Human Capital, Fertility, and Growth under Borrowing Constraints *

Erasmo Papagni†
Dipartimento di Diritto ed Economia, Università di Napoli II,
Piazza Umberto I, 81043 Capua.

March 2002

Abstract

In this paper we investigate economic growth in economies where households face liquidity constraints, and young agents rely on the family to finance their investments in education. We analyze the type of family aid in which youths can borrow because their parents guarantee the loan repayment with their income. In an OLG model of economic growth, it is shown how multiple equilibria can arise. A stable trap of low-development is characterized by high fertility rates and low investment in human capital. On the other hand, economies with a sufficiently low starting rate of fertility grow according to a process that may describe a demographic transition. In this case, borrowing constraints gradually vanish and the process of growth reaches a steady state characterized by the optimality of fertility and schooling choices. Econometric evidence on the significant roles of family

*During this research I was visiting two times at the Department of Economics of UCLA, which I thank for hospitality. I am grateful to Costas Azariadis for helpful discussion and suggestions. I also thank seminar partecipants at the Universities of Salerno and Parthenope for comments and suggestions. The usual disclaimers apply. Financial support from the National Research Council of Italy is acknowledged
†Tel.: 39771740845; Fax:39823622984

income and size, and credit constraints among the determinants of international secondary school enrollment rates is provided to support the main hypotheses of the model.

JEL codes: O41; O15; J13.

Keywords: Growth; Education; Borrowing Constraints.

1 Introduction

An important tradition in the theoretical and applied analyses of economic development concentrates on human capital and fertility. Human capital and the rate of population growth are economic and demographic phenomena deeply rooted in the family's organization and behavior. From this point of view economists (e.g., Becker, Murphy and Tamura, 1990; Tamura, 1996; Galor and Weil, 1998; Dahan and Tsiddon, 1998) analyze the demographic transition (Caldwell, 1976) from the state of low development with high fertility to that of a growing economy with low fertility rate[1].

In recent years, models of economic growth with multiple equilibria have been put forward to provide a comprehensive account of poverty traps (Azariadis, 1996). Aggregate externalities in human capital production and coordination failure (Lucas, 1988; Azariadis and Drazen, 1990) may hinder the development of an education system in poor countries. The same negative influence on human capital can be ascribed to capital market imperfections (De Gregorio, 1996). Among the most relevant are asymmetric information owned by banks and households, and the low quality of human capital as a collateral (Becker, 1993). An important strand of research (e.g., Banerjee and Newman, 1993; Galor and Zeira, 1993) assumes borrowing constraints on investments in education to analyze the evolution of income distribution and economic growth. Galor and Zeira (1993) assume credit markets in which there are costs that borrowers pay in terms of interest rates higher than lending rates. Due to this imperfection, individuals who invest in education rely on bequests of their parents, and economic growth depends on the initial distribution of wealth.

Credit market imperfections play a significant role in recent models of growth with endogenous determination of fertility and human capital. Galor

[1]Recent reviews of the literature on endogenous population and economic growth are Ehrlich and Lui (1997) and Nerlove and Raut (1997).

and Weil (2000) make an ambitious attempt to model the transition from a Malthusian regime of economic growth to a modern regime characterized by the negative relation between fertility and the rate of growth of per capita income. The crucial assumption in Galor and Weil (2000) is the complementarity of education and technological progress. The model gives rise to a virtuous circle between these two forms of knowledge production which raise wages and the return to child quality. Technical change causes substitution of child education for number of children, and triggers the onset of a demographic transition. Dahan and Tsiddon (1998) propose a model in which population is composed of skilled and unskilled agents. Due to capital market imperfections, children's education depends on bequests from the parents. The dynamics of population and human capital differ in the two classes of agents, and the demographic transition depends on income distribution.

In this paper we examine the role of the family in the finance of investments in education and, in particular, we investigate how altruistic parents who face borrowing constraints may reduce fertility to increase children's human capital. This is the main innovation of our model with respect to the existing macroeconomic literature on endogenous fertility which explains the decline in population growth as a by-product of economic growth. Such a modeling strategy downplays the active role of the family in the finance of investments in education and in economic growth. This role can be appreciated by putting together three important pieces of evidence arising from the applied literature on human capital, the family and economic growth. In fact, constraints on household borrowing have negative effects on aggregate educational attainment in cross-country econometric estimates presented in De Gregorio (1996). When credit is constrained, empirical studies of the economics of the family (e.g., Behrman, Pollak, and Taubman, 1989; Hanushek, 1992; Haveman and Wolfe, 1995), and of the literature on economic development (e.g., Schultz, 1988; Parish and Willis, 1993) show that individual educational attainment depends on family resources and family size. A significant negative parameter of family size is shown by Galor and Zang (1997) in cross-country regressions of the rate of growth of per capita income.

These econometric results strongly suggest a picture of the demographic transition in which the rate of fertility is an important instrument which altruistic parents may regulate to compensate for limited access to financial resources for children's education. Our model provides a new theoretical account of the endogenous behavior of the family in the demographic transition.

While most of the literature on liquidity constraints and human capital deals with bequests of parents to children, we examine the case in which youths can borrow because their parents guarantee the loan repayment with their income.

In the model that we propose the economic resources and size of the family become fundamental determinants of the educational choice because we assume that banks lend money proportionally to the parents' income. The family's income can be thought of as the collateral banks require to finance single family members. In an overlapping generations approach, adults are altruistic toward their children, therefore tending to tradeoff the positive effects on utility of one more child against the negative effects caused by tighter credit constraints and lower investment in education. Indeed, the family's income is a common resource that each young agent shares with other members. As family size grows, the amount of credit available to each member shrinks.

The dynamics of fertility and education are linked through the financial system. As a consequence of initial values of the rate of population growth, the model can describe economic growth as a low-development trap in high fertility economies characterized by heavy household borrowing constraints and low investment in knowledge. Conversely, a demographic transition could be experienced by economies that grow fast because parents - generation after generation – have fewer children and devote a growing amount of income to their education.

This paper shares with Galor and Zang (1997) the same focus on family size among the determinants of human capital and economic growth. In their article, under borrowing constraints, individual agents can finance schooling costs with a bequest that depends negatively on family size. Fertility is exogenous and has a negative influence both on the proportion of the labor force that becomes skilled workers, and on steady state per capita income. Our analysis seems complementary to that of Galor and Zang (1997) since it concerns a different type of funds for investment in education and concentrates on the dynamics of fertility.

This paper is organized as follows. Section 2 contains a general OLG model in which education is financed by loan contracts between young agents and banks with the guarantee of the parent's income. In section 3, the model is analyzed under the assumption of a small open economy and perfect credit markets. In section 4 we assume investment in human capital can be financed only with the aid of the family, and derive multiple equilibria from the model. Section 5 contains results of cross-country panel regressions of the secondary school enrollment rate againts variables proxying for family size and resources, and financial development. Conclusions follow in section 6.

2 The model

This section puts forward a general model of growth in which altruistic parents make decisions on consumption, savings, number and quality of children. The agent's life is summarized in three ages: childhood, middle age and old age; and in every period population is made up of three overlapping generations. Young agents acquire skills both from their parents and attending schools, which requires full time effort. Students finance their education by borrowing from banks. The credit market is imperfect and young individuals can borrow only up to an amount which banks establish commensurate to a collateral provided to children by their families. Adults are employed in a private sector that produces goods with physical and human capital inputs. Adults have children and allocate their time endowment net of leisure - normalized to one - to work and child care. Saving during the meddle age allows consumption in old age. The last generation owns physical capital.

2.1 Technology

Firms produce a single homogeneous good Y_t under perfectly competitive conditions, with physical capital K_t and labor in effective units as inputs. The labor force is composed by N_t adults, each of whom is endowed with e_t units of education; hence the maximum amount of efficiency units of labor is $L_t = e_t N_t$. However, each adult has $n_t = \frac{N_{t+1}}{N_t}$ children, and child rearing takes τ hours per child. Hence, the total amount of efficiency units of labor is $L_t(1 - n_t\tau)$, where $(1 - n_t\tau)$ represents hours spent on the job. A well-behaved concave production function with constant returns to scale describes production technology:

$$Y_t = F\left[K_t, L_t(1 - n_t\tau)\right] = L_t(1 - n_t\tau)y_t;$$

where

$$y_t = f(k_t) \text{ and } k_t = K_t/L_t(1 - n_t\tau);$$

Hence, y_t is output per unit of effective labor, and k_t is the ratio of capital on labor input measured in efficiency units. We assume that the capital stock depreciates fully in one generation.

Factors of production are paid their marginal contribution to production:

$$R(k_t) = f_k(k_t) \tag{1}$$

$$w(k_t) = f(k_t) - k_t f_k(k_t) \tag{2}$$

where $R(k_t)$ is the gross yield on loans, and $w(k_t)$ is the wage rate per efficiency unit of labor. The adult's earnings are denoted I_t:

$$I_t = e_t w(k_t)(1 - n_t\tau)$$

Human capital of young individuals born in period t - that we denote e_{t+1}- is produced with full time effort and two inputs: knowledge of the parents and physical goods b_{t+1} :

$$e_{t+1} = Ae_t^{1-a} b_{t+1}^a ; a \in (0,1) . \tag{3}$$

The role of parents' education in equation (3) is that of an intergenerational externality. Schooling activity requires material resources and students must finance the consequent expenditures by borrowing in the credit market. The parameter A represents the level of technology in human capital production. This technology shows constant returns to scale, and the stock of knowledge does not depreciate, hence, if enough resources are devoted to human capital accumulation it can proceed in the future without limit.

2.2 Adult behavior

Adult agents devote income earned on the job to the welfare of their family. Preferences of adults include their life-cycle consumption, number and education of children. Such preferences are represented by a concave intertemporal utility function:

$$U\left(c_t^{t-1}, c_{t+1}^{t-1}, n_t, b_{t+1}\right) \tag{4}$$

where c_j^i is consumption in period j of an individual born in period i. Investment in human capital enters the utility function of adults both because it approximates the future welfare of children and the noneconomic value of knowledge. The cost of education b_{t+1} includes consumption of the first generation.

Parents can affect the amount of resources children spend for human capital investment when, as a consequence of credit market imperfections, child borrowing is constrained and the family provides collateral in loan contracts of children. Otherwise, education can be financed without restrictions. Here, we denote with $b_{t+1}^o$the unrestricted choice of investment in education, and with $b_{t+1}^c$the constrained choice. The following household maximization program (5) can represent both these two alternatives :

$$\begin{aligned} & \underset{c_t^{t-1}, c_{t+1}^{t-1}, N_t}{MaxU} \left(c_t^{t-1}, c_{t+1}^{t-1}, n_t, b_{t+1}\right) \\ & s.t. \\ & c_t^{t-1} + \frac{c_{t+1}^{t-1}}{R(k_{t+1})} + b_t R(k_t) \leq e_t w\left(k_t\right)\left(1 - n_t \tau\right). \end{aligned} \tag{5}$$

The life-cycle budget constraint derives from adult's allocation of labour earnings to loan repayment and present and future consumption.

Problem (5) can be specified in two different ways according to the value of the investment in education: b_{t+1}^oor b_{t+1}^c.The main difference lies in the dependence of the constrained value b_{t+1}^c on the fertility rate n_t, since the optimal choice of human capital investment is a function of future fertility of children but does not depend on their family size. As will be clear in Section 4 when parents choose the number of children they take into account the negative effect of one more child on children's resources for education.

2.3 Child behavior

During childhood individuals acquire skills through their relations with the adult population. This process of knowledge transfer must be supported by expensive activities of learning. We can easily think about schooling, or less formal means for information and knowledge transmission which have some cost (books and magazines; computers and tvs; journeys; etc.). The main reason for investment in human capital is the return that children will gain in middle age in the labor market. Here we abstract from the important non market value of knowledge and culture that explains part of people's choices.

2.3.1 Optimal choice of human capital investment

During the first part of their life agents choose the level of education they will be endowed with when adult. Children are neutral to risk, and the optimal choice of human capital investment derives from the maximization of net returns:

$$\underset{b_{t+1}}{Max} \left\{ e_{t+1} w\left(k_{t+1}^e\right)\left(1-n_{t+1}^e \tau\right) - b_{t+1} R\left(k_{t+1}^e\right) \right\}; \tag{6}$$

where k_{t+1}^e and n_{t+1}^e denote expectation taken at time t on k_{t+1}, n_{t+1}. We assume that agents have perfect foresight. Given eq. (3), problem (6) is concave in b_{t+1} and the following first order condition is sufficient to maximize the net revenue from investment in education:

$$\frac{\partial e_{t+1}}{\partial b_{t+1}} w\left(k_{t+1}^e\right)\left(1-n_{t+1}^e \tau\right) - R\left(k_{t+1}^e\right) = 0 \tag{7}$$

From equation (7) the following function for the optimal choice of expenditure in human capital can be derived:

$$b_{t+1}^o = e_t \left[\frac{Aw\left(k_{t+1}^e\right) a}{R\left(k_{t+1}^e\right)} \right]^{\frac{1}{1-a}} \left(1-n_{t+1}^e \tau\right)^{\frac{1}{1-a}} \tag{8}$$

The optimal value b_{t+1}^o is a linear function of the parents' human capital, and an increasing function of the capital to labor ratio. A peculiar feature of the decision rule eq. (8) is its dependence on the size of the family that a young agent will build in the future. She has to foresee not only future wage and interest rate but also her choice as an adult with respect to the number of children. Higher fertility reduces the net returns of investment in education because less effort can be applied on the job.

2.3.2 The finance of educational expenditures

Education has a direct cost that students finance by borrowing on a credit market that is not perfect. Human capital has a high degree of illiquidity, slavery is prohibited and human capital cannot serve as collateral. To lend money, banks require concrete collateral, and income and wealth of the parents are actually among the most common. Here we assume that there is agreement in the family with respect to the educational programs of the children.

Loan contracts involving families could be rationalized as a case of group lending (e.g. Varian, 1990; Besley and Coate, 1995). In undeveloped economies, where banks have limited power against borrowers who do not repay their debts, contracts that involve the joint liability of a group of lenders can increase the repayment ratio. In a similar way we assume that altruistic parents may provide collateral to debt contracts of their children because they have tight control over repayment of loans (see Banerjee and Newman, 1993).

A child born at time t applies for a loan D_t that she will spend on her educational project: $D_t = b_{t+1}$. Even if from this investment she will get a certain return $V(D_t)$, she cannot borrow by pledging her future earnings. Parents care about her welfare, and provide a collateral C_t out of their income. The outcome of investments in human capital is not uncertain, but loan repayment can be under risk due to moral hazard arguments. Rational children always spend the amount they borrow in education, but when they become adult they might renege on this debt, for example leaving the country to find a job abroad. From the point of view of lenders, adult's income is a reliable collateral in an institutional context in which there is sufficient enforcement of property rights. On the other hand, parents have some control over children's behavior. If children do not repay a loan when they become adult, we assume that - with probability p - parents can punish them and recover the lost collateral.

As a consequence, lenders and parents make loans of the amount that ensures repayment. When children repay their debt, net returns are

$$V(D_t) - D_t R_{t+1};$$

while in the opposite choice they expect to earn:

$$V(D_t) - pC_tR_{t+1}.$$

Hence, loan repayment is the best decision when:

$$V(D_t) - D_tR_{t+1} \geq V(D_t) - pC_tR_{t+1};$$

which means: $D_t \leq pC_t$.

Children's borrowing depends on the value of collateral C_t that parents provide. In this imperfect market for household loans lenders and borrowers have asymmetric information, hence lenders estimate the household's reliability and sign contracts only up to a maximum amount of collateral. Lenders set the maximum amount of family's borrowing on the basis of its labor earnings, which is an information easily available to banks. Other information that is useful to lenders is the value of household debts. Banks do not share information on their customers, hence households can hide the level of their borrowing. Accordingly, we assume that the rule lenders follow is to accept collateral if it is lower than a ceiling commensurate to the average income of family members:

$$C_t \leq \phi \frac{I_t}{n_t}; \ \phi > 0,$$

hence:

$$D_t = b^c_{t+1} \leq p\phi \frac{e_t w(k_t)(1 - n_t\tau)}{n_t}. \tag{9}$$

Equation (9) gives the maximum value of children's borrowing as a function of the parent's income and family size. Hence, in this model the effective extent of liquidity constraint is an endogenous variable. Higher fertility has a negative effect on children's education when the optimal value of investment in education is higher than the value of the loan available on the credit market:

$$b^o_{t+1} > b^c_{t+1}$$

In the next sections we consider the case of a small open economy and perfect international capital mobility. Accordingly, capital accumulation is driven by the exogenous international rate of interest R^w, and domestic savings are unrelated to investments. This hypothesis also means that at any time t the capital stock per unit of effective labor takes the value that ensures equality between productivity and international interest rate:

$$R(k) = f^{'}(k) = R^w \tag{10}$$

Hence, in equilibrium

$$w(k) = f(k) - kf^{'}(k); k = f^{'-1}(R^w) \tag{11}$$

We assume that the economy of the rest of the world is in steady state with constant R^w. In the following sections symbols k, R, w without time subscript denote the corresponding variables in eqs. (10) and (11).

As a relevant consequence of this hypothesis on the international capital markets, both the optimal and constrained choices of human capital investment are function of parent's education and fertility rate:

$$b_{t+1}^o = e_t b(n_{t+1}); b_{t+1}^c = e_t b(n_t).$$

The comparison between the two alternatives depends on the future choice of fertility of a child and on her family size. Child borrowing could be constrained if children living in large families were to prefer a significant investment in their skills because when adult they will build a small family. If the size of children's family is too big, young individuals spend less than the optimal amount on their education and will not have the desired number of children.

Investment in education plays a crucial role in this model. Economic growth depends on the accumulation of human capital. In the case of sub-optimal expenditure for education, the rate of growth of per-capita income is low and the economy converges to a low state of development. The opposite regime of endogenous growth can be the outcome of economic dynamics driven by the unconstrained choices of fertility and education. The rate of fertility governs this change of regime of economic growth.

In the following section we analyze the model dynamics when family size is low enough to ensure that the financial system always provides enough resources to finance the optimal value of human capital investment.

3 Equilibrium growth with unconstrained human capital accumulation

The model describes an economy populated by individuals who, when young, choose the optimal amount of expenditure in human capital investment and find in the credit market the relative amount of resources. Adult individuals enjoy the returns of this investment and allocate labor earnings - net of debt repayment - to consumption, savings, number and quality of children. These decisions are consistent with the next generation's planned investment in education.

Given the optimal choice of education that children make, the following is the decisional problem of an adult:

$$\begin{aligned}
&\underset{c_t^{t-1},\, c_{t+1}^{t-1},\, n_t}{Max} \left\{ \log(c_t^{t-1}) + \beta \log(c_{t+1}^{t-1}) + \alpha \log(n_t) + \varphi \log(b_{t+1}) \right\} \\
&s.t. \\
&c_t^{t-1} + \tfrac{c_{t+1}^{t-1}}{R} + b_t R \leq e_t w \left(1 - n_t \tau\right); \\
&e_t = A e_{t-1}^{1-a} {b_t}^a; \\
&b_{t+1} = e_t \left[\tfrac{Awa}{R}\right]^{\frac{1}{1-a}} \left(1 - n_{t+1}^e \tau\right)^{\frac{1}{1-a}}
\end{aligned} \tag{12}$$

The first-order conditions give the following optimal rule for the number of children:

$$n_t \tau e_t w = \frac{\alpha}{1+\beta} \left[e_t w \left(1 - n_t \tau\right) - b_t R \right] \tag{13}$$

where agent's perfect foresight implies: $n_t^e = n_t$. Parents have children up to the number that equates a share $\frac{\alpha}{1+\beta}$ of the earned income net of debt repayment to the opportunity cost of children. After some algebra, from eq. (13) we get the optimal value of the fertility rate:

$$n_t = \frac{\alpha}{\tau(1+\beta+\alpha)} \frac{1 - aA}{1 - \frac{\alpha}{\tau(1+\beta+\alpha)} aA} \equiv n^l < \frac{1}{\tau} \tag{14}$$

n^l is a function of the parameters of preferences and technology, and it is always smaller than $\frac{1}{\tau}$, the maximum value of fertility when time endowment is allocated completely to child rearing.

We derive the dynamics of human capital from the production function eq. (3) in which we substitute the optimal choice of $b_{t:}$

$$\frac{e_t}{e_{t-1}} \equiv \gamma_t = \left[\frac{Awa}{R}\right]^{\frac{a}{1-a}} (1 - n_t \tau)^{\frac{a}{1-a}} \tag{15}$$

Given n_t from eq. (14), education evolves from generation to generation with a constant gross rate of growth γ_t which is greater than one if - among other parameters - the technological parameter A and the ratio w/R are large enough.

Equations (14) and (15) describe the dynamics of the model economy in the case of unconstrained borrowing of children. We can then characterize the competitive equilibrium of the economy. Given the initial values N_0, e_0, a dynamic equilibrium consists of sequences $\{k_t, n_t, e_t\}_{t=0}^{\infty}$ such that the capital market is in equilibrium - $k_t = k = f'^{-1}(R^w)$ - and fertility and education are determined by eq. (14) and eq. (15). Hence, at the initial period the economy instantly goes in equilibrium, and subsequently it grows along a unique steady state path. In fact we define a steady state equilibrium of this economy as a couple of stationary values $(\overline{n}, \overline{\gamma})$ which are given by equations (14) and (15).

Households choose the optimal rate of fertility which remains constant over time. When population grows according to this equilibrium rate, children can borrow on the credit market and can attain their planned level of human capital. Earned incomes grow with human capital and the economy provides further finance to investment in education. This virtuous process of economic growth unfolds into the future without limit.

4 Equilibrium growth with constrained human capital accumulation

A different regime of economic growth applies to the model economy when the optimal choice of investment in human capital cannot be fully financed by loans. The desired amount of expenditure for education is greater than the borrowing limit that lenders are willing to provide to each family member. In this context, a linkage arises between children's investment in human capital and the family's resources and size. Adult individuals are aware of the consequence of their fertility choice on the resources available for children's education and take it into account in their decisional rule concerning the number of children. This rule solves the following problem of an adult at time t:

$$\begin{aligned}
&\underset{c_t^{t-1},\, c_{t+1}^{t-1},\, n_t}{Max} \left\{ \log(c_t^{t-1}) + \beta \log(c_{t+1}^{t-1}) + \alpha \log(n_t) + \varphi \log(b_{t+1}), \right\} \\
&s.t. \\
&c_t^{t-1} + \tfrac{c_{t+1}^{t-1}}{R} + b_t R \leq e_t w \left(1 - n_t \tau\right); \\
&e_t = A e_{t-1}^{1-a} {b_t}^a; \\
&b_{t+1} = \tfrac{p \phi e_t w (1 - n_t \tau)}{n_t}.
\end{aligned} \tag{16}$$

The solution to problem (16) gives the following first order conditions:

$$\frac{1}{c_t^{t-1}} = \lambda; \tag{17}$$

$$\frac{\beta}{c_{t+1}^{t-1}} = \frac{\lambda}{R}; \tag{18}$$

$$\frac{\alpha}{n_t} - \frac{\varphi}{(1 - n_t \tau) n_t} = \lambda \tau e_t w \tag{19}$$

The left side of eq. (19) is the marginal utility of children which is the sum of a positive effect of fertility and a negative effect on utility due to the reduction of funds available for children's education. Hence, according to eq. (19) parents have children up to the number that equates marginal benefits to marginal costs measured in utils. In order to ensure non-negative marginal utility of children we set the following assumption:

- Assumption A1.

$$n_t. \leq \frac{1}{\tau}\left(1 - \frac{\varphi}{\alpha}\right)$$

Some algebra on the full set of first order conditions gives the following first order difference equation:

$$n_{t-1} = \frac{1}{\tau + \left(\frac{w^a A}{\theta^{1-a}}\right)^{\frac{1}{1-a}} \left[(1 - n_t\tau) - \frac{n_t\tau(1-n_t\tau)(1+\beta)}{\alpha(1-n_t\tau)-\varphi}\right]^{\frac{1}{1-a}}} = \Phi\left(n_t\right) \tag{20}$$

where $n_t, n_{t-1} \in \left[0, \frac{1}{\tau}\right)$

This nonlinear function $\Phi\left(n_t\right)$, defined in the domain $\left[0, \frac{1}{\tau}\right)$, assumes positive values, is continuous and increasing . If there exists an inverse function of $\Phi\left(n_t\right)$, we can derive a nice description of the equilibrium dynamics of the rate of fertility. This analytical development can be easily shown after the introduction of some new notation. Accordingly, we define a new variable x_t, hours of labor:

$$x_t \equiv (1 - n_t\tau);$$

and a new function:

$$g\left(x_t\right) \equiv \left[x_t - \frac{x_t\left(1-x_t\right)\left(1+\beta\right)}{\alpha x_t - \varphi}\right].$$

It can be shown that: $\frac{\partial g(x_t)}{\partial x_t} \equiv g_x\left(x_t\right) > 0;$ $\frac{\partial^2 g(x_t)}{\partial x_t \partial x_t} \equiv g_{xx}\left(x_t\right) < 0;$

The following assumption on $g\left(x_t\right)$ is sufficient to characterize $\Phi\left(n_t\right)$ as a convex function.

- Assumption A2.

$$-\frac{g_{xx}(x_t)\, g(x_t)}{g_x(x_t)} > \frac{a}{1-a}$$

Lemma. *Given Assumptions A1, A2, the function* $\Phi(n_t)$ *is continuous, monotone increasing and convex, and there exists its inverse function* $n_t = \Phi^{-1}(n_t) \equiv \Psi(n_{t-1})$*.* $\Psi(n_{t-1})$ *is continuous, monotone increasing and concave for values of* $n_{t-1} \in \left[0, \frac{1}{\tau}\right)$*.*

Proof. See Appendix A.

This Lemma provides a characterization of the function $\Psi(n_{t-1})$ that describes the dynamics of the rate of fertility. From inspection of equation (20) we derive the intersection of $\Psi(n_{t-1})$ with the horizontal axis. In fact it can be easily verified that $\Psi(n_{t-1})$ assumes positive values when n_{t-1} is greater than $\widehat{n}_{t-1} \geq 0$. Furthermore, when n_{t-1} takes the upper bound $n_{t-1} = \frac{1}{\tau}$, $\Psi(n_{t-1})$ takes a value lower than $\frac{1}{\tau}$. More precisely:

$$\begin{cases} \Psi(\widehat{n}_{t-1}) = 0 \Leftrightarrow \widehat{n}_{t-1} = \left[\tau + \frac{w^a A}{R\theta}\right]^{-1} \\ \\ \Psi(n_{t-1}) = \frac{1}{\tau}\frac{\alpha-\varphi}{1+\beta+\alpha} \Leftrightarrow n_{t-1} = \frac{1}{\tau} \end{cases}$$

The analysis of general equilibrium economic dynamics is based on the recursive system of difference equations:

$$\begin{cases} n_t = \Psi(n_{t-1}) \\ \\ \gamma_t = A\left[\frac{p\phi w(1-n_{t-1}\tau)}{n_{t-1}}\right]^a \end{cases}$$

Human capital grows along a path with a rate that is an inverse function of the rate of fertility. The kind of dynamics that fertility follows depends on the characterization of the difference function $n_t = \Psi(n_{t-1})$ in which we distinguish two cases: $\Psi(n_{t-1})$ crosses twice the 45 degree line; $\Psi(n_{t-1})$ lies below the 45° line.

Figure 1 pictures the former case. There is clear evidence of multiple steady states. The steady state in which fertility is high, n^h, can be thought of as a low-development trap. In fact, when the economy converges to this stable dynamic equilibrium the rate of fertility remains high and consequently the constrained investment in education is low.

When $\Psi(n_{t-1})$ crosses the 45° line from below, the steady state is unstable. On the left of this steady state, fertility moves towards the origin of the axes, but before reaching that value it meets the threshold value that ensures escape from credit constraints. At that lower threshold, fertility instantaneously takes the optimal value n^l and investment in education is determined by the optimal choices of children.

Figure 1 contains all important information on the different regimes of economic growth produced by the model economy. When the initial value of the fertility rate is low enough, a process of development can be described by the progressive reduction of fertility and the joint increase of investment in education by households who find an increasing amount of credit in the market. This process may account for a major component of demographic transition. In the case in which $\Psi(n_{t-1})$ lies below the 45° line there exists only one stable steady state at the optimal value of fertility n^l.

In our model, low-development trap is the outcome of high fertility and credit constraints which reinforce each other in economies whose initial fertility rate is high. This picture seems consistent with important features of the experiences of developing countries.

5 Econometric evidence

The main results of our theoretical analysis concern the existence of low and high growth equilibria and the crucial role of family size in the selection of the growth regime. In this section we provide some econometric evidence which focuses on the main mechanism that drives economic dynamics under both the regimes of growth: the extent of borrowing constraints and that of family resources determine children's investment in human capital. Most of the recent empirical literature on economic growth and human capital concentrates on the determinants of the rate of growth of per capita GDP. As asserted by Krueger and Lindahl at the end of their recent review article (Krueger and Linahl, 2001), in the literature there is robust evidence on the role of level and change in education in economic growth. This econometric evidence provides us with an important motivation for research on the human capital-economic growth nexus, but it is not so useful in the appreciation of the many aspects which lie behind this relation. The development of an educational system is one of the crucial dimensions that distinguish industrialized from less developed countries. Hence, there is important scope for investigation of factors which may hinder or promote human capital investment.

In spite of the extensive microeconometric literature on the educational attainment of children (for a review see Haveman and Wolfe, 1995), more scant is the existing econometric analysis of the determinants of education at the country level. De Gregorio (1996) estimates regressions of secondary and tertiary school enrollment ratios with respect to variables as per capita GDP in 1970, expenditure in education and some financial variables. He finds significant evidence of the effects of borrowing constraints on school enrollment ratios. Similar evidence emerges from the regressions presented in Benhabib and Spiegel (2000), where the dependent variable is the log difference in years of schooling per worker.

In the following econometric analysis we concentrate on the explanation of households' schooling decisions which are proxied by *Enrol-sec*, the secondary school gross enrollment ratio. The full set of explanatory variables includes some proxies of family background and size, and proxies of financial development and credit availability. Data refer to a set of 59 countries and to three years: 1970, 1980, 1990. As a consequence of missing data we performed separate regressions to evaluate the effects of family background and credit market imperfections. In this way, each regression refers to a sample of countries rather representative of the World.

From the World Bank database WISTAT we take *HHsize,* the average household size. This variable is pivotal in our model of schooling decision, hence it always appears in the regressions we performed. From the same database we draw data on the share of female among household heads *Fem-head* that is a good proxy for family strength. Actually, in many developing countries, women's economic conditions are much worse than those of men, hence families that rely on female heads differ greatly from the rest. This effect should be better appreciated by the inclusion of the product *HHsize*Fem-head* among regression variables. Data on *Fem-head* refer only to one year, 1990 or around.

Real per capita GDP, Y, is a proxy for the wealth of a country, and the product *Y*HHsize* is a proxy for the average household income. Parental education and the social environment are considered by the proxy variable

Sec25, the percentage of the population aged 25 and over that attained secondary school. These data are from Barro and Lee (2000). Schooling enrollment decisions are certainly affected by public policy. The state shares with households some of the costs of education, and we include the ratio of public expenditure on education as a percentage of GDP, *Geduc,* among the regression variables. Families also benefit from several kinds of public transfers. The share of expenditures on social security and welfare over GDP, *SocSec,* is the proxy for that form of public aid to the family.

The first set of regressions of the school enrollment rate concerns those variables proxying for family background. The selection of the estimation method is driven by the Hausman test of the null hypothesis of individual random effects versus the alternative of fixed effects.The possible influence of variable endogeneity on the estimated parameters seems less important if we consider that *Enrol-sec* could be thought of a predetermined variable in the regressions for household size and school attainment of the adult population. Actually, both variables, *HHsize* and *Sec25,* refer to decisions made in the past by several generations of adults and should be largely unaffected by schooling choices of the present generation. For example, the family size could depend on the enrollment rate through the effect of schooling on fertility, but data show a very lagged effect of fertility on *HHsize*. School enrollment could appear also in regressions of per capita income, but even in this case it is likely that this improvement in the quality of the labor force has a lagged effect on economic growth. These hints are confirmed by 2SLS estimates that show small differences of parameter estimates obtained with different methods.

Table 1 presents the results of estimates of school enrollment with respect to variables proxying for the family background. Panel data estimates are of the variance components model because this option follows from the Hausman test performed for each equation. At first glance, estimates confirm our hypothesis on the importance of the family as a determinant of aggregate schooling investment. Both the adult level of educational attainment, the size and income of the family always take significant parameters with the predicted sign. Gender of the family head contributes to the explanation of the enrollment rate. This variable also contributes to the estimates when multiplied to *HHsize,* signaling the significant difficulties that big families with female head may face in financing educational investment of children. From these regressions, the action of the state seems significant and positive when directed to the support of households' income (*SocSec*), but appears less influential when addressed to the public finance of education.

The second econometric model we specify for the secondary school enrollment rate includes variables proxying for financial development and credit issued to the private sector. The aim of this econometric exercise is a test of the joint effects of family background and borrowing constraints on investment in education. The set of regressors is enlarged with the inclusion of three proxies of the size and efficiency of the financial system: *Depth, Bank, Credit*. *Depth* is defined by the ratio of liquid liabilities of the financial sector to GDP. *Bank* is the ratio of deposit money bank assets and the sum of deposit money and central bank assets. *Credit* is the ratio of private credit by deposit money banks and other financial institutions to GDP. Data are from Beck, Demirgüç-Kunt and Levine (1999), who rely on IMF publications.

The econometric results are displayed in Table 2. The joint inclusion of household's characteristics and financial variables finds statistical support. *HHsize* enters in many cases with strongly significant parameters with negative sign. The financial proxies show significant parameters, even if we cannot investigate estimate robustness under more general specification due to the lack of data for many countries and variables. The results suggest the importance of the interaction of family structure and resources with the financial system for understanding country effort in human capital accumulation.

There already exists some evidence for the effects of financial development on human capital investment. De Gregorio (1996) regresses the secondary and tertiary school enrollment ratios against the following variables: consumer credit, the maximum loan-to-value ratio, the ratio between total credit from the banking system to the nonfinancial private sector and the GDP. Benhabib and Spiegel (2000) specify regressions of the change in the log of average years of schooling in the labor force with respect to the same variables we use plus income distribution. Our findings not only lend further support to their results, but highlight the crucial role of the family for the explanation of aggregate school enrollment.

6 Conclusions

This paper has put forward a theoretical analysis of the family as a non-market institution which - facing limited access to the credit market - provides their chidren collateral in loan contracts aiming to finance investments in education. This collateral comes out of the parents' income and is a decreasing function of family size. From this kind of educational funding multiple equilibria derive. A stable trap of low-development is characterized by high fertility rates and low investment in human capital. On the other hand, economies with a sufficiently low starting rate of fertility grow according to a process that may describe a demographic transition. In this case, borrowing constraints gradually vanish and the process of growth reaches a steady state characterized by the optimality of fertility and schooling choices.

Our theoretical analysis was complemented by some econometric evidence that provides support for the main assumption of the model. Family size and resources are among the most important determinants of human capital investment even at an aggregate country level.

However, the existing literature lacks deeper investigation of the relations between households and the financial markets. Further research could highlight ways in which families choose to gather and redistribute finances on imperfect capital markets (Cigno, 1993; Behrman, J. R. 1997). This issue could be an important part of models of economic growth and endogenous fertility.

APPENDIX A

Proof of Lemma. Let us define $\Phi\left(n_t\right)$ in the following way:

$$n_{t-1}=\frac{1}{\tau+\left(\frac{w^a A}{\theta^{1-a}}\right)^{\frac{1}{1-a}} g\left(x_t\right)^{\frac{1}{1-a}}} \equiv \frac{1}{Z\left(x_t\right)}.$$

The first and second derivatives of $Z\left(x_t\right)$ are

$$\frac{\partial Z\left(x_t\right)}{\partial x_t}=\left(\frac{w^a A}{\theta^{1-a}}\right)\left(\frac{1}{1-a}\right) g\left(x_t\right)^{\frac{a}{1-a}} g_x\left(x_t\right)>0;$$

$$\frac{\partial^2 Z\left(x_t\right)}{\partial x_t \partial x_t}=\left(\frac{w^a A}{\theta^{1-a}}\right)\left(\frac{1}{1-a}\right) g\left(x_t\right)^{\frac{a}{1-a}}\left[\frac{a}{1-a}+\frac{g_{xx}\left(x_t\right) g\left(x_t\right)}{g_x\left(x_t\right)}\right] \frac{g_x\left(x_t\right)}{g\left(x_t\right)}<0.$$

Hence, applying these two results into the first and second derivatives of $\Phi\left(n_t\right)$ the first part of the Lemma can be easily proved. The proof of concavity of the inverse function follows from the application of the implicit function theorem to the first and second derivatives of $\Psi\left(n_{t-1}\right)$:

$$\frac{\partial \Psi\left(n_{t-1}\right)}{\partial n_{t-1}}=\frac{1}{\Phi_{n_t}\left(n_t\right)}>0;$$

$$\frac{\partial^2 \Psi\left(n_{t-1}\right)}{\partial n_{t-1} \partial n_{t-1}}=-\frac{\Phi_{n_t n_t}\left(n_t\right)}{\left[\Phi_{n_t}\left(n_t\right)\right]^3}<0;$$

APPENDIX B

Enrol-sec Ratio of secondary school enrollment to the population of the age group that corresponds to the level of education, World Bank data.

Y Real GDP per capta in constant dollars, Penn World Tables 5.6.

Sec25 Percentage of secondary school attained in the total population aged 25 and over, Barro and Lee (2000).

HHsize Average households size, WISTAT database World Bank.

Fem-head Percentage of female household heads, WISTAT database World Bank.

Soc-sec Government expenditure on social security and welfare as a percentage of GDP, IMF from World bank data.

Geduc Government expenditure on education as a percentage of GDP, IMF from World bank data.

Depth Ratio of liquid liabilities to GDP, lines 34 and 35 of the IFS, Beck, T., A. D. Demirguc-Kunt, and R. Levine. (1999).

Bank Ratio of deposit money bank assets and the sum of deposit money and central bank assets, IFS, Beck, T., A. D. Demirguc-Kunt, and R. Levine (1999).

Credit Ratio of private credit by deposit money banks and other financial institutions to GDP, IFS, Beck, T., A. D. Demirguc-Kunt, and R. Levine (1999).

References

[1] Azariadis, C. (1996). The Economics of Poverty Traps: Part One: Complete Markets, *Journal of Economic Growth* 1, 449-496.

[2] Azariadis, C., and A. Drazen. (1990). Threshold Externalities in Economic Development, *Quarterly Journal of Economics* 105, 501-526.

[3] Banerjee, A. and A. Newman. (1993). Occupational Choice and the Process of Development, *Journal of Political Economy*,101, 274-298.

[4] Barro, R. J., and G. S. Becker. (1989). Fertility Choice in a Model of Economic Growth, *Econometrica* 57, 481-501.

[5] Barro, R. J. and J. Lee. (2000) International data on educational attainment: Updates and implications, Working paper, Center for International Development, Harvard University.

[6] Beck, T., A. D. Demirgüç-Kunt, and R. Levine. (1999) A New Database on Financial Development and Structure, World Bank mimeo.

[7] Becker, G. S. (1993). *Human Capital.* III ed., Chicago: University of Chicago Press.

[8] Becker, G., and H. G. Lewis. (1973). On the interaction between quantity and quality of children. *Journal of Political Economy*, 81, S279-S288.

[9] Becker, G. S., K. Murphy, and R. Tamura. (1990). Human capital fertility and economic growth, *Journal of Political Economy* 98, S12-S37.

[10] Behrman, J. R. (1997). Intrahousehold Distribution and the Family, in: M. R. Rosenzweig and O. Stark (eds.), *Handbook of Population and Family Economics*, vol. 1B, Amsterdam: Elsevier Science.

[11] Behrman J. R., R. A. Pollak, and P. Taubman. (1989). Family Resources, Family Size, and Access to Financing for College Education, *Journal of Political Economy* 97, 398-419.

[12] Benhabib, J and M. Spiegel. (2000). The role of Financial Development in Growth and Investment, *Journal of Economic Growth*, 5, 341-360.

[13] Besley, T., and S. Coate. (1995). Group Lending, Repayment Incentives and Social Collateral, *Journal of Development Economics* 46, 1-18.

[14] Caldwell, J. (1976). Toward a restatement of demographic transition theory, *Population and Development Review* , 321-366.

[15] Cigno, A. (1993). Intergenerational Transfers Without Altruism: Family, Market and State, *European Journal of Political Economy*, 7, 505-518.

[16] Dahan M., and D. Tsiddon. (1998). Demographic Transition, Income Distribution, and Economic Growth, *Journal of Economic Growth* 3, 29-52.

[17] De Gregorio, J. (1996). Borrowing Constraints, Human Capital Accumulation, and Growth, *Journal of Monetary Economics* 37, 49-71.

[18] Ehrlich I. and F. Lui (1997) The Problem of Population Growth: A review of the literature from Malthus to Contemporary Models of Endogenous Population and Endogenous Growth, *Journal of Economic Dynamics and Control*, 21, 205-242.

[19] Galor O., and D. N. Weil. (2000). Population, Technology, and Growth: From the Malthusian Regime to the Demographic Transition and Beyond, *American Economic Review*, 90, 806-828.

[20] Galor O., and H. Zang. (1997). Fertility, Income Distribution, and Economic Growth: Theory and Cross-Country Evidence, *Japan and the World Economy*, 9, 197-229.

[21] Galor O., and J. Zeira. (1993). Income Distribution and Macroeconomics, *Review of Economic Studies*, 60, 35-52.

[22] Hanushek E. A. (1992) The Tradeoff between Child Quantity and Quality. *Journal of Political Economy*, 100, 84-117.

[23] Haveman, R., and B. L. Wolfe. (1995). The Determinants of Children's Attainments: A Review of Methods and Findings. *Journal of Economic Literature*, XXXIII, 1829-1878.

[24] Krueger, A. B. and M. Lindahl. (2001) Education for Growth: Why and for Whom?, *Journal of Economic Literature*, XXXIX, 1101-1136.

[25] Lucas, R. E., Jr. (1988). On the Mechanics of Economic Development, *Journal of Monetary Economics* 22, 3-42.

[26] Nerlove M. and L. K. Raut (1997) Growth Models with Endogenous Population: a General Framework, in M. R. Rosenzweig and O. Stark (eds.) *Handbook of Population and Family Economics* Vol. 1B, Amsterdam: Elsevier Science Publishers.

[27] Parish, W. L., and R. J. Willis. (1993). Daughters, Education, and Family Budgets, *The Journal of Human Resources*, 28, 863-898.

[28] Schultz T. P., (1988). Education Investments and Returns, in H. Chenery and T. N. Srinivasan (eds.), *Handbook of Development Economics*, Amsterdam: Elsevier Science Publishers.

[29] Tamura, R. (1996). From Decay to Growth: A Demographic Transition to Economic Growth, *Journal of Economic Dynamics and Control* 20, 1237-1261.

[30] Varian, H. R. (1990). Monitoring Agents with other Agents, *Journal of Institutional and Theoretical Economics*, 146, 153-174.

Table 1. Regressions of the rate of enrollment at secondary schools.
The role of family and public resources. Dependent variable *Enrol-sec*.

Estimation	V. C.	V. C.	V. C.	V. C.	V. C.	2
	1	2	3	4	5	
Sec25	0.190	0.188	0.141	0.187	0.173	0.16
	(6.44)	(6.41)	(3.88)	(6.23)	(5.05)	(2.5
Y	0.233		0.219	0.205		0.41
	(4.50)		(3.20)	(3.49)		(2.6
*Y*HHsize*		0.067			0.065	
		(4.73)			(3.86)	
HHsize	-0.626	-0.881	-0.739	-0.849	-0.884	-0.6
	(-3.82)	(-6.20)	(-3.14)	(-4.77)	(-5.33)	(-2.
Fem-head			-0.523			-0.2
			(-2.00)			(-2.4
*HHsize*Fem-head*				-0.07		
				(-1.92)		
SocSec			0.127			
			(2.45)			
Geduc					0.056	
					(0.06)	
Observations	177	177	108	147	138	147
R^2	0.79	0.79	0.83	0.82	0.77	0.80
Hetero	1.25 (0.26)	1.46 (0.23)	1.82 (0.18)	0.76 (0.38)	0.67 (0.41)	
Hausman test	2.87 (0.41)	2.77 (0.43)	4.99 (0.17)	11.69 (0.02)	1.27 (0.87)	

Notes. Student's t are in parentheses. Hetero is a Lagrange multipliers test of heteroskedasticity. In parentheses of Hetero and Hausman are probability values.

Table 2. Regressions of the rate of enrollment at secondary schools.
The effects of financial variables. Dependent variable *Enrol-sec*.

Estimation	O. L. S.	O. L. S.	V. C.	V. C.	V. C.	V. C.
	1	2	3	4	5	6
Sec25	0.171					0.169
	(4.92)					(4.86)
Y	0.210					0.227
	(3.12)					(3.22)
*Y*HHsize*			0.184		0.190	
			(10.06)		(10.73)	
HHsize	-0.062	-0.151		-0.169		-0.064
	(-3.08)	(-8.85)		(-7.56)		(-2.71)
Depth		0.393	0.172			
		(3.63)	(2.46)			
Credit		-0.115		0.192	0.141	
		(-0.81)		(2.50)	(2.04)	
Bank	0.210	0.304				0.095
	(3.08)	(3.45)				(1.40)
Observations	117	132	132	132	132	117
R^2	0.80	0.66	0.69	0.58	0.70	0.79
Heterosked.	1.08 (0.30)	0.25 (0.62)	2.61 (0.11)	3.17 (0.07)	0.63 (0.43)	0.84 (0.3
Hausman test			1.13 (0.57)	8.11 (0.02)	3.40 (0.18)	6.32 (0.1

Notes. Student's t are in parentheses. Hetero is a Lagrange multipliers test of heteroskedasticity. In parentheses of Hetero and Hausman are probability values.

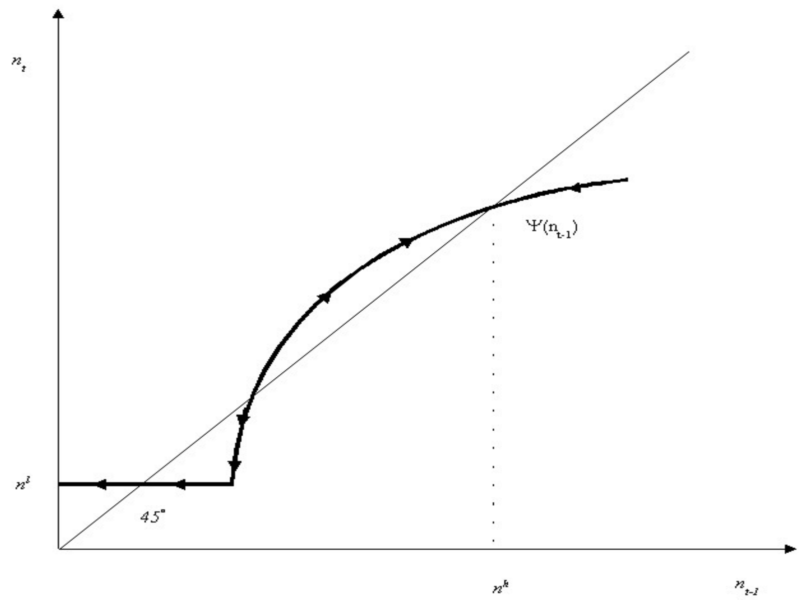

Figure 1: The dynamics of fertility under borrowing constraints