

Professional Advice*

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Abstract

Professional experts offer advice with the objective of appearing well informed. Their ability is evaluated on the basis of the advice given and the realized state of the world. We model this situation as a reputational cheap-talk game with continuous signal, state, and ability type spaces. Despite allowing a message space as rich as the signal space, at best two messages are sent in the most informative equilibrium. The expert can credibly transmit only the direction but not the intensity of the information possessed. Equilibrium forecasts are then systematically less precise than under truth-telling, and learning on the expert's ability is slow.

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1. Introduction

Economists have a bad name as forecasters. We argue that their desire to be perceived as good forecasters turns them into poor forecasters.¹ Our argument applies more generally to professional experts who are motivated by the market evaluation of their performance. For instance, consultants and business advisers are concerned about the reputation for the quality of their services, managers are interested in their career, and politicians wish to be re-elected. While strategic communication by informed agents with partisan bias is relatively well understood since Crawford and Sobel (1982), typically professionals are not interested in the decision made per se. Beyond the seminal investigation of Scharfstein and Stein (1990), little is known of cases where the adviser is simply an expert by profession, subject only to the implicit incentives provided by the labor market.

Following the lead of Holmström (1982), we assume that experts care about their future prospects as summarized by the reputation that they are able. More able experts have access to a more informative signal structure (“experiment”) in the sense of Blackwell. Specifically, an expert receives a continuous signal of ability-dependent precision about a continuum of states. The expert is then asked to reveal such information to two receivers (a decision maker and an evaluator), in a setting where no proof can be given to substantiate the claim. The expert’s payoff depends on the evaluator’s updated belief on expertise, whilst the decision maker’s payoff depends on an action taken under uncertainty about the state of the world.

This is a cheap-talk model, since the expert does not bear a direct cost from message sent, but only cares about the receiver’s response consisting in the evaluation of her ability. Some information can possibly be communicated in equilibrium when senders with different information rank differently the receiver’s evaluation of ability following the various messages sent. For this to be the case, in our model it is necessary that the evaluator receives ex post some additional information about the state. We assume with Scharfstein and Stein (1990) that the state of the world is eventually revealed regardless of the message sent or the decision taken. The evaluator can therefore rely on the message sent as well as on the realized state of the world to update the belief on the expert’s ability.

Contrary to naive intuition, forecasters who wish to be perceived accurate will not truthfully reveal their private information. Imagine a truthtelling equilibrium, whereby the signal of an expert can be inferred from the message reported. Typically, the expert would then wish to lie and claim to possess a signal which gives a higher expected posterior

¹See “Dustmen as Economic Gurus” (*The Economist*, 3 June 1995) on the good performance of a sample of London dustmen as forecasters of key economic variables.

reputational value. In the special case where the signal received does not convey any information on the expert's own ability, there is an incentive to bias the report towards the prior belief. More generally, the signal which guarantees the highest expected reputational payoff against naive beliefs is not the one privately possessed by the expert. Truth-telling cannot be sustained in equilibrium, so that professional advice cannot be taken at face value.

Similarly to the findings of Crawford and Sobel (1982) in the partisan model, communication has a coarse nature also in a reputational setting. The endogenous coarseness of the reputational cheap-talk equilibrium, independently observed also by Campbell (1998), is the point of departure of our analysis. In our specification of the model the most informative equilibrium is either binary, whereby the sender can communicate roughly the direction of her information but cannot accurately convey its intensity, or it is completely uninformative. In either case, a reported message pools many signals, and is therefore far less precise than the sender's true signal. The report not only garbles the information about the state of the world, but also the information about the expert's true ability.

The reputational cheap-talk framework has been fruitfully applied to decision making and recommendations in the financial industry. See Ehrbeck and Waldmann (1996) for an empirical investigation on forecasters of 3-month U.S. Treasury bills interest rates, Lamont (1995) on macroeconomic forecasters, Graham (1999) on investment newsletters, Welch (1996) and Hong, Kubik and Solomon (1998) on security analysts, and Chevalier and Ellison (1998) on mutual fund managers. See Hirshleifer (1993) for an extensive review of the literature on managerial reputation and corporate investment decisions.

Our model can be also applied to information revelation and decision making by politicians. If more competent politicians are more likely to be re-elected, politicians have the same incentives as our professional advisers. Similarly, the reputational objective is natural when modeling conversation among people who have common preferences over the different alternatives or have a negligible effect on the final decision to be taken. See Ottaviani and Sørensen (1998) for a model of political debate among heterogeneous experts motivated by their reputation as good forecasters.

The paper is organized as follows. Section 2 sets up our baseline model where the sender and the receiver share the same prior belief about the sender's true ability. Section 3 shows that such an expert cannot reveal any information in equilibrium when the prior on the state of the world is sufficiently concentrated, but will otherwise be able to transmit one of two messages. Extensions of the model are contained in Section 4. As shown in Section 4.1, the expert cannot communicate any information when the evaluator does not receive any external information on the state. Section 4.2 shows that when the expert knows privately her own ability type, it is always possible to sustain a binary informative equilibrium. In Section 4.3 we investigate whether competition resulting from relative reputational

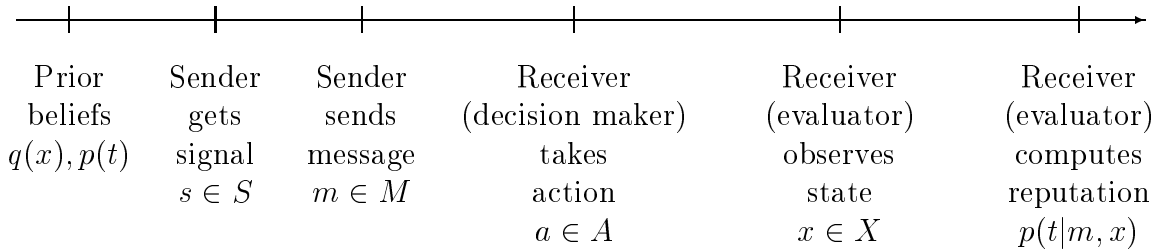


Figure 1: Time line for the model. The conditional density of the signal is $f(s|x, t)$. The decision maker’s payoff is $u(a, x)$, while the sender’s payoff is $\int_T v(t)p(t|m, x) dt$.

concerns can solve the communication difficulties. We identify a natural setting where this is not the case. A systematic review of the related literature is postponed to Section 5. Section 6 concludes. All proofs are collected in the Appendix.

2. Model

Setup and Timing. The sender of ability (or talent) type $t \in T$ privately receives an informative signal $s \in S$ on the state of the world $x \in X$ according to the conditional probability density function (p.d.f.) $f(s|x, t)$ specified below. Assume x and t are statistically independent, with common prior belief $q(x)$ on state and $p(t)$ on ability. Until Section 4.2, assume that the sender does not know her own ability type t . After observation of the non-provable signal, the sender decides which message $m \in M$ to send. The message space is as rich as the signal space, $M = S$. The message is observed by two receivers. The first receiver (the decision maker) updates the prior belief about the state x and decides which action $a \in A$ to take.² The second receiver (the evaluator) further observes the eventual realization of the state x , and uses both message and state realization to evaluate the sender’s expertise.³ The labor market is usually thought to perform this evaluation task.

Preferences. The decision maker’s preferences are represented by a von Neumann-Morgenstern utility $u(a, x)$. The evaluator computes the posterior reputation of the sender

²The decision maker has only a passive role in our game, being negatively affected if the message is less than fully informative. Furthermore, it is not necessary to assume that the decision maker be a separate party. For instance, the decision maker might be identified with the evaluator or with the sender. Scharfstein and Stein (1990) consider the latter case, where the decision taken on behalf of the principal also serves as message for the labor market.

³Notice that we depart from Holmström (1982) and Holmström and Ricart i Costa (1986) by assuming that the state x is observed regardless of the message sent.

$p(t|m, x)$.⁴ The sender's preferences over posterior reputations are represented by the increasing von Neumann-Morgenstern utility function $v(t)$. The sender's aim is to maximize $Ev(t)$ where the expectation is taken with respect to the evaluator's equilibrium posterior belief about t .⁵ The expected reputational payoff from message m for a sender with signal s is

$$V(m|s) = \int_X \int_T v(t)p(t|m, x)q(x|s) dt dx.$$

Information. Conditionally on the state $x \in X = [-1, 1]$ and ability type $t \in T = [0, 1]$, the signal is distributed according to the density

$$f(s|x, t) = \frac{1}{2}(1 + stx), \tag{2.1}$$

with $s \in S = [-1, 1]$.

We have not seen this “linear” statistical model formulated anywhere else, but the typical symmetric binary model has the same generalized density (2.1), with $S = X = \{-1, 1\}$ and $T = \{\underline{t}, \bar{t}\}$, where $0 \leq \underline{t} < \bar{t} \leq 1$. That is the model studied by Scharfstein and Stein (1990) and Trueman (1994). The linear model is a natural generalization of such a binary-signal model to allow for a continuum of states, signals, and ability types in a tractable way. It has two important ordering properties. First, $f(s|x, t)$ satisfies the monotone likelihood ratio property (MLRP) in s, x for any value of $t > 0$: The likelihood ratio $f(s|x, t)/f(s|x', t)$ is increasing in s for $x > x'$. Clearly, also $f(s|x) = f(s|x, Et)$ satisfies the MLRP. In other words, high s is good news for high x . Second, t appropriately parameterizes expertise, since the signal distribution for a lower type is a garbling of the distribution for a higher type. Specifically, consider the garbling $g(s|s) = p$ and $g(-s|s) = 1 - p$ for $p \in [1/2, 1)$. If s is distributed with density $f(s|x, t_1)$ we see that the garbled signal has density

$$pf(s|x, t_1) + (1 - p)f(-s|x, t_1) = \frac{1}{2}(1 + s[pt_1 - (1 - p)t_1]x) = f(s|x, t_2)$$

where $t_2 = t_1(2p - 1) < t_1$. Finally, this signal structure is boundedly informative about the state (the only exception to this being for $t = 1$, when signals $s = \pm 1$ rule out states $x = \mp s$ respectively).

⁴It can be useful to think of the evaluator as being rewarded for predicting as accurately as possible the ability t of the expert based on the information (m, x) available.

⁵The payoff to the sender depends entirely on the receiver's belief and may be intangible. Our formulation encompasses the case of a professional politician who derives a private benefit from being reappointed by an electorate assessing her competence. The payoff is tangible if, as in Holmström (1982), the increasing function $v(t)$ is derived from the value of the services provided in a future second and last period by the expert, when the expert would reveal truthfully her signal. The payoff to the decision maker in the second period would depend on the posterior distribution on ability. In particular, any first-order stochastic shift in such distribution would result in higher payoff for any decision maker.

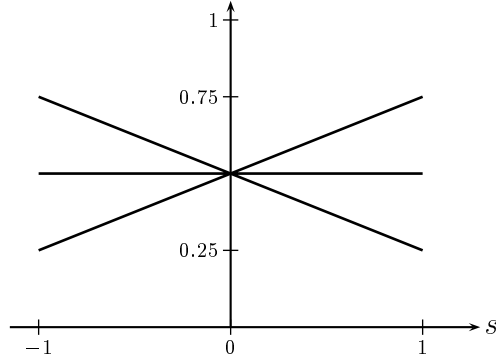


Figure 2: Graphs of the conditional densities $f(s|x, t) = (1 + stx)/2$ for fixed $t = 1/2$ and three values of $x = -1, 0, 1$. The downward sloping line corresponds to the case with $x = -1$, the flat one to $x = 0$, and the increasing one to $x = 1$. Intermediate values of x would give intermediate lines. Each line (other than the one corresponding to $x = 0$) becomes steeper as t increases.

Bayesian Updating and Reputational Payoff. With a slight abuse of notation, m denotes not only a message but also the (measurable) set of signals at which the expert sends that message. We write

$$f(m|x, t) = \int_m f(s|x, t) ds = \frac{1 + E_m[s]tx}{2} \lambda(m), \quad (2.2)$$

where $\lambda(m) = \int_m ds$, and $E_m[s] = \int_m s ds / \lambda(m)$ is the arithmetic mean of s over the set m . Similarly, $f(m|x) = \int_T f(m|x, t)p(t) dt$. The posterior belief of the receiver on the ability of the sender after receiving message m and observing the realized state x is

$$p(t|m, x) = \frac{f(m|x, t)}{f(m|x)} p(t) = \frac{1 + xtE_m[s]}{1 + xE_m[s]Et} p(t). \quad (2.3)$$

The reputational payoff of message m in state x is

$$W(m|x) \equiv \int_T v(t)p(t|m, x) dt. \quad (2.4)$$

For a sender with signal s the posterior belief on the state is

$$q(x|s) = \frac{f(s|x)}{f(s)} q(x) = \frac{1 + sxEt}{1 + sExEt} q(x), \quad (2.5)$$

and the expected reputational payoff of message m is

$$V(m|s) = \int_X W(m|x)q(x|s) dx. \quad (2.6)$$

3. Equilibrium

We first demonstrate the impossibility of a fully separating equilibrium in Section 3.1. Next, we characterize the equilibria. There can be only partition equilibria with endogenously coarse communication (Section 3.2) and the only informative equilibria are binary and typically multiple (Section 3.3). There is no informative equilibrium at all when the prior belief on the state is sufficiently concentrated. In Section 3.4 we discuss some empirical implications of this model.

3.1. Endogenous Coarseness: Why is Truthtelling not an Equilibrium?

We now argue that truthtelling cannot be an equilibrium in this model. In a fully separating equilibrium $E_m[s] = m$. When the receiver believes that messages truthfully reflect the signals, the expert can pretend to have received any signal. Maximizing (2.6) over m we obtain the first-order condition

$$V'_m(m|s) = 0. \tag{3.1}$$

for an expert with signal s to wish to send signal m . Without loss of generality, in a truthtelling equilibrium $m = s$. Simple calculations (in the Appendix) show that the ideal signal which the expert wishes to send in general differs from the observed signal.

Result 1 (No Truthtelling) *Truthtelling is not an equilibrium.*

The intuition is most transparent in the special case with $p(t|s) = p(t)$ for all s , so that the signal is uninformative about ability.⁶ In the linear model this happens when the prior on the state is such that $Ex = 0$ (a symmetric prior $q(\cdot)$ around $x = 0$ guarantees this). Any sender who reports truthfully is expecting the same value Ev . A sender with signal $s > 0$ has incentive to pretend to have a signal $s' \in (0, s)$, in between the neutral signal 0 (which results in a posterior distribution on the state equal to the prior, $q(x|s = 0) = q(x)$) and the actual signal possessed s . Since $s' > 0$, higher realizations of x are better news about ability when s' has been reported. Since $s > s'$, the sender with s believes more in higher realizations of x than does the sender with s' . Therefore, the sender with s has a higher expected reputational payoff from reporting such s' compared to that of the sender with signal s' , itself equal to the truthtelling value Ev . This result parallels Proposition 3.1 of Campbell (1998):

⁶This assumption was made by Scharstein and Stein (1990) and is maintained in Campbell (1998). Since in this case the sender does not learn anything about own ability by observing the signal, the message sent cannot signal any such knowledge. This assumption amounts to a degenerate restriction on the set of prior beliefs on the state.

Result 2 (Best Deviation) *When the signal is uninformative about ability ($p(t|s) = p(t)$ for all s , i.e. when $Ex = 0$), the best deviation against naive beliefs is to report a different signal s' in between the neutral signal 0 and the signal possessed s .*

If the message sent were verifiable and explicit incentives were allowed, truth-telling could be implemented by offering the reward schedule $R(m) = \int_{-1}^m V'_m(m'|m') dm'$ to the expert. This is seen immediately from equation (3.1). Correspondingly, the ex-ante cost of implementing truth-telling would be $\int_S R(s) f(s) ds$. Notice that the cost could be lower if the reward were allowed to depend also on the realization of the state. For the rest of this paper we exclude the possibility of monetary incentives.

3.2. Partition Equilibria

Having ruled out perfect separation as an equilibrium outcome, we now show that equilibria have a partition structure whereby connected sets of signals are pooled. This result relies on this single-crossing property of the cumulative distribution function P of the posterior belief on t :

$$\frac{\partial[P(t|s', x) - P(t|s, x)]}{\partial x} < 0 \text{ for all } t \text{ for } s' > s, \quad (3.2)$$

verified in the Appendix. According to this condition, higher signals are better news for ability the higher is the state of the world.

Consider two possible messages, m and m' where m' is higher than m in the sense that $E_{m'}[s] > E_m[s]$. Then (3.2) implies that $P(t|m', x) - P(t|m, x)$ is decreasing in x for all t . Thus a high message yields a better reputation, the higher the state. Since experts with higher signals believe more in higher states, we can establish the following monotonicity property:

Result 3 (Monotonicity) *$V(m'|s) - V(m|s)$ is increasing in s when $E_{m'}[s] > E_m[s]$.*

Incentive compatibility implies that if two messages have unequal arithmetic centers, $E_{m'}[s] > E_m[s]$, all expert types who send message m' have higher signals than the types who send m . This implies that:

Proposition 1 (Partitions) *All (Perfect Bayesian) equilibria are essentially connected partition equilibria.⁷*

⁷What does essentially mean? The only possibility for two messages sent over sets m and m' not ranked against each other is that the two sets have $E_m[s] = E_{m'}[s]$. In that case we note that the two corresponding messages, m and m' convey equal information about x, t . Indeed, $f(m|t, x) = f(m'|t, x) = (1 + E_m[s]tx)/2$ for all t, x . Thus the two messages might as well be pooled into one message. Conversely, given any partition equilibrium, there exist an infinity of essentially equivalent equilibria, whereby each set m of the equilibrium partition is further subdivided into sets with the same arithmetic mean $E_m[s]$.

As is typical in cheap-talk games, there exists always a completely uninformative (pooling or babbling) equilibrium. If the evaluator expects all messages to be uninformative, the senders have no choice but to pool on the relevant messages. Rather than discussing equilibrium selection, we characterize the set of all Perfect Bayesian equilibria in our reputational cheap-talk games.

3.3. Binary Equilibria

Binary equilibria have one message sent for $s \geq b$ and another for $s < b$, where $b \in (-1, 1)$. Our crucial finding in this model is that all informative equilibria are binary. The proof proceeds by contradiction. Suppose that more than two messages were sent in equilibrium. An expert with a signal at the border between two adjacent messages must be indifferent between them. The contradiction follows from the fact that the two indifference conditions for the extremes of an intermediate message are incompatible.

Proposition 2 (Necessity of Binary Equilibria) *All informative equilibria are binary.*

Next, we characterize binary equilibria. Let m denote the message sent for $s \in [-1, b]$ and m' the message sent in $(b, 1]$, with $-1 < b < 1$. We must have $V(m|b) = V(m'|b)$, which we rewrite with (2.6), using $E[v(t)|t] - E[v(t)]Et > 0$ (since v is increasing), as

$$\int_X \frac{x(1 + bxEt)}{[2 + (b-1)xEt][2 + (b+1)xEt]} q(x) dx = 0, \quad (3.3)$$

or equivalently

$$\int_X \frac{q(x|b)}{f(m'|x)} dx = \int_X \frac{q(x|b)}{f(m|x)} dx. \quad (3.4)$$

Equation (3.4) effectively characterizes binary equilibria. In any binary equilibrium, the threshold signal b must solve the equation. If some b solves the equation, it is a binary equilibrium to let the messages be $[-1, b]$ and $(b, 1]$. It is a peculiar property of the linear specification that the value function does not enter this indifference equation. We only relied on the property that v is increasing.

In the following, we first identify a prominent instance where a binary equilibrium exists. Then we argue that the number of binary equilibria is typically even. Finally, we find robust “herding” conditions under which no binary equilibrium exists.

When is there a symmetric equilibrium with $b = 0$? Inserting in (3.3) we find

$$\int_X \frac{x}{(2 - xEt)(2 + xEt)} q(x) dx = 0.$$

As the function $x/[4 - x^2(Et)^2]$ is anti-symmetric around 0, we conclude that the symmetric equilibrium exists when the distribution of x is symmetric around 0.

Proposition 3 (Existence of Binary Equilibrium) *When the prior on x is symmetric around 0, there is a symmetric binary equilibrium with messages $[-1, 0]$ and $[0, 1]$.*

Why is the sender willing to conform with messages $[-1, 0]$ and $[0, 1]$ in this case? Message $[-1, 0]$ gives a favorable posterior reputation when combined with observation of a negative state, and an unfavorable reputation when the state x turns out positive. Vice versa for message $[0, 1]$. The sender learns nothing about the state when receiving the neutral signal $s = 0$, so that with the symmetric prior and posterior on the state it looks equally attractive to send either the high or the low message. A positive signal indicates instead that positive states are more likely, making message $[0, 1]$ more attractive than $[-1, 0]$. Similarly, a negative signal makes $[-1, 0]$ look best.

More generally, it is difficult to characterize when (3.3) can be solved. At $b = -1$ and $b = 1$ the left-hand side of (3.3) is equal to $Ex/4$. The integral varies continuously with b , but this is not enough to prove existence of a b which solves (3.3) when $Ex \neq 0$. Nevertheless, this proves that the number of binary equilibria must be even (possibly zero) when $Ex \neq 0$:

Proposition 4 (Even Number of Binary Equilibria) *Generically in the prior belief, the number of binary equilibria is even.*

Finally, assume that the prior is highly concentrated near some $x > 0$. Any signal s is of bounded informativeness about states of the world, so the posterior $q(x|s)$ will still be concentrated around x . Whenever the state turns out positive, it is favorable to the expert's reputation to have reported a message with $E_m[s] > 0$. If it were possible to send a message with $E_m[s] > 0$, the expert would want to send this message, regardless of the signal s actually received. This cannot hold in equilibrium, since $E_m[s] = 0$ when all signals are pooled into one message:

Proposition 5 (No Informative Equilibrium) *When the prior distribution on the state is sufficiently concentrated on any $x \neq 0$, there is no informative equilibrium.*

Note that this result does not hold for $x = 0$, since Proposition 3 shows existence of equilibrium for (symmetric) priors arbitrarily concentrated on 0. Yet, whenever x turns out very close to 0, the messages convey very little information about ability.

Proposition 5 is an important building block in a model of herding forecasters. Experts who give advice in sequence, learn about the state of the world by listening to each others' recommendations. As more experts speak informatively, the beliefs of later experts become ever more concentrated at the true state x by the law of large numbers. According to Proposition 5, experts eventually stop being informative and learning grinds to a halt. See

Smith and Sørensen (1999) for a full exposition of this logic in the statistical rather than reputational model. Notice that here herding obtains as in Bikhchandani, Hirshleifer and Welch (1992), without any need for conditionally dependent signals across experts.

We have found that the most informative equilibrium is either binary or even completely uninformative. Since reported messages pool many signals, they are far less precise than the sender's true signal. The sender can communicate at most the direction of her information but cannot convey its intensity. There is pooling on the intensity dimension, since experts would always want to pretend to have more precise information. Rationality of the evaluator makes this incentive self defeating.

This is bad news for the decision maker who relies on the information about the state contained in message m to take action a . The resulting welfare loss of the decision maker is equal to $Eu(a(m), x|m) - Eu(a(s), x|s)$. By its own nature, the market evaluation of ability is accurate but introduces myopia in the model. Future employers of the expert are probably interested in learning as well as possible the true ability of the expert, so they would also have preferred less garbling of the signal. Finally, it will depend on convexity properties of $v(t)$ whether the expert herself benefits from market learning about her ability. If v were convex, all parties are hurt by the fact that information transmitted in equilibrium is coarser than the information possessed by the expert.

Example: Binary State of the World. Assume that the prior distribution of x is concentrated on -1 and $+1$, with q being the prior probability of state $+1$. Now (3.3) can be re-written as

$$\frac{q}{1-q} = \frac{1 - bEt/2 + (b-1)Et/2 + (b+1)Et}{1 + bEt/2 - (b-1)Et/2 - (b+1)Et}, \quad (3.5)$$

a third order polynomial equation in b . When $q = 1/2$, $b = 0$ is an equilibrium. The equation has one solution in $(-1, 1)$ when $q = 1/2$, two solutions in $(-1, 1)$ when q is sufficiently close to (but not equal to) $1/2$, one solution when q takes on a knife-edge value above (or below) $1/2$, and no solution in $(-1, 1)$ whenever q is beyond these values.

Equation (3.5) must have a solution for $b \in (-1, 1)$ in order for the expert to be credible. Let $g(b, Et)$ denote the right hand side of (3.5). Simple algebra proves that: $g(b, 0) = 1$ for any b ; $g(-1, Et) = g(0, Et) = g(1, Et) = 1$ for any Et ; $g(b, Et)$ is decreasing in Et for $b \in (-1, 0)$ and increasing in Et for $b \in (0, 1)$. This fact allows us to conclude that the range of values assumed by g (by varying b) is wider the larger is Et . Therefore, if (3.5) has a solution with $b \in (-1, 1)$ for some given Et , then it also has a solution for all smaller values of Et . This means that the better is the prior reputation of the expert (the larger is Et), the larger is the set of prior beliefs q on the state such that the expert can speak credibly. Better experts can speak informatively under more circumstances.

According to Proposition 5 there is a wide set of prior beliefs where no expert can speak

credibly, no matter how good is her prior reputation. The most extreme values taken by $g(b, Et)$ when $b \in [-1, 1]$ and $Et \in [0, 1]$ appear when $Et = 1$. Then $g(b, 1) = (3+b)/(3-b)$ which can vary (through b) from $1/2$ to 2 . Therefore, if $q < 1/3$ or $q > 2/3$ there can be no solution to (3.5) regardless of Et .

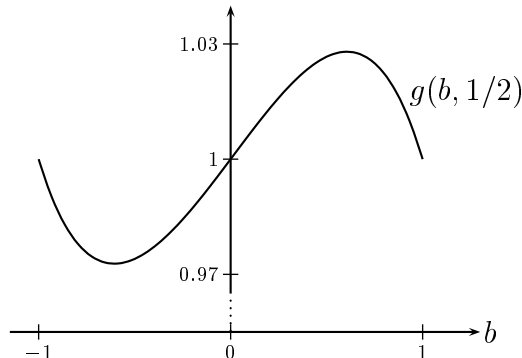


Figure 3: Graph of the right hand side of (3.5) when $Et = 1/2$, i.e. of $g(b, 1/2)$.

3.4. Predictions

For the application of this model to the predictions of professional forecasters, we need a clarification on how information is communicated. In equilibrium the receiver understands that signals in a certain interval ($s \in m$) are pooled on the same message by the sender. This message could be any signal from m , or senders might even be randomizing over signals. Given the arbitrariness of the cheap-talk language, an empirical comparison of the forecasters' literal statements with the outcome of the forecasted variable x is impossible.

Nevertheless, the receiver's beliefs $f(x|m)$ as well as the optimal decision $a(m) = \arg \max_a Eu(a, x|m)$ are unambiguously determined in equilibrium. The natural language of forecasters dictates them to communicate this belief (or its mean $E[x|m]$) or to recommend the action $a(m)$. Alternatively, under delegation the action taken serves as the message. Statements in such languages can be easily compared with the realized state.

According to our findings, the message sent is a strong coarsening of the signal. The belief $f(x|m)$ is unbiased in our model, being derived from Bayesian updating, but it is less accurate than the forecaster's private belief $f(x|s)$. If many experts are polled simultaneously, their forecasts will be very similar, concentrated on at most two different positions. Moreover, if their forecasts were replicated using knowledge of the economy, they would appear surprisingly inaccurate, since the replication gives the more informative $f(x|s)$. It would also seem that their forecast errors were excessively correlated. The problem is that the forecasters do not say what they think.⁸

⁸There is a large empirical literature on economic forecasters. See Zarnowitz (1967) for an early empirical study of the precision of forecasts and Keane and Runkle (1998) for a recent contribution on testing the rationality of forecasters.

4. Extensions

4.1. Maximizing Interim Reputation

In our baseline model we have assumed that the evaluator observes the state of the world. Consider now the case where the evaluator only observes the message, without any information on the realization of the state of the world. Analogously to the canonical cheap-talk model of Crawford and Sobel (1982), the receiver's evaluation action is based exclusively on the message reported by the sender. We find that the only equilibrium is pooling. The indifference condition in a reputational model implies that different messages cannot reveal different information about t . In the linear (as well as in the binary) model, no information about t implies no information about x , so that:

Proposition 6 (Interim Reputation) *In the interim reputation model where $Ex \neq 0$ there is no informative equilibrium, even allowing for mixed strategies.*

This case is also a continuous-signal reputational-objective analogue of Brandenburger and Polak's (1996) model. In their binary-signal model there is no pure-strategy informative equilibrium other than for a degenerate prior on the state (their Proposition 1), but there is an informative mixed-strategy equilibrium for a set of non-degenerate priors on the state (Proposition 2). Their mixed strategy equilibrium has the property that all messages are equally attractive to the sender. Yet, their messages convey some information about the state of the world, something impossible in our reputational model.

We conclude that for any information at all to be possibly communicated in equilibrium it is necessary that the evaluator receives some information about the state on top of the message sent by the adviser. Some sorting is necessary in signalling games for messages to be credible. In a similar variation of the model, the state x would be commonly known by both sender and receiver before the message was sent. Again, there would be no sorting and therefore no information about ability could be transmitted in equilibrium. In our baseline model we have considered the case where the state is observed by the evaluator, and this gave us the sorting condition (3.2).

4.2. Equilibrium with Known Own Ability

In our basic model, the expert receives a signal s which is possibly more informative about ability than the message m submitted in equilibrium. In non-trivial dynamic extensions, there would therefore be asymmetric information on ability between the sender and the receivers. We relax the common prior on ability by allowing the expert to know perfectly her own type, as first done by Trueman (1994). His analysis can be easily adapted to show that in a binary model there exists always an informative equilibrium, which often involves

some randomization by the least able of the two types. Clearly, coarseness of equilibrium is not an issue in his model with two signals and two ability types.⁹

Consider now an expert who privately knows not only the signal realization s but also her own ability type t . To avoid arbitrary restrictions on the message space we now allow $M = S \times T$. An expert of ability t who receives signal s has posterior on the state

$$q(x|s, t) = \frac{f(s|x, t)}{f(s|t)}q(x) = \frac{1 + stx}{1 + stEx}q(x). \quad (4.1)$$

Let m_t denote the subset of signals which go into message m for type t . Upon observation of message m and state x , the evaluator will arrive at the posterior reputation

$$p(t|m, x) = \frac{f(m_t|x, t)}{f(m|x)}p(t).$$

The expected reputational payoff of message m for a sender with signal s and known ability t is

$$V(m|s, t) \equiv \int_X \int_T v(t')p(t'|m, x) dt' q(x|s, t) dx. \quad (4.2)$$

Notice that we are assuming that value functions are not ability dependent. This is a strong assumption, because an expert with private information on her own ability knows better than the market how her reputation will be updated in later periods. In a full dynamic model an expert's prospects of future earnings would then depend on ability.

The problem is one of multi-dimensional signaling. Notice from (4.1) that all signal-ability type combinations with $st = k$ constant result in the same posterior belief on the state

$$q(x|s, t = k/s) = \frac{1 + kx}{1 + kEx}q(x). \quad (4.3)$$

The rectangular hyperbola $t = k/s$ represents such iso-posterior locus in the space $S \times T = [-1, 1] \times [0, 1]$. See Figure 4 for a map of iso-posterior curves.

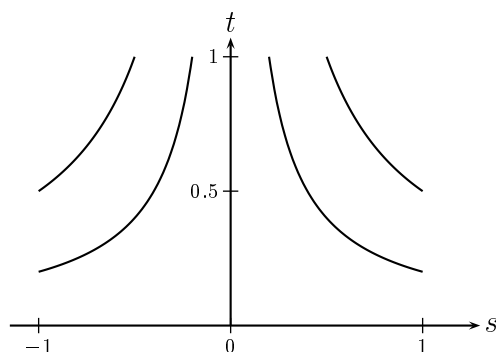


Figure 4: Iso-posterior curves with $k = -1/2$, $k = -1/5$, $k = 1/5$, and $k = 1/2$.

⁹Along these lines, Levy (1998) constructs a model with binary signals and states, and a continuum of ability types, but with a mixed reputational objective. Despite dealing with a continuum of types, Levy restricts exogenously the message (action in her language) space to be binary. Our analysis shows that with a pure reputational objective the coarseness of the message space arises endogenously.

We focus on equilibria where the threshold of indifference between a message and another are iso-posterior curves. In a binary equilibrium message m is sent for $-1/t \leq s \leq k/t$ and message m' is sent for $k/t \leq s \leq 1/t$. In contrast to the case when the expert does not know her ability, there is an informative equilibrium for *any* prior belief on the state.

Proposition 7 (Known Own Ability) *When the expert knows her own ability, there exists always a binary informative equilibrium.*

Why is there necessarily an informative equilibrium when the expert knows her own type? If some message were sent exclusively by the highest possible type, then it would give the highest possible reputation, regardless of the realized state of the world. More able experts are more confident in their prediction of the state of the world. As the prior belief on the state becomes very skewed, only the strongest types have the self-confidence required to send a message opposite to the prior. If a sufficiently small set of good types are sending the message, they signal that they are good and thereby secure a good minimum reputation, even when the state of the world turns out against them.

Finally, in a simple example (with $v(t) = t$ and $p(t)$ uniform on $[0, 1]$) there is only one equilibrium within the binary class, in contrast with the typical multiplicity of binary equilibria found in the case of unknown ability. Furthermore, with binary state and fair prior it can be verified that there are no symmetric iso-posterior equilibria with three messages.

4.3. Multiple Experts and Relative Reputational Concerns

Can competition between experts affect the amount of information credibly communicated? For instance, full information revelation results in equilibrium when consulting simultaneously multiple perfectly informed experts in the partisan cheap-talk model of Crawford and Sobel (1982). Consider instead multiple professional experts with conditionally independent signals. If they simultaneously report their messages and care only about their own reputation, the equilibrium is the same as in the single-expert model.¹⁰

In our reputational setting it is quite natural to allow for *relative* performance evaluation. Often, the market rewards those with better reputation more if they are more scarce. For instance, Laster, Bennett and Geoum (1999) build a model where economists who share the same noisy information are asked to pick one of a discrete number of forecasts. They assume that the prize of best forecaster is shared among those who are on the

¹⁰Krishna and Morgan (1999) investigate the case where partisan experts speak sequentially. Likewise, Ottaviani and Sørensen (1998) and Levy (1998) model sequential consultation of professional experts with absolute reputational concerns.

mark, so that arbitrage among forecasts leads to equilibrium differentiation. The same logic which drives their result could lead one to expect that in our setting more differentiation (and perhaps more information revelation in our model with privately information experts) would result when a concern for relative reputation is introduced. We now show that this is not the case when reputational preferences have a von Neumann-Morgenstern representation and experts have conditionally independent signals.

Consider this model where experts $i = 1, \dots, N$ report simultaneously. With relative reputational concerns, the von Neumann-Morgenstern payoff $u^i(t^i, t^{-i})$ to expert i depends also on the ability of all other experts $t^{-i} \equiv (t^1, \dots, t^{i-1}, t^{i+1}, \dots, t^N)$. We maintain the assumption that u^i is increasing in t^i , but it might well be decreasing in t^j for $j \neq i$. We make the natural assumption that the state of the world and the ability types of the experts are independently distributed. Conforming with the standard case of information economics, we further assume that the noisy signals of experts are *independent* conditionally on the state and ability draws. Notice that in this way we depart from the route taken by a large part of the reputational herding literature since Scharfstein and Stein (1990), who assumed that better managers have more correlated signals conditional on the state of the world.

The independence assumptions implies stochastic independence of posterior reputations of different experts updated after the reports and observation of the state of the world. Moreover, only an expert's own message (and the state of the world) influences the updating of the reputation of that expert. According to the martingale property of updated Bayesian beliefs, the expected posterior reputations of other experts equal the prior reputations. Finally, the von Neumann-Morgenstern payoff is linear in those beliefs. Thus we have arrived at the next theorem (from Ottaviani and Sørensen (1999)) which does not rest on our functional assumptions about $f(s|x, t)$, and which also holds if each expert privately knows her own ability:

Theorem 1 (Irrelevance of Relative Reputation) *Assume that the experts have von Neumann-Morgenstern payoffs, and that their signals are independent conditionally on state and ability. In equilibrium of the relative reputation model, expert i behaves as in the absolute reputation model with increasing value function*

$$v^i(t^i) = E_{t^{-i}}[u^i(t^i, t^{-i})]$$

where the expectation is taken with respect to the prior belief $p(t^{-i})$.

According to this theorem, in order to generate new and interesting results a relative reputations model must either assume that there is correlation of experts' signals conditionally on the state and ability draw or give up the von Neumann-Morgenstern formulation.

For an investigation of relative reputational concerns in a binary model with conditionally correlated signals see Effinger and Polborn (1998).¹¹ Notice that the von Neumann-Morgenstern formulation is rather restrictive in this setting. Naturally, the market might instead reward experts on the basis of a comparison of some summary statistics of their updated reputation. For instance, in analogy with Laster et al.'s (1999) winner-take-all specification, the expert with highest expected ability could receive all the rewards. This cannot be modeled with von Neumann-Morgenstern payoffs.

Back to the model with our functional assumption (2.1), we can conclude that the equilibrium behavior of each expert is unaffected by the introduction of relative reputation evaluation. This follows from the special property of the linear model that the equilibrium is invariant to monotonic transformations of the value function, as seen in equation (3.4).¹²

Proposition 8 (Equivalence of Equilibrium with Relative Reputation) *In the linear model with conditional independence across experts, the equilibrium in the relative reputation model is the same as in the absolute reputation model.*

5. Related Literature

In the cheap-talk model of Crawford and Sobel (1982) a privately-informed sender is interested in influencing the decision taken by a receiver. In contrast with the case of delegation considered by Holmström (1977), in cheap talk the receiver cannot commit to take any decision other than the one which is ex-post optimal given the information communicated by the sender. Communication is possible whenever the sender and the decision maker have sufficiently similar preferences. Equilibria are endogenously coarse, in the sense that messages sent pool intervals of signals. While in Crawford and Sobel's model the sender is always better informed than the receiver, in our setting the evaluator observe an additional signal (the realization of the state) before taking the action (evaluation of the sender). This feature makes our model fundamentally different from theirs. The partition structure of the equilibria in our professional model is similar to their finding in the partisan setting, but it is driven by different forces. In our baseline model, we have also found that communication cannot be finer than binary. This mirrors the fact that there is

¹¹A rather different issue is that of benchmarking and accuracy of the market's inference on managerial ability, addressed by Zwiebel (1995). In his model ability does not index the quality of information but the return from the investment available to a manager, as in Holmström and Ricart i Costa (1986). Assuming that the manager knows her own ability, Zwiebel considers a number of situations where managers of high and low ability invest in high return projects, while managers of intermediate ability opt for less profitable but more commonly adopted ones. Less profitable projects have the advantage of providing a more accurate benchmark for the market's evaluation of ability.

¹²This proposition applies only to the case of initial symmetric information about the expert's ability. When the expert has private information about ability, the equilibrium depends instead on the value function.

no natural notion of closeness between our expert’s objective and the receiver’s evaluation objective.¹³

To the best of our knowledge, the first formulation of a reputational cheap talk model where more able managers have access to more precise information about investment is contained in the second and less well known part of Holmström (1982), Section III. He considered two situations where the state of the world is realized only if the investment is undertaken and assumed that the manager does not know her own ability. In his first example, a risk averse manager prefers not to invest in order to shield herself from the risk associated with learning about ability which would follow the investment. In his second example, the manager is risk neutral and receives a non-verifiable signal with a continuous level of precision, much like in our model. Once again, the only equilibrium is pooling.

In their seminal investigation of herd behavior by rational agents, Scharfstein and Stein (1990) have constructed a multi-agent dynamic model of reputational cheap talk. Departing from Holmström, they assumed that the state of the world is eventually realized regardless of the decision made by the manager.¹⁴ Signals, states, and ability types are all binary in their model, and they fix a prior on the state such that there exists an informative equilibrium. Ours is a continuous version of the one-agent game of their model, and we treat the prior on the state parametrically.

Scharfstein and Stein maintained Holmström’s assumption that the sender does not know her own ability type, while Trueman (1994) looked at the other polar case of an expert who knows her own ability type perfectly in a variation of the binary model. In his model there is always an equilibrium (in mixed strategies) where some information is revealed, regardless of the prior on the state. In the pure strategy equilibria of our continuous model we have found the same qualitative difference between the cases where the expert knows and does not know her own ability.

Campbell (1998) has independently obtained results similar to ours. His paper is cast in a financial setting with an information-gathering money manager and a portfolio-choosing investor. Abstracting from the application, his model features two states of the world and two ability types and a continuum of signals, with the degenerate restriction that

¹³Piketty and Spector (1996) and Banerjee and Somanathan (1997) take the alternative route of endowing agents with common preferences but different prior beliefs, because agents with common priors would have no reason to lie. We refer to Banks (1991) and Austen-Smith (1992) for an introduction on applications of signaling and cheap talk to politics.

¹⁴When applying the model to the selection of real investment opportunities it is reasonable to assume with Holmström that the state of the world (i.e. outcome of the investment) remains unknown if no investment is undertaken. When instead modeling financial investment, Scharfstein and Stein’s assumption that the realization becomes known regardless of the decision of the individual investor is more adequate. Finally, our Proposition 6 considers the equilibrium with a reputational version of Brandenburger and Polak (1996)’s objective of maximizing the stock-market price prevailing before the state of the world is realized.

the signal received be uninformative about the manager’s own ability. Our analysis in Section 3.1 shows that endogenous coarseness of the equilibrium (his second proposition) results without these additional assumptions. Campbell also obtains a binary equilibrium, but does not discuss whether it is unique (the focus of our analysis in Section 3.3).

Prendergast and Stole (1996) construct a model where the manager (expert) cares both about the reputation about ability and the payoff attained with the decision taken. While in the cheap-talk framework the action taken by the receiver is constrained to be ex-post optimal in equilibrium, they assume that the decision is delegated to the manager. Equivalently, the receiver commits to implement the decision recommended by the sender, though it is not ex-post optimal. The equilibrium in their model is fully separating because the decision is delegated to the informed manager who primarily cares about the payoff attained by the principal. Nevertheless, the decision taken is not ex-post optimal, but it is distorted because of the reputational motive. This equilibrium would clearly not survive in a cheap-talk framework, as the ex-post optimal decision for the receiver would not be compatible with signal-to-signal incentive compatibility.

Empirically, Graham (1996) investigates how well economists can predict the direction of economic growth. Laster, Bennett and Geoum (1999) build and test a model where economists are motivated by the accuracy of their prediction and the publicity gained when their forecast is exactly on the mark. Differentiation of forecasts results from the desire to be alone with the perfect forecast. While they simply posit that experts aim for an accurate forecast, we give a full microeconomic foundation to the objective of professional advisers.

Finally, in other cheap-talk models senders are interested in a reputation but not for forecasting ability. For instance, Sobel (1985) studies reputation building by an informed advisor who could have opposite interests to the decision maker. Morris (1997) explores the implications of reputational concerns for the advice of experts who could be either “good” (i.e. concerned about both the payoff of the decision maker and their reputation to be good) or “bad” (i.e. with a partisan bias for one action). We instead assume that experts are concerned about their perceived ability, updated according to their recommendation and the realization of the state.

6. Conclusion

In light of the recent empirical confirmations of the importance of career concerns, communication by reputationally driven experts deserves further investigation. Compared to the partisan expert case, the analysis is complicated by the necessary presence of multi-dimensional learning on state and ability. We have been able to overcome these problems by formulating a simple but special model which is amenable to manageable analysis. We

have found that experts who are motivated by their reputation as good forecasters can only provide recommendations which are much less accurate than the information they possess. The imprecise statements made are appropriately discounted by the audience. We believe that these robust findings can shed some light on some commonly observed phenomena in communication by non-partisan informed parties.

The exact amount of information which can be credibly conveyed depends on the nature of the information possessed by the expert. Here we have provided only partial answers to this general problem, but we believe to have raised some important questions: Under which conditions does truthtelling result? Is the crucial sorting condition (3.2) satisfied in more general statistical models? In a companion paper (1999) we analyze the equilibrium of reputational cheap-talk games with alternative signal structures. For instance, truthtelling is an equilibrium in a model with continuous state space where the expert receives a perfectly informative signal with positive probability. This is not true with finitely many states. We also show that in the normal learning model with known own ability there is always a binary equilibrium and we provide examples of models where more than two signals can be sent credibly.

Increasing specialization of labor suggests that information be collected by professional experts. While the implicit incentives provided by the market discipline their behavior, they also increase the scope for strategic behavior in the revelation of given level of information. How will optimally designed explicit incentives overcome these problems? A natural starting point to investigate the interaction between explicit and implicit incentives is provided by Holmström and Ricart i Costa (1986), who analyze the simpler setting where ability just adds to the value of the output produced. It would also be interesting to extend our model to allow the expert to become more informed by acquiring costly signals. The point of departure would be Osband's (1989) study of explicit incentives for truthtelling and information acquisition by forecasters in the absence of reputational concerns.

Appendix A: Proofs

Result 1 (No Truthtelling) Truthtelling is not an equilibrium.

Proof. (3.1) computed at $m = s$ is

$$\int_X \int_T v(t) \frac{f'_s(s|x, t) f(s|x) - f(s|x, t) f'_s(s|x)}{[f(s|x)]^2} p(t) \frac{f(s|x)}{f(s)} q(x) dt dx = 0$$

or equivalently

$$\int_X \int_T v(t) \frac{t - Et}{2} p(t) dt \frac{xq(x)}{1 + sxEt} dx = 0.$$

Notice that $E[v(t)t] > E[v(t)]Et$ when t does not have a degenerate distribution and v is strictly increasing, since t and $v(t)$ have positive covariance, so that truthtelling requires that

$$\int_X \frac{x}{1 + sxEt} q(x) dx = 0 \quad (\text{A.1})$$

holds for all s .

We now show that (A.1) can only hold at one value of s . Assume that (A.1) holds at two signals $s' > s$. Subtracting the two versions of equation (A.1) yields the contradiction

$$0 = \int_X \frac{(s' - s)x^2 Et}{[1 + s'xEt][1 + sxEt]} q(x) dx > 0,$$

where the strict inequality follows from the integrand being positive for all x . \square

Result 2 (Best Deviation) When the signal is uninformative about ability ($p(t|s) = p(t)$ for all s , i.e. when $Ex = 0$), the best deviation against naive beliefs is to report a different signal s' in between the neutral signal 0 and the signal possessed s .

Proof. Observe that any sender who reports truthfully has expected value Ev :

$$V(s|s) = \int_T v(t) \int_X p(t|s, x) q(x|s) dx dt = \int_T v(t) p(t|s) dt = Ev(t).$$

Now, fix $s > 0$ without loss of generality. We argue that the sender with s can profitably deviate to any signal $s' \in (0, s)$. Reporting s' gives the expected reputational value

$$V(s'|s) = \int_X \int_T v(t) p(t|s', x) dt q(x|s) dx$$

to be compared with the truthtelling value $V(s|s)$. We will argue that $V(s'|s) > V(s|s)$ for any $s' \in (0, s)$, as this proves the incentive to deviate from s to s' . Since $V(s'|s') = V(s|s)$, we can equivalently show $V(s'|s) > V(s'|s')$.

Our comparison rests on two facts. First, since $s > s'$, $q(x|s)$ first-order stochastically dominates $q(x|s')$. Second, with the positive signal $s' > 0$, the higher is the state of the world, the more favorable the updated reputation, so that

$$W(s'|x) = \int_T v(t) p(t|s', x) dt$$

is an increasing function of x . This follows from Milgrom's (1981) Proposition 1 because: x is good news for t when $s' > 0$,

$$\frac{p(t'|s', x')}{p(t|s', x')} = \frac{1 + s't'x'}{1 + s'tx'} \frac{p(t')}{p(t)} > \frac{1 + s't'x}{1 + s'tx} \frac{p(t')}{p(t)} = \frac{p(t'|s', x)}{p(t|s', x)}$$

for $t' > t$ and $x' > x$; and $v(t)$ is increasing. Combining the two facts we reach the desired

$$V(s'|s) = \int_X \int_T v(t)p(t|s', x) dtq(x|s) dx > \int_X \int_T v(t)p(t|s', x) dtq(x|s') dx = V(s'|s').$$

Finally, we show that deviating to any s' outside the interval $(0, s)$ is not profitable. First, for $s' \geq s$, the first fact is reversed, since s' is better news than s for x . This in turn reverses the final inequality, making the deviation unattractive. Second, consider $s' \leq 0$. The second fact above is reversed, as higher x is worse news about ability when $s' \leq 0$. We can conclude that the best deviation is to some $s' \in (0, s)$. \square

Verification of Property (3.2). Integrating the posterior belief on t conditional on s and x ,

$$p(t|s, x) = \frac{f(s|x, t)}{f(s|x)}p(t) = \frac{1 + sxt}{1 + sxEt}p(t), \quad (\text{A.2})$$

one gets

$$P(t|s, x) = \frac{1 + sxE[t'|t' \leq t]}{1 + sxEt}P(t),$$

so that

$$\frac{\partial^2}{\partial s \partial x} P(t|s, x) = \frac{1 - sxEt}{(1 + sxEt)^3} P(t) \{E[t'|t' \leq t] - Et\} \leq 0$$

for any s, x, t . The inequality is strict when $P(t) \in (0, 1)$, verifying property (3.2). \square

Result 3 (Monotonicity) $V(m'|s) - V(m|s)$ is increasing in s when $E_{m'}[s] > E_m[s]$.

Proof. We first prove that $W(m'|x) - W(m|x)$ is increasing in x . We have

$$W(m'|x) - W(m|x) = \int_T v(t)[p(t|m', x) - p(t|m, x)] dt.$$

By inequality (3.2) and since v is increasing, the right-hand side is increasing in x according to Lemma 1 below. Next, we observe that

$$V(m'|s) - V(m|s) = \int_X [W(m'|x) - W(m|x)]q(x|s) dx.$$

Here $q(s|x)$ satisfies the MLRP and $W(m'|x) - W(m|x)$ is increasing in x , so that $V(m'|s) - V(m|s)$ increases in s by Proposition 1 of Milgrom (1981). \square

Lemma 1 Let $T = [t_0, t_1]$ be a real, compact interval. Assume that $H(t_0|x) = H(t_1|x) = 0$ for all x and that $\partial H(t|x)/\partial x \leq 0$ for all t . Letting $h(t|x) \equiv \partial H(t|x)/\partial t$, $\int_T v(t)h(t|x) dt$ is increasing in x for any increasing function $v(\cdot)$.

Proof. Integrating by parts, as in the standard case, one gets

$$\begin{aligned} \int_T v(t)h(t|x) dt &= [v(t)H(t|x)]_{t_0}^{t_1} - \int_T v'(t)H(t|x) dt \\ &= - \int_T v'(t)H(t|x) dt. \end{aligned}$$

The larger is x , the smaller is $H(t|x)$ for all t , and since $v'(t) \geq 0$, it follows that $\int_T v'(t)H(t|x) dt$ is smaller. \square

Proposition 2 (Necessity of Binary Equilibria) All informative equilibria are binary.

Proof. Suppose that three different messages were sent in signal intervals $[a, b]$, $[b, c]$, $[c, d]$. By distinctness, we know that $a \leq b < c \leq d$, or that $a < b \leq c < d$.

By incentive compatibility, an individual with signal $s = b$ must be indifferent between the $[a, b]$ and $[b, c]$ messages, i.e. $V([a, b]|b) = V([b, c]|b)$. Here $E_{[a,b]}[s] = (a + b)/2$ and $E_{[b,c]}[s] = (b + c)/2$, so by (2.6) the indifference condition can be rewritten as

$$\int_X \frac{2x(a - c) \{E[tv(t)] - E[v(t)]E[t]\}}{[2 + (a + b)xEt][2 + (b + c)xEt]} \frac{1 + bxEt}{1 + bExEt} q(x) dx = 0. \quad (\text{A.3})$$

Eliminating constants, (A.3) becomes

$$\int_X \frac{x}{[2 + (a + b)xEt][2 + (b + c)xEt]} (1 + bxEt) q(x) dx = 0. \quad (\text{A.4})$$

Indifference between messages $[b, c]$ and $[c, d]$ at signal c gives a condition analogous to (A.4). Subtracting this other condition from (A.4) and simplifying we find

$$\int_X \left[\frac{1 + bxEt}{2 + (a + b)xEt} - \frac{1 + cxEt}{2 + (c + d)xEt} \right] \frac{xq(x)}{2 + (b + c)xEt} dx = 0. \quad (\text{A.5})$$

Notice that

$$\begin{aligned} \frac{1 + bxEt}{2 + (a + b)xEt} &> \frac{1}{2} \Leftrightarrow x(b - a) > 0 \\ \frac{1 + cxEt}{2 + (c + d)xEt} &< \frac{1}{2} \Leftrightarrow x(d - c) > 0. \end{aligned}$$

Therefore, if $a < b$ or $c < d$, the square-bracketed term in (A.5) changes sign with x

$$\frac{1 + bxEt}{2 + (a + b)xEt} - \frac{1 + cxEt}{2 + (c + d)xEt} > 0 \Leftrightarrow x > 0$$

and thus the integrand in equation (A.5) is positive, providing a contradiction.

Consider the other case, $a = b < c = d$. By the above reasoning this can be consistent with a full partition equilibrium, only if $-1 = a = b$ and $1 = c = d$. But this still implies a probability 0 of seeing any informative message, so the equilibrium is babbling. \square

Proposition 5 (No Informative Equilibrium) When the prior distribution on the state is sufficiently concentrated on any $x \neq 0$, there is no informative equilibrium.

Proof. Define

$$h(b, Et, x) = \frac{x(1 + bxEt)}{[2 + (b - 1)xEt][2 + (b + 1)xEt]}$$

as the integrand of (3.3). Consider an arbitrary $\hat{x} > 0$. Then for all $b \in [-1, 1]$,

$$h(b, Et, \hat{x}) \geq \frac{1 - \hat{x}Et}{4(1 + \hat{x}Et)} > 0$$

since $1 + b\hat{x}Et \geq 1 - \hat{x}Et$, $2 \geq 2 + (b - 1)\hat{x}Et$ and $2(1 + \hat{x}Et) \geq 2 + (b + 1)\hat{x}Et$. By continuity, for x close enough to \hat{x} , $h(b, Et, x)$ is positive and bounded away from zero. When the prior on the state is sufficiently concentrated on \hat{x} , there is no solution to the indifference equation (3.3), since the left-hand side is positive for any b . Therefore, there is no informative equilibrium. The same conclusion is reached in the case the prior is concentrated near a negative state. \square

Proposition 6 (Interim Reputation) In the interim reputation model where $Ex \neq 0$ there is no informative equilibrium, even allowing for mixed strategies.

Proof. Let $\mu(m|s)$ denote mixed strategies. We have

$$f(m|t) = \int_S \mu(m|s)f(s|t) ds = \frac{1 + E_m[s]tEx}{2} \lambda(m),$$

where $\lambda(m) = \int_S \mu(m|s) ds$, and $E_m[s] = \int_m s\mu(m|s) ds / \lambda(m)$. Then

$$p(t|m) = \frac{f(m|t)}{f(m)} p(t) = \frac{1 + E_m[s]tEx}{1 + E_m[s]EtEx} p(t)$$

so that

$$V(m) = \int_T v(t)p(t|m) dt = \frac{E[v(t)] + E_m[s]E[tv(t)]Ex}{1 + E_m[s]EtEx}.$$

Consider an equilibrium where more than one message is sent with positive probability. It is necessary that all messages give the same reputational payoff, since otherwise only the message with highest payoff would be sent. Take any two such messages m and m' . The equilibrium condition $V(m) = V(m')$ is equivalent to

$$E_m[s](E[tv(t)] - E[v(t)]Et)Ex = E_{m'}[s](E[tv(t)] - E[v(t)]Et)Ex.$$

For the special case $Ex = 0$, anything is an equilibrium, including truthtelling which is very informative. Recall that $E[tv(t)] - E[v(t)]E[t] > 0$, so when $Ex \neq 0$ the different messages

sent in equilibrium must have the same arithmetic mean $E_m[s] = E_{m'}[s]$. This common mean must be zero, as the mean of the means must be the mean signal. In this equilibrium the expert can communicate the intensity but not the direction of the information, and no message conveys any information about t or x . Then any two messages might as well have been pooled into one. \square

Proposition 7 (Known Own Ability) When the expert knows her own ability, there exists always a binary informative equilibrium.

We establish this result through two Lemmas.

Lemma 2 *There exists a $k \in (-1, 1)$ such that $V(m|s, t = k/s) = V(m'|s, t = k/s)$ when m is sent by $st \leq k$ types and m' is sent by the others.*

Proof. First, we argue that for k sufficiently close to 1, the sender with $st = k$ prefers message m' over m . As k tends to one, the set m' of (s, t) satisfying $st \geq k$ shrinks towards the corner $(s, t) = (1, 1)$. So, $\lambda(m')$ tends to zero, while $\lambda(m)$ tends to one. Since m tends towards an uninformative message, we have $W(m|x)$ tending to $Ev(t)$ for all x . For any $\varepsilon > 0$ there exists a $k^* \in (0, 1)$ such that $V(m|s, t) < Ev(t) + \varepsilon$ for all $k > k^*$. Since v is increasing and the prior distribution of t is not degenerate, there exists some $t^* \in (0, 1)$ with $v(t^*) > Ev(t)$. When $k > t^*$, message m' is sent only by types who know $t > t^*$, so that $V(m'|s, t) > v(t^*)$. Let $\varepsilon = (v(t^*) - Ev(t))/2$ and choose the k^* defined above. When $k > \max\langle t^*, k^* \rangle$ then $V(m|s, t) < Ev(t) + \varepsilon < v(t^*) < V(m'|s, t)$. This is true for all pairs (s, t) , and in particular for all those with $st = k$.

By analogy, when k is sufficiently close to -1 , the sender with $st = k$ prefers message m over m' . When k changes, we continuously change $V(m|s, t = k/s)$ and $V(m'|s, t = k/s)$. We know that $V(m|s, t = k/s) - V(m'|s, t = k/s)$ is positive for k near -1 and negative for k near $+1$. There must be an intermediate equilibrating k . \square

For incentive compatibility, the next argument (similar to Result 3) proves that for fixed definitions of messages, $V(m|s, t) - V(m'|s, t)$ is monotonic across iso-posterior curves (we have already observed that it is constant on iso-posterior curves).

Lemma 3 *If the receiver believes that m is sent by the $st \leq k$ types and that m' is sent by the others, then $V(m'|s, t = l/s) - V(m|s, t = l/s)$ is increasing in l .*

Proof. First, we prove that $P(t|m, x) - P(t|m', x)$ increases in x . We consider the case $k \geq 0$, but an analogous argument works for $k < 0$. Since m_t is the set of s where $st \leq k$,

$$f(m_t|x, t) = \int_{m_t} f(s|x, t) ds = \int_{m_t} \frac{1 + stx}{2} ds = \frac{1 + I_{m_t}[s]tx}{2}$$

where

$$I_{m_t}[s] = \int_{m_t} s ds = \int_{-1}^{k/t} s ds = \begin{cases} 0 & \text{if } 0 \leq t \leq k \\ \frac{1}{2}(\frac{k^2}{t^2} - 1) & \text{if } k \leq t \leq 1. \end{cases} \quad (\text{A.6})$$

We then have

$$f(m|x) = \int_0^1 f(m_t|x, t)p(t) dt = \frac{1 + E[tI_{m_t}[s]]x}{2},$$

so that

$$p(t|m, x) = \frac{1 + I_{m_t}[s]tx}{1 + E[tI_{m_t}[s]]x}p(t)$$

and

$$P(t|m, x) = \frac{1 + E[t'I_{m_{t'}}[s] | t' \leq t]x}{1 + E[tI_{m_t}[s]]x}P(t). \quad (\text{A.7})$$

(A.6) implies that $I_{m_t}[s] \leq 0$, so that

$$\begin{aligned} 0 &\geq E[t'I_{m_{t'}}[s] | t' \leq t]P(t) = \int_0^t t'I_{m_{t'}}[s]p(t') dt' \\ &\geq \int_0^1 t'I_{m_{t'}}[s]p(t') dt' = E[tI_{m_t}[s]]. \end{aligned}$$

It is seen directly from (A.7) that $P(t|m, x)$ is increasing in x . Similar calculations for message m' show that $E[tI_{m_t}[s]] \geq E[t'I_{m_{t'}}[s] | t' \leq t]P(t) \geq 0$ and that $P(t|m', x)$ is decreasing in x . Therefore $P(t|m, x) - P(t|m', x)$ increases in x as claimed, so that

$$W(m'|x) - W(m|x) = \int_T v(t)[p(t|m', x) - p(t|m, x)] dt$$

is increasing in x . Next, note from (4.3) that an increase in l yields an FSD increase in $q(x|s, t = l/s)$, implying that

$$V(m'|s, t = l/s) - V(m|s, t = l/s) = \int_X [W(m'|x) - W(m|x)]q(x|s, t = l/s) dx.$$

is increasing in l . □

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Appendix B: Omitted Proof

Theorem 1 (Irrelevance of Relative Reputation) Assume that the experts have von Neumann-Morgenstern payoffs, and that their signals are independent conditionally on state and ability. In equilibrium of the relative reputation model, expert i behaves as in the absolute reputation model with increasing value function

$$v^i(t^i) = E_{t^{-i}}[u^i(t^i, t^{-i})]$$

where the expectation is taken with respect to the prior belief $p(t^{-i})$.

Proof. The reputational value of message m^i for expert i with signal realization s^i is

$$V^i(m^i|s^i) = \int_X W^i(m^i|x) q(x|s^i) dx \quad (\text{B.1})$$

where

$$W^i(m^i|x) = \int_{M^{-i}} \int_{T^i} \int_{T^{-i}} u^i(t^i, t^{-i}) p(t^{-i}|m, x) dt^{-i} p(t^i|m, x) dt^i f(m^{-i}|x) dm^{-i}.$$

By the assumption of independence of the ability of different experts, posterior reputations are again stochastically independent. In particular, the posterior reputation of an expert is stochastically independent of the message reported by another expert, $p(t^{-i}|m^i, m^{-i}, x) = p(t^{-i}|m^{-i}, x)$ and $p(t^i|m^i, m^{-i}, x) = p(t^i|m^i, x)$, so that

$$\begin{aligned} W^i(m^i|x) &= \int_{T^i} \int_{M^{-i}} \int_{T^{-i}} u^i(t^i, t^{-i}) p(t^{-i}|m^{-i}, x) dt^{-i} f(m^{-i}|x) dm^{-i} p(t^i|m^i, x) dt^i \\ &= \int_{T^i} v^i(t^i, x) p(t^i|m^i, x) dt^i, \end{aligned}$$

where

$$v^i(t^i, x) = \int_{T^{-i}} u^i(t^i, t^{-i}) \int_{M^{-i}} p(t^{-i}|m^{-i}, x) f(m^{-i}|x) dm^{-i} dt^{-i}.$$

The law of iterated expectations gives

$$\int_{M^{-i}} p(t^{-i}|m^{-i}, x) f(m^{-i}|x) dm^{-i} = p(t^{-i}|x) = p(t^{-i}),$$

where we also used the independence of t^{-i} and x . It follows that

$$v^i(t^i, x) = \int_{T^{-i}} u^i(t^i, t^{-i}) p(t^{-i}) dt^{-i}.$$

and thus $v^i(t^i, x) = v^i(t^i)$ does not depend on x . Furthermore, since u^i is increasing in t^i for any t^{-i} , we find that $v^i(t^i)$ is an increasing function of t^i . We are thus back to the original problem with absolute reputational concerns, with the individual objective function $v^i(t^i) = E_{t^{-i}} u^i(t^i, t^{-i})$.

If expert i knew her own type, equation (B.1) would be modified to $V^i(m^i|s^i, t^i) = \int_X W^i(m^i|x) q(x|s^i, t^i) dx$ but the rest of the proof would go through as before. \square