# Value Maximization As An *Ex-Post* Consistent Firm Objective When Markets are Incomplete

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**Abstract**

In competitive economies with private firm ownership, incomplete markets, and firm shareholders changing over time, several firm objectives have been proposed. Some are useful to understand efficiency of equilibria, and others are explicitly consistent with majority shareholder control or collective choice rules, but it is not always clear if versions of each type are consistent with versions of the other type. This paper shows that *ex-post*, value maximizing rules, (including those proposed by Drèze (1974), and Grossman and Hart (1979),) are consistent with shareholder preferences in such economies; that is, along the equilibrium path, in every period and state of the world, every coalition of a firm's shareholders in that period and state approves a value maximizing production plan. This result applies to cases when shareholders within a firm and across firms can form coalitions, and when stock trading can be ex-dividend or cum-dividend, and with a combination of both. This result does not resolve the problem of inefficiency of stock market equilibria, or that of *ex ante* disagreement among shareholders. It can help understand when firm objectives with some desirable properties are consistent with a particular version of shareholder control, and it provides a stability criterion (in terms of robustness to shareholder coalitions) for organizing productive resources in such economies.

# 1 Introduction

Several firm objectives have been proposed when competitive markets are incomplete. For some examples, see Radner (1972), Drèze (1974), Ekern and Wilson (1974), Radner (1974), Grossman and Hart (1979), DeMarzo (1993), Kelsey and Milne (1996), and Bonnisseau and Lachiri (2003). Some of these objectives are useful to understand the efficiency of equilibria when markets are incomplete; see, for example, Diamond (1967), Stiglitz (1982), Drèze (1985), and Geanakoplos, Magill, Quinzii, and Drèze (1990). Others are useful to understand firm objectives consistent with a process of majority shareholder control or some collective choice rule; see, for example, DeMarzo (1993), Kelsey and Milne (1996), Cres and Tvede (2001), and Tvede and Cres (2005).

Formal connections between these two types of objectives have not been explored in much detail. For example, it is unclear if objectives such as those by Drèze (1974), or Grossman and Hart (1979) are consistent with some form of shareholder control, or if shareholder control such as that proposed by DeMarzo (1993) is consistent with a Drèze-type rule. Such connections can help understand when firm objectives with desirable efficiency properties are consistent with a decentralized process of shareholder control.

Using a particular process of *ex post* shareholder control, this paper provides a justification for the consistency of value maximization (including versions proposed by Drèze and Grossman and Hart) with shareholder control. It is shown that *ex post*, value maximization is consistent with shareholder preferences when markets are incomplete and shareholders are changing over time, in the sense that along the equilibrium path, in every time period and state of the world, it is individually rational for each shareholder to approve a value-maximizing production plan. More generally, using ideas from Core theory, it is shown that along the equilibrium path, every coalition of a firm's shareholders in that period and state approves a value-maximizing production plan.[1] Intuitively, if shareholders are separate from managerial control, and present shareholders do not know the identity of future shareholders and cannot credibly affect future plans, then it is in all shareholders' best interest that firm managers maximize firm value.[2]

For some types of economies, such as private ownership economies with anonymous stock market trade over time (so changing shareholders over time), and with ownership separate from management, a credible evaluation and change of production plans, and of management, is more natural *ex post*, and not necessarily *ex ante*. For example, with anonymous stock market trade over time, firm shareholders cannot be assumed to know identities of other shareholders, *ex ante*. At most, shareholders in a particular period can be assumed, *ex post*, to know identities of other shareholders in that period, perhaps in a shareholder meeting for that period. Moreover, firm shareholders in a particular period can affect firm decisions in that period, (by voting to remove management, if necessary,) and they can block a production plan inconsistent with their preferences even if such a plan is preferred by different shareholders in a different period. Therefore, shareholders in a particular period cannot credibly assume that a firm manager is able to implement a future production plan consistent with existing shareholder preferences. Shareholders can, at most, credibly affect

[1] In particular, shareholders in every time period and state unanimously approve such a plan.

[2] The technical intuition for this result is similar to that for the result for Arrow-Debreu economies; a value-maximizing production plan provides a maximal extension of every consumer-shareholder's budget set in each period and state in which a consumer-shareholder can affect production choices, and therefore, if a consumer-shareholder is optimizing over such a budget set, another production plan cannot improve upon this outcome.

the choice of a production plan at a node at which they are firm shareholders. Consequently, for a particular production plan to satisfy a consistent firm objective, such a plan should survive shareholder blocking in every period (more generally, every node). Such a process is formalized below.[3]

It is noteworthy that this paper does not consider the problem of efficiency of equilibrium allocations when markets are incomplete. As is well-known, when markets are incomplete, competitive equilibria and stock market equilibria might fail to be Pareto efficient (Hart (1975), Stiglitz (1982)). Moreover, with particular definitions of constrained efficiency, competitive equilibria can be constrained Pareto efficient (Diamond (1967), Grossman (1977)), can fail to be constrained Pareto efficient (Dierker, Dierker, and Grodal (2002)), and can generically fail to be constrained Pareto efficient (Geanakoplos and Polemarchakis (1986), Geanakoplos, Magill, Quinzii, and Drèze (1990)).

Moreover, this paper does not eliminate the problem of *ex-ante* disagreement among firm shareholders in equilibrium. The analysis here can be viewed as providing an *ex-post* stability criterion (in terms of robustness to shareholder coalitions) for organizing productive resources in decentralized economies with incomplete markets and sequential trade. It remains important to explore avenues for improving efficiency of equilibrium outcomes and improving risk-sharing in incomplete markets when consumer equilibrium rates of substitution are non-collinear. Selection of particular versions of value maximization can be based on their efficiency properties, and that avenue remains open for study in economies considered

[3]Notice that an *ex post* evaluation may not be appropriate in all cases. For example, if a collection of entrepreneurs know each other and run a firm themselves, (perhaps as a sole proprietorship or private partnership,) then anonymity of future shareholders may not be maintained, and an *ex ante* evaluation is a reasonable assumption.

here. For some results regarding efficiency effects of financial innovation and taxation, see Elul (1995), Cass and Citanna (1998), and Turner (2003), among others.

Furthermore, this paper focuses on competitive markets. Analysis of firm objectives under oligopolistic markets is presented in Dierker and Grodal (1996), and in Dierker and Grodal (1999).

The paper proceeds as follows. The next section shows the consistency of value maximization as a firm objective when markets are incomplete. This is shown formally in the context of Radner-GEI economies, and with natural assumptions about shareholder preferences and shareholder control. The section after that presents some extensions of this basic result.

## 2 Value Maximization as a Consistent Firm Objective

The seminal paper by Radner (1972) formalizes a model of an economy in which markets are incomplete in the sense that in each period and state of the world, all commodities for delivery in that period and state of the world can be traded, but some commodites for future delivery cannot be traded. In such an economy there are finitely many consumers, firms, and commodities. Firms are privately owned by consumer-shareholders, and shares of firms are traded in stock markets. In each period and state, a consumer-shareholder can trade only in commodities and firm shares for which markets exist in that period and state. Consumers use firm shares to move income among different time periods, and among different states of the world to finance a consumption plan that they desire most. Radner's model of an economy, and some of its extensions are also referred to as a model of general equilibrium with incomplete markets (GEI model). Such an economy is formalized below.[4]

A Radner-GEI economy is formalized as follows. Trade takes place over time, and there is uncertainty. Economic activity takes place over a finite number of elementary time periods, indexed $t = 1, \ldots, T$, and a finite number of states of the world, indexed $s = 1, \ldots, S$. Each state $s$ is a particular history of the environment from period 1 through period $T$. The events observable in period $t$ are given by a partition $\mathcal{S}_t$ of $\{1, \ldots, S\}$. To reflect dependence of actions in period $t$ on events observable in that period, a function on $\{1, \ldots, S\}$ is said to be $\mathcal{S}_t$-measurable if it is constant on each event $E_t \in \mathcal{S}_t$. To reflect the additional availability of information as time goes on, the sequence of partitions, $\mathcal{S} = (\mathcal{S}_t)_{t=1}^T$, is taken to be nondecreasing in fineness, and it is termed an information structure. For $t = 1, \ldots, T$, let $\mathcal{R}_t = \left\{ (\xi_t(s))_{s=1}^S \in \Re^S \,|\xi_t(\cdot) \text{ is } \mathcal{S}_t\text{-measurable} \right\}$ be the subspace of $\Re^S$ consisting of vectors which are $\mathcal{S}_t$-measurable and let $\mathcal{R} = \mathop{\times}\limits_{t=1}^{T} \mathcal{R}_t$.

There are a finite number of privately-owned firms, indexed $j = 1, \ldots, J$, a finite number of consumer-shareholders, indexed $i = 1, \ldots, I$, and in each period $t$, state $s$, a finite number of commodities, indexed $\ell = 1, \ldots, L$.

The production technology for firm $j$ is $Y^j \subset \mathop{\times}\limits_{t=1}^{T} \mathcal{R}_t^L$. A production plan for firm $j$ is an element $y^j = (y_t^j)_{t=1}^T \in Y^j$, and it entails an input-output (or netput) of $y_t^j(s)_\ell$ units of good $\ell$ in period $t$, state $s$.[5] A production profile is a collection $y = (y^j)_{j=1}^J$ where for each $j$, $y^j$ is a production plan for firm $j$. When convenient, $y_t^j(s)$ denotes a production plan for firm $j$ in period $t$, state $s$, and $(y_t^j(s))_{j=1}^J$ denotes a production profile in period $t$, state $s$. As usual, each $Y^j$ is closed, convex, includes the possibility of inaction, $0 \in Y^j$, and

[4]Additional details about such economies are given in Grossman and Hart (1979), and in Magill and Quinzii (1996).

[5]As usual, negative entries in a production plan are inputs, and positive entries are outputs.

satisfies free disposal, $Y^j \supset \left( -\mathop{\times}\limits_{t=1}^{T} (\mathcal{R}_t^L)_+ \right)$. Moreover, as usual, there is no aggregate free lunch, $\left(\sum_j Y^j\right) \cap \left( \mathop{\times}\limits_{t=1}^{T} (\mathcal{R}_t^L)_+ \right) = \{0\}$, and there is aggregate irreversibility of production, $\left(\sum_j Y^j\right) \cap \left(-\sum_j Y^j\right) = \{0\}$.

Each firm is privately owned by consumer-shareholders. Firm ownership provides a shareholder a claim on firm profits proportional to her firm shareholding, and firm ownership can be traded in stock markets. Therefore, both a consumer-shareholder's shareholding, and a firm's shareholders can change over time. The shareholding space in period $t \leq T-1$ is $\Theta_t = \left\{ \theta_t \in \mathcal{R}_t^J \mid 0 \leq \theta_t \leq 1 \right\}$, that in period $t = T$ is $\Theta_T = \{0\} \subset \mathcal{R}_T^J$, and the shareholding space is $\Theta = \mathop{\times}\limits_{t=1}^{T} \Theta_t$. A shareholding plan for consumer-shareholder $i$ is denoted $\theta^i$, it is an element of $\Theta$, and it entails holding a $\theta_t^{i,j}(s)$ share of firm $j$ in period $t$, state $s$. The shareholding profile space in period $t \leq T-1$ is $\Theta_t^{\Delta} = \left\{ \theta_t \in \Theta_t^I \,\middle|\, \text{for every } j, s, \ \sum_i \theta_t^{i,j}(s) = 1 \right\}$, that in period $t = T$ is $\Theta_T^{\Delta} = \Theta_T^I$, and the shareholding profile space is $\Theta^{\Delta} = \mathop{\times}\limits_{t=1}^{T} \Theta_t^{\Delta}$. A shareholding profile is an element of the shareholding profile space, it is denoted $\theta = (\theta^i)_{i=1}^I$, where $\theta_t^{i,j}(s)$ is the shareholding of consumer-shareholder $i$ in firm $j$ in period $t$, state $s$.

The consumption space in period $t$ is $X_t = (\mathcal{R}_t^L)_+$, and the consumption space is $X = \mathop{\times}\limits_{t=1}^{T} X_t$. A consumption plan for consumer-shareholder $i$ is an element $x^i = (x_t^i)_{t=1}^T \in X$, and it entails consumption of $x_t^i(s)_\ell$ units of good $\ell$ in period $t$, state $s$. A consumption profile is a collection $x = (x^i)_{i=1}^I$ where for each $i$, $x^i$ is a consumption plan for consumer-shareholder $i$. When convenient, $x_t^i(s)$ denotes a consumption plan for consumer-shareholder $i$ in period $t$, state $s$, and $(x_t^i(s))_{i=1}^I$ denotes a consumption profile in period $t$, state $s$. Each consumer-shareholder $i$ has a preference relation ( $\succeq^i \subset X \times X$) that is complete, reflexive, transitive, convex, continuous, and strongly monotone ($x > \acute{x} \Rightarrow x \succ^i \acute{x}$).[6] Each consumer-shareholder

[6]The notions of indifference ($\sim^i$) and strict preference ($\succ^i$) are the usual ones.

$i$ has a consumption endowment $w^i = (w^i_t)^T_{t=1} \in X$, and a shareholding endowment $\theta^i_0 \in \Theta_1$, such that the collection of consumption endowments satisfies $\inf_i w^i \gg 0$, and the collection of shareholding endowments satisfies $\theta_0 = (\theta^i_0)^I_{i=1} \in \Theta^\Delta_1$.

Prices of commodities and firms are defined as follows. The price space in period $t \leq T-1$ is $\Delta_t = \left\{(p_t, q_t) \in (\mathcal{R}^{L+J}_t)_+ \,\middle|\, \text{for every } s,\ \sum_\ell p_t(s)_\ell + \sum_j q_t(s)_j = 1\right\}$, and that in period $t = T$ is $\Delta_T = \{(p_T, 0) \in (\mathcal{R}^{L+J}_T)_+ \mid \text{for every } s,\ \sum_\ell p_T(s)_\ell = 1\}$. The price space is $\Delta = \underset{t=1}{\overset{T}{\times}} \Delta_t$. A price system is an element $(p,q) \in \Delta$, with $p_t(s)_\ell$ the price of a unit of good $\ell$ in period $t$, state $s$, and $q_t(s)_j$ the price of firm $j$ in period $t$, state $s$.

Periodic discount rates for firms are denoted as follows. For firm $j$, the discount rate in period $t = 1$ is $\delta^j_1 \equiv 1 \in \mathcal{R}_1$, and in period $t \geq 2$, it is $\delta^j_t \in \mathcal{R}_t$, where for every $s$, $0 < \delta^j_t(s) \leq 1$. A discount rate system for firm $j$ is $\delta^j = (\delta^j_t)^T_{t=1}$, and a discount rate system is $\delta = (\delta^j)^J_{j=1}$. Notice that this formulation includes criteria by Drèze, and by Grossman and Hart. That is, in a two-period model, the Drèze criterion posits a discount rate system for firm $j$ in period 2 as the weighted average of inter-temporal rates of substitution of shareholders in period 2, and in a multi-period model, the Grossman-Hart criterion posits a periodic discount rate in each period as the weighted average of appropriate inter-temporal rates of substitution of shareholders in period 1.

Consider a price system $(p,q)$, discount rate system $\delta$, and a production plan $y^j$ for firm $j$. For every $t$, let $p_t \Box y^j_t \equiv (p_t(s) y^j_t(s))^S_{s=1}$. The $\delta^j$-value of firm $j$ in period $t$ is $\delta^j_t p_t \Box y^j_t \equiv \sum_s \delta^j_t(s) p_t(s) y^j_t(s)$, and the $\delta^j$-value of firm $j$ is $\delta^j p \Box y^j \equiv \sum^T_{t=1} \delta^j_t p_t \Box y^j_t$. A production plan $y^j$ maximizes $\delta^j$-value over $Y^j$ if $\delta^j p \Box y^j = \max_{\acute{y}^j \in Y^j} \delta^j p \Box \acute{y}^j$.

For a price system $(p,q)$, a production profile $y = (y^j)^J_{j=1}$, and a consumption-shareholding plan $(x^i, \theta^i)$ for consumer-shareholder $i$, the disposable income of $i$ in period $t$, state $s$ is

$$W^i(p,q,y)_t(s) \;=\; p_t(s)w^i_t(s) \;+\; q_t(s)\theta^i_{t-1}(s) \;+\sum_j \theta^{i,j}_{t-1}(s)p_t(s)y^j_t(s).$$

A consumption-shareholding plan $(x^i, \theta^i)$ is $(p,q,y)$-affordable for consumer-shareholder $i$, if in every period $t$, state $s$, $p_t(s)x^i_t(s)+q_t(s)\theta^i_t(s) \;\leq\; W^i(p,q,y)_t(s)$.[7] A consumer-shareholder's budget set consists of all consumption and shareholding plans that are affordable, and her demand set consists of those plans in the budget set that are optimal with respect to her preference relation. Formally, for a price system $(p,q)$, and a production profile $y = (y^j)^J_{j=1}$, the budget set for consumer-shareholder $i$ is

$$B^i(p,q,y) = \left\{(x^i,\theta^i) \in X \times \Theta \,\middle|\, (x^i,\theta^i) \text{ is } (p,q,y)\text{-affordable } \right\},$$

and the demand set for consumer $i$ is

$$D^i(p,q,y) = \left\{(x^i,\theta^i) \in B^i(p,q,y) \,\middle|\, (\acute{x}^i,\acute{\theta}^i) \in B^i(p,q,y) \Rightarrow x^i \succeq^i \acute{x}^i \right\}.$$

A ***Radner-GEI economy*** is a collection

$$\left\{\mathcal{S}, (Y^j)^J_{j=1}, (\succeq^i, w^i, \theta^i_0)^I_{i=1}\right\},$$

where $\mathcal{S} = (\mathcal{S}_t)^T_{t=1}$ is an information structure, $Y^j$ is a production technology for firm $j$, and $(\succeq^i, w^i, \theta^i_0)$ is the preference relation and consumption-shareholding endowment of consumer-shareholder $i$. A ***Radner equilibrium*** is a collection $(p,q;\delta, y = (y^j)^J_{j=1}; (x^i,\theta^i)^I_{i=1})$, where

[7] Notice that as in Radner (1972), and in Drèze (1974) and Grossman and Hart (1979), a $\theta^{i,j}_{t-1}(s)$ shareholding by consumer-shareholder $i$ in firm $j$ in period $t-1$, state $s$ provides her with a $\theta^{i,j}_{t-1}(s)$ share of profits of firm $j$ in period $t$, state $s$, (and she continues to receive a share of profits in future periods to the extent that she continues shareholding in this firm.) This implies, of course, that profits in each period are distributed to shareholders as dividends. Moreover, this is consistent with ex-dividend stock trading. The results here remain true with cum-dividend stock trading, as explained in the next section.

$(p, q)$ is a price system, $\delta$ is a discount rate system,[8] for every $j$, $y^j$ maximizes $\delta^j$-value over $Y^j$, for every $i$, $(x^i, \theta^i) \in D^i(p, q, y)$, $\sum_{i=1}^I x^i = \sum_{i=1}^I w^i + \sum_{j=1}^J y^j$, and $\theta = (\theta^i)_{i=1}^I \in \Theta^\Delta$. In other words, a Radner equilibrium is a collection of prices, one set for each period and state when markets for trade are open, a discount rate system, a production profile, and a consumption and shareholding profile, such that firms are maximizing value, consumers are maximizing preferences, and in every period and state, all consumption markets are clearing, and all firm shares are held by consumer-shareholders.[9] When convenient, a consumption and production profile $((x^i)_{i=1}^I, (y^j)_{j=1}^J)$ is an allocation if $\sum_{i=1}^I x^i = \sum_{i=1}^I w^i + \sum_{j=1}^J y^j$, and a consumption and production profile $((x_t^i(s))_{i=1}^I, (y_t^j(s))_{j=1}^J)$ in period $t$, state $s$ is an allocation in period $t$, state $s$, if $\sum_{i=1}^I x_t^i(s) = \sum_{i=1}^I w_t^i(s) + \sum_{j=1}^J y_t^j(s)$. With this terminology, a consumption and production profile $((x^i)_{i=1}^I, (y^j)_{j=1}^J)$ in a Radner equilibrium is an allocation, and consequently, it is an allocation in every period $t$, state $s$.

The main result in this paper is that in economies with private firm ownership, when markets are incomplete, and firm shareholders can change over time, value maximization is, *ex-post*, a firm objective consistent with shareholder preferences. The formal analysis

[8] To maintain generality, further restrictions on the discount rate system are not specified. This permits an application of the results developed here to different specifications of the discount rate system; for example, as prescribed by Drèze (1974), Grossman and Hart (1979), and Bonnisseau and Lachiri (2003). It is useful to keep in mind that when discount rate systems depend on shareholdings and inter-temporal rates of substitutions, then to prove existence of equilibrium, some restrictions on the discount rate system, such as continuity in shareholdings and marginal utilities, would have to be specified.

[9] This paper does not focus on existence of equilibrium. For existence of equilibrium with bounds on short sales, see Radner (1972), and Grossman and Hart (1979), and for generic existence of equilibrium without bounds on short sales, see Duffie and Shafer (1985), Duffie and Shafer (1986), and Magill and Quinzii (1988). For a result on non-existence with unbounded short sales, see Momi (2001).

includes postulating shareholder control over firm decisions, deriving shareholder preferences for production plans from consumer preferences, and showing that along the equilibrium path in a Radner equilibrium, no coalition of shareholders wants to deviate from a value-maximizing production plan. Details are as follows.

To see that a value-maximizing production plan can be aligned with shareholder preferences, it is important, as a first step, to understand the process by which shareholders might be able to exercise control over firm decisions, and to formalize an idea of when a production plan might be preferred by a coalition of consumer-shareholders. With anonymous trading in stock markets, shareholders in a given period cannot be assumed to know the identities of other shareholders, except possibly when they interact with other shareholders in a shareholder meeting. In this sense, exercise of shareholder control in a given period (and state) is limited, at most, to shareholders in that period (and state). Moreover, as future firm shareholders can affect future firm choices, existing shareholders can be credibly assumed only to affect the firm's current period production plan. Therefore, to formalize this concept of shareholder control, it is assumed that (1) there is a shareholder meeting in every period (and state), and at this meeting, firm shareholders in that period (and state) can form voting coalitions and vote on a firm's future production plan, (2) a shareholder in a given period (and state) does not know if another consumer will be a shareholder in a different period (or state), (3) a single future shareholder cannot affect a firm's future production plan, and consequently, (4) shareholders in a given period can credibly affect only the firm's current period production plan.

Shareholder control is exercised as follows. For expositional clarity, suppose, in this paragraph, that there is no uncertainty. As mentioned above, for a period $\hat{t} = 1, \ldots, T$, a

$\theta^{i,j}_{\hat{t}-1}$ shareholding by consumer-shareholder $i$ in firm $j$ provides her with a $\theta^{i,j}_{\hat{t}-1}$ share of firm $j$ profits in period $\hat{t}$. This is consistent with ex-dividend stock trading; that is, a seller of firm share in period $\hat{t}-1$ keeps her share of firm profits for period $\hat{t}-1$, and a buyer of firm share in period $\hat{t}-1$ receives a share of firm profits for period $\hat{t}$. To aid interpretation, this can be viewed as stock market trade at the end of a period. At the beginning of each period $\hat{t}$, there is a shareholder meeting of existing shareholders; that is, shareholders who bought firm shares at the end of period $\hat{t}-1$. These shareholders are termed period $\hat{t}$ shareholders. At this meeting, managers of firm $\hat{j}$ present period $\hat{t}$ shareholders the production results for period $\hat{t}-1$, and their current and future production plan, denoted $(y^{\hat{j}})^{T}_{t=\hat{t}}$. Intuitively, a subset of period $\hat{t}$ shareholders prefers a feasible production plan $\acute{y}^{\hat{j}}_{\hat{t}}$ to production plan $y^{\hat{j}}_{\hat{t}}$, if they can re-distribute among themselves their period $\hat{t}$ endowments and production plan $\acute{y}^{\hat{j}}_{\hat{t}}$ to get new consumption plans for period $\hat{t}$ so that when compared to the consumption plan each could have achieved with her period $\hat{t}$ endowment and her share of profits from production plan $y^{\hat{j}}_{\hat{t}}$, no shareholder in this subset is made worse off with her new consumption plan, some shareholder in this subset is made strictly better off, and shareholders not in this subset are unaffected.[10] For a subset of shareholders to approve production plan $y^{\hat{j}}_{\hat{t}}$, it must be that

[10] Shareholder preferences can be formulated by considering an additional budget-feasible constraint; that is, shareholder preferences for production plans can be further restricted to be appropriately affordable for each consumer-shareholder in a coalition. The result here is in the familiar tradition of coalition re-distributions that are feasible with coalition endowments without additional constraints regarding budget sets. Of course, in either case, a value-maximizing plan is appropriately affordable in equilibrium, and moreover, to the extent a budget constraint imposes additional restrictions on candidate plans that might dominate a value-maximizing plan, the result here can be viewed as a stronger result in that it holds with a potentially greater number of candidate production plans.

there is no production plan $\acute{y}_{\hat{t}}^{\hat{j}}$ that this subset of shareholders prefers to $y_{\hat{t}}^{\hat{j}}$. These ideas are formalized in the following concepts about the consumption preferences of a consumer-shareholder in a given period, and the production preferences of subsets of shareholders, and it is shown that when $y^j = (y_t^j)_{t=1}^T$ maximizes value over $Y^j$, every subset of shareholders in every period approves this plan.[11]

The idea of making a consumer-shareholder better off in a given period and state can be motivated by considering a consumer-shareholder $i$ with preference relation $\succeq^i$, and a consumption plan $x^i$, and naturally deriving a preference for consumption in a given period and state as follows. In a particular period $t$, state $s$, consumer-shareholder $i$ prefers a bundle of commodities $\acute{x}_t^i(s)$ to $x_t^i(s)$, if she prefers the consumption plan that is derived from $x^i$ by replacing $x_t^i(s)$ with $\acute{x}_t^i(s)$ to the consumption plan $x^i$. More formally, let $x^i$ be a consumption plan for consumer-shareholder $i$, and let $\succeq^i$ be the preference of consumer-shareholder $i$. If $\acute{x}_{\hat{t}}^i(\hat{s})$ is a consumption plan for consumer-shareholder $i$ in period $\hat{t}$, state $\hat{s}$, then ***consumer-shareholder*** $i$ ***prefers*** $\acute{x}_{\hat{t}}^i(\hat{s})$ ***to*** $x_{\hat{t}}^i(\hat{s})$ ***in period*** $\hat{t}$***, state*** $\hat{s}$, denoted $\acute{x}_{\hat{t}}^i(\hat{s}) \succeq_{\hat{t}}^i (\hat{s}) \; x_{\hat{t}}^i(\hat{s})$, if $\hat{x}^i \succeq^i x^i$, where $\hat{x}^i$ is defined as follows: $\hat{x}_t^i(s) = \acute{x}_t^i(s)$ if $t = \hat{t}$ and $s = \hat{s}$, and $\hat{x}_t^i(s) = x_t^i(s)$ otherwise, and ***consumer-shareholder*** $i$ ***strictly prefers*** $\acute{x}_{\hat{t}}^i(\hat{s})$ ***to*** $x_{\hat{t}}^i(\hat{s})$ ***in period*** $\hat{t}$***, state*** $\hat{s}$, denoted $\acute{x}_{\hat{t}}^i(\hat{s}) \succ_{\hat{t}}^i (\hat{s}) \; x_{\hat{t}}^i(\hat{s})$, if $\hat{x}^i \succ^i x^i$. Let $\succeq^i$ be the preference

[11]Notice that the results presented here do not depend on the particular timing of shareholder meetings and stock trade mentioned above. For stock market trade as in Radner (1972), the only interpretive requirement would be that a shareholder meeting is held at least an instant before stock trade, so that shareholders who are to receive profits in a period are the ones voting on firm plans for that period. Conditional on this requirement, shareholder meeting and stock trade can be held at any time during a period. In particular, this applies for an elementary time period, which is also viewed as a node. Similar interpretations naturally hold for cum-dividend stock trading.

of consumer-shareholder $i$, and let $x^i$ be a consumption plan for consumer-shareholder $i$. The preference $\succeq_t^i(s)$ is weakly monotone in period $t$, state $s$, if for every consumption plan $\acute{x}^i$, $\acute{x}_t^i(s) \gg x_t^i(s) \Rightarrow \acute{x}_t^i(s) \succ_t^i (s) \; x_t^i(s)$. Notice that if $\succeq^i$ is strongly monotone, then $\succeq_t^i(s)$ is weakly monotone in period $t$, state $s$, for every period $t$, state $s$. Moreover, if $\succeq_t^i(s)$ is weakly monotone in period $t$, state $s$, for every period $t$, state $s$, then $\succeq^i$ is weakly monotone.

Production preferences of subsets of shareholders are formalized as follows. Let $y = (y^j)_{j=1}^J$ be a production profile, and $(x^i, \theta^i)_{i=1}^I$ be a consumption and shareholding profile. A consumer $i$ is a shareholder of firm $j$ in period $t$, state $s$ if $\theta_{t-1}^{i,j}(s) > 0$. For each firm $j$, period $t$, state $s$, let $S_t^j(s) = \left\{ i \left| \theta_{t-1}^{i,j}(s) > 0 \right. \right\}$ be the collection of shareholders of firm $j$ in period $t$, state $s$. A ***firm*** $j$ ***coalition in period*** $t$, ***state*** $s$, denoted $C_t^j(s)$, is a non-empty subset of $S_t^j(s)$. Let $C_{\hat{t}}^{\hat{j}}(\hat{s})$ be a firm $\hat{j}$ coalition in period $\hat{t}$, state $\hat{s}$, let $\acute{y}_{\hat{t}}^{\hat{j}}(\hat{s})$ be a production plan for firm $\hat{j}$ in period $\hat{t}$, state $\hat{s}$, and define $C_{\hat{t}}^{\hat{j}}(\hat{s})$ ***prefers*** $\acute{y}_{\hat{t}}^{\hat{j}}(\hat{s})$ ***to*** $y_{\hat{t}}^{\hat{j}}(\hat{s})$, if

- the production plan $\acute{y}^{\hat{j}}$ formed by setting $\acute{y}_t^{\hat{j}}(s) = \acute{y}_{\hat{t}}^{\hat{j}}(\hat{s})$ if $t = \hat{t}$, and $s = \hat{s}$, and $\acute{y}_t^{\hat{j}}(s) = y_t^{\hat{j}}(s)$ otherwise, is feasible, (that is, $\acute{y}^{\hat{j}} \in Y^{\hat{j}}$,)[12] and

- for every consumer-shareholder $i \in C_{\hat{t}}^{\hat{j}}(\hat{s})$, there is a consumption plan $\acute{x}_{\hat{t}}^i(\hat{s})$ in period

[12]Notice that feasibility of production plans restricts the types of alternative productions plans that can be considered in a period and state. For good produced periodically, say, t-shirts produced in a given period, (or any number of other goods, such as boxes of cereal, pairs of jeans, sandwiches, pizzas, paper towels, DVD players, air conditioners, tires, cars, computers, trucks, periodic labor services, bricks, light bulbs, and so on,) this has a natural interpretation of using a different production plan for a given period and state. (This can be viewed as a type of separability in production over time.) But there may be some types of plans that cannot be changed much over one period, say, producing large ships, or planes, or large rockets. In such cases, once a production plan has been commissioned, there might not be very many alternative production plans that are feasible for a given period.

$\hat{t}$, state $\hat{s}$ such that the production profile $(\acute{y}^j_{\hat{t}}(\hat{s}))_{j=1}^J$ in period $\hat{t}$, state $\hat{s}$ formed by setting $\acute{y}^j_{\hat{t}}(\hat{s}) = \acute{y}^{\hat{j}}_{\hat{t}}(\hat{s})$ if $j = \hat{j}$, and $\acute{y}^j_{\hat{t}}(\hat{s}) = y^j_{\hat{t}}(\hat{s})$ otherwise, and the consumption profile $(\acute{x}^i_{\hat{t}}(\hat{s}))_{i=1}^I$ in period $\hat{t}$, state $\hat{s}$ formed by setting $\acute{x}^i_{\hat{t}}(\hat{s}) = \acute{x}^i_{\hat{t}}(\hat{s})$ if $i \in C^{\hat{j}}_{\hat{t}}(\hat{s})$, and $\acute{x}^i_{\hat{t}}(\hat{s}) = x^i_{\hat{t}}(\hat{s})$ otherwise, together form an allocation in period $\hat{t}$, state $\hat{s}$, (that is, $\sum_{i=1}^I \acute{x}^i_{\hat{t}}(\hat{s}) = \sum_{i=1}^I w^i_{\hat{t}}(\hat{s}) + \sum_{j=1}^J \acute{y}^j_{\hat{t}}(\hat{s})$,) and

- for every consumer-shareholder $i \in C^{\hat{j}}_{\hat{t}}(\hat{s})$, $\acute{x}^i_{\hat{t}}(\hat{s}) \succeq^i_{\hat{t}} (\hat{s})\ x^i_{\hat{t}}(\hat{s})$, and for some consumer-shareholder $i \in C^{\hat{j}}_{\hat{t}}(\hat{s})$, $\acute{x}^i_{\hat{t}}(\hat{s}) \succ^i_{\hat{t}} (\hat{s})\ x^i_{\hat{t}}(\hat{s})$.

The first condition ensures that a candidate for a preferred production plan is feasible, the second condition ensures that if the initial production and consumption profiles form an allocation in a given period and state, then the candidate production and consumption profiles remain an allocation in that period and state, and therefore, the candidate consumption and production profiles form a re-distribution that affects at most the shareholders of a particular firm that are in a coalition, and the third condition ensures that nobody in the coalition is made worse off, and someone is made strictly better off. For a firm $j$ coalition in period $t$, state $s$, $C^j_t(s)$, a production plan $y^j$ is $C^j_t(s)$***-proof*** if there is no $\acute{y}^j_t(s)$ such that $C^j_t(s)$-prefers $\acute{y}^j_t(s)$ to $y^j_t(s)$. A production plan $y^j$ is ***coalition proof*** if for every period $t$, state $s$, and for every firm $j$ coalition in period $t$, state $s$, $C^j_t(s)$, $y^j$ is $C^j_t(s)$-proof.[13]

The corollary following the lemma and theorem below formalizes the role of value maximization as a firm objective consistent with shareholder preferences when markets are incomplete and firm shareholders are changing over time.

**Lemma.** *Let* $(p, q)$ *be a price system,* $y = (y^j)_{j=1}^J$ *be a production profile,* $(x^i, \theta^i) \in D^i(p, q, y)$*,*

[13]Notice that definitions regarding coalitions include shareholders of a particular firm. This can be weakened to allow for coalitions of shareholders across firms, as explained in the next section.

*and $\acute{x}^i$ be another consumption plan for $i$. In period $\hat{t}$, state $\hat{s}$,*

$$\text{if} \quad \acute{x}^i_{\hat{t}}(\hat{s}) \ \succeq^i_{\hat{t}}(\hat{s}) \ x^i_{\hat{t}}(\hat{s}), \quad \text{then} \quad p_{\hat{t}}(\hat{s})\acute{x}^i_{\hat{t}}(\hat{s}) + q_{\hat{t}}(\hat{s})\theta^i_{\hat{t}}(\hat{s}) \ \geq \ W^i(p,q,y)_{\hat{t}}(\hat{s}), \quad \text{and}$$

$$\text{if} \quad \acute{x}^i_{\hat{t}}(\hat{s}) \ \succ^i_{\hat{t}}(\hat{s}) \ x^i_{\hat{t}}(\hat{s}), \quad \text{then} \quad p_{\hat{t}}(\hat{s})\acute{x}^i_{\hat{t}}(\hat{s}) + q_{\hat{t}}(\hat{s})\theta^i_{\hat{t}}(\hat{s}) \ > \ W^i(p,q,y)_{\hat{t}}(\hat{s}).$$

**Proof**. To prove the first statement, suppose its hypothesis is true, and suppose that $p_{\hat{t}}(\hat{s})\acute{x}^i_{\hat{t}}(\hat{s}) + q_{\hat{t}}(\hat{s})\theta^i_{\hat{t}}(\hat{s}) \ < \ W^i(p,q,y)_{\hat{t}}(\hat{s})$. Then there is $\hat{x}^i_{\hat{t}}(\hat{s}) \gg \acute{x}^i_{\hat{t}}(\hat{s})$ such that $p_{\hat{t}}(\hat{s})\hat{x}^i_{\hat{t}}(\hat{s}) + q_{\hat{t}}(\hat{s})\theta^i_{\hat{t}}(\hat{s}) \ \leq \ W^i(p,q,y)_{\hat{t}}(\hat{s})$. Therefore, if the consumption plan $\hat{x}^i$ is defined as $\hat{x}^i_t(s) = \hat{x}^i_{\hat{t}}(\hat{s})$ if $t = \hat{t}$ and $s = \hat{s}$, and $\hat{x}^i_t(s) = x^i_t(s)$ otherwise, then $(\hat{x}^i, \theta^i) \in B^i(p,q,y)$. Moreover, if the consumption plan $\tilde{x}^i$ is defined as $\tilde{x}^i_t(s) = \acute{x}^i_t(s)$ if $t = \hat{t}$ and $s = \hat{s}$, and $\tilde{x}^i_t(s) = x^i_t(s)$ otherwise, then $\hat{x}^i_{\hat{t}}(\hat{s}) \gg \acute{x}^i_{\hat{t}}(\hat{s})$ implies $\hat{x}^i \succ^i \tilde{x}^i$, and $\acute{x}^i_{\hat{t}}(\hat{s}) \ \succeq^i_t(s) \ x^i_{\hat{t}}(\hat{s})$ implies $\tilde{x}^i \succeq^i x^i$. Therefore, $\hat{x}^i \succ^i x^i$, contradicting the optimality of $x^i$.

To prove the second statement, suppose its hypothesis is true, and suppose that $p_{\hat{t}}(\hat{s})\acute{x}^i_{\hat{t}}(\hat{s}) + q_{\hat{t}}(\hat{s})\theta^i_{\hat{t}}(\hat{s}) \ \leq \ W^i(p,q,y)_{\hat{t}}(\hat{s})$. In this case, if the consumption plan $\tilde{x}^i$ is defined as $\tilde{x}^i_t(s) = \acute{x}^i_t(s)$ if $t = \hat{t}$ and $s = \hat{s}$, and $\tilde{x}^i_t(s) = x^i_t(s)$ otherwise, then $(\tilde{x}^i, \theta^i) \in B^i(p,q,y)$. Moreover, $\acute{x}^i_{\hat{t}}(\hat{s}) \ \succ^i_{\hat{t}}(\hat{s}) \ x^i_{\hat{t}}(\hat{s})$ implies $\tilde{x}^i \succ^i x^i$, contradicting the optimality of $x^i$. ∎

**Theorem.** *Let $(p,q)$ be a price system, $\delta$ be a discount rate system, $y = (y^j)_{j=1}^J$ be a production profile, $(x^i, \theta^i)_{i=1}^I$ be a consumption and shareholding profile, and for every $i$, $(x^i, \theta^i) \in D^i(p,q,y)$.*

*For every firm $j$, if $y^j$ maximizes $\delta^j$-value over $Y^j$, then $y^j$ is coalition proof.*

**Proof**. Fix $\hat{j}$, suppose $y^{\hat{j}}$ maximizes $\delta^{\hat{j}}$-value over $Y^{\hat{j}}$, and suppose $y^{\hat{j}}$ is not coalition proof. Let $\hat{t}$ be a period, $\hat{s}$ be a state, $C^{\hat{j}}_{\hat{t}}(\hat{s})$ be a firm $\hat{j}$ coalition in period $\hat{t}$, state $\hat{s}$, and $\acute{y}^{\hat{j}}_{\hat{t}}(\hat{s})$ be a production plan for firm $\hat{j}$ in period $\hat{t}$, state $\hat{s}$ such that $C^{\hat{j}}_{\hat{t}}(\hat{s})$ prefers $\acute{y}^{\hat{j}}_{\hat{t}}(\hat{s})$ to $y^{\hat{j}}_{\hat{t}}(\hat{s})$. For every consumer-shareholder $i \in C^{\hat{j}}_{\hat{t}}(\hat{s})$, let $\acute{x}^i_{\hat{t}}(\hat{s})$ be a consumption plan in period $\hat{t}$, state $\hat{s}$ such that the production profile $(\acute{y}^j_{\hat{t}}(\hat{s}))_{j=1}^J$ in period $\hat{t}$, state $\hat{s}$ formed by setting

$\acute{y}_{\hat{t}}^{j}(\hat{s}) = \acute{y}_{\hat{t}}^{\hat{j}}(\hat{s})$ if $j = \hat{j}$, and $\acute{y}_{\hat{t}}^{j}(\hat{s}) = y_{\hat{t}}^{j}(\hat{s})$ otherwise, and the consumption profile $(\acute{x}_{\hat{t}}^{i}(\hat{s}))_{i=1}^{I}$ in period $\hat{t}$, state $\hat{s}$ formed by setting $\acute{x}_{\hat{t}}^{i}(\hat{s}) = \acute{x}_{\hat{t}}^{i}(\hat{s})$ if $i \in C_{\hat{t}}^{\hat{j}}(\hat{s})$, and $\acute{x}_{\hat{t}}^{i}(\hat{s}) = x_{\hat{t}}^{i}(\hat{s})$ otherwise, together form an allocation in period $\hat{t}$, state $\hat{s}$, $(\sum_{i=1}^{I} \acute{x}_{\hat{t}}^{i}(\hat{s}) = \sum_{i=1}^{I} w_{\hat{t}}^{i}(\hat{s}) + \sum_{j=1}^{J} \acute{y}_{\hat{t}}^{j}(\hat{s}),)$ and for every consumer-shareholder $i \in C_{\hat{t}}^{\hat{j}}(\hat{s})$, $\acute{x}_{\hat{t}}^{i}(\hat{s}) \succeq_{\hat{t}}^{i} (\hat{s})\ x_{\hat{t}}^{i}(\hat{s})$, and for some consumer-shareholder $i \in C_{\hat{t}}^{\hat{j}}(\hat{s})$, $\acute{x}_{\hat{t}}^{i}(\hat{s}) \succ_{\hat{t}}^{i} (\hat{s})\ x_{\hat{t}}^{i}(\hat{s})$. Then, by the lemma above, for every consumer-shareholder $i \in C_{\hat{t}}^{\hat{j}}(\hat{s})$, $p_{\hat{t}}(\hat{s})\acute{x}_{\hat{t}}^{i}(\hat{s}) + q_{\hat{t}}(\hat{s})\theta_{\hat{t}}^{i}(\hat{s}) \geq W^{i}(p,q,y)_{\hat{t}}(\hat{s})$, and for some consumer-shareholder $i \in C_{\hat{t}}^{\hat{j}}(\hat{s})$, $p_{\hat{t}}(\hat{s})\acute{x}_{\hat{t}}^{i}(\hat{s}) + q_{\hat{t}}(\hat{s})\theta_{\hat{t}}^{i}(\hat{s}) > W^{i}(p,q,y)_{\hat{t}}(\hat{s})$. Therefore,

$$
\begin{aligned}
&\textstyle\sum_i p_{\hat{t}}(\hat{s})w_{\hat{t}}^{i}(\hat{s}) + \sum_j p_{\hat{t}}(\hat{s})\acute{y}_{\hat{t}}^{j}(\hat{s}) + \sum_i q_{\hat{t}}(\hat{s})\theta_{\hat{t}}^{i}(\hat{s}) \\
&= \textstyle\sum_i \left(p_{\hat{t}}(\hat{s})\acute{x}_{\hat{t}}^{i}(\hat{s}) + q_{\hat{t}}(\hat{s})\theta_{\hat{t}}^{i}(\hat{s})\right) \\
&> \textstyle\sum_i W^{i}(p,q,y)_{\hat{t}}(\hat{s}) \\
&= \textstyle\sum_i \Big(p_{\hat{t}}(\hat{s})w_{\hat{t}}^{i}(\hat{s}) + q_{\hat{t}}(\hat{s})\theta_{\hat{t}-1}^{i}(\hat{s}) + \sum_j \theta_{\hat{t}-1}^{i,j}(\hat{s})p_{\hat{t}}(\hat{s})y_{\hat{t}}^{j}(\hat{s})\Big) \\
&= \textstyle\sum_i p_{\hat{t}}(\hat{s})w_{\hat{t}}^{i}(\hat{s}) + \sum_i q_{\hat{t}}(\hat{s})\theta_{\hat{t}-1}^{i}(\hat{s}) + \sum_j p_{\hat{t}}(\hat{s})y_{\hat{t}}^{j}(\hat{s}),
\end{aligned}
$$

whence $p_{\hat{t}}(\hat{s})\acute{y}_{\hat{t}}^{\hat{j}}(\hat{s}) > p_{\hat{t}}(\hat{s})y_{\hat{t}}^{\hat{j}}(\hat{s})$, and consequently, $\delta^{\hat{j}}p\Box\acute{y}^{\hat{j}} > \delta^{\hat{j}}p\Box y^{\hat{j}}$, contradicting the hypothesis that $y^{\hat{j}}$ maximizes $\delta^{\hat{j}}$-value over $Y^{\hat{j}}$. ∎

**Corollary.** *Let* $(p,q;\delta,y = (y^{j})_{j=1}^{J};(x^{i},\theta^{i})_{i=1}^{I})$ *be a Radner equilibrium. For every* $j$*,* $y^{j}$ *is coalition proof, if, and only if,* $y^{j}$ *maximizes* $\delta^{j}$*-value over* $Y^{j}$*.*

The *if* part follows from the theorem, and the *only if* part is trivial by definition of a Radner equilibrium. Moreover, notice that the proofs above have a flavor similar to that of proving that an equilibrium allocation in an Arrow-Debreu economy lies in the Core. In the case here, an *ex post* evaluation is a node-wise evaluation, and as markets at a node are (relatively) complete, ideas similar to those used in Walrasian economies are applied at a node.[14] Therefore, the result here depends on the justification of a node-wise evaluation,

[14]Indeed, the proof works with a version of node-wise value maximization.

which in this paper, has been motivated by a particular process of shareholder control. To the extent that process does not apply, the results here would not hold. In particular, this proof would not work, if shareholders could form coalitions over time, for example, if shareholders could commit to remaining shareholders over time. In this sense, the result here does not resolve the problem of *ex ante* disagreements among shareholders, or inefficiency of stock market equilibria.

The corollary above formalizes value maximization as a firm objective consistent with shareholder preferences when markets are incomplete and firm shareholders are changing over time. The consistency of this objective can be viewed from the perspective of sequential shareholder evaluation, as follows. When presented with a $\delta^j$-value-maximizing production plan for firm $j$, $y^j = (y_t^j)_{t=1}^T$, shareholders given by the shareholding endowment unanimously approve $y_1^j(s)$ as a production plan in period 1, state $s$, and when presented with $(y_t^j)_{t=2}^T$, shareholders in period 2, state $s$ unanimously approve $y_2^j(s)$ as a production plan in period 2, state $s$, and continuing in a similar manner, when presented with $(y_t^j)_{t=\hat{t}}^T$, shareholders in period $\hat{t}$, state $s$ unanimously approve $y_{\hat{t}}^j(s)$ as a production plan in period $\hat{t}$, state $s$. This is true regardless of which state of the world is realized. In this sense, a value-maximizing production plan survives sequential shareholder evaluation along the equilibrium path.

It is noteworthy that the result presented above is independent of particular voting requirements; that is, this result is independent of what share of the firm is required of a coalition before it can affect firm decisions, and consequently, this result applies even when shareholders with a small share of the firm (for example, members of a Board) can affect firm decisions.

# 3 Some Extensions

The consistency of value maximization is shown above for the case where shareholders of a particular firm in a particular period (and state) can form coalitions, and for the case where shareholders have a claim on next period's (same state's) production plan. Both these conditions can be relaxed, so that value maximization remains a consistent firm objective when shareholders of different firms in a particular period (and state) can form a coalition across firms, and when shareholders in a firm in a particular period (and state) have a claim to that period's (and state's) production plan, as follows.

Value maximization remains a consistent firm objective when shareholders can form coalitions across firms. This can be seen by an appropriate modification of the definition of a coalition, and a re-formulation of production preferences of coalitions of shareholders across firms, as follows.

Let $y = (y^j)_{j=1}^J$ be a production profile, and $(x^i, \theta^i)_{i=1}^I$ be a consumption and shareholding profile, and for each firm $j$, period $t$, state $s$, let $S_t^j(s)$ be the collection of shareholders of firm $j$ in period $t$, state $s$, and $C_t^j(s)$ be a (possibly empty) firm $j$ coalition in period $t$, state $s$. A ***coalition in period*** $t$***, state*** $s$ is a non-empty union of firm coalitions in period $t$, state $s$, denoted $C_t(s) = \bigcup_j C_t^j(s)$. Let $C_{\hat{t}}(\hat{s})$ be a coalition in period $\hat{t}$, state $\hat{s}$, let $(\acute{y}_{\hat{t}}^j(\hat{s}) : j \text{ satisfies } C_{\hat{t}}^j(\hat{s}) \neq \emptyset)$ be a collection of production plans in period $\hat{t}$, state $\hat{s}$, and define $C_{\hat{t}}(\hat{s})$ ***prefers*** $(\acute{y}_{\hat{t}}^j(\hat{s}) : j \text{ satisfies } C_{\hat{t}}^j(\hat{s}) \neq \emptyset)$ ***to*** $(y_{\hat{t}}^j(\hat{s}) : j \text{ satisfies } C_{\hat{t}}^j(\hat{s}) \neq \emptyset)$, if the production profile $(\acute{y}^j : j \text{ satisfies } C_{\hat{t}}^j(\hat{s}) \neq \emptyset)$ formed by setting for every $j$ such that $C_{\hat{t}}^j(\hat{s}) \neq \emptyset$, $\acute{y}_t^j(s) = \acute{y}_{\hat{t}}^j(\hat{s})$ if $t = \hat{t}$, and $s = \hat{s}$, and $\acute{y}_t^j(s) = y_t^j(s)$ otherwise, is feasible, (that is, for every $j$ such that $C_{\hat{t}}^j(\hat{s}) \neq \emptyset$, $\acute{y}^j \in Y^j$,) and for every consumer-shareholder $i \in C_{\hat{t}}(\hat{s})$, there is a consumption plan $\acute{x}_{\hat{t}}^i(\hat{s})$ in period $\hat{t}$, state $\hat{s}$ such that the production profile $(\acute{y}_{\hat{t}}^j(\hat{s}))_{j=1}^J$

in period $\hat{t}$, state $\hat{s}$ formed by setting $\acute{y}^j_{\hat{t}}(\hat{s}) = \acute{y}^{\hat{j}}_{\hat{t}}(\hat{s})$ if $j$ satisfies $C^j_{\hat{t}}(\hat{s}) \neq \emptyset$, and $\acute{y}^j_{\hat{t}}(\hat{s}) = y^j_{\hat{t}}(\hat{s})$ otherwise, and the consumption profile $(\acute{x}^i_{\hat{t}}(\hat{s}))^I_{i=1}$ in period $\hat{t}$, state $\hat{s}$ formed by setting $\acute{x}^i_{\hat{t}}(\hat{s}) = \acute{x}^i_{\hat{t}}(\hat{s})$ if $i \in C_{\hat{t}}(\hat{s})$, and $\acute{x}^i_{\hat{t}}(\hat{s}) = x^i_{\hat{t}}(\hat{s})$ otherwise, together form an allocation in period $\hat{t}$, state $\hat{s}$, (that is, $\sum_{i=1}^I \acute{x}^i_{\hat{t}}(\hat{s}) = \sum_{i=1}^I w^i_{\hat{t}}(\hat{s}) + \sum_{j=1}^J \acute{y}^j_{\hat{t}}(\hat{s})$,) and for every consumer-shareholder $i \in C_{\hat{t}}(\hat{s})$, $\acute{x}^i_{\hat{t}}(\hat{s}) \succeq^i_{\hat{t}} (\hat{s})\ x^i_{\hat{t}}(\hat{s})$, and for some consumer-shareholder $i \in C_{\hat{t}}(\hat{s})$, $\acute{x}^i_{\hat{t}}(\hat{s}) \succ^i_{\hat{t}} (\hat{s})\ x^i_{\hat{t}}(\hat{s})$. The first condition ensures that a candidate for a preferred collection of production plans is feasible for every firm that has a shareholder in the coalition, the second condition ensures that if the initial production and consumption profiles form an allocation in a given period and state, then the candidate production and consumption profiles remain an allocation in that period and state, and therefore, the candidate consumption and production profiles form a re-distribution that affects at most shareholders in a coalition, and the third condition ensures that nobody in the coalition is made worse off, and someone is made strictly better off. For a coalition in period $t$, state $s$, $C_t(s)$, a production profile $(y^j)^J_{j=1}$ is $C_t(s)$***-proof*** if there is no $(\acute{y}^j_{t+1}(s) : j$ satisfies $C^j_t(s) \neq \emptyset)$ such that $C_t(s)$ prefers $(\acute{y}^j_{t+1}(s) :$ $j$ satisfies $C^j_t(s) \neq \emptyset)$ to $(y^j_{t+1}(s) : j$ satisfies $C^j_t(s) \neq \emptyset)$. A production profile $(y^j)^J_{j=1}$ is ***coalition proof*** if for every period $t$, state $s$, and for every coalition in period $t$, state $s$, $C_t(s)$, $(y^j)^J_{j=1}$ is $C_t(s)$-proof.

With this notation and terminology, the statement and proof of the lemma presented above remain true. The first line in the statement of the theorem remains the same, and the second line is modified to read, "*If* $(y^j)^J_{j=1}$ *maximizes* $\delta$*-value over* $(Y^j)^J_{j=1}$*, then* $(y^j)^J_{j=1}$ *is coalition proof.*" Here $(y^j)^J_{j=1}$ maximizes $\delta$-value over $(Y^j)^J_{j=1}$ means that for every $j$, $y^j$ maximizes $\delta^j$-value over $Y^j$. The proof of the theorem needs obvious modifications; one useful modification is to notice that in the last line, after "*whence*" the words "*for some* $\hat{j}$

*such that* $C_t^{\hat{j}}(s) \neq \emptyset$" should be added. The second line of the corollary is then modified to state, "$(y^j)_{j=1}^J$ *is coalition proof, if, and only if,* $(y^j)_{j=1}^J$ *maximizes* $\delta$*-value over* $(Y^j)_{j=1}^J$."

Value maximization remains a consistent firm objective with cum-dividend stock trading. This can be seen by an appropriate modification of a consumer-shareholder's budget set, and the definition of coalition proof, as follows.

A consumer-shareholder's budget set is modified as follows. For a price system $(p, q)$, a production profile $y = (y^j)_{j=1}^J$, and a consumption-shareholding plan $(x^i, \theta^i)$ for consumer-shareholder $i$, the disposable income of $i$ in period $t$, state $s$ is

$$W^i(p, q, y)_t(s) \;=\; p_t(s)w_t^i(s) \;+\; q_t(s)\theta_{t-1}^i(s) \;+ \sum_j \theta_t^{i,j}(s)p_t(s)y_t^j(s).$$

In this setting, a $\theta_t^{i,j}(s)$ shareholding by consumer-shareholder $i$ in firm $j$ in period $t$, state $s$ provides her with a $\theta_t^{i,j}(s)$ share of profits of firm $j$ in the same period $t$, state $s$, (and she continues to receive a share of profits in future periods to the extent that she continues shareholding in this firm.)[15] A consumption-shareholding plan$(x^i, \theta^i)$ is $(p, q, y)$-affordable for consumer-shareholder $i$, if in every period $t$, state $s$, $p_t(s)x_t^i(s) + q_t(s)\theta_t^i(s) \;\leq\; W^i(p, q, y)_t(s)$. The budget set and demand set are now defined as in the previous section, as are the notion of a Radner-GEI economy, and a Radner equilibrium.

Shareholder control is formalized with the same assumptions as in the previous section, except that for the formal analysis, shareholders in a particular period who can credibly affect the firm's production plan in that period are the after-trade shareholders. (It is these

[15]Notice that for $t = T$, $\theta_T^{i,j}(s)$ is identically 0, so a coalition (a non-empty subset of shareholders with positive shareholding) is not defined. There are some obvious alternatives in this case; as $q_T(s)$ is also identically zero, one alternative is to restrict the definition of coalition proof to $t \leq T-1$; another alternative is to restrict a firm's planning horizon to $T-1$, so that for every $j$, $y_T^j(s)$ is identically zero.

shareholders who receive firm profits in a given period.) To aid interpretation, this can be viewed as stock-market trade at any moment before a shareholder meeting. Shareholder control is exercised in a manner similar to that described in the previous section. Production preferences of subsets of shareholders are formalized as above, but with the modification that the collection of shareholders of firm $j$ in period $t$, state $s$ is given by $S_t^j(s) = \left\{ i \left| \theta_t^{i,j}(s) > 0 \right. \right\}$. The remaining notation and terminology for coalition proof is unaffected. With this new notation and terminology, the statement and proof of the lemma remain true. The statement of the theorem remains the same, and it is proved with the obvious modification in the time index for shareholdings. The corollary is unaffected.

It is easy to appropriately extend the result to the case where short sales in firm shares are permitted, and when assets in addition to firm shares are available. Moreover, the extensions mentioned above can also be combined to prove, for example, the consistency of value maximization with coalitions of shareholders across firms and with cum-dividend stock trading.

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