North-South Trades and Growth Miracles*

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Abstract

This paper proposes a two-country "economic" model (in the sense that it contains utility and profit maximization motives), in which a low-income economy enjoys a high growth rate relative to a high-income economy, thanks to importing technologies (or "machines") invented in the high-income economy. Following Romer (1990), the growth of an economy is sustained by increasing varieties of inputs; while a high-income economy (and a closed economy) should invest in R&D to invent new inputs (or "machines"), an open, low-income economy may trade with the high-income economy to import them, which reduces the cost of productivity advances. The model can generate the growth paths of the U.S. and the South Korea.

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1. Introduction

Recent studies have pointed out that economies with low income levels – especially those open to international trade – tend to grow faster than economies with high income levels. In light of the findings in the empirical literature, a desirable growth theory should be able to explain the following stylized facts:

- Openness of an economy is important for the growth of income or productivity. (Frankel and Romer (1999) and Alcalá and Ciccone (2004).) Open, low-income economies grow faster than high-income economies or closed, low-income economies. (Sachs and Warner (1995) and Choi (2005).)
- The trade in intermediate products and capital equipment is particularly important in economic growth, among other types of trades. (Coe, Helpman and Hoffmaister (1997).)
- The trading partner is also important. The trade with advanced economies, rather than with low-income economies, helps an economy to accelerate growth. (Chuang (1993, 1997).)
- Most of R&D investments or technological inventions are made in a few high-income economies. That is, most economies adopt the knowledge diffused from these economies, rather than discover it by themselves. (Eaton and Kortum (1996, 1999) and Keller (2004).)

However, many of the models explaining a relatively higher growth of low-income economies have been "mechanical." That is, economists often assume that the productivity growth of an economy directly depends on something else, for example, the productivities of other (advanced) economies. (See Klenow and Rodríguez-Clare (2005) for a survey on such models.) This approach has a difficulty in explaining why international trade affects economic growth of low-income economies, because there is no economic reasoning on why open economies should be different from others in terms of productivity growth.[1]

[1] An interesting approach to relate trade to growth is suggested by successive studies of Alvarez and Lucas (2005a, 2005b). They emphasize higher levels of capital accumulation made possible by international trade, while this paper views the adoption of advanced technology as a main source of "growth miracles."

The goal of this paper is to "endogenize" technological progresses of open, low-income economies. The key observation is as follows. The introduction of new technologies (or inputs, or "machines") – for example, fertilizer, tractor, typewriter, tansistor, computer and internet – increases the productivity of an economy. Completely "isolated" economies, as well as open economies with top levels of productivities, should invest in R&D to invent new "machines" for these productivity advances. However, open economies with lower levels of productivities may benefit from interactions with advanced economies. For example, if a sub-Saharan economy does not own any computers but wishes to introduce some, it can develop its own technologies to invent them, which is perhaps costly and time-consuming. But it may also choose to seek for a help from an advanced economy that knows how to build them. That is, the sub-Saharan economy may (i) import the computers, (ii) attract the technologies (that are required to build them) through patents or foreign direct investments, and (iii) even send the students to learn the required knowledge, etc. All of these, which are probably cheaper than inventing computers without other's help, will allow low-income economies to advance their productivity levels at a higher speed.

This paper is based on the "love of variety" model, used by Romer (1990), Aghion and Howitt (1992) and Koren and Tenreyro (2005). For simplicity, I assume the introduction (either in form of invention or import) of new machines is the only source of economic growth.[2] There is a free entry to the machine service market (in which the machine service is supplied to final good producers), so the investment in R&D is made under the condition that the cost of machine invention equals the value of a machine.

Now, the key assumption is that the economy with a high level of technology can (re)produce at a cheaper price what it has already invented. The export price of a reproduced machine is assumed to be determined in a competitive market. Also, potential machine owners of a low-income economy are allowed to import the machines which are new to the economy, by exporting (or paying) the final goods in response. We can generate the calibration result in which an open, *high*-income economy grows at about 2%, while an open, *low*-income economy enjoys a higher growth rate especially when their income gap is wider. Although the model is

[2] Here, "machines" are not limited to the literal meaning, but may include patented technologies or other immaterial products.

simple, it can explain all of four stylized facts listed above.

This paper is organized as follows. Section 2 derives the growth path of a closed economy. Section 3 extends the model to a two-country case and describes how two economies develop. Section 4 concludes.

2. Basic Model

2.1. Technology

To obtain the growth path of an economy, it requires to discuss how production decision is made in each period. Consider an economy that currently holds n machines at a given period. All machines are different from each other and can be substituted only to a constant elasticity of substitution (CES). The output of an economy is given by

$$Y = \left(\sum_{i=1}^{n} X_i^{\sigma} \right)^{1/\sigma}, \tag{1}$$

where $0 < \sigma < 1$ is a given parameter in which $1/(1-\sigma)$ becomes the elasticity of substitution, and X_i is the units of "services" (for example, hours of operations) that machine i provides for final good producers. For simplicity, assume that the owner of machine i can produce one unit of machine "service" by hiring one unit of labor. That is,

$$X_i = l_i, \tag{2}$$

where l_i is the units of labor hired by the owner of machine i. I further assume that machines do not depreciate, so that they can contribute to the production process forever.

As we may conjecture from the shape of production function (1), it turns out that all n machines provide the same units of services (by symmetricity) in an equilibrium defined later. Then, it becomes easy to see how input diversity affects the productivity. Assuming $X_i = X$ for all $i = 1, ..., n$, we can write (1) as

$$Y = n^{1/\sigma} X. \tag{3}$$

Let us denote by L the size of labor force, so that $L = \sum_{i=1}^{n} l_i$ in the labor market clearance. Then, from (2) and (3),

$$Y = n^{\phi} L, \tag{4}$$

where $\phi \equiv 1/\sigma - 1$. Since $0 < \sigma < 1$, ϕ satisfies $\phi > 0$, so (4) implies that the per-capita income Y/L (or equivalently, the productivity) is increasing in the number of machines n. This is the "love of variety" effect discussed in Romer (1990) or Gross and Helpman (1991).

2.2. Production

Machine owners act as monopolistic competitors: Each of them sets the unit price of the service provided by her machine, but takes the overall prices of services by other machines as given. On the other hand, I assume that the final good sector is perfectly competitive. The consumers (and investors at the same time) of this economy equally share the mutual fund of all the existing machines, so they earn the capital income (from this mutual fund) as well as the labor income (from working for machine owners). The equilibrium of this economy in the production procedure can be defined as follows:

Definition 1 *Given the number of machines n and the size of labor force L, an equilibrium in the production procedure consists of the wage w, the input prices $\{\chi_i\}_{i=1}^{n}$, the quantities supplied of machine services $\{X_i\}_{i=1}^{n}$, such that*

(a) Final good producers maximize the profits in a competitive market, that is, they solve given $\{\chi_i\}_{i=1}^{n}$,

$$\max_{\{X_i\}} \left(\sum_{i=1}^{n} X_i^{\sigma} \right)^{1/\sigma} - \sum_{i=1}^{n} \chi_i X_i, \tag{5}$$

(b) The owner of machine i maximizes the profit in a monopolistically competitive market, that is, she solves given w,

$$\max_{\chi_i} (\chi_i - w) X_i(\chi_i), \tag{6}$$

where $X_i(\chi_i)$ emphasizes that X_i depends on χ_i, and

(c) The labor market clears, that is,

$$\sum_{i=1}^{n} l_i = L. \tag{7}$$

Notice that in (5), the price of this final good is normalized to 1. Solving the model, we have the following results while the proof can be found in the appendix.

Proposition 2 *Given the number of machines n and the size of labor force L of an economy, the equilibrium in the production procedure (defined in Definition 1) has the following properties:*

(a) Wage: $w = \sigma n^{\phi}$

(b) Unit Price of Machine Service i: $\chi_i = n^{\phi}$ for all i,

(c) Quantity Supplied of Machine Service i: $X_i = L/n$ for all i,

(d) Output Produced: $Y = n^{\phi} L$,

(e) Profit of Machine Owner i:

$$\pi_i = (1-\sigma)n^{\phi-1}L, \text{ for all } i. \tag{8}$$

2.3. Growth in a Closed Economy

Now we are ready to analyze how a closed economy grows over time. In particular, we are interested in a balanced growth in which the consumption (C_t), income (Y_t) and machine variety (n_t) grow at constant rates.

1. Assume that the preference of a representative consumer has a constant relative risk aversion with coefficient θ:

$$U_t = \sum_{\tau=0}^{\infty} \beta^{\tau} u(C_{t+\tau}) = \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{C_{t+\tau}^{1-\theta}}{1-\theta}, \tag{9}$$

   where C_t is the consumption at period t, and β is the period discount value. The consumer's decision is made in the following procedure. At the beginning of period t, the number of machines n_t is given from the investment decision of the previous period. The consumer works for a period, and at the end of period t, the labor income (which is $w_t \times L_t$) and the capital income (or "dividend" or "profit", which is $n_t \times \pi_t$) are paid to the consumer. Then, she consumes C_t and invest the rest in R&D to build new machines. Based on the investment decision, n_{t+1} is determined, which is n_t plus the number of new machines invented. Throughout this paper, I suppose there is no change in the size of labor force, so that $L_t = L$ for all t.

2. I further assume that the invention of a new machine takes $\gamma \times L$ units of final goods, where γ is a constant.[3] Notice that as in many other studies, the cost of invention is scaled by L because otherwise an economy with a larger population should develop machine varieties faster.[4]

3. Now recall that the machine service market is monopolistically competitive. In view of this assumption, suppose there is a free entry to this market, so that machines are introduced at a fixed cost (of γL) until the owners break even. That is, denoting by v_t the (symmetric) "value" of any machine at the end (i.e., after dividend payment) of period t, we may write

$$v_t = \gamma L. \tag{10}$$

Rather than maximizing (9) with respect to the budget constraint directly, it is convenient to consider an existing machine at period t and to apply the fundamental equation of asset pricing. By definition, this machine has a value v_t. In the next period, it pays the dividend π_{t+1} to the representative consumer, and also has a new value v_{t+1}. In an equilibrium, the units of decreased marginal utility from holding this asset today should be equal to the discounted units of increased marginal utility tomorrow. Mathematically, we can write this as $v_t u'(C_t) = \beta(\pi_{t+1} + v_{t+1})u'(C_{t+1})$, or equivalently,

$$\left(\frac{C_{t+1}}{C_t}\right)^{\theta} = \beta \frac{\pi_{t+1} + v_{t+1}}{v_t}. \tag{11}$$

We are interested in the case in which the consumption grows at a constant rate. Let us define $\mu_c \equiv C_{t+1}/C_t$ for all t. Then, using (8) and (10), we can write (11) as

$$\mu_c = \left(\frac{\beta[(1-\sigma)n_t^{\phi-1} + \gamma]}{\gamma}\right)^{1/\theta}. \tag{12}$$

For μ_c to be a constant, it requires that n_t is a constant (which is not an interesting case for our purposes) or that the following assumption is true.[5]

[3] As in Grossman and Helpman (1991, Chapter 3) or Koren and Tenreyro (2005), we may assume that γ is a specific (decreasing) function of n. In this case, we may release Assumption 3 (which is introduced later).

[4] This assumption means, for example, the installation cost of new types of computers in all offices will be higher in China than in Sri Lanka because of China's huge population.

[5] Barro and Sala-i-Martin (1995, Chapter 6) also put similar restrictions. Gallaway, McDaniel and Rivera (2003) report that the estimate of elasticity of substitution (based on the time series of U.S. imports) is about 1 or 2.

Assumption 3 *The elasticity of substitution ($1/(1-\sigma)$) in production function (1) is 2. (This implies that $\sigma = 1/2$ and $\phi = 1$.)*

Under this assumption, we may rewrite (4), (8) and (12) as

$$Y_t/L = n_t, \tag{13}$$

$$\pi_{it} = (1-\sigma)L, \tag{14}$$

$$\mu_c = \left(\frac{\beta[1-\sigma+\gamma]}{\gamma}\right)^{1/\theta}. \tag{15}$$

Notice that (13) implies that the per-capita income (or equivalently, productivity) is the same as the number of machines. This result turns out to be useful in a later discussion.

To obtain how n_t grows in this growth path, we need another condition that comes from the resource constraint. In this economy, the number of current machines is the only state variable, so let us write the consumption C_t as a function of n_t, that is, $C_t = C(n_t)$.

4. Notice that the final output Y_t is either consumed or invested, so $Y_t = C_t + (n_{t+1} - n_t)\gamma L$, or equivalently,

$$n_t L = C(n_t) + (n_{t+1} - n_t)\gamma L, \tag{16}$$

by (13).

The appendix solves equation (16) (with (15)) for a functional form of C. Then, the growth rate of n_t can be easily obtained from (15). The following proposition summarizes the results, while the proof is in the appendix.

Proposition 4 *In a balanced growth of the model described above under Assumption 3, the output level Y_t, the number of machines n_t, and the consumption level C_t all grow at the same rate,*

$$\mu = \left(\frac{\beta[1-\sigma+\gamma]}{\gamma}\right)^{1/\theta}. \tag{17}$$

With this result and other conditions obtained in this section, the consumption, per-capita income (or productivity) and machine variety at any given period can easily be obtained, given the initial values for these variables.

3. Trade and Growth

3.1. Cost of Machine Introduction

In this section, I consider two economies A and B that hold n^A and n^B machines, respectively. Economy A is more developed in the sense that it has invented more machines, that is, $n^A > n^B$. For tractability, I assume that the introduction of machines should follow a given "order" (just as the internet cannot be invented before the computer is invented). That is, if the 513th machine in economy A is a tractor, the same is true in economy B.

For economy A to introduce a new machine (i.e., $(n^A + 1)$-th machine), it needs to invent it through the R&D investment (as in the previous section). However, economy B can introduce a new machine (i.e., $(n^B + 1)$-th machine) through *either invention or import*. Economy A already knows how to build $(n^B + 1)$-th machine, so it is less costly for A to reproduce one than for B to invent it. If A and B are allowed to trade, B may choose to import a new machine from A. Instead, economy B should export something else (say, final goods) to A to pay for this.

Let us be specific. Recall that economy B's invention of a new machine requires $L^B\gamma$ units of final goods, where L^B is the (constant) population size of B. Assume that for an advanced economy A that already holds $n^A(> n^B)$ machines, installing this machine for economy B requires

$$L^B W(n^A/n^B) \tag{18}$$

units of final goods, where $W(x)$ is defined on $x \geq 1$.[6] Here, n^A/n^B captures the technological advances of economy A relative to economy B. The more A is advanced to B, the lower A's cost of reproducing machines for B is, so I assume W is decreasing, i.e., $W'(x) < 0$ for $x \geq 1$. Furthermore, since the per-capita cost of economy A's producing $(n^A + 1)$-th machine (that is new even to economy A) is γ, it is natural to assume that $W(1) = \gamma$. Based on these assumptions, $n^A > n^B$ implies that

$$W(n^A/n^B) < \gamma. \tag{19}$$

For simplicity, I disregard the difference of cost in economy A's reproducing $(n^B + 1)$-th machine and $(n^B + k)$-th machine $(k > 1)$ as long as they are reproduced in the same period.

[6] Notice that the cost is scaled by B's population because this is an installation for B. Again, China requires more computers than Sri Lanka.

That is, once n^A/n^B is given for a specific period, the cost (18) is applied for any machines that A reproduces for B in that period.[7]

3.2. Trade Procedure

This subsection specifies how international trade between economies A and B is proceeded. At the end of period t, investors in two economies make contracts regarding the trades, which determines how many machines to be built by A for B and how many units of final goods to be exported from B to A. The final goods are paid immediately after the contract is made, and can be (i) consumed, (ii) invested in R&D to invent new machines, or (iii) used to reproduce existing machines immediately by economy A. Based on the contract, machine exporters in economy A starts to reproduce and install new machines for economy B, which is completed at the beginning of $t+1$ (just as newly invented machines are ready to be operated at the beginning of $t+1$ while the investment decision is made at the end of t.) The procedure is described in Table 1 in detail.

Just as there is a free entry to the machine service market (as is assumed in the previous section), I assume there is a free entry to machine export market, so anyone in economy A can join this market as long as there is a positive profit.[8] This implies that anyone in economy A is willing to be engaged in machine export market as long as the price of a machine (paid by B) is no less than $L^B W(n^A/n^B)$. Since there is a free entry, the export price of a machine is determined to be equal to the marginal cost (which is $L^B W(n^A/n^B)$), so the profit by exporting a machine becomes zero in the equilibrium.

[7]A rigorous treatment should define the cost function $\widetilde{W}(n^A, k)$, which is the cost of economy A's reproducing kth machine. However, the final result is not affected so much because the key point here is that this per-capita cost is lower than γ.

[8]Recall that the machine service market is monopolistically competitive, so any machines are owned monopolistically. However, I assume those machines can be reproduced by anyone in economy A because all the consumers/investors in economy A share the mutual fund over all machines installed in economy A.

	End of Period t	Beginning of Period $t+1$
Economy A	(1) Trade contract is made. Final goods (that can be used immediately) are received from economy B accordingly. (2) With final goods produced in A and received from B, the investment decision is made. Final goods are consumed, invested in R&D to invent new machines, or used to reproduce machines for B according to the contract.	Newly invented machines are ready to be operated. Production process starts.
Economy B	(1) Trade contract is made. Final goods are sent to economy A accordingly. (2) With final goods produced in B minus those sent to A, the investment decision is made. Final goods are either consumed or invested in R&D.	The machines imported from A or invented by B are ready to be operated. Production process starts.

Table 1: **Trade Procedure of Two Economies**

This implies that regardless of the trade volume, nothing changes for economy A's growth path. Even though the economy is now open for North-South trade, economy A will still make the same investment on the invention of new machines decision because the profit from exporting machines is zero anyway. So the growth paths of income, consumption and machine variety are all the same as the case of a closed economy.[9] The remaining question is, then, how economy B is affected by this North-South trade.

3.3. Gain from Trade

By the free entry condition in the machine service market, potential machine owners in B are free to either invest in R&D to invent a new machine (which costs $L^B\gamma$) or make a contract with machine exporters in A to import one (which costs $L^BW(n^A/n^B)$). By (19), it is always domi-

[9]If technology is monopolistically owned, the model may also generate gains for country A, too. Also, there should be welfare gains from trading different final goods in other types of model.

nant to take the latter strategy, which explains why R&D expenditure is concentrated mainly on a few high-income economies.

The representative consumer of economy B decides how many machines to *import*, just as a closed economy (or an open, high-income economy A) decides how many machines to *invent*. Since there is a free entry to the machine service market, the cost (paid at the end of t) of importing a machine (to be operated at the beginning of $t+1$) should be the same as the value of a machine (to continue to be operated at the beginning of $t+1$) after dividend payment of period t. That is,

$$v_t^B = L^B W(n_t^A/n_t^B), \tag{20}$$

which is analogous to (10) in a closed economy. Notice that (11) (which comes from the fundamental equation of asset pricing) still holds. So, (11) together with (14) and (20) yields

$$\frac{C_{t+1}^B}{C_t^B} = \left(\frac{\beta[1-\sigma+W(n_{t+1}^A/n_{t+1}^B)]}{W(n_t^A/n_t^B)}\right)^{1/\theta}, \tag{21}$$

which is analogous to (15). The final output is either consumed or exported to A (so that machines can be imported), so

$$n_t^B L^B = C_t^B + (n_{t+1}^B - n_t^B)L^B W(n_t^A/n_t^B). \tag{22}$$

Two equations (21) and (22) describe how an open, low-income economy grows over time. At each period t, the representative consumer decides how much to consume (C_t^B) and how many machines to be operated in the next period (n_{t+1}^B), given the current number of machines (n_t^B).

It is not easy to solve the system (21) and (22). As the appendix suggests, a possible approximation is to assume $n_{t+1}^A/n_{t+1}^B \approx n_t^A/n_t^B$ at each t, that is, the ratio of machine varieties does not change a lot in one period.[10] If we take this approximation, the system (21) and (22) becomes similar to (15) and (16) except that a machine can now be introduced at a cheaper price, $W(n_t^A/n_t^B)$, rather than γ. The solution is given in the following proposition.

Proposition 5 *In the two-country model described above, the growth path of economy A follows the path described in Proposition 4. On the other hand, the growth rate of income and machine*

[10] Recall that from Proposition 2, the national income is proportional to the number of machines. So the assumption behind this approximation implies that the ratio of incomes between two economies does not change a lot, say, in one year.

variety in economy B is given by, under the approximation described above,

$$\mu^B(n_t^A/n_t^B) \approx \left(\frac{\beta[1-\sigma+W(n_t^A/n_t^B)]}{W(n_t^A/n_t^B)}\right)^{1/\theta} \tag{23}$$

3.4. Calibration

By the calibration exercise, let us see how the North-South trade affects the growth of an open, low-income economy. Recall that Assumption 3 takes $\sigma = 1/2$. Further assume that the relative risk aversion (θ) is 2 and the annual time discount value (β) is 0.95. Let us take the per-capita cost of machine invention (γ) to be 5.[11] Then, by (17), the growth rate of income in a closed economy becomes about 1.02 (or 2%).

Now let us consider two open economies, high-income and low-income. Assume an open, high-income economy starts with the per-capita income of $Y_0^A/L^A = 10^4$. (Notice that by (13), economy A's initial number of machines, n_0^A, is 10^4.) On the other hand, an open, low-income economy has an initial income of $Y_0^B/L^B = 10^{2.5} \approx 316$. Finally, assume

$$W(x) = \gamma x^{\alpha},\ \alpha < 0,\ \text{for } x \geq 1, \tag{24}$$

so that $W(1) = \gamma$ and $W'(x) < 0$ for $x \geq 1$. We may try several values for α, but for now let us take $\alpha = -0.35$.

The growth paths of two economies based on Proposition 5 are illustrated in Figure 1. Country A grows at a constant rate of about 2%, and the per-capita income of \$10,000 becomes \$88,360 in 100 years. Country B, with imports of machines from country A, grows at about 12% at the beginning, and this growth rate gradually decreases as its income level catches up country A. The per-capita income of \$316 becomes \$55,697 (176 times) in 100 years. If country B grows as a closed economy (i.e., at about 2%), this would be only \$2,794, or 5% of the open economy path.

[11] This implies that \$5 investment today provides a new machine tomorrow, which pays \$1 per period forever.

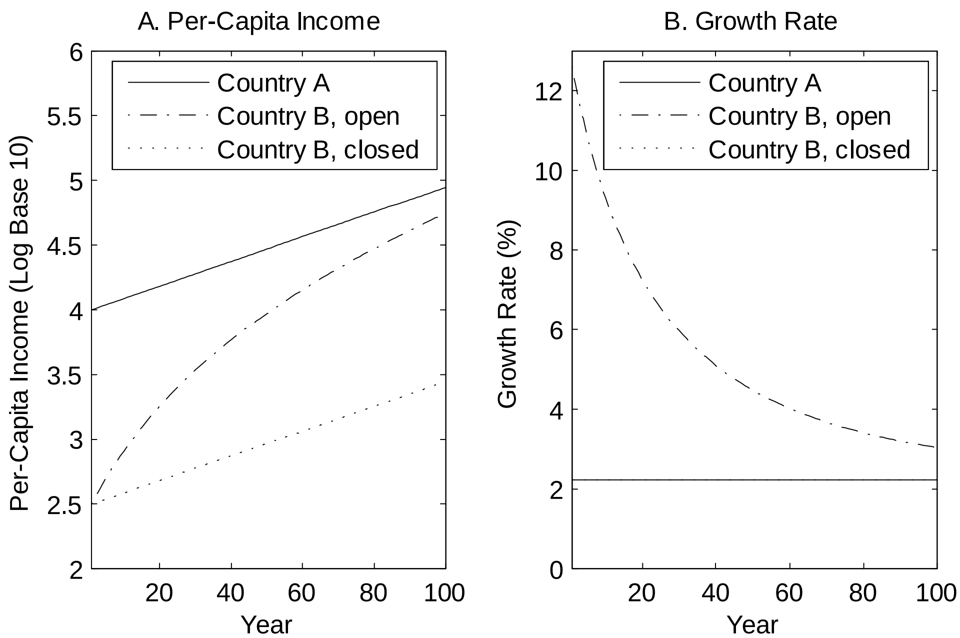

Figure 1: **Growth Paths of Countries A (high-income) and B (low-income)** The two figures show the paths of per-capita income and growth rate for 100 years, in which a high-income economy and a closed, low-income economy grow at about 2%, and an open, low-income economy grows at a higher rate, thanks to machine imports from a high-income economy.

Our model is a two-country model, so it may not be appropriate to apply it for multi-country reality. However, we may still get some implication as long as low-income economies gain from North-South trades in reality. Let us take, for example, the United States as a high-income economy and the South Korea as a low-income economy. According to the Wacziarg and Welch (2005) data of Sachs and Warner's (1995) openness dummies, the South Korea has been open to international trade since 1968. Figure 2 uses the same calibration numbers as before to predict Korea's growth path since 1968. The figure shows that the model can properly explain the growth path of Korea. If Korea has never been open, its per-capita income in 2003 would be about $3,200, which is less than 1/3 of the actual income of the same year.[12]

[12]Of course, the figure shows that even before 1968, Korea was growing at a considerably high rate. Although we are taking 1968 as an initial year of openness, the economy should have been opening gradually and interacting with the rest of the world even before 1968.

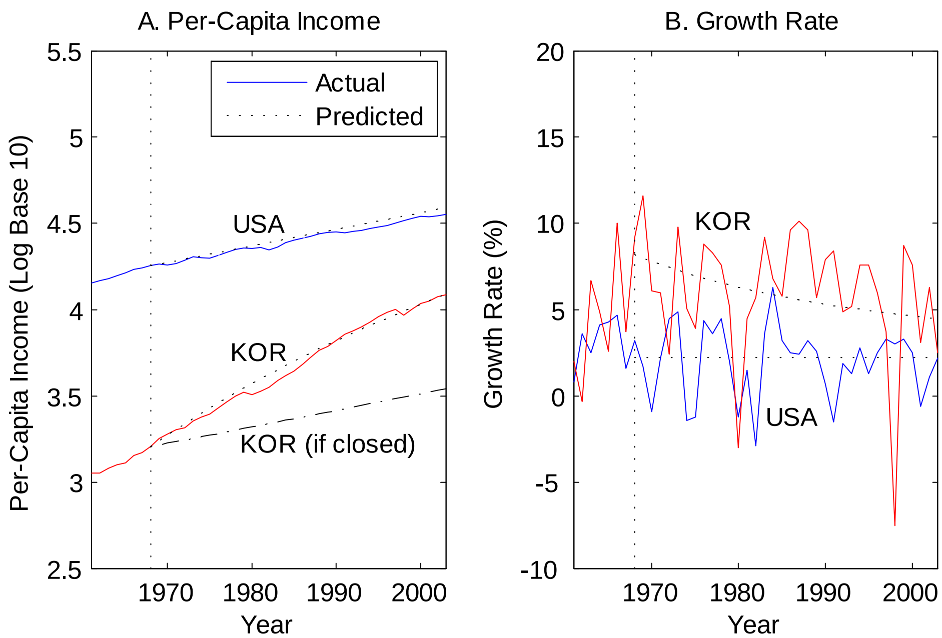

Figure 2: **Growth Paths of the U.S. and the South Korea** The two figures show the paths of per-capita income and growth rate in the U.S. and the South Korea. Korea is assumed to be open since 1968.

4. Conclusion

This paper proposes a two-country "economic" model (in the sense that it contains utility and profit maximization motives), in which a low-income economy enjoys a high growth rate relative to a high-income economy, thanks to importing technologies (or "machines") invented in the high-income economy. The result that the trade in "machines" with advanced economy accelerates the technology adoption of an open, low-income economy is related to the four stylized facts listed in the introduction. (i) Openness is important for growth, and open, low-income economies grow faster than open, high-income economies. (ii) Trade in "machines" is important in growth. (iii) Trade with advanced economy is also important. (iv) R&D investment is mostly made in advanced economies. While the model concentrates on generating growth path of an open, low-income economy, an extension of this paper should be able to describe the case of many countries, with explanation on the growth paths of consumption, investment, and volume of trades including South-South and North-North trades.

In essence, this paper answers one of the challenges raised by Lucas (2000): Why does an economy that started growing in a later period grow faster? The model explains that perhaps the interaction between economies – especially through international trade – is one of the main reasons why technology diffuses and low-income economies grow faster. Another question – why are the starting gate are opened in an earlier period only for some economies – still remains to be answered. Perhaps, some combination of this paper and Parente and Prescott (2005) can provide an insight.

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Appendix

Proof of Proposition 2. The first order condition for the problem of final good producers (5) is $X_i^{\sigma-1}\left(\sum X_i^{\sigma}\right)^{(1-\sigma)/\sigma} = \chi_i$, which can be written as

$$X_i = \chi_i^{1/(\sigma-1)} Y. \tag{25}$$

Then, the machine owner i's problem (6) can be written by $\max_{\chi_i}(\chi_i - w)\chi_i^{1/(\sigma-1)}Y$, and its first order condition is

$$\chi_i = w/\sigma. \tag{26}$$

Finally, the market clearing condition (7) requires that $\sum X_i = L$, but (25) and (26) imply that X_i does not depend on i, so

$$X_i = L/n, \tag{27}$$

which is result (c). Result (d) is straightforward from result (c) and the shape of production function (1). To obtain result (a), write (25) with (26) and result (c) as $X_i = (w/\sigma)^{1/(\sigma-1)} n^{1/\sigma-1} L$, and apply result (c) again. Result (a) and equation (26) provide result (d). Finally, result (e) can be obtained by the definition of profit $\pi_i = X_i(\chi_i - w)$ with results (a), (b) and (c). We can also show that the profits of final good producers are always zero. ■

Proof of Proposition 4. The consumption growth rate is obtained in (15). To solve (16) with (15), guess and verify that $C(n) = C_0 n$. Equation (15) can be written as $n_{t+1}/n_t = (\beta(1-\sigma+\gamma)/\gamma)^{1/\theta}$, which is a balanced growth rate of machine variety. Also, we can rewrite (16) as $C_0 = (1-(\mu-1)\gamma)L$, so our guess is verified. By (13), the output level grows at the same rate as machine varieties. ■

Proof of Proposition 5. Assuming $n_{t+1}^A/n_{t+1}^B \approx n_t^A/n_t^B$, (21) is approximated by

$$\frac{C_{t+1}^B}{C_t^B} \approx \left(\frac{\beta[1-\sigma+W(n_t^A/n_t^B)]}{W(n_t^A/n_t^B)}\right)^{1/\theta}. \tag{28}$$

Guess and verify that $C_t^B = C_0(n_t^A/n_t^B) \times n_t^B$. Using $n_{t+1}^A/n_{t+1}^B \approx n_t^A/n_t^B$, (28) implies

$$\frac{n_{t+1}^B}{n_t^B} \approx \left(\frac{\beta[1-\sigma+W(n_t^A/n_t^B)]}{W(n_t^A/n_t^B)}\right)^{1/\theta}, \tag{29}$$

which provides the growth rate of machine variety, given the numbers of machines of two economies in t. By (13), this also provides the growth rate of income in economy B. We can write (22) as

$$C_0\left(\frac{n_t^A}{n_t^B}\right) = \left[1 - \left[\left(\frac{\beta[1-\sigma+W(n_t^A/n_t^B)]}{W(n_t^A/n_t^B)}\right)^{1/\theta} - 1\right] W\left(\frac{n_t^A}{n_t^B}\right)\right] L^B \tag{30}$$

to verify our guess. ■