Economic Growth and the Financial Economics of Capital Accumulation under Shifting Technological Change[1]

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This paper develops a simple and practical framework for characterizing (long-run) economic growth and fluid capital accumulation under shifting technological change. The framework specifies a technological change that depends on exogenous and endogenous factors as well as the interaction between the two sets of factors, and therefore allows for the evaluation of the (a) output/capital ratio via capital stock (balance sheet) adjustments and intensities, (b) value added stream of income via changes in cash flows (income statements) at some rates of change constrained to be compatible with (c) financial economics of the feasible accumulation of capital investment (sources and uses of funds statement). The model is simple - it is only a single-equation reduced form of a transcendental production function - yet one which can be estimated either as (1) a single equation, (2) three single equations, or (3) a system of three seemingly unrelated equations.

Keywords: economic growth and fluid capital accumulation, economic growth under shifting technological change,

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[1]This paper extends Appendix 2 of a research proposal for evaluating dynamic gains from NAFTA which the authors wrote back in 1994/1995 when they were, respectively, student and professor in the Mineral Economics Program at the University of Arizona. Amavilah is currently an adjunct instructor of economic principles at Glendale College, and the one-man principal of REEPS, which consults and publishes occasional papers on resources, technology, and the engineering economy of small open economies. Dr. Richard Newcomb, formerly Professor of Mineral Economics in the Department of Mining and Geological Engineering, is also Professor Emeritus of Agricultural and Resource Economics, both at the University of Arizona. Dr. Newcomb generated the ideas and concepts presented in the third section of paper. However, Amavilah alone bears all sunk and opportunity costs of the views expressed in this draft version of the paper.

Solow-Swan residual, Arrow-learning effect, Lucas-Romer endogenous growth models, neo-Solow and post-Solow growth models, new growth models, new new growth models

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This paper develops a transcendental production function (trans-function) model capable of depicting the interactions of resource formation and technical change in determining economic growth. The model pays attention to the interactive mechanisms not only between accumulation (saving) and investment capital of the conventional growth theory, but also on the growth of the stock of *fluid resources* that enhance productive sectors and labor skills according to the more recent theory of endogenous technical change. The presentation proceeds as follows: Section 1 provides a brief background to the subject matter. Section 2 describes the general framework. Section 3 highlights the financial economics of capital accumulation under shifting technological change, and is followed by the fourth section that summarizes the comparative rates of growth and technical change. Section 5 makes short concluding remarks.

1. Background

In a Solow *originale* economy growth beyond investment and capital is determined by exogenous, neutral or labor-biased technical change (Solow, 1956, 1957).[2] Swan (1956, 2002) assumes the rates of growth of labor and technical change to be exogenous such that long-run economic growth depends on the growth of output-capital ratio. For Lewis (1954, 1965) economic growth, and hence economic development, is possible through the growth of surplus labor. Vintage and learning-by-doing models such as those by Solow (1960), Arrow (1962), and others modified Solow, Swan, and other early growth models in ways that lowered the Solow residual. However, these attempts were unsuccessful in providing a lasting relief from the concern that diminishing returns to net capital ultimately occur as the Harrod-Domar model predicted. The main reason for this is that labor in vintage and learning models became just another machine, albeit a smart one, in the sense that effective workers learn how to use certain tools efficiently but without those tools they are just workers. Barro and Sala-I-Martin (1990, 1991), Holtz-Eakin (1993), and Mankiw, Romer and Weil (1992), and Romer (1987) refocused the debate on the labor side in attempts to differentiate skilled labor (L_2) from Solow's "effective labor" (L) and the Arrow-learning effect. In all these efforts, however, the process of capital accumulation continued to depend on the saving rate, while L_2-formation went largely unexplained (see Tamura, 1991). Consequently, technological spill-overs suggested by these models tend to zero as nations converge to a steady state equilibrium growth.

Current growth theory continues to takes Solow-Swan as its starting point in stating the production function as

$$Y=Ae^{\delta t}X^{\rho}, \tag{1}$$

[2]We identify three Solows in this paper: *Solow originale* refers to Solow's two papers - the theoretical one that appeared in the *QJE* (1956) and its empirical companion published in the *Amer. Review of Economics and Statistics* (1957). These two papers focuses on the relationship between (the rates of growth of) technical change and output. *Neo-Solow* includes early vintage models such as Solow (1960) and learning models such as Arrow (1962) which generalized the aggregate production function. In *post-Solow* we include Solow (1997) and Lucas-Romer type of endogenous new growth models.

where Y is output produced, A is the level of technology evolving over the time index t at an exogenously-given rate of technical change (δ), X is a vector of resources determining Y, ρ is a vector of constant coefficients.[3] In (1) the scale of production $\sum_i \rho = 1$ so that capital increases with rising savings, but if the ratio of capital to labor is also rising, then the marginal productivity of capital must fall in the long run, because the rate of technical change increases only in response to exogenous changes in investments in knowledge and/or in equipment (De Long and Summers, 1991).

While, on one hand, neo-Solow growth models focus on the determinants of technical change, especially on the fact that the onset of diminishing returns to capital are postponed temporarily by policy induced externalities, on the other hand, post-Solow or neo-Schumpeter growth models consider technical change endogenous, and give explicit recognition to the importance of intermediate goods and services in the production process (Jones, 1995, Lucas, 1988, 1993, Grossman and Helpman, 1990, 1991, Romer, 1989, 1990, 1992, 1993, 1994a, 1994b). Still capital has remained the catch-all term vindicating Kaldor's early criticism that capital as a single number ignores its own fluidity (Kaldor, 1957, 1961, Kaldor and Mirrles, 1962, Kaldor, 1966, 1972, 1975).[4] Taylor (2000) and Newcomb (1976) argue for capital as a fluid pool of stocks consisting of sunk or fixed (exogenous) capital stock and fluid (endogenous) capital. Fixed capital affects real time production function, investment, and long-run growth only through depreciation. Moreover, the effect of depreciation is limited by the allowable schedule, so that fluid capital is essentially cumulative net investment plus allowable depreciation. As a basis for finance, surplus is augmented by new produced means of production (intermediates and planned inventories). Now, since depreciation and foreign-sourced intermediate inputs are themselves exogenous, and since domestic intermediate inputs depend on domestic investment, it follows that fluid capital is the upper limit of the investment function in the sense that long-run growth is subject to constant domestic investment (I) constraints and *in-situ* production capacity (Taylor, 2000).

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[3]While assuming Solow-Swan as a starting point is economical, it is unfortunate because both Solow and Swan sought to develop models that are neither form nor assumption-specific. As Solow put it: "When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect"(p.).

[4]What Nick Kaldor had in mind was that prior to Solow the conventional wisdom of dominant models, such as the Harrod-Domar model, was that long-run economic growth depended mainly on capital. So it was natural that in attacking Solow some critics pointed to model specification error (wrong functional form), while others saw the problem as mis-specification error (wrong variables included or correct variables excluded). The latter set of criticism has not gone away to-date; the former is not peculiar to Solow because it has been raised with respect to the restrictiveness of the original Cobb-Douglas production function (Douglas, 1948). To join the debate, the interested reader may want to review a recent retrospective and a reply on the "Cambridge capital controversies" by Cohen and Harcourt (2003a, 2003b) and the comments it provoked from Pasinetti (2003), Fisher (2003), Filipe and McCombie (2003), and Greenfield (2003). Maurice Scott (1991) offers another perspective on how to deal with the investment function in the production function framework, and Haavelmo (1960) and Lester Taylor (2002) give others.

A key difference between Solow and neo-Solow growth models on one hand, and post-Solow models on the other hand, is of both focus and perspective. Whereas the former stresses the evolution of technical change ($e^{\delta t}$), the latter emphasizes the determination of technological change ($Ae^{\delta t}$), the direction of its bias, and its implications for factor substitutability as well as factor complementarity as well. From the neo-Solow paradigm $Y = f(K, Ae^{\delta t} L, H)$ - Mankiw, Romer, and Weil; $Y = Af(e^{\delta t} K, L)$ - Arrow; and $Y = f(K,[(Ae^{\delta t} K)L], H)$ - Barro, and others. Grossman and Helpman's perspective is different from new Solow only to the degree (not minor degree, mind you!) it accounts for the direct effects on growth of intermediate inputs (Z) within a CES production formula, i.e., $Y = Ae^{\delta t} f(L, K, Z)$. Either way factor-biased new Solow output growth rate will be something like

$$\dot{Y} = (1-\alpha)\dot{A} + \alpha\dot{L} + \beta\dot{K} + \delta t \qquad (2a)$$
$$\dot{Y} = (1-\beta)\dot{A} + \alpha\dot{L} + \beta\dot{K} + \delta t \qquad (2b) \qquad (2)$$

where (2a) is labor-biased, (2b) is capital-biased, and the dot indicates the growth of the variable over which it stands. Thus, (2) shows that neo-Solow specifications reduce the magnitude of unexplained variations in output growth (or Solow residual); however, they do not say, or say little about, how and what brings about A-dot.

2. General Framework

In this section we extend (1) further one step at a time. First, we imposed a transcendental functional form and rewrite it as:

$$Y = Ae^{\delta t} X^{\rho} e^{\rho' X}. \qquad (3)$$

Second, we argue that aggregate output is function of exogenous (fixed) inputs (X_1) and endogenous (fluid) inputs (X_2), and rewrite (3) as

$$Y = Ae^{\delta t}(X_1)F(X_2)e^{F^*(X_1, X_2)}, \qquad (4)$$

where A(.) is Solow, F(.) is new growth, exp[F*(.)] depicts interactions between A(.) and F(.), for A' > 0, F' > 0 and $F^{*'} < 0$. As a result of these additional assumptions, *function A(.)F(.)exp(F*(.))* is not immutably form-specific. For instance, assuming F*(.) = 0, if A(.) = 1, (4) becomes a pure endogenous function, and if F(.) = 1, it is a pure Solow function. We take another wack at this later. For now we note that key among exogenous inputs (X_1) are: raw labor (L_1), fixed capital (K_1) less depreciation

(ΔK_1) as a part of *in-situ* capacity to produce (Φ_y) which is a function of capacity to invest (Φ_I). Exogenous inputs are of a Solow-type and they rise and run over time in accordance with Solow and so does the output they produce. Hence, exogenous inputs can be interpreted as representing Romer's "objects" or "classical" inputs. Endogenous variables (X_2) include fluid capital (K_2) plus depreciation allowance a la Hotelling (1925 or Darnell, 1990), and human capital (L_2); they carry the dynamic motion of growth and represent "ideas" or "reproducible" inputs, and change at their own endogenously-determined rates, in addition to the exogenously-determined Solow rate.[5] The presence (absence) of fluid capital formation guarantees a surplus (deficit) that reflects asset growth (decay) and net return (loss) on fluid capital in any time period. Unlike in conventional theory, here the endogenous portion of the production function is not necessarily subject to constant returns to scale *a priori*, but it allows for scale (dis)economies, and overall (weak) constancy of scale, depending on whether exogenous or endogenous variables dominate the determination of Y.

This unification of classical, neoclassical, and current endogenous growth theories is hardly new; but among its main advantages is that it allows us to introduce a single-equation reduced form representation that can be estimated either (a) as three single equations, (b) as a seemingly unrelated system of three equations, or (c) as a simple dynamic reduced form in which the relevant interactions are accounted for transcendentally, i.e.,

$$Y=Ae^{\delta t}L^{\alpha}K^{\beta}\exp(\alpha' L+\beta' K),$$
$$\alpha > 0,\ \beta > 0,\ \delta > 0;\ \alpha' < 0, \beta' < 0; \exp(.)=e^{F^*(.)}. \tag{5}$$

From (5) average labor productivity is

$$y=Y/L=Ae^{\delta t}k^{\beta}\exp(\alpha'+\beta' k). \tag{5'}$$

Now define total current capacity to produce as a product (multiple) of capacities to produce using exogenous and endogenous variables, i.e., $\Phi = A * F$. Next divide labor (L) into raw labor (L_1) and human capital (L_2), and split physical capital (K) into fixed capital (K_1) and fluid capital (K_2) such that

$$Y=\Phi L_1^{\alpha_1}L_2^{\alpha_2}K_1^{\beta_1}K_2^{\beta_2}\exp[\alpha_1' L_1+\alpha_2' L_2+\beta_1' K_1+\beta_2' K_2+\delta t] \tag{6}$$

In this case we can evaluate the output intensity of either raw labor (Y/L_1), human capital (Y/L_2), or aggregate labor (Y/L). For example, average labor productivity (Y/L) is

[5]As shown later note that from Taylor (2000) short-run capacity to produce is fixed, but in the long run it varies with changes in the capacity to investment, which is a function of the financial system.

$$y=\Phi \ell_i^{\alpha_i} k_i^{\beta_i} \exp[\alpha_i' \ell_i + \beta_i' k_i + \delta t], \tag{6'}$$

where $\ell_i = L_i/L$ and $k_i = K_i/L$. Close inspection shows that (6) encompasses Lucas as a special case, and suggests that (productivity) growth rate is determined by more than just the rates of investment in physical resources. While it is not radical to presume that labor grows at a rate approximately equal to the rate of population growth, the rate of the human capital part of labor surely cannot be assumed to equal that of population, for it depends on both population growth and physical capital (investment in schools, hospitals, factories, etc.).[6] In other words, learning, technology, surplus generation, and fluid capital formation are all critical determinants of economic growth. Recent efforts to modify Solow by making economic growth and technical progress both endogenous while preserving the convergence property of the original model are impressive in the sense that the productivity of effective labor can expand even in the absence of technological advances, and its growth rate is determined by the rates of investment and output (Lucas, 1988; Tamura, 1991; Romer, 1986; 1987, Barro and Sala-i-Martin, 1990; 1991; Mankiw, Romer and Weil, 1991). However, only in the newer versions of endogenous growth theories is it clear that not only are investment rate, labor, technological progress, depreciation, and human K-accumulation predictors of the productivity of effective workers, but also that the impacts of these determinants are disaggregated so as to permit technological change to be financed by the public, private, or both, which leads to no or slow convergent growth rates among economies (Lucas, 1993, Romer, 1994).

Now if we indexed labor and capital constraints in (6) as**:**

$$L=L_1^{\theta_1} L_2^{\theta_2} e^{\mu};\ K=K_1^{\gamma_1} K_2^{\gamma_2} e^{\mu}, \tag{7}$$

then ignoring the error term (e^{μ}) for simplicity and solving (7) for L_1 and K_1 we get

$$L_1=(L*L_2^{-\theta_2})^{\theta_1^*},\ \theta_1^*=1/\theta_1;\quad K_1=(K*K_2^{-\gamma_2})^{\gamma_1^*},\ \gamma_1^*=1/\gamma_1. \tag{7'}$$

Plugging L_1 and K_1 from (7') into (6) leads to

$$Y=\Phi L^{\theta_1^*\alpha_1} L_2^{-\theta_2\theta_1^*\alpha_1+\alpha_2} K^{\gamma_1^*\beta_1} K_2^{-\gamma_2\gamma_1^*\beta_1+\beta_2} e^{(\theta_1^*\alpha_1)'L-(\theta_2\theta_1^*\alpha_1+\alpha_2)'L_2+(\gamma_1^*\beta_1)'K-(\gamma_2\gamma_1^*\beta_1+\beta_2)'K_2+\delta t}. \tag{8}$$

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[6]Note that although it is reasonable to assume the rates of growth of labor and population to be exogenous Kosobud and O'Neil (1974) and Becker, Glaesser and Murphy (1999) show that making population endogenous to growth alters Malthusian the long-run equilibrium path.

Clearly (8) is too busy; it says what needs said only with too much. It is also exposed to statistical problems, the most obvious of them being multicolinearity due to the contemporaneity of K and K_2 in the power term of the model, and L and L_2, and K and K_2 in the exponential term of the expression. One way of dealing with these problems, is to note from (7) that K_2 is some proportion (ψ) of K, i.e., $K_2 = \psi K \Rightarrow K = \psi^{-1} K_2$, which we can use to replace K in (8) since our interest is in K_2. But first let $\alpha_1^* = \theta_1^* \alpha_1, \alpha_2^* = -\theta_2 \theta_1^* \alpha_1 + \alpha_2, \beta_1^* = \gamma_1^* \beta_1, \beta_2 = -\gamma_2 \gamma_1^* \beta_1 + \beta_2$, so that we can rewrite (8) as

$$Y = \Phi L^{\alpha_1^*} L_2^{\alpha_2^*} \psi^{-\beta_1} K_2^{\beta_1^* + \beta_2^*} \exp[(\alpha_1^{*\prime} L + \alpha_2^{*\prime} L_2 + (\beta_1^* + \beta_2^*)' K_2 + \delta t], \tag{9}$$

which is a lot simpler than (8). And if we further define $\beta_3^* = \beta_1^* + \beta_2^*$, and we know that $\psi^{-\beta_1}$ is a constant, then letting $\Pi = \Phi \psi^{-\beta_1}$, the labor intensive form of (9) becomes

$$y = \Pi \ell_2^{\alpha_2^*} k_2^{\beta_3^*} \exp[\alpha_1^{*\prime} + \alpha_2^{*\prime} \ell_2 + \beta_3^{*\prime} k_2 + \delta t] \tag{9'}$$

In moving from (9) to (9') the framework of this paper focuses on capital-labor ratio and is therefore Solow.

The second way we can deal with multicolinearity in (8) is to go Swan (1956, 2002 cf. Dixon, 2003) by stressing output-capital ratio and thereby getting rid off of K as (10) below shows:

$$Y = \Pi L^{\alpha_1^*/\beta_1^*} L_2^{\alpha_2^*/\beta_1^*} K_2^{\beta_2^*/\beta_1^*} \exp[(\alpha_1^*/\beta_1^*)' L - (\alpha_2^*/\beta_1^*)' L_2 - (\beta_2^*/\beta_1)' K_2 + \delta t], \tag{10}$$

which considering labor productivity is

$$y = \Pi \ell_2^{\alpha_2^*/\beta_1^*} k_2^{\beta_2^*/\beta_1^*} \exp[(\alpha_1^*/\beta_1^*)' - (\alpha_2^*/\beta_1^*)' \ell_2 - (\beta_2^*/\beta_1)' k_2 + \delta t]. \tag{10'}$$

Again (9') and (10') have the same form, but the former derives from Solow while the latter comes from Swan. In either case, Halter, Carter and Hocking (1957), Beatie and Taylor (1985), Baraim (1994), and Amavilah (1996, 1998), among others have shown that the variable elasticities of inputs (ξ_i) and production scale (ϵ) for (10) are:

$$\xi_{L_2}=\alpha_2+\alpha_2' L_2;\xi_{K_2}=\beta_2+\beta_2' K_2;\epsilon=\xi_{L_2}+\xi_{K_2}. \quad (11)$$

The marginal input productivities are variable as well and from (11) that is an easy tautology to check. What is not tautological is that this seemingly complicated set-up above can be estimated easily by OLS, GLS, or other methods because all the variables are linear in the parameters. The combined advantages of flexibility and data economy make this framework favorable for use where data availability is limited.

3. The Economics of Capital Accumulation under Shifting Technological Change

The general framework outlined above represents the accounting system that is capable of estimating empirically separate rates of growth in productivity attributable to the generation of:

(i) conventional resource rents associated with exogenous change and
(ii) innovation rents associated with endogenous change, both subject to
(iii) a financially feasible balanced growth rate.

Thus, (9)-(10) above measure technical change as it enters into (a) the evaluation of the output/capital ratio via capital stock (balance sheet) adjustments and intensities, or alternatively as (b) the value added stream of Y via changes in cash flows (income statements) at some rates of change constrained to be compatible with (c) the financial economics of growth as the feasible accumulation of capital investment (sources and uses of funds statement). This framework can then be used to produce estimates of the parameters of interest for analyzing and forecasting behavior and growth on both levels of stock and flow adjustments as well as their interactions. One can, for instance, use this approach to examine the impacts on growth of switching from reliance on rents generated in resource sectors (A) to innovation rents associated with new technologies (F) and their interactions (F*). In other words, this set up distinguishes:

(1) Exogenous shifts in productivity of balance sheet stocks X: A(t);
(2) Changes in flows in the income statement from endogenous improvements: F(x,t); and
(3) Interactive growth dynamics which display the optimal balanced growth path of aggregate production consistent with the financial constraints of a banking system committed to price and exchange rate stability: F*(X, x, t).

In examining *function* A(.)F(.)F*(.) this framework offers nothing new; but in a related area it offers an economical way of decomposing (5) into its components for accounting and parametric estimation of resource and innovation rents and their dynamic interaction in accelerating growth. We discuss each in turn in relation to Solow's, Swan's, and the new growth, models, beginning with Solow. Solow's measure of accumulating surplus is a function of capital and labor stocks impacted by (neutral) exogenous technical change. Assuming the stock of labor is fully employed, dA/dt permits the estimation of technical change as the shift in the productivity of balance sheet assets X given the labor

force (L) and resources of land (oil and minerals, etc.) as off-balance sheet constraints. Assuming F*(.) = 0, it is clear from (9') that the Solow residual appears as resource rents, i.e.,

$$\exp[\ln(A(X,L)t)] = \exp[lny - \alpha_2^* \ln\ell_2 - \beta_3^* \ln k_2], \quad \alpha_2^* + \beta_3^* = 1; \ \beta_3^* = \beta_1^* + \beta_2^*, \tag{12}$$

which, supposing an addi-log function for ease of exposition, implies that the rate of technical change

$$\frac{d\exp(\ln\Pi + \delta t)}{dt} = \delta \tag{12'}$$

If the fixed capital stock X_1 (in equipment, plants, mines, farms, etc.) remained constant and no dividends are paid out, then the growth in liquid assets X_2, including accumulated surplus, approximates the *rate of return to capital (R*), and A(t) is just the unexplained residual that plugs the asset (X) and liability (W) sides of the balance sheet, such that X = W. Alternatively, under Solow's accounting assumptions of constant returns to scale (CRTS), indexing to avoid aggregation problems, the fixed assets (resource) sector production function permits a rough estimate of the productivity of fluid capital as a numeraire, meaning

$$v = A(q)t, \tag{13}$$

where $v = Y / K_2$ is output-to-fluid capital ratio, or income to debt ratio, and $q = K_1 / K_2$ is fluid capital intensity, or incremental debt/equity ratio. Hence,

$$v = A(t)f(q). \tag{13'}$$

Viewed from the income statement, accounting for dividends as well as investment growth and surplus accumulation, the Solow residual (gap) reduces to the familiar Solomon-Gordon measure of the productivity of capital or the return to capital as earning per share (*E/S*) plus growth (*g*) such that

$$v = A(t)f(q) = R + g, \ R = E/S \Rightarrow X = W. \tag{13"}$$

Versions of measuring learning in endogenous factor flows and intermediate produced means of production, such as Lucas's, suggest that since A(t) represents an exogenous shift in the balance sheet, it is as much a "measure of ignorance" as it is of technical change. To improve on Solow, Jorgenson and Lau (197), Jorgenson and Grilliches (1967), among others, expand *f* into the KLEM or KL(EM) format in order to include variable (income statement) inputs or flows to explain as much aggregate

growth as possible in terms of endogenous factors. On the other hand, Solow's (1960) and Jorgenson's (1972) responses to critics of Solow *originale* focused on the relationship between investment and technical progress using vintage models (Cf. Michel,1995, Desai, 1995). These extensions made possible applications of non-algebraic functions that combine stocks and flows, leaving CRTS, decreasing returns to scale (DRTS) or IRS open to empirical test. In other words, it allowed writing the general production function as

$$Y=\Pi(K_1L_1)^{\alpha_1}(L_2L)^{\alpha_2}K_2^{\beta}e^{\delta t}. \tag{14}$$

The form of (14) is alright, but it is a potential source of identification problems insofar as its intensive form can be written as either,

$$y=\Pi\ell_1^{\alpha_1}\ell_2^{\alpha_2}k_2^{\beta}e^{\delta t}=\Pi\ell^{\alpha_1+\alpha_2}k_2^{\beta}e^{\delta t} \tag{14'}$$

or as,

$$y=\Pi\ell_2^{\alpha_2}k_1^{\beta_1}k_2^{\beta_2}e^{\delta t}=\Pi\ell_2^{\alpha_2}k^{\beta_1+\beta_2}e^{\delta t} \tag{14"}$$

where in (14') $\ell_1 = K_1L_1/L$ is ratio, in (14") $k_1 = K_1L_1/L$ is conventional capital-labor ratio, $\ell_2 = L_2L/L$ is human capital intensity of labor. Thus, inspection should show that (14) is similar to (9), except that it excludes exp(F*(.)). So, compared to (12), Lucas technical change is

$$\begin{aligned}&\exp(\ln\Pi+\delta t)=\exp[lny-\alpha_3\ln\ell-\beta\ln k_2],\quad \alpha_3+\beta=1;\ \alpha_3=\alpha_1+\alpha_2.\\ &\qquad\qquad\vee\\ &\exp(\ln\Pi+\delta t)=\exp[lny-\alpha\ln\ell-\beta_3\ln k_2],\quad \alpha+\beta_3=1;\ \beta_3=\beta_1+\beta_2.\end{aligned} \tag{15}$$

Thus, Lucas's rate of technical change, given by

$$\frac{d\exp(\ln\Pi+\delta t)}{dt}=\delta(X_1), \tag{15'}$$

depends on exogenous inputs (X_1), while the overall economic growth is endogenous. What that means is that fixing the level of K_2-productivity v as a function of capacity (Φ), and viewing the proportion

of skilled labor to total labor as ℓ_2, we have Lucas in (14). On the other hand, Romer's growth accounting model for technical change is endogenous, representing the importance of the knowledge embodied in investment at the margin in new high tech equipment associated with high tech industries, i.e.,

$$Y=vL_2^{\alpha}K_2^{\beta}e^{\delta t}, \tag{16}$$

the intensive form of which is

$$\begin{aligned} y&=v\ell_2^{\alpha}k_2^{\beta}e^{\delta t} \quad (a)\\ y&=vk_2^{\beta}e^{\delta t} \quad (b) \end{aligned} \tag{16'}$$

where in (16a) y = Y/L, and in (16b) y = Y/L_2. The corresponding technical change and its rate of change are

$$\begin{aligned} &\exp[(\ln ve^{\delta t}]=\exp(\ln y-\alpha\ln\ell_2-\beta k_2),\ v=R+g,\ R=E/S\\ &\frac{d\exp[\ln ve^{\delta t}]}{dt}=\frac{d\exp[\ln[(R+g)e^{\delta t}]]}{dt}=\delta(X_2). \end{aligned} \tag{17}$$

The rates of technical change and economic growth are both endogenous.

Taylor (2000) adds another perspective in which production depends on "financial capital" in the Wicksell-Schumpeter fashion, or "equity capital" according to Newcomb (1976).[7] Financial capital or equity capital consists of the fixed and physical portion made up of produced means of production (K_1) and the fluid pool of capital that supports labor in production (K_2). Since *in-situ* K_1 is already a part of existing production structure, only the K_2 part of K matters to production, i.e.

[7]Taylor is correct in the assessment that economists refuse to make "money" an argument in the production function. Takayama (1993: 423-429) shows that thinking through the money-growth research motivated by Tobin (1965), and which is the basis of Solow's (1956) Equation 15, p. 92, reveals that physical capital accumulation process is driven by money. In a lecture on the circular flow model, one of the authors was asked by a student: "Where is the bank in that diagram?". Of course, the initial response was a goffle-de-gook; subsequent explanations turned out too difficult for a beginning student with no interest in studying economics. The point: it is counterintuitive that liquidity is not given proper place in the production.

$$Y=\Phi(K_1)[L,K_2,t]=\Phi L^{\alpha}K_2^{\beta}e^{\delta t}, \tag{18}$$

so that the rate of technical change is

$$\frac{d\exp[\ln\Phi(K_1;K_2,t)e^{\delta t}]}{dt}=\Phi'(.)+\delta. \tag{18'}$$

Combining the insights of (12)-(18) into a financial "balanced growth" model along conventional lines by associating y and v with the constraints of the financial system, we have

$$\begin{aligned} &y=v\ell^{*\alpha}k^{*\beta}\exp[\alpha'\ell^{*}+\beta k^{*}+\delta t],\\ &\ell^{*}=(L_1/L_2)/L,\ k^{*}=(K_1/K_2)/L. \end{aligned} \tag{19}$$

Consequently technical change,

$$\begin{aligned} &\exp[\ln(R+g)e^{\delta t}]=\exp[\ln y-\alpha\ell^{*}-\beta k^{*}-\alpha'\ell^{*}-\beta k^{*}],\ \alpha'<0,\ \beta'<0,\\ &\exp[\ln(R+g)+\delta t-\alpha'\ell^{*}-\beta k^{*}]=\exp[\ln y-\alpha\ell^{*}-\beta k^{*}], \end{aligned} \tag{19'}$$

and its rate of change is

$$\frac{d\exp[\ln(R+g)e^{\delta t}]}{dt}=\delta(X_1,X_2,X_1X_2) \qquad (a)$$

$$\frac{d\exp[\ln(R+g)+\delta t-\alpha'\ell^{*}-\beta k^{*}]}{dt}=\delta(X_1,X_2,X_1X_2). \qquad (b) \tag{19"}$$

Eq. (19") shifts with fluid capital accumulation[8], which suggests that the integration of Solow-Swan, as well as Lucas-Romer into a general equilibrium meeting financial constraints via conventional balanced growth theory requires that we represent A(.), F(.), and F*(.) in a functional form like (18)

[8]Amavilah (1996, 1998) has interpreted (19") as combining Hicks-Solow neutrality with the Arrow -earning effect. Using a translog form Han, Kalijaran and Singh (2002) suggest another interpretation.

that combines the three dimensions (i)-(iii). This is an accounting representation of (9) which encompasses the separate effects of exogenous and endogenous technical change, and tracing as the economy shifts from resource rents in traditional primary sector to innovation rents in secondary or higher sectors. The interaction of these sectors represents the additional (third dimension) role of dynamics on the total productivity measure. Again this set-up can be estimated as a single equation, as three single equations, or as a system of three seemingly unrelated equations:

$$\begin{aligned} Y_1 &= A(X_1, L_1; L) \\ Y_2 &= F(Y_1, X_2, L_2; L) \\ Y_3 &= e^{F^*(Y_1, Y_2, \ell_2^*, k_2^*)}. \end{aligned} \tag{20}$$

As we said repeatedly (20) is nothing other than (9).

4. Economic Growth Rates under Alternative Rates of Technical Change

If we assume that exp(F*(.)) = 0 in (9'), then economic growth rate can be stated compactly as

$$\dot{y} = \delta + \alpha_2^* n + \beta_3^* m, \tag{21}$$

where n and m are growth rates of labor and fluid capital, respectively. The growth rate of labor is exogenous because it is given by the rates of growth of the adult population (mainly immigration in the USA) and employment - it is exogenous. What is the rate of capital growth? As alluded to above, there are many competing explanations, perhaps as many as there are definitions of capital as the Cambridge capital debates reveal (Taylor, 2000, and footnote #4). For Swan (1956, 2000) m is the growth of the output-fluid-capital ratio, i.e., $m = s(v \equiv Y / K_2) = sv = E / S + g = R + g,$ such that

$$\dot{y} = \delta + \alpha_2^* n + \beta_3^* [R + g], \tag{22}$$

which, since $\beta_3^* = \beta_1^* + \beta_2^*$, expands to

$$\dot{y} = \delta + \alpha_2^* n + \beta_1^* R + \beta_1^* g + \beta_2^* R + \beta_2^* g. \tag{22'}$$

It is then a logical case that in Swan's equilibrium $m = \dot{y}$, and the long-run growth is

$$\dot{y}^* = \frac{\delta}{1-\beta_1^*-\beta_2^*} + \frac{\alpha_2^*}{1-\beta_1^*-\beta_2^*} n = \frac{\delta}{\alpha_2^*} + n, \ \alpha_2^* = 1-\beta_1^*-\beta_2^*. \tag{22"}$$

Figure 1 shows that for Swan technical progress is simply a shifter, and in that sense it is comparable to increasing returns to scale ($\alpha_2^* + \beta_3^* > 1$) in its effect on economic growth. The basic result is that before optimal $k = K/L$, $m > n$, so that an increase in saving/investment, increases k and y, but decreases $v = Y/K$ and $q = K_1/K_2$. Overall the economy grows with rising m , but growth is checked by falling v and q.

For Solow $m = s(k \equiv K/L) = \dot{y} = g,$ and $\alpha_2^* = \alpha_1^*$ and $\beta_3^* = \beta_1^*$, because $\beta_2^* = 0$. In that case the long-run equilibrium growth rate is

$$\dot{y}^* = \frac{\delta}{\alpha_1^*} + n. \tag{23}$$

Mankiw, Romer, and Weil (1992) extension of Solow replaces α_2^* with α_3^*, everything else remains Solow (see Gemmell, 1995). Still it is important to note that although (22") and (23) look the same, they are different in essence. Judging from (22'), for instance, economic growth rate is Solow *iff* $\beta_1^* R + \beta_2^* R + \beta_2^* g = 0$, otherwise it is Swan. Compare Figures 1 and 2.

Next let us take Lucas and Romer as examples of endogenous growth models. From Lucas's (1993) point of view

$$\dot{y} = \delta(X_1) + \alpha_3 n + \beta_3^* \dot{y} \Rightarrow m = \dot{y}, \tag{24}$$

so that the long-run equilibrium growth is

$$\dot{y}^* = \frac{\delta(X_1)}{1-\beta_3^*} + \frac{\alpha_3}{1-\beta_3^*} n = \frac{\delta(X_1)}{1-\beta_3} + \frac{\alpha_1}{1-\beta_2^*} n_1 + \frac{\alpha_2}{1-\beta_1^*} n_2, \ \alpha_3 = \alpha_1 + \alpha_2, \ 1-\beta_3^* \neq 1-\beta_2^* \neq 1-\beta_1^*. \tag{24'}$$

This seems to suggest that economic growth is endogenous in human capital, whereas technical change is endogenous in conventional capital.

According to Romer (1990)

$$\dot{y}=\delta(L_2)+\alpha_3^* n+\beta_3^* m \Rightarrow \dot{y}^*=\frac{\delta(L_2)}{1-\beta_3^*}+\frac{\alpha_3^*}{1-\beta_3^*}n,\ \textit{iff}\ m=\dot{y}. \tag{24"}$$

This meaning that economic growth depends of technological change, which depends on publicly or privately funded research and development, which depends on ideas, which depends on human capital (see Gemmell, 1995: 170-171). This reading is consistent with Romer's objects-ideas dichotomy. In their extension of Romer, Graca, Jafarey and Philippopoulos (1995) study K-L_2 interactions and conclude that in the early stages of economic development, economic growth is driven by objects so that the rate of technical change is Lucas. In the late stages of development human capital drives both K_1 accumulation and economic growth such technical change is Romer. If they are correct, the conclusion means that stimulating economic growth requires accumulation of objects, while sustaining economic growth requires accumulation of ideas.

Finally, Taylor's growth rate equation is

$$\dot{y}^*=\delta(X)+\alpha n+\beta_3^*\xi,\ \xi=\sum_i^N \frac{R_i-C_i}{1+\iota},\ \xi \le P \neq m=\dot{y}. \tag{25}$$

Eq.(25) is different in its interpretation of capital growth; ξ is the short-run marginal cost of setting-up and running *in-situ* production capacity (Φ), and ι (iota) is the discount rate = Keynes marginal efficiency of capital = Fisher's internal rate of return. Since ξ is financed from saving and investment, it cannot exceed the spot product price (P), if investment is to be justified.[9]

As Figure 3 indicates a modest contribution of this paper is the recognition that technical change depends of fixed and endogenous resources, and their interactions in determining the rates of fluid and human capital accumulation, as well as economic growth, i.e.,

$$\dot{y}=\delta(X_1,X_2,X_1X_2)+(\alpha_2^*+\alpha_2^{*\prime})n+(\beta_3^*+\beta_3^{*\prime})m. \tag{26}$$

But in this case $m \neq g$ as in Solow unless $\alpha_2^{*'}=0$ and $\beta_3^{*'}=0$, i.e., (3) cannot be Solow, but it could be Swan so that

[9]In his book Taylor reviews and interprets definitions by Schumpeter, Keynes, Fisher, Bohn-Bawerk, Jevon, Wicksel, and brief comments by Samuelson and Solow. He states that his own definition of fluid capital is influenced mainly, not exclusively, by Schumpeter, Bohm-Bawerk, and Wicksell.

$$\dot{y}=\delta(X_1,X_2,X_1X_2)+(\alpha_2^*+\alpha_2^{*\prime})n+(\beta_3^*+\beta_3^{*\prime})[R+g]. \qquad (26')$$

For $R + g = m = \dot{y}$, the long-run growth rate is

$$\dot{y}^*=\frac{\delta(X_1,X_2,X_1X_2)}{\beta_3^*+\beta_3^{*\prime}}+\frac{\alpha_2^*+\alpha_2^*}{\beta_3^*+\beta_3^{*\prime}}n \;-\; QED. \qquad (26'')$$

Eq. (25)-(26) imply that in the short-run *n* is exogenous, and δ and *m* shift with X_2, given L, and hence with α and β. In the long-run δ changes with X, and α and β, and $n = m$. Thus, novel models of (26) and their level counterparts dealt with in the previous section, describe with familiar theory economic growth and capital formation under conditions of shifting technological change. As such it encompass and enrich, Solow-Swan and company, and Lucas-Romer and others as special cases.

4. Concluding Remarks

Drawing upon familiar insights this paper developed a flexible framework capable of portraying interactions among resources, technical change, and economic growth. The model is a simple single equation reduced form of a transcendental production function, which can be estimated either as a single equation, three single equations, or a system of three seemingly unrelated equations. The first (A(.)) of the three equations focuses on exogenous factors and is therefore associated with a Solow technical change. The change in A(.) lifts economic growth (measured as labor productivity) at a constant rate of technical change (δ). The second equation, F(.), depends on endogenous variables, mainly fluid capital and human capital. The change in F(.) amplifies the change in A(.) and sends economic growth in a higher than before orbit (trajectory) and at a faster rate of technical change than previously enabled by a change in A(.) alone. The third and final equation represents the interactions between endogenous and exogenous variables. Its effect is to crank up the technical efficiency of parameters and of the production technology itself.

By paying attention to the mechanisms of the interactions not only between accumulation (saving) and investment capital of the conventional growth theory, but also on the growth of the stock of *fluid resources* that enhances productive sectors and labor skills according to the more recent theory of endogenous technical change, the model developed here shows that in the end A(.) is function of objects so that ΔA lifts up the economy, but once the economy is in orbit, with only sY force behind it, its Newtonian law of motion is slowed down by the friction of diminishing returns to net capital investment (sY). But Φ = A*F is a function of both objects and ideas. Consequently the force of ΔF sends the economy into a higher orbit and at a faster rate of technical change than a force of ΔA alone. The combined force of changes in A(.) and F(.), i.e., Δ(A*F), offsets, or at least postpones, the onset of diminishing returns by releasing the investment constraint via fluid capital accumulation. There is no additional lift to the system due to $\Delta\Pi = \Delta(A*F*F^*)$ because there is no new investment. Even so,

ΔF^* sets the path for financiable long-run growth, because the presence (absence) of fluid capital formation guarantees a surplus (deficit) that reflects asset growth (decay) and net return (loss) on fluid capital in any time period. In other words, the endogenous portion of the production function is not necessarily subject to constant returns to scale *a priori*; it allows for scale (dis)economies, and overall (weak) constancy of scale, depending on whether exogenous or endogenous variables dominate the determination of *y*. This suggests further that according to the newer versions of endogenous growth theories not only are investment rate, labor, technological progress, depreciation, and human K-accumulation predictors of the productivity of effective workers, but also that the impacts of these determinants are disaggregated so as to permit technological change to be financed by the public, private, or both, which then leads to no or slow convergent growth rates among economies.

Representing the general framework as an accounting system enables empirical estimation of separate rates of growth in productivity attributable to the generation of (i) conventional resource rents associated with exogenous change, and (ii) innovation rents associated with endogenous change, both subject to (iii) a financially feasible balanced growth rate. In other words, this approach provides a means for examining the impacts on growth of, say, switching from reliance on rents generated in resource sectors (A) to innovation rents associated with new technologies (F) and their interactions (F*). In doing so, it distinguishes:(1) exogenous shifts in productivity of balance sheet stocks A(t); (2) changes in flows in the income statement from endogenous improvements; and (3) interactive growth dynamics which display the optimal balanced growth path of aggregate production consistent with the financial constraints of a banking system committed to price and exchange rate stability: F*(X, x, t).

Thus, the integration of Solow-Swan, as well as Lucas-Romer into financial economics as a general equilibrium meeting financial constraints via conventional balanced growth theory requires that we represent A(.), F(.), and F*(.) in a general functional form like (20) that combines the three dimensions (i)-(iii). This is an accounting representation of (9) which encompasses the separate effects of exogenous and endogenous technical change. The interaction represents the additional role of dynamics on the total productivity measure. Because the set-up requires a minimum qualitative restrictions, it can be estimated easily as a single equation, as three unrelated equations, or as a system of three seemingly unrelated equations.

Figure 1 - Swan: Economic Growth, Capital Accumulation, and Technical Change

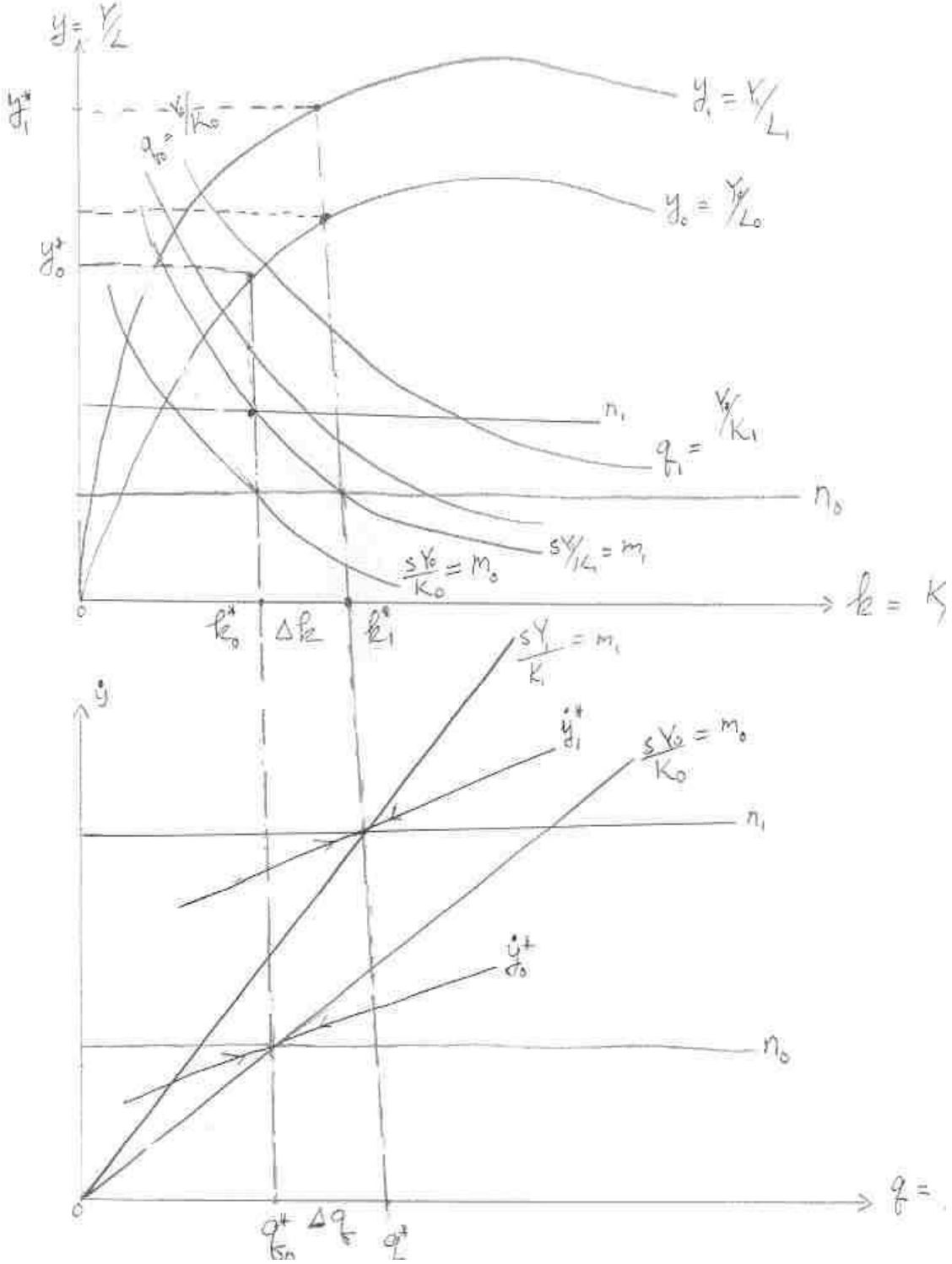


Figure 2 - Solow: Economic Growth, Capital Accumulation, and Technical Change

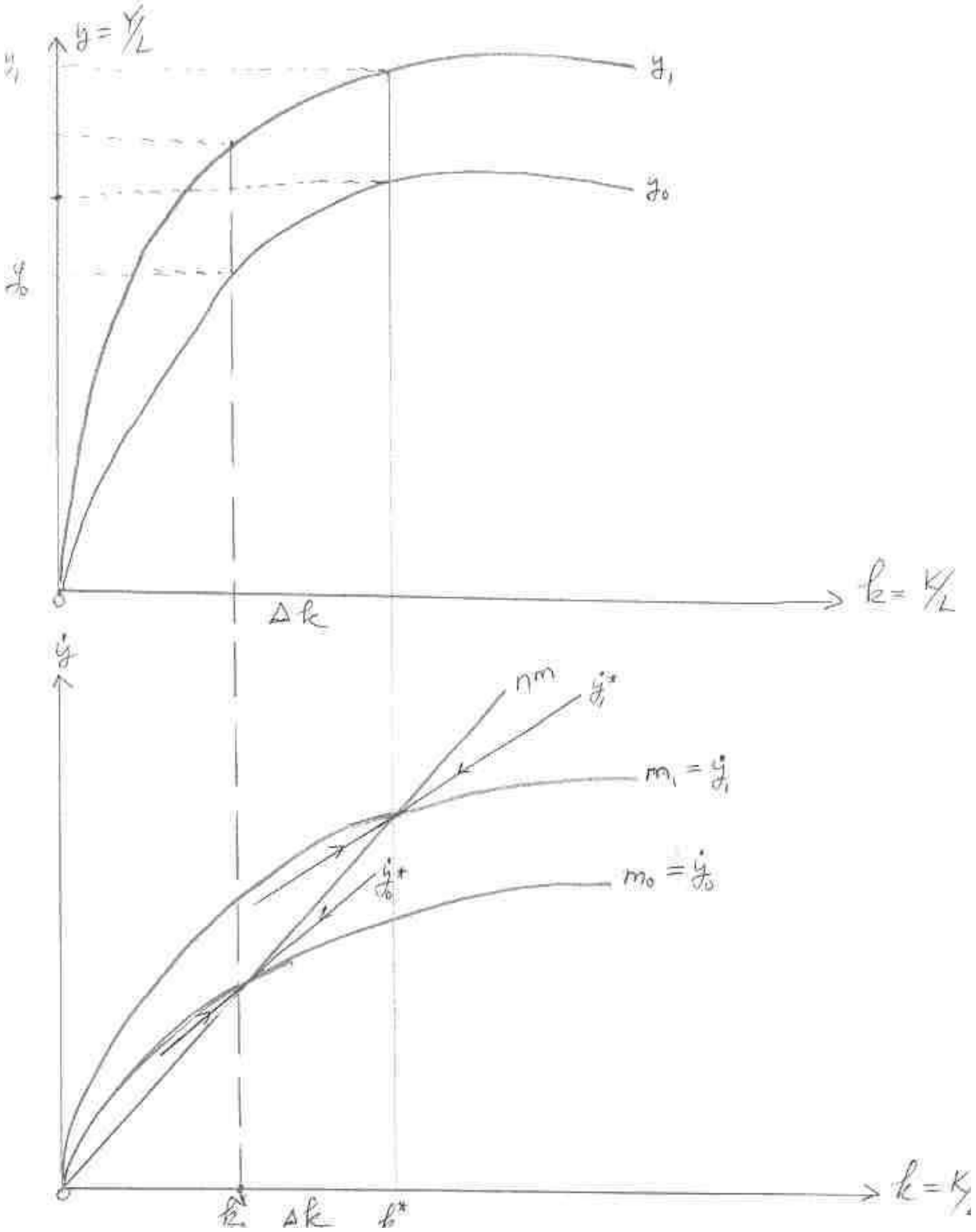
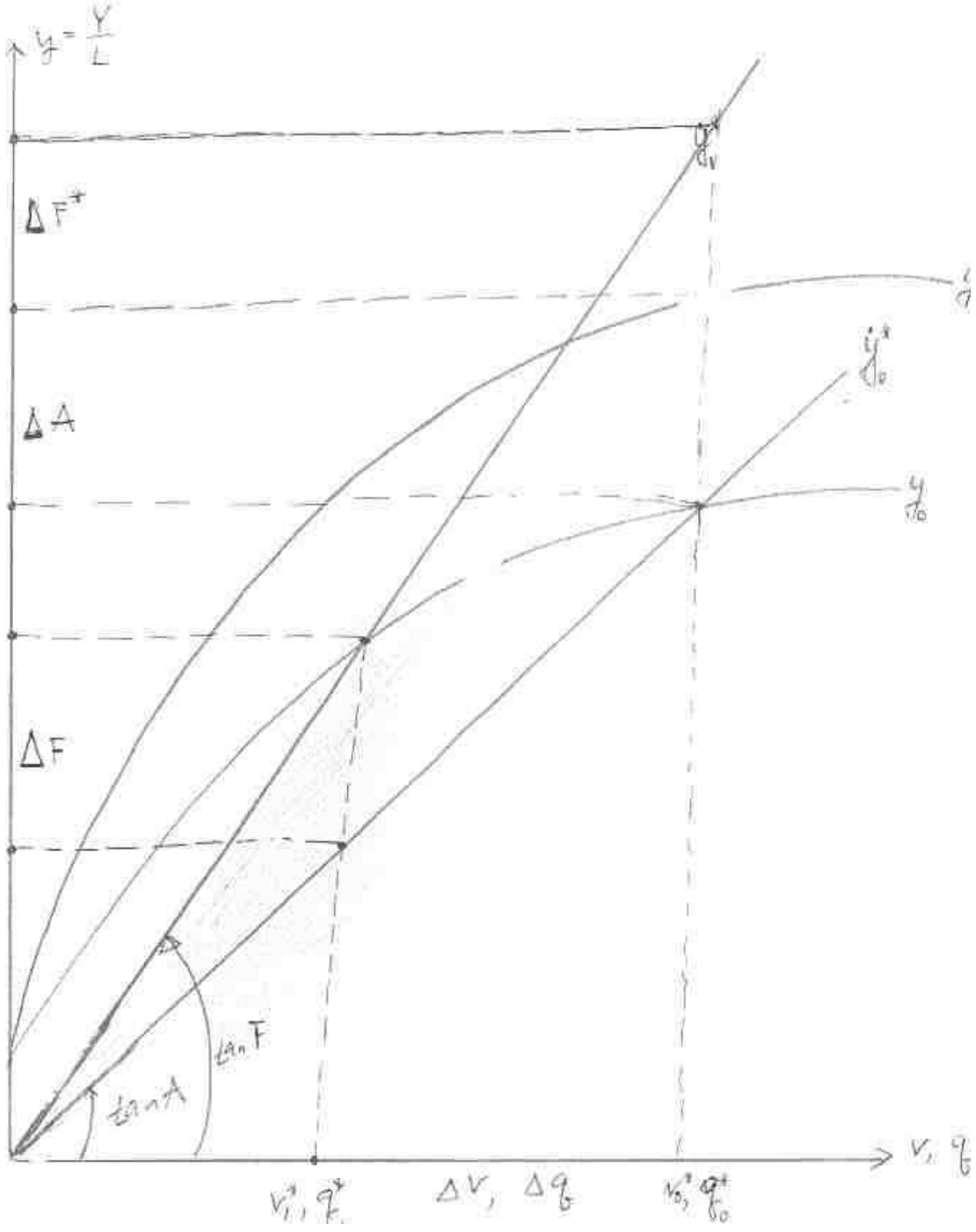


Figure 3 - Economic Growth, Capital Accumulation, and Shifting Technical Change



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