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Entrepreneurial Innovation

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Abstract

This paper constructs an equilibrium model of entrepreneurial innovation where individuals differ in their attitude toward uncertainty. Unlike previous models of innovation, the firm-formation process is endogenous. An entrepreneur, who owns residual profits, utilizes an uncertain technology and hires a worker who may only be partially isolated from uncertainty. While the available production technologies are exogenously specified, the technologies that operate in equilibrium are endogenous, depending on both the entrepreneur's prior beliefs about the profitability of the technology, as well as the worker's willingness to work with the uncertain technology. The general equilibrium setting allows us to explore the impact of innovation on the nature of the firm. The relationship between technological uncertainty and the nature of the firm is able to explain the commonly observed S-shaped diffusion profile. As uncertainty falls, firms evolve from being *entrepreneurial* to *corporate*, finally becoming *bureaucratic*.

JEL Codes: D5, D8, L2, J24, M13, O31
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1 Introduction

With a burgeoning dot-com entrepreneurial class fuelling technological expansion, growth and dizzying market cycles, innovative entrepreneurship has captured the interest of policy-makers. Tales – apocryphal and otherwise – of entrepreneurs such as Microsoft's Bill Gates and Apple's Steve Jobs, who launched new firms in untried and uncertain industries, spur others to seek their own fortunes through entrepreneurial innovation. Behind these start-ups is a visionary entrepreneur, who abandons his safe job and stakes all on the success of the fledgling enterprise. Such innovations are poorly captured in the economics literature, which has tended to consider a more cautious type of innovation, where existing firms enter new, but closely related, markets. Concepts such as "leadership" and the "entrepreneurial vision" of individuals – which are central to the management literature on innovation – are largely absent from these economic models, despite the significant impact that a handful of individuals have had in shaping the new technology of developed economies.[1]

The present paper has two objectives. The first is to construct an equilibrium model of innovation and firm formation in the presence of uncertainty, and propose a new equilibrium concept which is suitable for studying commodity innovation. Although the economy is a stripped-down affair, with an emphasis on the labor markets, it offers a simple and flexible model of entrepreneurial innovation and diffusion where agents differ in their attitude to uncertainty.

The second objective is to integrate the literature on the entrepreneurial firm with that on the diffusion of innovations; re-establishing the entrepreneur as central to understanding the process of firm formation and innovation. We show that *innovation proof equilibria*, can aid us in understanding real-world phenomena such as the S-shaped diffusion curve.

By innovation, we mean the implementation of a new productive technology: either a new method of production for an existing product, or the production of an entirely new product. Standard models in the Arrow-Debreu spirit, exogenously specify the commodities that will be marketed, therefore precluding the role of an entrepreneur as an innovator.

Innovative enterprises will typically exhibit highly uncertain profitability, in the Knightian (1921) sense. Therefore, the attitude of agents to situations where outcomes are governed by stochastic processes without objective probabilities is central to understanding the motivation for innovation. As Schumpeter (1947, p.152) argues:

> "It is in most cases only one man or a few men who see the new possibility and are able to cope with the resistance and difficulties which action always meets with outside the rut of established practice."

In the Bayesian world of subjective expected utility (SEU), agents arbitrarily choose a single distribution from those consistent with the available information, reducing objective uncertainty to subjective risk. However, decision-makers might instead prefer to adopt an holistic view of the set of possible distributions, and incorporate *all* of them into the decision-making process

[1] Of course, it did not take the recent NASDAQ gymnastics to make economists aware of the problem: Baumol (1968) made a similar lament some 30 years ago. However, economists have been extremely slow to respond, so the criticism has lost little of its force over the intervening decades.

[2]. In our framework, agents follow the latter approach. They utilize either the *upper* or *lower envelope* of the set of distributions to guide their decisions.[3] We designate the former type *Bulls*, and their more timid counterparts *bears*.

This deviation from Bayesianism is undertaken for several reasons. First, evidence against the descriptive realism of SEU is now legion.[4] Second, the framework of Dempster (1967) allows the uncertainty of a process to be specified using exogenous "parameters". This facilitates comparative statics exercises. We can vary the degree of uncertainty surrounding a new technology to examine its effects on equilibrium. Since the process of innovation diffusion inevitably leads to uncertainty reductions, this feature of the model is extremely useful for studying diffusion. Finally, our model of behavior allows us to define "optimism" and "pessimism" in the face of uncertainty in a consistent way. A Bull, for example, is optimistic in a consistent fashion about the outcome of *any* uncertain process in *any* market of *any* economy. An SEU decision-maker's degree of "optimism" must be separately specified with respect to every distinct uncertain process. Thus, we can study an infinite range of different economies using only our two types of agent – Bulls and bears – which greatly simplifies the analysis.

In our model, the uncertainty attitude of the individual is central to innovation. There are a continuum of agents, a fraction α of whom are Bulls, and a finite number of production technologies, summarized by reduced-form stochastic revenue functions. However, there is no exogenous specification of firms. Any agent may choose to start a firm using any of the available technologies. Each firm employs a single worker. Employer liability is limited, however, so wages may not be fully paid in all states. Each technology has its own labor market, since the wage default contingencies may vary across two firms offering the same wage but using different technologies. Agents choose which labor market to participate in, and which side to take.

The inclusion of entrepreneurship and firm formation into a standard GE framework provides a particular challenge. All agents are assumed to be price takers in those markets with a non-zero density of participants, and wages must clear all markets simultaneously. However, in addition, the equilibrium must be *innovation-proof.* Since the singular role of the innovative entrepreneur is to *establish a new (labor) market*, it is meaningless to speak of an equilibrium wage rate for that market, which the entrepreneur takes as given. Instead, the entrepreneur must assess for herself the wage offers that are sufficient to attract a worker from his current job into the new enterprise. Therefore, an equilibrium is innovation-proof if it is not possible for an entrepreneur to start a firm with a new (currently unused) technology, pay a wage adequate to attract a worker, and generate an expected return that improves upon her own equilibrium utility. Hence, in equilibrium, all profitable entrepreneurial opportunities have been exploited.

Therefore, innovation requires the existence of an entrepreneur and a worker who are willing to share the uncertainties of the new enterprise in a manner which improves on the next-best occupational options of both. Indeed, the existence of a new technology, even if not taken up, can influence wages in other industries, as existing labor markets absorb the pressure presented by the new entrepreneurial option now available to all agents. This phenomena is in fact observed in the dot-com sector, where there is pressure on employers to retain staff by paying higher salaries and matching the working conditions of start-ups, such as casual dress codes and

[2]Compare with the methodology of *robust Bayesian inference* discussed in Huber (1981)

[3]See Dempster (1967).

[4]See Camerer (1995) for a survey.

greater work flexibility. As one journal notes "with barriers to entry crumbling, easier access to capital for start-ups and support networks for new businesses in place, savvy employees have never been so empowered".[5] In other words, the existence of new technology improves the outside options of workers in an established industry. The notion of an "innovation-proof entrepreneurial equilibrium", presented in section 3, is able to capture this effect.

Our focus on individual heterogeneity in attitudes to uncertainty reflects both the popular notion of the innovative entrepreneur as someone who is less averse to uncertainty, as well as the views of economists such as Schumpeter (1928), who suggest that innovation is fuelled by a distinct type of person, who takes pleasure in "doing what has not been done before" (*ibid.*, p.380). In a recent NZ study, Pinfold (1998) observes that only 42.5% of start-up ventures established in NZ during 1988 and 1989 survived for five years or more. However, a 1997 survey of start-up owners revealed that: "They rated the chances of their business surviving its first five years at 75.7%, which was 23.5% higher than they rated the chances of similar start-ups." (*ibid.*, p.1) Clearly, optimism in the face of uncertainty seems characteristic of such entrepreneurs.

Branson, the founder of Virgin Records and Virgin Airlines, is an archetypal innovative entrepreneur. He claims that being an adventurer and an entrepreneur are similar, in that both are "willing to go where most people wouldn't dare".[6] However, he dismisses the notion that they are risk-seekers, claiming instead that they are more comfortable than most with uncertainty. The psychology literature (surveyed in Wärneryd (1988)) supports this view. It reveals that owner-managers exhibit levels of *risk aversion* which are not significantly lower than those of salaried managers, leading psychologists to speculate that any differences between the two groups may indeed lie in the perception of non-objective risks (uncertainties).

Having recognized the importance of uncertainty to the individual, the challenge is to determine its impact on the innovative behavior of firms. Traditionally, firm objectives have been considered separately from the decision to innovate. Jensen (1982) considers the manner in which heterogeneous prior beliefs of firms can explain an S-shaped diffusion curve. We depart from this convention by allowing the firm's objective to arise from the attitude of the entrepreneur and worker. This enables us to examine the interactions between the nature of the technology that is operated, and the characteristics of members of the firm who operate the technology. The latter determines what "sort" of firm it is: its objectives and mode of operation. As Drèze (1985, p.5) points out, firms, unlike human beings, have no "visceral reaction to uncertainty". He argues that in the face of uncertainty and incomplete markets, the objectives of the firm need to be induced from the objectives of the owners and workers. While Kihlstrom and Laffont (1979) and others have recognized this in single technology models, its added significance in economies with multiple technologies does not appear to be fully appreciated. Access to jobs in firms employing other technologies means that workers must tacitly agree to the firm's objectives, even if not explicitly bearing any of the uncertainty. An employer, for example, might personally prefer a highly uncertain enterprise about which they are extremely optimistic, but choose to run a more cautious one due to the lack of potential workers who share their optimism (or demand an exorbitant wage to in order to buy into the entrepreneur's "folly").

[5] "Leave the identity crisis to corporations," *Management Today* (10 May 2000).
[6] "What it takes to Start a Startup," *Fortune* (July 1999) **139(11):** pp.135-140.

Indeed, after the crash of dot-com stocks, internet companies reported difficulty recruiting workers, and many hurried to eliminate "dot-com" from their names.[7] As workers in these companies found themselves facing redundancies, recruitment consultants in the industry recommend that

> "job hunters act like venture capitalists and evaluate prospective employers' business plans. This means not just looking at the job itself, but also taking stock of how experienced the leaders are, what level of competition the company faces, how deep its pockets are and whether the company has any unique value to bring to the market beyond an advertising campaign."[8]

Many dot-com ideas are thought to flounder, not for want of entrepreneurial vision, but for want of workers who share that vision. Indeed, it is often observed that when a firm changes its "strategic direction" executives often leave the firm, because of their lack of confidence in the new direction. In our model, one of the costs of innovating is hiring workers willing to work in a highly uncertain industry.

The evolution of new industries in practice closely reflects Schumpeter's ideas on innovation and diffusion. The familiar pattern of an S-shaped diffusion curve, where the adoption rate of new technology is initially slow and then becomes more rapid as the industry evolves, has been discussed by a number of authors, who offer a variety of explanations as to why some firms wait before adopting a new technology, while others jump in early. We show that an S-shaped diffusion profile can be motivated by the changing objectives of the firm. We argue that as uncertainty is resolved, the very nature of entrepreneurship and the employment relationship changes.

Firms utilizing new technology move through three distinct phases. If uncertainty is sufficiently large, then it pervades the whole organization, with workers and employers affected alike. These infant industries are characterized by owners and workers who share a similar optimism in the new industry – both owners and workers are *Bulls* – and share the expected profits equally. We label this firm *entrepreneurial*, since it has a number of the hall-marks of new ventures and start-ups, where workers are remunerated through options and are driven by the same vision as the owner.

As the new industry matures and the chances of success are known more precisely, more staid and cautious *bears* enter the market as workers, which radically alters changes the nature of the firm. The new structure may be described as *corporate.* Only the owners are optimists, while the workers are cautious types whose wage contract shields them from much of the uncertainty. Hence, in comparison to an entrepreneurial firm, the *Bullish* owners of the corporate enterprise face lower labor costs. The corporate structure gives owners additional surplus in exchange for bearing extra uncertainty.

The switch to the corporate phase provides added impetus to diffusion. Bulls are freed up from labor duties to start new firms; a greater proportion of the population is able to effectively participate in the innovative sector; and wages are reduced, as *bears* offer cheaper labor than

[7] For example, Shopnow.com changed to Network Commerce Inc. ("The shame of dot-coms," *Computerworld* (August 7, 2000) **34(32)**: 52-53).

[8] *Ibid.*

Bulls, whose optimistic views about the returns to firm ownership imply a higher reservation wage. This corresponds to the steep part of the diffusion curve.

As uncertainty falls further, we eventually enter a third phase in which *bears* are happy to take on the role of firm ownership in the new sector. This new type of firm, with pessimists at all levels is referred to as *bureaucratic*.

The paper is organized as follows. The next section describes the economy and the decision-making framewor we employ. Section 3 is the more technical part of the paper; first, we define an equilibrium and show it exists under failry mild conditions; then we introduce innovation-proof equilibria, and show they exist under no additional requirements. Section 4 discusses some properties of innovation-proof equilibria. Section 5 discusses diffusion of innovations in our economy. Section 6 concludes.

2 The Entrepreneurial Economy

The entrepreneurial economy consists of several industries. An industry's production function will be more-or-less suppressed in the analysis. Each industry's income possibilities are subject to risk, although agents are not fully informed about this process. Therefore, the attitude of agents to situations where outcomes are governed by stochastic processes with vague information is central to understanding the motivation for innovation.

2.1 Decision making under uncertainty

There is a finite set Θ of *payoff-relevant states*. To describe the uncertainty surrounding the realization of the payoff-relevant state, we introduce a set S of *fundamental states*, endowed with a σ–algebra Σ and a commonly known probability measure p^S. We take S to be the unit interval $[0, 1]$, Σ the usual Borel σ–algebra on S, and p^S the Lebesgue measure on (S, Σ), unless otherwise specified. The payoff–relevant states are related to the fundamental states by a measurable *information correspondence* $\Gamma : S \twoheadrightarrow \Theta$. If fundamental state s is realized, the available information implies that the payoff–relevant state is some $\theta \in \Gamma(s)$, but nothing else is known about the relative likelihoods of states in $\Gamma(s)$. Hence, if Γ is singleton-valued (a *function*), then knowledge of p^S and Γ implies a commonly known probability on Θ. Otherwise, the available information about the stochastic process by which payoff-relevant states are realized may be too vague to yield a well-defined probability.

Example 1 Suppose that $\Theta = \{\theta_1, \theta_2, \theta_3\}$ and define Γ as follows:

$$\Gamma(s) = \begin{cases} \{\theta_1\} & \text{if} \quad s \in \left[0, \frac{1}{3}\right] \\ \{\theta_1, \theta_3\} & \text{if} \quad s \in \left(\frac{1}{3}, \frac{2}{3}\right] \\ \{\theta_2, \theta_3\} & \text{if} \quad s \in \left(\frac{2}{3}, 1\right] \end{cases}$$

What is the probability of event $\{\theta_1\}$? One can identify a lower and upper bound, $\frac{1}{3}$ and $\frac{2}{3}$ respectively, since $s \in \left[0, \frac{1}{3}\right]$ guarantees the realization of θ_1, while $s \in \left(\frac{2}{3}, 1\right]$ precludes it. However, there is no basis for choosing any particular point in this interval as being uniquely "correct". Similarly, one can identify lower and upper bounds for any other subset of Θ.

This formalization of vagueness of information is due to Dempster (1967).[9] Because the source and structure of uncertainty is precisely specified, one can examine the comparative static effects of changes to the nature or degree of this uncertainty. For example, if $\Gamma(s) \subseteq \hat{\Gamma}(s)$ for all $s \in S$, then it is natural to say that Γ describes a situation involving less uncertainty than $\hat{\Gamma}$.

Following Dempster, we say the collection of upper bounds for subsets of Θ induced by Γ is the *upper probability* on $\{\Theta, \mathcal{P}(\Theta)\}$, and the collection of lower bounds is the *lower probability*.[10] Formally, the upper probability of event $E \in \mathcal{P}(\Theta)$ is given by

$$\overline{v}(E) := p^S \left(\{s \in S \mid \Gamma(s) \cap E \neq \emptyset\}\right)$$

and the lower probability is

$$\underline{v}(E) := p^S \left(\{s \in S \mid \emptyset \neq \Gamma(s) \subseteq E\}\right).$$

Unless Γ is a function, these objects need not be probabilities because they may violate the usual additivity condition. They are examples of a more general class of objects called *capacities*, which were extensively studied by Choquet (1953-4). When Γ is singleton-valued, the upper and lower probabilities coincide, and their common value is an ordinary probability measure.

Example 1 (continued) The lower probability of $\{\theta_1\}$ is $\frac{1}{3}$, that of $\{\theta_2, \theta_3\}$ is similarly $\frac{1}{3}$, while the lower probability of Θ is 1. This illustrates the fact that lower probabilities are *superadditive*. Conversely, the upper probabilities of $\{\theta_1\}$ and $\{\theta_2, \theta_3\}$ are $\frac{2}{3}$, illustrating that upper probabilities are *subadditive*. For a full characterization of the properties of these capacities, see Chateauneuf and Jaffray (1989), Dempster (1967) and Shafer (1976).

Upper and lower probabilities may also usefully be described by their *Möbius inverse*(see Chateauneuf and Jaffray (1989) and Shafer (1976)). This inverse is the mapping $m : \mathcal{P}(\Theta) \to [0, 1]$ defined as follows

$$m(E) = p^S \left(\{s \in S \mid \Gamma(s) = E\}\right) \qquad \forall E \in \mathcal{P}(\Theta).$$

Thus, m is the probability induced by p^S on the power set $\mathcal{P}(\Theta)$. Furthermore:

$$\underline{v}(E) = \sum_{A \subseteq E} m(A)$$

and

$$\overline{v}(E) = \sum_{\{A \mid A \cap E \neq \emptyset\}} m(A) \; = \; 1 - \underline{v}(E^c).$$

Let $f : \Theta \to \mathbb{R}$ be a random variable with range $\{r_1, r_2, \ldots, r_n\}$, indexed such that $r_1 < r_2 < \cdots < r_k$. How do agents evaluate these potentially vague revenue lotteries? Dempster

[9] See also Mukerji (1997) for a useful discussion in the context of economic decision-making.

[10] $\mathcal{P}(\Theta)$ denotes the *power set* of Θ; i.e. the set of all subsets of Θ.

(1967) suggests employing the *Choquet integral.* Define $E_i = f^{-1}(r_i)$, for $i = 1, 2, \ldots, k$. The *Choquet expected value of f with respect to the capacity v* is

$$\mathbb{E}_v f = r_1 + \sum_{j=2}^{k} (r_j - r_{j-1}) v \left(\bigcup_{i=j}^{k} E_i \right) \tag{1}$$

When v is additive (i.e. a probability measure) this is equivalent to a standard expected value calculation. For general capacities v, the Choquet integral satisfies the condition

$$\mathbb{E}_v (af + b) \;=\; a \left[\mathbb{E}_v f\right] + b$$

for arbitrary random variable f and constants a and b. However, it will *not* be the case in general that

$$\mathbb{E}_v (f + g) \;=\; \mathbb{E}_v f \;+\; \mathbb{E}_v g.$$

A sufficient condition for additivity of the Choquet expectation operator is *comonotonicity* of f and g (Schmeidler (1986)). The variables f and g are comonotonic if there do *not* exist states $\theta, \theta' \in \Theta$ such that $f(\theta) > f(\theta')$ and $g(\theta) < g(\theta')$.

Bulls and bears

We assume that all agents are risk neutral: that is, their objective is to maximize the Choquet expected value of their income. We also assume there are two types of agents: *Bulls* and *bears.* When calculating Choquet expectations, Bulls employ the upper probability, and bears employ the lower probability induced by Γ on $\{\Theta, \mathcal{P}(\Theta)\}$.[11] For any random variable $f : \Theta \to \mathbb{R}$, denote by $\overline{\mathbb{E}}f$ its Choquet expectation with respect to the upper probability, and by $\underline{\mathbb{E}}f$ its Choquet expectation with respect to the lower probability. Observe in particular that

$$\overline{\mathbb{E}}f \;\geq\; \underline{\mathbb{E}}f$$

for any f. Bulls and bears clearly have beliefs which are equally consistent with the underlying information structure, but Bulls choose to interpret the available information in a more optimistic fashion than bears.

Finally, let us offer one further perspective on this decision-making process. The set of additive probabilities on $\{\Theta, \mathcal{P}(\Theta)\}$ that are consistent with p^S and Γ is known as the *core* of the lower probability $\underline{v}$.[12] Formally,

$$\begin{aligned} \text{core}(\underline{v}) \;&=\; \{\pi \in \Delta(\Theta) \mid p(E) \geq \underline{v}(E) \quad \forall E \subseteq \Theta\} \\ &=\; \{\pi \in \Delta(\Theta) \mid p(E) \leq \overline{v}(E) \quad \forall E \subseteq \Theta\} \end{aligned}$$

where $\Delta(\Theta)$ denotes the set of all additive probability measures on $\{\Theta, \mathcal{P}(\Theta)\}$. A natural definition of Bullish optimism from this perspective would be the tendency to evaluate any random variable $f : \Theta \to \mathbb{R}$ by calculating its expected value using the most "favorable"

[11] These are special cases of the axiomatic decision model of Jaffray and Wakker (1994).
[12] See Schmeidler (1989).

probability from core($\underline{v}$). In fact, such a definition of Bullish preferences is entirely consistent with our model, as it is well known that

$$\overline{\mathbb{E}}f = \max_{\pi \in \text{core}(\underline{v})} \mathbb{E}_\pi f.$$

Similarly:

$$\underline{\mathbb{E}}f = \min_{\pi \in \text{core}(\underline{v})} \mathbb{E}_\pi f.$$

Given the unfamiliarity of most economists with the process of Choquet integration, this alternative characterization of Bullish and bearish preferences can be a comforting aid to one's intuitive understanding of their behavior. For example, if $\Gamma(s) \subseteq \hat{\Gamma}(s)$ for all $s \in S$, then, in obvious notation, core($\underline{v}$) $\subseteq$ core($\hat{\underline{v}}$). That is, the core shrinks as uncertainty reduces, which is what one would expect. This causes bearish and Bullish behavior to converge.

2.2 The Economy

The economy consists of a continuum of agents, divided into Bulls and bears, and I industries. Agents simultaneously choose occupations based on common information about the processes determining revenues in the various industries, a given vector of industry wage rates, and common expectations about the equilibrium densities of firms in each industry. Each industry's production function will be more-or-less suppressed in the analysis. We are also not specifically interested in contracting within the firm[13]. As Baumol (1968, 1993) points out, the entrepreneurial function is distinct from the managerial one. The latter is more concerned with day-to-day matters such as fine-tuning the input mix in production. These decisions are concealed within a reduced form revenue function and a simplifying assumption that each firm hires precisely one worker.[14]

A firm's revenue depends on the density of firms in each industry plus some stochastic factors. Formally, the revenue function of a firm in industry i is $R_i(\theta, \delta)$, where $\theta \in \Theta$ is the payoff-relevant state, and δ is a vector of densities, δ_i being the total density of firms in industry i. Since each firm hires exactly one worker, the density δ_i of firms in industry i is an element of $\left[0, \frac{1}{2}\right]$.

The object of our analysis is the following abstract economy.

Definition 1 *An* economy *is an object*

$$\mathcal{E} = \left\{ (S, \Sigma, p^S), (\Theta, \mathcal{P}(\Theta)), \Gamma, \left\{ R_i : \Theta \times \left[0, \frac{1}{2}\right]^I \to \mathbb{R}_{++} \right\}_{i=1}^{I}, \alpha \right\}$$

[13]Kelsey and Spanjers (1997) focus on this issue.

[14]By contrast, Kihlstrom and Laffont (1979) explicitly include a managerial decision about the number of workers to hire. It should also be noted that decisions about the internal organisation of the firm arguably fall on the boundary between the managerial and entrepreneurial function, as these determine how information is processed and uncertainty managed within the firm. Our simplified firm structure ignores these issues. Addressing them is a matter demanding further attention in future research.

where (i) Γ is a measurable correspondence from S to Θ; (ii) R_i gives the revenue of a typical firm in industry i when it hires exactly one worker; and (iii) $\alpha \in [0,1]$ divides the unit interval into Bulls, agents with indices in the sub-interval $[0,\alpha)$, and bears, agents with indices in the sub-interval $[\alpha,1]$.

Revenues

The revenue function summarizes all input choices other than the hiring of the single worker. It also summarizes the process of output price determination. With a continuum of firms, the most natural assumption is that firms are price-takers on these output markets, and prices are set to match aggregate demand to the supply implied by the vector δ. However, strategic interaction within or across industries in the determination of output prices is also compatible with the model.

Note that R_i depends on the entire vector of industry densities. It is natural to suppose that R_i is weakly decreasing in δ_i. If different industries produce substitute goods, or if there is a fixed consumer pool for which all I industries compete, then R_i may also be weakly decreasing in each δ_j. However, complementarity of inter-industry demand systems can also be accommodated.

An important special case in which it is natural to think of R_i decreasing in each δ_j occurs when the model is used to describe I producers of differentiated products *within the same industry.* This interpretation allows us to consider issues of product innovations within an industry, as opposed to the innovative introduction of an entirely new product category.

We shall make the following technical assumption throughout:

Assumption 1 *Each R_i is bounded and is also continuous in δ.*

This assumption is needed to guarantee existence of equilibria – see Theorem 3.1.

Given a density vector δ, industry i's revenue is a random variable on $\{\Theta, \mathcal{P}(\Theta)\}$. Note that although each revenue function R_i is expressed as a function of the same θ, this does not imply that each is subject to the same uncertainty.[15] For example, it may be that $\theta \in \mathbb{R}^I$, and for each i, the value of R_i is independent of θ_j for every $j \neq i$. In this case, the uncertainty in industry i depends on the precision of the available information about the ith component of θ, and this may be quite different to the uncertainty surrounding the jth component. For example, one might wish to distinguish "established" industries, whose stochastic revenues are known precisely, from currently inactive industries (potential innovations) for which there is great uncertainty about profitability. Our framework allows this distinction to be modelled in a natural way.

3 Equilibrium

In equilibrium, labor markets clear, and the density of firms in each industry is required to match agents' common expectations. The latter is analogous to the more familiar assumption

[15] Correlation of random shocks across technologies matters only at the macro level. For the decisions of individual agents it matters not, since each agent can work in only one firm. Correlation would affect their occupation choice only if they could divide their time among several jobs.

that agents correctly anticipate output prices in the various output markets. However, as our interest is in the supply side – labor markets and firm formation – we suppress these output prices in the analysis.[16]

Each agent in an economy $\mathcal{E}$ has $2I$ occupational options: wage-earning or firm ownership in one of the I industries. An agent selects exactly one occupation: they cannot divide their time among a portfolio of jobs. In each industry i, firm owners obey the following:

Assumption 2 *Each owner employs one worker. An employment contract specifies a wage level $w_i \geq 0$ and the following limited liability clause: if state $\theta \in \Theta$ is realized, the entrepreneur pays the worker*

$$\min\{w_i,\, R_i(\theta,\, \delta)\}$$

The limited liability clause implies that wage earning need not generate a sure income.[17]

To evaluate returns from the various occupational options, agents need to know the vector $w = (w_1, w_2, \ldots, w_I)$ of equilibrium wage rates for each industry, and the equilibrium vector of industry densities $\delta = (\delta_1, \delta_2, \ldots, \delta_I)$. For given values of w and δ, agents choose occupations that maximize their Choquet expected income. In making these calculations, bears employ the lower probability, and Bulls the upper probability.

Given (w, δ), a Bull obtains utility

$$\overline{\mathbb{E}}\,[R_i(\theta,\, \delta) \,-\, \min\{w_i,\, R_i(\theta,\, \delta)\}]$$

from owning a firm in industry i, and utility

$$\overline{\mathbb{E}}\,[\min\{w_i,\, R_i(\theta,\, \delta)\}]$$

from being a wage-laborer in the same industry. The utility obtained by bears from these occupations may be described similarly. Let $\mathcal{O} = \{1, 2, \ldots, 2I\}$ be a set of indices for occupations in $\mathcal{E}$. We shall index the occupation of being a firm owner in industry i by $(2i - 1) \in \mathcal{O}$, and the occupation of being a wage earner in industry i by $2i \in \mathcal{O}$.

Definition 2 *Given an economy $\mathcal{E}$ and vectors (w, δ), the Bulls'* optimal occupation set $BR^{\mathcal{E},B}(w,\, \delta) \subseteq \mathcal{O}$ *is the set of occupations that maximize a Bull's Choquet expected income. The bears'* optimal occupation set $BR^{\mathcal{E},b}(w,\, \delta) \subseteq \mathcal{O}$ *is defined similarly.*

In addition to the vectors (w, δ), an equilibrium must also specify the occupation of each agent. This is described using an *allocation function.*

[16]In particular, we implicitly assume that demand conditions on the output markets are independent of the equilibrium incomes of the agents in the model. This assumption is made for simplicity, and it is implicit in Kihlstrom and Laffont (1979).

[17]Although we have not done so, it would be straightforward to elaborate the model so that agents also have heterogeneous endowments of initial wealth, and must draw on personal wealth to pay wages if necessary (as in Kihlstrom and Laffont (1979)). Then, wealthier agents may be able to pay lower wages in equilibrium because of the lower likelihood of default. Formally, this effect is similar to a credit constraint in a model where owners must invest in capital in order to start their businesses, and credit markets are imperfect. In each case, personal wealth has a positive effect on the profitability of business ownership. Hence, wealthier agents are more likely to become owners, as some empirical evidence – such as Evans and Jovanovic (1989) – suggests.

Definition 3 *An* allocation function *for the economy $\mathcal{E}$ is a Lebesgue measurable function $\phi : [0,\, 1] \to \mathcal{O}$.*

Definition 4 *The triplet $(w,\, \delta,\, \phi)$ is an* entrepreneurial equilibrium *of $\mathcal{E}$ if*

(i) $\phi(j) \in BR^{\mathcal{E},B}(w,\, \delta) \quad \forall j \in [0,\, \alpha)$*;*

(ii) $\phi(j) \in BR^{\mathcal{E},b}(w,\, \delta) \quad \forall j \in [\alpha,\, 1]$*; and*[18]

(iii) $\text{Leb}\left[\phi^{-1}(2i)\right] = \text{Leb}\left[\phi^{-1}(2i-1)\right] = \delta_i$ *for each* $i \in \{1, 2, ..., I\}$.

Thus, an entrepreneurial equilibrium specifies wages, industry densities and individual occupations such that, when all agents anticipate (w, δ) and each chooses his or her occupation optimally, all labor markets clear and the common expectations of δ are confirmed. It is quite possible that some industries fail to operate in equilibrium: that is, the vector δ may have some zero components. Hence, the equilibrium endogenously determines which industries operate (for example, which innovations are implemented), as well as the wage rates and firm densities in each operating industry.

Theorem 3.1 *Under Assumptions 1 and 2, every economy $\mathcal{E}$ has an equilibrium $(w,\, \delta,\, \phi)$.*

Proof: Let $\Phi\,(w, \delta)$ denote the set of allocation functions for $\mathcal{E}$ consistent with (w, δ). Denote the associated set of occupational density vectors as

$$Z\,(w, \delta) = \left\{ \left(Leb\left[\phi^{-1}(i)\right]\right)_{i=1}^{2I} \;\middle|\; \phi \in \Phi\,(w, \delta) \right\}$$

Notice that elements of $Z\,(w, \delta)$ belong to the unit simplex $\Delta^{2I-1} \subseteq \mathbf{R}_{+}^{2I}$. Now let us define the $2I \times 2I$ matrix $C = [c_{ij}]$ as follows:

$$c_{ij} = \begin{cases} 1 & if \qquad j = 2i-1 \quad or \quad j = 2(i-I)-1 \\ -1 & if \qquad j = 2i \quad and \quad i \leq 2I \\ 0 & otherwise \end{cases}$$

Letting[19] $X\,(w, \delta) = C\,Z\,(w, \delta)$, we see that each $x \in X\,(w, \delta)$ is a 2I-vector whose first I components give the *net excess density of owners* in each industry associated with some allocation consistent with (w, δ), and whose second I components give the *total density of owners* in each industry associated with the same consistent allocation. This transformation induces a one-to-one mapping between $Z\,(w, \delta)$ and $X\,(w, \delta)$ since C is non-singular. Alternatively, one can recover $z \in Z\,(w, \delta)$ from $x = Cz \in X\,(w, \delta)$. Therefore, $X\,(w, \delta)$ is a subset of $B := C\Delta^{2I-1}$.

Next, extend each R_i to the domain $\Theta \times [0,\, 1]^I$ as follows:

$$R_i\,(\theta, \delta) := R_i\left(\theta, \left(\min\left\{\frac{1}{2}, \delta_i\right\}\right)_{i=1}^{I}\right)$$

[18] In (iii), *Leb* denotes Lebesgue measure.

[19] To interpret the following transformation, assume that points in Euclidean space are *column* vectors.

These extended R_i functions are still strictly positive, and continue to satisfy Assumption 3. In particular, since the extended functions are bounded, we may define

$$\overline{W} = \max_i \sup_{(\theta,\delta)\in\Theta\times[0,1]^I} R_i(\theta,\delta)$$

For arbitrary vectors $y \in \mathbf{R}^{2I}$, let $proj_I\, y$ denote the projection of y onto its first I components; and $proj_{-I}\, y$ the projection of y onto its second I components. Then one may observe that

$$proj_{-I}\, B = \left\{ y \in \mathbf{R}^I_+ \;\middle|\; \sum_{k=1}^{I} y_k \leq 1 \right\} \subseteq [0,1]^I$$

The foregoing observations and definitions confirm that $D := [0, \overline{W}]^I \times [0,1]^I \times B$ is a compact and convex subset of Euclidean space. Let the correspondence $\xi : D \rightrightarrows D$ be defined as follows:

$$\xi(w,\delta,x) = \left\{ \arg \max_{\tilde{w}\in[0,\overline{W}]^I} \tilde{w}\cdot proj_I\, x \right\} : \times : \{proj_{-I}\, x\} : \times : X(w,\delta)$$

One can easily verify that ξ is well-defined on its domain, and that its range is contained in D.

To see that any fixed point of ξ determines an equilibrium of $\mathcal{E}$, suppose that $(w,\delta,\phi) \in \xi(w,\delta,\phi)$. We must therefore have

$$\delta = proj_{-I}\, x \tag{2}$$

Then, suffices to show that

$$x^{(I)} := proj_I\, x = 0 \tag{3}$$

since (3) implies that all labor markets can clear given expectations (w,δ). From (2), we thus obtain that δ gives the corresponding vector of industry densities (which coincides, by (3), with the vector of owner densities). Finally, (3) and (2) jointly imply that

$$\delta \in \frac{1}{2}\Delta^{I-1}$$

as required. Hence, (w,δ,ϕ) is an equilibrium for any $\phi \in \Phi(w,\delta)$ generating x.

We now verify that (3) holds. Suppose instead that $x_i^{(I)} > 0$ for some i. The definition of ξ and the fact that (w,δ,ϕ) is a fixed point imply $w_i = \overline{W}$. But then all agents *strictly* prefer being wage-earners in industry i than being firm owners in industry i. This follows because R_i is strictly positive by assumption: no matter how pessimistic agents' beliefs, wage-earners in industry i must expect a strictly positive income when $w_i = \overline{W}$. Since $x \in X(w,\delta)$, this means $x_i^{(I)} \leq 0$, which is a contradiction. Assuming $x_i^{(I)} < 0$ leads to a contradiction by a symmetric argument. Equation (3) is therefore confirmed.

Summarising, we defined the correspondence $\xi : D \rightrightarrows D$, and deduced that for any fixed point $(w,\, \delta,\, x)$, there is a ϕ such that (w,δ,ϕ) is an equilibrium of $\mathcal{E}$. The final step is to show that ξ does indeed have a fixed point. But ξ satisfies all the conditions of Kakutani's Fixed Point Theorem. The arguments are somewhat lengthy, and may be found in Appendix A. This completes the proof. □

Discussion of the equilibrium concept

Unfortunately, Definition 4 is not entirely satisfactory as it stands. In particular, it may preclude innovation on the basis of irrational expectations about labor costs. Suppose, for example, that $\delta_i = 0$ in equilibrium. How do we interpret the common wage expectation w_i? A potential entrant into industry i would presumably assess the wage costs of operating in that industry to be equal to the minimum wage necessary to attract some other agent away from his or her current occupation. However, there is no reason why w_i should correspond to this wage rate in general.

Example 3 Let the fundamental state space be $S = S_1 \cup S_2$, where S_1 and S_2 are disjoint, and define the information mapping

$$\Gamma(s) = \begin{cases} \{\theta_1\} & \text{if } \; s \in S_1 \\ \{\theta_2, \, \theta_3\} & \text{if } \; s \in S_2 \end{cases}$$

with payoff-relevant state space

$$\Theta = \{\theta_1, \, \theta_2, \, \theta_3\}$$

Finally, assume the measure p^S satisfies $p^S(S_j) = \frac{1}{2}$ for each $j \in \{1, 2\}$. Thus, the lower and upper probabilities of the event $\{\theta_1\}$ are both $\frac{1}{2}$; while events $\{\theta_2\}$ and $\{\theta_3\}$ each have lower probability 0, and upper probability $\frac{1}{2}$.

Consider a two industry economy ($I = 2$) with revenue functions

$$R_i(\theta_k, \, \delta) = 1 \qquad i, k \in \{1, 2\}$$

$$R_1(\theta_3, \, \delta) = 4 \quad \text{and} \quad R_2(\theta_3, \, \delta) = \frac{9}{2}$$

Thus, firm density has no impact on revenue, and industry 2 weakly dominates industry 1 as a generator of revenue.

Suppose agents anticipate the following wages and densities:

$$(\hat{w}, \, \hat{\delta}) \;=\; \left((1, \, 2), \, \left(\frac{1}{2}, \, 0\right)\right)$$

We claim that $(\hat{w}, \, \hat{\delta}, \, \phi)$ is an equilibrium for the allocation function ϕ which assigns all Bulls to firm ownership in industry 1, and all bears to be workers in industry 1. To see why, observe that

$$\overline{\mathbb{E}}\left[R_1(\theta, \, \hat{\delta}) - \min\{1, R_1(\theta, \, \hat{\delta})\}\right] \;=\; \overline{\mathbb{E}}\left[\min\{2, R_2(\theta, \, \hat{\delta})\}\right] \;=\; \frac{3}{2}$$

$$\overline{\mathbb{E}}\left[R_2(\theta, \, \hat{\delta}) - \min\{2, R_2(\theta, \, \hat{\delta})\}\right] \;=\; \frac{5}{4}$$

$$\overline{\mathbb{E}}\left[\min\{1, R_1(\theta, \, \hat{\delta})\}\right] \;=\; \underline{\mathbb{E}}\left[\min\{1, R_1(\theta, \, \hat{\delta})\}\right] \;=\; \underline{\mathbb{E}}\left[\min\{2, R_2(\theta, \, \hat{\delta})\}\right] \;=\; 1$$

$$\mathbb{E}\left[R_1(\theta,\,\hat{\delta}) - \min\{1, R_1(\theta,\,\hat{\delta})\}\right] \;=\; \mathbb{E}\left[R_2(\theta,\,\hat{\delta}) - \min\{2, R_2(\theta,\,\hat{\delta})\}\right] \;=\; 0$$

So $BR^{\mathcal{E},B}(\hat{w},\,\hat{\delta}) = \{1, 4\}$ and $BR^{\mathcal{E},b}(\hat{w},\,\hat{\delta}) = \{2, 4\}$.

However, an equilibrium in which industry 2 does not operate seems a matter for concern. Suppose an enterprising Bull from the above equilibrium contemplates closing her current firm in industry 1 and opening a monopoly in industry 2. This Bull could offer her current (industry 1) worker the chance to work in the new enterprise for a wage of $\tilde{w}_2 = 1$. The worker will be happy to accept, as the offered wage provides precisely the same expected income as he is currently earning in industry 1. The Bull owner of the industry 2 monopoly anticipates a profit of

$$\overline{\mathbb{E}}\left[R_2(\theta,\,\hat{\delta}) - \min\{1, R_2(\theta,\,\hat{\delta})\}\right] \;=\; \frac{7}{4}$$

which strictly exceeds her current expected profit in industry 1.

Part of the problem lies with the standard definition of equilibrium. As noted by Makowski (1980) in a different context, the standard notion of Walrasian equilibrium is not entirely satisfactory when the commodoties sold in equilibrium are determined endogenously. In that framework, the prices of "non-produced" commodities are free to be anything. On the other hand, some of these prices may leave incentives for firms to open the corresponding market and starting the production of that particular good. In that framework, Makowski defined a "full walrasion equilibrium" as the case in which this profit-making opportunities to innovation are absent. In the following, we impose an equilibrium refinement in a similar spirit.

3.1 Innovation-Proofness

Equilibria which leave such profitable opportunities unexploited are clearly unacceptable. Definition 5 refines the equilibrium concept to eliminate such possibilities. It requires that no lucrative entrepreneurial opportunity is left unexploited.

Definition 5 *Consider an economy $\mathcal{E}$, and let $(w,\,\delta,\,\phi)$ be an entrepreneurial equilibrium of $\mathcal{E}$. This equilibrium is* innovation-proof *if there does* **not** *exist an industry i with $\delta_i = 0$, a potential entrepreneur $k \in [0,\,1]$, a potential worker $k' \in [0,\,1]$ $(k \neq k')$, and a wage level $\hat{w}_i > 0$ such that*

(a) $\mathbb{E}_{v_{k'}}\left[\min\{\hat{w}_i, R_i(\theta,\,\delta)\}\right]$ *is no less than the utility k' obtains from his current occupation $\phi(k')$ (where $v_{k'}$ denotes the capacity of agent k'); and*

(b) $\mathbb{E}_{v_k}\left[R_i(\theta,\,\delta) - \min\{\hat{w}_i, R_i(\theta,\,\delta)\}\right]$ *strictly exceeds the utility k obtains from her current occupation $\phi(k)$ (where v_k denotes the capacity of agent k).*

The concept of innovation-proofness represents a conceptual modification of the standard logic of price-taking equilibrium. Potential innovators are assumed not only to know the wages in existing industries, but also the reservation wage necessary to start a firm in any new industry. The (unmodelled) process by which this reservation wage information is promulgated within a market economy is clearly different to the (unmodelled) process by which wages in currently active labor markets are made known.

Formulating conjectures about reservation wages in inactive labor markets is fundamental to the process of innovative entrepreneurship. However, unlike writers in the Austrian tradition such as Kirzner, we do not differentiate agents in terms of their ability to formulate accurate conjectures of this sort (their "alertness" to entrepreneurial opportunity). Instead, we assume that all agents have perfect knowledge of reservation wage levels in all potential industries, and equal awareness of the revenue functions of inactive industries. Relaxation of this assumption may be useful, but is beyond the scope of the present paper. Innovation in our model is therefore driven not by differences in agents' awareness of the entrepreneurial opportunities, but by differences in their responses to the uncertainties that may surround revenue levels in new industries.

In summary, innovation-proofness ensures that equilibria are robust to lucrative innovations when all agents have equal access to information about potential revenue functions, and assess the implicit wage rates in inactive industries at the reservation wage of the "cheapest" potential worker.

The following Theorem verifies the internal consistency of our model.

Theorem 3.2 *Under Assumptions 1 and 2, every economy $\mathcal{E}$ possesses an innovation-proof entrepreneurial equilibrium.*

Proof: See Appendix A.1.

4 Optimism and occupational choice

Kihlstrom and Laffont (1979) (henceforth KL79) obtained a psychological profile of entrepreneurs as the less risk-averse members of the community. We have already mentioned that this psychological characterization does not stand up to empirical scrutiny.

Moreover, KL79 employs a model with only a single technology (labor market), and in which wages must be paid with certainty, out of the employer's personal resources if necessary. Neither assumption is realistic, and both seriously hamper the model's capacity to say anything about entrepreneurial innovation. The identity of entrepreneurs, and in particular of innovators, will depend in potentially complex ways on general equilibrium pressures in a multi-technology economy, and on the institutional framework in which labor contracting takes place. Indeed, in Kihlstrom and Laffont (1982, 1983), the earlier (negative) correlation between risk aversion and entrepreneurship is lost through the introduction of a richer contracting framework.

The present model departs slightly from KL79's labor market institutions, by allowing wage default. This reflects the obvious fact that employees in new start-ups may often face as much uncertainty as the owner – if the firm fails, all are out of pocket. However, we refrain from introducing more flexible contracting arrangements, as our focus is on the general equilibrium effects from the existence of multiple technologies.[20] Despite the uncertainties of wage labor, it remains true in our model that Bulls have a greater affinity for firm ownership, and bears a greater propensity towards wage-earning roles. More precisely:

[20] See Rigotti and Ryan (2000) for a model in which fully state-contingent employment contracts are allowed. In that model, however, there is only one technology.

Lemma 4.1 *(a) If bears prefer to be firm owners rather than workers, so do Bulls. Formally:*

$$\underline{\mathbb{E}}\,[R_i(\theta,\,\delta)\;-\;\min\{w_i,\,R_i(\theta,\,\delta)\}]\;\geq\;\underline{\mathbb{E}}\,[\min\{w_i,\,R_i(\theta,\,\delta)\}]$$

implies

$$\overline{\mathbb{E}}\,[R_i(\theta,\,\delta)\;-\;\min\{w_i,\,R_i(\theta,\,\delta)\}]\;\geq\;\overline{\mathbb{E}}\,[\min\{w_i,\,R_i(\theta,\,\delta)\}]\,.$$

(b) If Bulls prefer to be workers rather than firm owners, so do bears. Formally:

$$\overline{\mathbb{E}}\,[\min\{w_i,\,R_i(\theta,\,\delta)\}]\;\geq\;\overline{\mathbb{E}}\,[R_i(\theta,\,\delta)\;-\;\min\{w_i,\,R_i(\theta,\,\delta)\}]$$

implies

$$\underline{\mathbb{E}}\,[\min\{w_i,\,R_i(\theta,\,\delta)\}]\;\geq\;\underline{\mathbb{E}}\,[R_i(\theta,\,\delta)\;-\;\min\{w_i,\,R_i(\theta,\,\delta)\}]\,.$$

Neither converse is true in general.

Proof: It is straightforward to verify that $R_i(\theta,\,\delta) - \min\{w_i,\,R_i(\theta,\,\delta)\}$, $\min\{w_i,\,R_i(\theta,\,\delta)\}$ and $2\min\{w_i,\,R_i(\theta,\,\delta)\}$ are pairwise comonotone. Hence

$$\underline{\mathbb{E}}\,[R_i(\theta,\,\delta)\;-\;\min\{w_i,\,R_i(\theta,\,\delta)\}]\;\geq\;\underline{\mathbb{E}}\,[\min\{w_i,\,R_i(\theta,\,\delta)\}] \tag{4}$$

if and only if

$$\underline{\mathbb{E}}\,[R_i(\theta,\,\delta)]\;\geq\;2\underline{\mathbb{E}}\,[\min\{w_i,\,R_i(\theta,\,\delta)\}] \tag{5}$$

$$\Leftrightarrow\;\;\underline{\mathbb{E}}\,[R_i(\theta,\,\delta) - 2\min\{w_i,\,R_i(\theta,\,\delta)\}]\;\geq\;0 \tag{6}$$

Therefore, using (6), it follows that (4) implies

$$\overline{\mathbb{E}}\,[R_i(\theta,\,\delta) - 2\min\{w_i,\,R_i(\theta,\,\delta)\}]\;\geq\;0,$$

which is equivalent to

$$\overline{\mathbb{E}}\,[R_i(\theta,\,\delta)\;-\;\min\{w_i,\,R_i(\theta,\,\delta)\}]\;\geq\;\overline{\mathbb{E}}\,[\min\{w_i,\,R_i(\theta,\,\delta)\}]\,.$$

This proves (a). Case (b) is proved in similar fashion. Counter-examples to the converses are easy to construct. □

An obvious corollary of Lemma 4.1 is that when $I = 1$ and $\alpha \in (0,1)$, there will always be some Bulls who own firms and some bears allocated to laboring jobs in an entrepreneurial equilibrium (unless relevant uncertainty is absent; e.g. Γ is a function).[21] This is the analogue of KL79's result on entrepreneurial psychology. Hence, our focus on uncertainty rather than risk, and our Assumption 2 do not, on their own, cause significant departures from KL79. However, when embedded in our multi-technology general equilibrium model, they do have interesting implications for the process of innovation diffusion. We shall illustrate these implications via an example in the next section.

[21] ¿From the proof of Lemma 4.1, relevant uncertainty is present provided

$$\overline{\mathbb{E}}\,[R_1(\theta,\,\delta) - 2\min\{w_1,\,R_1(\theta,\,\delta)\}]\;>\;\underline{\mathbb{E}}\,[R_1(\theta,\,\delta) - 2\min\{w_1,\,R_1(\theta,\,\delta)\}]\,.$$

5 Uncertainty and the diffusion of innovations

Much of the existing literature on innovation is concerned with the process of diffusion of new products and techniques. An aspect of this process which is discussed by a number of existing models is the effects of uncertainty reduction on diffusion. As firms innovate, they release information about their novel product or technology. This reduces the uncertainty surrounding the value of the innovation, and hence will tend to promote or discourage further "innovators", depending on the nature of the information released.

Jensen (1982) and Vettas (1998) are notable examples. In the latter paper, entry leads to learning by both potential suppliers and consumers, and therefore facilitates further entry. In Vettas, however, firms are exogenously given, with no role for the entrepreneur. The learning on the consumer side is by observing other consumers buying the same good twice. For example, as he suggests in footnote 4, the effect of observing ones neighbor purchasing a car the second time around has an effect. However, the reliance on this sort of signal means that learning will often be very slow since consumer durables tend to have a high life expectancy.

Schumpeter, for example, saw entrepreneurs as pioneers into uncertain endeavors, whose efforts cleared a path along which "swarms" of more conservative business-people would follow, should the reconnaissance reports prove favorable. This accords well with the empirical evidence of S-shaped diffusion curves for successful innovations. However, it also posits both a mechanism of diffusion (uncertainty reduction), and a central agent (the innovative entrepreneur) driving the process. The combined role of uncertainty and entrepreneurship has been given scant attention in the formal theoretical literature on diffusion.

Dempster's model of uncertainty is ideally suited to filling this gap. When the graph of Γ is properly contained in that of Γ', then the former represents a situation of less uncertainty than the latter. This allows us to study the effects of uncertainty reduction on equilibrium.

Importantly, the preferences of each agent may be consistently defined across these uncertainty-ordered economies: Bulls continue to use the upper – and bears the lower – envelope of the (shrinking) set of probabilities generated by the information correspondences. Under a Bayesian model of behavior, such as SEU, individuals choose a single probability from this set. If the set shrinks so as to exclude their chosen probability, then a new probability – hence entirely new preferences – must be specified.

Let us therefore consider a simple example of the diffusion of a successful innovation. Set $I = 2$, and define $\Theta = \{1, 2\}$, $R_1(\theta, \delta) = 10 - 2\delta_1$ and

$$R_2(\theta,\, \delta) \;=\; \begin{cases} 21 \;-\; 4\delta_2 & \text{if } \;\theta = 1 \\ 1 & \text{if } \;\theta = 2 \end{cases}$$

Technology 1, which we may think of as the basis for a well-established industry, exhibits no uncertainty whatsoever. Technology 2, however, which we shall suppose to correspond to a newly available potential innovation, fails completely in state 2.[22]

Let $\varepsilon \in [0, 1]$ index both time and the level of certainty about technology 2. At time ε, uncertainty is generated by the information correspondence Γ_ε with Möbius inverse m_ε such that $m_\varepsilon(\{1\}) = \frac{3\varepsilon}{4}$, $m_\varepsilon(\{2\}) = \frac{\varepsilon}{4}$, and $m_\varepsilon(\{1, 2\}) = 1 - \varepsilon$. Denote by $\underline{v}_\varepsilon$ and $\overline{v}_\varepsilon$ the lower and upper probabilities respectively that are induced by Γ_ε.

[22]Think of its having a scrap value of 1 in this eventuality.

It is easily checked that $\Gamma_{\varepsilon'}(s) \subseteq \Gamma_\varepsilon(s)$ for all s when $\varepsilon' > \varepsilon$. Hence, as ε increases, uncertainty reduces. In the limit – when $\varepsilon = 1$ – uncertainty vanishes, as Γ_1 induces a unique probability: $m_1(\{1\}) = \frac{3}{4}$ and $m_1(\{2\}) = \frac{1}{4}$. The innovative technology 2 is "successful" in the sense that its limiting expected revenue exceeds that of technology 1:

$$\mathbb{E}_{\underline{v}_1}[R_2] \; = \; 16 \; - \; 3\delta_2$$

which strictly exceeds R_1 for all δ_1 and $\delta_2 = \frac{1}{2} - \delta_1$.

For each ε, the economy has a *unique* innovation-proof equilibrium, as we shortly verify. This allows us to study the process of diffusion. Beginning with $\varepsilon = 0$, we show that some agents do choose to innovate (operate technology 2). We assume that this releases information, causing ε to rise over time. We may then examine how the equilibrium changes as this uncertainty is gradually reduced. As we shall see, a rich diffusion structure emerges.[23]

Initially, only Bulls enter industry 2, both as firm owners and wage-earners. We may think of these initial entrants as small-scale start-ups, in which all firm participants face considerable financial uncertainty. The culture of the firms in these infant industries is optimistic, both workers and bosses share the optimistic belief that their fledgling product is bound for greatness, and both share expected revenues equally. We label such a firm *entrepreneurial.*

A recent PBS television special "Triumph of the Nerds: The Rise of Accidental Empires" charted the diffusion of PCs. An early participant in the industry commented: "Most of the people in the industry (at that time) were young because the guys who had any real experience were too smart to get involved in all these crazy little machines."[24]

After some time, these entrepreneurial enterprises release sufficient information to attract bears into the industry – first as workers, and later as firm owners. As uncertainty falls below a critical level, the possibility of operating industry 2 firms along more corporate lines, with uncertainty-averse workers enjoying reasonable financial security, emerges. This gives firm owners in the new sector access to a large, and relatively cheap, workforce, providing additional stimulus to innovation. This spurt in diffusion may be interpreted as the steep portion of the S-shaped empirical diffusion curve, and corresponds to Schumpeter's "swarm" of conservative imitators.

To make good on these claims, let us first observe that the absence of uncertainty in industry 1 implies that any participant in that industry – either firm owner or wage earner – will obtain exactly

$$\frac{1}{2}R_1(\theta,\, \delta) \; = \; 5 \; - \; \delta_1 \tag{7}$$

For industry 2 we have

$$\begin{aligned}\mathbb{E}_{\overline{v}_\varepsilon}[R_2(\theta,\, \delta)] \quad &= \quad 1 \, + \, \overline{v}_\varepsilon(\{1\})(20 - 4\delta_2) \\ &= \quad 1 \, + \, (4 - \varepsilon)(5 - \delta_2)\end{aligned} \tag{8}$$

and

$$\begin{aligned}\mathbb{E}_{\underline{v}_\varepsilon}[R_2(\theta,\, \delta)] \quad &= \quad 1 \, + \, \underline{v}_\varepsilon(\{1\})(20 - 4\delta_2) \\ &= \quad 1 \, + \, 3\varepsilon(5 - \delta_2)\end{aligned} \tag{9}$$

[23]Note that while our analysis is essentially a comparative static exercise, it may be shown to be equivalent to a dynamic model where agents are forward looking.

[24]Transcript available on http://www.pbs.org/nerds/.

Hence
$$\frac{1}{2}\mathbb{E}_{\overline{v}_\varepsilon}\left[R_2(\theta,\,\delta)\right] \;>\; 5 \;\geq\; \frac{1}{2}R_1(\theta,\,\delta)$$
for all δ and all ε, so there is always an occupational option in industry 2 that Bulls strictly prefer to either occupation in industry 1. Hence, in equilibrium, all Bulls must be in industry 2, which implies $\delta_2 \geq \frac{\alpha}{2}$. Therefore, the economy is gaining experience of industry 2 continuously from time $\varepsilon = 0$, justifying our use of ε to index both time and the degree of uncertainty reduction.

Conversely, from (7) and (9), it is clear that initially, when ε is near zero, no bears occupy roles in industry 2. In particular, if $\varepsilon = 0$, it is clear that the unique innovation-proof equilibrium has all Bulls in industry 2 and all bears in industry 1. High uncertainty is particularly attractive to Bulls, because of the potential it allows for high rewards. For bears, on the other hand, it conveys only the perils of possible disaster. As uncertainty reduces (ε rises), Bullish dreams of untold riches are forced within more realistic bounds, while bearish anxieties are alleviated. Hence, the former group become less sanguine about the opportunities in industry 2, while the latter become less averse to participating in innovative enterprises.

In order to describe the equilibria for $\varepsilon > 0$, let us define the following three functions. First, define $\overline{w}_2(\delta_2, \varepsilon)$ as the wage in industry 2 at which Bulls are indifferent between firm ownership and wage-laboring in industry 2. In other words, $\overline{w}_2(\delta_2, \varepsilon)$ is the solution to[25]
$$\frac{1}{2}\left[1 \,+\, (4-\varepsilon)(5-\delta_2)\right] \;=\; \mathbb{E}_{\overline{v}_\varepsilon}\left[\min\left\{w_2,\, R_2(\theta,\,\delta)\right\}\right] \tag{10}$$
Since the left-hand side exceeds 1, we must have $w_2 \geq 1$ to satisfy (10). It is also clearly necessary that $w_2 \leq 21 - 4\delta_2$. Therefore, if (10) holds, w_2 is fully paid in state $\theta = 1$, and workers receive \$1 in state $\theta = 2$. Hence:
$$\begin{aligned}\mathbb{E}_{\overline{v}_\varepsilon}\left[\min\left\{w_2,\, R_2(\theta,\,\delta)\right\}\right] &= 1 + \overline{v}_\varepsilon(\{1\})(w_2 - 1) \\ &= 1 + \frac{(4-\varepsilon)(w_2-1)}{4}\end{aligned}$$
from which it follows that
$$\overline{w}_2(\delta_2, \varepsilon) \;=\; 11 \,-\, \delta_2 \,-\, \frac{2}{(4-\varepsilon)} \tag{11}$$
If $w_2 > \overline{w}_2(\delta_2, \varepsilon)$, then neither type wishes to own a firm in industry 2. This is incompatible with equilibrium, since we already know that $\delta_2 \geq \frac{\alpha}{2}$. Hence $w_2 \leq \overline{w}_2(\delta_2, \varepsilon)$.

We next derive a lower bound for w_2. Define $\underline{w}(\delta_1, \varepsilon)$ as the wage at which bears are indifferent between owning a firm and laboring in industry 2. Thus, $\underline{w}(\delta_1, \varepsilon)$ solves
$$\frac{1}{2}\left[1 \,+\, 3\varepsilon(5-\delta_2)\right] \;=\; \mathbb{E}_{\underline{v}_\varepsilon}\left[\min\left\{w_2,\, R_2(\theta,\,\delta)\right\}\right] \tag{12}$$
If $w_2 < 1$, then bears will strictly prefer an occupation in industry 1 to wage-laboring in industry 2, so we shall assume for the purposes of solving (12) that $w_2 \geq 1$. In this case,
$$\begin{aligned}\mathbb{E}_{\underline{v}_\varepsilon}\left[\min\left\{w_2,\, R_2(\theta,\,\delta)\right\}\right] &= 1 + \underline{v}_\varepsilon(\{1\})(w_2 - 1) \\ &= 1 + \frac{3\varepsilon(w_2-1)}{4}\end{aligned}$$

[25] Equation (10) follows by the logic leading to equation (5) in the proof of Lemma 4.1.

so (12) implies

$$\underline{w}_2(\delta_2, \varepsilon) \;=\; 11 \;-\; \delta_2 \;-\; \frac{2}{3\varepsilon} \tag{13}$$

If $w_2 < \underline{w}_2(\delta_2, \varepsilon)$, then neither type will accept a wage earning position in a firm in industry 2. Therefore, $w_2 \geq \underline{w}_2(\delta_2, \varepsilon)$ in equilibrium.

Finally, define $\hat{w}_2(\delta_2, \varepsilon)$ to be the wage at which bears are indifferent between working in industry 1 and accepting a wage-earning position in industry 2. That is, $\hat{w}_2(\delta_2, \varepsilon)$ solves

$$\mathbb{E}_{\underline{v}_\varepsilon}\left[\min\left\{w_2,\, R_2(\theta,\, \delta)\right\}\right] \;=\; 5 \;-\; \delta_1$$

$$\Leftrightarrow \quad 1 \;+\; \frac{3\varepsilon(w_2 - 1)}{4} \;=\; 5 \;-\; \left(\frac{1}{2} \;-\; \delta_2\right) \tag{14}$$

(using the assumption $w_2 \geq 1$ as before). Solving (14) gives

$$\hat{w}_2(\delta_2, \varepsilon) \;=\; 1 \;+\; \frac{14 + 4\delta_2}{3\varepsilon} \tag{15}$$

Figure 1 graphs these three functions for given δ_2.

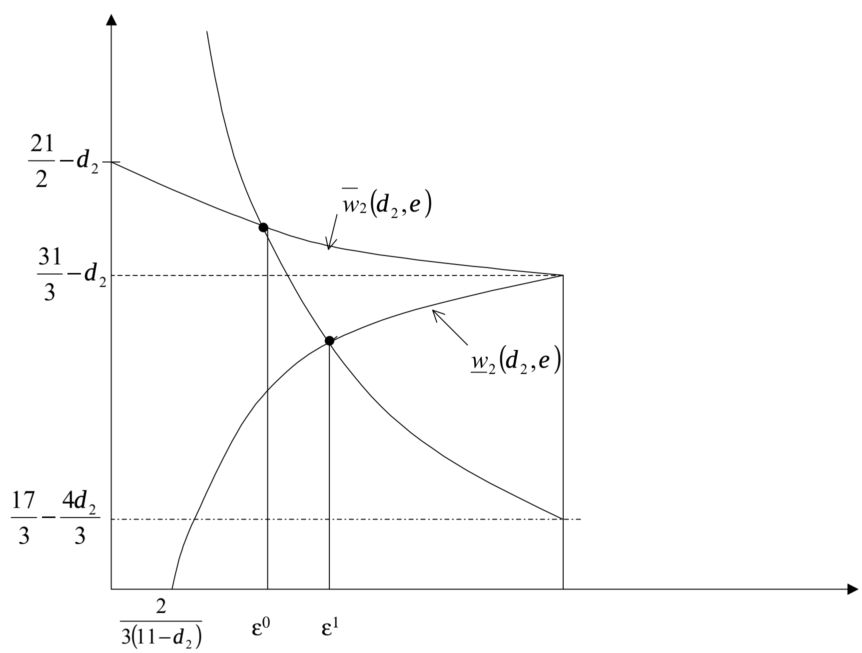

Figure 1: Critical w_2 bounds

Observe that for each δ_2, there is a unique ε^0 for which $\underline{w}_2(\delta_2, \varepsilon^0) = \hat{w}_2(\delta_2, \varepsilon^0)$, and a unique $\varepsilon^1 > \varepsilon^0$ for which $\hat{w}_2(\delta_2, \varepsilon^1) = \overline{w}_2(\delta_2, \varepsilon^1)$.

Recall that $\underline{w}_2(\delta_2, \varepsilon) \leq w_2 \leq \overline{w}_2(\delta_2, \varepsilon)$ in equilibrium. Let us consider three exhaustive cases: (i) $\underline{w}_2(\delta_2, \varepsilon) < w_2 < \overline{w}_2(\delta_2, \varepsilon)$, (ii) $w_2 = \underline{w}_2(\delta_2, \varepsilon) < \overline{w}_2(\delta_2, \varepsilon)$, and (iii) $\underline{w}_2(\delta_2, \varepsilon) < \overline{w}_2(\delta_2, \varepsilon) = w_2$.

If $\underline{w}_2(\delta_2, \varepsilon) < w_2 < \overline{w}_2(\delta_2, \varepsilon)$, then all Bulls wish to own firms in industry 2. Equilibrium therefore requires that bears are happy to work in these firms. Since firm ownership in industry

2 is not optimal for bears, and $\alpha < \frac{1}{2}$, some bears must remain in industry 1. Therefore, we must have $w_2 = \hat{w}_2(\delta_2, \varepsilon)$ so that bears are indifferent between industry 1 occupations and wage-earning in industry 2. Thus, case (i) requires

$$\underline{w}_2(\delta_2, \varepsilon) < w_2 = \hat{w}_2(\delta_2, \varepsilon) < \overline{w}_2(\delta_2, \varepsilon) \tag{16}$$

Next take case (ii). Once again, all Bulls are owners of industry 2 firms. Now bears are equally happy owning or working in such firms. Since some bearish workers are necessary, we must have $w_2 \geq \hat{w}_2(\delta_2, \varepsilon)$. Thus, case (ii) requires

$$\hat{w}_2(\delta_2, \varepsilon) \leq w_2 = \underline{w}_2(\delta_2, \varepsilon) \tag{17}$$

Finally, in case (iii), any bears in industry 2 are workers, while Bulls are indifferent between the two industry 2 occupations. Since $\alpha < \frac{1}{2}$, we cannot have all bears in industry 2, so it is necessary that $w_2 \leq \hat{w}_2(\delta_2, \varepsilon)$. Hence:

$$w_2 = \overline{w}_2(\delta_2, \varepsilon) \leq \hat{w}_2(\delta_2, \varepsilon) \tag{18}$$

Combining (16)–(18) with Figure 1, we see that for each (δ, ε), there is a unique candidate equilibrium value for w_2. This is illustrated in Figure 2.

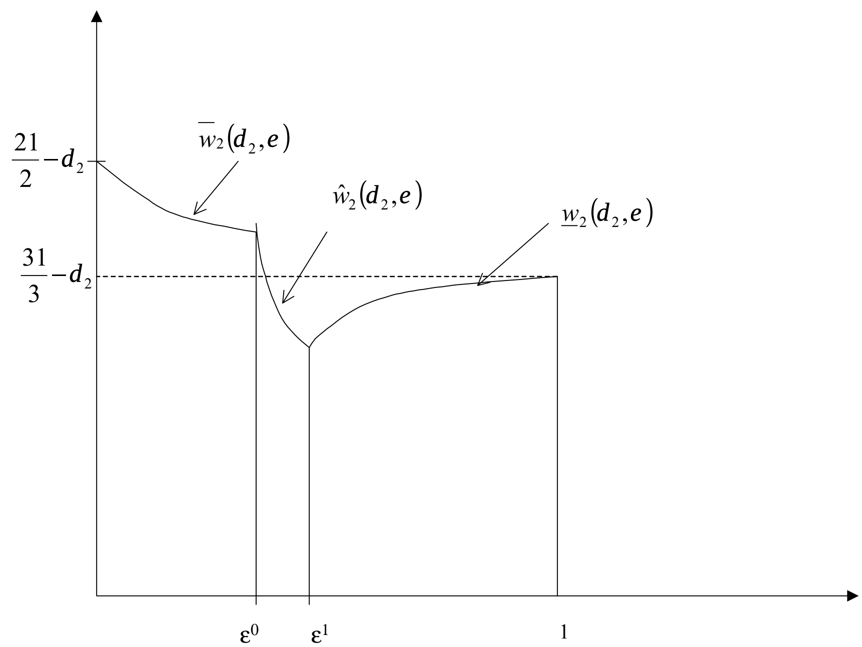

Figure 2: Candidate equilibrium w_2 values

Thus, given δ_2, if $\varepsilon \leq \varepsilon^0$, then $w_2 = \overline{w}_2(\delta_2, \varepsilon)$. In other words, if uncertainty is very high (i.e. early on in the diffusion process), then bears are unwilling to enter industry 2, so w_2 must leave Bulls indifferent between ownership and wage-labor in industry 2. Also, it is clear that $\delta_2 = \frac{\alpha}{2}$ in this case.

As uncertainty falls to $\varepsilon \in (\varepsilon^0, \varepsilon^1)$, bears become attracted by the industry 2 wage. Since they are not yet emboldened sufficiently to own firms in industry 2, some bears must remain in industry 1, so $w_2 = \hat{w}_2(\delta_2, \varepsilon)$. Access to this comparatively cheaper bearish labor force causes wages to drop more sharply during this phase of diffusion. Bulls are an expensive labor force because of their perceived lucrative outside opportunity as owners in industry 2. Since bears are less optimistic about the returns to this occupation, they have a lower reservation wage. Over this phase, $\delta_2 \in \left(\frac{\alpha}{2}, \alpha\right)$.

Finally, when $\varepsilon \geq \varepsilon^1$, uncertainty – and wages – have become so low that bears are happy to own industry 2 firms. Since a labor force is still required, we must have $w_2 = \overline{w}_2(\delta_2, \varepsilon)$ in equilibrium. Over this final phase, $\delta_2 \in \left[\alpha, \frac{1}{2}\right]$, and wages are rising. This represents the maturity of the new industry, as its comparative advantage over the old industry 1 is revealed, and the increasing expected returns must be shared with the labor force.

The foregoing description of the diffusion process is broadly accurate, but some details remain to be tidied. Figure 2 is drawn for given δ_2, while δ_2 is clearly changing, at least through the last two phases of the process. Our equilibrium reasoning must take this into account. In fact, doing so changes matters only slightly: there is period between phases 2 and 3 at which $\delta_2 = \alpha$ is constant. Once density α of bears are working in industry 2, time must pass and uncertainty reduce further before bears become happy to own such firms and the next phase can begin. Figures 3–5 illustrate the complete equilibrium analysis.

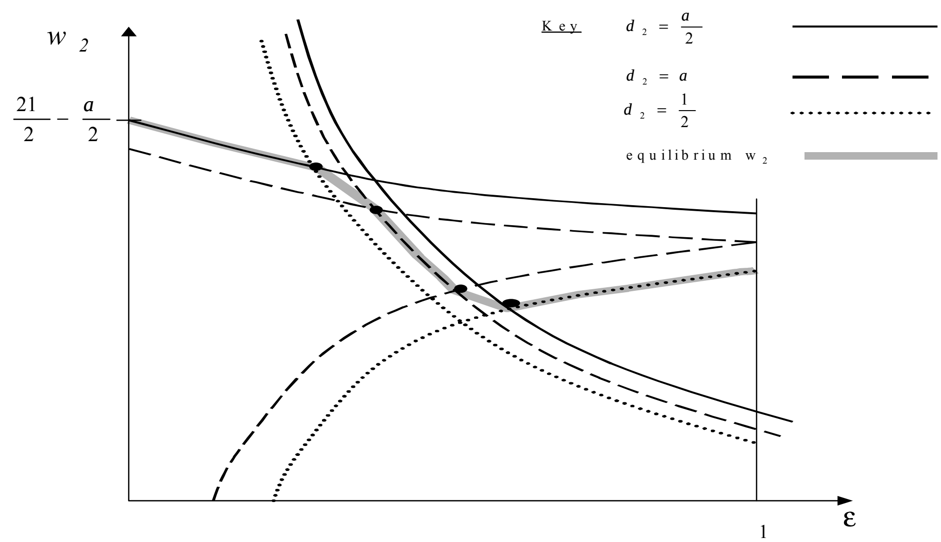

Figure 3: Constructing the equilibrium time path for w_2

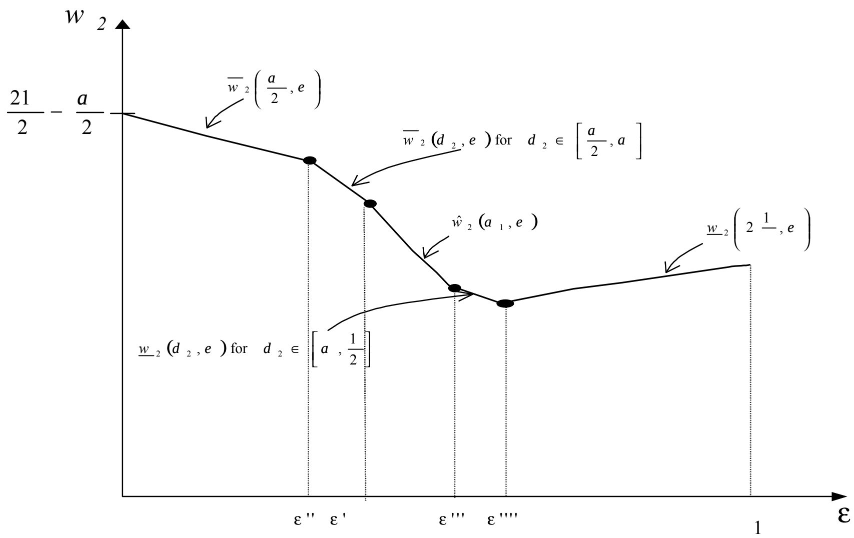

Figure 4: Equilibrium w_2 for each $\varepsilon \in [0, 1]$

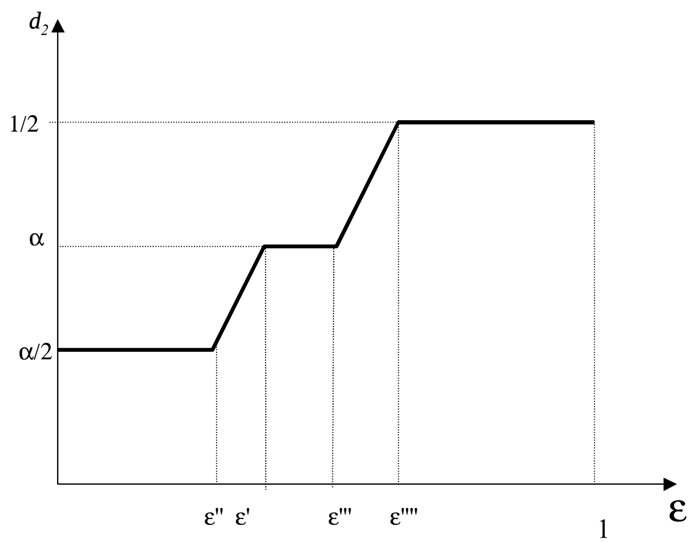

Figure 5: Equilibrium δ_2 for each $\varepsilon \in [0, 1]$

Observe that Figures 4 and 5 imply a particular pattern of correlation between wages and firm density in the new industry. This raises the possibility of empirical testing of the model. Also, from Figure 5 we may discern the basis for an S-shaped diffusion curve. The flat portion in the middle, where $\delta_2 = \alpha$, is largely an artefact of our assuming only two types of agent. In reality, many will exist, with Bulls and bears occupying the two extremes of the spectrum. Adding in these additional types is likely to smooth the curve into an S. Initially, a small group of the most optimistic entrepreneurial types will test the waters, and diffusion will accelerate only once uncertainty has reduced enough that the bulk of the economy's more conservative agents are sufficiently convinced of its value. Observe that as this happens, the character of industry 2 firms will change. It will no longer be necessary to employ potential entrepreneurs in these firms, as in the early stages of innovation. Instead, a larger labor force, made up of uncertainty-averse individuals after steady, secure jobs, will become available. These will be employed in preference to optimists, as they are cheaper. Firms in the innovative sector mature from entrepreneurial operations, to corporate enterprises. As potential entrepreneurs are relieved of having to be workers, diffusion now proceeds apace. Eventually some firms in the new sector become bureaucratic, with *bears* employing *bears.*

It is also worth noting that at time $\varepsilon = 0$, when innovation first takes place, there is likely to be a sudden change in the character of firms in the old industry. Were we to introduce a small amount of uncertainty in these firms, then we would observe Bulls (mostly) employing bears in this industry prior to the arrival of the second technology. When the new technology arrives at $\varepsilon = 0$, all the Bulls switch to the new sector. This forces more bears to take on ownership roles in the original sector, so these firms switch from corporate to bureaucratic. In this sense, then, the innovation process cannot be viewed as the diffusion of *firms* from one sector to another. Crucially, innovation involves the break-up of existing firms and the re-constitution of new ones. If α is near $\frac{1}{2}$, then prior to $\varepsilon = 0$, almost all firms have optimistic objective functions; but at $\varepsilon = 0$, less then half do.

6 Concluding Remarks

The objective of this paper is to exhibit a simple framework that embodies the notion of entrepreneurial innovation, motivated by the recognition that many important innovations have been initiated by individuals, not firms. Both Gates and Ford, as individuals, held optimistic beliefs about their respective industries, and built firms to pursue their vision. To talk of Microsoft moving into the software market, or Ford moving into the motor vehicle market is meaningless. Yet, this is exactly how the formal economic models treat innovation – as a 'hard-headed' sideways shift by an existing firm into a new and uncertain industry.

Where uncertainty predominates, and there are limitations on the ability of the entrepreneur to shield workers from this uncertainty, firms' objectives ought to be treated as endogenous. The firm captures more than simply a production technology: it is also a mechanism for resolving difference in attitudes to uncertainty. When an industry is very new and uncertainty is high, both workers and owners share the same uncertainty attitude. As uncertainty is resolved, there are gains from trading across differences in attitude to uncertainty. Therefore, the nature of the firm is shown to change with the maturation of the industry. It is this change in the nature of the firm in the face of resolving uncertainty that creates the S-shaped diffusion curve in our model.

The PC industry is an excellent example of this process of diffusion and maturation. The early innovators were optimistic entrepreneurs who worked in small partnerships. In the late seventies, the large established IBM eventually recognized the potential in the PC and developed the IBM acorn. This computer was launched in 1981. IBM predicted that it would sell half a million by 1984. It sold 2 million!

A Properties of the correspondence ξ

To complete the proof of Theorem 3.1 it is sufficient to show that the correspondence ξ is upper hemi-continuous (u.h.c.), and has non-empty, compact and convex values. It will then follow by Kakutani's Fixed Point Theorem that ξ has a fixed point.

In the course of the arguments, it will be necessary to consider limits of sequences of allocation functions. Although such functions are points in an infinite-dimensional space, the topological arguments are simplified by observing that only finitely many properties of these functions are relevant to the analysis.

Given any allocation function ϕ, define the following $4I$ (Lebesgue) measurable sets:

$$E_i^B(\phi) = \phi^{-1}(i) \cap [0,\, \alpha) \qquad i = 1, 2, \ldots, 2I$$
$$E_i^b(\phi) = \phi^{-1}(i) \cap [\alpha,\, 1] \qquad i = 1, 2, \ldots, 2I$$

These sets identify the agents of each type assigned to each of the $2I$ occupations by ϕ. For our purposes, the relevant features of ϕ are simply the Lebesgue measures of these sets. In particular, we need to know whether or not ϕ is *consistent* with some given (w, δ); and the implied total densities of agents in each occupation. We shall therefore associate with each ϕ a finite vector $t(\phi)$ which will summarize these properties.

Each $t(\phi)$ will be a vector in

$$\mathcal{T} \;=\; \left\{ t \in [0,\, 1]^{4I+1} \;\middle|\; 0 = t_1 \le t_2 \le \cdots \le t_{2I+1} = \alpha \le t_{2I+2} \le \cdots \le t_{4I+1} = 1 \right\}$$

In particular, $t(\phi) = \hat{t} \in \mathcal{T}$ if and only if

$$\hat{t}_i = \sum_{k<i} Leb\left[E_k^B(\phi)\right] \qquad \forall i \le 2I+1$$

and

$$\hat{t}_i = \alpha + \sum_{k<i-2I} Leb\left[E_k^b(\phi)\right] \qquad \forall i \in \{2I+2, 2I+3, \ldots, 4I+1\}$$

That is, if $t(\phi) = \hat{t} \in \mathcal{T}$, then for each $i \in \mathcal{O}$

$$\begin{aligned} Leb\left[E_i^B(\phi)\right] &= \hat{t}_{i+1} - \hat{t}_i \\ Leb\left[E_i^b(\phi)\right] &= \hat{t}_{2I+i+1} - \hat{t}_{2I+i} \end{aligned}$$

Conversely, given any $t \in \mathcal{T}$, we may construct an allocation function $\phi_{[t]}$ as follows:

$$\phi_{[t]}(j) = i \iff \begin{cases} j \in [t_i,\, t_{i+1}); \quad \textit{or} \\ j \in [t_{i^*},\, t_{i^*+1}] \quad \textit{and} \;\; i = i^* - 2I; \quad \textit{or} \\ j \in (t_k,\, t_{k+1}] \quad \textit{and} \;\; i = k - 2I > i^* - 2I \end{cases}$$

where $i^* = \min\{i \mid t_{i+1} > \alpha\}$. It should be clear that, for every $i \in \mathcal{O}$

$$Leb\left[E_i^B(\phi)\right] \;=\; Leb\left[E_i^B(\phi_{[t(\phi)]})\right]$$

and

$$Leb\left[E_i^b(\phi)\right] \;=\; Leb\left[E_i^b(\phi_{[t(\phi)]})\right]$$

Let us now return to our consideration of the properties of ξ. Firstly, it is obvious that:

Lemma A.1 *$\xi(w, \delta, x) \neq \emptyset$ for every $(w, \delta, x) \in D$.*

We next show:

Lemma A.2 *$\xi(w, \delta, x)$ is compact for every $(w, \delta, x) \in D$.*

Proof. Boundedness is immediate, so it suffices to prove closedness. The sets

$$\left\{ \arg \max_{\tilde{w} \in [0, \overline{W}]^I} \ \tilde{w} \cdot proj_I x \right\}$$

and $\{proj_{-I}\, x\}$ are clearly closed; so we need only consider $X(w, \delta)$.

Let $\{x^n\}_{n=1}^{\infty}$ be a sequence in $X(w, \delta)$ with limit $\overline{x}$. Then $\left\{z^n \equiv C^{-1}x^n\right\}_{n=1}^{\infty}$ is a sequence in $Z(w, \delta)$ with limit $\overline{z} = C^{-1}\overline{x}$. Associated with each z^n will be an allocation function $\phi_n \in \Phi(w, \delta)$ such that

$$z^n = \left(Leb\left[\phi_n^{-1}(i)\right]\right)_{i=1}^{2I}$$

Consider the sequence $\{t(\phi_n)\}_{n=1}^{\infty}$ in $\mathcal{T}$. Since $\mathcal{T}$ is clearly compact, this has a convergent subsequence with limit $\overline{t} \in \mathcal{T}$. If we can show that

$$\phi_{[\overline{t}]} \in \Phi(w, \delta) \tag{19}$$

and

$$\overline{z} = \left(Leb\left[\phi_{[\overline{t}]}^{-1}(i)\right]\right)_{i=1}^{2I} \tag{20}$$

then the lemma is proved.

Suppose that (15) does *not* hold. Then, there must exist some $k \in \{B, b\}$ and some $i \in \mathcal{O}$ such that $i \notin BR^{\mathcal{E},k}(w, \delta)$ and

$$Leb\left[E_i^k\left(\phi_{[\overline{t}]}\right)\right] > 0$$

Consider the case $k = B$ (the argument is similar if $k = b$). Then for some

$$i \notin BR^{\mathcal{E},B}(w, \delta)$$

we have

$$\begin{aligned} & Leb\left[E_i^k\left(\phi_{[\overline{t}]}\right)\right] > 0 \\ \Rightarrow\ & \overline{t}_{i+1} - \overline{t}_i > 0 \\ \Rightarrow\ & t_{i+1}^n - t_i^n > 0 \end{aligned}$$

for sufficiently large n along the subsequence defining $\overline{t}$. But this contradicts the fact that $\phi_n \in \Phi(w, \delta)$ for all n. Hence, (15) must hold.

To see (16), suppose that m indexes the subsequence defining $\overline{t}$. Then

$$\begin{aligned} z^m &= \left(Leb\left[E_i^k\left(\phi_{[t^m]}\right)\right]\right)_{i=1}^{2I} \\ &= \left(\left(t_{i+1}^m - t_i^m\right) + \left(t_{2I+i+1}^m - t_{2I+i}^m\right)\right)_{i=1}^{2I} \\ &\to \left(\left(\overline{t}_{i+1} - \overline{t}_i\right) + \left(\overline{t}_{2I+i+1} - \overline{t}_{2I+i}\right)\right)_{i=1}^{2I} \\ &= \left(Leb\left[E_i^k\left(\phi_{[\overline{t}]}\right)\right]\right)_{i=1}^{2I} \end{aligned}$$

and (16) follows. □

Lemma A.3 *$\xi(w, \delta, x)$ is convex for every $(w, \delta, x) \in D$.*

Proof. Again, the sets

$$\left\{ \arg \max_{\tilde{w} \in [0, \overline{W}]^I} \ \tilde{w} \cdot proj_I x \right\}$$

and $\{proj_{-I}\, x\}$ are obviously convex. Let $\hat{x}$, $\tilde{x} \in X(w, \delta)$ and consider

$$x \,=\, \lambda \hat{x} \,+\, (1-\lambda)\tilde{x}$$

for some $\lambda \in (0, 1)$.

Define $\hat{z} = C^{-1}\hat{x}$, $\tilde{z} = C^{-1}\tilde{x}$, and

$$z \,=\, C^{-1}x \,=\, \lambda \hat{z} \,+\, (1-\lambda)\tilde{z}$$

Let $\hat{\phi}$ and $\tilde{\phi}$ be allocation functions in $\Phi(w, \delta)$ satisfying

$$\hat{z} = \left(Leb \left[\hat{\phi}^{-1}(i) \right] \right)_{i=1}^{2I}$$

$$\tilde{z} = \left(Leb \left[\tilde{\phi}^{-1}(i) \right] \right)_{i=1}^{2I}$$

Now observe that $\mathcal{T}$ is a convex set, and hence consider the $t \in \mathcal{T}$ defined as

$$t \,=\, \lambda \hat{t} \,+ (1-\lambda)\tilde{t}$$

where $\hat{t} = t(\hat{\phi})$ and $\tilde{t} = t(\tilde{\phi})$. Since

$$\begin{aligned} Leb\left[E_i^B(\phi_{[t]})\right] &= t_{i+1} - t_i \\ &= \lambda(\hat{t}_{i+1} - \hat{t}_i) \,+\, (1-\lambda)(\tilde{t}_{i+1} - \tilde{t}_i) \end{aligned}$$

and

$$\begin{aligned} Leb\left[E_i^b(\phi_{[t]})\right] &= t_{2I+i+1} - t_{2I+i} \\ &= \lambda(\hat{t}_{2I+i+1} - \hat{t}_{2I+i}) \,+\, (1-\lambda)(\tilde{t}_{2I+i+1} - \tilde{t}_{2I+i}) \end{aligned}$$

it is obvious that $\phi_{[t]} \in \Phi(w, \delta)$. We now deduce that for every $i \in \mathcal{O}$

$$\begin{aligned} Leb\left[\phi_{[t]}^{-1}(i)\right] &= (t_{i+1} - t_i) \,+\, (t_{2I+i+1} - t_{2I+i}) \\ &= \lambda\left((\hat{t}_{i+1} - \hat{t}_i) \,+\, (\hat{t}_{2I+i+1} - \hat{t}_{2I+i})\right) \,+ \\ &\qquad (1-\lambda)\left((\tilde{t}_{i+1} - \tilde{t}_i) \,+\, (\tilde{t}_{2I+i+1} - \tilde{t}_{2I+i})\right) \\ &= \lambda\, Leb\left[E_i^B(\phi_{[\hat{t}]}) \cup E_i^b(\phi_{[\hat{t}]})\right] + (1-\lambda)\, Leb\left[E_i^B(\phi_{[\tilde{t}]}) \cup E_i^b(\phi_{[\tilde{t}]})\right] \\ &= \lambda \hat{z}_i \,+\, (1-\lambda)\tilde{z}_i \end{aligned}$$

Thence $z \in Z(w, \delta)$ and the proof is complete. □

Lemma A.4 *ξ is u.h.c.*

Proof. Since ξ is the Cartesian product of three compact-valued correspondences (Lemma A.2), it suffices by Proposition 11.25 of Border (1985) to prove upper hemi-continuity for each of the three component correspondences. The second component correspondence is trivially u.h.c., and the first is u.h.c. by standard arguments, so let us consider the third component of ξ.

$X(w, \delta)$ will be u.h.c. if and only if (Border (1985, Proposition 11.11)), for every sequence $\{(w, \delta)^n\}_{n=1}^{\infty}$ in $[0, \overline{W}]^I \times [0,1]^I$ converging to some limit $(\overline{w}, \overline{\delta})$ (in $[0, \overline{W}]^I \times [0,1]^I$), and every sequence $\{x^n\}_{n=1}^{\infty}$ satisfying

$$x^n \in X\left((w, \delta)^n\right) \qquad \forall n$$

there is a convergent subsequence of $\{x^n\}_{n=1}^{\infty}$ with limit in $X(\overline{w}, \overline{\delta})$.

Let $\{(w, \delta)^n\}_{n=1}^{\infty}$, $(\overline{w}, \overline{\delta})$, and $\{x^n\}_{n=1}^{\infty}$ satisfy the conditions of the preceding paragraph. Define the sequence

$$\left\{z^n = C^{-1}x^n\right\}_{n=1}^{\infty}$$

and let $\{\phi^n\}_{n=1}^{\infty}$ be an associated sequence of allocation functions. Then the sequence

$$\{t^n = t(\phi^n)\}_{n=1}^{\infty}$$

has a convergent subsequence with limit $\overline{t} \in \mathcal{T}$.

We claim that $\phi_{[\overline{t}]} \in \Phi(\overline{w}, \overline{\delta})$. To see this, observe that the correspondences $BR^{\mathcal{E},B}$ and $BR^{\mathcal{E},b}$ are clearly upper hemi-continuous by the continuity of each type's objective function in (w, δ). In particular, the Choquet integral is continuous, as is clear from equation (1). Therefore, if $1 \leq i \leq 2I$ and $i \notin BR^{\mathcal{E},B}(\overline{w}, \overline{\delta})$, then it must also be the case that $i \notin BR^{\mathcal{E},B}(w^n, \delta^n)$ for all sufficiently large n. Hence, we cannot have $\overline{t}_{i+1} > \overline{t}_i$. Analogous conclusions hold for $2I+1 \leq i \leq 4I$ and $(i-2I) \notin BR^{\mathcal{E},b}(\overline{w}, \overline{\delta})$. So

$$\phi_{[\overline{t}]} \in \Phi(\overline{w}, \overline{\delta})$$

as claimed.

Finally, to complete the proof of the lemma, we must show that

$$Leb\left[\phi_{[\overline{t}]}^{-1}(i)\right] = \overline{z}_i \qquad \forall i$$

But this may be seen as follows:

$$\begin{aligned} Leb\left[\phi_{[\overline{t}]}^{-1}(i)\right] &= \lim_{n\to\infty} Leb\left[\phi_{[t^n]}^{-1}(i)\right] \\ &= \lim_{n\to\infty} z_i^n \\ &= \overline{z}_i \end{aligned}$$

This completes the proof. □

A.1 Proof of Theorem 3.2

The following Lemma is useful as preliminary result.

Lemma A.5 *Suppose economy $\mathcal{E}$ is such that the information correspondence Γ induces upper and lower probabilities $\overline{v}$ and $\underline{v}$ having the following property:*

$$\forall E \in \mathcal{P}^*(\Theta) \equiv \mathcal{P}(\Theta) \setminus \{\emptyset\} \qquad \overline{v}(E) > 0 \;\Rightarrow\; \underline{v}(E) > 0$$

Then, any equilibrium of $\mathcal{E}$ is innovation-proof.

Proof: Suppose that $(w,\, \delta,\, \phi)$ is an equilibrium of $\mathcal{E}$. Suppose further that there exists an industry i with $\delta_i = 0$, such that a wage $\hat{w}_i \geq 0$ and agents $k, k' \in [0,\, 1]$, $k \neq k'$, may be found satisfying

$$\mathbf{E}_{v_k}\left[R_i(\theta,\, \delta) : - : \min\{\hat{w}_i,\, R_i(\theta,\, \delta)\}\right] \geq y^{\mathcal{E},\beta_k}(w,\, \delta)$$

and

$$\mathbf{E}_{v_{k'}}\left[\min\{\hat{w}_i,\, R_i(\theta,\, \delta)\}\right] \geq y^{\mathcal{E},\beta_{k'}}(w,\, \delta)$$

with at least one strict inequality.

Wage rate $\widehat{w}_i$ must be different from w_i since otherwise $(w,\, \delta,\, \phi)$ would not have been an equilibrium in the first place. Suppose that $\hat{w}_i > w_i$. In this case, we claim that agent k cannot be strictly better off after the re-allocation. This is because occupation $2i-1$ was weakly less desirable to agent k than k's occupation under ϕ when firm owners in industry i had to pay wage w_i. Therefore, owning a firm in industry i and paying the strictly higher wage $\hat{w}_i$ cannot make k strictly better off than in the original equilibrium. Therefore

$$\begin{aligned} y^{\mathcal{E},\beta_k}(w,\, \delta) &= \mathbf{E}_{v_k}\left[R_i(\theta,\, \delta) - \min\{\hat{w}_i,\, R_i(\theta,\, \delta)\}\right] \\ &= \mathbf{E}_{v_k}\left[R_i(\theta,\, \delta) - \min\{w_i,\, R_i(\theta,\, \delta)\}\right] \end{aligned} \tag{21}$$

Since

$$R_i(\theta,\, \delta) - \min\{c,\, R_i(\theta,\, \delta)\}$$

and

$$\min\{c,\, R_i(\theta,\, \delta)\}$$

are *comonotonic* in θ for any $c \in \mathbf{R}_+$, we may conclude that ([**?**])

$$\mathbf{E}_{v_k}\left[R_i(\theta,\, \delta)\right] = \mathbf{E}_{v_k}\left[R_i(\theta,\, \delta) - \min\{c,\, R_i(\theta,\, \delta)\}\right] + \mathbf{E}_{v_k}\left[\min\{c,\, R_i(\theta,\, \delta)\}\right]$$

$$\Rightarrow \quad \mathbf{E}_{v_k}\left[R_i(\theta,\, \delta) - \min\{c,\, R_i(\theta,\, \delta)\}\right] = \mathbf{E}_{v_k}\left[R_i(\theta,\, \delta)\right] - \mathbf{E}_{v_k}\left[\min\{c,\, R_i(\theta,\, \delta)\}\right]$$

for any $c \in \mathbf{R}_+$. Therefore, (21) implies

$$\mathbf{E}_{v_k}\left[\min\{\hat{w}_i,\, R_i(\theta,\, \delta)\}\right] = \mathbf{E}_{v_k}\left[\min\{w_i,\, R_i(\theta,\, \delta)\}\right]$$

which in turn implies

$$v_k\left(\{\theta \in \Theta \mid R_i(\theta,\, \delta) > w_i\}\right) = 0 \tag{22}$$

Because k' must be strictly better off under the reallocation, we have

$$\begin{aligned}\mathbf{E}_{v_{k'}}\left[\min\left\{w_i,\, R_i(\theta,\,\delta)\right\}\right] &\leq y^{\mathcal{E},\beta_{k'}}(w,\,\delta) \\ &< \mathbf{E}_{v_{k'}}\left[\min\left\{\hat{w}_i,\, R_i(\theta,\,\delta)\right\}\right]\end{aligned}$$

and hence

$$v_{k'}\left(\{\theta \in \Theta \mid R_i(\theta,\,\delta) > w_i\}\right) > 0 \tag{23}$$

But (22) and (23) imply that $v_k = \underline{v}$ (i.e. $\beta_k = b$) and $v_{k'} = \overline{v}$ (i.e. $\beta_{k'} = B$). Thus, we have a contradiction to the assumption of the lemma.

The case $\hat{w}_i < w_i$ yields a contradiction by an analogous argument. □

Corollary A.1 *Suppose economy $\mathcal{E}$ is such that the information correspondence Γ induces the lower probability $\underline{v}$ whose Möbius inverse m satisfies*

$$m(\{\theta\}) > 0 \quad \forall \theta \in \Theta$$

Then, any equilibrium of $\mathcal{E}$ is innovation-proof.

The assumption on m implies $\underline{v}(E) > 0$ for all $E \in \mathcal{P}^*(\Theta)$ and the result follows from Lemma 4.1. □

We are now ready to prove that under Assumptions 1–3, every economy $\mathcal{E}$ possesses an innovation-proof equilibrium.[26]

Proof of Theorem 3.2

Let $\overline{v}$ and $\underline{v}$ denote the upper and lower probabilities induced by Γ. For the purposes of the present argument it is convenient to make the assumption that all "redundant" elements have been removed from Θ; that is, all θ such that

$$\underline{v}(\{\theta\}) = \overline{v}(\{\theta\}) = 0$$

If m, the Möbius inverse of $\underline{v}$, satisfies the condition in Corollary 4.1 we are done. Suppose not; that is, $m(\{\theta\}) = 0$ for some $\theta \in \Theta$.

Consider the space of functions

$$\mathcal{M} = \left\{ f : \mathcal{P}(\Theta) \to [0,\,1] \;\middle|\; f(\emptyset) = 0, \quad \sum_{E \in \mathcal{P}(\Theta)} f(E) = 1 \right\}$$

Each $f \in \mathcal{M}$ is the Möbius inverse of some lower probability on $(\Theta,\, \mathcal{P}(\Theta))$; conversely, each lower probability on $(\Theta,\, \mathcal{P}(\Theta))$ has a Möbius inverse in $\mathcal{M}$ (see Shafer (1976)). We may identify $\mathcal{M}$ with the unit simplex $\Delta^{|\mathcal{P}^*(\Theta)|-1}$. In particular, $\mathcal{M}$ is compact and convex.

Choose some $\tilde{\pi} \in ri[core(\underline{v})]$. Then, $\tilde{\pi}(E) = \underline{v}(E)$ if and only if $\underline{v}(E) = \underline{v}(E)$, and $\tilde{\pi}(E) > \underline{v}(E)$ otherwise. In particular, letting $\tilde{m} \in \mathcal{M}$ denote the Möbius inverse of $\tilde{\pi}$, we must have $\tilde{m}(\{\theta\}) > 0$ for every $\theta \in \Theta$ by our non-redundancy assumption.

[26] Some of the notation used in the following proof is defined in Appendix A.

Now let us define

$$m^n \;=\; \frac{1}{n}\,\tilde{m} \,+\, \left(1-\frac{1}{n}\right)\, m$$

The sequence $\{m^n\}_{n=1}^{\infty}$ in $\mathcal{M}$ clearly converges to m and satisfies $m^n(\{\theta\}) > 0$ for every $\theta \in \Theta$. If $\{\underline{v}^n\}_{n=1}^{\infty}$ is the associated sequence of lower probabilities, then $\underline{v}^n \to \underline{v}$ as $n \to \infty$, and for each n

$$\underline{v}^n(E) \geq \underline{v}(E) \quad \forall E \subseteq \Theta$$

¿From $\underline{v}^n$ we may construct the associated upper probability $\overline{v}^n$ as follows (see Dempster (1967)):

$$\overline{v}^n(E) = 1 - \underline{v}^n(E^c) \quad \forall E \in \mathcal{P}(\Theta)$$

For each n, define $\mathcal{E}^n$ as the economy with identical α and revenue functions to $\mathcal{E}$, but in which Bulls evaluate their employment options using the capacity $\overline{v}^n$, and bears evaluate their options using the capacity $\underline{v}^n$. For the purposes of the following argument it is unnecessary to define a sequence of information correspondences generating these beliefs. By Corollary 4.1, each $\mathcal{E}^n$ has an innovation-proof equilibrium $(w^n,\, \delta^n,\, \phi_n)$. Let $(w,\, \delta,\, t)$ denote the limit of a convergent subsequence of $\{(w^n,\, \delta^n,\, t\,(\phi_n))\}_{n=1}^{\infty}$ (retaining n as index of the convergent subsequence for notational convenience). We claim that $\left(w,\, \delta,\, \phi_{[t]}\right)$ is an innovation-proof equilibrium of $\mathcal{E}$.

For every $i \in \{1, 2, \ldots, I\}$, note that

$$\begin{aligned} Leb\left[\phi_{[t]}^{-1}(2i-1)\right] \;&=\; (t_{2i} - t_{2i-1}) \,+\, (t_{2I+2i} - t_{2I+2i-1}) \\ &=\; \lim_{n\to\infty} \left[\left(t_{2i}^n - t_{2i-1}^n\right) \,+\, \left(t_{2I+2i}^n - t_{2I+2i-1}^n\right)\right] \\ &=\; \lim_{n\to\infty} Leb\left[\phi_n^{-1}(2i-1)\right] \\ &=\; \lim_{n\to\infty} \delta_i^n \\ &=\; \delta_i \end{aligned}$$

and

$$Leb\left[\phi_{[t]}^{-1}(2i)\right] = \delta_i$$

similarly. So $\phi_{[t]}$ clears all labor markets and generates industry density vector δ.

To complete the proof it suffices to show that $\phi_{[t]} \in \Phi(w,\, \delta)$, and that $(w,\, \delta,\, \phi_{[t]})$ is innovation-proof. Agents of type $\beta \in \{B,\, b\}$ have objective functions which are continuous in $(w^n,\, \delta^n,\, v_\beta^n)$, where v_β^n denotes type β's belief capacity (recall (1) and Assumption 3). Hence

$$\forall \beta \in \{B,\, b\} \qquad \limsup_{n\to\infty}\; BR^{\mathcal{E}^n,\beta}(w^n,\, \delta^n) \subseteq BR^{\mathcal{E},\beta}(w,\, \delta)$$

If $t_{i+1} - t_i > 0$ for some $i \notin BR^{\mathcal{E},B}(w,\, \delta)$, then $t_{i+1}^n - t_i^n > 0$ must also hold for sufficiently large n. Furthermore, if $t_{2I+i+1} - t_{2I+i} > 0$ for some $i \notin BR^{\mathcal{E},b}(w,\, \delta)$, then again $t_{2I+i+1}^n - t_{2I+i}^n > 0$ must also hold for sufficiently large n. In each case we have a contradiction to $\phi_n \in \Phi(w^n,\, \delta^n)$. Hence, $\phi_{[t]} \in \Phi(w,\, \delta)$.

Suppose that $\left(w,\, \delta,\, \phi_{[t]}\right)$ is *not* innovation-proof. That is, there exist agents k and k', an industry i with $\delta_i = 0$, and a wage $\hat{w}_i$ such that if k owns a firm in industry i and employs k'

as a worker at $\hat{w}_i$, both do as well as under the candidate equilibrium, and at least one of them does strictly better. Suppose agent k does strictly better (the argument is similar if we choose k' instead). Then

$$\mathbf{E}_{v_k}\left[R_i(\theta,\,\delta) \,-\, \min\{\hat{w}_i,\, R_i(\theta,\,\delta)\}\right] \;>\; y^{\mathcal{E},\beta_k}(w,\delta) \tag{24}$$

where β_k and v_k are k's type and belief capacity respectively. Clearly, it must be the case that $w_i > \hat{w}_i$. Furthermore, (7) and the fact that

$$v^n_{k'}(E) > 0 \qquad \forall E \in \mathcal{P}^*(\Theta)$$

imply the existence of some $\eta > 0$ such that, for every n,

$$\mathbf{E}_{v^n_{k'}}\left[\min\{c,\, R_i(\theta,\,\delta)\}\right]$$

is *strictly* increasing in c when $c \in [\hat{w}_i, \hat{w}_i + \eta]$. On the other hand, since

$$\mathbf{E}_{v_{k'}}\left[\min\{\hat{w}_i,\, R_i(\theta,\,\delta)\}\right] \;\geq\; y^{\mathcal{E},\beta_{k'}}(w,\delta) \;\geq\; \mathbf{E}_{v_{k'}}\left[\min\{w_i,\, R_i(\theta,\,\delta)\}\right]$$

we see that

$$\mathbf{E}_{v_{k'}}\left[\min\{c,\, R_i(\theta,\,\delta)\}\right] \;=\; y^{\mathcal{E},\beta_{k'}}(w,\delta) \quad \forall c \in [\hat{w}_i,\, w_i]$$

In particular, we must have $v_{k'} \equiv \underline{v}$ and

$$m(B) = 0 \quad \forall B \subseteq \{\theta \in \Theta \mid R_i(\theta,\,\delta) > \hat{w}_i\} \;\neq\; \emptyset$$

Recalling that $\underline{v}^n \geq \underline{v}$, we deduce

$$\mathbf{E}_{v^n_{k'}}\left[\min\{\hat{w}_i,\, R_i(\theta,\,\delta)\}\right] \;\geq\; \mathbf{E}_{v_{k'}}\left[\min\{\hat{w}_i,\, R_i(\theta,\,\delta)\}\right]$$

for each n. Letting $\tilde{w}_i \in (\hat{w}_i, \hat{w}_i + \eta)$, we therefore have

$$\mathbf{E}_{v^n_{k'}}\left[\min\{\tilde{w}_i,\, R_i(\theta,\,\delta)\}\right] \,-\, y^{\mathcal{E},\beta_{k'}}(w,\delta) \;>\; \mathbf{E}_{v_{k'}}\left[\min\{\tilde{w}_i,\, R_i(\theta,\,\delta)\}\right] \,-\, y^{\mathcal{E},\beta_{k'}}(w,\delta)$$

for all n. Hence

$$\mathbf{E}_{v^n_{k'}}\left[\min\{\tilde{w}_i,\, R_i(\theta,\,\delta^n)\}\right] \,-\, y^{\mathcal{E}^n,\beta_{k'}}(w^n,\delta^n) \;\geq\; \mathbf{E}_{v_{k'}}\left[\min\{\tilde{w}_i,\, R_i(\theta,\,\delta)\}\right] \,-\, y^{\mathcal{E},\beta_{k'}}(w,\delta)$$

for all sufficiently large n, using the continuity of the functions on the left-hand side of the inequality. Therefore, for all large n,

$$\mathbf{E}_{v^n_{k'}}\left[\min\{w^n_i,\, R_i(\theta,\,\delta^n)\}\right] \,-\, y^{\mathcal{E}^n,\beta_{k'}}(w^n,\delta^n) \;>\; \mathbf{E}_{v_{k'}}\left[\min\{\tilde{w}_i,\, R_i(\theta,\,\delta)\}\right] \,-\, y^{\mathcal{E},\beta_{k'}}(w,\delta)$$

since $\mathbf{E}_{v^n_{k'}}\left[\min\{c,\, R_i(\theta,\,\delta^n)\}\right]$ is strictly increasing in c at $c = \tilde{w}_i$ when n is sufficiently large. But observe that the right-hand side of this inequality is equal to zero, so

$$\mathbf{E}_{v^n_{k'}}\left[\min\{w^n_i,\, R_i(\theta,\,\delta^n)\}\right] \;>\; y^{\mathcal{E}^n,\beta_{k'}}(w^n,\delta^n)$$

which is a contradiction. This completes the proof. □

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