An inquiry into the multiplier process in IS-LM model

Author: Li ziran

Address:

Li ziran,
Room 409, Building 38#, Peking University,
Beijing 100.871,PRC.
Phone: (86) 010-62763074
Internet Address: jefferson@water.pku.edu.cn

Abstract

The multiplier theory is still an important analytical tool in many macroeconomic textbooks. For example, a number of textbook authors use the theory to explain the process of growth in goods market by expanding the multiplier process into a geometric series, and thus obtain the route of economic growth. Then some students raise an interesting question: Can we induce the dynamics of the monetary and fiscal transmission mechanism of IS-LM model through the multiplier process? (None of the textbooks involve this problem; alternative solutions are available in economics journals, but go beyond the scope of our students' knowledge.) If not, what is the problem in the analysis of the multiplier process? Here I first show some professor's deduction of the "monetary transmission mechanism", and then analyze the main problems and discuss the multiplier theory. Finally, I propose a generalization of the monetary transmission mechanism approach. I trace the change of demand and output in the process of increase respectively, and use a stochastic series of variables to reflect regularity in the their relationship, and obtain another two curves in IS-LM model representing their relationship In conclusion, I demonstrate theoretically that the economy will ultimately reach its equilibrium point, following the route of LM curve between one static equilibrium point to another.

Deduction 1

For the sake of numerical calculation, we analyze the multiplier process in a three-department economy.

We have two important curves in IS-LM model representing the equilibrium contrail of the goods market and money market respectively:

IS curve**:**

$$\textbf{(1) } Y= \frac{C_0 + cTR_0 + I_0 + G_0 - br}{1 - c(1 - t)} = \frac{A_0 - br}{1 - c(1 - t)} \quad (A_0 = C_0 + cTR_0 + I_0 + G_0) \qquad (1)$$

LM curve**:**

$$\textbf{(2) } \frac{M}{P} = kY - hr, \qquad (2)$$

The intersection of the two curves determines the equilibrium output of our economy:

$$\textbf{(3) } Y_0 = \frac{A_0 + \frac{bM}{hP}}{1 - c(1 - t) + \frac{bk}{h}} \qquad (3)$$

Where Y is output or national income; C_0 is autonomous consumption; I represents investment spending r is the interest rate and b measures interest response of investment, I_0 is autonomous investment spending; G is government purchase of goods and services; TR_0 is transfer to the private sector; t is tax rate; c measures marginal propensity to consume out of disposable income; h is the interest elasticity of money demand; k is the output elasticity of money supply.

Some professors hold the following deduction of the monetary transmission mechanism in IS-LM model:

[I] Initiative effects:

(1)The government increases real money stock by □M/p.

(2)Good markets adjust relatively slow and Y will not change in a short period, while asset market adjusts quickly and thus interest rates will be down by □M/(hp) (**M/p=kY-hr ,□M/p=kY-h□r** .Otherwise the equation M/p=kY-hr will not hold after an increase on its left side),.

(3)Lower interest rate will stimulate the investment demand and consequently increase the aggregate demand. And, the output and national income begin to rise.

The above stages can be demonstrated by the following graph:

$\Delta M/p \xrightarrow{\Delta R=\Delta M/hP} I\uparrow \xrightarrow{\Delta I=b\Delta M/hP=\Delta AD} AD\uparrow \xrightarrow{\Delta AD=\Delta Y} Y\uparrow$

[II] Induced effects:

Round 1

$Y\uparrow \xrightarrow{\Delta R=k\Delta Y/h} R\uparrow \xrightarrow{\Delta I=-bk\Delta Y/h} I\downarrow \xrightarrow{\Delta Y_I=-bk\Delta Y/h} Y\downarrow$

Notes: M/p=kY-hr, M/p will remain stable after the monetary policy is applied, and therefore, in order to hold the equation, r will rise as a result of the increase in Y. And this increase can be denoted as Y_i

$Y\uparrow \xrightarrow{\Delta Y_d=(1-t)\Delta Y} Y_d\uparrow \xrightarrow[\Delta C=c\Delta Y_d]{\Delta Y_d=(1-t)\Delta Y} C\uparrow \xrightarrow{\Delta AD=\Delta C} AD\uparrow \longrightarrow Y\uparrow$ (This increase in Y is induced by increased consumption demand, and thus can be denoted as Y_c)

In round 1, the combination of effects induced by the initiative increase in Y is Y_c+Y_i, we may denote Y_c+Y_i by Y_1, then $\mathbf{Y_1=[c(1-t)-bk/h]\Delta Y}$ .Similarly, In round 2, the combination of effects induced by the change of Y in round 1(denoted by Y_1) is $\mathbf{[c(1-t)-bk/h]Y_1}$,or to be more exact, $\mathbf{[c(1-t)-bk/h]^2\Delta Y}$(denoted by Y_2).And in round 3, $\mathbf{[c(1-t)-bk/h]^3\Delta Y}$……….

If $|c(1-t)-bk/h|<1$, the successive terms in the series become progressively smaller, we can write out the successive rounds of increased output, starting with the initial increase in output, we may obtain a geometric series: $\mathbf{\Delta Y}$, $\mathbf{[c(1-t)-bk/h]^1\Delta Y}$, $\mathbf{[c(1-t)-bk/h]^2\Delta Y}$ …..

Moreover, the total change can be obtained by adding them up:

$$\sum Y_i=\Delta Y\{1+[c(1-t)-bk/h]^1+[c(1-t)-bk/h]^2+\cdots\}=\sum\Delta Y/[1-c(1-t)+bk/h]=\frac{\frac{b\Delta M}{hp}}{1-c(1-t)+\frac{bk}{h}}$$

Problems:

1. This deduction seeks to use the multiplier theory to explain the mechanism. It implies that an increase in demand will immediately induce a same amount of increase in Y. If the economy is in recession how can the output gain such a sharp increase?

2. The linear discrete dynamical systems can behave in some surprising ways. When $|c(1-t)-bk/h|<1$, a sequence obtained from a linear discrete dynamical system bounces around the equilibrium point and the bounces get smaller so that the sequence approaches the equilibrium point; but when $|c(1-t)-bk/h|>1$, it bounces around the equilibrium point but the bounces get larger so that the sequence does not approach the equilibrium point. If $|c(1-t)-bk/h|>1$ the initial effect is $\mathbf{b\Delta M/(hP)}$, while the predicted equilibrium output is $\dfrac{C_0+cTR_0+I_0+G_0+\dfrac{b(M+\Delta M)}{hp}}{1-c(1-t)+\dfrac{bk}{h}}$, and the predicted final total increase of outcome is $\dfrac{\dfrac{b\Delta M}{hp}}{1-c(1-t)+\dfrac{bk}{h}}$. Obviously, $\dfrac{\dfrac{b\Delta M}{hp}}{1-c(1-t)+\dfrac{bk}{h}} < \mathbf{b\Delta M/(hP)}$. That means after the initial increase output exceeds that of the equilibrium. We can also figure out that in the second period, the output is below that of the equilibrium, and in the third period, the output exceeds that of the equilibrium again…. Just like a cobweb graph. If the economy is in recession and output is below that of the sufficient employment, how can the output gain such a sharp increase immediately after the increase in money stock?

A number of macroeconomic text-book authors examine the government purchase multiplier as follows[1]: Suppose the government increases purchase of goods by $\mathbf{\Delta G}$, the multiplier process $\mathbf{\Delta G/(1-c)}$, where c is the marginal propensity to consume, can be expanded into $\mathbf{\Delta G + c\,\Delta G + c^2\Delta G +\ldots+\ldots}$ for $\mathbf{0< c < 1}$, and thus we obtain the route of economic growth. Nevertheless, this applies only for an economy with a sufficiently large excess capacity to produce, for instance, an economy with only one customer and dozens of barbers sitting around. If we think about output being say Boeing 747s, the analysis is more complex, and the geometric series will fail, simply because it takes time to build a plane.

The multiplier theory has its strength, but its assumptions are very strict: (1) sufficient large excess capacity to produce (2) output should exactly meet the surplus demand in each round. (3) One market. Instructors usually teach the theory (or they themselves accept it literally) as axioms and leave out these assumptions. Thus, it is reluctant for them to use the theory to explain the case where these assumptions are slightly violated. Then how can we analyze the multiplier process?

The Generalization of the Multiplier Process Approach

When we explain the multiplier process, the geometric series only give us a logical deduction .In fact; the number value of the increase of each round is indefinite. We can deduce the multiplier process in another way. Here I trace the change of demand and output in the process of increase respectively:

The government increases purchases of goods by $\Delta \mathbf{G}$, which increases the aggregate demand in our economy by $\Delta \mathbf{G}$. In the first round, we suppose the goods market does not adjust quickly enough to meet the excessive demand, and income (or output) only increases by $\Delta \mathbf{Y_1}$ ($<<\Delta G$). Then consumption <u>demand</u> will increase by $\mathbf{c}\Delta \mathbf{Y_1}$. Since demand still exceeds output, output will continue to increase. Then we obtain an increase of $\Delta \mathbf{Y_2}$ in the second round, and a relevant increase of $\mathbf{c}\Delta \mathbf{Y_2}$ in consumption <u>demand</u>. This cycle repeats until output meet demand. (Note that in each round output increases more than demand: $\Delta \mathbf{Y_i} > \mathbf{c}\Delta \mathbf{Y_i}$, and thus output will ultimately meet demand.)Thus we can obtain the equation on equilibrium point: $\sum_{i=1}^{+\infty} \Delta Y_i = \Delta G + c\Delta Y_1 + c\Delta Y_2 + c\Delta Y_3 + \ldots = \Delta G + c\sum_{i=1}^{+\infty} \Delta Y_i$,

and derive the number value of the total income increase: $\sum_{i=1}^{+\infty} \Delta Y_i = \Delta G/(1-c)$.

The above analysis shows an alternative method of explaining the multiplier process, and the result is the same as we use the traditional method of geometric series. However, there is great difference if we analyze complicated market. Now let us look at the multiplier process in IS-LM model:

Application in IS-LM Model

If we expand the monetary multiplier process $\frac{\Delta G}{1-c(1-t)+\frac{bk}{h}}$ (where Δ**G** is the increased government purchase of goods) into a geometric series and specify a definite change of each round: Δ**G** , **[c(1-t)-bk/h]**Δ**G** , **[c(1-t)-bk/h]^2**Δ**G**…. ,we cannot explain the case of **c(1-t)-bk/h** $\in$ **(-∞, -1)**.

When we apply the multiplier theory to explain IS-LM model, which is a model involving both the money market and goods market, result is unsatisfactory. This is partly because the multiplier $\frac{\Delta G}{1-c(1-t)+\frac{bk}{h}}$ incorporates the process of change on two separate markets into a single course, in which individuals behave on money market and goods market separately. This time the multiplier process fails again. In fact, the aggregate output and aggregate demand of the nation, not the multiplier process, determine the equilibrium point in an economy. If AD>AS, output will increase; if AD<AS, output will decrease; if AD=AS, the economy reaches its equilibrium point. Many macroeconomics textbook authors do not emphasize the distinction between demand and output in IS-LM model, and literally take the equations as axioms. In fact, IS curve represents combination of interest rates and income at which the goods market clears. Thus the math symbols Y, G, I, C etc. represent both the demand and output, since they are equal on the equilibrium point. When we trace the process of growth, which is an out-of -equilibrium course, we should consider the two concepts separately.

Now we use the generalization of the multiplier process approach to deduce the dynamics of the government purchase multiplier:

First, the government purchase of goods increases AD by $\Delta \mathbf{G}$, and the output will increase as a result of the stimulation of excessive demand. This time we abandon the ambition to predict the exact value of increase in each round. Let $\Delta \mathbf{Y_1}$ denote an uncertain increase in <u>output</u> in the first round, thus the change in income is equal to $\Delta \mathbf{Y_1}$. Then the income increase will induce an increase of $\mathbf{c(1\text{-}t)\ \Delta Y_1}$ in <u>consumption demand</u> and a decrease of $\mathbf{bk\Delta Y_1/h}$ in <u>investment demand</u>. Similarly, we can analyze the change in the second round: an increase of $\Delta \mathbf{Y_2}$ in <u>output</u> and a change of $\mathbf{[c(1\text{-}t)\text{-}\ bk\ /h]\ \Delta Y_2}$ in <u>demand</u>. And this cycle repeats until AD=AS. The range of c, t and k is (0 , 1), while the range of b and k is (0 ,+∞).Thus $\mathbf{c(1\text{-}t)\text{-}bk/h \in (-\infty, 1)}$.If $\mathbf{c(1\text{-}t)\text{-}bk/h \in (0 , 1)}$, then with the increase in AS, AD will consequently increase but on a smaller scale.($\Delta \mathbf{Y_i} > \mathbf{[c(1\text{-}t)\text{-}bk/h]\Delta Y_i}$) If $\mathbf{c(1\text{-}t)\text{-}bk/h \in (-\infty,0)}$,with an increase of $\Delta \mathbf{Y}$ in AS, AD will decrease by $\mathbf{[bk/h\text{-}c(1\text{-}t)]\Delta Y}$. All in all, AS will ultimately meet AD.

Table 1 shows the detail of the transmission mechanism, where GAP_i denotes the gap between AD and AS in each round.

We obtain a stochastic series of the increase in output: $\Delta \mathbf{Y_1}$, $\Delta \mathbf{Y_2}$, $\Delta \mathbf{Y_3}$,….. and a relevant series of the change in demand $\mathbf{[c(1\text{-}t)\ \text{-}bk/h]\Delta Y_1}$, $\mathbf{[c(1\text{-}t)\ \text{-}bk/h]\Delta Y_2}$,… When GAP_i=0, the economy reaches its equilibrium point. From the equation $\Delta \mathbf{G}\ \text{-}[1\text{-}c(1\text{-}t)\ \text{-}bk/h] \sum_{i=1}^{k} \mathbf{Yi} = \mathbf{0}$, we derive the total increase in income:

$$\sum_{i=1}^{k} \Delta Yi = \frac{\Delta G}{1 - c(1-t) + \frac{bk}{h}}$$

and total increase in interest rate: **(k/h)** $\sum_{i=1}^{k} \Delta Yi = \dfrac{\frac{k\Delta G}{h}}{1 - c(1-t) + \frac{bk}{h}}$

Conclusion: the economy will ultimately reach its equilibrium point following the route of E_1 E_2 along the LM curve.[2] (Figure 1 shows the route of the change in demand and output on condition that $\mathbf{c(1\text{-}t)\ \text{-}bk/h>0}$ **,** while Figure 3 shows the case of $\mathbf{c(1\text{-}t)\ \text{-}bk/h<0}$)

The case of the monetary transmission mechanism is similar (Figure 2):

An increase of Δ**M/p** in the real money stock shifts the LM schedule to the right. The money market adjusts immediately, and interest rates decline between point E_1 and E_3[3] by Δ**M/(Ph)** according to the following equations:

$$\mathbf{M/p=kY-hr \dashrightarrow \Delta M/p=-h\Delta r}$$

The lower interest rate stimulates an increase of **b**Δ**M/(Ph)** in investment demand, and then the output begins to rise. $\Delta \mathbf{Y_1}$ denotes an indefinite increase of output in the first round. The income increase will induce a decrease of $\mathbf{bk\Delta Y_1/h}$ in investment demand by raising the interest rate and an increase of $\mathbf{c(1\text{-}t)\Delta Y_1}$ in consumption demand, according to the following equations:

$$\mathbf{M/p=kY-hr \dashrightarrow 0=k\Delta Y_1-h\Delta r,\ \Delta r=k\Delta Y_1/h}$$

$$\mathbf{I=I_0-br,\ \Delta I=-b\Delta r}$$

$$\dashrightarrow \mathbf{\Delta I=-bk\Delta Y_1/h}$$

$$\mathbf{C=C_0+c[(1-t)Y+TR_0] \dashrightarrow \Delta C=c(1-t)\Delta Y_1}$$

Similarly, we can analyze the change in the second round: an increase of $\Delta \mathbf{Y_2}$ in output and a change of $\mathbf{[c(1\text{-}t)\text{-}bk/h]\Delta Y_2}$ in demand. This cycle repeats until AD=AS. Finally, we obtain a series of the increase in output: $\Delta \mathbf{Y_1}$, $\Delta \mathbf{Y_2}$, $\Delta \mathbf{Y_3}$,… and a relevant series of the change in demand: $\mathbf{[c(1\text{-}t)\text{-}bk/h]\Delta Y_1}$, $\mathbf{[c(1\text{-}t)\text{-}bk/h]\Delta Y_2}$, $\mathbf{[c(1\text{-}t)\text{-}bk/h]\Delta Y_3}$….

On the equilibrium point, we have:

$$\mathbf{\Delta M/(hp)-[1-c(1-t)-bk/h]}\sum_{i=1}^{k}\Delta Y_i=0, \qquad \sum_{i=1}^{k}\Delta Y_i=\frac{\frac{b\Delta M}{hp}}{1-c(1-t)+\frac{bk}{h}}$$

Advantages of this Generalization Approach

First, the method still holds if we consider other parameters that determine the IS-LM model. For example, in an open economy, we can simply add a equation to modify the deduction: **X=g-mY-nr** , where **X** is net export, **m** is the marginal propensity to export

Then the equilibrium income determined by IS-LM model is:

$$Y=\frac{C_0+cTR_0+I_0+G_0+g}{1-c(1-t)+m+\frac{(b+n)k}{h}} \qquad (4)$$

and the series will be more complex to demonstrate the mechanism, but the result is similar.

Second, it can, if not very reluctantly, explain the effect of monetary policy with a horizontal IS curve and the effect of fiscal policy with a vertical LM curve:

(1)If $b \to \infty$, we obtain a horizontal IS curve .Now we trace the monetary policy: The lower interest rate stimulates an infinitely large increase of $\mathbf{b\Delta M/(Ph)}$ in investment demand, $\mathbf{\Delta Y_1}$ denotes an increase in output in the first round .The income increase will induce a decrease of $\mathbf{bk\Delta Y_1/h}$ in investment demand by raising the interest rate, and an increase of $\mathbf{c(1\text{-}t)\Delta Y_1}$ in consumption demand. Here we have a flat (or almost horizontal)" demand curve"AE_2 with a negative slope, and a relatively steeper "supply curve". Since $\mathbf{\Delta Y_1}$ only generates infinitely large changes in investment demand, and thus avoid the embarrassment of explaining an infinitely large decrease of $\mathbf{bk\Delta M/(ph)}$ in output when applying the traditional method of geometric series. We obtain:

$$\lim_{b\to\infty}\sum_{i=1}^{k}\Delta Y_i=\frac{\frac{b\Delta M}{hp}}{1-c(1-t)+\frac{bk}{h}}=\frac{\Delta M}{kp} \qquad (5)$$

(2)If $h \to 0$, we obtain a vertical LM curve. Let us trace the effect of fiscal policy: First, the excessive government purchase of goods increases AD by $\mathbf{\Delta G}$.$\mathbf{\Delta Y_1}$ denotes an increase of output in the first round. Then an increase of $\mathbf{c(1\text{-}t)\Delta Y_1}$ in consumption demand and a decrease of $\mathbf{bk\Delta Y_1/h}$ in investment demand. Since an infinitely small $\mathbf{\Delta Y}$ can be large enough to make the decrease of investment demand counteract ΔG, here we have a vertical "supply curve" E_1E_2 and a "demand curve"AE_2 with a negative slope. Thus we avoid the problem of explaining an infinitely large decrease of $\mathbf{bk\Delta G/h}$ in output. Finally we obtain:

$$\lim_{h\to 0}\sum_{i=1}^{k}\Delta Y_i=\frac{\Delta G}{1-c(1-t)+\frac{bk}{h}}=0 \qquad (6)$$

NOTES

1. For example, the following textbooks expand the multiplier process into a geometric series: Mankiw (1996), Gordon (1993), Hall and Taylor (1993), Dornbusch and Fischer (1994)

2. It seems that the effects on consumption and investment both occur within the same round, in fact, they are two separate processes. For example, suppose the effect on investment lags *m* rounds behind that on consumption demand and thus GAP_k equals $\Delta\mathbf{G}$ **-[1-c(1-t)]** $\sum_{i=1}^{k} \mathbf{Y_i}$ **-bk/h** $\sum_{i=1}^{k-m+1} \Delta \mathbf{Y_i}$. With the increase of output ,GAP still becomes progressively smaller .We can consider the series in another way: increased consumption demand **c(1-t)**$\Delta\mathbf{Y_1}$ **, c(1-t)**$\Delta\mathbf{Y_2}$ **, c(1-t)**$\Delta\mathbf{Y_3}$**…** change of investment demand **-bk/h**$\Delta\mathbf{Y_1}$ **, -bk/h**$\Delta\mathbf{Y_2}$ **,-bk/h**$\Delta\mathbf{Y_3}$ …,increase of output $\Delta\mathbf{Y_1}$**,**$\Delta\mathbf{Y_2}$**,**$\Delta\mathbf{Y_3}$… In the long run, we can still add up the aggregate effect of each round and get the same result: $\sum_{i=1}^{k} \Delta \mathbf{Y_i} = -\mathbf{bk/h} \sum_{i=1}^{k} \Delta \mathbf{Y_i} + \mathbf{c(1-t)} \sum_{i=1}^{k} \Delta \mathbf{Y_i} + \Delta\mathbf{G}$

3. Robert J. Gordon explained that: "Finding themselves with more money than they need…. This raises the prices of stocks and reduces the interest rate. The initial decline in interest rate is called \`liquidity effect\` of a monetary expansion. The lower interest rate raises the desired level of autonomous consumption and investment spending requiring an increase in production. This is the \`income effect\` of a monetary expansion."(Gordon 1990) Rudiger Dornbusch summarizes the stages in the transmission mechanism as follows:(1) Change in real Money supply (2) Portfolio adjustments lead to a change in asset prices and interest rates (3) Spending adjusts to the change in interest rate (4) Output adjust to the change in aggregate demand (Dornbusch and Fischer 1994.)

REFERENCES

(1) Dornbusch, R., and S. Fischer. 1994. *Macroeconomic*s. 6th. ed. New York: McGraw-Hill.

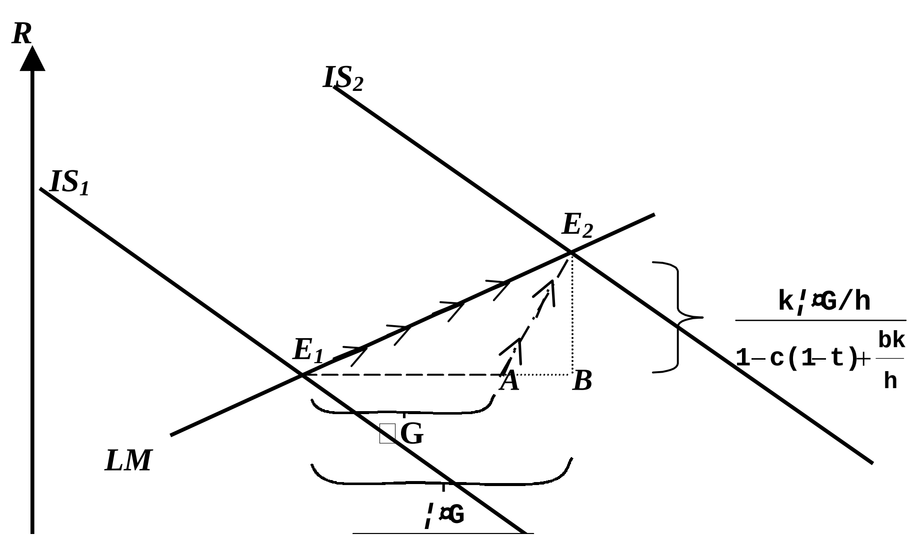
(2) Froyen, R. 1996. *Macroeconomics: Theories and policies*. 5th. ed. Upper Saddle River, N.J.: Pren-tice-Hall.

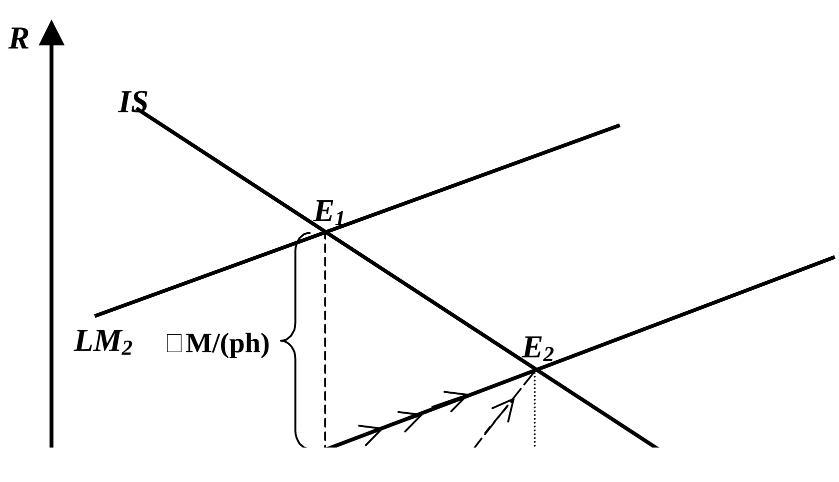
(3) Gordon, R. J. 1990. *Macroeconomics*, 5th ed. New York: Harper Collins College Publishers.

(4) Mankiw, N. G.1996. *Macroeconomics*. 3rd. ed. New York: Worth.

Table 1

Round	Effect on C	Effect on I	Combination effects of each period	GAP$_i$
0	0	0	0	□G
1	c(1-t)□Y_1	(-bk/h)□Y_1	c(1-t)□Y_1-bk/h□Y_1	□G -[1-c(1-t) +bk/h]□Y_1
2	c(1-t)□Y_2	(-bk/h)□Y_2	c(1-t)□Y_2-bk/h□Y_2	□G -[1-c(1-t) +bk/h] (□Y_1+□Y_2)
…	…	…	…	…
m	c(1-t)□Y_m	(-bk/h)□Y_m	c(1-t)□Y_m-bk/h□Y_m	□G -[1-c(1-t) +bk/h $\sum_{i=1}^{m} \Delta Y_i$
….	…	…	…	…
k	c(1-t)□Y_k	(-bk/h)□Y_k	c(1-t)□Y_k-bk/h□Y_k	□G -[1-c(1-t) +bk/h] $\sum_{i=1}^{k} \Delta Y_i$




R
IS
E1
E2
LM2
□M/(ph)

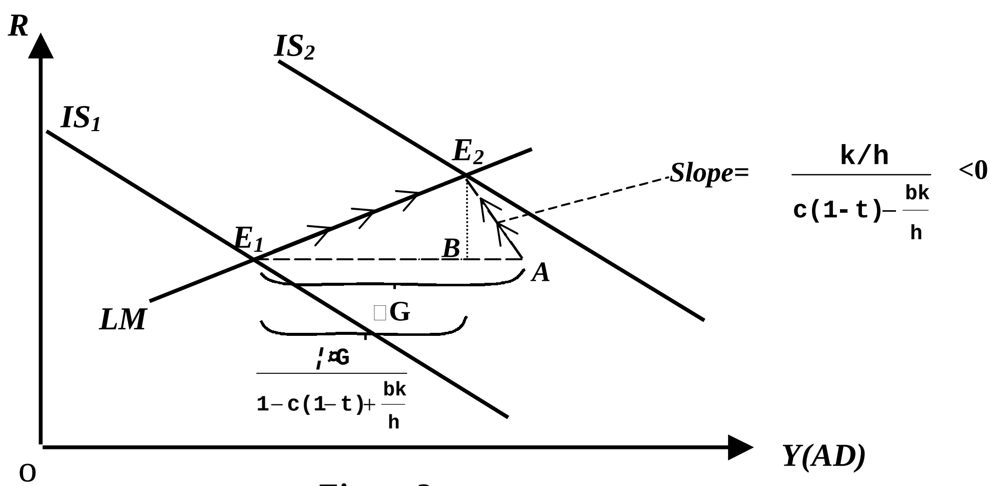


Figure 3

***The income increase is smaller than the government spending if* c(1-t)-bk/h <0**