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FURTHER REMARKS ON WALRAS' LAW AND NONOPTIMAL EQUILIBRIA*

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ABSTRACT

The objective of this note is to show that the positively valued excess supplies which Aiyagari (1992) connects with Pareto inefficiency for overlapping generations economies represent an economic opportunity that can potentially be exploited by government or by a private financial intermediary through the issuance of unsecured debt. We demonstrate that, when unsecured debt is issued, Walras' Law does not fail in the sense described by Aiyagari. However, the mere issuance of unsecured debt does not ensure Pareto efficiency. We show that Pareto efficiency is achieved if and only if the opportunity to issue unsecured debt is optimally exploited, for example, by an *earnings-driven* private financial intermediary.

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1. Introduction

In a recent paper, Aiyagari (1992) demonstrates a connection between the failure of Walras' Law and nonoptimal equilibria in overlapping generations economies. The significant implication of Walras' Law in finite economies, given all prices are positive and all consumers are locally nonsatiated, is that an excess supply (in value terms) cannot exist for some subset of goods without an excess demand (in value terms) existing for some other subset of goods. Aiyagari defines the failure of Walras' Law as a situation in which this implication of Walras' Law does not hold. His basic (and interesting) result is to show that "a competitive equilibrium is nonoptimal if and only if the above implication of Walras' Law fails in its neighborhood."

Aiyagari (1992, section 2) clearly demonstrates that Walras' Law can fail for an OG economy in this sense. But shouldn't we be skeptical of a model in which positively valued excess supplies can occur in equilibrium? After all, where do the excess supplies go? Nonsatiated consumers would not simply throw the excesses away.

The objective of this note is to show that the positively valued excess supplies which Aiyagari connects with Pareto inefficiency represent an unexploited economic opportunity. Moreover, it is an opportunity that can potentially be exploited by government or by a private financial intermediary through the issuance of unsecured debt. We demonstrate below that, when unsecured debt is issued, Walras' Law does not fail in the sense described by Aiyagari. However, the mere issuance of unsecured debt does not ensure Pareto efficiency. We show that Pareto efficiency is achieved if and only if the opportunity to issue unsecured debt is exploited to its fullest extent, for example, by an *earnings-driven* private financial intermediary.

2. An OG Economy With No Unsecured Debt Issue

Consider a pure exchange overlapping generations (OG) economy which begins in period 1 and extends into the infinite future. One perishable consumable resource exists, which is distinguished in period t as "good t." At the beginning of each period t, a single two-period lived "generation t consumer" is born. The generation t consumer is endowed with $w^y > 0$ units of good t and $w^o > 0$ units of good $t+1$. His preferences over consumption profiles (c_t^y, c_{t+1}^o) are represented by a utility function $U(c_t^y, c_{t+1}^o)$ that is twice continuously differentiable, strictly quasi-concave, strictly increasing, and satisfies $U(c_t^y, 0) = U(0, c_{t+1}^o) = U(0, 0)$ and

$$MRS(w^y, w^o) \; \equiv \; U_1(w^y, w^o)/U_2(w^y, w^o) \; < \; 1 \, . \tag{1}$$

It is also assumed that consumer preferences satisfy gross substitutability.[1]

The population of the economy in period 1 consists of the young generation 1 consumer and one old "generation 0 consumer." The generation 0 consumer has an endowment of $w^o > 0$, prefers more consumption to less, and dies at the end of period 1.

Intertemporal trades are facilitated by a price system $\boldsymbol{p} = (p_1, p_2, \ldots)$, where p_t denotes the price of good t in terms of a unit of account. Given this price system, the lifetime utility maximization problem faced by each generation t consumer, $t \geq 1$, takes the form:

$$\max \; U(c_t^y, c_{t+1}^o) \tag{2}$$

with respect to (c_t^y, c_{t+1}^o) subject to the budget and nonnegativity constraints

$$p_t c_t^y + p_{t+1} c_{t+1}^o \; = \; p_t w^y + p_{t+1} w^o \; ; \tag{3}$$

$$c_t^y \geq 0, \quad c_{t+1}^o \geq 0 \, . \tag{4}$$

Given the stated restrictions on consumer preferences, any solution to this utility maximization problem must satisfy

$$MRS(c_t^y, c_{t+1}^o) \; = \; p_t/p_{t+1} \, . \tag{5}$$

Finally, the consumption level of the generation 0 consumer is given by

$$p_1 c_1^o = p_1 w^o. \tag{6}$$

[1] In the model at hand, gross substitutability implies that an increase in the rate at which the generation t consumer can trade good t for good $t+1$ results in an increase in his optimal savings, $t \geq 1$.

Let $\boldsymbol{c} = (c_1^0, (c_1^y, c_2^y), (c_2^y, c_3^o), \ldots)$ denote an *allocation* for the economy. A nonnegative allocation $\boldsymbol{c}$ is *feasible* if and only if the market for good t clears in each period $t \geq 1$, in the sense that

$$w^y + w^o \ \geq\ c_t^y + c_t^o\ ,\quad t \geq 1\ . \tag{7}$$

Following Aiyagari (1992, Section 2.1), an *equilibrium* for the economy is an allocation $\boldsymbol{c} \geq 0$ and a price system $\boldsymbol{p} > 0$ that satisfy conditions (3), (4), (5), (6), and (7).

If the market clearing conditions (7) were required to hold as equalities, then it is straightforward to show that the unique equilibrium allocation for the economy would be the "autarkic" allocation in which each consumer directly consumes his endowment profile and the price sequence $\boldsymbol{p}$ satisfies $p_t/p_{t+1} = MRS(w^y, w^o)$ for all $t \geq 1$. However, because the market clearing conditions (7) allow for excess supply, the autarkic allocation is *not* the only equilibrium allocation for the economy. In fact, there are an infinite number of equilibrium allocations. What is unique about the autarkic allocation is that it is the only equilibrium allocation for which the value of excess supply is zero in every market. For all other equilibrium allocations, at least one market has a positively valued excess supply.

To illustrate, consider Figure 1 where an offer curve is drawn for the generation t consumer along with three lifetime budget constraints. For simplicity, consider only the stationary equilibria for which the (gross) rate of return p_t/p_{t+1} takes on a constant value ρ for all $t \geq 1$, with $MRS(w^y, w^o) \leq \rho \leq 1$. The constant rate of return implies that each generation t consumer consumes the same consumption profile $(c_t^y, c_{t+1}^o) = (c^y, c^o)$. Moreover, the lifetime budget constraint (3) reduces to $\rho c^y + c^o = \rho w^y + w^o$ for all $t \geq 1$, implying that

$$w^y + w^o - c^y - c^o = [1 - \rho][w^y - c^y] \geq 0. \tag{8}$$

Three possible cases will now be considered: $\rho = MRS(w^y, w^o)$; $\rho = 1$; and $MRS(w^y, w^o) < \rho < 1$. These three cases correspond to the three lifetime budget constraints depicted in Figure 1.

If $\rho = MRS(w^y, w^o)$, then $(c_t^y, c_{t+1}^o) = (w^y, w^o)$ for each $t \geq 1$, impying that the market clearing conditions (7) holds as equalities for all $t \geq 2$. Since $p_1 > 0$, it follows from (6) that the generation 0 consumer consumes his endowment (i.e., $c_1^o = w^o$), hence market clearing also holds as an equality for $t = 1$. Consequently, as noted above, no excess supply exists in this autarkic equilibrium.

If $\rho = 1$, then (c_t^y, c_{t+1}^o) equals the "golden rule" consumption profile $(\bar{c}^y, \bar{c}^o)$ for each $t \geq 1$, and condition (8) implies that the market clearing conditions (7) hold as equalities for all $t \geq 2$. In period 1, the generation 0 consumer consumes his endowment (i.e., $c_1^o = w^o$), while the generation 1 consumer consumes less than his endowment (i.e., $c_1^y = \bar{c}^y$). Thus, there is an excess supply of good 1 in period 1 (i.e., $\bar{c}^y + c^o < w^y + w^o$). Since $p_1 > 0$, this period 1 excess supply is positively valued, meaning Walras' Law fails.

If $\rho = \hat{\rho}$, where $MRS(w^y, w^o) < \hat{\rho} < 1$, then $(c_t^y, c_{t+1}^o) = (\hat{c}^y, \hat{c}^o)$ for each $t \geq 1$, where $\hat{c}^y < w^y$. Together with condition (8), this implies that an excess supply of good t exists in each period $t \geq 2$. In period 1, the generation 0 old consumer consumes his endowment (i.e., $c_1^o = w^o$) while the generation 1 young consumer consumes less than his endowment (i.e., $c_1^y = \hat{c}^y$). Thus, an excess supply of good 1 exists in period 1 (i.e., $\hat{c}_1^y + c_1^o < w^y + w^o$). Since $p_t > 0$ for all $t \geq 1$, the excess supply present in each period $t \geq 1$ is positively valued, meaning Walras' Law fails.

Nonstationary equilibria exist for this model as well. However, the results presented here are sufficient to demonstrate the meaning of Aiyagari's assertion that Walras' Law fails for the OG model. Considering all equilibria for the economy presented here, stationary and nonstationary, it can be shown that a positively valued excess supply exists in every equilibrium except the autarkic equilibrium.

To this point, the intermediation process for the economy has not been explicitly articulated. The precise form of this intermediation process would not be significant if positively valued excess supplies did not occur in equilibrium. However, positively valued excess supplies do occur; and because it does not make sense that nonsatiated consumers would discard valuable resources, it is important to consider what happens to these excess supplies. If the intermediary were a central clearing house, then the clearing house would hold any excess supplies after the trades had been made. Recognizing this, it is evident that there is an opportunity associated with intermediation in this model that is not being recognized. What consumer would not want to own the clearing house?

In the next section, we show that the issuance of unsecured debt allows this intermediation opportunity to be exploited, and Walras' Law (in Aiyagari's sense) no longer fails.

3. An OG Economy With Unsecured Debt Issue

Suppose the economy described in section 2 is now modified by having the generation 0 old consumer issue unsecured debt in amount D_0. Let this unsecured debt be taken as the unit of account, so that the price p_t denotes the number of units of unsecured debt necessary to buy one unit of good t in period t. Under these assumptions, the budget constraint of the generation 0 old consumer becomes

$$c_1^o = w^o + \frac{D_0}{p_1} \, . \tag{9}$$

Consumers in generations $t \geq 1$ are not allowed to issue debt, but they are allowed to purchase old debt and then to resell it. Debt can also be sold short, allowing consumers to borrow. No other intermediation options are available. Under these assumptions, the lifetime utility maximization problem faced by the generation t consumer, $t \geq 1$, takes the form:

$$\max \ U(c_t^y, c_{t+1}^o) \tag{10}$$

with respect to (c_t^y, c_{t+1}^o, D_t) subject to the budget and nonnegativity constraints

$$c_t^y \ = \ w^y - \frac{D_t}{p_t} \, ; \tag{11}$$

$$c_{t+1}^o \ = \ w^o + \frac{D_t}{p_{t+1}} \, ; \tag{12}$$

$$c_t^y \geq 0, \ \ c_{t+1}^o \geq 0 \, . \tag{13}$$

Let $\boldsymbol{D} = (D_0, D_1, D_2, \ldots)$ denote the sequence of unsecured debt holdings for the economy. An *equilibrium* for the economy is then a triplet $(\boldsymbol{c}, \boldsymbol{p}, \boldsymbol{D})$ consisting of an allocation $\boldsymbol{c} \geq 0$, a price system $\boldsymbol{p} > 0$, and a debt sequence $\boldsymbol{D}$ that satisfy conditions (5), (7), (9), (11), (12), and (13), together with the following market clearing condition for the unsecured debt:

$$D_{t-1} \;\geq\; D_t \;\text{ for all }\; t \geq 1 \,. \tag{14}$$

Recall that unsecured debt is issued only once, in period 1, hence the supply of unsecured debt available in each period $t \geq 1$ is given by the unsecured debt D_{t-1} held by generation $t-1$.

The young age and old age budget constraints (11) and (12) together generate the lifetime budget constraint (3). Thus, there are only two essential differences between the economy presented here and the economy presented in section 2. First, the generation 0 consumer can here receive a wealth windfall from the issuance of unsecured debt, whereas no such windfall was previously possible. Second, the medium of exchange is here explicitly identified as being unsecured debt, whereas the medium of exchange was not previously specified.

Although Walras' Law was shown to fail for the economy without unsecured debt, it cannot fail for the present economy. To understand this, note that by combining the young age budget constraint for generation t with the old age budget constraint for generation $t-1$ one obtains

$$p_t[w^y + w^o - c^y_t - c^o_t] + [D_{t-1} - D_t] \;=\; 0 \,, \;\; t \geq 1 \,. \tag{15}$$

Using condition (15), a positively valued excess supply of good t implies an excess demand for unsecured debt in period t, a violation of the market clearing condition (14). Thus, no positively valued excess supply of any good $t \geq 1$ can exist in equilibrium, meaning Walras Law cannot fail.

This restoration of Walras' Law rules out some of the inefficient equilibria obtained for the section 2 economy. For example, because an excess supply is no longer possible in equilibrium, none of the allocations for the section 2 economy that were associated with rates of return ρ satisfying $MRS(w^y, w^o) < \rho < 1$ can now be supported as equilibria.

Nevertheless, Pareto efficiency is still not ensured. As is known from Gale (1973), Pareto efficiency for the present economy depends upon the real value of the period 1 unsecured debt. If the period 1 price p_1 is such that the initial real debt level is given by $D_0/p_1 = [\bar{c}^o - w^o]$, then the economy has a unique stationary Pareto efficient equilibrium allocation in which each generation t consumer consumes the golden rule consumption profile $(c_t^y, c_{t+1}^0) = (\bar{c}^y, \bar{c}^o)$, the generation 0 consumer consumes $c_1^0 = \bar{c}^o$, and the rate of return in each period $t \geq 1$ is given by $\rho = 1$. Alternatively, if $p_1 = +\infty$ so that $D_0/p_1 = 0$ (i.e., the unsecured debt is worthless), then the only possible equilibrium allocation is the Pareto inefficient autarkic allocation in which each consumer simply consumes his own endowment in each period t. If $0 < D_0/p_1 < \bar{c}^o - w^o$, then a nonstationary Pareto inefficient equilibrium allocation results, with the consumption profile of the generation t consumer converging to the endowment profile as t becomes arbitrarily large. Finally, increasing the initial unsecured debt level above $[\bar{c}^o - w^o]$ puts the economy on a path to economic collapse, for the real demand for unsecured debt exceeds the total endowment of the economy in finite time.

In summary, in the present economy, Pareto efficiency is obtained if and only if full advantage is taken of the gain which can be had from issuing unsecured debt. In Pingle and Tesfatsion (1991a,b;1993) it is shown that the opportunity to obtain a wealth windfall by issuing unsecured debt can be exploited by earnings-driven private intermediaries (e.g., through the issue of corporate debentures) as well as by a government (e.g., through the issue of fiat money). Indeed, in Pingle and Tesfatsion (1991b) it is shown that the earnings objective of the private corporate intermediary is satisfied if and only if price conditions hold which are analogous to the Cass-Balasko-Shell transversality condition elaborated in Balasko and Shell (1980), a necessary and sufficient condition for Pareto efficiency. But as elaborated in Pingle and Tesfatsion (1993), these price conditions push to economy to the very brink of economic collapse, in the sense that any slight increase in the rate of return in any period t pushes the economy onto an explosive infeasible path. Consequently, there is still much to learn concerning unsecured debt issue (public and private), efficiency, and economic instability in OG economies.

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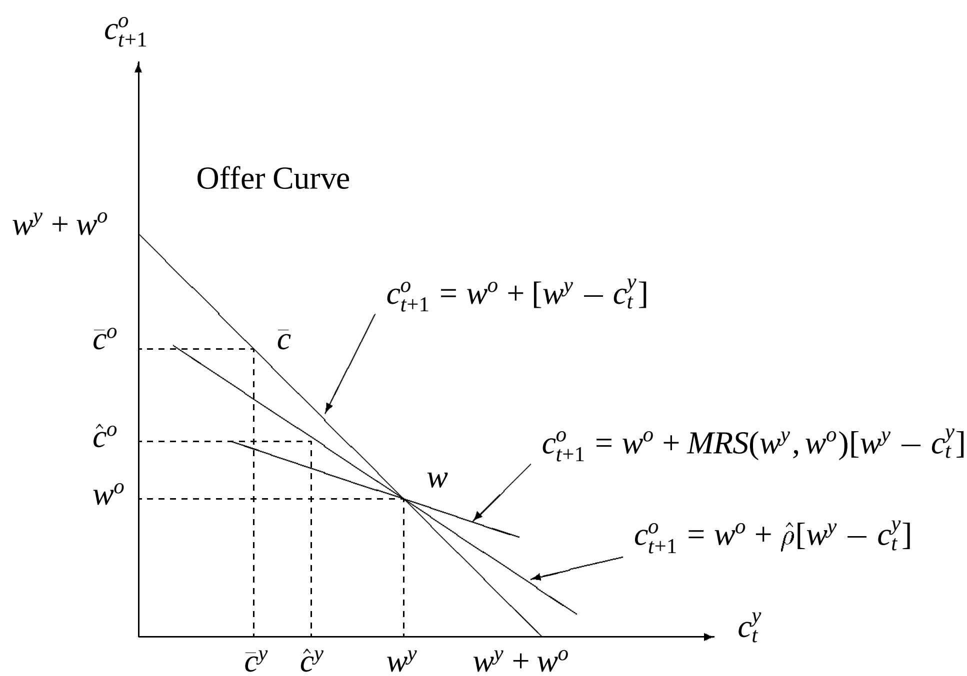

c^o_{t+1}
Offer Curve
$w^y + w^o$
$c^o_{t+1} = w^o + [w^y - c^y_t]$
$\bar{c}^o$
$\bar{c}$
$\hat{c}^o$
$c^o_{t+1} = w^o + MRS(w^y, w^o)[w^y - c^y_t]$
w
w^o
$c^o_{t+1} = w^o + \hat{\rho}[w^y - c^y_t]$
c^y_t
$\bar{c}^y$
$\hat{c}^y$
w^y
$w^y + w^o$

Figure 1