Biproportional Techniques in Input-Output Analysis: Table Updating and Structural Analysis

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ABSTRACT. *This paper introduces the rest of this issue, which is dedicated to the contributions of Sir Richard Stone, Michael Bacharach, and Philip Israilevich. It starts out with a brief history of biproportional techniques and related matrix balancing algorithms. We then discuss the RAS algorithm developed by Sir Richard Stone and others. We follow that by evaluating the interpretability of the product of the adjustment parameters, generally known as R and S. We then move on to discuss the various formal formulations of other biproportional approaches and discuss what defines an algorithm as "biproportional". After mentioning a number of competing optimization algorithms that cannot fall under the rubric of being biproportional, we reflect upon how some of their features have been included into the biproportional setting (the ability to fix the value of interior cells of the matrix being adjusted and of incorporating data reliability into the algorithm). We wind up the paper by pointing out some areas that could use further investigation.*

Keywords: biproportion, RAS, matrix balancing, interpretation, algorithms, extensions.

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1. The Early History of Biproportional Matrix Balancing

At least as early as the 1930s, researchers documented biproportional adjustment techniques—also known as "iterative proportional fitting" or "raking" (Ireland & Kullback, 1968). The two first documented pieces were by Kruithof (1937), who estimated telephone communications traffic and called it "the method of twin factors", and—according to Bregman (1967)—by Leningrad architect G.V. Sheleikhovskii, who estimated transportation traffic flows.

Nevertheless, it was not until Deming & Stephan (1940), who used the approach to derive expected contingency tables, that the approach became accessible in the English language as well as to social scientists. Since then, not only has the technique been refined and extended but an abundance of applications has emerged. They include the estimation of birth and death rates (Chandrsekar & Deming, 1949), migration flows (Chilton & Poet, 1973; Schoen & Jonsson, 2003), transportation flows (e.g., Furness, 1965), international and interregional trade (Bénard, 1963; Canning & Wang, 2003), transition probabilities (Theil & Rey, 1966; Moffit, 1993; Kendall & Nichols, 2002), voting patterns (Johnston *et al*., 1982, Balinski & Gonzalez, 1997),[1] and the incidence of fire reporting (Greene *et al*., 2001). One of the most sophisticated families of techniques for filling in missing data, however, are expectation maximization algorithms based on Markov chain Monte Carlo methods with a Gibbs sampler originally developed by Geman & Geman (1984) to reconstruct electronic images.[2] A somewhat related line of so-called "bound and collapse" algorithms developed by Ramoni & Sabastiani (1998) also has been gaining momentum. Despite the ever-increasing sophistication of techniques for estimating nondisclosed or missing data, however, simple biproportional approaches probably remain most popular.

Because of its simplicity and popularity, we opted to focus this issue of *Economic Systems Research* upon biproportional approaches in the development of input-output tables and similar economic accounts. Nevertheless, a few contributions herein surely reveal approaches that are strong divergences from techniques that are strictly biproportional.

Interestingly it was none other than the founder of input-output analysis who first used biproportional techniques in this setting. Indeed, Leontief (1941) used it to identify the sources of intertemporal change in the cells of a given nation's input-output tables. Nonetheless, it was Sir Richard Stone (Stone *et al*., 1942; Stone, 1961, 1962; Stone & Brown, 1962) who waved the banner on behalf of this technique within the field.

Given this background, in the next section we review the general biproportional formulation posed by Stone (1961, 1962) in an input-output context. Next, we develop brief reviews of other algorithms that have been used since Bacharach (1970) to solve problems that could be handled by RAS. We classify them into two types: alternative biproportion formulations and those that can strictly be formulated as mathematical programs. In the course of summarizing the alternative biproportional formulations, in Section 3 we attempt to unify them. In Section 4, we point both linear and nonlinear programming algorithms that have or could be used in the context of balancing input-output tables. While we review the material in Sections 3 and 4 we point out why some of the algorithms were employed, besides their unadulterated novelty. As a result, we follow with Section 5, which briefly discusses published extensions to the biproportional approaches that mend certain problems inherent to Stone's original formulation. We conclude this paper by discussing areas within the field of biproportional techniques that are ripe for additional research within the field of input-output analysis. Throughout the introduction to this special issue on biproportional techniques in interindustry analysis, we contextualize this issue.

2. The Basic Problem and an Early Algorithm

The fundamental problem to which biproportional techniques are applied is rather simple and can be stated as follows. Consider two rectangular matrices, an "initial" matrix $\bar{\mathbf{Z}}$ of dimension n by m where $a_{ij} \in \mathbb{R}^{+}$ and an unknown "target" matrix $\bar{\mathbf{Z}}^{*}$ with the same dimensions and other mathematical properties. (Overbars denote an array with the dimension of a full set of input-output transactions, including value added and final demand.) Indeed, only the margins (the row and column sums) of $\bar{\mathbf{Z}}^*$ are known. The problem is to find a third matrix $\tilde{\tilde{\mathbf{Z}}}$ not only with the same dimension and mathematical properties as $\bar{\mathbf{Z}}^*$ but also with the same known margin, i.e.,

$$(1) \qquad \Sigma_{i=1}^{n} \bar{z}_{ij}^{*} = \Sigma_{i=1}^{n} \tilde{\tilde{z}}_{ij} \text{ and } \Sigma_{j=1}^{m} \bar{z}_{ij}^{*} = \Sigma_{j=1}^{m} \tilde{\tilde{z}}_{ij}$$

Usually an objective of this problem is to obtain a $\tilde{\tilde{\mathbf{Z}}}$ that is as close as possible to $\bar{\mathbf{Z}}^*$, following defined criteria. While the margins of $\bar{\mathbf{Z}}^*$ are used to obtain $\tilde{\tilde{\mathbf{Z}}}$ via the structure of $\bar{\mathbf{Z}}$, it is typically only possible to make ex-post observations about the closeness of $\tilde{\tilde{\mathbf{Z}}}$ to $\bar{\mathbf{Z}}^*$. And, of course, more often than not, $\bar{\mathbf{Z}}^*$ remains forever unknown.

It may not be immediately clear from this set up, but there are infinite solutions to this problem. Indeed, since the initial working by Deming & Stephan (1940), numerous algorithms have been developed to solve it. Stone (1962) and Stone & Brown (1962) developed a particular biproportional procedure that has become known as "RAS", due, it seems, to the notation Stone used in a series of papers on the topic. While the **A** in the name "RAS" clearly denoted the direct requirements matrix in a Leontief setting, one could immediately surmise through observation that the matrices, which Stone used to pre- and post-multiply **A** in his papers, are labeled suspiciously similar to the author's first and last initials. Below, we diverge slightly from Stone's notation, although we do not wish to confuse the reader, as we discuss operations on flows and not direct coefficients.

RAS has two main advantages over competitive algorithms. First, it is a relatively simple algorithm that assures no negative values can be achieved.[3] Second, it demands a minimum of data. RAS is an iterative procedure where the rows and columns of preliminary estimates of $\tilde{\tilde{\mathbf{Z}}}$ are iteratively changed using proportions that are based on the known margins of $\bar{\mathbf{Z}}^*$. If we let angled brackets $\langle \cdot \rangle$ denote a diagonal matrix, with the denoted vector on its diagonal and zeros elsewhere, and **e** a summation vector of 1s of appropriate dimension, the algorithm can be described as follows:[4]

Step 0 (Initialization): Set $p = 0$. Let the initial value of the estimate be the known values of the prior, i.e., $\tilde{\bar{\mathbf{Z}}}^{(0)} = \bar{\mathbf{Z}}$.

Step 1 (Row Scaling)**:** Let $p = p + 1$, and $\hat{\bar{\mathbf{R}}}^{(p)} = \left\langle \bar{\mathbf{Z}}^{*}\mathbf{e} \right\rangle \left\langle \tilde{\bar{\mathbf{Z}}}^{(p-1)}\mathbf{e} \right\rangle^{\mathbf{-1}}$ and $\tilde{\bar{\mathbf{Z}}}^{(p\text{-}\frac{1}{2})} = \hat{\bar{\mathbf{R}}}^{(p)} \tilde{\bar{\mathbf{Z}}}^{(p\text{-}1)}$.

Step 2 (Column Scaling): Next let $\hat{\bar{\mathbf{S}}}^{(p)} = \left\langle \mathbf{e}'\bar{\mathbf{Z}}^{*} \right\rangle \left\langle \mathbf{e}'\tilde{\bar{\mathbf{Z}}}^{(p-\frac{1}{2})} \right\rangle^{\mathbf{-1}}$ and $\tilde{\bar{\mathbf{Z}}}^{(p)} = \tilde{\bar{\mathbf{Z}}}^{(p\text{-}\frac{1}{2})}\hat{\bar{\mathbf{S}}}^{(p)}$.

In RAS, Steps 1 and 2 constitute a full iteration and are repeated until, once a full iteration is complete, both $\hat{\bar{\mathbf{R}}}^{(p)}$ and $\hat{\bar{\mathbf{S}}}^{(p)}$ for a given iteration are extremely close to their respective identity matrices. Typically this algorithm will kick out its estimate of $\bar{\mathbf{Z}}^{*}$ either when all diagonal elements of both $\hat{\bar{\mathbf{R}}}^{(p)}$ and $\hat{\bar{\mathbf{S}}}^{(p)}$ are within a certain pre-specified tolerance (say, 1.00000000 ± 0.0000001) or after a specific number of iterations (say, k) have passed. Indeed, Bacharach (1970) demonstrated that there is a unique equilibrium solution to this algorithm. That is, $\lim_{k\to\infty} \tilde{\tilde{\bar{\mathbf{Z}}}}^{(k)} = \tilde{\tilde{\bar{\mathbf{Z}}}}$. Further, Bacharach noted that $\tilde{\tilde{\bar{\mathbf{Z}}}}^{(k)} = (\Pi_{p=1}^{k}\hat{\bar{\mathbf{R}}}^{(p)})\bar{\mathbf{Z}}(\Pi_{p=1}^{k}\hat{\bar{\mathbf{S}}}^{(p)})$. Hence, by letting $\hat{\bar{\mathbf{R}}} = (\Pi_{p=1}^{k}\hat{\bar{\mathbf{R}}}^{(p)})$ and $\hat{\bar{\mathbf{S}}} = (\Pi_{p=1}^{k}\hat{\bar{\mathbf{S}}}^{(p)})$, $\tilde{\bar{\mathbf{Z}}} = \hat{\bar{\mathbf{R}}}\bar{\mathbf{Z}}\hat{\bar{\mathbf{S}}}$, which the analyst hopes very close to, if not the same as, $\bar{\mathbf{Z}}^{*}$.

2.1. Interpretation of $\hat{\bar{\mathbf{R}}}$ and $\hat{\bar{\mathbf{S}}}$

In this setting, Stone *et al*. (1963) interpreted $\hat{\bar{\mathbf{R}}}$ as embodying interindustry or intercommodity substitution effects. This interpretation arises because $\bar{\mathbf{Z}}$ is pre-multiplied by $\hat{\bar{\mathbf{R}}}$, which means that it acts on $\bar{\mathbf{Z}}$ strictly rowwise. Hence, not only does $\hat{\bar{\mathbf{R}}}$ "correct" $\bar{\mathbf{Z}}$ by applying an overall *shift* factor that raises the scalar growth rate of the entire economy,[5] $\bar{\delta} = [\mathbf{e}'(\bar{\mathbf{Z}}^{*}\mathbf{e})]/[\mathbf{e}'(\bar{\mathbf{Z}}\mathbf{e})]$, it can be articulated as the product of the scalar shift factor $\bar{\delta}$ and a diagonal matrix $\hat{\bar{\mathbf{T}}}\left(= \hat{\bar{\mathbf{R}}}/\bar{\delta}\right)$, which redistributes *shares* of gross output rowwise on the elements of the prior matrix $\bar{\mathbf{Z}}$. That is, one can identify the economywide change in input mix as $\hat{\hat{\mathbf{T}}}$. This means that through $\hat{\hat{\mathbf{T}}}$ the RAS algorithm increases the average economywide use of one sector's production as an input at the apparent expense of another's—by definition an economywide substitution effect.[6] Stone *et al.* (1963) also used such shift-share thinking to arrive at an economic interpretation of $\hat{\hat{\mathbf{S}}}$ when $\overline{\mathbf{Z}}$ and $\overline{\mathbf{Z}}^*$ are full sets of national economic accounts including noncompetitive imports. $\hat{\hat{\mathbf{S}}}$ is decomposed into the same scalar shift effect, $\overline{\delta}$, and a similarly composed economywide share effect by industry $\hat{\hat{\mathbf{W}}} = \hat{\hat{\mathbf{S}}} / \overline{\delta}$ that almost exclusively embodies adjustments to the initial economy's industry mix.

2.2. Matrices of Direct Requirements and a "Reinterpretation" of $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$

To move this discussion of the interpretation of RAS along, let us first digress briefly into a more typical application of RAS. That is, RAS is usually not effected upon the full input-output transactions matrix. Typically in the literature, researchers have focused use of the RAS technique on $\mathbf{Z}$ and $\mathbf{Z}^*$, which are intermediate industry accounts only. That is, researchers have tended to assume that "current year" vectors of total intermediate inputs, $\eta_j^* = \Sigma_{i=1}^n z_{ij}^*$, and total intermediate outputs, $\upsilon_i^* = \Sigma_{j=1}^m z_{ij}^*$, are known or knowable. In such cases, $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$, articulate differences *within* the intermediate matrix only. Hence, $\hat{\hat{\mathbf{T}}}$, $\hat{\hat{\mathbf{W}}}$, and δ similarly denote the vectors of input mix, industry mix, and shift factors common across intermediate industries only, not economywide. We should note that the tendency has been for researchers to focus upon the direct requirements matrix $\mathbf{A} = \mathbf{Z}\hat{\mathbf{X}}^{-1}$ - where $\mathbf{x} = \overline{\mathbf{Z}}\mathbf{e} = (\mathbf{e}'\overline{\mathbf{Z}})'$ - and not the intermediate transactions matrix $\mathbf{Z}$.[7] Nonetheless, all RAS operations actually are acted upon margins of the intermediate transactions matrix, and the termination tests employed are grounded in comparisons of the vectors $\mathbf{Z}^{(p)}\mathbf{e}$ and $\mathbf{e}'\mathbf{Z}^{(p)}$ to the vectors $\boldsymbol{\eta}^*$ and υ^*, respectively, although $(\mathbf{e}'\mathbf{Z}^{(i)})/(\mathbf{e}'\mathbf{Z}^*)$ sometimes is compared to $\upsilon / \mathbf{x}$ for the case of columnwise adjustments.[8] As a result, the economic interpretation of industry mix effects in the total set of interindustry accounts boils down to one of productivity effects when only intermediate accounts are RASed.[9]

Moreover, it has been shown that any interpretation of r_i and s_j (for arbitrary i and j) is questionable since these terms are not identified in the context of the RAS algorithm. That is, RAS is hyperbolically homogenous since, for an arbitrary scalar α, $\hat{\mathbf{R}}\mathbf{Z}\hat{\mathbf{S}} = (\alpha\hat{\mathbf{R}})\mathbf{Z}(\hat{\mathbf{S}}/\alpha)$. Or, more generally, for an arbitrary diagonal matrix $\hat{\boldsymbol{\alpha}} > \mathbf{0}$, the diagonal matrix $\hat{\boldsymbol{\beta}}$ is such that $\hat{\mathbf{R}}\mathbf{Z}\hat{\mathbf{S}} = (\hat{\boldsymbol{\alpha}}\hat{\mathbf{R}})\mathbf{Z}(\hat{\boldsymbol{\beta}}\hat{\mathbf{S}})$. ("Hyperbolically homogenous" means that $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\beta}}$ necessarily must be the inverse of one another.) We use the minimum discrimination of information approach (explained in Section 3.2) to explain what we mean through an example. Assume

$$\mathbf{Z} = \begin{bmatrix} 5 & 2 & 1 \\ 3 & 9 & 4 \end{bmatrix},\ \mathbf{Z}^* = \begin{bmatrix} 11 & 3 & 2 \\ 2 & 8 & 6 \end{bmatrix}$$

and that $\bar{r}_i^{(0)} = 1$ for all i. This yields:

$$\hat{\mathbf{R}} = \begin{bmatrix} 1.464 & 0 \\ 0 & 0.738 \end{bmatrix},\ \hat{\mathbf{S}} = \begin{bmatrix} 1.364 & 0 & 0 \\ 0 & 1.150 & 0 \\ 0 & 0 & 1.811 \end{bmatrix}, \text{and } \tilde{\mathbf{Z}} = \begin{bmatrix} 9.98 & 3.37 & 2.65 \\ 3.02 & 7.63 & 5.35 \end{bmatrix}.$$

On the other hand, if we change only one value of the algorithm's initializing vector, $\mathbf{r}^{(0)}$, we arrive at different values for $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$. Letting $\bar{r}_2^{(0)} = 2$, we get:

$$\hat{\mathbf{R}} = \begin{bmatrix} 2.323 & 0 \\ 0 & 1.171 \end{bmatrix},\ \hat{\mathbf{S}} = \begin{bmatrix} 0.859 & 0 & 0 \\ 0 & 0.724 & 0 \\ 0 & 0 & 1.142 \end{bmatrix}, \text{and } \tilde{\mathbf{Z}} = \begin{bmatrix} 9.98 & 3.37 & 2.65 \\ 3.02 & 7.63 & 5.35 \end{bmatrix}.$$

So $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$ do not remain the same: in mathematical parlance this means they are not identified. Interestingly, however, the solution is the same in the two problems. Moreover, across the two examples, the ratio r_2/r_1 remains equal to 0.504, and the ratios s_2/s_1 and s_3/s_1 keep fixed at 0.843 and 1.576, respectively. So while the absolute levels of the elements of $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$ change, their relative values within each vector do not. From this it is clear the product $\hat{\mathbf{R}}\mathbf{Z}\hat{\mathbf{S}}$ may possibly be interpreted as the result of industry and input substitution effects. That is, although the values of $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$ are not identified, biproportional approaches force the effects of $\hat{\mathbf{R}}$ to be offset by the effects of $\hat{\mathbf{S}}$ and vice versa.

Unfortunately since $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$ are not identified, it is definitely not possible to give a straightforward interpretation to them in terms of absolute values, at least without some other transformation(s). The fact that the elements within each of these two vectors have constant relative values offers a potential solution. A typical mathematical solution in such cases is normalization. In this case, due to issues of degrees of freedom, normalization can be affected on either $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$, not both! Due to the iterative nature of the procedure, interpretation of such normalization is not so simple. Normalization without some sort of rationale predicated in economic theory (e.g., simply letting $r_1 = 1$ or $\Sigma_{i=1}^{n} r_i = 1$) can increase the interpretation problem rather than solve it (de Mesnard, 2002, 2004*a*). An interesting exception to this may be a rationale posed by van der Linden & Dietzenbacher (1995) who suggested normalizing such that the average of elements r_i (using appropriate weights) equals one, which can be interpreted as a global substitution effect of zero. Nonetheless, any normalization amounts to imposing a constraint on a Lagrangian since, as can be inferred from Section 3.2, the $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$ terms are themselves Lagrange multipliers. This is because they are derived by imposing $r_i = \mathrm{e}^{-(1+\lambda_i)}$ where the λ_i are the multipliers associated with the constraints $\Sigma_{j=1}^{m} z_{ij}^{*} = \Sigma_{j=1}^{m} \tilde{z}_{ij}$ and $s_j = \mathrm{e}^{-(1+\mu_j)}$ where the μ_j are the multipliers associated with the constraints $\Sigma_{i=1}^{n} z_{ij}^{*} = \Sigma_{i=1}^{n} \tilde{z}_{ij}$. This makes such normalization an odd sort of mathematical operation to undertake.

To summarize, it is only the relative value of any given element within each of the $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$ vectors that is interpretable. As a result, normalizing not only is possible, it is the only proper way to interpret the elements of $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$ since their absolute values *cannot* be interpreted.[10]

2.3. On the Gap between the Projection and the Target

Using their form of normalization, van der Linden & Dietzenbacher (1995, 2000) note that there surely must be other "sector-specific" substitution effects not accounted in a decomposition of economic change like that detailed above and that a lack of accounting for such sector-specific factors may well be the reason that the above economic interpretation of RAS was not convincing to some, most notably Lecomber (1975*a*, *b*) and Miernyk (1977, 2004). Hence, in addition to $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$ acting on $\mathbf{Z}$ to derive and estimate of $\mathbf{Z}^*$, van der Linden & Dietzenbacher suggest that a third factor should be used to identify sources of economic change, essentially rediscovering work of Kouevi (1965) mentioned in Lecomber (1975a, p. 17). This element-specific factor they define as $\Delta_{ij} = a_{ij}^* / \tilde{a}_{ij}$, the ratio of a given "actual" element of the direct matrix to its value estimated via RAS.[11] Hence,[12]

$$a_{ij}^* = \tilde{a}_{ij}\Delta_{ij} = \tilde{r}_i a_{ij} \tilde{s}_j \Delta_{ij} .$$

With this in mind, it was de Mesnard (1988, 1990, 2004*b*) who appears to have first extended the economic interpretation of RAS by recognizing the gap between $\mathbf{Z}^*$ and $\tilde{\mathbf{Z}}$ in terms of structural change.[13] Since $\tilde{\mathbf{Z}}$ has the same column and row margins as $\mathbf{Z}^*$ after the proportional projection, $\tilde{\mathbf{Z}}$ is directly comparable to $\mathbf{Z}^*$, removing the effect of differences in sector size—the case between $\mathbf{Z}$ and $\mathbf{Z}^*$. When applying this principle, one applies a *biproportional filter*, a generalized version of the well-known shift-share method discussed earlier (for more information, see, e.g., Stevens & Moore, 1980). The direct comparison of $\tilde{\mathbf{Z}} = RAS(\mathbf{Z}, \mathbf{Z}^*)$ and $\mathbf{Z}^*$ is done by computing the gap between $\tilde{\mathbf{Z}}$ and $\mathbf{Z}^*$ and various norms: $\left\|\mathbf{Z}^* - \tilde{\mathbf{Z}}\right\|$ for the whole economy, $\left\|\mathbf{Z}_i^* - \tilde{\mathbf{Z}}_i\right\|$ for rows *i*, $\left\|\mathbf{Z}_j^* - \tilde{\mathbf{Z}}_j\right\|$ for columns *j* and even $\left|z_{ij}^* - \tilde{z}_{ij}\right|$ for all *i*, *j*. These Frobenius norms can be interpreted in terms of distance between the projected matrix $\tilde{\mathbf{Z}}$ and the target matrix $\mathbf{Z}^*$: this is handy when evaluating the structural change. Note that this generates some difficulties as $\left\| RAS\left(\mathbf{Z},\mathbf{Z}^*\right)-\mathbf{Z}^*\right\| \neq \left\| RAS\left(\mathbf{Z}^*,\mathbf{Z}\right)-\mathbf{Z}\right\|$. Hence, in *Biproportional Methods of Structural Change Analysis: A Typological Survey*, featured in this issue, *Louis de Mesnard* presents an interpretation that can be given to the gap, some difficulties for measuring it, and some methods to circumvent these difficulties.

3. Alternative Biproportional Techniques

It is our objective in this introductory piece to demonstrate the new directions that matrix-balancing techniques are taking. So we will not rehash Polenske's (1997) review of findings on the performance of RAS. Thus, for background, we will quickly review here instead the line of research on matrix-balancing techniques. The reason we do so, is that these alternative techniques typically are used to overcome one or more of RAS's apparent shortcomings. Indeed, it is by overcoming such shortcomings that biproportion techniques have advanced most during the past two decades. Interestingly, while some researchers have been extending these alternate approaches, others have been advancing their apparent similarities to RAS. Hence, in this manner we will come full circle in the subsequent section by showing how some researchers have modified the traditional RAS algorithm, presented earlier, to overcome its own shortcomings.

Matrix-balancing techniques can be classified broadly into two categories: scaling algorithms (like RAS) and optimization algorithms. Scaling algorithms are identified by the way they balance matrices—they iteratively multiply rows and columns of a prior matrix by positive constants to derive a series of candidate solution matrices. Optimization algorithms minimize functions that measure distance between a specified characteristic of candidate matrices and the prior matrices. These so-called objective or penalty functions induce the problems' solutions to be as close as possible to their priors. The constraints of the optimization problem are the balancing conditions that force the solution from being the same as the prior.

3.1. The Diagonal Similarity Scaling Algorithm

Osborne (1960)[14] and, later, Grad (1971) applied a somewhat different scaling algorithm than RAS. It is the so-called "diagonal similarity scaling" (DSS) algorithm which follows:

Step 0 (Initialization): Set p=0. Let the initial value of the estimate be the known values of the prior, i.e., $\tilde{\mathbf{Z}}^{(0)} = \mathbf{Z}$.

Step 1 (Select the index): Let q be defined as the minimum index satisfying the following: $|\mathbf{e}'\mathbf{z}^{p}_{q\bullet} - \mathbf{z}^{p}_{\bullet q}\mathbf{e}| = \max_{1 \le i \le n} |\mathbf{e}'\mathbf{z}^{p}_{i\bullet} - \mathbf{z}^{p}_{\bullet i}\mathbf{e}|$.

That is, scan the matrix to find the row-column pair (row i and column i) for which the row sum is most different from the column sum.

Step 2 (Scale): With scaling parameter $\alpha^{p} = [(\mathbf{z}^{p}_{q\bullet}\mathbf{e})/(\mathbf{e}'\mathbf{z}^{p}_{q\bullet})]^{1/2}$ let

$$z^{p+1}_{ij} = \begin{cases} \alpha^{p} z^{p}_{ij}, \text{ if } i = q, j \neq q \\ \dfrac{z^{p}_{ij}}{\alpha^{p}} \quad , \text{ if } j = q, i \neq q \\ z^{p+1}_{ij}, \qquad \text{otherwise.} \end{cases}$$

Existence and uniqueness results for the DSS algorithm are available in Eaves *et al*. (1985) and Schneider (1989, 1990).

The approach gets its name because each iteration derives a new candidate matrix by applying a single diagonal matrix to both sides of the last candidate matrix. This diagonal matrix $\mathbf{D}^{p}$ has the value of the identity matrix in all but one element—that diagonal element where $i = q$. In that one case, $d^{p}_{qq} = \alpha^{p}$—the scaling parameter. As a result, Step 2 of the algorithm can be alternately formulated as $\mathbf{Z}^{p+1} = \mathbf{D}^{p}\mathbf{Z}^{p}\left(\mathbf{D}^{p}\right)^{-1}$. This formulation gives it the appearance of being a special case of RAS. Moreover, Lamond & Stewart (1981) and Cottle *et al*. (1986) likened DSS and RAS to the coordinate descent algorithm applied to the dual of minimum discrimination of information problem (presented next).

An advantage of this approach over standard RAS is an extension provided by Schneider (1989, 1990) that enables it to handle upper and lower bounds on the margin totals. But its main drawback is that it requires column sums to equal row sums, which is assured in the case of input-output analysis only for gross output. In this case gross value added and gross final demand would have to be known. Of course, balancing of the components of these two particular accounts could not be performed within the context of this algorithm. In any case, the minimum discrimination approach provides the same solution with greater flexibility.

3.2. The Minimum Discrimination of Information Approach

One of the earliest and most prominent scaling techniques besides RAS is the minimum discrimination of information approach. Building on important work by Shannon (1948*a*, *b*) and Kullback & Liebler (1951) on information theory, Uribe *et al*. (1965) associated the RAS solution with the optimal solution of the following minimization problem:[15]

$$\text{Minimize} \qquad I = \Sigma_{i=1}^{n}\Sigma_{j=1}^{m}\tilde{z}_{ij}\log(\tilde{z}_{ij}/z_{ij})$$

$$\text{subject to} \qquad \Sigma_{j=1}^{m}\tilde{z}_{ij} = \Sigma_{j=1}^{m}z_{ij}^{*} \quad \forall i \quad \text{and} \qquad \Sigma_{i=1}^{n}\tilde{z}_{ij} = \Sigma_{i=1}^{n}z_{ij}^{*} \quad \forall j$$

The advantage of this minimum discrimination of information approach, as it has become known, is that it can be more readily applied when less than perfect information is available than is required either by RAS or DSS. For example, it has been used when only some of the row and column totals are known or all of it is known with some specified degree of reliability.

Interestingly, its solution is precisely $\tilde{\mathbf{Z}} = \hat{\mathbf{R}}\,\mathbf{Z}\,\hat{\mathbf{S}}$, where $r_i = z_{i\bullet}^{*}/\Sigma_{j=1}^{m}z_{ij}s_j$ for all *i*, and $s_j = z_{\bullet j}^{*}/\Sigma_{i=1}^{n}r_i z_{ij}$, for all *j*. Hence, the algorithm can be solved iteratively by giving initial values, say $\mathbf{r}^{(0)} = \mathbf{e}$ or $\mathbf{s}^{(0)} = \mathbf{e}$:[16] $r_i^{(p+1)} = z_{i\bullet}^{*}/\Sigma_{j=1}^{m}z_{ij}s_j^{(p+1)}$ for all *i,* and $s_j^{(p+1)} = z_{\bullet j}^{*}/\Sigma_{i=1}^{n}r_i^{(p)}z_{ij}$ for all *j*.

As in the case of RAS, this is computed iteratively until equilibrium is reached. Hobson (1969) and Snickars & Weibull (1977) explored the properties of this approach and have shown that the solution exists, is unique, and converges, provided its conditions are respected. It is rather obvious: the series $r_i^{(p+1)} = z_{i\bullet}^{*}/[\Sigma_{j=1}^{m}(z_{\bullet j}^{*}/\Sigma_{i=1}^{n}r_i^{(p)}z_{ij})z_{ij}]$ for all *i* has necessarily a fixed point, unique, existing and converging because the function *I* is a convex and continuously derivable function defined on a compact set.

3.3. Entropy Maximization

The same result can also be generated from the entropy maximization principle (Jaynes, 1957*a*, *b*; Wilson, 1970):

$$\text{Maximize} \quad H = \Sigma_{i=1}^{n} \Sigma_{j=1}^{m} \tilde{z}_{ij} \log \tilde{z}_{ij}$$

$$\text{subject to} \quad \Sigma_{j=1}^{m} \tilde{z}_{ij} = \Sigma_{j=1}^{m} z_{ij}^{*} \quad \forall i \quad \text{and} \quad \Sigma_{i=1}^{n} \tilde{z}_{ij} = \Sigma_{i=1}^{n} z_{ij}^{*} \quad \forall j$$

$$\text{and to} \ \Sigma_{i=1}^{n} \Sigma_{j=1}^{m} c_{ij} \tilde{z}_{ij} = K \ ,$$

where c_{ij} is a unit cost of exchange per monetary unit exchanged (including transportation costs, transaction costs, legal fees, bank charges, etc.) on arc $\{i, j\}$ and *K* is the total cost of exchange over all the arcs of the network.

One can readily obtain the minimum discrimination of information approach and vice versa by posing $z_{ij} = \mathrm{e}^{(-\gamma c_{ij})}$, or equivalently $c_{ij} = -(\log z_{ij})/\gamma$ for all *i*, *j,* where *γ* is the multiplier associated with the last constraint. Note that $-1/\gamma$ could be interpreted as a scale factor. Moreover, when *K* is minimized, this algorithm retrieves the result of the so-called “transport problem” (provided all variables are real).

3.4. Other Biproportional and Biproportion-like Problems in Economics

The number of problems in economics that lead, more or less, to a biproportional form is very large. Examples are: the gravity model (Nijkamp, 1975); the probabilistic multinomial model (Choukroun, 1975); the interaction minimization principle (Watanabe, 1969; Guiasu, 1979); the theory of movements (Alonso, 1978; Ledent, 1981); utility theory (Niedercorn & Bechdolt, 1969; Niedercorn & Moorhead, 1974); and Gumbel’s method (Thionet, 1976).

Vermot-Desroches (1986) and de Mesnard (1988, 1990) suggest that after a bit of transformation these alternatives should yield the same result as either the minimization discrimination of information or entropy maximization approaches. Many algorithms could potentially generate this result—some yet to be invented. If they fail to yield the same result as RAS, the concept of biproportion could be in doubt. Fortunately, de Mesnard (1994) demonstrated that all algorithms with a biproportional form (provided they respect constraints (1)) do, in fact, yield the same results. This provided a substantial basis toward refuting the reputation of biproportional methods, especially RAS, for being strictly empirical.

4. Non-biproportional Approaches

A third family of algorithms also largely developed in the 1970s and 1980s. We call members of this rather broad family non-biproportional approaches. Current they are handled strictly by mathematical programming software like LINDO and MINOS. They are all general constrained matrix problems that cannot be solved using the simpler set of scaling techniques. But they are closely related to matrix balancing problems (Bacharach, 1970; Byron, 1978). Again, the objective function of these optimization problems can take many forms. Generally they minimize the sum some measure of distance between all elements of the two matrices (the prior and the estimated projection).

Of this genre, the most commonly applied objective function is that first presented to an input-output audience by Almon (1968), the minimization of the square of the Euclidean distance between $\tilde{\mathbf{Z}}$ and $\mathbf{Z}$, i.e., $\Sigma_{i=1}^{n}\Sigma_{j=1}^{m}(\tilde{z}_{ij}-z_{ij})^2$. It is of particular interest because it has statistical properties that permit parallels to ordinary least squares (Durieux & Payen, 1976).

Among other methods there are:

- The *minimization of the Hölder norm* at the power θ: $\Sigma_{i=1}^{n}\Sigma_{j=1}^{m}|\tilde{z}_{ij}-z_{ij}|^{\theta}$.

When θ = 1, minimizing the Hölder norm becomes the *minimization of the absolute differences*, $\Sigma_{i=1}^{n}\Sigma_{j=1}^{m}|\tilde{z}_{ij}-z_{ij}|$,

- The weighted absolute differences (Lahr 2001*a*, *b*), i.e., $\Sigma_{i=1}^{n}\Sigma_{j=1}^{m}(z_{ij}\,|\,\tilde{z}_{ij}-z_{ij}\,|)$.
- The normalized absolute differences used by Matuszewski *et al*. (1964), i.e., $\Sigma_{i=1}^{n}\Sigma_{j=1}^{m}(|\,\tilde{z}_{ij}-z_{ij}\,|\,/\,z_{ij})$,
- Pearson's χ^2 or the normalized square of differences used by Deming & Stephan (1940) and Friedlander (1961), i.e. $\Sigma_{i=1}^{n}\Sigma_{j=1}^{m}[(\tilde{z}_{ij}-z_{ij})^2/z_{ij}]$,
- Neyman's χ^2: $\Sigma_{i=1}^{n}\Sigma_{j=1}^{m}[(\tilde{z}_{ij}-z_{ij})^2/\tilde{z}_{ij}]$, although the lack of knowledge about $\tilde{\mathbf{Z}}$ makes this objective less desirable from the perspective of the typical problem posed by input-output analysts.
- The weighted square differences, i.e., $\Sigma_{i=1}^{n}\Sigma_{j=1}^{m}[z_{ij}(\tilde{z}_{ij}-z_{ij})^2]$.

One of the main characteristics of this class of methods is that it can generate negative-valued elements in $\tilde{\mathbf{Z}}$ even if there are none in **Z**: that is, sign is not always preserved. While this can be a "good" thing, it is generally annoying since negative flows are difficult to interpret in economic terms (e.g. what is a negative flow $\tilde{z}_{ij}$? If it is the flow of commodity *i* sent by sector *j* to sector *i*, then it isodd since sector *j* is not the producer of commodity *i*. And if it is the negative flow of commodity *i* sent by sector *i* to sector *j*, what does that mean? Is it something like the return of books from a college bookstore after the end of a semester?) It is always possible to introduce an additional set of nonnegativity constraints ($\tilde{z}_{ij} \geq 0$ for all *i*, *j*) but remember that the solutions accumulate at the borders of the convex set of these constraints (each time where the solution would be negative). Hence, many zeros may appear in $\tilde{\mathbf{Z}}$.[17]

The family of nonlinear optimization algorithms that can be applied to the general constrained matrix problem is large. Hence, it would be nigh unto impossible to elucidate them within the confines of this introduction. In addition to traditional linear programs, methods used to balance matrices include nonlinear network methods (see, e.g., Bachem & Korte 1978, 1980, 1981a,b,c; Dembo *et al*. 1989), conjugate gradient algorithms (Byron, 1978), Lagrangian relaxation techniques, and successive overrelaxation algorithms. Many of the algorithms have yet to be applied to economic accounts.

Dembo *et al*. (1989) and Schneider & Zenios (1990) provide surveys of large-scale network optimization algorithms, and Zenios *et al*. (1989) do so with an eye upon the context of social accounting matrices. These algorithms are so classified because they exploit the network structure inherent in the matrices upon which they operate—one *node* for every row and for every column of the matrix, one *arc* for every nonzero entry, and *backarcs* to convert transportation into a circulation network and to enforce the condition that the total flow into a column is the same as the total flow into the corresponding row. Although Bachem & Korte presented what eventually became their 1981 paper in *Metrika* at the Seventh International Conference on Input-Output Techniques held in Innsbruck in April of 1979, it was Florian & Los (1982) and van der Ploeg (1982, 1984, 1988) who apparently first delivered this concept to the eyes of input-output analysts.

Most of the literature in this field applies quadratic or entropy penalty functions. The quadratic function is justified because of its ready statistical interpretation as a generalized least-squares estimator, and the entropy function because of its basis in information theory as mentioned earlier.

Several of the objective functions that had been published in the literature are not linear and must be linearized before finding a solution. Recall that linearization is not the panacea for such nonlinear problems because it is only worthwhile if the solution is in the neighborhood of the hyperplane of linearization.

In *Alternate Input-Output Matrix Updating Formulations* in this issue, *Randall W. Jackson & Alan T. Murray* compare the performances of several of these algorithms to that of RAS, two of them bypassing biproportion's sign-preserving features using a suggestion by Junius & Oosterhaven (2003). Interestingly, the authors find RAS is a top performer in their simulations, especially with regard to speed. But it is not consistently best at yielding an accurate projected matrix. Moreover, without sign-preserving features some of the other algorithms also perform well. But in these cases, at least at present, computational efficiency is extremely poor compared to that of RAS.

5. Extensions of Biproportion Techniques

RAS, in particular, and biproportional techniques, in general, seemed to lose favor among competitive methods used by researchers during the 1980s and 1990s. Nonetheless, it appears now that extensions at least were incubating. Indeed, a number of the extensions of biproportional techniques demonstrated in this special issue of *Economic Systems Research*, recall earlier works published and unpublished that harken to works from this period.

5.1. Biproportion with Known Interior Information

Perhaps one of the earliest extensions of RAS known to the editors of this issue was one Lecomber (1975a) mentioned and that was performed by Omar (1967) who removed the sign-preserving feature of RAS, simply by setting all cells in the prior matrix so that they were zero. The negative numbers inherent to the prior tables were fortunately sufficiently small, so resulting tables apparently were not sensitive to such alteration (Schneider, 1965; Omar, 1967).

Philip Israilevich, one of three major contributors to whose memories this issue is dedicated—the others being Sir Richard Stone and Michael Bacharach—developed something called "ERAS". Israilevich's work was informed about the fundamentals of RAS provided by his professors Ronald E. Miller and Peter Blair (see Miller & Blair, 1985) as well as his familiarity with advances in matrix balancing techniques founded upon optimization techniques via Stavros Zenios and his colleagues. In his Ph.D. dissertation entitled *Biproportional Forecasting of Input-Output Tables*, Israilevich (1986) developed a RAS-based approach that fixes interior cell values of the "forecasted" matrix to known values. This approach helped Szyrmer (1989) develop an interesting piece that showed more known information in such a setting will improve RAS's projection capability.[18] Lahr (2001a) supported this notion as well, by measuring the gain in accuracy when adding selected perfect information to a nonsurvey table. The data added in Lahr's work were based on an algorithm that identified sectors that had the greatest linkage power and then the elements within that sector to which the Leontief inverse was most sensitive.

According to Paelinck & Waelbroeek (1963), one can incorporate known interior information matrix information in a RAS setting. When projecting to a desired table, one can account for the values of known interior cells of such a table by setting the particular known cell values to zero and subtracting the known values from the corresponding row and column margins. One can obtain the solution by proceeding with the standard RAS procedure and placing the known values back into their cells when the algorithm terminates. In this way, rather than adding to constraints (1), which would obviate the possibility of biproportional techniques and often was a reason why mathematical programs were used, this operation basically requires a mere realigning of them. Of course, too many zeros can possible pose a problem since zero cells serve to block the ability of the RAS procedure to distribute excess value from one sector to another.

Gilchrist & St. Louis (1999) described an algorithm they call "TRAS" (three-stage RAS) after Bacharach's (1970, pp. 93-99) two-stage algorithm. Like Israilevich's (1986) approach, of which these authors were unaware, TRAS extends RAS by including, in the table to be projected, "known" information beyond that for the column and row totals. In particular, using a perspective not unlike that presented by Oosterhaven *et al*. (1986), who updated a set of regional tables based upon a new national table, Gilchrist & St. Louis (1999) cite the case of provincial tables in Canada where some elements of published tables are suppressed at one level of sectoral detail and not suppressed at a more-aggregated level of sectoring. The object of their algorithm then is to fill in the detailed matrix's censored data using information from its more aggregated but uncensored equivalent. The so-called "third stage" of TRAS is one adjusting for differences in sectoral detail between the two matrices. Using a Monte Carlo experiment for all Canadian provinces, the authors showed that being informed by the more aggregated tables improved the accuracy of the detailed tables substantially compared to a standard RAS approach. This was especially the case for the Make tables as opposed to the sparser Use tables.

In *An Algorithm for the Consistent Inclusion of Partial Information in the Revision of Input-Output Tables* in this issue *Donald Gilchrist & Larry V. St. Louis* revisit TRAS, limiting themselves to the construction of a relatively detailed table for Alberta only. This time they opt for a stronger test of the marginal value of TRAS over

RAS. They do so by using the results of the RAS adjustments as the initial coefficients in subsequent TRAS adjustments. They then measured the marginal gain of TRAS over RAS, by regressing the result from the RAS and the incremental gain from TRAS against the actual coefficients for five different tables: the make coefficients, use coefficients, market share, direct coefficients, and Leontief inverse matrices. The incremental gain from TRAS is shown to be statistically significantly different from the information obtained via RAS in all but the market share matrix. While both RAS's and TRAS's performance in estimating elements of the make coefficients and market share tables was not very good and their ability to replicate the Use coefficients matrix was fair, their ability to replicate the direct coefficients and Leontief inverse was remarkable. Hence, it is clear that timely, accurate input-output tables can be produced with approaches like TRAS.

Timely annual estimates of national accounts are not only desired, but, thanks to enhanced computer power, are becoming a reality in many corners of the world. Accordingly, the U.S. Bureau of Economic Analysis (US BEA) recent embarked on a research venture to speed up their procedures for producing annual tables at the same level of sectoral detail at which they supply their benchmark tables. In their contribution to this issue entitled *Increasing the Timeliness of U.S. Annual I-O Accounts,* US BEA researchers *Mark Planting & Jiemin Guo* reveal some results from the BEA investigation. As might be expected, they implement price information as did Barker (1975) and Snower (1990) and find the value of including known information about value added by industry. Like many previous studies, they find that adding known information tends to improve the quality of the annual tables resulting from the automated updating procedure.[19] They also find that accuracy of tables with enhanced sectoral detail is more heavily influenced by the addition of the estimated value added data. But perhaps most surprising, however, is their finding that difference in the relative accuracy between tables (of the same dimension) with and without known value added precisely accounted is not that great.

5.2. Biproportion Incorporating the Reliability of "Known" Data

As mentioned earlier, one reason that researchers seem to have moved away from RAS was that it was difficult to incorporate information on the relative reliability of data that is "known". Needless to say, balancing matrices composed of data with different qualities can be an imposing challenge. As Lahr (1993) and others have noted, analysts typically draw upon a variety of data sources in constructing an input-output table. This is especially true for the case of national accounting agencies, which can even have multiple sources for a particular data element.

This apparent focus away from RAS due to data reliability issues occurred, however, despite an early RAS-based concept for attacking this problem (Stephan, 1942; Stone *et al*. 1942) and a promising generalization of RAS published by Allen & Lecomber (1975). The latter pair of authors essentially applied RAS only to a matrix that embodied "any views on the relative accuracy of cells" (Allen & Lecomber, 1975, p. 48) in an initial matrix that was a hybrid of the prior matrix and "known" exogenous new data. Perhaps the departure from RAS for accounting for data reliability was simply due to the fact that optimization techniques could handle reliabilities relatively explicitly using weak inequality constraints and objective functions that incorporate penalties for deviations from some "desired" value. On the other hand, it may well have been that procedures for documenting data reliability at the time simply were not up to the task.

Although Allen & Lecomber (1975) performed tests on their generalized version of RAS, they did not document how they had identified the relative reliability of their data. Indeed, explicit discussion of how to measure data reliability in an input-output setting was not really documented until the debate between Gerking (1976; 1979*a*, *b*), Miernyk (1976, 1979), and Gerking & Pleeter (1977), which Gerking started due to the general subjective treatment of data reliabilities in a West Virginia input-output table.[20] Jensen & McGaurr (1976) may well have been the first to document explicitly how they accounted for data reliability in the development of input-output tables.

Recent work by Robinson *et al.* (2001) briefly summarizes some of the efforts undertaken via constrained optimization approaches to handle differing data reliabilities. Most notable among them is that by Golan *et al*. (1994), who, following suggestions made in the pioneering work of Stephan (1942), Stone *et al*. (1942) and Byron (1978), develop a specification where each "known" element of a projected matrix is assumed to have a different degree of robustness associated with it.

Lahr (2001*a*) returned to RAS as a means of handling data reliability issues, albeit only for deviations from "known" margins. He did so by adjusting the RAS algorithm so that the termination criterion for the differences of intermediate margin estimates from "known" margins (so-called "tolerances") were pre-specified percentages of the known margins themselves. He designed these pre-specified percentages to reflect reliability of the "known" margins: the greater the reliability, the lower the percentage.

Based upon an algorithm developed in Stone (1984), in this issue *Ebsen Daalgard & Christian Gysting* describe *An Algorithm for Balancing Commodity-flow Systems*. Their algorithm includes a biproportional row and column pass at its outset, and can handle commodity-flow systems with six different sets of prices (RAS can handle only one). Perhaps more importantly, unlike RAS, margin totals in their algorithm do not have to be balanced at the start. A key feature of the algorithm is its ability to incorporate information on the relative reliability of different data elements in a target vector. Comparing the results of a fully automated pass of their algorithm at an aggregated level to an official manual one balanced at a more detailed level, the authors find few differences of any magnitude worthy of mention. Interestingly, their results were quite robust with respect to substantial variations in the reliability parameters. Given the large successes the authors show, it will be interesting to see how their approach performs in another setting, especially with greater sectoral detail.

5.3. The Concept of Frame Shifting

Conway (1990) developed an alternative approach to circumvent RAS's shortcomings in updating input-output models—the so-called regional econometric input-output model (REIM) approach. The REIM approach uses principles of Marshallian output equilibrium to circumvent the need for updated prices to rebalance an input-output table's rows in value terms. Without prices, columnwise constraints cannot be imposed; as a result much of value added is considered a residual, as typical in national accounts.

By anyone's standard it would be a bit of stretch to consider the REIM approach "biproportional" since it only adjusts across rows once. Nonetheless, we mention it here because Israilevich (2002)[21] conceived an extension of Conway's innovation that amounts to a single adjustment for both the row and column of an input-output table. He called it "frame shifting". He overcame the lack of prices by suggesting that price equations could be built using Sheppard's Lemma with the direct input-output coefficients as input shares as in Hudson & Jorgenson (1974) without explicitly articulating physical quantities.

Unaware of Israilevich's work at the outset, *Kurt Kratena & Gerold Zakarias* implement it after a fashion in *Input Coefficient Change Using Bi-proportional Econometric Adjustment Functions* in this issue. The application is one for Austria, and varies from Israilevich's concept by accounting for actual rather than implicit energy prices and by not necessarily ensuring that the row adjustment conforms with the output vector. As in the case of several other instances in this issue of *ESR*, RAS outperforms the novel technique in forecasting a known input-output table. Nonetheless, this approach shows much promise, especially from the perspective of economic rationale. Hence, it is not surprising that these authors suggest several ways in which one could improve upon their technique.

6. Conclusion

Searching for a better estimate of a matrix based on the structure of a prior and margins of another matrix is a very general problem. As we point out at the start of this paper, it is applied in many fields outside the developing application to National Accounting for the purposes of estimating regional flows, interregional flows, international flows, financial flows, migration flows, voting behavior,[22] etc. Biproportional methods are efficient where information is missing, unavailable, and when econometric estimation is at least difficult if not impossible—typically where phenomena are represented by matrices.

If attendance at the sessions we organized at recent professional meetings is any measure, biproportional and related techniques have regained the popularity they once owned in the production of input-output accounts and social accounting matrices. This may largely be due to the ready availability of data and very low-cost of computing power, especially relative to that of labor. Hence, advancing these techniques clearly remains a vital, potent theme in our field. Moreover, as papers in this issue bear witness, this is a topic in which academics have as much to learn from researchers in national accounting offices as vice versa. As a result it is clear that we shall be learning more from practitioners about their innovations in balancing social accounting matrices or systems of national accounts. We look forward to learning more particulars about how they incorporate differing data reliabilities and the complexities of subjective reliability assessments. Moreover, it will be useful to learn if the enhancements in accuracy and labor savings yield returns in making these accounts more timely.

Clearly there are other topics from which we should hear some news in the near future. Applications of Geman & Geman’s (1984) expectation algorithm and/or Ramoni & Sabastiani’s (1998) “bound and collapse” algorithm seem to have some potential if the nature of the “neighborhood” surrounding the missing accounts data can be specified without much difficulty.

It is clear there is a need for greater intensity in the use of biproportional techniques in certain settings. The increasing ease of data access and the mounting evidence for maintaining sectoral detail in official accounts will make a single complex series of different biproportional procedures a necessity for those who construct multiregional input-output accounts. First one would estimate interregional trade along the lines suggested by Canning & Wang (2003). Then regional data like that pointed out as being critical by Lahr (2001*b*) would itself have to be “filled-in” for disclosure problems before it is integrated into an intermediate set of accounts that subsequently would have to be balanced itself.

In any case, biproportional techniques and related algorithms are alive and well with a great prognosis in the future of interindustry accounting. We look forward to seeing more experiences with them published within the pages of this journal.

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Endnotes

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[1] Estimating voting patterns can require working on integer numbers instead of real numbers, which generates some difficulties.

[2] The origins of this line of work are quite different from that of standard biproportional algorithms, stemming from the likes of Metropolis *et al*. (1953) and Hastings (1970). It is also related to the expectation maximization algorithm developed by Dempster *et al*. (1977) as well as algorithms developed by King *et al*. (2001), Gelfand & Smith (1990), and Tanner & Wong (1987), which are used to fill in missing data. For more details, see King *et al*. (2001), Little & Rubin (2002), or Schafer (1997).

[3] Junius & Oosterhaven (2003) have introduced a version of RAS, that they call "Generalized RAS" that accepts negative entries in $\mathbf{Z}$ and $\mathbf{Z}^*$. Also remember that a zero in $\mathbf{Z}$ is projected as a zero in $\tilde{\mathbf{Z}}$ with RAS.

[4] This presentation is for any set of real non-negative matrices, not only for input-output matrices.

[5] This growth rate incorporates both real and inflationary growth if the family of $\mathbf{Z}$ matrices is not in real terms.

[6] This substitution matrix would also include information on real and inflationary growth as well as relative prices changes if the family of $\mathbf{Z}$ matrices is not in real terms.

[7] This is mostly due to academic interest in the changing structure of the direct requirements matrices since with some care they can be interpreted as representing the state of an economy's technological development.

[8] Both of these operations are Hadamard (element-by element) division.

[9] However, it should be noted that results from the direct requirement matrices approach and of the transaction matrices approach are not necessarily comparable.

[10] This problem is not apparent in RAS, because it always starts with the same initial values, although the values of $\hat{\mathbf{R}}$ and $\hat{\mathbf{S}}$ could depend upon whether one starts RAS with a row or column adjustment operation (steps 1 and 2 above are exchanged). The reader could find a parallel to this issue in the question of using relative or absolute prices in the general equilibrium theory.

[11] It must be noted that van der Linden and Dietzenbacher (1995, 2000) actually call this ratio a "sector-specific change" and not until Dietzenbacher & Hoekstra (2002), who used the difference $|a_{ij}^* - \tilde{a}_{ij}|$ instead, was it given the more appropriate nomenclature of the "cell-specific change".

[12] This is comparable to de Mesnard's (in this issue) concept of computing on the flow matrices $|z_{ij}^* - \tilde{z}_{ij}|$. Note that the difference approach makes possible the computation of the relatively simple Frobenius norm, either globally as in, columnwise as $[\Sigma_i (z_{ij}^* - \tilde{z}_{ij})^2]^{1/2}$, or rowwise as $[\Sigma_j (z_{ij}^* - \tilde{z}_{ij})^2]^{1/2}$, while the ratio approach necessitates computing a less common geometric norm.

[13] Indeed, van der Linden & Dietzenbacher (1995, 2000) and Dietzenbacher & Hoekstra (2002) also acknowledge this in the course of their work. Toh (1998), Andréosso-O'Callaghan & Yue (2000) and Jalili (2000) build on it as well.

[14] Osborne (1960) developed this technique to enhance the accuracy of computing eigenvalues of a matrix.

[15] Bregman (1967) and Theil (1967) often tend to be cited for this contribution.

[16] Nothing obliges one to initialize this algorithm with a vector of ones. Any other set of starting values will do, even a vector with different values for each element *i*. Moreover, doing so does not alter the values of the all-important product of the **r** and **s** vectors at equilibrium! Hence, one clear advantage over RAS is this approach's lack of need for known initial starting values. Interestingly, if one does start with a vector of ones for **s**, after half of an iteration this approach produces intermediate candidate matrices that are identical to those produced via RAS. It remains that this approach is more general than RAS, or, RAS is a particular case of it.

[17] This should not be confused with the advantages provided by Junius & Oosterhaven's (2003) generalized RAS (GRAS): in GRAS, both $\mathbf{Z}$ and $\mathbf{Z}^*$ contain negative terms.

[18] Lahr (2001*a*) as well as some unpublished experiments conducted by de Mesnard in consultation with Ronald E. Miller indicate that the extent of any biproportional "improvement" depends in part on the axiomatic chosen to measure the improvement: indeed, in some cases, the improvement could be negative.

[19] Planting & Guo measure table quality simply by counting the number of large coefficients differences.

[20] Although,Gerking did not mention it in his arguments, Stone *et al*. (1942) support his contention that it is important to formally account for data reliability.

[21] This is a post humous publication, cobbled together by his good friend and colleague Geoffrey J.D. Hewings. Philip Israilevich passed away in early December 1997, after a three-year bout with retinal cancer.

[22] Balinski & Gozalez (1997) apply an integer biproportional method to estimate voting patterns.