Growth and Employment Effects of Unions in a Simple Endogenous Growth Model*

Jörg Lingens†

3rd July 2002

Abstract

In this paper we analyse the effects of simultaneous union wage bargaining in a simple two sector growth model. We show that the overall employment effect of unionisation is ambiguous and depends on the relative sectoral wage. Besides the employment effects we analyse how unionisation changes the rate of growth. It is shown that both research sector unionisation and intermediate sector unionisation lower the rate of growth, although the effect of research sector unionisation is more fierce. Moreover we analyse the impact of parameter changes on the growth differential, i.e. the rate of growth in the competitive case over the rate in the unionised case. We can show that whether various parameter changes narrow or widen the growth differential crucially hinges on the wage differential in the unionised case.

JEL: O4, J5

Keywords: Labour Unions, Unemployment, Growth, R&D

---

*I am indebted to Jochen Michaelis, Martin Debus and Nikolai Ziegler for valuable comments.
†University of Kassel, Nora-Platel Str.4, 34127 Kassel, Germany. Email: Lingens@wirtschaft.uni-kassel.de
1 Introduction

One way of modelling endogenous growth is by assuming that firms invest in research and development and that this process leads to a more efficient economy. So more goods can be produced with an unchanged resource base. The two theoretical approaches to model R&D is either to assume that a research sector produces blueprints of new varieties of an intermediate good and that these intermediate goods are used to produce a consumption good (e.g. Romer (1990)) or to assume some kind of creative destruction (Aghion/Howitt (1992)) where the research firms produce qualitative better intermediate goods and drive the incumbent intermediate monopolist out of the market. Although it seems that both modelling strategies lead to very different models both are (qualitatively) very much the same. Both have, among other things, in common that the equilibrium (steady state) growth rate is determined by the allocation of an exogenous resource pool to production and research. The labour market drives this allocation making it very important for the determination of the rate of growth. Usual assumptions concerning the labour market are that it is perfectly competitive and that by arbitrage the wage differential between the two sectors of the economy is zero.

Are these assumptions about the institutional organisation of the labour market a good proxy of the labour markets we can observe? Labour markets in most economies are far from being perfectly competitive. Union wage bargaining is the common way of wage determination, especially in continental Europe. Although union density (union members as part of the work force) is steadily declining\(^1\), union wage coverage (the part of the workforce which is payed according to a union wage agreement) is rather stable on a very high level (between 69% in Denmark and 98% in Austria (see the CESifo DICE database). The static effects of union wage bargaining are rather well analysed and established see, e.g. Layard et al. (1991) or Booth (1995). Usually it is assumed, that unions are able to capture rents and bargain a wage exceeding the competitive outcome. So unions might cause steady state unemployment. But in the context of R&D growth models, unions might also affect the equilibrium allocation of the resource pool of the economy and thereby change the growth rate of an economy. The assumption of no intersectoral wage differential is contradicted by the evidence that substantial wage differentials exist, even when controlling for individual and workplace characteristics (see Edin/Zetterberg (1992)). So it seems that these wage differentials should be implemented into these kind of growth models.

We analyse a simple endogenous growth model where growth is driven by the development of new varieties of intermediate goods which is extended by union wage bargaining. The main problem when analysing union wage bargaining in a two sector economy is that the two wage bargains are interdependent. A high wage (and hence low employment) in one sector has negative effects on the whole economy, because it will be harder to find a job once unemployed. In the few models in the literature implementing union wage bargaining

\(^1\)For an explanation of this phenomenon see Acemoglu et al. (2001).
into R&D growth models, this problem of two sector bargaining is usually solved by either assuming that the factor covered by bargaining is only used in one sector of the economy (e.g. as done in Lingens (2001)) or by assuming a one sector bargaining in an economy with (artificial) mobility constraints (e.g Quang/Vousden (1999), where labour must decide in which sector to work ex ante and does not have the chance to move into the other sector ex post). The approach in this paper is different, because we model a simultaneous two sector wage bargaining variant of Layard and Nickell (1990) as, e.g. put forward by Kolm (1998) and Holmlund (1997). In this framework jobs are exogenously split and workers enter the unemployment pool and find another job somewhere in the economy. The union bargains the wage at the firm level (taking all macroeconomic parameters as given) so to maximise lifetime utility of its members. The consequences of the wage bargain are that the wage in the intermediate goods sector as well as in the research sector will be higher than in the comparable competitive framework and, more importantly for the growth effects of the wage bargain, the wages in the two sectors need not to be identical. Hence there is room for wage differentials. This is due to the fact that workers cannot switch directly from one sector to another. So one union can bargain a higher wage without fearing competition caused by migration from the other sector.

The growth effect of union wage bargaining are the following. With the unionisation of the research sector labour market, employment in that sector will unambiguously decline. This is because the union in the research sector will bargain a wage that exceeds the competitive outcome. This will result in lower research sector employment. Moreover the relative labour demand in the research sector will decline. So unionisation of the research sector will dampen the rate of growth of the economy (which is not surprising). The interesting point is that unionisation of the intermediate sector labour market lowers economic growth although the unionisation has two countervailing effects. On the one hand wage demands will increase (not only in the intermediate sector, but also in the research sector because outside opportunities are better). This effect is bad for growth. But on the other hand the relative labour demand in the research sector will increase, because the relative sectoral wage declined. The latter effect will c.p. foster research sector employment. Nevertheless it is shown that the growth dampening effect will always dominate the good growth effect.

Moreover the growth differential, defined as research sector employment in the competitive case over research sector employment in the unionised case is analysed. We show the effect of parameter changes on this growth differential. It will be shown that the question wether the growth differential narrows or widens crucially hinges on the relative wage z.

The remaining paper is organised as follows. In the next section the benchmark case of the growth model will be presented briefly. In the third section we will introduce wage bargaining and determine the equilibrium of the unionised economy. In the fourth section the unemployment effects are analysed. The fifth section is devoted to the analysis of the effects of
unionisation on the growth rate of the economy. The last section summarises and concludes.

2 The Growth Model

We consider a standard growth model which follows Grossman/Helpman (1997), where the consumption good sector uses intermediate good varieties as input factors. The number of these varieties rise and so, due to specialisation (see Ethier (1982)) it is possible to produce a bigger amount of the consumption although the primary production factors are fixed.

2.1 The Consumption Good

The production function of consumption goods takes the following form:

\[ Y_t = \left( \int_0^{n_t} x_t(i)^{\alpha} \, di \right)^{\frac{1}{\alpha}}, \]

(1)

where \( Y_t \) is the amount of the consumption good produced at \( t \), \( x_t(i) \) is the intermediate variety \( i \) used at \( t \) and \( n_t \) is the number of intermediate good varieties available at time \( t \). By profit maximisation the demand for variety \( i \) of the intermediate good follows as:

\[ P_{x(i)} = \left( \int_0^{n_t} x_t(i)^{\alpha} \, di \right)^{\frac{1-n}{\alpha}} \cdot x(i)^{\alpha-1} \cdot P_Y, \]

(2)

where \( P_{x(i)} \) denotes the price of an intermediate good of variety \( i \) and \( P_Y \) denotes the price of the consumption good. For ease of exposition we take the latter good as the numéraire, hence \( P_Y = 1 \).

2.2 The Intermediate Good Sector

A variety of the intermediate good is produced using labour with a CRS technology, hence the production function is linear and reads:

\[ x(i) = L_{x(i)}, \]

(3)

where \( L_{x(i)} \) denotes the amount of labour used in an intermediate good firm. The intermediate good producer is monopolist of the variety \( i \). The profit an intermediate producer earns reads:

\[ \Pi_{x(i)} = P_{x(i)} \cdot x(i) - w_x^L \cdot L_{x(i)} - r \cdot P_n. \]

(4)

Note that a monopolist can only produce when a blueprint of a new intermediate good variety has been bought. To get a blueprint the monopolist has to borrow money on the capital
market to buy the blueprint at price $P_n$. He has to repay the debt infinitely, so the repayment every period is $r \cdot P_n$. With profit maximisation the labour demand of a monopolist is given by:

$$w^L_x = \alpha \cdot P_{x(i)}.$$  \hspace{1cm} (5)

The elasticity of this labour demand curve is given by $\frac{1}{\alpha - 1}$. This is straightforward, because of the linear production function. The (physical) marginal product will not decline, but the price will decline due to higher production. So the elasticity of the labour demand curve and the price elasticity of the intermediate good demand curve will coincide.\(^2\) Because all intermediate good firms are symmetric (they face the same demand function, produce under the same technology and so on), the same amount is produced in every intermediate good firm. Hence $x(i) = x$ and $P_{x(i)} = P_x$. Labour demand of an intermediate good firm in a symmetric equilibrium reads:

$$w^L_x = \alpha \cdot (n_t)^{\frac{1-\alpha}{\alpha}}.$$  \hspace{1cm} (6)

### 2.3 The Research Sector

The research sector produces blueprints using labour and some fixed factor which could be thought of as human capital (or high-skilled labour)\(^3\) that is only used in the research sector. We assume the production function of the research sector being Cobb Douglas:

$$\Delta n_t = n_t \cdot (L^n \cdot H^{1-\lambda}),$$  \hspace{1cm} (7)

where $\Delta n_t$ denotes the number of new blueprints produced in a time interval of length one and $H_n$ is the amount of human capital used in the research sector. Note that we have incorporated the usual research spill-over by assuming that the number of new varieties produced depends on the number of already existing ones. So it is easier (in terms of labour used) to produce a new blueprint when the number of already existing ones is high. With profit maximisation, labour demand in the research sector is given by:

$$w^L_n = P_n \cdot n_t \cdot \lambda \cdot L_{n}^{\lambda-1} \cdot H_{n}^{1-\lambda}.$$  \hspace{1cm} (8)

\(^2\)All these results are derived under the assumption that $n_t$ is large enough, that $\frac{d(x(i) e \cdot t)}{d(x(i))} = 0$.

\(^3\)Assuming this factor being constant over time is a bit unsatisfactory, but we want to concentrate on the effect of unions on one component of growth. Nevertheless it would be a nice extension to endogenise human capital accumulation.
The demand for human capital is given by:

\[ w^L_n = P_n \cdot n_t \cdot (1 - \lambda) \cdot L_n^\lambda \cdot H_n^{-\lambda}. \] (9)

### 2.4 Households

The two types of households in the economy (households that supply labour and households that supply human capital) consume final goods and have to decide how much to save and how much to consume. The lifetime utility function (independently of the type of household) is given by\(^4\):

\[ V = \sum_{t=0}^{\infty} (1 + \rho)^{-t} \cdot ln(C_t^j), \] (10)

where \(\rho\) is the rate of time preference and \(C_t^j\) is consumption at \(t\) of household of type \(j\), where \(j \in [H; L]\). Households maximise (10) subject to an intertemporal budget constraint which is given by:

\[ A_{t+1} = (1 + r) \cdot A_t + w^L_t(j) - C_t^j, \] (11)

where \(A_t\) is an asset of a household at time \(t\). In the aggregate this asset is the debt given to the intermediate firms (in the aggregate consumption loans cancel out). By solving the intertemporal decision problem of households, we get an equation relating the growth rate of consumption to the difference between the rate of interest and the rate of time preference (using the simplification of \(log(1 + x) = x\) for small \(x\)),

\[ g_{C_t^j} = r - \rho, \] (12)

where \(g_{C_t^j}\) is the growth rate of consumption of a household of type \(j\). Equilibrium in the goods market implies that \(C_t^L + C_t^H\) equals \(Y_t\) at every point in time. So for the growth rates of output and the consumption of the two types of households the following must hold:

\[ g_{Y_t} = \frac{C_t^L}{Y_t} \cdot g_{C_t^L} + \frac{C_t^H}{Y_t} \cdot g_{C_t^H}. \] (13)

Equation (13) reveals that the growth rate of consumption (which is by equation (12) the same for both types of households) must be equal to the rate of output growth (so \(g_{Y_t} = g_{C_t^L} = g_{C_t^H}\)).

\(^4\)The assumption that both types of households have the same utility function simplifies the algebra but does not change the results.
2.5 Steady State Equilibrium

In a steady state the allocation of labour between the intermediate good and the research sector is constant. Hence $g_n$, is constant over time as well as aggregate production in the intermediate sector $X = n_t \cdot x$. The steady state growth rate of final output is given by $g_n = \frac{1 - \alpha}{\alpha} \cdot g_n$. To determine the equilibrium allocation of the stock of labour (which determines production and the rate of growth) we use a slightly different approach as, e.g. applied in Grossman/Helpman (1997). Let us divide the labour demand in the research sector (8) by the labour demand in the intermediate sector (5) and denote the relative research sector wage (the research sector wage in terms of the intermediate sector wage) $\frac{w^L}{w_x}$ by $z$. This yields:

$$z = \frac{P_n \cdot n_t \cdot \lambda \cdot L_n^{\lambda-1} \cdot H_n^{1-\lambda}}{\alpha \cdot n_t \cdot \frac{1}{1-\alpha}^{\frac{1}{\alpha}}}$$ \hspace{1cm} (14)

Note that we assume free entry into the intermediate goods market, so in an equilibrium profits in this sector will be zero. This zero profit condition determines the price of blueprints as:

$$P_n = \frac{p_x \cdot x - w_x^L \cdot L_x}{r}$$ \hspace{1cm} (15)

Euler's theorem implies that $p_x \cdot x - w_x^L \cdot L_x = (1 - \alpha) \cdot p_x \cdot x$. Furthermore the rate of interest is given by (12) and the price of an intermediate good in a symmetric equilibrium is given by $P_x = n^{\frac{1}{1-\alpha}}$. So the equation determining the equilibrium allocation (14) changes to:

$$z = X \cdot (1 - \alpha) \cdot \frac{\lambda \cdot L_n^{\lambda-1} \cdot H_n^{1-\lambda}}{\alpha \cdot g_n \cdot \frac{1}{1-\alpha} + \rho}$$ \hspace{1cm} (16)

Because we assume (in this competitive framework) that labour is perfectly mobile between sectors the relative wage, $z$, must be equal to one. Furthermore we assume that human capital is supplied inelastically with the consequence that it is always fully employed at the exogenous given level $H_n$. So equation (16) establishes a relation between the amount of labour used in the production process and the amount of labour devoted to the research sector. To close the model we need an additional equation which is the resource constraint of the economy (the amount of labour employed in both sectors of the economy cannot exceed the resource base):

$$\bar{L} = L_x + L_n,$$ \hspace{1cm} (17)

so there is only an exogenous amount of labour $\bar{L}$ to allocate to both sectors. Combining equations (17) and (16) and substituting $X$ and $g_n$ by the production functions (3) and (7), we get the following equilibrium condition for employment in the research sector (and this
also determines (indirectly) the rate of growth of the economy:

\[
n \cdot L_n^\lambda \cdot \bar{H}_n^{1-\lambda} \cdot \frac{1 - \alpha}{\alpha} + \rho = \frac{1 - \alpha}{\alpha} \cdot (\bar{L} - L_n) \cdot \lambda \cdot L_n^{\lambda - 1} \cdot \bar{H}_n^{1-\lambda}
\]  

Equation (18) gives equilibrium employment in the research sector. Due to the assumed structure of production, equation (18) is not analytically solvable for the levels of employment in the two sectors. But we can log-linearise the equation and can argue for relative changes. Log linearisation of equation (18) (where a \( \sim \) over a variable denotes a relative change, hence \( \bar{x} = \frac{dx}{x} \)) gives:

\[
\left( 1 - \lambda + \lambda \cdot b_1 + \frac{L_n}{\bar{L} - L_n} \right) \cdot \bar{L}_n = (1 - \lambda) \cdot b_2 \cdot \bar{H}_n + \frac{\bar{L}}{\bar{L} - L_n} \cdot \bar{L} - b_2 \cdot \bar{\rho} - \frac{b_2}{1 - \alpha} \cdot \bar{\alpha},
\]

where the coefficients \( b_1 \) and \( b_2 \) are given in the appendix A3 (both are smaller than one, positive and add up to one, hence \( b_1 + b_2 = 1 \)). With equation (19) at hands we can analyse the impact of the various parameters on the growth rate of the economy. A higher stock of human capital will foster the rate of growth. This is because with a higher stock of human capital, marginal productivity of labour in the research sector will rise.\(^5\) A higher stock of labour will also raise employment in the research sector and therewith growth, because a higher stock of labour will be allocated evenly to the two sectors of the economy, hence production and research output will rise. A more impatient economy (higher \( \rho \)) will have less employment in the research sector and hence will grow slower. This is because in an economy with impatient inhabitants savings are low and this tends to increase the (steady state) rate of interest. Thus, borrowing is expensive and so there will be less entrepreneurs in the intermediate goods sector. So the price for patents will be low causing the marginal value product of labour in the research sector to decline. This dampens employment in the research sector. Very similar arguments can be applied when explaining the effect of a change of \( \alpha \). An increase in \( \alpha \) implies that the market for intermediate goods is more competitive.\(^6\) So profits of an intermediate goods firm decline implying that fewer entrepreneurs enter the market and demand for blueprints (and hence the price for blueprints) will be low. This will lower again employment in the research sector.

\(^5\)A higher stock of human capital has also a negative effect on the employment of labour in the research sector, as with a higher rate of growth (due to the positive effect) the rate of interest will rise which dampens the positive effect. Nevertheless the positive effect will always dominate.

\(^6\)Remember that the elasticity of substitution between two intermediate good varieties is given by \( \frac{1}{1-\alpha} \). Hence with a higher \( \alpha \) the intermediate goods become better substitutes.
3 The Unionised Economy

In this section we derive the equilibrium of the economy when we have union wage bargaining in the research as well as in the intermediate goods sector.\(^7\) We assume, starting from an arbitrary steady state equilibrium with perfectly competitive labour markets, that there is union formation in every firm in the economy. All employed workers in a firm are members of the union and membership is exogenous to the union. Every union bargains at the firm level a wage for its members which maximises their life time utility given by equation (10). Because we consider a perfectly decentralised wage setting, all macroeconomic effects are taken as exogenous during the bargain. Life time utility of a typical union member depends (via the indirect utility function) solely on the state dependent income of that member. The union members might experience different states, namely being employed in the intermediate sector, being employed in the R&D sector or being unemployed.\(^8\) The value attached to those states is given by the following equations:

\[
V^{x\{i\}}_t = \left( w^{x\{i\}}_t + D_t + a \cdot V^u_{t+1} + (1 - a) \cdot V^{x\{i\}}_{t+1} \right) \cdot \frac{1}{1 + \rho} \tag{20}
\]

\[
V^{n\{i\}}_t = \left( w^{n\{i\}}_t + D_t + e \cdot V^u_{t+1} + (1 - e) \cdot V^{n\{i\}}_{t+1} \right) \cdot \frac{1}{1 + \rho} \tag{21}
\]

\[
V^u_t = \left( B_t + D_t + c \cdot V^{n\{i\}}_{t+1} + c \cdot V^x_{t+1} + (1 - c - e) \cdot V^U_{t+1} \right) \cdot \frac{1}{1 + \rho}, \tag{22}
\]

where \(V^{x\{i\}}, V^{n\{i\}}\) is the value of working in a specific intermediate firm and a specific research firm, respectively. \(V^x\) and \(V^n\) are the values of working in the intermediate sector and in the research sector, respectively. \(V^u\) is the value of being unemployed, \(B_t\) is the unemployment benefit, \(D_t\) are the dividends a union member gets (independently of the employment status) and \(a, e, c\) are the probabilities of losing a job in the intermediate sector, loosing a job in the research sector and finding a job in the intermediate or the research sector, respectively. Note that we assume that the probability of finding a job in one of both sectors is the same. One might argue that, once unemployed, it is harder to find a job in the research sector, because more sector specific skills are needed, which are lost during the time being unemployed. This would not change the results in any way, but would make the algebra a bit more complicated. Note that the value of being employed (in any firm) is growing in a steady state, because the wage grows. The growth rate of the wage is given by \(\frac{1}{\alpha} \cdot g_n = g_Y\) (see equation (5)). In the following we assume that the unemployment benefit is indexed to the wage in the intermediate good sector and that this ratio is constant \(\left( \frac{B_t}{w^{n\{i\}}_t} \right)\). Hence the unemployment benefit must also grow at the above given rate. As in the competitive case we concentrate on steady states,

\(^7\)The modelling of this two sector wage bargain follows rather closely Holmlund (1997) and Kohn (1998).

\(^8\)It is important to note that a direct switch from one sector to another is not possible without frictions, i.e. being unemployed. This is because both sectors are dominated by union wage bargaining. So an agent who quits a job in one sector cannot be sure to find a job in the other immediately because the union rations jobs.
which implies that $V_{t+1}^x = (1 + g_Y) \cdot V_t^x$; $V_{t+1}^n = (1 + g_Y) \cdot V_t^n$ and $V_{t+1}^U = (1 + g_Y) \cdot V_t^U$.

3.1 Wage Bargaining in the Intermediate Sector

The union bargains the wage with a firm in the intermediate sector. Union's utility function is given by:

$$U^{\text{union}} = L_{x(i)} \cdot V_t^x + (m - L_{x(i)}) \cdot V_t^U,$$  

so when bargaining the wage, the union maximises life time utility of all its members $m$ taking into account that some might get unemployed. To determine the wage in an intermediate sector firm we apply the (asymmetric) Nash bargaining solution. The Nash product is given by:

$$\Omega_x = (U^{\text{union}} - \bar{U}^{\text{union}})^\beta \cdot (\Pi_x - \bar{\Pi}_x)^{1-\beta},$$  

where $\beta$ is the bargaining power of the labour union, $\Pi_x$ is the profit of the intermediate firm and $\bar{U}^{\text{union}}$ is the union's utility in the case that no agreement is reached (in this case all the union members "get" the value of being unemployed). $\bar{\Pi}_x$ is the profit of the firm if there is no agreement. In such a situation the firm is not able to produce output, so revenue is zero, but because the firm had to buy a patent in advance (to get the chance of bargaining with the union and start production) there will be costs of $r \cdot P_n$. Maximising the Nash product $\Omega_x$ with respect to the wage and subject to the labour demand curve (5) gives the bargained wage (see the appendix A1):

$$w_{x(i)}^L = \phi_x \cdot \left( (\rho - g_Y) \cdot V_t^U - D_t \right),$$  

where $\phi_x = \left( \frac{(1-\beta) \cdot a + \beta}{a} \right)$. The bargained wage is a mark up over the present value of being unemployed (net of dividends). The mark up depends (positively) on the bargaining power of unions and negatively on the price elasticity of the intermediate good demand (which in turn determines monopoly profits, so the higher these profits are, the higher will be the wage). Furthermore equation (25) reveals that the competitive case is nested in the union case. When bargaining power $\beta$ is zero, $\phi_x$ will be one and the wage will be equal to the competitive wage.

3.2 Wage Bargaining in the R&D Sector

To determine the wage in the R&D sector, we follow the same approach, we have already applied to determine the wage in the intermediate sector. We assume that the union in the

---

9 We could also have assumed that the firm would only buy a patent when agreement is reached or equivalently that there is a secondary market for patents so that the union could sell the patent in case of a dispute, hence $\bar{\Pi}_x$ would be zero. In such a case the wage, which is agreed on would be lower. This would not change the main results, but it would complicate the algebra.
The research sector has the same utility function as the union in the intermediate sector.\(^\text{10}\) The following Nash product determines the bargained wage:

\[
\Omega_n = (U^{\text{union}} - \bar{U}^{\text{union}})\mu \cdot (\Pi_n - \bar{\Pi}_n)^{1-\mu}. \tag{26}
\]

In this case \(\mu\) denotes the bargaining power of the union, \(\bar{U}^{\text{union}}\) is again the utility of the union when there is no agreement and this is, as in the intermediate sector, \(m \cdot V^U\). The profit of a research firm in the case of a dispute, \(\bar{\Pi}_n\), is \(-W_n^H \cdot H_n\). So we assume that in the case of a dispute with the labour union nothing can be produced (which is obviously a result of the assumed production function), but that human capital must be paid accordingly. This last assumption can be justified on the basis that we assume a bargaining structure where on the first stage the whole stock of human capital is hired and so the firm’s hands are tied when bargaining with the union.\(^\text{11}\) The wage that maximises the Nash product (which is the bargained wage) is given by (see the appendix A2):

\[
w_n^{L(i)} = \phi_n \cdot (\rho - g_V) \cdot (V^U - D), \tag{27}
\]

where \(\phi_n = \left(\frac{(1-\mu)\lambda + \mu}{\lambda}\right)\). As in the intermediate sector the bargained wage in the research sector will be a mark up over the value of being unemployed (net of dividends). This mark up depends positively on the bargaining power of unions in the research sector. The wage depends negatively on the labour demand elasticity (reflected by the parameter \(\lambda\)). The economic intuition is, as in all of the bargaining models of this kind, that with a flatter labour demand curve (hence the more elastic it is) the bargained wage will be lower, because a given wage hike of the union will be punished by a greater loss of employment. So in this case the union is reluctant to bargain a high wage.

### 3.3 General Equilibrium

Up to this point we have only derived the bargained wage on the firm level for the research and the intermediate sector. What we want to do is to aggregate these wage equations to the macroeconomic level. This is done by deriving an explicit equation for the value of being unemployed and assuming symmetry within the research and the intermediate goods sector, hence \(w_{x(i)}^L = w_x^L\) and \(w_n^{L(i)} = w_n^L\). To derive an explicit equation for \((\rho - g_V) \cdot V^U\) we use the

---

\(^{10}\)Since low skilled labour is homogenous this seems to be a straightforward assumption.

\(^{11}\)Another, may be more plausible argument for this assumption would be that we assume that human capital owns the firms and receives the difference between revenues and labour costs as income. This would result in the same Nash product, assuming that the firm wants to maximise income and cannot produce without labour.
above given value equations (20), (21), (22). From these equations we get:

\[
V^x_t - V^U_t = \left( \frac{1}{\rho - g_Y + a \cdot g_Y + c \cdot g_Y} \right) \cdot \left( w^L_x - B - c \cdot g_Y \cdot (V^n_t - V^U_t) \right) \\
V^n_t - V^U_t = \left( \frac{1}{\rho - g_Y + e \cdot g_Y + c \cdot g_Y} \right) \cdot \left( w^L_n - B - c \cdot g_Y \cdot (V^x_t - V^U_t) \right).
\]

(28)

(29)

These two equations can finally be solved for \( V^x_t - V^U_t \) and \( V^n_t - V^U_t \). The solutions can be plugged into equation (22). This yields an equation for the present value of being unemployed net of dividends as a function of the unemployment benefit, the wages in the two sectors of the economy, the exogenous probabilities of finding jobs and getting unemployed, respectively and as a function of the wage growth and the discount rate of agents. This equation would be very messy and little is lost if we assume that the discount rate approaches the steady state growth rate (so we analyse the model for a special valued discount rate).\(^\text{12}^\) Doing this we get the following equation for the value of being unemployed net of dividends:

\[
\lim_{\rho \to g_Y} (\rho - g_Y) \cdot (V^U_t - D) = B_t \cdot \left( 1 - c \cdot \frac{a + e}{\Gamma - (c \cdot g_y)^2} \right) + g_Y \cdot c \cdot g_Y \cdot \frac{a}{\Gamma - (c \cdot g_y)^2} \cdot w^L_x + g_Y \cdot c \cdot g_Y \cdot \frac{e}{\Gamma - (c \cdot g_y)^2} \cdot w^L_n,
\]

\[
(30)
\]

with \( \Gamma \equiv (g_Y \cdot a + g_Y \cdot c) \cdot (g_Y \cdot e + g_Y \cdot c) \). Because we focus on steady states, we also assume that there is a flow equilibrium on the labour market. This implies that \( a \cdot L^x = c \cdot U \) and \( e \cdot L^n = c \cdot U \). Using this to substitute for the probabilities of changing between states from equation (30) and plugging this into equation (25) we get the following equation\(^\text{13}^\):

\[
\frac{w^L_x}{L} = \left( \frac{(1 - \beta) \cdot \alpha + \beta}{\alpha} \right) \cdot \left( \frac{U}{L} \cdot B + \frac{L_n}{L} \cdot w^L_n + \frac{L_x}{L} \cdot w^L_x \right)
\]

(31)

Factoring out \( w^L_x \) on the RHS of equation (31) and collecting terms we get the following (note that we also used the following definition of unemployment \( U = L - L_n - L_x \)):

\[
L_x = L \left( \frac{\alpha}{(1 - \beta) \cdot \alpha + \beta - b} \right) \cdot \frac{1}{1 - b} - L_n \cdot (z - b) \cdot \frac{1}{1 - b}
\]

\[
(32)
\]

where \( z \) is the relative research sector wage (as defined in the preceding section) and \( b = \frac{B}{w^L_x} \), so \( b \) is the replacement ratio in terms of the intermediate sector wage. By equations (27) and (25) the relative research sector wage is given by:

\[
z = \frac{\alpha}{(1 - \beta) \cdot \alpha + \beta} \cdot \frac{(1 - \mu) \cdot \lambda + \mu}{\lambda} = \frac{\phi_n}{\phi_x}.
\]

\[
(33)
\]

\(^{12}\) This simplification is also applied in Holmlund (1997) or Kolm (1998).

\(^{13}\) Note that we could have also plugged it into equation (27) without any substantial change of the equations. Which approach to take to determine the equilibrium is a matter of taste.
The relative research sector wage in the unionised economy is only a function of the markups in the two sectors. This is because all other parameters affect the bargaining situations symmetrically and hence will not change the relative wage. We assume in the following that \( z > b \). If this would not be the case no one would like to work in the research sector, because the wage in this sector would be lower than the unemployment benefit. Note that in the unionised case \( z \) needs not to be equal to one like in the competitive one. When labour markets are competitive any wage differential would result in worker flows until the wage in both sectors is equalised. When unions bargain the wage in both sectors there is unemployment in the economy and every worker who quits a job expects to be unemployed for a certain amount of time. So there is no perfect mobility between the two sectors with the result that arbitrage can only equalise expected life time utilities between the two states (being employed in the intermediate goods and in the research sector).\(^{14}\) As shown in equation (33) the wage differential will be a function of the parameters \( \alpha, \beta, \lambda, \mu \). The higher the union bargaining power, the lower the price elasticity in the intermediate goods market, the higher will be the bargained wage in the intermediate sector and the lower will be \( z \). The opposite is true when the bargaining power in the research sector is high and the wage elasticity of the labour demand curve is low. In this case the bargained wage in the research sector will be high and \( z \) will c.p. be high.

Equation (32) determines a curve which is in the one sector union literature (e.g. Carlin and Sośkice (1990)) usually referred to as the wage setting curve. The wage setting curve in the one sector models depicts the bargained wage as a function of employment. A rather similar interpretation can be applied for equation (32). As there is an additional sector this equation reflects the bargained wage as a function of employment opportunities in both sectors of the economy, given the bargained wage in the other sector. Or, which is actually the same interpretation, the bargained relative wage as a function of employment. With some modifications equation (32) gives a relation between employment in both sectors given the wage differential. The wage setting curve is downward slopping in the \( L_x-L_n \)-space, because higher employment in the research sector raises the value of being unemployed. So the union in the intermediate sector would bargain a higher wage which in turn would lower employment in this sector.

The equilibrium in the economy is determined by the wage setting curve (given by equation (31)) and the relative labour demand in the two sectors given by equation (16) (which is the pendant to the price setting curve in one sector models). The price setting curve is upward slopping in \( L_x-L_n \)-space. In figure 1 the equilibrium of the economy and the resource constraint are shown.

In this figure \( PS \) denotes the price setting curve, \( WS \) the wage setting curve and \( Resource \) the resource constraint. The interception of the wage and the price setting curve constitutes

\(^{14}\) A crucial assumption is obviously that workers cannot search on the job, but have to quit their old job to switch between sectors.
the equilibrium in the economy (point $A$ in the figure). Obviously only points to the left of resource constraint are feasible. Moreover the resource constraint depicts another important point. The distance between the resource constraint and the wage setting curve shows the level of unemployment in the economy caused by unionisation of the low skilled labour markets.

As equations (31) and (16) are not explicitly solvable in level we again log-linearise the equations and get:

$$\tilde{z} = \tilde{L}_x - \left( \frac{b_2}{1 - \alpha} \right) \cdot \tilde{\alpha} - (1 - \lambda + b_1 \cdot \lambda) \cdot \tilde{L}_n + ((1 - \lambda) \cdot b_2) \cdot \tilde{H}_n - b_2 \cdot \tilde{\rho} \quad (34)$$

$$\tilde{L}_x = c_1 \cdot \tilde{L} - c_2 \cdot \tilde{\beta} + c_3 \cdot \tilde{\alpha} - \frac{U}{L_x} \cdot \tilde{b} - c_4 \cdot \tilde{z} - c_5 \cdot \tilde{L}_n \quad (35)$$

where the coefficients $c_1$, $c_2$, and $c_3$ are given in the appendix A3. Note that all coefficients are positive. Finally using (35) and (34) we can determine equilibrium employment in the intermediate and the research sector$^{15}$:

$$\tilde{L}_n \cdot (1 - \lambda \cdot b_2 + c_5) = \left( c_3 - \frac{b_2}{1 - \alpha} \right) \cdot \tilde{\alpha} + ((1 - \lambda) \cdot b_2) \cdot \tilde{H}_n$$

$$- b_2 \cdot \tilde{\rho} + c_1 \cdot \tilde{L} - c_2 \cdot \tilde{\beta} - \frac{U}{L_x} \cdot \tilde{b} - (c_4 + 1) \cdot \tilde{z} \quad (36)$$

$$\tilde{L}_x \cdot \left( \frac{1}{1 - \lambda \cdot b_2} + \frac{1}{c_5} \right) = \frac{c_1}{c_5} \cdot \tilde{L} - \frac{c_2}{c_5} \cdot \tilde{\beta} + \left( \frac{b_2}{1 - \alpha} \cdot \frac{1}{1 - \lambda \cdot b_2} + \frac{c_3}{c_5} \right) \cdot \tilde{\alpha}$$

$$- \frac{U}{L_x \cdot c_5} \cdot \tilde{b} - \left( \frac{c_4}{c_5} - \frac{1}{1 - \lambda \cdot b_2} \right) \cdot \tilde{z} - \frac{b_2}{1 - \lambda \cdot b_2} \cdot \tilde{H}_n + \frac{b_2}{1 - \lambda \cdot b_2} \cdot \tilde{\rho} \quad (37)$$

$^{15}$Actually we determine the relative change of equilibrium employment in the two sectors.
4 Equilibrium Unemployment

In the preceding section we have determined equilibrium employment in both sectors given simultaneous union wage bargaining. In this section we will determine unemployment in the economy and analyse in which way parameter changes affect unemployment. By definition the relative change of unemployment is given by:

$$\bar{U} = \frac{\bar{L}}{\bar{U}} \cdot \frac{\bar{L}}{\bar{U}} \cdot \bar{L} - \frac{L_x}{\bar{U}} \cdot \frac{L_x}{\bar{U}} \cdot \bar{L} - \frac{L_n}{\bar{U}} \cdot \bar{L}$$  \hspace{1cm} (38)

Plugging equations (37) and (36) into equation (38) we get equilibrium unemployment in the economy. This is given by:

$$\bar{U} = \left( \frac{\bar{L}}{\bar{U}} - \frac{1}{1 - b \cdot \lambda + c_5} \cdot d_5 \right) \cdot \bar{L} + \frac{1}{1 - b \cdot \lambda + c_5} \cdot \left( -d_1 \cdot \tilde{\alpha} + d_2 \cdot \tilde{\beta} + d_3 \cdot \tilde{z} + d_4 \cdot \tilde{\rho} + d_6 \cdot \tilde{b} + d_7 \cdot \tilde{H}_n \right)$$  \hspace{1cm} (39)

Note that $\tilde{z}$ is an endogenous variable given by the relative change of the mark-ups in the two sectors. To economise on space we did not plug in for $z$. But when analysing changes of, e.g. union bargaining power we have to keep in mind that this has direct effects, but also indirect effects via the change of $z$. The coefficients $d_1$ to $d_7$ are given in the appendix A3.

Firstly we analyse the effects of changes of the resource base of the economy on unemployment. A higher stock of low skilled labour has ambiguous effects on employment. On the one hand a higher $\bar{L}$ will increase employment in both sectors, because a union member that is unemployed will face longer unemployment spells, because there are more people competing for jobs. So there will be wage moderation. On the other hand also the employment pool will rise. Hence, when the employment increase will overcompensate the increase in $\bar{L}$, unemployment will decrease. This will be the case when the following holds:

$$\frac{\bar{L}}{\bar{U}} - \frac{1}{1 - b \cdot \lambda + c_5} \cdot d_5 < 0$$  \hspace{1cm} (40)

The sign of the coefficient is indeterminate, as is the overall effect on unemployment. Diagrammatically a higher stock of labour results in an outward shift of the wage setting curve and to an outward shift of the resource constraints.

A higher stock of human capital implies that the marginal productivity of labour in the research sector increases. Hence, (given the mark-ups) the relative labour demand increases; the price setting curve shifts downwards. As figure 1 depicts this downward shift will raise unemployment when the wage setting curve is flat, because in this case the distance between the equilibrium and the resource constraint increases. Using the definition of $c_5$ in $d_7$ gives $\frac{\bar{L}}{\bar{U}} \cdot \left( \frac{\tilde{z} \cdot b}{1 \cdot \tilde{b} \cdot z} - 1 \right)$. Hence when $z$ exceeds one, unemployment will rise with a bigger stock of human
capital and unemployment will decrease when \( z \) is smaller than one. What is the economic intuition behind this property? A \( z \) smaller than one implies that the research sector labour market is more competitive than the intermediate sector one. So with a higher demand in the research sector, resources are shifted into the sector with the less unionised labour market. But this on the other hand implies that the employment loss in the intermediate sector is overcompensated by the employment gain in the research sector. So overall employment will increase (the opposite is true when \( z \) is bigger than one).

The effect of a higher rate of time preference on unemployment is quite the same the effect of a lower stock of human capital. A higher \( \rho \) results in an upward shift of the price setting curve, because labour demand in the research sector has fallen. This is because the marginal value product of labour has fallen as the price of blueprints declined (see the competitive case). We observe a reallocation of resources from the research into the intermediate sector. Using \( c_5 \) we get \( d_4 = (\frac{f}{\rho} \cdot (1 - \frac{\alpha}{1 - \beta}) \)). When \( z \) is bigger than one, unemployment will decrease. When it is smaller than one, unemployment will increase when \( \rho \) increases. The rationale behind this is the following. When \( z \) is bigger than one, the intermediate sector will be the less unionised of the two sectors. So a higher \( \rho \) implies a reallocation of resources into the more competitive sector. This will increase overall employment.

A higher replacement ratio will increase unemployment. This is the standard results also present in one sector models. With a higher replacement ratio, being unemployed is less costly in terms of foregone income. So unions in both sectors will push for higher wages and this will dampen employment. Diagrammatically a higher \( b \) implies an inward shift of the wage setting curve. Inspecting figure 1 implies that in this case the distance between the wage setting curve and the resource constraint (and hence unemployment) increases.

The last parameters influencing unemployment in the economy are the competitiveness of the intermediate goods market, the bargaining power of intermediate sector unions and the relative sectoral wage \( z \). The relative wage is an endogenous variable and is given by the following equation:

\[
\tilde{z} = \frac{\beta}{(1 - \beta) \cdot \alpha + \beta} \cdot \tilde{\alpha} - \frac{(1 - \alpha) \cdot \beta}{(1 - \beta) \cdot \alpha + \beta} \cdot \tilde{\beta} + \frac{(1 - \lambda) \cdot \mu}{(1 - \mu) \cdot \lambda + \mu} \cdot \tilde{\mu}
\]  

(41)

First of all consider a change of the competitiveness in the intermediate goods market. A higher \( \alpha \) has two direct effects. On the one hand the price setting curve will shift upward implying a reallocation of labour towards the intermediate sector. This is because with a higher \( \alpha \) intermediate goods are better substitutes and hence monopoly profits of intermediate sector firms decrease. But this decrease implies that the demand for blueprints decreases too with the consequence that the price for blueprints declines. Hence labour demand in the research

\[16\] We assume the production function in the research sector as being sector hence we do not consider changes of \( \lambda \) in the analysis.
sector will decrease compared to that of the intermediate sector. On the other hand the wage setting curve will shift outwards, because with higher \( \alpha \) the bargained wage in the intermediate sector declines as the monopoly profits the union can capture are lower than before. But this also implies that the outside opportunities in the research sector decline. So the bargained wage in that sector will decrease, too.

In addition to these direct effects, the change of \( \alpha \) also affects the relative sectoral wage \( z \) which again shifts the wage setting as well as the price setting curve. The overall coefficients governing the effect of \( \alpha \) changes on unemployment in the economy is given by:

\[
-d_1 + \frac{\beta}{(1 - \beta) \cdot \alpha + \beta} \cdot d_3
\]  

(42)

The sign of this coefficient is ambiguous (as was already shown that a change of \( \alpha \) has several partly countervailing effects). Nevertheless the following can be shown: when \( z \) exceeds one (hence the wage in the research sector is bigger than in the intermediate sector) and is big enough, it is more likely that a higher \( \alpha \) will lead to declining unemployment. When \( z \) is bigger than one, the coefficient \(-d_1\) showing the direct effect of a change in \( \alpha \) is unambiguously negative. The rationale behind this is the following. Due to the outward shift of the wage setting curve unemployment will unambiguously decline. There is only a countervailing effect to this unemployment dampening effect when the change of the relative labour demand causes a reallocation towards the more unionised sector. As the change of the labour demand shifts resources into the intermediate sector, the direct effect will dampen unemployment only when the labour market in this sector is more competitive. This is the case when \( z \) exceeds one. The indirect effect of a change of \( \alpha \) on unemployment via the change of \( z \) is ambiguous. When \( z \) is very low a change of \( z \) will cause unemployment to increase. This is due to the following. Inspecting equations (19) and (34) shows that a higher \( z \) will shift the price setting curve upwards (because relative labour demand in the intermediate sector increases) but will shift the wage setting curve inwards. The overall effect is more likely to be positive when \( z \) is high.

A change of the bargaining power of unions in the intermediate sector has also ambiguous effects. Again we have direct effects and effects via a change of the relative wage. The overall effect is depicted by the following equation:

\[
d_2 - \frac{(1 - \alpha) \cdot \beta}{(1 - \beta) \cdot \alpha + \beta} \cdot d_3
\]  

(43)

First of all unemployment will decrease when bargaining power of intermediate sector unions declines. This is because with a lower \( \beta \) the wage setting curve will shift outward. This clearly enhances employment. On the other hand a lower \( \beta \) implies that \( z \) will increase. This indirect effect shifts the wage setting curve inwards and the price setting curve upwards. When \( z \) is smaller than one, a higher \( z \) will lead to more unemployment. So when \( z \) is smaller than one the employment effects of lower intermediate sector bargaining power will be ambiguous. On
the other hand a higher $z$ makes it more likely that unemployment decreases when $\beta$ decreases.

Finally a change of the bargaining power of research sector unions only affects unemployment via a change in $z$. Lower bargaining power lowers $z$ and shifts the price setting curve downwards (as relative labour demand in the research sector increases) and shifts the wage setting curve outwards. Using $d_3$ it can be shown that unemployment will unambiguously decrease in the event of a lower $z$, when $z$ is smaller than one. In this case resources will be shifted into the research sector which is more competitive and so employment will increase.

5 Growth Effects of the Wage Bargain

In this section we analyse the impact of unionisation on the (steady state) rate of growth of the economy. We have seen that the (steady state) rate of growth of the economy is given by the rate of the evolution of new intermediate good varieties. But the growth rate of intermediate good varieties is a function of the resources (labour and human capital) devoted to the research sector. As the supply of human capital is perfectly inelastic, employment of human capital is fixed (at full employment level) and is not affected by the unionisation of the labour market. When we want to know the impact of unionisation on the rate of growth we have to determine the impact of the wage bargaining in the two sectors on employment in the research sector. In the last section the overall employment effects have been analysed. In the following we focus on the effects on research sector employment. Moreover we want to compare the rate of growth in the union case and in the non-union case. This means that we analyse how parameters change the rate of growth in the union case, but also how parameters change the growth differential between the unionised and the non-unionised economy.\textsuperscript{17}

First of all we analyse the effects of parameters that only affect research employment in the unionised case (to distinguish effects on the rate of growth and the growth differential). Inspecting equations (19) and (36) it can be seen that these are only the replacement ratio $b$, union bargaining power in the intermediate sector $\beta$ and changes of the relative wage (where we focus on the effect of changes of the bargaining power of research sector unions and changes of $\alpha$). A higher replacement ratio and/or greater bargaining power of research sector unions will unambiguously dampen employment in the research sector. This is straightforward as this will lead to higher bargained wages in the research sector. This causes a reallocation of the labour pool to unemployment (in the case of a higher $b$) or to the intermediate goods sector (in the case of a higher $\mu$). An interesting point is the impact of higher union bargaining power in the intermediate sector (hence a rise in $\beta$). This has direct and indirect effects on employment in the research sector and hence on the growth rate of the economy. The direct effect is that the wage setting curve will shift inwards when $\beta$ increases. This obviously

\textsuperscript{17}We define the growth differential as the growth rate in the non-union case over the growth rate in the union case. So we can use the difference between equations (19) and (36) to analyse the impact of parameters on the growth differential.
dampens research sector employment. The indirect effect is that a higher $\beta$ lowers the wage in the research sector in terms of the intermediate sector wage ($z$ decreases). On the one hand this will result in a downward shift of the price setting curve, because labour demand in the research sector increases relative to labour demand in the intermediate sector. This implies a reallocation of resources to the research sector. On the other hand the wage setting curve will shift outwards. The union in the intermediate sector will moderate wage demands (given research sector employment), so that employment would increase. Taking the direct and the indirect effects into account, the overall effect of a higher $\beta$ on research sector employment is (using equation (36) and (41)) given by:

$$-c_2 + (c_4 + 1) \cdot \left( \frac{(1 - a) \cdot \beta}{(1 - \beta) \cdot \alpha + \beta} \right)$$

When this coefficient exceeds zero, an increase in the bargaining power of intermediate sector union power will result in higher research sector employment and hence in a higher rate of growth. Using the definition of $c_2$ and $c_4$ given in the appendix we get the following condition that must be satisfied for research employment to increase when $\beta$ rises:

$$-\bar{L} + \frac{1}{\phi_n} \cdot L_n + (1 - b) \cdot L_x > 0$$

As equation (45) reveals, this condition will never be satisfied because both $\frac{1}{\phi_n}$ and $(1 - b)$ are smaller than one and with the resource constraint $L_n + L_x$ cannot exceed $\bar{L}$. So unionisation of the intermediate sector labour market will unambiguously dampen the rate of growth. But nevertheless the decline of the rate of growth will be lower, the higher is the mark up in the research sector and the lower is the replacement ratio. The economic intuition for these properties is the following. A lower replacement rate implies that the inward shift of the wage setting curve is small so that the employment dampening effect is small. That a higher mark up in the research sectors lowers the decline of the rate of growth seems at first glance unorthodox. The intuition for this strange property is that a higher mark up in the research sector ($z$ is high) implies that the same relative decrease of $z$ due to higher bargaining power in the intermediate sector is accompanied by a very high absolute decrease of the relative wage. With the absolute decrease of $z$ being high, the positive effects of a high relative labour demand in the research sector and lower wage demands by the intermediate sector unions (because unemployment is more costly) are very high. So the inward shift of the wage setting curve is moderate and the shift of the price setting curve is strong. Both effects dampen the decline of the rate of growth due to a unionisation of the intermediate sector.

Another interesting parameter is the change of the price elasticity of the demand of intermediate goods $\alpha$. This change has two effects on research sector employment. On the one hand a higher $\alpha$ implies that labour demand in the research sector declines as the price for
blueprints declines.\textsuperscript{18} With a lower relative labour demand in the research sector, employment and hence the rate of growth will decrease. This is the standard effect also present in the competitive case. In addition to this effect, the wage demands in the intermediate sector decrease so that the relative research sector labour demand decreases. So the negative effect on the price setting curve is even stronger in the unionised case. On the other hand a higher \( \alpha \) will shift the wage setting curve outward. The economic intuition for this property is the following. The mark up intermediate sector unions will set is lower when \( \alpha \) increases, but, as the relative wage \( z \) increases (being unemployed is less costly, because the unemployed have a chance of getting a job in the research sector, which is relatively better payed than before) and so the value of being unemployed increases and hence the bargained wage. So the effect of a change of \( \alpha \) on the wage setting curve is ambiguous. What we can show is that if the latter effect on the wage setting curve dominates (the wage setting curve will shift inward as \( \alpha \) increases), the overall effect on research sector employment of higher \( \alpha \) is definitely negative.

The coefficient determining the overall effect of a change of \( \alpha \) on research sector employment is (using (36) and (41)) given by:

\[
c_3 - \frac{b_2}{1 - \alpha} - (c_4 - 1) \cdot \frac{\alpha}{(1 - \beta) \cdot \alpha + \beta}
\]  

(46)

The sign of this coefficient is indeterminate, as the preceding analysis already suggested. Nevertheless it can be shown (using the definitions of \( c_3 \) and \( c_4 \)) that the coefficient is more likely to be negative (a higher \( \alpha \) will result in a lower rate of growth) when:1.) employment in the research sector is high, 2.) the mark up in the research sector is high, 3.) union bargaining power in the intermediate sector is low and 4.) \( \alpha \) is big. The economic intuition for the latter two properties is that in these cases the absolute decline of the mark up in the research sector will be low and hence the (employment enhancing) outward shift of the wage setting curve will be low. The first two properties imply that the wage decrease in the intermediate sector will be low as the increase of the outside opportunity will dominate. The consequence is again that the shift of the wage setting curve will be low and hence it will be more likely that the bad effect due to lower labour demand in the research sector will dominate.

But not only the affect of union wage bargaining is of interest, besides also (as already mentioned) how the parameters that determine research sector employment in both the unionised and the competitive case, affect the growth differential, defined as research sector employment in the competitive case over research employment when unions bargain the wage. Using equations (36) and (19) and assuming that the change of all parameters that only affect research employment in the competitive or in the union case are zero \( (\bar{b} = \bar{\mu} = \bar{\beta} = 0) \), we get the

\textsuperscript{18}The argument goes again as follows. Higher \( \alpha \) implies lower monopoly profits in the intermediate goods sector. Hence there are fewer entrepreneurs entering the intermediate goods market with the consequence that blueprint demand decreases. This dampens blueprint prices.
linearised growth differential ($\tilde{g}$) as:

$$\tilde{g} = e_1 \cdot \tilde{L} + e_2 \cdot \tilde{H}_n - e_3 \cdot \hat{\rho} + e_4 \cdot \hat{a} \tag{47}$$

Equation (47) reveals that changes of the stock of human capital will narrow the growth differential as long as $c_5$ is smaller than $\frac{L_n}{L_m}$. This is true as long as $z$ is smaller than one, i.e. unionisation in the research sector is "weaker" than in the intermediate sector. The intuition for this property is that in both cases (when labour markets are competitive and when they are unionised) research employment will increase (as the marginal productivity of labour will increase). But in the unionised case there will be an additional reallocation of the labour pool towards the research sector as long as it is more competitive (which is the case when $z$ is smaller than one). Hence the overall increase of the rate of growth due to an increase of the stock of human capital will be bigger in the unionised than in the competitive case. So the impact of a changing stock of human capital on the rate of growth of the economy will be more severe in the unionised economy as long as $z$ is smaller than one. Very similar arguments hold true when analysing the effect of a change of the rate of time preference on the growth differential. When $z$ is smaller than one, a decline of the rate of time preference of agents implies a bigger increase in research sector employment in the unionised than in the competitive case. Hence the growth differential will decrease. Speaking in graphical terms, the wage setting curve is flatter than the resource restriction (in $L_x-L_n$-space). Hence any shift of the price setting curve will have a stronger effect on research sector employment.

The coefficient showing the impact of a change of the labour pool on the growth differential exceeds one as long as the following holds:

$$\frac{L}{L - L_n} \cdot \frac{1}{c_1} = \frac{1 - \lambda + \lambda \cdot b_1 + \frac{L_n}{L - L_n}}{1 - \lambda + \lambda \cdot b_1 + c_5} \tag{48}$$

It can be shown (using the definition of $c_1$) that the first term of equation (48) always exceeds one. So as long as the second term is unambiguously smaller than one, the coefficient will be positive. This will be the case as long as $z$ exceeds one (and so the mark up in the research sector is bigger than in the intermediate sector).

With a positive coefficient the growth differential will rise when the stock of labour rises. Hence the increase of research employment will be bigger in the competitive case than in the unionised case. The intuition is that the allocation of the bigger resource pool between the two sectors of the economy will be biased in the unionised economy as labour demand will be higher in the sector where the relative wage is lower. Hence the increase of research employment will be low in the unionised case when the mark up in that sector is high.

---

19It is true that the coefficient is positive when $z$ exceeds one. But it is obviously not true that the coefficient is negative when $z$ is smaller than one. There will be an intermediate case with $z$ smaller than one but a positive coefficient. We will not consider this intermediate case.
The sign of the coefficient depicting the impact of $\alpha$ on the growth differential is indeterminate. This is because the effect of changes of $\alpha$ on research employment is indeterminate in the unionised case (as was already analysed in the preceding part), whereas in the competitive case higher $\alpha$ will always lead to lower research sector employment.

6 Conclusion

In this paper union wage bargaining has been integrated into a standard growth model of expanding product variety. There will be simultaneous wage bargaining in the research as well as in the intermediate sector. As the union rations labour (by bargaining a wage that exceeds the competitive outcome) there will be unemployment in the economy. Moreover there will be a wage differential between the two sectors of the economy. This wage differential cannot be competed away, because workers cannot switch directly from one job to another. So if they want to change sectors they have to give up their current employment and try to find a job in the other sector. But as unions ration jobs there will be a probability that they do not get employed and stay unemployed for a certain amount of time. So the allocation of the labour pool is no longer driven by arbitrage but by the wage bargain of unions.

First of all the impact of unionisation on unemployment has been analysed. It was shown that a higher stock of labour will raise employment in both sectors of the economy. An interesting point is that although overall employment will increase, it is possible that the unemployment rate will increase. The employment effects of a higher stock of human capital and a lower rate of time preference are ambiguous and depend on the relative wage $z$. When $z$ is lower than one a higher stock of human capital or a lower rate of time preference implies a reallocation to the research sector. As this is less unionised when $z$ is lower than one overall employment will increase, too. It was also shown that the employment effects of higher union bargaining power is ambiguous and also hinges crucially on the wage differential. Another interesting point is that lower intermediate sector competitiveness will not necessarily lead to neither higher employment in the unionised economy nor to a lower rate of growth (as is sometimes suggested in the standard labour and growth literature (see e.g. Aghion and Howitt (1999) or Layard et al. (1991)).

Moreover we analysed the impact of unionisation on the rate of growth of the economy. Not surprisingly the unionisation of the research sector labour market is unambiguously bad for the rate of growth. Higher bargaining power in the research sector will dampen research sector employment and hence the rate of growth. The effect of the unionisation of the intermediate goods sector is growth dampening, too. Although in this case there are two countervailing effects (higher bargaining power increases the rationing effect of the union, but there is also a reallocation effect, because the relative labour demand in the research sector increases) it can be shown that the rate of growth will always decline.
Eventually the impact of parameters changes on the growth differential, defined as research employment in the competitive case over research employment in the unionised case have been analysed. It was shown that these parameter changes (e.g. changes in the stock of human capital, the labour pool or the rate of time preference and the competitiveness parameter $a$) affect the rate of growth in the two cases asymmetrically. It could be shown that the impact of parameter changes is more severe in the unionised economy as long as $z$ is smaller than one and hence unionisation in the research sector is weaker than in the intermediate sector.

A Appendix

A.1 The bargained wage in the intermediate goods sector

Maximising the Nash product given by equation (24) subject to the labour demand curve in the intermediate goods sector yields the following first order condition (remember that the wage elasticity of the intermediate goods sector labour demand is identical with the price elasticity of the intermediate goods demand, because of the linear production function):

$$\beta \cdot \left( \frac{1}{\alpha - 1} + \frac{1}{\rho - g_Y + a \cdot (1 + g_Y)} \cdot \frac{w^L_{x(i)}}{V^x_{i(t)} - V^U_t} \right) = (1 - \beta) \cdot \frac{\alpha}{1 - \alpha} = 0 \quad (A1)$$

From equation (20) we know that in a steady state the following holds:

$$(\rho - g_Y + a \cdot (1 + g_Y)) \cdot V^x_{i(t)} = w^L_{x(i)} + D_t + a \cdot (1 + g_Y) \cdot V^U_t \quad (A2)$$

Manipulation of (A2) shows that along the steady state the following will hold:

$$(\rho - g_Y + a \cdot (1 + g_Y)) \cdot \left( V^x_{t} - V^U_t \right) = w^L_{x(i)} + D_t - (\rho - g_Y) \cdot V^U_t \quad (A3)$$

Plugging (A3) into (A1) and collecting terms yields the wage equation stated in the text.

A.2 The bargained wage in the research sector

Maximising the Nash product given in equation (26) with respect to the wage and subject to the labour demand curve in the research sector (given by equation (8)) this gives the following relation:

$$\frac{\mu}{\rho - g_Y + e \cdot (1 + g_Y)} \cdot \frac{w^L_n}{V^n_t - V^U_t} = \frac{(1 - \mu) \cdot \gamma + \mu}{1 - \gamma} \quad (A4)$$
From equation (21) we know that the following holds:

\[
(\rho - g_Y + e \cdot (1 + g_Y)) \cdot (V_t^n - V_t^U) = w_n^L + D_t - (\rho - g_Y) \cdot V_t^U
\]

(A5)

Plugging (A5) into (A4) we can solve for the bargained wage in the research sector as given in the text.

A.3 Coefficients of the linearised equilibrium Conditions

This table shows the coefficients of the linearised price setting and wage setting curve. Whenever possible it is shown whether the coefficient exceeds one or not.

<table>
<thead>
<tr>
<th>( b_1 )</th>
<th>( \frac{g_n \cdot \frac{1 - \alpha}{L_n}}{g_n \cdot \frac{1 - \alpha}{1 + \rho}} &lt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_2 )</td>
<td>( \frac{g_n \cdot \frac{1 - \alpha}{L_n}}{g_n \cdot \frac{1 - \alpha}{1 + \rho}} &lt; 1 )</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &lt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; 1 )</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( c_4 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; 1 )</td>
</tr>
<tr>
<td>( c_5 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; 1 )</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( d_3 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( d_4 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( d_5 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( d_6 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( d_7 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( e_1 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( e_3 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
<tr>
<td>( e_4 )</td>
<td>( \frac{L \cdot (1 - \beta \cdot \frac{1 - \alpha}{1 - \alpha})}{L_n} &gt; \frac{1 - \alpha}{1 - \alpha} )</td>
</tr>
</tbody>
</table>
References


   "http://www.cesifo.de/pls/diceguest/search.process_simple.search_page"


