A General Equilibrium Analysis of the Demand for Money

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Abstract

A model of capital service is constructed on stylized consumer behaviour and the total quantity of financial resources. The demand for money becomes a function of the wage rate under agreement as well as interests rates and overall economic activity.

I. Introduction

An expansion of the reasons why people hold money can generate implications for the level of employment. Traditionally, money has been defined as any good that acts as a medium of exchange, unit of account, and store of value (Harris, 1981). The definition itself suggests the reasons why people hold money: demand for transactions, ease of communication of value in trade, and diminishing marginal utility in consumption at any given moment in time. To this traditional list, one can also add cultural reasons. For the purposes of monetary theory, culture will be conceived as a set of standards which evolve to solve problems. In the vast array of cultural traits, technology is the fastest evolving inasmuch as technical progress implies ever more efficient solutions and displacement of previous standards. So, an expectation of cultural change in general, and technological change in particular, can bear directly on expectations of wages under agreement and work its way into a general equilibrium model. To show how the culture of ever-evolving technology fits into the model and integrates with wage agreements, we must first set a simple stage. We will assume that capital markets exist, that labour is not the only input, that the economy is fully decentralized with atomistic producers and consumers, and that all agents hold complete information in their transactions. We will also assume that production can be used for consumption or subsequent investment, and that the salary level is determined under agreement at any given moment in time.

In this system, a quantity of fiduciary currency exists which is available based on the representative agent agreement. Currency in the form of wages allows the consumer to acquire the desired quantity of products. In other words, currency serves as medium of exchange as well as a measure and store of value. We will also assume that consumers have an infinite life horizon and demand a certain amount of money based on their expectation of future purchasing power, which also includes a buffer for unexpected expenses. Based on such stylised behaviour, one can rigorously generate a demand function for money with three salient variables: nominal interest rate, overall economic activity, and the wage rate under agreement.

II. Economic model

With the stage now set, the objective function of the prototypical consumer is determined by his or her demand for consumption, for leisure, and for real balances. Equation (1) captures such behaviour and accords with established theory as long as it is continuous and differentiable. Over possibilities of leisure and real balances for the prototypical consumer, preferences are assumed well-behaved and nested preferences, being homogeneous of degree "λ".

The second equation represents the budget constraint for the consumer in the frontier of efficient expenses, consistent to a regime of effective property.

The basic intertemporal calculation is:

$$Max. \quad V = (q_{c1}, q_{c2}, \cdots, q_{ct}, \cdots, s_1, s_2, s_3, \cdots, s_t, \cdots) \qquad (1)$$

Subject to:

$$\begin{aligned}
&m_0 + w_1\tau_1 + \pi_1 = p_1 q_{c1} + w_1 s_1 + m_1; \qquad (2)\\
&m_1 + e(w_2)\tau_1 + \pi_2 = e(p_2)q_{c2} + e(w_2)s_2 + m_2;\\
&\vdots\\
&m_{t-1} + e(w_t)\tau_1 + \pi_t = e(p_t)q_{ct} + e(w_t)s_t + m_t;\\
&\vdots
\end{aligned}$$

where:

V = the utility function;

q_{ci} = the demand for consumption in period i;

w = the nominal wage;

τ = the biological maximum time to work (approximately 16 hours);

s = the leisure = $(\tau - t_o)$;

t_o = the labour supply;

π = the non salary income;

e = expectations;

p = the price of the product;

$\mathbf{m_t} = m_a + m_b + m_k$

where:

$\mathbf{m_a}$ = dividends;

$\mathbf{m_b}$ = fringe benefits;

$\mathbf{m_k}$ = fees for capital services.

Fees for capital services depend on real financing that the prototypical consumer channels through the system and the total quantity of financial resources that he or she contributes to the economy, in which the aggregate capital is financed during the period following production (Noriega, 1994). In this case, the resources that the prototypical consumer reserves for savings, in terms of real balances, during period “t” will, in turn, finance part of the capital demand by companies in the subsequent period. Similarly, the consumer’s current revenues for capital investment in “t” have been the result of his or her savings in “t-1”. Currency as a store of value is supported in the system by the remaining balances, once the demand for consumption is satisfied.

Since S = (ι–to), the constraint in the maximization problem can be re–written as:

$$
\begin{aligned}
& m_0 + w_1 t_o + \pi_1 = p_1 q_{c1} + m_1; \qquad (2) \\
& m_1 + e(w_2) t_{o1} + \pi_2 = e(p_2) q_{c2} + m_2; \\
& \vdots \\
& m_{t-1} + e(w_t) t_{ot} + \pi_t = e(p_t) q_{ct} + m_t; \\
& \vdots
\end{aligned}
$$

For the basic intertemporal calculation, currency is introduced to the utility function under the following assumptions:

1. f (.) is an indirect utility function of the complete intertemporal calculation of the consumers;
2. The variable m_t is explained by all future demands, if it persists recursively in the future;
3. The agents exhibit expectations of unitary price-elasticity;
4. To have been formed beginning with well-known magnitudes:

$$e(w_t) = \alpha_t(w_1), \quad e(p_t) = \beta_t(w_t), \qquad \forall\, t > 1 \qquad (3)$$

Now, let:

$$q_t^{(e)} = \alpha_t q_{c1} \qquad (4)$$

$$s_t^{(e)} = \beta_t s_1 \qquad (5)$$

where (4) is a proportional function of unitary elasticity, and (5), is the expectation of leisure. Once q_{c1} and s_1 have been determined, the expectations of every period emerge. In other words,

$$Max. \quad V = \left[q_{c1}, s_1, (\alpha_2 q_{c1}), (\beta_2 s_1), (\alpha_3 q_{c1}), (\beta_3 s_1), \cdots\right] \quad (6)$$

$$Max. \quad V = \left[q_{c1}, s_1, q_{c2}^{(e)}, s_2^{(e)}, q_{c3}^{(e)}, s_3^{(e)}, \cdots\right] \qquad (6')$$

And constrained, the maximization problem becomes

$$Max. \quad V = \left[q_{c1}, s_1, q_{c2}^{(e)}, s_2^{(e)}, q_{c3}^{(e)}, s_3^{(e)}, \cdots\right] \qquad (6')$$

Subject to:

$$m_0 + w_1\tau_1 + \pi_1 = p_1 q_{c1} + w_1 s_1 + \sum_{t=2}^{\infty}\left(p_t^{(e)} q_t^{(e)} + w_t^{(e)} s_t^{(e)}\right) \qquad (7)$$

Recognizing that s = (τ–t_o),

$$m_0 + w_1 t_{o1} + \pi_1 = p_1 q_{c1} + \sum_{t=2}^{\infty} \left(p_t^{(e)} q_t^{(e)} \right) \qquad (7')$$

which does not change the result of the demand function. The expectations of the agents are based on present information. The quantity of resources they hope to obtain is reflected in real balances which, in turn, reflect both belief and confidence that consumption will be realized in the future. So,

$$m^*/p \quad = \left(q_{c2}^{(e)}, s_2^{(e)}, q_{c3}^{(e)}, s_3^{(e)}, \cdots \right) \qquad (8)$$

$$m^* = \sum_{t=2}^{\infty} \left(p_t^{(e)} q_t^{(e)} + w_t^{(e)} s_t^{(e)} \right) \qquad (9)$$

And using $s = (\tau - t_o)$,

$$m^* = \sum_{t=2}^{\infty} p_t^{(e)} q_t^{(e)} \qquad (9')$$

where:

m * is the demand of monetary balances;

m*/p is the demand of real monetary balances.

Now we shall consider irrelevant any distinction of periods:

$$Max. \quad V = (q_c, s, m^*/p) \qquad (10)$$

Subject to:

$$m_0 + w\tau + \pi = pq_c + ws + m^*; \qquad (11)$$

or

$$m_0 + w_1 t_{o1} + \pi_1 = p_1 q_{c1} + m^* \qquad (12)$$

Equation (12) can be interpreted as the *ex post* relationship among the total revenues perceived by the consumers and their consumption. Since the firms channel total wages, benefits, and the capital services to the consumers, starting from (12), a relationship between the value of the total production and the distribution of the consumers can be established:

$$pq = pq_c + m^* \qquad (13)$$

where:

$$pq = m_0 + wt_o + \pi$$

From equation (13) the following is obtained:

$$p(q - q_c) = m^* \qquad (14)$$

that is to say:

$$\frac{m^*}{p} = q_{i+1} \qquad (14')$$

where,

q = the magnitude of the production;

q_i = the physical capital determined by capital markets, in which the consumers, when dedicating their savings to the purchase of stocks, channel resources to the firms so that they can have a part of the production generated by themselves in the current period to use as an input in the subsequent period of the productive process.

As the expected demand $e(q_d)$ is assumed to be proportional to the current demand, the capital demand that firms carry out during the current period in order to resume productive processes for the following period can be expressed as:

$$q_{i+1} = \chi^{-1}(\delta - 1)\varphi\left(\frac{w}{r*}\right) f\left(q, \frac{r*}{w}\right) \qquad (15)$$

where

$r^*=(1+r)P_{0,}$ P_0 is the price of product q_i in the previous period

r = the interest rate which represents, in its magnitude, the consumption sacrifice reward to the consumers

"f(q,r*/w)" = a function with form and degree "f^{-1} (.)", which is defined on the proportional functions of demand expectations and of interest rate–wage ratio.

The parameter "f" is a positive real number which defines the proportional function of expectation "f(w/r *)" on the price of the factors. The assumption of proportional expectations allows us to express the demand of future capital as a function of the present period.

Replacing (15) in (14 '):

$$\frac{m*}{p} = \chi^{-1}(\delta - 1)\varphi\left(\frac{w}{r*}\right) f\left(q, \frac{r*}{w}\right) \qquad (16)$$

the demand for real monetary balances has an inverse relationship with the interest rate. With a positive interest rate and given a certain level of p, the consumer will transform monetary balances into stocks of the company.

Since "f (.)" is a homogeneous function which has the same characteristics as the inverse of the production function, the economic problem can be represented by the supply and demand of inputs for the producers:

$$Max(1+\theta) = pq(wt + r*q_i)^{-1} \qquad (17)$$

Subject to:

$$q = g\left[(t - t*), q_i\right] \qquad (18)$$

where

"t" = the total quantity of work used by a company;

t * = the volume of necessary work to make possible the installation of the company in the economy;

θ = the rate of benefits which is similar to $pq(wt)^{-1}-1$.

Considering t, we see that

$$t = f^{-1}\left[q, \left(\frac{w}{r*}\right)\right] \qquad (19)$$

which allows us to re-write (16) as

$$\frac{m*}{p} = \chi^{-1}(\delta - 1)\varphi f*\left(q, \frac{w}{r*}\right) \qquad (20)$$

where f * is homogeneous of degree "(ν^{-1}-1)".

One can also consider an excess of demand of for money as

$$M_o = pq_c + pq_{i+1} \qquad (21)$$

where M_0 is the monetary supply. The product demand function, is given by

$$q_c = (1+\phi_1+\phi_2)^{-1}(m_0 - w\tau + \pi)p^{-1} \qquad (22)$$

and one can plug in (19) to obtain

$$M_o = \left[\left(1+\phi_1+\phi_2\right)^{-1}\left(m_0 - w\tau + \pi\right) + \chi^{-1}(\delta - 1)\varphi f*\left(q, \frac{w}{r*}\right)\right]p \qquad (23)$$

The interpretation of equation (23) is that the demand for money has a positive relationship with overall economic activity and the nominal wage rate, and an inverse relationship with $r^*=(1+r)P_0$.

III. Conclusion

The demand for money is based on a general equilibrium model where markets depend on real capital financing as seen when a prototypical consumer ploughs his or her savings back into the system given the agreed wage. The logical deduction of the model accords to traditional theory: a direct relationship between the money demand and overall economic activity, and an inverse relationship between the demand for money and the interest rate. What is novel to the model is the introduction of proportional expectations and agreed salaries. One sees that the demand for money becomes a function of nominal wages, which tend to adjust to the demand of money correspondent to a given price level. If the wage rate and incomes from capital remunerations are both known, the possibility of full employment will depend not just on the interest rate and price level, but also demand expectations.

IV. References

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