Analysis of convergence process of East German economy on the base of a two-region growth model

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ABSTRACT

The main purpose of the paper is to analyse the speed of regional convergence of unified Germany on the basis of a two-region growth model with public productive spending. The model explains the dynamics of convergence, taking into account high fixed wages in East Germany, recent dynamics of public productive spending, government subsidies for private investments and a structure of human capital in both parts of Germany. The paper detects a new possible cause of the recent halt of East German economic convergence, namely the influence of pre-unification educational infrastructure dynamics. Using the model, we estimate numerically different reform strategies, which have the goal to increase the speed of convergence.

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INTRODUCTION

The process of regional convergence has been one of the most important issues in German economic development in recent years. East Germany has inherited a 'backward' economy from the communist era. Moreover, German unification in 1990 was followed by the dramatic decline of output in the eastern federal states. The depression period ended in 1991 with the East German GDP per capita at 35% of the western level. In 1996 this indicator reached 56%, and some optimistic forecasts suggested full convergence in 20-25 years. However, since 1996 the convergence process has stalled. Recent studies provide different explanations for the current stagnation. Sinn (2000) argues that the current halt of convergence results from excessively high East German wages, which have been agreed on by trade unions and employers organisations. Sinn (2000) also considers the termination of the investment subsidy law (Fördergebietsgezetz) as a possible cause of the problem. Klodt (2000) treats an underutilization of capacity of East German firms as the cause of the convergence standstill. The present paper sets up the model, which explains the dynamics of convergence, taking into account high fixed wages in the East, recent dynamics of public productive spending, government subsidies for private investments and a structure of human capital in both parts of Germany. This model allows to analyse causes of the current stagnation and to estimate whether they will continue to retard the convergence. The paper detects a new possible cause of deficient labor productivity in East Germany, namely the influence of pre-unification educational infrastructure dynamics.

A number of studies define reforms, which can help accelerate the much-needed convergence. Some studies offer a number of measures, which allow for wages undercutting productivity growth. According to Sinn (2000) these measures include a new system of social assistance, opt-out classes and wage asset swaps. Wurzel (2001) supports the idea that the scope for plant level agreements should be widened further. Funke and Strulik (2000) as well as May (2001) assert that the speed of convergence depends on the effort in infrastructure accumulation in the new states. Gundlach (2001) gives an opinion that namely educational infrastructure spending can provide further convergence. The present paper compares these measures and defines the conditions that allow for the above-mentioned reforms. Both the convergence speed and unemployment level in the whole Germany are taken into account when analysing possible results of the reform strategies. In some cases excessively high convergence speed can lead to an undesirable increase in West German unemployment. This paper sets up a dynamical approach for policy recommendations. The main model of the paper and the relevant numerical solutions obtained from it show how the government can optimally react to the results of wage negotiations. The main numerical finding is that the

government should increase public infrastructure spending in the new states only if the speed of wage convergence exceeds 0.4% per year.

Formally, the main model of this paper can be characterised as a two-region growth model with government productive spending and human capital accumulation. Some parts of the model are based on the ideas of Lucas (1988), Barro (1990), Ono/Shibata (1992), and Funke/Strulik (2000). The novelty of the model consists in the introduction of a fixed wage ratio between regions, application of a monopoly trade union model in the two-region dynamic model, adoption of a continuous form of collateral capital restrictions as well as disaggregating labor force by a graduation date. Such a disaggregating has made a decisive contribution to the structure of the model, because it allows to explain the current standstill in economic convergence of the new states through decreasing proportion of the educational infrastructure between East and West Germany before unification.

The paper is organized as follows: the first section discusses the empirical basis of the problem; the second section, entitled "Formalization," introduces the basic model; the third section analyses the causes of the recent halt of East German economic convergence; and the final section compares four different strategies for possible reforms of the East German economy.

1. EMPIRICAL BASIS

Characteristic features of the model are based on recent dynamics of basic indicators for the convergence process in East Germany. Tables 1 and 2 summarize the empirical base for the development of the model.

Table 1. Basic economic indicators in East Germany in percent of West German level

	1997	1998	1999	2000	2001	2002
Gross domestic product per capita	56.1	55.5	55.7	55.2	55.7	56.2
Capital stock per capita	54.8	57.3	60.6	63.1	65.5	67.0
Average wage	69.8	69.8	70.0	70.0	70.0	70.0
Traffic infrastructure stock per capita	44.7	45.5	46.1	46.8	47.6	48.4

Source: *Statistisches Bundesamt Deutschland, own calculations*

Table 2. Unemployment rates in percent

	1997	1998	1999	2000	2001	2002
West Germany	10.0	10.0	9.9	9.8	10.0	10.0
East Germany	15.9	17.0	16.2	16.1	16.0	16.0

Sources: *Statistisches Bundesamt Deutschland, own calculations*

At first glance, tables 1 and 2 show paradoxical time series. The first paradox is the constant output ratio between East and West Germany. The permanent growth of capital stock and traffic infrastructure stock in the new states comparing to the old states must lead to an increase in output convergence rate. However, in reality the output grows at the equal speed in both East and West Germany. In the present paper this paradox is solved by introducing a detailed formalization of human capital dynamics and by disaggregating the labor force variable by the date of graduation. Section 3 provides detailed analysis of this paradox in the framework of the model. The negative rate of growth of human capital proportion between the two parts of Germany will be explained. However, right after such an explanation, the second paradox appears. Consider the permanent decrease of human capital proportion counterbalances the permanent increase in traffic infrastructure stock proportion so that output proportion remains constant. For this condition it is expected that private capital demand proportion between the new and the old states will not change and the capital stock proportion will remain constant. Despite of this expectation the capital stock proportion between East and West Germany continues to grow. This paradox is solved by introducing collateral capital restrictions in the model and by assuming that the potential demand for private investments in East Germany[1] significantly exceeds the real private investments in the new states. Therefore, we assume that the growth rate of capital stock proportion between the two parts of Germany is defined essentially by collateral capital restrictions, in addition to the other production factors growth rates.

2. FORMALIZATION

Consider a two-region economy. The model includes equations for households, firms and government behaviour. The production function has the form

$$Y_i = A_i G_i^{\alpha(i)} K_i^{\beta(i)} (H_i L_i)^{1-\alpha(i)-\beta(i)} \overline{H}_i^{\chi(i)},\ 0 < \alpha, \beta, \chi < 1,\ 0 < \alpha + \beta < 1, \qquad (2.1)$$

where $i = W, E$ denotes West or East Germany, $G_i(t)$ is the capital stock of public traffic infrastructure, $K_i(t)$ represents the private capital, $L_i(t)$ is the size of the work force actually employed at time t, $H_i(t) = \overline{H}_i(t)$ stands for the efficiency of human capital, and A is the productivity parameter. The bar over H indicates that this quantity is given for each individual maximizer. The variables $\overline{H}_i$ serve to include the external effect of human capital in the model. It is assumed

$$A_E = A_W = A, \qquad (2.2)$$

[1] Potential demand for private investments – demand for private investments in case collateral capital restriction are not active.

therefore, we suppose that the cause of the productivity gap between the new and the old federal states is the "backward character" of East German traffic infrastructure and human capital level.

The firm sector in region $i = W, E$ is described by:

$$\int_0^\infty e^{-rt}\left\{(1-\tau)(1-\tau_i)\left[A_i G_i^{\alpha(i)} K_i^{\beta(i)} (H_i L_i)^{1-\alpha(i)-\beta(i)} \overline{H}_i^{\chi(i)} - \omega_i L_i\right] - I_i + B_i - \frac{1}{\overline{K}_i - I_i}\right\} dt \to \max_{I_i, L_i}, \quad (2.3)$$

subject to

$$\dot{K}_i = I_i - \delta K_i, \quad (2.4)$$

$$I_i < \overline{K}_i, \quad (2.5)$$

where $\omega_i(t)$ denotes the average wage level, $I_i(t)$ is private investments at time t, δ is depreciation rate, $r(t)$ is interest rate, and $B_i(t)$ is investment subsidies, τ is the rate of corporate tax (Körperschaftsteuer), τ_i is the average rate of trade tax (Gewerbesteuer) in region $i = E, W$, and $\overline{K}_i$ is an exogenous constant, which defines collateral capital restrictions. The reciprocal of $(\overline{K}_i - I_i)$ was introduced in the firms' objective function (2.3) in order to keep the continuity of the model structure.

The federal government subsidizes private investments in the new states in order to support the reconstruction of the East German economy. The East German firms receive additional bonuses from the government. In general, the size of the bonus is directly proportional to the investment volume:

$$B_i = h_i I_i, \quad h_W = 0, \quad 0 \le h_E \le 1. \quad (2.6)$$

Further, we refer to h_E as the "subsidies rate". Now we can rewrite (2.3)

$$\int_0^\infty e^{-rt}\left\{(1-\tau)(1-\tau_i)\left[A_i G_i^{\alpha(i)} K_i^{\beta(i)} (H_i L_i)^{1-\alpha(i)-\beta(i)} \overline{H}_i^{\chi(i)} - \omega_i L_i\right] - (1-h_i) I_i - \frac{1}{\overline{K}_i - I_i}\right\} dt \to \max_{I_i, L_i}. \quad (2.7)$$

The Hamiltonian of (2.7) and (2.4) reads:

$$H = (1-\tau)(1-\tau_i)\left[A_i G_i^{\alpha(i)} K_i^{\beta(i)} (H_i L_i)^{1-\alpha(i)-\beta(i)} \overline{H}_i^{\chi} - \omega_i L_i\right] - (1-h_i) I_i - \frac{1}{\overline{K}_i - I_i} + \xi(I_i - \delta K_i). \quad (2.8)$$

The first-order conditions are

$$\xi_i - (1-h_i) - \frac{1}{(\overline{K}_i - I_i)^2} = 0, \quad (2.9)$$

$$(1-\tau)(1-\tau_i)\left\{\left[1-\alpha(i)-\beta(i)\right] A_i G_i^{\alpha(i)} K_i^{\beta(i)} H_i^{1+\chi(i)-\alpha(i)-\beta(i)} L_i^{-\alpha(i)-\beta(i)} - \omega_i\right\} = 0. \quad (2.10)$$

The co-state equation is

$$\dot{\xi}_i = r\xi_i - (1-\tau)(1-\tau_i)\beta(i) A_i G_i^{\alpha(i)} K_i^{\beta(i)-1} H_i^{1+\chi(i)-\alpha(i)-\beta(i)} L_i^{1-\alpha(i)-\beta(i)} + \delta\xi_i. \quad (2.11)$$

To formalize the dynamics of consumption we describe a representative household. The

utility functional of the representative household in each region is defined as

$$U_i = \int_0^\infty \frac{c_i^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \qquad (2.12)$$

where $c_i(t)$, $i = W, E$ denotes west or east consumption per capita at time t, ρ is the time preference rate, and $\sigma^{-1} < 1$ is the intertemporal elasticity of substitution. A representative household maximizes U_i subject to the budget constraint

$$(1+\tau_{IND})c_i + \dot{a}_i = r\, a_i + (1-\tau_L(\omega_i))\omega_i + z_i, \qquad (2.13)$$

where $a_i(t)$ represents the value of real wealth of the household, $z_i(t)$ represents transfers from the government, τ_{IND} is the sum of average rates of all indirect taxes, and $\tau_L(\omega_i)$ is the average rate of wage tax (Lohnsteuer) in the region $i = E, W$. The rate of wage tax is defined as a function of the wage level, since this tax is progressive one.

The Hamiltonian of this problem is

$$H = \frac{c_i^{1-\sigma} - 1}{1-\sigma} + \eta\left[ra_i + (1-\tau_L(\omega_i))\omega_i + z_i - (1+\tau_{IND})c_i\right]. \qquad (2.14)$$

The first-order condition is

$$c_i^{-\sigma} - \eta(1+\tau_{IND}) = 0, \qquad (2.15)$$

The co-state equation is

$$\dot{\eta} = (\rho - r)\eta. \qquad (2.16)$$

From (2.15) and (2.16), one obtains the Ramsey rule

$$\frac{\dot{c}_i}{c_i} = \frac{1}{\sigma}(r - \rho). \qquad (2.17)$$

Therefore,

$$\frac{\dot{C}_i}{C_i} = \frac{1}{\sigma}(r - \rho), \qquad (2.18)$$

where $C_i(t)$, $i = W, E$ denotes total west or east consumption at time t,

The next step of the formalization is the description of the labor market. The formalization is based on the fact that East German wages depend upon West German wages. The most important wage negotiations for the new states have been completed in 1991. Only West German entrepreneurs participated in these negotiations, because at that time private firms in the new states were at the stage of creation. West German negotiators strived to avoid an emergence of high-competitive East German firms. The negotiations have defined the proportion between East and West German wages. We express this fact formally by

$$\omega_E = \gamma\omega_W \quad , \quad 0 < \gamma < 1, \qquad (2.19)$$

where $\gamma(t)$ is the exogenously determined wage ratio.

The next assumption about the labor market is that trade unions have the following utility function:

$$u_i = (L_i - \Psi_i)^{\Lambda}\left(\frac{\omega_i}{[1-\alpha(i)-\beta(i)]A_i G_i^{\alpha(i)} K_i^{\beta(i)} H_i^{1+\chi(i)-\alpha(i)-\beta(i)}}\right)^{\mathrm{P}}, \qquad (2.20)$$

$$u = D_E u_E + D_W u_W, \qquad (2.21)$$

The employment level $L_i(\omega_W)$ is defined from the solution of the optimisation problem (2.7), (2.4). The trade unions choose the wage level to maximize the utility function

$$u(\omega_W) \to \max_{\omega_W}, \qquad (2.22)$$

The following equations describe intra-regional migration

$$\dot{D}_W = m\left[\left(\frac{L_W}{D_W}\right)^{\mu} \omega_W^{1-\mu} - \left(\frac{L_E}{D_E}\right)^{\mu} \omega_E^{1-\mu}\right], \qquad (2.23)$$

$$\dot{D}_E = -\dot{D}_W, \qquad (2.24)$$

where D_i is the active population in each region, m and μ are preferences parameters of the immigrants.

The next part of the model is the description of public and private capital accumulation. We assume that a part of tax earnings is spent on public infrastructure accumulation. The total income of federal government (Bund), states (Länder) and communities (Gemeinden) is[2]

$$\begin{aligned} J = {} & (\tau_W + \tau - \tau_W \tau)\left[A_W G_W^{\alpha(W)} K_W^{\beta(W)} (H_W L_W)^{1-\alpha(W)-\beta(W)} \overline{H}_W^{\chi(W)} - \omega_W L_W\right] + \\ & + (\tau_E + \tau - \tau_E \tau)\left[A_E G_E^{\alpha(E)} K_E^{\beta(E)} (H_E L_E)^{1-\alpha(E)-\beta(E)} \overline{H}_E^{\chi(E)} - \omega_E L_E\right] + \\ & + \tau_{IND} C + \tau_L(\omega_W)\omega_W L_W + \tau_L(\omega_E)\omega_E L_E, \end{aligned} \qquad (2.25)$$

where

$$C = C_W + C_E. \qquad (2.26)$$

We denote $0 \le q_i \le 1$ as the share of public income, which is spent on traffic infrastructure accumulation in each region. Then we describe accumulation of traffic infrastructure by

$$\dot{G}_i = q_i J - \delta\, G_i, \quad i = W, E. \qquad (2.27)$$

Similarly

$$\dot{N}_i = n_i J - \delta\, N_i, \quad i = W, E, \qquad (2.28)$$

[2] We consider only most important taxes. The real value of public income exceeds the value of J, but its dynamics generally coincides with the dynamics of J.

where $N_i(t)$ denotes the current stock of educational infrastructure in each region and n_i is the share of public income which is spent on educational infrastructure accumulation in each region.

The dynamics of total capital stock K are represented by

$$\begin{aligned} C + \dot{K} &= AG_W^{\alpha(W)} K_W^{\beta(W)} (H_W L_W)^{1-\alpha(W)-\beta(W)} \overline{H}_W^{\chi(W)} + \\ &+ AG_E^{\alpha(E)} K_E^{\beta(E)} (H_E L_E)^{1-\alpha(E)-\beta(E)} \overline{H}_E^{\chi(E)} - (1-\varepsilon)J - \delta K \end{aligned}, \qquad (2.29)$$

where εJ is the total amount of transfers from government to households.

To complete the model we describe accumulation of human capital. We suppose the following rule

$$H_i(t) = \int_{t-T}^{t} H_i(s,t)ds, \qquad (2.30)$$

$$\dot{H}_i(s,t) = \frac{\psi L_i(t) H_i(s,t)}{D_i(t)}, \qquad (2.31)$$

$$H_i(s,s) = \lambda \int_{s-S}^{s} \frac{N_i(x)}{D_i(x)} dx. \qquad (2.32)$$

The equation (2.31) describes the "learning by doing" effect. The equation (2.32) describes the influence of educational infrastructure stock on the accumulation of human capital. Parameters ψ and λ are the weights of these two effects. The function $H_i(s,t)$ denotes the average level of human capital in the year t for the group of population s. This group consists of people, who have finished their education in the year s.

3. BASIC SCENARIO

In this section the basic solution of the model is considered. The dynamics of the fixed wage level in East Germany are still not fully predetermined. Therefore, we must consider different variants of an exogenously determined trajectory of $\gamma(t)$. As a checkpoint for further analysis we use the trajectory of fixed wage proportion, which provides a constant value of convergence rate

$$\theta = \frac{Y_E / D_E}{Y_W / D_W}, \qquad (3.1)$$

after 2003. The basic solution of the model generally coincides with the real trajectory of German economic indicators in 1997-2002. It verifies reasonable parameterisation of the model.

Table 3 summarizes model parameterisations. The result of the solution appears in Figure1.

Table 3. Model parameterisation.

Parameter	Description	Value
$G_W(0)$	Infrastructure in old states in 1997[1]	602.7
$G_E(0)$	Infrastructure in new states in 1997[1]	63.7
$K(0)$	Total private capital stock in 1997[1]	10372
$D_W(0)$	West German active population in 1997[2]	322
$D_E(0)$	East German active population in 1997[2]	76
$C(0)$	Total consumption in 1997[1]	1933
$\overline{K}_W$	Collateral capital restrictions in the old states[1]	1410
$\overline{K}_E$	Collateral capital restrictions in the new states[1]	155
σ	Reciprocal of intertemporal elasticity of substitution	2.5
$\tau_L(\omega)$	Average wage tax rate	$0.024\ \omega + 0.025$
τ	Proportion between corporate tax earnings and total profit of private firms before taxation	0.03
τ_W	Proportion between trade tax earnings and total profit of private firms before taxation in the old states	0.0306
τ_E	Proportion between trade tax earnings and total profit of private firms before taxation in the new states	0.0216
τ_{IND}	Sum of average rates of all indirect taxes	0.1714
$\alpha(W)$	Production elasticity for traffic infrastructure in the old states	0.05
$\beta(W)$	Production elasticity for private capital in the old states	0.39
$\chi(W)$	Production elasticity for external effect of human capital in the old states	0.2
$\alpha(E)$	Production elasticity for traffic infrastructure in the new states	0.03
$\beta(E)$	Production elasticity for private capital in the new states	0.39
$\chi(E)$	Production elasticity for external effect of human capital in the new states	0.2
A	Productivity parameter	0.95
δ	Depreciation rate	0.05
ρ	Time preference rate	0.02
q_E	Rate of government traffic infrastructure investments in the new states	0.007
q_W	Rate of government traffic infrastructure investments in the old states	0.054
n_E	Rate of government educational infrastructure investments in the new states	0.0025
n_W	Rate of government educational infrastructure investments in the old states	0.0026
h_E	Subsidies rate	0.4
$\gamma(t)$	Fixed wage ratio	See Fig. 1
Ψ_W	Parameter of trade unions utility function	145
Ψ_E	Parameter of trade unions utility function	61.5
Λ	Parameter of trade unions utility function	0.5
P	Parameter of trade unions utility function	2.2727
ε	Share of transfers from government to households in the total government spending	0.218
T	Average duration of personal economic activity	30

S	Average duration of education	10
λ	Weight of educational infrastructure influence	0.0424
ψ	Weight of "learning by doing effect" influence	0.1
m	Preferences of immigrants	0.1
μ	Preferences of immigrants	0.5
$N_W(t)$, $t \in [1957;1997]$	Educational infrastructure in the old states	$0.4554\,t - 879.37$
$N_E(t)$, $t \in [1957;1997]$	Educational infrastructure in the new states	3.9917

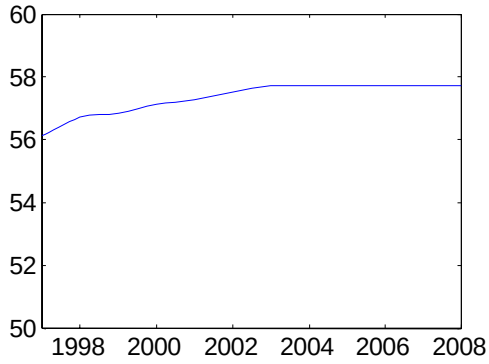
Sources: Statistisches Bundesamt Deutschland, own calculations.

Notes: 1.) Mrd DM in prices of 1995

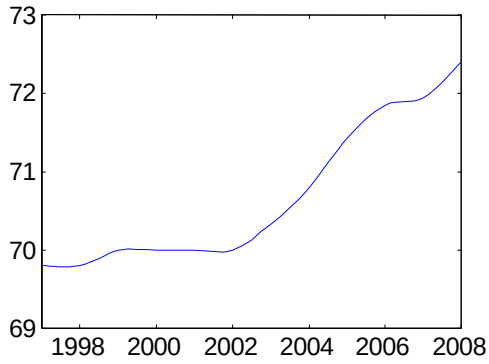
2.) 10^5 *inhabitants*

Figure 1. Basic solution.

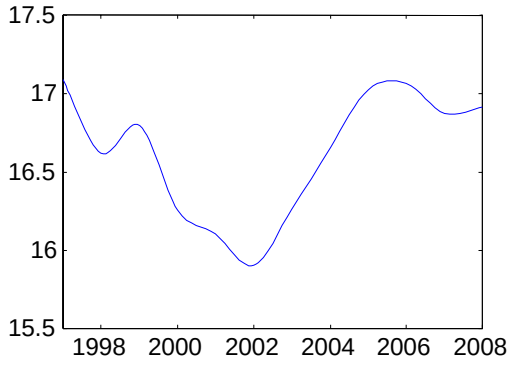
$\theta(t) \times 100\%$ - East German output per capita in % of West German level



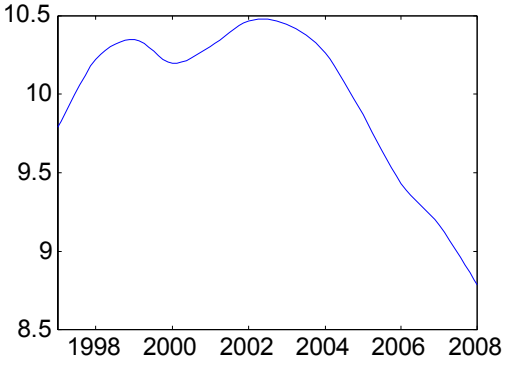
$\gamma(t) \times 100\%$ - fixed level of East German wages in % of West German level



Unemployment rate in East Germany(%)



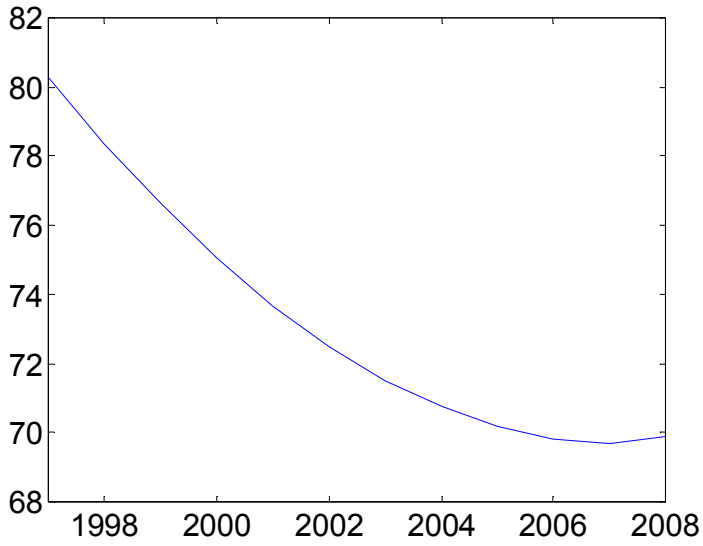
Unemployment rate in West Germany(%)



In the framework of the model and the basic solution, the dynamics of the current halt of convergence can be described as follows: Subsidization of the East German firms raises the demand for the private investments in the new states. This demand is extremely high, but it cannot be fully satisfied because of collateral capital restrictions. Nevertheless, East German capital stock grows at a great speed. On the other hand, before unification there was a gap in growth rates of educational infrastructure between the two parts of Germany. This gap still has an influence on the rate of human capital convergence between East and West

Germany because the labour force with educations obtained before 1991 still accounts for a major proportion of the East German population (equation (2.30)). Due to the fact that before unification the educational infrastructure stock grew more rapidly in West Germany than in East Germany, the difference in human capital levels between the two parts of the country will continue to increase until the proportion of the population with education obtained after 1991 remains above a critical level. As seen in figure 2, this critical moment corresponds to the year 2007.

Figure 2. Basic solution.
East German human capital level in % of West German level



Such a behaviour of human capital levels in the new and old states explains the current halt of East German economic convergence. The convergence rate can be factorised as follows:

$$\frac{Y_E / D_E}{Y_W / D_W} = \theta = \theta_1 \theta_2 \theta_3 \ , \qquad (3.2)$$

$$\theta_1 = \frac{H_E^{1+\chi(E)-\alpha(E)-\beta(E)}}{H_W^{1+\chi(W)-\alpha(W)-\beta(W)}} \ , \qquad (3.3)$$

$$\theta_2 = \frac{G_E^{\alpha(E)} K_E^{\beta(E)}}{G_W^{\alpha(W)} K_W^{\beta(W)}} \ , \qquad (3.4)$$

$$\theta_3 = \frac{A_E L_E^{1-\alpha(E)-\beta(E)} D_W}{A_W L_W^{1-\alpha(W)-\beta(W)} D_E} \ . \qquad (3.5)$$

With this factorisation one notes that the component θ_1 has decreased in recent years, the component θ_2 has increased, and the component θ_3 has remained approximately constant. Moreover the technological coefficients and current values of economic indicators have led to approximately equal growth speeds of $(-\theta_1)$ and θ_2. This implies a constant level of the convergence rate and explains the current halt of East German economic convergence.

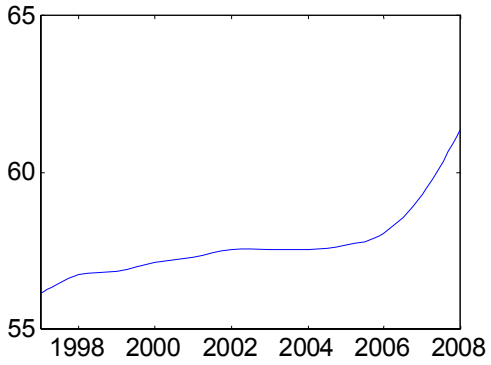
4. APPLICATION OF THE MODEL FOR POLICY RECOMMENDATIONS

The current stagnation of the regional convergence can be overcome. To accomplish this, at least one of the four factors of production must be affected in order to raise the output in the new states. Therefore, there are at least four strategies that will allow economic convergence to continue. The East German traffic infrastructure stock and human capital level can be raised by enlargement of public investments in the corresponding infrastructure component. In terms of the model this means an increase in parameters q_E or n_E. The growth rate of capital stock in the new states can be accelerated by increasing government subsidies to private firms. This strategy corresponds to an increase in the subsidies rate h_E. The employment level in East Germany can also be raised. The reduction of East German wages is a possible way to attain higher employment. In terms of the model, this variant of reform corresponds to a decrease of the growth speed of γ.

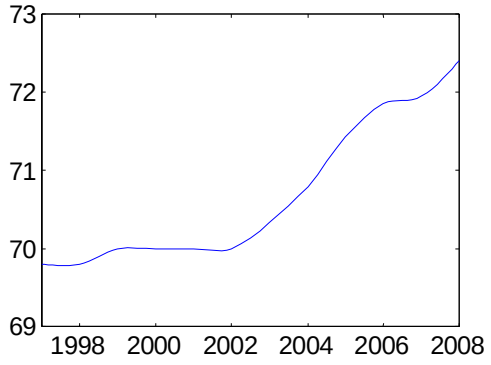
The model allows us to estimate numerically the results of the described reforms. Below we compare the efficiencies of each reform strategy. For this purpose, four additional numerical solutions of the model were obtained. Three of these solutions show the results of the possible enlargement of public transfers to the East German economy with 2 Mrd DM per year. This sum is supposed to be spent to raise three different parameters q_E, n_E or h_E. Each solution corresponds to an increase in one from these parameters. The increase in public expenditures is supposed to take place during 2003. Before this year the considered solutions coincide with the basic solution. The results of these solutions appear in Figures 3-5.

Figure 3. Numerical solution for the case of enlargement of traffic infrastructure investments in East Germany since 2003

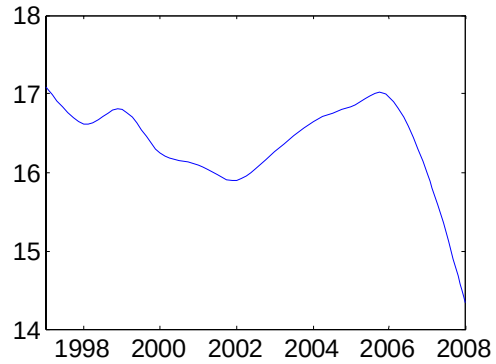
$\theta(t)\times 100\%$ - East German output per capita in % of West German level



$\gamma(t)\times 100\%$ - fixed level of East German wages in % of West German level



Unemployment rate in East Germany(%)



Unemployment rate in West Germany(%)

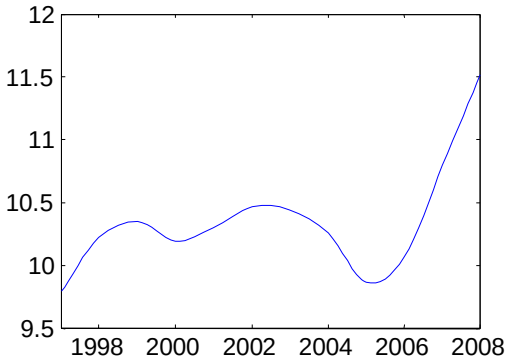
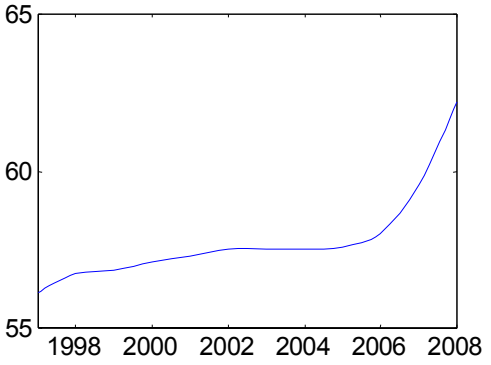
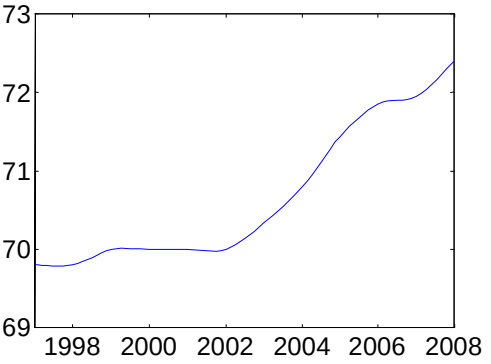


Figure 4. Numerical solution for the case of enlargement of educational infrastructure investments in East Germany since 2003

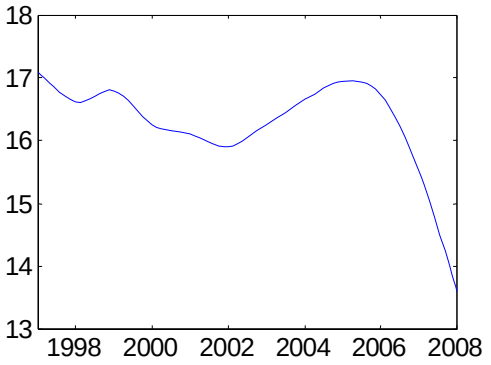
$\theta(t)\times 100\%$ - East German output per capita in % of West German level



$\gamma(t)\times 100\%$ - fixed level of East German wages in % of West German level



Unemployment rate in East Germany(%)



Unemployment rate in West Germany(%)

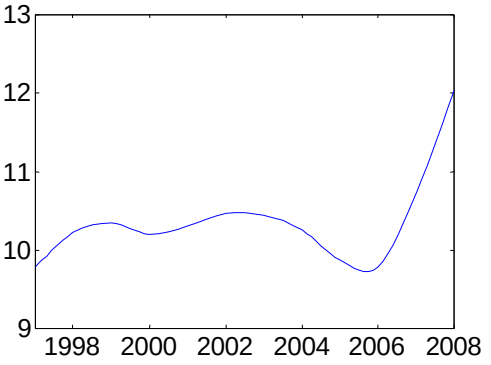
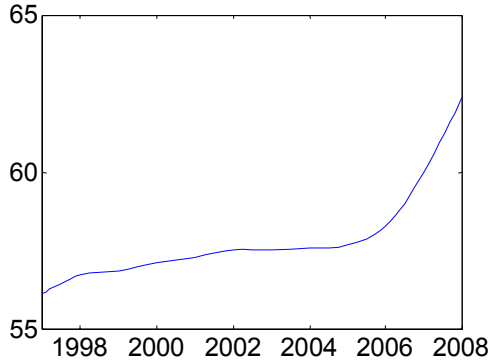
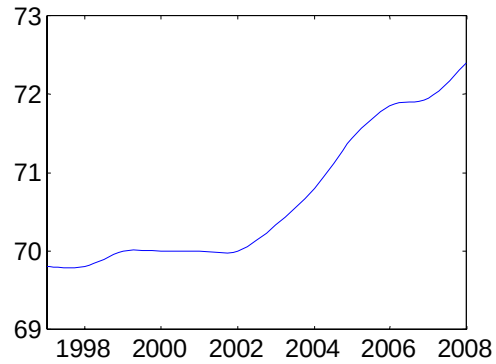


Figure 5. Numerical solution for the case of increase in subsidies rate since 2003

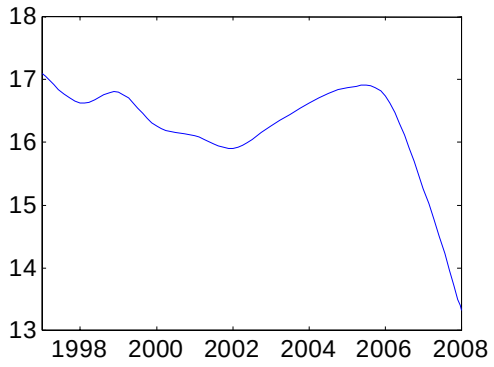
$\theta(t)\times 100\%$ - East German output per capita in % of West German level



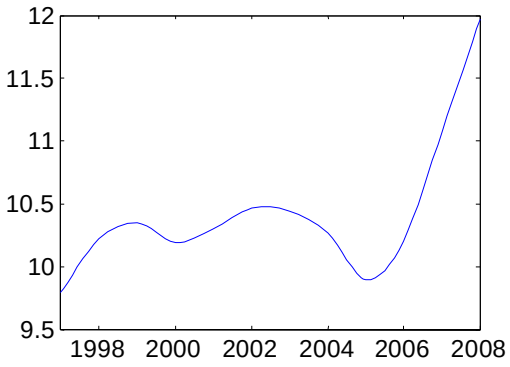
$\gamma(t)\times 100\%$ - fixed level of East German wages in % of West German level



Unemployment rate in East Germany(%)



Unemployment rate in West Germany(%)

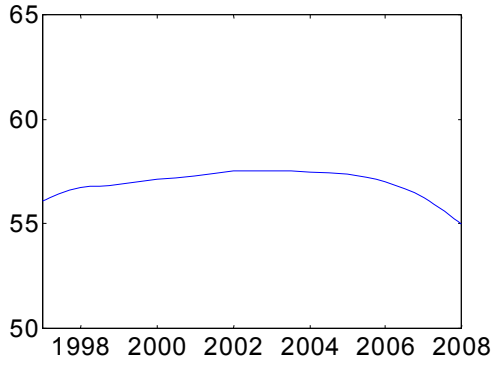


The results of these numerical solutions show that these three strategies all have the same disadvantage, namely a significant increase in unemployment in the old states.

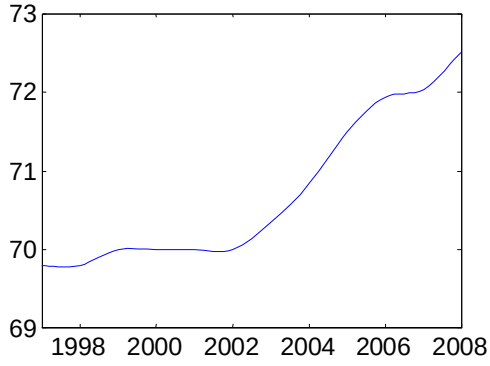
The next numerical solution demonstrates the results of changing the exogenously determined fixed wage ratio $\gamma(t)$. The solutions of the modified model appear in Figures 6-7.

Figure 6. Numerical solution for the case of additional acceleration of wage level growth in East Germany since 2003

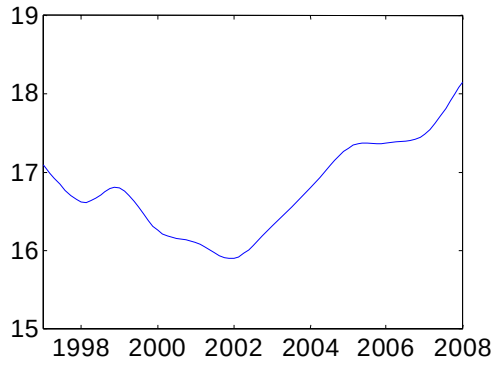
$\theta(t)\times 100\%$ - East German output per capita in % of West German level



$\gamma(t)\times 100\%$ - fixed level of East German wages in % of West German level



Unemployment rate in East Germany(%)



Unemployment rate in West Germany(%)

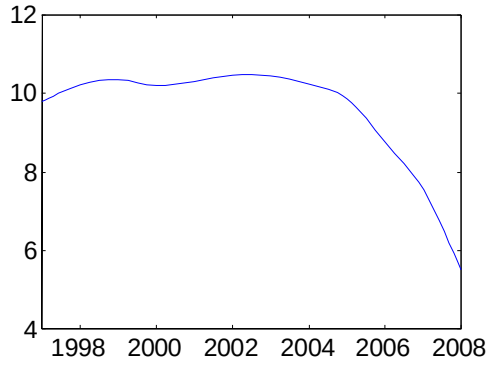
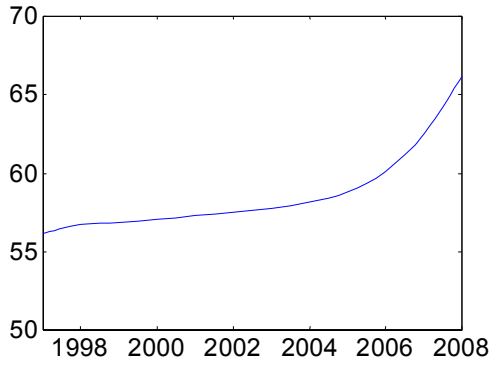
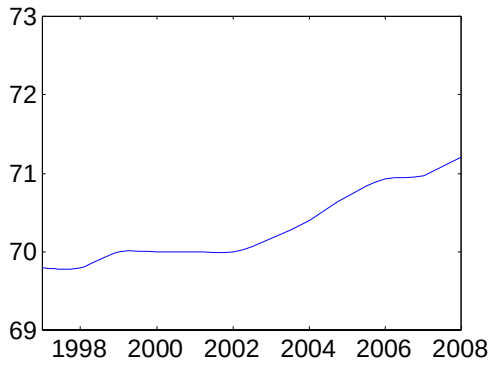


Figure 7. Numerical solution for the case of low growth rate of wage level in East Germany since 2003

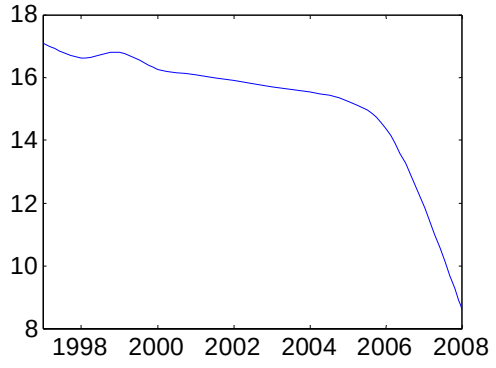
$\theta(t)\times 100\%$ - East German output per capita in % of West German level



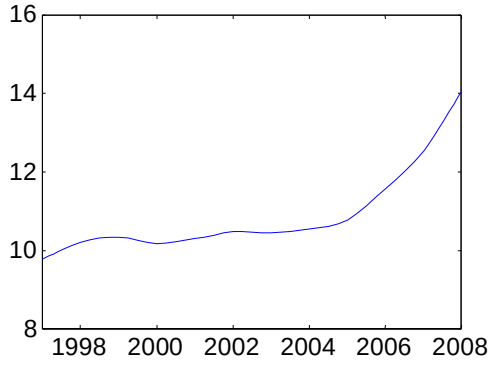
$\gamma(t)\times 100\%$ - fixed level of East German wages in % of West German level



Unemployment rate in East Germany(%)



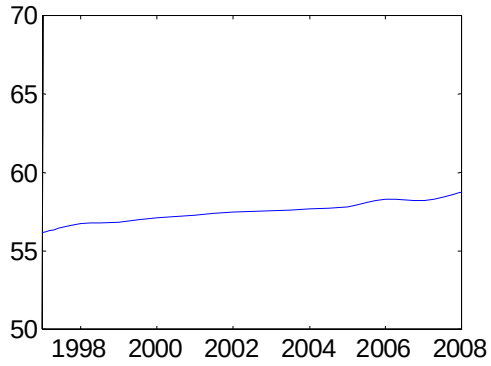
Unemployment rate in West Germany(%)



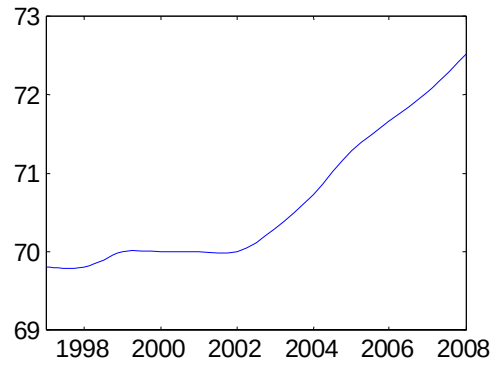
These solutions demonstrate that an additional acceleration of wage level convergence will lead to an increase in unemployment in the new states and regression in output convergence. A rate of wage convergence that is too low can also be harmful, because it will result in high unemployment in the old states. The results show the necessity for a moderate decrease in the growth rate of wage levels in East Germany in comparison to the basic solution between 2003 and 2006. On the other hand, after overcoming the critical level of East German human capital in 2007, the wage convergence should be accelerated. The example of such wage proportion trajectory $\gamma(t)$ is given by the next numerical solution presented in Figure 8.

Figure 8. Numerical solution for the case of recommended strategy of wage level growth in East Germany since 2003

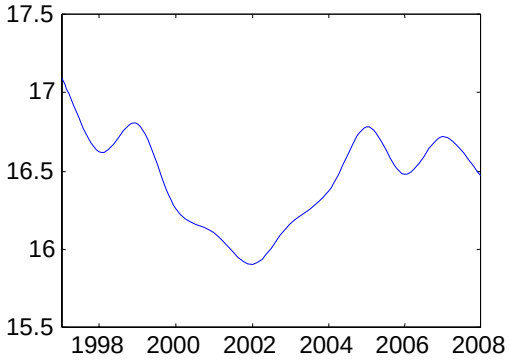
$\theta(t)\times 100\%$ - East German output per capita in % of West German level



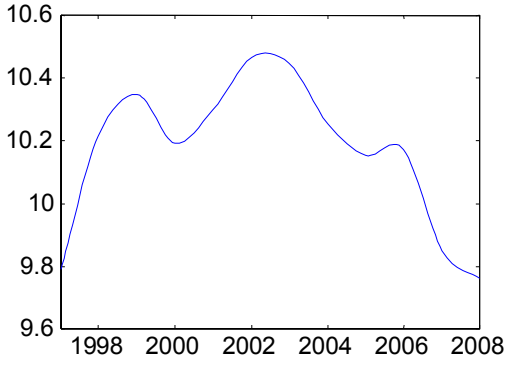
$\gamma(t)\times 100\%$ - fixed level of East German wages in % of West German level



Unemployment rate in East Germany(%)



Unemployment rate in West Germany(%)



Generally, a government cannot directly control wage determination, therefore the recommended strategy does not provide direct instructions on quantitative values of policy instruments. Nevertheless, the solution provides information on the necessary government reaction to future results of wage negotiations. If East German wages grow slower than in the recommended strategy, the government must decrease the volume of support for East Germany in order to prevent an increase in unemployment in West Germany. However, if the future wage negotiations result in more rapid wage convergence, the government must enlarge public infrastructure spending in the new states.

CONCLUSIONS

Despite the current standstill, economic convergence of East Germany can be continued in the succeeding years. The cause of the temporary halt of convergence in 1996-2002 is the deficient growth rate of educational infrastructure in East Germany between 1980 and 1990. The labor force with an education obtained in the eighties accounts for a major part of labor force at present. Therefore, the educational infrastructure level in 1980-1990 has a direct influence on the present East German economic convergence.

As can be seen from the introduced model, there are a number of measures, which can help to accelerate the much-needed convergence. The mathematical approach, which have been developed and explained in this paper, is a practical way to clearly compare these measures. A significant increase in output convergence speed would be harmful because of the increase in unemployment this can create in the old states. Maintaining the wage convergence speed at approximately 0.4% per year appears to be the optimal variant of development, as this does not lead to an increase in West German unemployment. Since controlling wage level can be difficult for federal government, the estimation of optimal wage convergence can serve as an orientation point for the choice of government support volume for the new states. If East German wages grow slower than 0.4% per year, the government must decrease the volume of support for East Germany in order to prevent an increase in unemployment in West Germany. However, in case the future wage negotiations result in more rapid wage convergence, the government must increase public infrastructure spending in the new states in order to provide further output convergence between East and West Germany.

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