak technology. Finally, if $\alpha > \mu/(1 - \mu)$ $(\eta > 1)$ explosive growth will occur (depending on initial conditions). Formally, this problem resembles the planner's problem discussed in our presentation of Romer's (1986) physical capital externality model in section 4.

In the special case $\alpha = \mu/(1 - \mu)$ $(\eta = 1)$ the balanced growth path can be explicitly characterized. In this case, the first order condition for the planner's problem is given by:

$$\gamma^{\sigma} = \beta[1 + bA].$$

Moreover, from the law of motion of z, if x_t/z_t is a constant, x/z, we have that

$$\gamma = z_{t+1}/z_t = 1 + b(x/z).$$

Finally, we have that $c/z + x/z = A$.

Thus, the usual comparative statics results hold in this setting, with slightly different interpretations.

For example, as can be seen, an increase in productivity in production of either intermediate goods (i.e., a), or investment (i.e., b), or final output (i.e., M) leads to an increase in the growth rate. This occurs due to an increase in investment on the extensive margin, z. One can think of this as corresponding to research and development and that z is the "level" of technology.

Extensions

A. Process Separability and Aggregate Non-Convexities. In those cases in which $\eta > 1$, the model analyzed here has an aggregate non-convexity on the technological side. This can be easily seen in the final, reduced form of the planner's problem. Although it is clear at that level, it is not clear where this arises in the original formulation of the problem.

The simplest interpretation of the model presented in this section is that there is an externality in specialization across processes. To see how this is the case, consider an alternative specification in which there are no interactions across different processes each of which uses an intermediate good. That is, if we think of combining labor, $n_1(i)$, with an intermediate goods of type i, x(i), to produce output through the process i, a production function of the form

$$(5.2) \qquad y_t = M \int_0^z (n_1(i))^{1-\alpha}(x(i))^{\alpha}di$$

is a natural candidate. Again, for a given level of z, this technology is homogeneous of degree one in $(n_1(\bullet), x(\bullet))$.

Using the same technology for producing x(i) from labor as above (i.e.,

$x(i) = an_2(i))$ gives the technological restrictions:

$$y_t = M a^{\alpha} \int_0^{z_t} (n_1(i))^{1-\alpha} (n_2(i))^{\alpha} di,$$

and,

$$\int_0^{z_t} n_1(i)\, di + \int_0^{z_t} n_2(i)\, di = 1.$$

Imposing symmetry, this is reduced to

$$y_t = Ma^{\alpha} z_t n_1^{1-\alpha} n_2^{\alpha} \text{ and } z(n_1 + n_2) = 1.$$

Thus, given a level for z, it follows that the optimal allocation of labor across the two activities is for $n_{1t}(i) = (1 - \alpha)/z_t$, $n_{2t}(i) = \alpha/z_t$ for all $i \in [0, z_t]$.

Given this, the planner's problem in this case is given by

$$\max \Sigma \beta^t \frac{c_t^{1-\sigma}}{1 - \sigma}$$

$$\text{s.t.} \quad c_t + x_t \leq y_t = M a^{\alpha} z_t \left(\frac{1 - \alpha}{z_t}\right)^{1-\alpha} \left(\frac{\alpha}{z_t}\right)^{\alpha} = M a^{\alpha} (1 - \alpha)^{1-\alpha} \alpha^{\alpha}$$

$$z_{t+1} = z_t + bx_t.$$

Clearly, since increases in z do not change output (a given amount is just "shuffled" across the i's), the solution to this problem has no investment--$x_t = 0$ and <u>no</u> growth.

The difference between these two models clearly lies in the choice of the aggregate technology, y_t. In the original formulation, (5.1), there is an "inter-process" externality while this is not the case in (5.2). That is, notice that, in the formulation in (5.1) if x(i) is increased, not only is output increased directly due to the heavier utilization of process i, the productivity of x(j) $j \neq i$ is also increased. It is this cross-effect (which is not present in the formulation in (5.2)) that is the source of the non-convexity in the problem. (Moreover, this interprocess externality is economy-wide since it occurs in the aggregate production function.)

B. Planner's and Market Allocations. Although we have followed a planning approach to the problem to this point, an alternative, market based approach is possible. Because the technology as a whole is not convex, the usual decentralization arguments will not hold here. Typically, some model of imperfect competition is used to decentralize this system. The details differ across authors. The interested reader should consult the original papers. Since these theories in general give rise to inefficient outcomes, it is not surprising that this holds here as well.

In particular, it can be shown that equilibrium allocations that grow more slowly than at the optimum are possible.

6. Overlapping Generations

In this section we take a slightly different perspective on the growth problem, changing the structure of individual preferences. As we have seen repeatedly in the previous sections, in the setting with infinitely lived agents, as long as the interest rate is sufficiently high (e.g., from a high marginal product of capital), growth will occur. Stated another way, as long as $\beta[1 - \delta + \lim_{k\to\infty} F_k(k,n)] > 1$, growth will occur, even in simple convex representations of technology.

In an OLG setting, a high rate of interest guarantees that consumption follows a growth trajectory during any individual's lifetime. However, this does not guarantee that the consumption of subsequent generations is higher than that of the current one. In the context of a model of overlapping generations, of course, this just what is necessary for growth of aggregate consumption to occur.

The remarkable fact that we will show here is that in contrast to the results in Section 2, growth cannot occur in equilibrium with an OLG structure (see Jones and Manuelli (1992) and Boldrin (1992)) in a one sector, convex growth model. This means that one of the other models of growth (i.e., those in sections 3, 4, or 5) must be used in these settings. All have been followed in the literature. Examples include Becker, Murphy and Tamura (1990), Bencivenga and Smith (1991) and (1993), Chou and Shy (1991), Persson and Tabellini (1991), Buiter and Kletzer (1992), Fisher (1992a) and (1992b), Glomm and Ravikumar (1992), (1993) and (1994), Japelli and Pagano (1992), Uhlig and Yanagawa (1992), Bencivenga, Smith and Starr (1993), Yanagawa and Grossman (1993), Asilis and Ghosh (undated).

To begin, suppose that the representative consumer for generation t solves:

$$\begin{aligned} &\text{Max } u^t(c_t^t, c_{t+1}^t) \\ &\quad \text{s.t.} \quad c_t^t + s_t \le w_t \\ &\qquad\qquad c_{t+1}^t \le w_{t+1} + (1 + r_{t+1})s_t \end{aligned}$$

where w_t is the wage rate in period t (we have assumed that each individual in each generation is endowed with one unit of inelastically supplied labor), c_i^t is the consumption in period i of the generation born at time t and s_t is saving during period t.

Equilibrium with two period lives requires that $s_t = k_{t+1}$. Since $c_t^t \ge 0$, this implies that $k_{t+1} \le w_t$.

Suppose now that k_t is continually growing without bound. Then,

$$1 \le k_{t+1}/k_t \le w_t/k_t.$$

However, if markets are competitive, $w_t = \partial F(k_t, n_t)/\partial n$. Finally, it can be shown that if F is continuously differentiable and homogeneous of degree one,

$$\lim_{k\to\infty} \partial F(k,n)/\partial k = 0 \text{ for all } n.$$

Thus, if $k_t \to \infty$, $w_t/k_t \to 0$, a contradiction. Because of this fact, it follows that if $k_t \to \infty$, it is necessarily the case that income generated by wages must converge to zero as a fraction of GNP. However, this is the only real source of income for the future generations since, investment is by its nature a zero profit activity. This implies that savings out of net income must necessarily go to zero as a fraction of output limiting the size of investment and the capital stock.

Thus, even if $1 - \delta + \lim_{k\to\infty} F_k(k,n) > 1$, growth will not occur in equilibrium for any specification of individual preferences.

Note that this failure of growth in this environment is not a sign of an inefficiency. Since the technology is so productive, it follows that the interest rate is high enough that the equilibrium is in fact, efficient (even though it does not grow).

How do we rationalize this result with the fact that we do observe growth occurring? There are several answers to this question that have appeared in the literature.

The simplest and most direct method is to introduce a dynastic structure into preferences with a generational consumption externality following Barro (1974). This effectively moves us back to the infinitely lived setting with all of the results from that literature.

A second method is to go to the 2-sector model analyzed in Section 3. In this case w_t as a fraction of GNP does not decrease to zero. In addition the price of capital relative to wages (or consumption) is decreasing. Thus, the relevant constraint on saving is $w/p_k k$ and this need not go to zero and growth is possible. The conditions for growth are much more complex than those presented in Section 2, however. (See Jones and Manuelli (1992) and Fisher (1992) for details.)

A third method is to go to the externality formulation as described in the presentation of physical capital externalities in Section 4. Again, w_t/k_t need not converge to zero as $k \to \infty$ and growth is possible in equilibrium. This is elaborated on in Boldrin (1992) and Jones and Manuelli (1992). Again, the conditions for growth are considerably complicated by the OLG structure.

In a similar vein, "endowment externalities" provide an alternative route (e.g., see Boldrin (1992), Buiter and Kletzer (1992) and Glomm and Ravikumar (1992) and (1993)). Here it is assumed that the quality of labor that individuals are endowed with depend on the investment decisions of past generations.

Alternatively, it is possible to show that through government policy, it is possible to generate growth as an outcome. In particular, a system of redistributing income from the old to the young will accomplish this. In this case, the problem faced by the generation t agent is

$$\max u(c_t^t, c_{t+1}^t)$$
$$\text{s.t.} \quad c_t^t + s_t \leq (1 - \tau)w_t + T_{1t}$$
$$c_{t+1}^t \leq (1 - \tau)w_{t+1} + (1 + (1 - \tau)r_{t+1})s_t + T_{2t+1},$$

where τ is the tax rate on income and T_{1t} and T_{2t} are the (lump sum) transfers given to the young and old generations, respectively. We require that the government balance the budget period by period:

$$T_{1t} + T_{2t} = \tau 2w_t + \tau(\partial F(k_t, 2)/\partial k - \delta)k_t.$$

If preferences are of the form $\frac{c_t^{1-\sigma}}{1-\sigma} \cdot \beta\frac{c_{t-1}^{1-\sigma}}{1-\sigma}$, it can be shown that growth will occur for some choices of parameters. In particular, if all redistribution is to the young ($T_{2t} = 0$ for all t), the equilibrium growth rate is non-monotonic in the tax rate. This is in sharp contrast to the results on the other models where increasing taxes lowers rates of growth (an exception to this is Barro

(1990) and the papers based on this model). In particular, when τ is low (close to zero) or high (close to one) no growth occurs in equilibrium. For moderate values of τ, however, the income effect of T_{1t} on savings offsets the negative incentive effect of the higher taxes, and growth occurs.

Note that if income (and k_t) grows here, it follows that the government is promising an ever increasing sequence of transfers to the young. This is reminiscent of a program in which the old are taxed to supply education for the young and, hence, has a natural interpretation.

This is something that is quite different from the results presented above. The point of this is quite simple, in an OLG model something extra (beyond a high rate of return on capital) is necessary for growth to be generated as an equilibrium. It is necessary for the new families that are born every period to be able to afford the ever increasing amounts of capital that must be transferred across generations. This additional restriction limits the class of economies that can display long run growth (relative to the infinitely-lived families analyzed above) and provides new avenues for growth effects of government policies.

Finally note that it is of course necessary, even in this case, that the marginal product of capital is sufficiently high for growth to occur. However, even though the undistorted allocation displays no growth, it is Pareto Optimal if the growth condition is satisfied. The reader can check that the distorted, growing allocation benefits all generations sufficiently far in the future at the cost of those early on in the process.

7. Final Comments

The literature on the the properties of models featuring endogenous growth has made much progress in recent years, yet there is still much work to be done.

Although some work has been done on the differences between growth in quantity and growth in quality (see Stokey (1988) and (1991) and Grossman and Helpman (1991a), (1991b) and (1991c) and the references in section 5 above), these papers are very special both in terms of their approach to the quality side of preferences and in their treatment of technology. Thus, this seems an area where there is room for fruitful new research.

While the non-stochastic versions of the models of endogenous growth have been worked out in substantial detail, comparatively little work has been done on stochastice models of endogenous growth. (Exceptions are King, Plosser and Rebelo (1988) and Jones, Manuelli and Stacchetti (1993).) Since this gives the opportunity of endogenizing the properties of the Solow residuals in the real business cycle literature and since one interpretation of growth in this literature is through innovation (a common interpretation of many of the endogenous growth models) this seems to be an area with a high payoff for further research.

Finally, most of the work on the relationship between government policy and growth has concentrated on macro policies (e.g., taxation, government spending and trade policies at the macro level). Much of the new literature on growth gives the opportunity for further study on the relationship between micro policy and growth and is an area of considerable interest. Examples of this include the relationship between regulation of specific industries (i.e., the banking or credit sector - because of the special role played by investment in these models), patent policy and its interaction with trade policy and growth.

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