Transitional Growth and Income Inequality: Anything Goes[1]

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Keywords: Inequality, Growth, Ramsey models
JEL-codes: O4, D31, D91

Revised March 2004

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Abstract

The effect of initial income inequality on growth is the subject of a large literature. We show, both analytically and with simulation experiments, that the same level of initial income inequality can be associated with very different income developments, depending on the source of the inequality. We consider three sources: differences in asset ownership, in productivity and in shocks. For these three sources the monotonicity, the persistence and even the sign of the resulting income changes can differ. We stress the implications for empirical work.

1 Introduction

The possible effect of income inequality on economic growth is the subject of a huge literature, partly inspired by the policy question whether income redistribution affects growth positively or negatively.[1] Theoretical contributions include models such as Kaldor (1956) in which inequality favours growth because the rich have higher marginal saving rates and models such as Alesina and Rodrik (1994) in which inequality leads to political instability, thereby reducing growth. The empirical literature has introduced measures of within-country income inequality in cross-country growth regressions. Until recently it has often found a growth enhancing effect of greater equality. Recent studies, however, have found a *positive* effect of income inequality on growth (e.g. Lundberg and Squire, 2003).

How inequality affects growth cannot be investigated by simply adding a measure of income inequality in a cross-country growth regression (e.g. Persson and Tabellini, 1994): income inequality is endogenous. For example, in Ramsey-type growth models individual accumulation decisions are driven by the difference between an agent's initial capital stock and its steady state value. The resulting transitional dynamics involve both economic growth and (as a result of convergence) endogenous changes in inequality. Clearly, these distributional changes cannot be analysed as the *determinants* of the accompanying economic growth: changes in inequality and growth are jointly determined by other factors such as the initial distribution of assets (Banerjee and Duflo, 2003, and Lundberg and Squire, 2003).

However, it *is,* of course, meaningful to ask how a change in the exogenous determinants of income inequality (e.g. inequality in the initial asset distribution) affects income growth. This is the question we address in this paper. Our point is a simple one: we show that the same change in income inequality can be associated with very different changes in growth in the same model, depending on the source of the underlying inequality. A given change in income inequality can be associated with changes in economic growth which differ not only in magnitude but also in persistence, monotonicity and even in sign.

The structure of the paper is as follows. In the next section we derive analytical results for two exogenous determinants of income inequality: heterogeneity of agents in terms of initial assets or in terms of productivity. In section 3 we extend these results, using simulation experiments and con-

[1] For recent surveys see Aghion *et al.,* 1999, and Thorbecke and Charumlind, 2002.

sider a third source of inequality, idiosyncratic shocks. We show that for the same initial income equality the economy can follow very different growth trajectories depending on the source of the inequality. Section 4 concludes, stressing implications for empirical work.

2 Income Inequality in a Ramsey Model

Asset Inequality

We start with the distribution of assets as the determinant of income inequality. Consider a single-good economy with two Robinson Crusoe agents (there are no factor, credit or insurance markets) in which income is determined by a common strictly concave production function:

$$y_{it} = f(k_{it})$$

where y is income, k the capital stock and the subscripts denote agent i and time t. An agent's optimal accumulation path is given by a policy function φ:

$$k_{it+1} = \varphi(k_{it}).$$

We assume that φ is strictly concave[2] with a steady state $k^* = \varphi(k^*)$, $\varphi(0) = 0$, $\varphi'(0) > 1$. Note that there exists a level $\widehat{k}$ $(0 < \widehat{k} < k^*)$ so that $\varphi'(k) \gtreqless 1$ for $k \lesseqgtr \widehat{k}$. An increase in inequality in the sense of a mean preserving spread of the distribution of k_{i0} will have no effect on k_0 (by construction). However, it will reduce k_1 (by Jensen's inequality and the strict concavity of φ). In the long run there is no effect since k_{it} converges to k^* for all i (convergence), irrespective of the initial distribution of assets. Hence inequality reduces growth in the short run (measured as the change from k_0 to k_1) but there is no long run effect on growth (cf. Banerjee and Duflo, 2003).

This is a neat result but not very helpful in practice since in empirical work growth is invariably measured as *income* growth rather than as a change in the aggregate capital stock k. The distinction matters since a mean preserving spread of the distribution of k_{i0} will have no effect on k_0 but it will, of course, change the value of initial aggregate income y_0. So how does a change in asset inequality affect *income* growth? Remarkably, this question does not seem to have been addressed in the literature.

[2] This is not implied by standard assumptions on the utility and production functions.

Let $h(y)$ denote the inverse of the production function, hence

$$f(h(y)) \equiv y,$$

from which

$$h''(y) \equiv -h'(y)^2 \frac{f''(h(y))}{f'(h(y))}. \tag{1}$$

The one-period change in output is $f(\varphi(h(y_0))) - y_0$, or $(f \circ \varphi \circ h)(y_0) - y_0$, where the operator '$\circ$' denotes composition of functions. Starting from $y_0+\Delta$ rather than from y_0 results in output growth $(f \circ \varphi \circ h)(y_0 + \Delta) - (y_0 + \Delta)$. Consequently, a mean preserving spread of initial income from (y_0, y_0) to $(y_0 - \Delta, y_0 + \Delta)$ will lead to a change DG in aggregate one-period growth of

$$\begin{aligned} DG &= (f \circ \varphi \circ h)(y_0 + \Delta) - (y_0 + \Delta) + (f \circ \varphi \circ h)(y_0 - \Delta) - (y_0 - \Delta) \\ &\quad -2\left((f \circ \varphi \circ h)(y_0) - y_0\right) \\ &= (f \circ \varphi \circ h)(y_0 + \Delta) - (f \circ \varphi \circ h)(y_0) + (f \circ \varphi \circ h)(y_0 - \Delta) \\ &\quad -(f \circ \varphi \circ h)(y_0) \\ &\approx (f \circ \varphi \circ h)''(y_0)\Delta^2, \end{aligned}$$

where the last step is based on a second-order Taylor expansion around y_0. The effect of a mean-income preserving spread in initial endowments therefore depends on the sign of $(f \circ \varphi \circ h)''(y_0)$, which may be written as

$$\begin{aligned} (f \circ \varphi \circ h)''(y) &= f''(\varphi(h(y)))\left(\varphi'(h(y))h'(y)\right)^2 \\ &\quad +f'(\varphi(h(y)))\varphi''(h(y))h'(y)^2 \\ &\quad +f'(\varphi(h(y)))\varphi'(h(y))h''(y). \end{aligned}$$

The first two terms of this expression are negative and the third term is positive, since $h(y)$ is convex. The sign of $(f \circ \varphi \circ h)''(y_0)$ is therefore ambiguous and examples can be constructed which go either way.[3] We will give sufficient conditions for DG to be negative for arbitrary y_0, and positive for rich economies (y_0 close to the steady state).

High initial income

For $y_0 = y^*$ we find (with $k^* = h(y^*)$)

$$\begin{aligned} (f \circ \varphi \circ h)''(y^*) &= f''(k^*)\left(\varphi'(k^*)h'(y^*)\right)^2 + f'(k^*)\varphi''(k^*)h'(y^*)^2 \\ &\quad +f'(k^*))\varphi'(k^*)h''(y^*). \end{aligned}$$

[3] In the 'canonical' case of the Ramsey model treated in many textbooks: log utility, Cobb-Douglas production function, and full annual depreciation, the term is always negative.

Hence, using equation (1):

$$(f \circ \varphi \circ h)''(y^*) = f''(k^*)\varphi'(k^*)h'(y^*)^2 \left(\varphi'(k^*) - 1\right) + f'(k^*)\varphi''(k^*)h'(y^*)^2.$$

The first term of this expression is positive, since $\varphi(k)$ intersects the 45° line from above, so that $\varphi'(k) < 1$ in the neighbourhood of the steady state. Hence, if $\varphi(k)$ is not too concave at the steady state, *i.e.* if $|\varphi''(k^*)|$ is sufficiently small, then $(f \circ \varphi \circ h)''(y^*) > 0$. If $(f \circ \varphi \circ h)''(y)$ is continuous in y, then in a (left) neighbourhood of y_0 also $(f \circ \varphi \circ h)''(y) > 0$.

Arbitrary initial income

Using again equation (1) but grouping terms differently, we find

$$\begin{aligned}(f \circ \varphi \circ h)''(y) &= f''(\varphi(h(y)))\left(\varphi'(h(y))h'(y)\right)^2 \\ &\quad + f'(\varphi(h(y)))h'(y)^2\varphi'(h(y))\left\{\frac{\varphi''(h(y))}{\varphi'(h(y))} - \frac{f''(h(y))}{f'(h(y))}\right\}.\end{aligned}$$

The first term of this expression is negative, the second term is negative if $\varphi(k)$ has stronger curvature than $f(k)$, or, more precisely, if $\varphi'(k)/f'(k)$ is non-increasing in k.[4]

Note that $\varphi'(h(y))$ enters the first term squared and only linearly in the second term. Since $\varphi'(h(y)) > 1$ for poor economies (with y_0 close to 0) one expects the first term to dominate for low initial incomes. Consequently we expect a mean-income preserving spread in initial endowments to reduce growth for poor economies.

The extension to a longer period is straightforward. Define $\varphi_1(k) = \varphi(k)$ and $\varphi_t(k) = \varphi(\varphi_{t-1}(k))$, $t = 2, 3, ...$ Then the function φ_t is strictly concave and $y_t = \varphi_t(k_0)$. Note that if $\varphi'(k)/f'(k)$ is decreasing in k then, by induction, $\varphi_t'(k)/f'(k)$ is also non-increasing in k. Hence a mean-income preserving spread in the distribution of assets reduces growth, not just for $t = 1$ but for all finite t.

Proposition 1 *If the distribution of k_0 is made less equal through a mean-income preserving spread and the policy function φ is strictly concave then income growth over a finite period is reduced if $\varphi'(k)/f'(k)$ is non-increasing in k. For an economy with agents close to the steady state a mean-income preserving spread* increases *growth if $|\varphi''(k^*)|$ is sufficiently close to* 0.

[4] In the canonical case $\varphi'(k)/f'(k)$ is constant and the second term vanishes.

Note that the second part of the proposition states that asset inequality can have a positive effect on income growth. This may seem to contradict the result of Banerjee and Duflo (2003). However, they consider asset rather than income growth.

Productivity Differences in a Ramsey Model

Now consider an economy with n agents. We assume that each agent is described by a stochastic Ramsey model (see Elbers *et al.*, 2003). Agent i solves:

$$\max_{\{c_{it},k_{it+1}\}} E_0 \sum_{t=0}^{\infty} u(c_{it})/(1+\rho)^t \tag{2}$$

subject to

$$\begin{aligned} k_{i,t+1} &= y_{it} + (1-\delta)k_{it} - c_{it} \\ y_{it} &= s_{it}a_{it}f(k_{it}) \\ \text{for } t &= 0,1,2,.. \text{ and } k_{i0}, s_{i0} \text{ given} \end{aligned}$$

where c denotes consumption, y income, u the instantaneous utility function, s a shock, a an agent-specific measure of total factor productivity (ability), ρ the discount rate, and δ the depreciation rate. The distribution of shocks is stationary. The function u is strictly concave and we assume that $\rho > 0$.

Unlike in the original Ramsey model, the agent is exposed to risk: $af(k)$ is affected by an idiosyncratic shock s with unitary mean $Es = 1$. When the agent decides on c_t and k_{t+1} both k_t and the realization s_{it} are known. Future shocks are, of course, unknown but the agent does know the distributions of these shocks.

In this section we ignore shocks to focus on heterogeneity in terms of productivity. Initially agents differ only in terms of a (hence k_{i0} is the same for all i) but these initial differences will, of course, be reflected in different accumulation paths (k_{it}). Since income satisfies $y = af(k)$ growth in income will be affected by changes in the distribution of ability both directly (through changes in a) and through the induced changes in k. In the steady state the capital stock is given by the familiar equilibrium condition:

$$a_i f'(k_i^*) = \rho + \delta.$$

This implicitly defines k_i^* as an increasing function of a_i. Suppressing the subscript i the slope of this function is given by:

$$\frac{dk^*}{da} = \frac{-f'}{af''} > 0.$$

Income in the steady state is given by $y^* = af(k^*(a))$. Hence

$$\frac{dy^*}{da} = f + af'\frac{dk^*}{da} = f - (f')^2/f'' > 0$$

and

$$\frac{d^2y^*}{da^2} = [f' - \frac{2(f'')^2 f' - (f')^2 f'''}{(f'')^2}]\frac{dk^*}{da}.$$

It follows that y^* is strictly convex in a iff

$$f'(f'')^2 > 2f'(f'')^2 - (f')^2 f'''$$

or iff

$$f' f''' > (f'')^2, \tag{3}$$

with the inequality reversed for strict concavity. An increase in inequality[5] (in terms of a mean preserving spread of a_i) will therefore increase growth, measured as the change from y_0 to $\lim_{t\to\infty} y_t = \sum_i y_i^*$, if and only if condition (3) is satisfied.

Proposition 2 *An increase in inequality in the form of a mean preserving spread of the distribution of a_{i0} increases (reduces) growth (measured as the change from y_0 to the limit value of y_t) iff $f'f''' > (f'')^2$ ($f'f''' < (f'')^2$).*

Hence the effect on growth of heterogeneity in terms of productivity depends on the curvature of the production function. It is useful to consider some special cases. In the case of a quadratic production function $f''' = 0$ so that inequality reduces growth. Conversely, in the Cobb-Douglas case $f(k) = k^\alpha$, $0 < \alpha < 1$, condition (4) is satisfied so that the more unequal the distribution of skills the higher the aggregate income level in the steady state and hence the higher the (transient) growth rate.[6]

Finally, consider income inequality as a result of idiosyncratic shocks. Risk affects growth in two ways. Exposure to risk will affect an agent's policy function φ. In addition to this *ex ante* effect there is an *ex post* effect: actual shocks affect growth (Elbers *et al.*, 2003). The *ex post* effect introduces heterogeneity if shocks are idiosyncratic. The sign of this effect

[5] Note that the issue of how the spread affects aggregate income does not arise here since income is linear in a.

[6] It is important to note that Proposition 2 is derived for the steady state. If we relaxed the assumption that agents cannot trade with each other and assumed instead a perfect capital market within a closed economy our conclusion would be unaffected: the interest rate r would be equal to ρ.

	impact on income growth	
	short run	long run
source of initial income inequality:		
1. inequality in initial assets		
$\varphi'(k)/f'(k)$ non-increasing	-	0
rich economy and $\lvert\varphi''(k^*)\rvert$ small	+	0
2. inequality in productivity		
$f'f''' > (f'')^2$	+/-	+
$f'f''' < (f'')^2$	+/-	-
3. idiosyncratic shocks	+/-	+/-

Table 1: Transitional Growth under Increasing Inequality

is ambiguous. (Elbers *et al.* using a very similar Ramsey model found a strongly negative effect.)

Table 1 summarizes our results thus far. It may be noted that income inequality can have either a positive or a negative effect on growth and that the effect may persist or wear off. The outcome depends not only on characteristics of the economy such as the curvature of the production function or the initial level of wealth ($\overline{k}_0$) but also, and more interestingly, on the cause of the initial income inequality. Two economies which are identical except that the initial inequality in the distribution of income reflects productivity differences in the one case and inequality in the distribution of assets in the other case, will behave very differently. This suggests that the empirical work on the relation between inequality and growth may suffer from specification error.

3 Simulations

In this section we use a version of the Ramsey model for simulation experiments. We assume a discount rate of 8%, a Cobb-Douglas production function $f(k) = 2.5k^{0.7}$ and $\delta = 1$. In the deterministic case the steady state

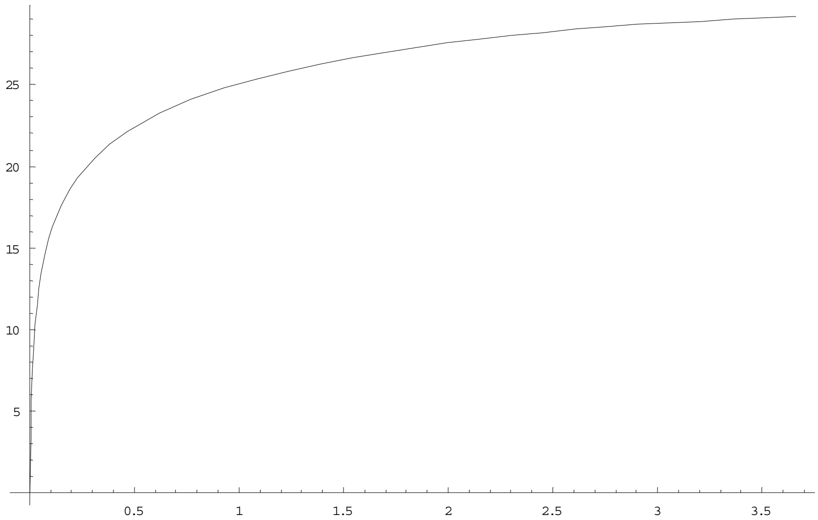

Figure 1: Utility Function for Simulation Experiments

value of k equals 5. We choose a policy function

$$\begin{aligned}\varphi(k) &= \gamma_1 f(k) \text{ for } k \leq k^A \\ &= 1 + 0.8k \text{ for } k^A \leq k \leq k^B \\ &= \gamma_2 f(k) + \gamma_3 \text{ otherwise}\end{aligned}$$

where $k^A = 2.91666$, $k^B = 6$, $\gamma_1 = 0.630257$, $\gamma_2 = 0.782523$, $\gamma_3 = -1.05714$. For these parameter values φ is continuously differentiable. The corresponding utility function can be obtained by integrating the Euler equation. This function is shown in Figure 1.

We first consider the effect of inequality of distribution of assets (k_{0i}). Initially we analyse this for a very poor economy with $y_0 = f(k_0)$ where $k_0 = 0.1$ (very far below the steady state value of 5). We compare the accumulation path for the case of full equality and the case where half the agents start from $f(k_{0i}) = 1.4y_0$ and the other half from $f(k_{0i}) = 0.6y_0$. Hence agents in this second economy differ in terms of their starting position k_{0i} but aggregate income y_0 is the same in the two economies. For both economies we calculate aggregate income y_t. In Figure 2 we show the development of y_t^U/y_t^E (aggregate income in the economy with inequality relative to that in the economy with full equality) over time. By construction there is no effect at $t = 0$. Thereafter the effect of inequality is initially negative and it disappears in the limit. Note that if growth is measured as the change

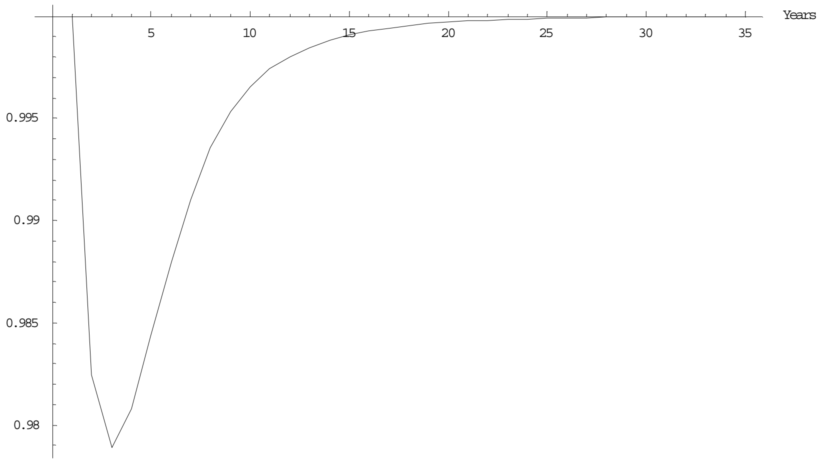

Figure 2: The Effect of Inequality in the Distribution of Assets on Aggregate Income (Poor Economy)

income ($y_t/y_0 - 1$) over a finite period then the effect of inequality on growth is negative.[7]

Figure 3 shows the same experiment but now for a much richer economy: $k_0 = 3.5$. In the case of inequality half the agents start from $f(k_{0i}) = 1.2y_0$ and the other half from $f(k_{0i}) = 0.8y_0$. As the Figure shows the effect of inequality is in this case positive.

These results show that the effect of inequality on growth need not be monotonic. This has an important implication for empirical work on the effect of inequality on growth. If initial inequality is used as a regressor in a growth regression it should be interacted with initial income.

Now consider income inequality arising from heterogeneity in terms of productivity. In the case of equality the production function is $f(k) = 2.5k^{0.7}$ for all agents while in the case of inequality $f(k) = 1.1 * 2.5k^{0.7}$ for half the agents and $f(k) = 0.9 * 2.5k^{0.7}$ for the other half. In all cases $k_0 = 0.5$. Figure 4 compares these two economies. Recall from the previous section that we could show (but only for the steady state) that the effect of inequality was positive. The Figure shows that the effect is again non-monotonic: it is positive for very poor economies and also near the steady

[7] This is growth in the sense of transitional dynamics; there is no growth in the steady state.

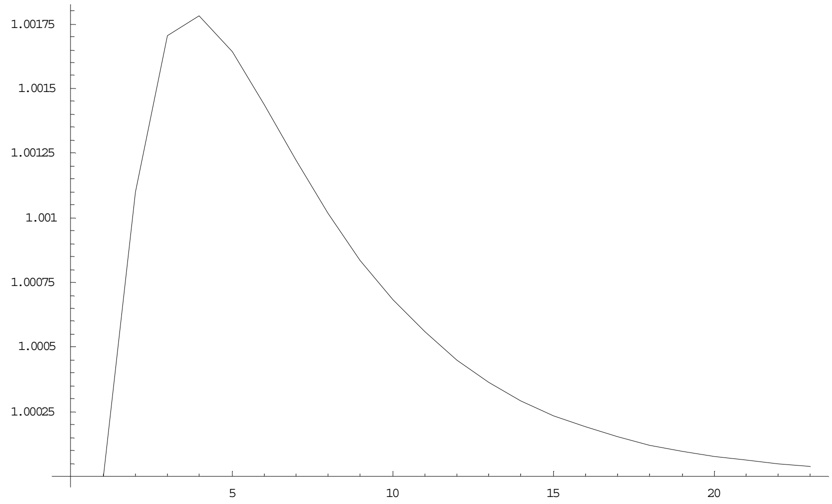

Figure 3: The Effect of Inequality in the Distribution of Assets on Aggregate Income (Rich Economy)

state, but negative for some intermediate values. As in the case of asset inequality the implication in growth regressions initial inequality should be interacted with income.

Finally, we consider inequality resulting from shocks. In this final simulation experiment income is given by $y = sf(k) = s2.5k^{0.7}$ where s takes the values 0.5 and 1.5 with equal probability. We simulate 10,000 paths and calculate the mean value of k_t. This mean is shown in Figure 5. Note that the long run mean is 3.5, well below the deterministic steady state value of 5. Hence the mean effect of shocks on growth is in this case strongly negative.

4 Conclusion

The relation between income inequality and economic growth has received enormous attention, particularly in the empirical literature. This literature addresses (at least implicitly) the question whether the effect of (initial) income inequality on growth is positive or negative. In this paper we argue that this question cannot be investigated in this form. Rather than asking how income inequality affects growth we should investigate what exogenous determinants drive changes in both inequality and growth. In this paper we have addressed that question for three exogenous determinants:

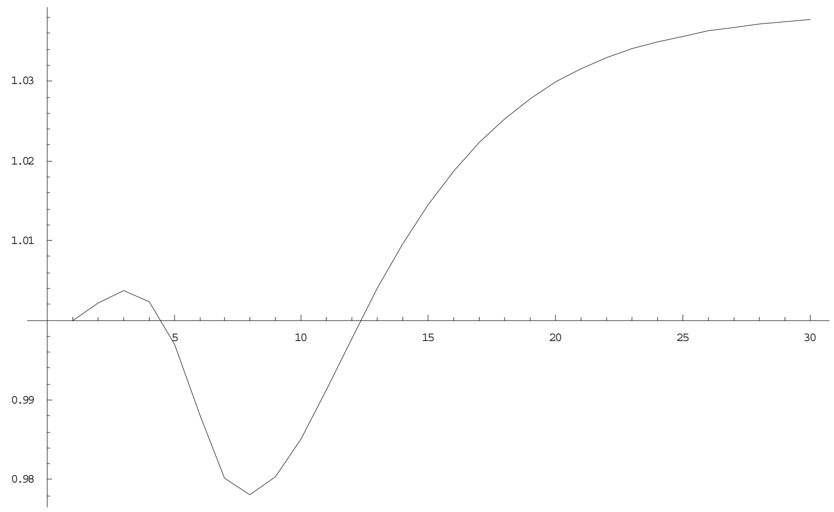

Figure 4: The Effect of TFP Inequality on Aggregate Income

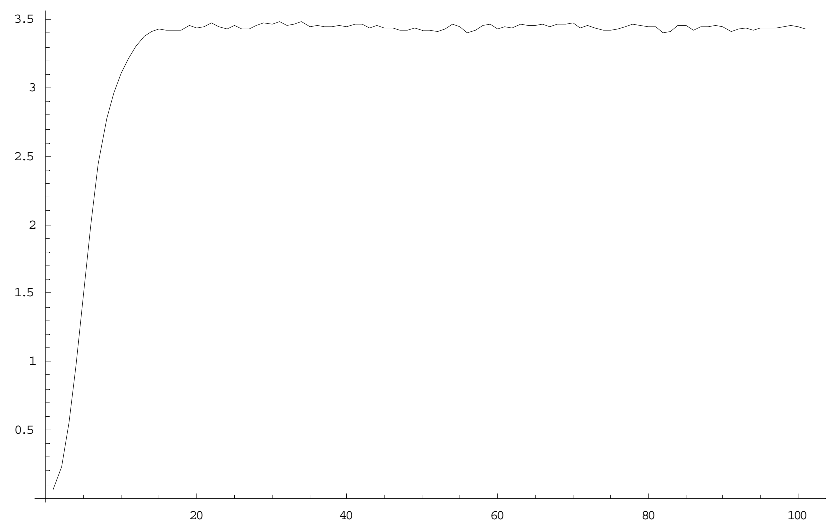

Figure 5: Capital Stock (Mean Value over 10,000 Simulation Paths) under Shocks

heterogeneity across agents in terms of (initial) asset ownership; in terms of (permanent) differences in productivity; and in terms of shocks.

We have used a very simple Ramsey model and rule out many of the mechanisms stressed in the literature, e.g. in political economy models (see Aghion *et al.*, 1999). Nevertheless, "anything goes": we find that the three underlying sources of inequality lead to very different changes in mean income over time. In particular, the *same* change in initial income inequality (a variable often used as regressor in cross-country growth regressions) can be associated with either an increase or a reduction in subsequent growth, and that the change in growth can be either transient or persistent.

Some of these results were derived analytically. In particular we showed that the short-run effect of asset inequality can be positive for a concave policy function (contrary to the result of Banerjee and Duflo, 2003) and that the long-run effect of productivity differences can be either positive or negative, depending on the production technology. This "anything can happen" conclusion was supported by our simulation results. Moreover, the effect of inequality can be non-monotonic in income and it depends on the underlying source of inequality.

Our results suggest that the growth regressions evidence on the effect of inequality on growth may be misleading since these regressions do not distinguish the different sources of income inequality. This can be problematic. For example, if income inequality in a very poor economy reflects both asset inequality (which is likely to have a negative effect on growth) and productivity differences (which can have a positive effect) then the effect of asset redistribution on income growth is likely to be underestimated.

What are the implications of our results for empirical work on inequality and growth? At a minimum it is advisable to control for the non-monotonicity in income by interacting income inequality measures (such as the Gini coefficient) with initial income levels. Ideally, rather than using a measure of income inequality one should include measures for the various sources of income inequality (such as inequality of the distribution of land ownership or measures of within-country TFP heterogeneity) as separate regressors

The literature is notoriously ambiguous on the relationship between inequality and growth. Our analysis suggests that this reflects substantial cross-country heterogeneity in terms of the underlying sources of inequality.

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