A Non-parametric Bootstrap Simulation Study in ESTAR (1) Model

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ABSTRACT

Smooth Transition Autoregressive (STAR) model has been employed in a number of current studies dealing with non-linearities. The usefulness of this model has been documented in these studies. However, the population statistical properties of the parameters in this model remain unknown. This study attempts to investigate these properties through a non-parametric bootstrap simulation study. The exponential STAR model of order one, which is sufficient in providing us the necessary information on the linear and non-linear parameters as well as the speed of transition of the STAR model despite its simplicity, is employed in this study. This study also investigates the size effect of the bootstrapped estimators and their confidence intervals by varying the number of bootstrap replications. Results of this study show that their empirical distribution are asymmetrical in nature with the linear parameter being positively skewed and the non-linear and transition parameters being negatively skewed. Besides, we find that the normality theory over rejects the significance of the estimated transition parameter. Another interesting point worth mentioning is that the minimum number of replications needed for the bootstrap confidence interval to be a good approximate of the standard normal one is 500.

Keywords: non-parametric bootstrap, ESTAR model, confidence intervals, statistical distribution

INTRODUCTION

Smooth Transition Autoregressive (STAR) model (Luukkonen *et al*., 1988) is a useful non-linear model in the field of econometric modeling and forecasting. The theoretical issues as well as illustration of the linearity tests, model specification and estimation procedure are documented in, among others, Luukkonen *et al*. (1988) and Teräsvirta (1994). This model has currently received much attention from economic researchers as it is viewed very suitable for modeling the non-linearities in economics variables such as exchange rates (Taylor and Peel, 1997; Sarantis, 1999; Sarno, 2000; Taylor and Peel, 2000 and Baum *et al*., 2001). The forecasting performance of STAR exchange rate model had been investigated in Liew (2002); Liew and Baharumshah (2002) and Liew *et al*. (2002) and they concluded that the out-of-sample forecasts from STAR model are significantly more accurate than both the linear Autoregressive (AR) and Simple Random Walk (SRW) models. To date, there is no doubt on the usefulness of STAR model. Nonetheless, the population statistical properties of the underlying parameters of this model remain unknown. This paper attempts to shed light on this issue using the non-parametric bootstrap simulation study. We employ the exponential STAR model of order one or briefly ESTAR (1) model in this study, which is adequate in providing us the necessarily information on the linear and non-linear parameters as well as the speed of transition despite its simplicity.

Consider a series y_t with mean u and variance σ_y^2. An ESTAR (1) model with known delay lag length d for this time series can be expressed as (see Teräsvirta, 1994):

$$y_t = u + a\,(y_{t-1} - u) + b\,(y_{t-1} - u)\,[1 - \exp(-\gamma(y_{t-d} - u)/\sigma_y^2] + \varepsilon_t \qquad (1)$$

where a and b stand for linear and non-linear AR parameter respectively; γ denotes the transition parameter, which governs the speed of transition from one regime to the other in the ESTAR model. These three unknowns have to be estimated by non-linear least square procedure. Meanwhile, $\varepsilon_t \sim$ iid (0, σ_ε^2) by assumption, which means the model's residuals are expected to follow a white nose process with mean zero and a constant variance σ_ε^2.

Theoretically, the value of the linear AR parameter (a) is allowed to take any absolute value greater than one (although value smaller or equal to one is also possible). Nevertheless, to meet the requirement of global stationarity in the model, the sum of the linear and non-linear AR parameters (a+b) should be some value less than one in magnitude; see, for example, Taylor and Peel (2000) for a discussion in the context of general STAR model of order p. Before estimating the non-linear model as specified in Equation 1, one must reject the null of linearity in the formal linearity tests beforehand; see Luukkonen *et al*. (1988) and Teräsvirta (1994) for details of such tests. If this requirement is met, the statistical significance of the non-linear parameters b or γ or both in Equation 1 implies further evidence in flavour of the non-linearities in y_t.

In this current study, we investigate the unknown population statistical properties of the parameters a, b and γ using the non-parametric bootstrap simulation procedure. These properties include the distributions, means, standard deviations, variance, skewness, kurtosis, and confidence intervals of the above parameters. Applying Equation 1 to the US dollar based Japanese yen (YEN/USD) return series and the non-linear least square procedure, we first find all the estimators of u, a, b, γ, σ_y^2 and σ_ε^2, denoted $\hat{u}$, $\hat{a}$, $\hat{b}$, $\hat{\gamma}$, $\hat{\sigma}_y^2$ and $\hat{\sigma}_\varepsilon^2$, in that order. Bootstrap samples are then simulated through the process of resampling the original sample data with replacement (Efron and Tibshirani, 1993); see also Choo *et al*. (2001) for a similar study in the Exponential Generalized Autoregressive Conditional Heteroscedasticity model. Discussions on the methodology and the results of the current bootstrap simulation are presented in the next two sections, whereas the final section contains our conclusions.

BOOTSTRAP SIMULATION STUDY

Our YEN/USD sample data is collected from various issues of International Financial Statistics (IFS) published by the International Monetary Fund (IMF). The data are recorded at quarterly interval covering the range of the first quarter of 1968 to the third quarter of 2001. The return series (consisting of 134 observations) is computed from this sample by using the formula:

$$y_t = p_t - p_{t-1} \tag{2}$$

where y_t and p_t stand for exchange rate return at time t and exchange rate at time t respectively.

The sample mean and variance of this series has been easily estimated as – 1.811 and 110.293 respectively. Teräsvirta (1994) linearity test has been performed on the resulting series and the results are summarised in Table 1. From Table 1, it is obvious that the null of linearity for this series has been rejected at 1% significance level. Moreover, test results also indicate the suitability of an ESTAR (1) model of delay lag length *d* = 3 in characterizing the behaviour of this return series.

Table 1
Linearity Test Results for the Japanese yen – US dollar Return Series[a]

	Delay Parameter, *d*							
	1	2	3	4	5	6	7	8
F Statistic[b]	0.2022	5.8009	10.2102	1.0742	0.3619	1.0859	3.1648	1.2429
MSV[c]	0.6537	0.0174	0.0018	0.3019	0.5485	0.2994	0.0776	0.2669

Notes: [a] The optimal lag length *p* is determined to be 1 by the Akaike bias-corrected information criterion (Hurvich and Tsai, 1989).

[b] The F statistic tests the null hypothesis of linearity against the alternative of non-linearity. Specifically, this test has the power against Exponential Smooth Transition Autoregressive (ESTAR) model (Teräsvirta, 1994).

[c] MSV denotes marginal significance value of the F statistic. The optimal delay parameter *d* is determined such that $\hat{d}$ = argmin MSV for $1 \leq d \leq 12$. See for instance (Teräsvirta, 1994) for the argument behind this rule of minimising the marginal significance value.

We then employ the non-linear least square procedure to estimate the ESTAR (1) model for this return series. The estimators $\hat{a}$, $\hat{b}$, $\hat{\gamma}$ and $\hat{\sigma}_{\varepsilon}^{2}$ are accordingly, 0.437, –0.425 and 1.226 and 105.223. Substituting these estimates into Equation 1 yields the following model:

$$y_t = -1.1811 + \underset{(0.177)^{**}}{0.437}\,(y_{t-1} + 1.1811)$$

$$-\ 0.425(y_{t-1} + 1.811)\,[1 - \exp(-1.226(y_{t-d} + 1.811)/110.293] + \varepsilon_t \qquad (3)$$

(0.239)* (0.895)

where figures in parentheses denote standard errors of estimates and asterisks * and ** imply significance at 5% and 10% respectively. This resulting model is utilised to generate bootstrap replications to analyse the statistical properties of the ESTAR estimators.

Bootstrap procedures

In order to accomplish our task in this study, we replicate N= {50, 100, 500, 1000, 5000, 10000} independent bootstrap estimators for each of the parameter Ω^*; $\Omega^* = \{\hat{a}*, \hat{b}*, \hat{\gamma}^*\}$ using the following estimation procedures:

1. Repeat the loop from Step 2 to Step 4 for replication number $i = 1, ...,N$.
2. Simulate the ith set of bootstrap random errors, ε_{it}^* from $\hat{\varepsilon}_t \sim G(0, 105.223)$ for t = 1, ..., 3s, where s is the sample size for each loop, which has the same number of observations as the original return series. $G(\cdot)$ stands for unknown empirical distribution in our case.
3. This ε_{it}^* sample together with the pre-sample estimates of u, a, b, γ and σ_y^2 are substituted into Equation 1 to recursively generate the bootstrap sample of y_{it}, denoted by y_{it}^* starting from an arbitrary chosen initial value of $y_{i0}^* = 0$. To eliminate the effects of starting values, we generate 3s bootstrap sample but use only the last s observations.

4. Similar to Equation 1, the simulated series in the preceding step is estimated with the non-linear least square procedure. We denote the related parameter by $\hat{a}_i^*$, $\hat{b}_i^*$ and $\hat{\gamma}^*$ to signify that there are estimators of the *i*th set bootstrap data.
5. Retrieve the resulting estimators $\hat{a}_i^*$, $\hat{b}_i^*$ and $\hat{\gamma}^*$ for i = 1, ..., N for the purpose of statistical analysis.

Statistical Analysis

The N bootstrap estimators for the parameter Ω^* = { $\hat{a}$*, $\hat{b}$*, $\hat{\gamma}^*$) are subjected to statistical properties analysis. The bootstrap estimates of the mean, $\overline{\Omega}^*$ and the standard error, $s_{\Omega*}$ are accordingly computed as:

$$\overline{\Omega}^* = (1/n)\sum_{i=1}^{N}\Omega_i^* \qquad \text{for } \Omega_i^* = \{\hat{a}*,\ \hat{b}*,\ \hat{\gamma}^*) \tag{4}$$

and

$$s_{\Omega*} = \left[(1/(n-1)\sum_{i=1}^{N}(\Omega_i^* - \overline{\Omega}_i^*)^2\right]^{1/2} \tag{5}$$

Meanwhile, the bootstrapped 100(1 – 2α)% confidence interval for the series Ω^* can be obtained by first sorting the series in ascending order and then locate the 100αth and the 100(1 –α)th percentiles. This yields the required confidence interval, $[\Omega_\alpha^*, \Omega_{(1-\alpha)}^*]$ where Ω_α^* and $\Omega_{(1-\alpha)}^*$ stand for the corresponding Ω^* values of the

100αth and the $100(1-\alpha)$th percentiles respectively. This study computes the 90%. 95% and 99% confidence intervals for Ω^*. In order to further examine the distribution of the series Ω^*, we also compute the length and shapes of the confidence intervals, using the definitions:

$$length = \Omega^*_{(1-\alpha)} - \Omega^*_{\alpha} \tag{6}$$

and

$$shape = (\Omega^*_{(1-\alpha)} - \overline{\Omega}^*)/(\overline{\Omega}^* - \Omega^*_{\alpha}) \tag{7}$$

Confidence interval with a shorter length implies a more accurate estimate of parameter and thus is always preferred to a longer one. While plotting the series Ω^*_i helps in showing the skewness of the distribution of the series Ω^*, the shape as defined in Equation 7 provides statistical information on the symmetrical property of the distribution. Shape $\neq$ 1 implies asymmetry, whereas shape = 1 implies symmetrical distribution.

Another important aspect of this paper is the study of the replication size effect of the bootstrap parameter Ω^*. This may be done by calculating the confidence intervals for different replication size N (50, 100, 500, 1000 and 5000, in addition to 10000 replications) with sample size s = 134. We compute the 90% and 95% confidence intervals for these replication sizes.

RESULTS AND DISCUSSIONS

Statistical Properties

The statistical properties of the bootstrap ESTAR parameters for different bootstrap sizes ranging from 50 to 10000 are summarized in Table 2. Two important conclusions could be drawn from this table. First, there is a clear sign that all parameter estimates approach their corresponding *true* values as shown in Equation 3. A typical example is given by the bootstrap estimates of the transition parameter γ. Table 2 depicts that its estimate for 50, 100, 500, 1000, 5000 and 10000 bootstrap replications are, in that order, –1.6327, –1.6632, –1.4989, –1.2463, –1.281 and –1.2495 (Panel 2C). Tracing from this trend, it is reasonable for one to expect that the bootstrap estimates of this transition parameter will converge to its *true* value, –1.2255 as the bootstrap size approaches infinity. The bootstrap estimates of both the linear parameter (Panel 2A) and non-linear parameter (Panel 2B) also give similar indication. One major implication of this finding is that the bootstrap estimates are *consistent*. Second, there is no clear pattern that may allow us to conclude that the bootstrap ESTAR estimates in this study follow the normal distribution. The kurtosis (skewness) values of linear parameter a, non-linear parameter, b and transition parameter γ are 0.8157 (–0.0314), 0.0773 (0.2681) and 0.2681 (–0.1651) respectively for the maximum bootstrap size in this study, that is 10000. It is well understood that the kurtosis and skewness values are 3 and zero respectively for a standard normal distribution. Thus, our results indicate that these estimates do not follow the standard normal distribution even when the replication size is as large as 10000. One major implication of this finding is that the estimates of exchange rate return series models need not be normal in nature and any interpretation based on normality assumption should be handle with care.

Table 2
Statistical Properties of Bootstrap ESTAR Parameters

Bootstrap Size	Mean	Variance	Kurtosis	Skewness
Panel 2A: Linear Parameter, *a* (*True value* = 0.4372)				
50	0.5375	0.0462	2.0636	1.2443
100	0.5378	0.0493	12.5077	2.7538
500	0.4759	0.0262	0.6373	0.5207
1000	0.4669	0.0405	1.0025	–0.0103
5000	0.4634	0.0374	1.3097	0.1916
10000	0.4620	0.0273	0.8157	–0.0314
Panel 2B: Non-linear Parameter, *b* (*True value* = –0.4253)				
50	–0.8012	0.7293	2.6508	–0.9248
100	–0.6909	0.1994	8.1625	–2.4458
500	–0.5592	0.0819	1.8644	–0.7908
1000	–0.5479	0.1228	0.3573	–0.2705
5000	–0.5509	0.0831	2.7539	0.6352
10000	–0.5475	0.0616	0.0773	0.2681
Panel 2C: Transition Parameter, γ (*True value* = –1.2255)				
50	–1.6327	1.5189	2.6508	–0.9248
100	–1.6632	3.0283	8.7408	–2.2064
500	–1.4989	1.2936	2.4014	–0.7909
1000	–1.2463	0.6534	0.3579	–0.0103
5000	–1.2801	0.6826	0.5838	–0.4137
10000	–1.2495	0.5901	0.2681	–0.1651

Note: The data generating process of our ESTAR (1) model is constructed as

$$y_t^* = u + a\,(y_{t-1}^* - u) + b\,(y_{t-1}^* - u)\,[1 - \exp(-\gamma(y_{t-d}^* - u)/\sigma_y^2\,] + \varepsilon_t^* \qquad (8)$$

where $u = -1.1811$, $a = 0.4372$, $b = -0.4253$, $\gamma = -1.2255$, $d = 3$, $\sigma_y^2 = 110.2930$ and $\varepsilon_t^* \sim G\,(0,\ 105.223)$. This specification actually follows that of the estimated ESTAR (1) model for the YEN/USD return series. See text for more details.

In order to gain a clearer picture on the distribution of the population parameters (visualised by the bootstrap parameters of this study), we plot the histograms, together with the normal curves, of the estimated parameters for each bootstrap size *N*. Figure 1 shows the various distributions of the bootstrap linear parameter *a* of the ESTAR model with respect to the bootstrap size. The plots show that this parameter is positively skewed for *N* ranges from 50 to 10000. Meanwhile, the distributions of the bootstrap non-linear parameter *b* and transition parameter γ are found to be negatively skewed in general, as depicted in Figure 2 and Figure 3 respectively. These observations of non-normality in the distribution of the estimated parameters are in line with our preceding statistical analysis.

Figure 1
Histograms and Normal Curves for Bootstrapped Linear Parameter

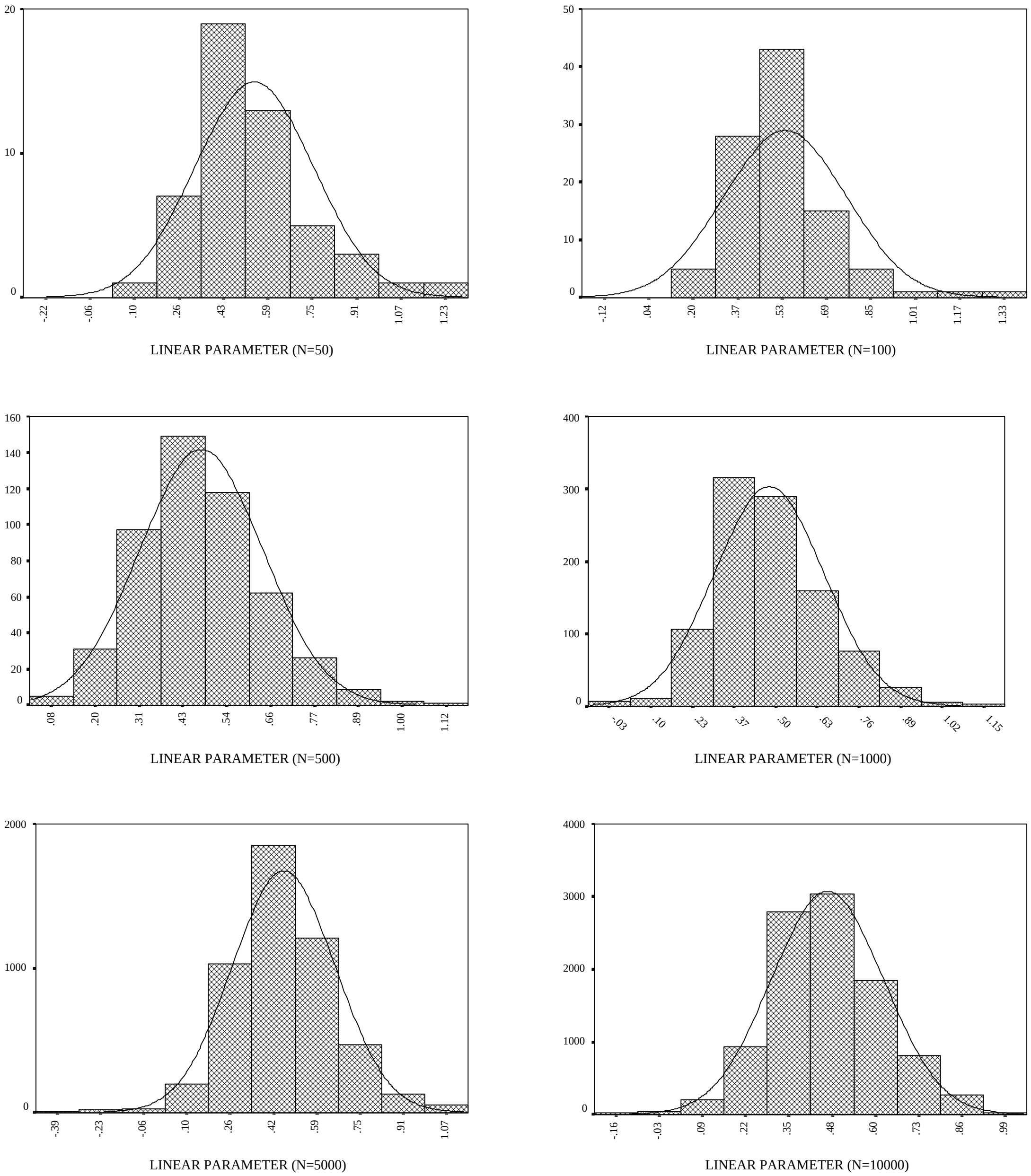


Figure 2
Histograms and Normal Curves for Bootstrapped Non-linear Parameter

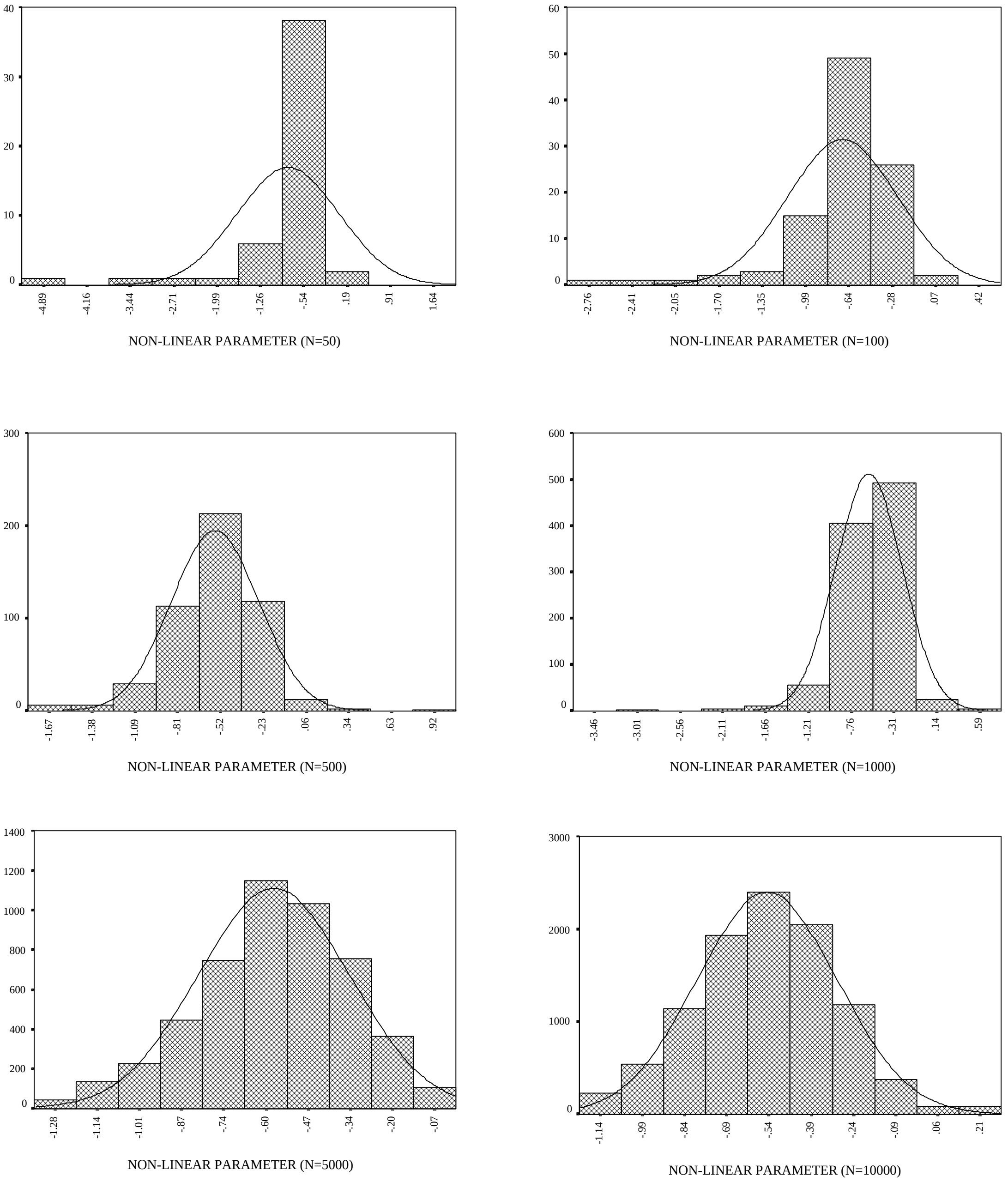
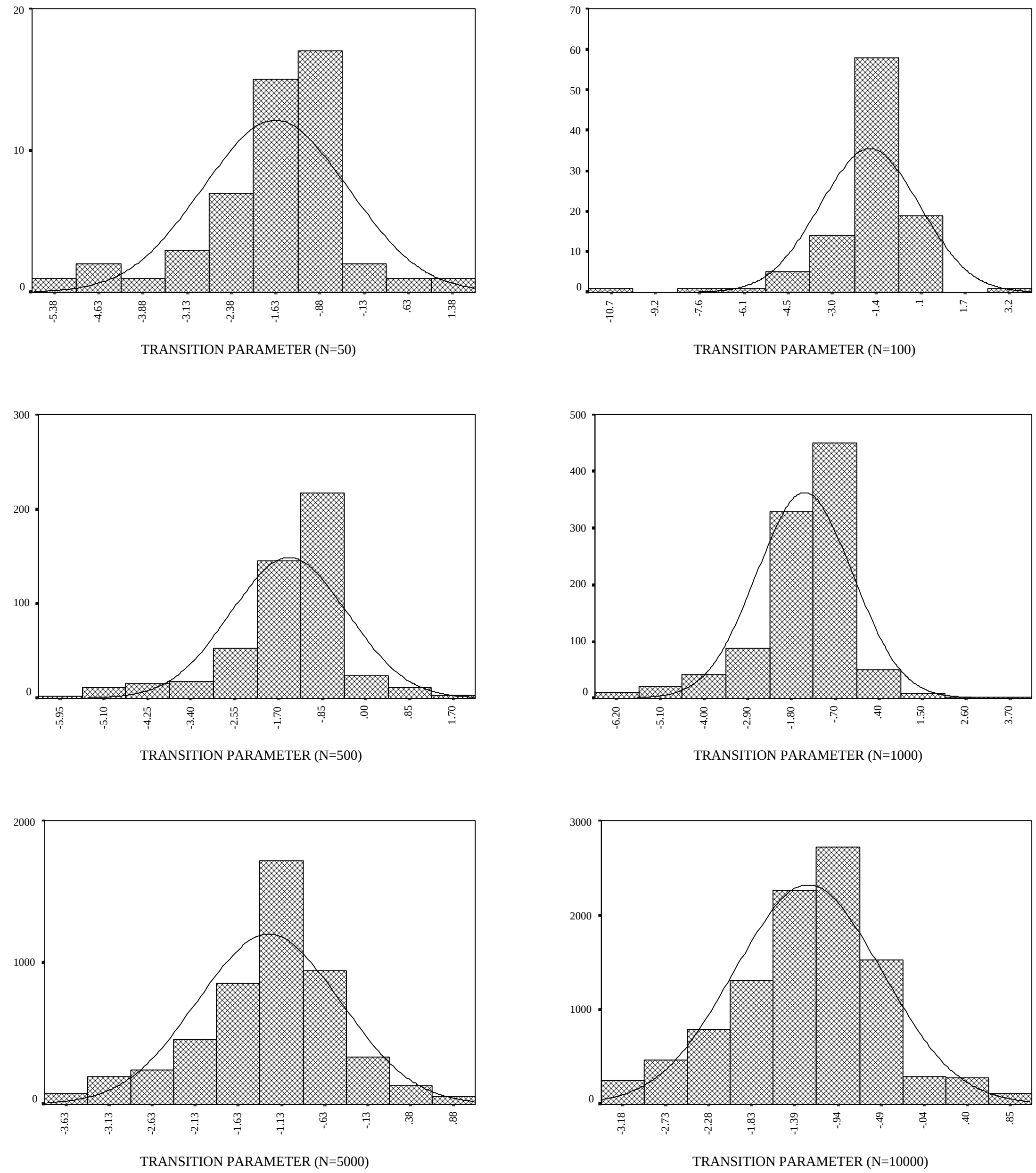


Figure 3

Histograms and Normal Curves for Bootstrapped Transition Parameter



Confidence Interval

The 90% and 95% confidence intervals for the linear, non-linear and transition parameters are computed for various bootstrap sizes and the results are tabulated in Table 3 and Table 4 respectively. Results of estimation show that the length of bootstrap confidence interval (both 90% and 95%) for the linear (transition) parameter is the shortest (longest) amongst the three parameters regardless of the bootstrap replications. For instance, the length of bootstrap 90% (95%) confidence intervals for the linear, non-linear and transition parameters are accordingly, 0.549 (0.6546), 0.8742 (0.1014) and 2.8829 (3.6955). This implies that it is most difficult for us to estimate the transition parameter with precise accuracy, as compare to the other two parameters. This finding verifies previous empirical reports that the transition parameter is difficult to estimate (see for instance, Taylor and Peel, 2000). Tables 3 and 4 also reveal that all the bootstrap ESTAR estimates follow asymmetrical distribution, since the shape of the confidence intervals in all cases are not equal to one. This finding is not surprising, as we have already found earlier in this study that bootstrap parameters are not normally distributed.

Table 3
90% Confidence Interval of the Parameter Estimates of ESTAR Model

Bootstrap Size	Mean	Standard Deviation	Lower	Upper	Length	Shape
Panel 3A: Linear Parameter, *a* (*True value* =0.4372)						
50	0.5375	0.2149	0.3101	0.9231	0.6130	1.6839
100	0.5378	0.2210	0.3101	0.9189	0.6088	1.6737
500	0.4759	0.1615	0.2348	0.7706	0.5358	1.2159
1000	0.4669	0.1724	0.2434	0.7760	0.5326	1.2947
5000	0.4634	0.1919	0.1833	0.7031	0.5198	0.8181
10000	0.4620	0.1653	0.2082	0.7391	0.5309	1.0918
Panel 3B: Non-linear Parameter, *b* (*True value* =–0.4253)						
50	–0.8012	0.8540	–1.4389	–0.1937	1.2452	0.9526
100	–0.6909	0.4465	–1.1411	–0.2156	0.9255	1.0558
500	–0.5592	0.2861	–1.0534	–0.1733	0.8801	0.8466
1000	–0.5479	0.3504	–1.0915	–0.1752	0.9163	0.7214
5000	–0.5509	0.2883	–1.1087	–0.1289	0.9798	0.7565
10000	–0.5475	0.2481	–1.0206	–0.1464	0.8742	0.8865
Panel 3C: Transition Parameter, γ (*True value* =–1.2255)						
50	–1.6327	1.5188	–4.2956	–0.3618	3.9338	0.4773
100	–1.6632	2.9981	–4.7232	0.3558	5.0790	0.6598
500	–1.4989	1.2910	–3.9169	–0.0494	3.8675	0.5995
1000	–1.2463	1.4588	–3.8426	0.3063	4.1489	0.5980
5000	–1.2801	0.6801	–2.9375	0.0283	2.9658	0.7894
10000	–1.2495	0.6770	–2.8543	0.0286	2.8829	0.7964

Note: Refer to Table 2.

Table 4
95% Confidence Interval of the Parameter Estimates of ESTAR Model

Bootstrap Size	Mean	Standard Deviation	Lower	Upper	Length	Shape
Panel 4A: Linear Parameter, *a* (*True value* = 0.4372)						
50	0.5375	0.2149	0.2770	1.0177	0.7407	1.8434
100	0.5378	0.2210	0.2770	1.1692	0.8922	2.4210
500	0.4759	0.1615	0.1868	0.8151	0.6283	1.1733
1000	0.4669	0.1724	0.2026	0.8394	0.6368	1.4094
5000	0.4634	0.1919	0.1185	0.9135	0.7950	1.3050
10000	0.4620	0.1653	0.1517	0.8063	0.6546	1.1096
Panel 4B: Non-linear Parameter, *b* (*True value* = –0.4253)						
50	–0.8012	0.8540	–2.6699	–0.1358	2.5341	0.3561
100	–0.6909	0.4465	–1.7935	–0.1471	1.6464	0.4932
500	–0.5592	0.2861	–1.2333	–0.0730	1.1603	0.7213
1000	–0.5479	0.3504	–1.3319	–0.0837	1.2482	0.5921
5000	–0.5509	0.2883	–1.1127	0.0665	1.1792	1.0990
10000	–0.5475	0.2481	–1.1218	–0.0804	1.0414	0.8133
Panel 4C: Transition Parameter, γ (*True value* = –1.2255)						
50	–1.6327	1.5188	–3.4779	0.9612	4.4391	1.4058
100	–1.6632	2.9981	–3.6799	0.7002	4.3801	1.1719
500	–1.4989	1.2910	–4.6725	0.4892	5.1617	0.6264
1000	–1.2463	1.4588	–4.7747	0.6064	5.3811	0.5251
5000	–1.2801	0.6801	–3.2291	0.3973	3.6264	0.8606
10000	–1.2495	0.6770	–3.3034	0.3921	3.6955	0.7993

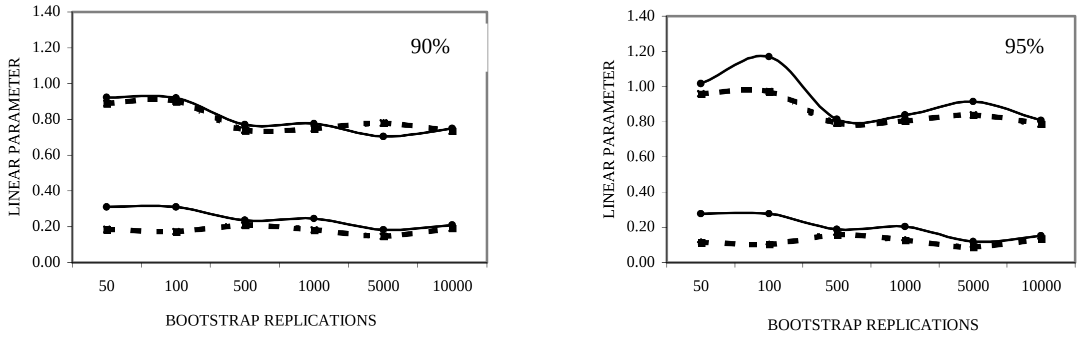
Note: Refer to Table 2.

In order to gain further insight on the impact of asymmetrical distribution of the ESTAR parameters on the confidence intervals, we compute the standard normal confidence intervals for comparison. The results of this exercise are plotted in Figure 4. The continuous and dotted lines in Figure 4 represent the bootstrap and standard normal confidence interval respectively. It is obvious from this figure that for bootstrap replications beyond $N = 500$, the two confidence intervals tend to converge. This is especially true for the linear and non-linear ESTAR parameters, whereby the two intervals achieve convergence when $N = 10000$. This phenomenon is not seen in the case of transition parameter. Rather, both the 90% and 95% standard normal confidence intervals ($N = 10000$) are contained in their corresponding bootstrap confidence intervals. This indicates that the standard normal over rejects the significance of the estimated transition parameter in empirical studies. This is in accordance with the work of Godfrey (1998), which among others, reports that bootstrap estimates appear to greatly reduce the problem of over-rejection of *true* models. Hence, bootstrap estimates are more reliable than standard normal confidence interval.

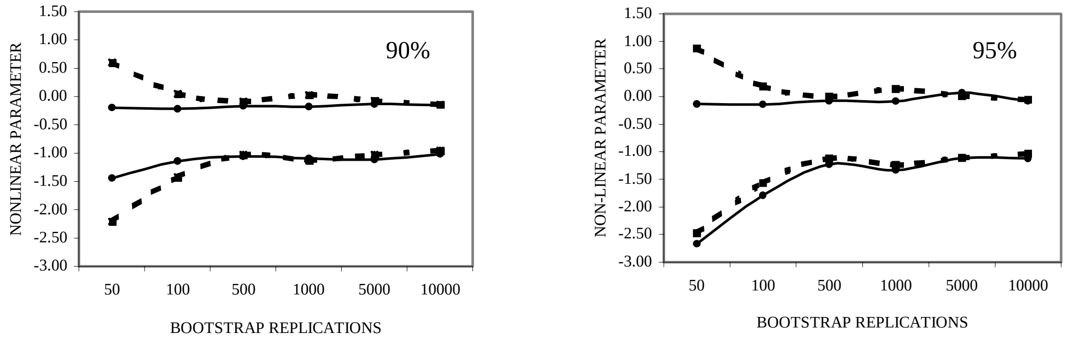
Figure 4

90% and 95% Bootstrap and Standard Normal Confidence Intervals

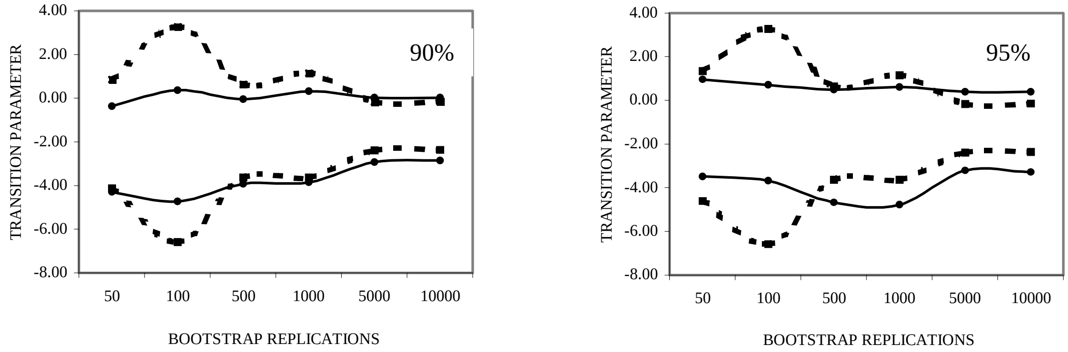
Linear Parameter



Non-linear Parameter



Transition Parameter



Note: Continuous and dotted lines denote bootstrap and standard normal confidence interval respectively.

CONCLUSIONS

This study uses bootstrap simulation to investigate the population statistical properties of the parameters in the recent widely applied exponential Smooth Transition Autoregressive (ESTAR) model. Based on the estimated ESTAR (1) model for YEN/USD exchange rate return series, we generate bootstrap series ranging from 50 replications to 10000 replications. ESTAR estimation procedure is reapply to each of these series to obtain the bootstrap parameter estimates. The parameters of interest include the linear, non-linear and transition parameters. Results of this study among others indicate that all bootstrap estimates approach their *true* values as the bootstrap replications increase. This bootstrap estimates may be regarded as a consistent estimates for the population parameters. However, there is no clear pattern that the population distribution of the ESTAR parameters will follow the standard normal distribution. In fact this study finds that their empirical distribution are asymmetrical in nature with the linear parameter being positively skewed and the non-linear and transition parameters being negatively skewed. Hence, any interpretation based on normality assumption on the ESTAR model must be handle with care. In particular, we find that the normality theory over rejects the significance of the estimated transition parameter. Another interesting point worth mentioning is that the minimum number of replications needed for the bootstrap confidence interval to be a good approximate of the standard normal one is 500.

REFERENCES

Baum, C. F., Barkoulas, J. T. and Caglayan, M. 2001. Non-linear Adjustment to Purcahsing Power Parity in the Post-Bretton Woods Era. *J. International Money and Finance*. 20 : 379–399.

Choo, W. C., Ahmad, M. I. and Midi, H. 2001. A Parametric Bootstrap Simulation Study in EGARCH Model. Pertanika J. Sci & Technol. 9(2): 169 – 188.

Efron, B. and Tibshirani, R. J. 1993. *An Introduction to the Bootstrap*. New York: Chapman and Hall.

Hurvich, C. M. and Tsai, C. L. 1989. Regression and Time Series Model Selection in Small samples. Biometrika. 76: 297 – 307.

Godfrey, L. G. (1998) Tests of Non-nested Regression Models: Some Results on Small Sample Behaviour and the Bootstrap, J. Econometrics. 84: 59 – 74.

Liew, K. S. 2002. Modelling and Forecasting of Exchange Rate Using Smooth Transition Autoregressive Models. Unpublished Master Thesis.

Liew, K. S. and Baharumshah, A. Z. 2002. How Well Ringgit-Yen Rate Fits the Nonlinear Smooth Transition Autoregressive and Linear Autoregressive Models. Pertanika J. Social Sci. & Humanities. Under Review.

Liew, K. S., Baharumshah, A. Z. and Roh, J. 2002. Forecasting Performance of Exponential Smooth Transition Autoregressive Exchange Rate Models. Manuscript.

Luukkonen, R. Saikkonen, P. and Teräsvirta, T. 1988. Testing linearity against Smooth Transition Autoregressive Models. *Biometrika*. 75 : 491– 499.

Sarantis, N. 1999. Modeling Non-linearities in Real Effective Exchange Rates. J. Int. Money & Finance. 18: 27 – 45.

Sarno, L. 2000. Real exchange rate behaviour in high inflation countries: empirical evidence from Turkey, 1980 – 1997. *Applied Economics Letters*. 7 : 285–291.

Taylor, M. P. & Peel, D. A. 1997. Nonlinearities in Real Exchange Rate Adjustment during the Recent Float: Empirical Evidence and Monte Carlo Analysis, *Discussion Paper*, Institute of Economics and Statistics, University of Oxford.

Taylor, M. P. & Peel, D. A. 2000. Non-linear adjustment, long-run equilibrium and exchange rate fundamentals. *J. International Money and Finance*. 19 : 33–53.

Teräsvirta, T. 1994. Specification, estimation, and evaluation of Smooth Transition Autoregressive Models. *J. Amer. Statist. Assoc.* 89(425) : 208–218.