A Model of Expertise*

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Abstract

We study a model in which two perfectly informed experts offer advice to a decision maker whose actions affect the welfare of all. Experts are biased and thus may wish to pull the decision maker in different directions and to different degrees. When the decision maker consults only a single expert, the expert withholds substantial information from the decision maker. We ask whether this situation is improved by having the decision maker consult a cabinet of (two) experts. We first show that there is no perfect Bayesian equilibrium in which full revelation occurs. When both experts are biased in the same direction, it is never beneficial to consult both. In contrast, when experts are biased in opposite directions, it is always beneficial to consult both. Finally, a cabinet of extremists is of no value.

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1 Introduction

The power to make decisions rarely resides in the hands of those possessing the necessary specialized knowledge. Instead experts are often solicited for advice by decision makers. Thus, a division of labor has arisen between those who have the relevant expertise and those who make use of it. The diverse range of problems confronted by decision makers, such as corporate CEOs or political leaders, almost precludes the possibility that they themselves are experts in all relevant fields and hence, the need for outside experts naturally arises. CEOs routinely seek the advice of marketing specialists, investment bankers and management consultants. Political leaders rely on a bevy of economic and military advisors. Investors seek tips from stockbrokers and financial advisors.

These and numerous other situations share some common features.

First, the experts dispensing advice are by no means disinterested. Differing objectives among the parties may lead the experts to attempt to influence the decision maker in ways that are not necessarily in the latter's best interests. Investment banks stand to gain from new issues and corporate mergers, decisions about which they regularly offer advice. The political future of economic and military advisors may be affected by the decisions on which they give counsel. Stockbrokers are obviously interested in the investment decisions of their clients.

Second, in nearly all of these cases decision makers are bombarded with advice from numerous experts, with possibly different agendas. Moreover, experts may strategically tailor their advice to counter that offered by other, rival, experts. For instance, hawks may choose more extreme positions on an issue if they know that doves are also being consulted, and vice-versa. Thus the decision maker faces the daunting task of sifting through the mass of sometimes conflicting opinions that are offered and coming to a conclusion as to the best course of action. Indeed, this ability is routinely touted as the mark of a good leader.

Thus, the decision maker, in determining the size and composition of her "cabinet" of advisors, must carefully consider the following questions: Is it possible to extract all information relevant to the decision from a cabinet? Is it better to actively consult a number of advisors or only a single, well chosen, advisor? Is it better to form a cabinet with diverse opinions about what is the "correct" decision, or does a cohesive cabinet lead to better advice? Is an advocacy system, where the decision maker appoints experts with opposing viewpoints, helpful in deciding on the correct action? How do experts with extreme views affect the advice offered to the decision maker? These questions form the central focus of our paper.

To get at these questions, we use a simple model in order to analyze the interplay among a single decision maker and *two* interested experts who have superior information. The experts offer advice to the decision maker in order to influence the decision in a way that serves their own, possibly differing, objectives. We ask how a decision maker should integrate the opinions of experts when faced with this situation.

In our model, an expert's preferences are parametrized by a measure of his inherent bias relative to the decision maker. The experts may differ both in terms of how biased they are and in which direction. They may be biased in opposite directions (opposing bias): one expert may wish to pull the decision maker to the left and the other to the right. Alternatively, both may wish to pull in the same direction but to differing degrees (like bias). The absolute value of the bias parameter indicates how "loyal" an expert is to the decision maker. Of course, a more loyal expert's objectives are more closely aligned with those of the decision maker.

If the decision maker had the option of consulting only one of the two experts for advice, it seems natural that he would consult only the more loyal expert, and indeed that is the case in our model. Nevertheless, *a priori* it may be beneficial to *combine* the advice from the two experts. It is easy to see that if the advice were solicited in a way that each expert was ignorant of the fact that the other was also offering advice, the decision maker would surely benefit relative to consulting only one expert. We study how this conclusion is affected if each expert were aware that the other was also offering advice.

We first establish that, even though the information possessed by the experts is perfectly correlated, the lack of congruence in incentives between the decision maker and the experts *always* leads to a withholding of information on the part of the experts. That is, it is impossible to form a cabinet such that the experts always reveal their information. To assess whether it might still be beneficial to combine the advice of experts, we examine separately the case where experts have like biases and the case where biases are opposing.

Like biases. When the two experts have like biases, we find that the decision maker would derive *no* benefit relative to consulting only one expert. Thus, despite the fact that the two experts have identical information and the more loyal expert does not fully divulge what he knows, the advice offered by the less loyal expert is of no additional value. Moreover, *ex ante* all parties, including the less loyal expert, would agree that the best course of action is for the decision maker to consult only the more loyal expert.

Opposing biases. When the two experts have opposing biases, the decision maker always derives some benefit from consulting both experts relative to consulting only one. Indeed, we show that even when experts would reveal no information if consulted alone, combining the information of the experts leads them to completely reveal their information over a portion of the state space, and this is beneficial relative to consulting only a single expert. However, this conclusion holds only if at least one of the experts is not an "extremist." If both of the experts are extremists, no information is revealed – either when they are consulted separately or when information is combined.

Related Work The advice that experts offer does not have any direct economic effect; at best it only influences economically relevant decisions. Thus experts' advice has the nature of "cheap talk." Indeed our basic model is closely related to the model of Crawford and Sobel (1982) of strategic information transmission between two parties, one of whom has information useful for the other (see also Green and Stokey (1980)).

Our model differs from the Crawford and Sobel (1982), hereafter referred to as CS, in that we allow for *multiple* sources of information. This context leads to important strategic considerations absent in the single expert analysis. Now an expert must consider not only how his advice will directly influence the decision maker but also what information is coming from other experts and how his mere presence will affect that. Likewise, the decision maker now has the option of listening to only a subset of the panel of experts. As will become apparent, the additional strategic considerations that arise with multiple experts lead to technical complications not present with a single expert. Differences between the single expert model of CS and our model are highlighted in later sections.

As is well known, models with cheap talk suffer from a plethora of equilibria and efforts to identify some as salient has led to the development of a substantial literature on refinements in this context (Matthews, Okuno-Fujiwara and Postlewaite (1991) and Farrell (1993)). Farrell and Rabin (1996) present a concise survey. The models we consider also have multiple equilibria; however, for the most part, our focus is on the "most informative" equilibrium.

Closely related are papers by Gilligan and Krehbiel (1989), who examine the case of opposing biases and by Austen-Smith (1990), who examines the case of like biases. Gilligan and Krehbiel (1989) are concerned with the effect on legislative outcomes of having two "expert" committees with opposing biases restrict the set of legislative alternatives that may be implemented. They show that the restrictive "closed rule" system of determining the set of alternatives, does not lead to different legislative outcomes compared to the "open rule" where the set of alternatives is unrestricted. Austen-Smith (1990) examines the effect of debate, modeled as cheap-talk, on legislative outcomes when there are three legislators. His model is substantially different from ours in that the "expert" legislators vote on what legislation is to be passed. Thus, the separation between the experts and the decision maker is absent in his model.

The problem of multiple experts has also been considered by Ottaviani and Sorensen (1997). Both their model and concerns, however, are different from ours. In their model the experts are not directly affected by the decisions but care only about making recommendations that are validated *ex post*. Thus experts care only about their reputation for "being on the mark." Ottaviani and Sorensen (1997) then show that a kind of "herd" effect results when experts are consulted sequentially: experts may well neglect their own information in order to appear correct. Also related are papers by Banerjee and Somanathan (1997) and Friedman (1998), which examine information transmission in a setting in which there is a continuum of potential experts with differing prior beliefs, at most one of whom receives an informative signal about the state. Finally, the effects of combining information provided by experts with opposing incentives has also been examined by Shin (1994) in the context of persuasion games (see Milgrom and Roberts (1986)), and by Dewatripont and Tirole (1998) in the context of a moral hazard model.

Our work is also somewhat related to the problem of information transmission between a decision maker and a single expert when the decision maker is uncertain about the bias of the expert. Sobel (1985) and Morris (1997) focus on reputational considerations in the expert's advice in analyzing this problem. Morgan and Stocken (1998) consider this problem in a static CS-like setting and focus on information transmission by equity analysts.

The remainder of the paper proceeds as follows. Section 2 outlines the basic model.

In Section 3, we establish the impossibility of complete information transmission as well as a structural property of a monotonic equilibria of the two expert game. Section 4 examines the like bias case and shows that the addition of a less loyal expert conveys no additional information relative to simply consulting the more loyal expert alone. In Section 5, we examine the opposing biases case and show that combining information can be beneficial even when neither expert will reveal any information when consulted alone. There are, however, limits to information gains from combining: when both of the experts are extremist or when an extremist expert is consulted last combining experts' advice is not beneficial. Section 6 examines possible extensions of the simple model. Finally, Section 7 concludes. All proofs are collected in Appendix A. A second appendix, Appendix B, takes up issues related to the existence of non-monotonic equilibria.

All of our results are illustrated by means of a series of examples. The examples offer the essential intuition for the general results without some of the technical details of the formal proofs. Indeed, all of the main propositions of the paper can be understood by means of the examples.

2 Preliminaries

In this section we sketch a simple model of decision making when there are multiple experts. We do not model any of the examples mentioned in the introduction explicitly. Rather our model is a stylized representation of the interaction among decision makers and experts across a broad range of institutional settings. The overall structure extends the model of CS to a setting with multiple experts.

Consider a *decision maker* who takes an action $y \in \mathbf{R}$, the payoff from which depends on some underlying state of nature $\theta \in [0,1]$. The state of nature θ is distributed according to the density function $f(\cdot)$. The decision maker has no information about θ, but there are two *experts* each of whom observes θ.

The two experts then offer "advice" to the decision maker by sending messages $m_1 \in [0,1]$ and $m_2 \in [0,1]$, respectively. After observing the state, messages are sent *sequentially* and *publicly*. First, expert 1 offers his advice, which is heard by both the decision maker and expert 2. Expert 2 then offers his advice, and the decision maker takes an action. The decision maker is not in any way bound by the advice of the experts. Instead, she is free to interpret the messages however she likes as well as to choose any action.[1]

The payoff functions of all three agents are of the form $U(y,\theta,b_i)$ where b_i is a parameter which differs across agents. For the decision maker, agent 0, b_0 is normalized to be 0. For the experts, agents 1 and 2, $b_i \neq 0$ and $b_1 \neq b_2$. We write $U(y,\theta) \equiv U(y,\theta,0)$ as the decision maker's payoff function. We suppose that U is twice continuously differentiable and satisfies $U_{11} < 0$, $U_{12} > 0$, $U_{13} > 0$. Since $U_{13} > 0$ the parameter b_i measures how closely the expert i's interests are aligned with those of the decision maker and it is useful to think of b_i as a measure of how *biased* expert i is, relative to the decision maker. We

[1] In the political science literature, this is referred to as the "open rule" (Gilligan and Krehbiel (1989)).

also assume that for each i, $U(y,\theta,b_i)$ attains a maximum at some y. Since $U_{11}<0$, the maximizing action is unique. The biases of the two experts and the decision maker are commonly known.

These assumptions are satisfied by "quadratic loss functions." In this case, the decision maker's payoff function is

$$U(y,\theta) = -(y-\theta)^2 \tag{1}$$

and expert i's payoff function is

$$U(y,\theta,b_i) = -(y-(\theta+b_i))^2 \tag{2}$$

where $b_i \neq 0$. An important case, first introduced by CS, combines quadratic loss functions with the assumption that the state θ is uniformly distributed on $[0,1]$. We will refer to this as the "uniform-quadratic" case.

In studying the multiple experts problem, we divide the analysis into two cases. If both experts are biased in the same direction, that is, both $b_1, b_2 > 0$, then the experts are said to have *like biases*. If the experts are biased in opposite directions, that is, $b_i > 0 > b_j$, then the experts are said to have *opposing biases*.[2]

Define $y^*(\theta) = \arg\max_y U(y,\theta)$ to be the *ideal* action for the decision maker when the state is θ. Similarly, define $y^*(\theta,b_i) = \arg\max_y U(y,\theta,b_i)$ be the ideal action for expert i. Since $U_{13}>0$, $b_i>0$ implies that $y^*(\theta,b_i) > y^*(\theta)$; and since such an expert always prefers a higher action than is ideal for the decision maker, we will refer to him as being *right-biased*. Similarly, if $b_i<0$ then $y^*(\theta,b_i) < y^*(\theta)$ and we refer to such an expert as being *left-biased*.

Notice that with quadratic loss functions, the ideal action for the decision maker is to choose an action that matches the true state exactly: for all θ, $y^*(\theta)=\theta$. The ideal action for an expert with bias b_i is $y^*(\theta,b_i) = \theta + b_i$.

A word of caution is in order. Our results fall into two categories. Some concern the structure of equilibria of the multiple experts game and are derived under the assumptions given above. Others concern welfare comparisons among equilibria and require an additional assumption. This is no different from the single expert model considered by CS and like them we need their Assumption M (p. 1444 of CS) in order to derive unambiguous welfare results (specifically, Propositions 2 and 3). This assumption, while not so transparent, is satisfied by the uniform-quadratic case. Thus in the interests of exposition, we have chosen to derive the welfare results only for the uniform-quadratic specification. The reader should be aware that these results can be derived more generally under Assumption M of CS.

3 Equilibrium with Experts

Single Expert Before studying equilibria of the model with two experts it is instructive to recall the structure of equilibria of the model with a single expert as derived by

[2]The case where both $b_1, b_2 < 0$ is qualitatively no different from the case where both $b_1, b_2 > 0$.

Crawford and Sobel (1982).

In the single expert game a strategy for the expert μ specifies the message $m = \mu(\theta)$ that he sends in any state θ. A strategy for the decision maker y specifies the action $y(m)$ that she takes following any message m by the expert. Let $P(\cdot|m)$ denote the cumulative distribution function that specifies posterior beliefs about the state held by the decision maker after the message m.

In a *perfect Bayesian equilibrium* (1) for all messages m, the decision maker's action $y(m)$ maximizes her expected payoff given her posterior beliefs $P(\cdot|m)$; (2) the beliefs $P(\cdot|m)$ are formed using the expert's strategy μ by applying Bayes' rule wherever possible; (3) given the decision maker's strategy y, for all states θ, $\mu(\theta)$ maximizes the expert's payoff.

CS show that every equilibrium of the single expert game has the following structure.[3] There are a finite number of equilibrium actions $y_1, y_2, ..., y_N$. The equilibrium breaks the state space into N disjoint intervals $[0, a_1)$, $[a_1, a_2)$, ... , $[a_{n-1}, a_n)$, ... , $[a_{N-1}, 1]$ with action y_n resulting in any state $\theta \in [a_{n-1}, a_n)$. The equilibrium actions are monotonically increasing in the state, that is, $y_{n-1} < y_n$. Finally, at every "break point" a_n the following "no arbitrage" condition

$$U(y_n, a_n, b) = U(y_{n+1}, a_n, b) \tag{3}$$

is satisfied. In other words, in state a_n the expert is indifferent between the actions y_n and y_{n+1}. Since $U_{12} > 0$, for all $\theta < a_n$, the expert strictly prefers y_n to y_{n+1} and for all $\theta > a_n$, the reverse is true. Thus (3) serves as an incentive (or self-selection) constraint.

Multiple Experts In the multiple experts game a pure strategy for expert 1 is a function $\mu_1(\theta)$ mapping states into messages and a pure strategy expert 2 is a function $\mu_2(\theta, m_1)$ mapping states and messages m_1 from expert 1 into messages. A (pure) strategy for the decision maker is a function $y(m_1, m_2)$ mapping messages into actions. Let $P(\cdot|m_1, m_2)$ denote the cumulative distribution function that specifies posterior beliefs about the state held by the decision maker after messages m_1 and m_2.

In the multiple expert game a (pure strategy) *perfect Bayesian equilibrium* (PBE) entails: (1) for all pairs of messages m_1 and m_2, the decision maker's action $y(m_1, m_2)$ maximizes her expected payoff given her beliefs $P(\cdot|m_1, m_2)$; (2) the beliefs $P(\cdot|m_1, m_2)$ are formed using the experts' strategies μ_1 and μ_2 by applying Bayes' rule wherever possible; (3) given the decision maker's strategy y, for all states θ and messages m_1 sent by expert 1 $\mu_2(\theta, m_1)$ maximizes expert 2's payoff; and (4) given the decision maker's strategy y and expert 2's strategy μ_2, for all states θ, $\mu_1(\theta)$ maximizes expert 1's expected payoff.[4]

Given a PBE we will denote by Y the *outcome function* that associates with every state

[3] CS actually characterize the set of *Bayesian* equilibrium outcomes. In the single expert game this is the same as the set of *perfect Bayesian* equilibrium outcomes. As we show in Section 3 this equivalence does not hold in the multiple expert game.

[4] The formal definition of a PBE requires only that the various optimality conditions hold for *almost every* state and pair of messages. This would not affect any of our results.

the resulting equilibrium action. Formally, for each θ

$$Y(\theta) = y(\mu_1(\theta), \mu_2(\theta, \mu_1(\theta))).$$

Denote by $Y^{-1}(y) = \{\theta : Y(\theta) = y\}$. Given an outcome function Y we can determine the resulting *equilibrium partition*

$$P = \left\{Y^{-1}(y) : y \text{ is an equilibrium action}\right\}$$

of the state space. The partition P is then a measure of the informational content of the equilibrium. If the equilibrium partition P is finer than P', then the informational content of P is greater than that of P', since the former allows the decision maker to discern among the states more effectively.

A PBE always exists. In particular, there are always equilibria in which all messages from both of the experts are completely ignored by the decision maker, or in other words, both experts "babble." To see that this is a PBE, notice that since the messages of the experts contain no information, the decision maker correctly disregards them in making her decision. Likewise, from the perspective of each of the experts, since messages will always be ignored by the decision maker, there is no advice giving strategy that improves payoffs relative to babbling. Obviously, information loss is most severe in a babbling equilibrium. Typically, there are also other, more informative, equilibria.

Example 1 Let the state θ be distributed uniformly on $[0,1]$, and let the payoff functions be of the quadratic loss kind specified in (1) and (2). This is the uniform-quadratic case introduced earlier.

Suppose that $b_1 = \frac{1}{40}$ and $b_2 = \frac{1}{9}$ so that the experts have *like* biases and expert 1 is less biased than is expert 2. A PBE for this game is depicted in Figure 1, where the states $a_1 = \frac{1}{180}$, $a_2 = \frac{22}{180}$, $a_3 = \frac{61}{180}$ and the actions $y_1 = \frac{1}{360}$, $y_2 = \frac{23}{360}$, $y_3 = \frac{83}{360}$, $y_4 = \frac{241}{360}$.

In the figure, the outcome function Y is the step function depicted by the dark lines. The lower dotted line depicts expert 1's ideal actions $y^*(\theta, b_1) = \theta + b_1$ and similarly, the upper dotted line depicts expert 2's ideal actions $y^*(\theta, b_2) = \theta + b_2$. In equilibrium, the information available to the decision maker is that the state lies in one of four intervals $[0, a_1)$, $[a_1, a_2)$, $[a_2, a_3)$ or $[a_3, 1]$. The action y_1 is then optimal for the decision maker given that he knows that $\theta \in [0, a_1)$, y_2 is optimal given $\theta \in [a_1, a_2)$, etc.

To see that this is an equilibrium configuration, notice that in state a_2 expert 1 is exactly indifferent between actions y_2 and y_3 since $(a_2 + b_1) - y_2 = y_3 - (a_2 + b_1)$. (In the figure this indifference is indicated by the vertical double pointed arrow centered on $a_2 + b_1$.) Expert 1 strictly prefers y_2 to y_3 in all states $\theta < a_2$ and prefers y_3 to y_2 in all states $\theta > a_2$. Thus given the decision maker's strategy he is willing to distinguish between states $\theta < a_2$ and states $\theta > a_2$.

Similarly, in state a_3 expert 2 is indifferent between y_3 and y_4 and is willing to distinguish between states $\theta < a_3$ and states $\theta > a_3$.

Thus in states a_2 and a_3 the "no arbitrage" condition (3) from CS holds for either expert 1 or expert 2.

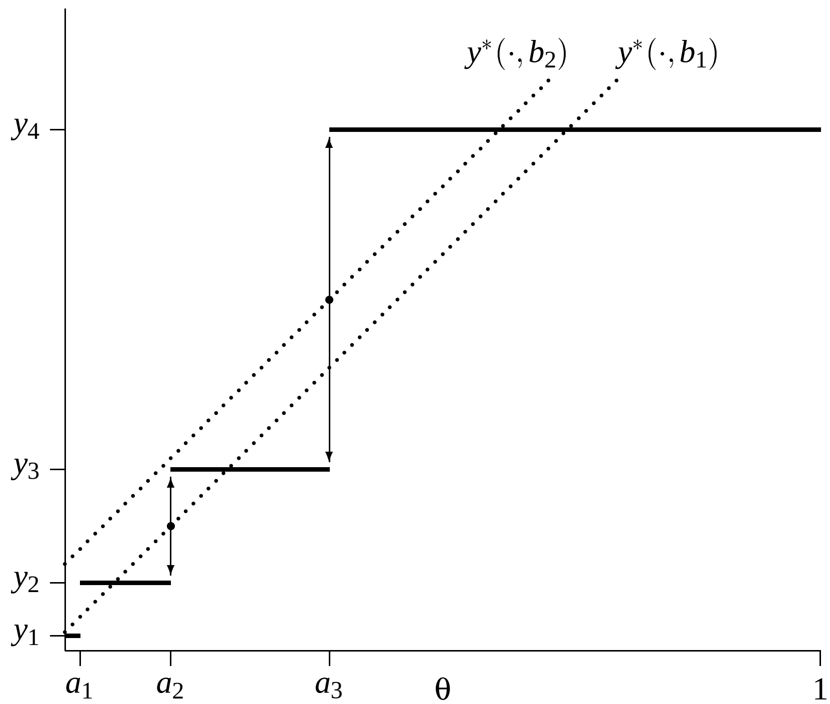

Figure 1: A PBE with Like Biases

In state a_1, however, neither expert is indifferent between y_1 and y_2. Indeed, expert 1 strictly prefers y_1 to y_2 in state a_1. (Notice that expert 1's ideal action $y^*(a_1, b_1)$, is closer to y_1 than y_2.) Expert 2, on the other hand, strictly prefers y_2 to y_1. The equilibrium calls for expert 1 to "suggest" action y_1 by sending a message $m_1 = y_1$ and for expert 2 to "agree" and also send the message $m_2 = y_1$. Expert 2 has the option of "disagreeing" with expert 1 and inducing action y_3. The equilibrium is constructed so that expert 2 is indifferent between y_1 and y_3 in state a_1 and so strictly prefers y_3 to y_1 if $\theta > a_1$. Thus, even though in states just above a_1, expert 1 would strictly prefer to switch from the equilibrium action y_2 to y_1, were he to actually suggest action y_1, expert 2 would disagree, resulting in action y_3. Since y_2 is preferred to y_3 by expert 1 in these states, expert 1 will choose not to deviate. Here we see how the strategic interaction of the two experts creates the possibility of "disciplining" the experts in a manner not possible for the single expert case.[5]

A Mechanism Design Interpretation Since our focus is on finding the most informative equilibrium in the multiple experts game, the following "mechanism design" interpretation of the decision maker's problem will sometimes prove helpful. Viewed in this light, finding

[5] A detailed specification of the equilibrium strategies and beliefs for this and all other examples in the paper may be obtained from the authors.

the most informative equilibrium may be viewed as a type of implementation problem where the "planner is a player" (see Baliga, Corchon and Sjöström, (1997)) but where the set of feasible game forms that the planner can propose is substantially constrained.

Specifically, suppose that the decision maker were free to assign a "meaning" to each of the messages that the experts might issue provided that the assigned meaning was consistent in equilibrium. In effect, the decision maker is choosing a *language.* This is equivalent to the decision maker announcing her beliefs $P(\cdot|m_1, m_2)$ for all message pairs (m_1, m_2). Such an announcement immediately implies an action $y(m_1, m_2)$ associated with each message pair. Given an announcement of beliefs $P(\cdot|m_1, m_2)$ and a message m_1, expert 2 chooses a strategy $\mu_2(\theta, m_1)$ to maximize his payoff. Similarly for expert 1. Finally, for the announced beliefs to be consistent requires that the announced beliefs correspond to posterior beliefs obtained by applying Bayes' rule for all message pairs $(\mu_1(\theta), \mu_2(\theta, \mu_1(\theta)))$. Thus the decision maker's problem is to choose a language that is incentive compatible.

The problem of choosing the most informative equilibrium is formally equivalent to choosing beliefs $P(\cdot|m_1, m_2)$ to maximize her *ex ante* expected payoff subject to the constraints that (μ_1, μ_2, y) form a PBE.

With this reformulation in mind, we turn to the question of whether there exists an announced set of beliefs satisfying the above constraints such that the state is always completely revealed. We shall refer to this as full revelation. Obviously, such a fully revealing equilibrium would maximize the decision maker's expected payoff.

3.1 Full Revelation

Full revelation means that in every state, the equilibrium action is the same as the decision maker's ideal action, that is, for all θ, $Y(\theta) = y^*(\theta)$. (Equivalently, the associated equilibrium partition $\mathcal{P}$ consists of singleton sets.) With only a single expert, CS show that full revelation is *not* a *Bayesian equilibrium (BE)*.

With multiple experts, however, full revelation can occur in a BE. We demonstrate this by looking at the case where the experts are biased in the same direction, that is, $b_1 > 0$, $b_2 > 0$. Both experts then prefer a higher action than is optimal for the decision maker: $y^*(\theta, b_i) > y^*(\theta)$. Suppose that the decision maker announces the beliefs $P(\theta = \min\{m_1, m_2\} | m_1, m_2) = 1$. The associated strategy of the decision maker is then $y(m_1, m_2) = y^*(\min\{m_1, m_2\})$. Let expert 1 follow the strategy $\mu_1(\theta) = \theta$ of revealing the state and expert 2 follow the strategy of also revealing the state regardless of what expert 1's message is: $\mu_2(\theta, m_1) = \theta$. In state θ, both experts send messages $m_1 = m_2 = \theta$, and the action taken is $y^*(\theta)$ which, from the perspective of either expert, is better than any action $y < y^*(\theta)$. Reporting an $m_i < \theta$ will only decrease i's payoff whereas reporting an $m_i > \theta$ will have no effect given that the other expert follows μ_j.

Thus with perfectly informed experts, there exists a BE in which the decision maker can extract all the information and achieve a first-best outcome.

The equilibrium constructed above, however, involves non-optimizing behavior on the part of expert 2 off the equilibrium path. Specifically, in state $\theta \in [0, 1)$ if expert 1 were to choose a message $m_1 = \theta + \varepsilon$, for $\varepsilon > 0$ small enough, it is no longer optimal for expert 2 to play $\mu_2(\theta, m_1) = \theta$. Indeed, he is better off also deviating to $m_2 = m_1$. Thus the full revelation BE constructed above is not a PBE.

The reason the Bayesian equilibrium constructed above does not survive the stronger PBE notion should be familiar. Expert 2 cannot credibly commit to reveal the state regardless of expert 1's message. Expert 1 can exploit this by exaggerating the true state slightly, confident in the knowledge that expert 2 will follow his lead.

More generally:

Proposition 1 *There does not exist a fully revealing PBE.*

3.2 Monotonic Equilibria

The outcome function associated with full revelation, as well as that associated with a babbling equilibrium, and with all equilibria where only a single expert is consulted have in common the property that the equilibrium action induced in a higher state is at least as large as that induced in a lower state. Formally, we will say that a PBE (μ_1, μ_2, y) is *monotonic* if the corresponding outcome function $Y(\cdot)$ is a non-decreasing function. Notice that the PBE constructed in Example 1 also shares this property.

For the remainder of the paper, our analysis will concern itself with monotonic equilibria. Recall that in the case of a single expert we know from CS that all equilibria are monotonic. This is not true with multiple experts as an example in Appendix B shows. For the case of like bias, we also provide sufficient conditions to ensure that all equilibria are monotonic equilibria (Proposition 5 in Appendix B). In the case of opposing biases, our conclusions remain unaltered if we also admit the possibility of non-monotonic equilibria.

The following result identifies some simple necessary conditions satisfied by monotonic equilibria.

Lemma 1 *Suppose Y is monotonic. If Y has a discontinuity at θ and*

$$\lim_{\varepsilon \downarrow 0} Y(\theta - \varepsilon) = y^- < y^+ = \lim_{\varepsilon \downarrow 0} Y(\theta + \varepsilon)$$

then

$$U\left(y^-, \theta, \min\{b_1, b_2\}\right) \geq U\left(y^+, \theta, \min\{b_1, b_2\}\right), \text{ and} \tag{4}$$

$$U\left(y^-, \theta, \max\{b_1, b_2\}\right) \leq U\left(y^+, \theta, \max\{b_1, b_2\}\right). \tag{5}$$

Viewed from a mechanism design perspective, the inequalities (4) and (5) are in the nature of incentive constraints: at any discontinuity, the expert with bias $\min\{b_1, b_2\}$ weakly prefers the lower action y^- whereas the expert with bias $\max\{b_1, b_2\}$ weakly prefers the higher action y^+. As we pointed out earlier, Example 1 shows that these inequalities may be strict for both players (for instance, at the first point of discontinuity, a_1) and thus the incentive compatibility constraint of each of the experts holds with slack. This is to be contrasted with the single expert case where the incentive constraints (3) must hold with equality.

4 Experts with Like Biases

In this section, we examine a situation in which a decision maker can consult with a group of like biased experts. We focus on two questions: first, what is the information content of advice offered by a given panel of such experts; and second, how should a decision maker determine the composition of such a panel. We begin by establishing limits to information transmission by a group of like biased experts.

Our first result shows that when the experts have like biases there can be at most a finite number of equilibrium actions played in any monotonic PBE.[6] In particular, it rules out the possibility of full revelation in the case of like biases (Case 2 in the proof of Proposition 1) since a fully revealing equilibrium must be monotonic and involves an infinite number of equilibrium actions.

Lemma 2 *Suppose experts have like biases and Y is monotonic. Then there are a finite number of equilibrium actions.*

The intuition for Lemma 2 is that if two equilibrium actions are sufficiently close to one another, then there will be some state where the lower action is called for, but both experts prefer the higher action. As a consequence, the first expert can deviate and send a message inducing the higher action confident that expert 2 will follow his lead. Put differently, it is impossible to satisfy the incentive constraints of Lemma 1 if equilibrium actions are too close together.

A consequence of Lemma 2 is that the information transmitted with like biased experts is severely limited: only a finite number of actions occur in equilibrium. The multiple expert setting shares this qualitative feature with the single expert model of CS despite the fact that the information possessed by the two experts is perfectly correlated.

We next turn to a more precise examination of the potential informational benefits of adding an expert.

4.1 Choosing a Cabinet

Suppose that there is a decision maker who must choose a panel of like biased experts, a cabinet, to advise her. She is aware of the biases of each of the experts that she might select. What is the optimal composition of the panel?

Proposition 2 shows the solution to the problem of determining the optimal panel is strikingly simple. Specifically, it is optimal to have a one member panel that consists of the least biased expert. We show that it is always the case that in the most informative partition equilibrium the more biased expert's advice has no value. In other words, the more biased expert is *redundant*.

[6] In Appendix B, we show (Lemma 4) that *all* PBE consist of only finite equilibrium actions for the uniform-quadratic case.

To answer this question requires that we make welfare comparisons among the set of monotonic PBE in a multiple experts setting. However, we remind the reader that to obtain unambiguous welfare comparisons even among single expert equilibria as in CS, we require an additional assumption (their Assumption M, p. 1444 of CS) to guarantee that a rightward shift in one break point leads to a rightward shift in all break points. The most frequently employed specification where this assumption is satisfied is the uniform-quadratic case given in Example 1. The transparency of the argument is also much improved by considering this case; thus, for the remainder of the section, our arguments will reflect the uniform-quadratic specification. However, one can show that our main result for this section (Proposition 2) holds generally when Assumption M is satisfied.

Example 2 It is useful to illustrate the information transmission properties of the multiple expert game by continuing to study the uniform-quadratic case from Example 1 where $b_1 = \frac{1}{40}$ and $b_2 = \frac{1}{9}$.

If the decision maker solicited only expert 1 for advice, the Pareto superior (and also most informative) equilibrium results in the partition P_1 of $[0,1]$ being communicated to the decision maker.

$$\begin{array}{cccccccccccccccc} & & 1 & & & 1 & & & & 1 & & & & & & \\ \vdash & - & + & - & - & + & - & - & - & + & - & - & - & - & - & \dashv \\ 0 & & \frac{1}{10} & & & \frac{3}{10} & & & & \frac{6}{10} & & & & & & 1 \end{array}$$

This means that if the true state θ lies in the interval $\left[0, \frac{1}{10}\right]$, the expert sends a message $m_1 = \frac{1}{20}$ advising the decision maker to choose action $y_1 = \frac{1}{20}$. Similarly, if $\theta \in \left[\frac{1}{10}, \frac{3}{10}\right]$ he suggests $y_2 = \frac{4}{20}$, if $\theta \in \left[\frac{3}{10}, \frac{6}{10}\right]$, he suggests $y_3 = \frac{9}{20}$ and if $\theta \in \left[\frac{6}{10}, 1\right]$ he suggests $y_4 = \frac{16}{20}$.

We will refer to points such as $\frac{1}{10}$, $\frac{3}{10}$ and $\frac{6}{10}$ as "break points" in that they determine how the set of states is broken up in such an equilibrium. In the figure given above the superscript above each break point labels the expert whose message distinguishes states below the break point from states above the break point. Thus, the decision maker only knows which of these four intervals contains the true state. As a result, his expected utility is -0.0083. Notice that in this case, there is no slack in the incentive constraints of expert 1 at any point of discontinuity.

Similarly, if the decision maker solicited only expert 2 for advice, the most informative equilibrium partition is P_2:

$$\begin{array}{cccccccccccccccc} & & & & & 2 & & & & & & & & & & \\ \vdash & - & - & - & - & + & - & - & - & - & - & - & - & - & - & \dashv \\ 0 & & & & & \frac{5}{18} & & & & & & & & & & 1 \end{array}$$

resulting in a payoff of -0.0332 to the decision maker. Notice that expert 2 withholds more information than does expert 1, in the sense that the variance of the true state of the world, given the equilibrium partition, is higher with expert 2 than expert 1. Intuitively, since expert 2 wishes the decision maker to choose a larger value of y than does expert 1, he withholds more information than does expert 1. Put differently, in announcing beliefs, the decision maker is faced with a much more daunting problem in satisfying incentive compatibility for the more biased expert 2. As a result, the most informative equilibrium consulting only expert 2 leads to considerably more information withholding.

If there were no further strategic considerations, that is neither expert knew of the other's existence, the decision maker could *combine* the reports of the two experts to obtain the partition $P_1 \wedge P_2$:

$$\begin{array}{cccccccccccccccc} & & 1 & & & 2 & 1 & & & & 1 & & & & & \\ \vdash & - & + & - & - & + & + & - & - & - & + & - & - & - & - & \dashv \\ 0 & & \frac{1}{10} & & & \frac{5}{18} & \frac{3}{10} & & & & \frac{6}{10} & & & & & 1 \end{array}$$

which is the coarsest common refinement (*join*) of P_1 and P_2. The decision maker's expected utility is now $-.0081$. Thus, it seems plausible that the addition of another expert, even an expert more biased than expert 1, might be helpful in overcoming the problem of strategic information withholding.

Of course, this ignores strategic interaction among the experts. That is, each expert acts as though he or she were the only source of information available to the decision maker. Indeed, the specification above is not a PBE in the multiple experts game. One profitable deviation is for expert 1 to induce a higher equilibrium action for states near $\frac{1}{10}$.

Now suppose that the decision maker solicits both experts for advice regarding the true state, and the experts are aware of each other's presence. What is the most informative monotonic PBE?

One such equilibrium was described in Example 1 with the following information partition.

$$\begin{array}{cccccccccccccccc} & & 1 & & & 2 & & & & & & & & & & \\ \vdash & + & + & - & - & + & - & - & - & - & - & - & - & - & - & \dashv \\ 0 & \frac{1}{180} & \frac{22}{180} & & & \frac{61}{180} & & & & & & & & & & 1 \end{array}$$

Recall that at the point $\theta = \frac{1}{180}$ neither expert is indifferent between y_1 and y_2 and so there is slack in both incentive constraints. In particular, expert 1 strictly prefers the *lower* equilibrium action for states near $\frac{1}{180}$ and for states near $\frac{61}{180}$. The decision maker's expected utility in this equilibrium is $-.0250$.

An analogous PBE when we eliminate the slack in expert 1's incentive constraint at the first point of discontinuity results in the equilibrium partition Q :

$$\begin{array}{cccccccccccccccc} & 1 & 1 & & & 2 & & & & & & & & & & \\ \vdash & + & + & - & - & + & - & - & - & - & - & - & - & - & - & \dashv \\ 0 & \frac{1}{72} & \frac{23}{180} & & & \frac{41}{120} & & & & & & & & & & 1 \end{array}$$

where expert 1 is exactly indifferent at the first two break points and expert 2 is indifferent at the third break point. This results in expected utility of $-.0247$ to the decision maker. This is better than the equilibrium of Example 1. The intuition for this result is that, by shifting the first break point to the right, informativeness is improved since all of the other break points shift to the right as well. Moreover, as there is slack in expert 1's incentive constraint, such a rightward shift is possible. Thus, one can show that Q is the most informative PBE in which there are three break points and both experts' messages are relevant. It can also be shown that it is not possible to create a fourth (or more) interior break point.

Even though the partition size is limited to that of the most loyal expert, it might still be possible that combining messages from the experts represents an informational improvement for the decision maker.

Comparing Q to P_1 we see that the more biased expert 2 distorts the third break point to the left, from $\frac{6}{10}$ to $\frac{41}{120}$. This reduces the information content in the right-most interval by introducing slack into expert 1's incentive constraint. Moreover, the leftward shift by expert 2 shifts all of the other break points to the left; thus it also distorts expert 1's break points to the left, from $\frac{3}{10}$ down to $\frac{23}{180}$ and from $\frac{1}{10}$ to $\frac{1}{72}$. The aggregate effect of these distortions is to reduce the expected utility of the decision maker and that of both experts.

Thus, we observe that *in the case of like biases the most informative monotonic equilibrium is generated by consulting the most loyal expert alone.*

To see why this argument generalizes, notice that for any PBE, if there is a break point where expert 1's incentive constraint holds with slack, it is possible to shift this break point to the right while still preserving incentive compatibility. By Assumption M, all of the other break points shift to the right as well and rightward shifts improve informativeness. Repeated application of this argument implies that the most informative equilibrium is one where there is no slack in any of expert 1's incentive constraints as points of discontinuity, but this is exactly the equilibrium condition for the single expert case given in CS. Thus, we have that the addition of one (or more) less loyal experts in the case of like biases can never help information transmission.

Formally,

Proposition 2 *Suppose that expert i has bias $b_i > 0$. Then the addition of another expert with bias $b_j \geq b_i$ is* ***never*** *informationally superior.*

Despite the fact that the messages of one expert can be used to discipline the incentives of the other expert to deviate, in the case of monotonic equilibria with like biased experts, this disciplining only has the effect of causing the incentive constraints to hold with slack. As we saw in the example, slack in the incentive constraints effectively shifts all of the break points to the left in any monotonic PBE and hence *reduces* information transmission.

Notice that this redundancy effect holds regardless of whether the committee is cohesive, in the sense the that the biases of the two experts are close to one another, or extremely diverse, in the sense that the less loyal expert is much more biased expert than the loyal expert.

5 Experts with Opposing Biases

Previously, we observed that a cabinet composed of two experts with like biases is no more effective than simply consulting the more loyal expert alone. In this section, we examine whether it is helpful for the decision maker to choose a cabinet where the experts biases oppose one another. Specifically, we study the case when the experts are biased in "opposite directions," that is, $b_i > 0 > b_j$. Now, while expert i still prefers a higher action than is ideal for the decision maker, expert j prefers a *lower* action: for all θ, $y^*\left(\theta, b_j\right) < y^*(\theta) < y^*(\theta, b_i)$. In effect, the experts want to tug the decision maker in opposite directions.

Recall from Proposition 1 that fully revealing PBEs do not exist. We argue below, however, that when experts have opposing biases it is possible to construct monotonic equilibria which are "semi-revealing" in the sense that the decision maker gets to know the true state over a *portion* of the state space. We then show that semi-revealing equilibria are informationally superior to the most informative single expert equilibrium. This construction requires, however, that the single expert not be an "extremist."

Extremists We will say that a right biased expert ($b_i > 0$) is an *extremist* if for all θ, $U(y^*(\theta), \theta, b_i) \leq U(y^*(1), \theta, b_i)$. Similarly, a left biased expert ($b_j < 0$) is an extremist if for all θ, $U\left(y^*(0), \theta, b_j\right) \geq U\left(y^*(\theta), \theta, b_j\right)$.

A right biased extremist is an expert whose bias is so high that no matter what the state he prefers the highest ideal action $y^*(1)$ to the ideal action $y^*(\theta)$. Similarly, a left biased extremist prefers the lowest ideal action $y^*(0)$ to $y^*(\theta)$.

In the uniform-quadratic case an expert is an extremist if $|b_i| \geq \frac{1}{2}$. Notice that if an extremist were to be consulted alone he would reveal no information whatsoever: in the single expert game the unique equilibrium involves only babbling.

5.1 Semi-Revealing PBE

With opposing biases there exist monotonic equilibria that are semi-revealing: a *continuum* of equilibrium actions are induced. Consider the uniform-quadratic case and suppose $b_1 < 0 < b_2 < \frac{1}{2}$.

Figure 2 depicts the outcome function Y associated with a PBE. As illustrated, in this equilibrium the state is completely revealed when it is below $1 - 2b_2$ and completely concealed otherwise. Thus for all states $\theta < 1 - 2b_2$, $Y(\theta) = \theta = y^*(\theta)$, the ideal action for the decision maker.

For all states $\theta \leq 1 - 2b_2$, the equilibrium strategies call for expert 1 to send the "true" message $m_1 = \theta$ and for expert 2 to "concur" by sending message $m_2 = m_1$. As long as expert 2 sends a message $m_2 < m_1 + 2b_2$ the decision maker follows expert 1's advice and chooses $y = m_1$. If expert 2 "disagrees" with expert 1 and sends a message $m_2 \geq m_1 + 2b_2$, the decision maker follows 2's advice and chooses $y = m_2$.

Notice, however, that if expert 1 were to "suggest" a lower action $y < \theta$ in state $\theta \leq 1 - 2b_2$ by sending the message $m_1 = y$, expert 2 would disagree. If $m_1 \leq \theta - b_2$ then expert 2 would disagree since he can induce his ideal action $y(\theta, b_2) = \theta + b_2$ by sending message $m_2 = \theta + b_2$. If $m_1 > \theta - b_2$ then expert 2 can induce $m_1 + 2b_2$ by disagreeing. This is indeed the best outcome 2 can obtain by disagreeing and is preferred to the action m_1 (since $m_1 + 2b_2 - (\theta + b_2) < (\theta + b_2) - m_1$). Hence, in either case, any attempt by expert 1 to deviate by suggesting a lower action will fail since expert 2 will disagree, thus

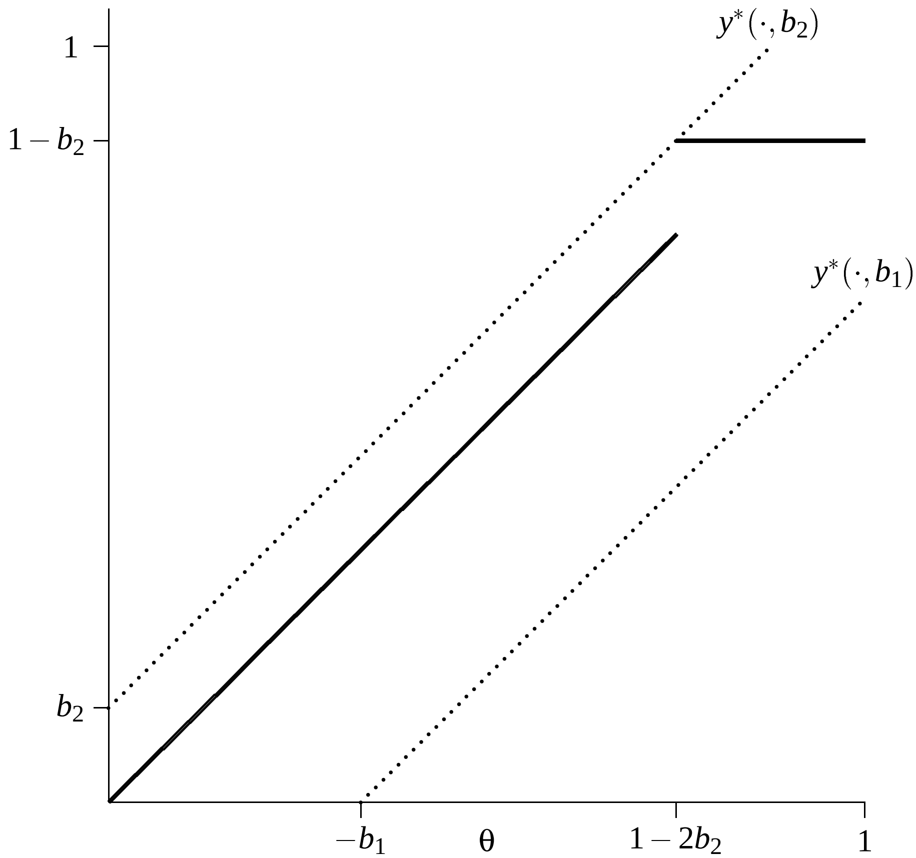

Figure 2: A PBE with Opposing Biases

saddling him with an even *higher* action than that called for in equilibrium. In this manner, it is possible to play the experts off against one another to obtain complete revelation in the interval $[0, 1-2b_2)$.

For states $\theta > 1-2b_2$, however, this construction fails since now there is no rationalizable action $z \leq 1$ such that expert 2 is indifferent between $y = \theta$ and z. Thus in states $\theta > 1-2b_2$ complete revelation is not possible. The equilibrium strategies call for expert 1 to suggest $m_1 = 1-b_2$. The decision maker follows 1's advice whenever $m_1 > 1-2b_2$ and expert 2's advice is irrelevant. If $m_1 \leq 1-2b_2$ then 2 disagrees and can induce the action 1.

It is useful to contrast the PBE constructed above with the equilibrium construction of Gilligan and Krehbiel (1989) (hereafter, GK). GK consider the uniform-quadratic case for experts with *equal* but opposing loyalties ($b_1 = -b_2$). Their construction also differs from ours in that they examine benefits to combining in a model in which experts' advice is given *simultaneously*. For extremely biased experts, $b_2 \in \left(\frac{1}{4}, \frac{1}{2}\right)$, the GK equilibrium construction yields a completely non-informative equilibrium: both experts babble. We have demonstrated that, when experts speak sequentially, it is still possible to obtain full revelation over the interval $[0, 1-2b_2]$.

With this result in hand, we turn to the general question of when a cabinet of advisors with opposing biases is helpful.

5.2 Choosing a Cabinet

We now show that the semi-revealing equilibrium constructed above is informationally superior to the most informative equilibrium with a single expert of bias b_2 as long as $0 < b_2 < \frac{1}{2}$.

Recall from CS (p. 1441) that any equilibrium partition P consists of the intervals $[a_0, a_1)$, $[a_1, a_2)$, ..., $[a_{n-1}, a_n)$, ..., $[a_{N-1}, a_N]$ where

$$a_n = \frac{n}{N} - 2n(N-n)\,b_2.$$

For $N \geq 2$, it is convenient to define

$$a_{N-1}(b_2) = \frac{N-1}{N} - 2(N-1)\,b_2$$

as the last break point in the partition P when the expert has bias b_2.

We will argue that for all N, $1-2b_2 > a_{N-1}$. This is certainly true for $N=1$ and for all $N \geq 2$ it is routine to verify that:[7]

$$\begin{aligned} 1-2b_2 &> \frac{N-1}{N} - 2(N-1)\,b_2 \\ &= a_{N-1}(b_2) \end{aligned}$$

The information partition Q generated by the semi-revealing equilibrium consists of singleton sets $\{\theta\}$ for all $\theta \leq 1-2b_2$ together with the set $(1-2b_2, 1]$. Any equilibrium information partition P generated by a single expert with bias b_2 consists of the intervals $[0, a_1)$, $[a_1, a_2)$, ..., $[a_{n-1}, a_n)$, ..., $[a_{N-1}, 1]$. Since $1-2b_2 > a_{N-1}(b_2)$, Q is clearly finer than P.

Finally, observe that the strategies of the semi-revealing equilibrium depend only on b_2 and are valid for all $b_1 < 0$ as long as $0 < b_2 < \frac{1}{2}$. The exact value of b_1 plays no role in the construction.

We have thus established:

Proposition 3 *Suppose that expert i has bias $b_i > 0$ and is not an extremist. Then the addition of another expert with $b_j < 0$ is* ***always*** *informationally superior.*

[7] Since $\frac{N-1}{N} < 1$ and $2(N-1)\,b_2 \geq 2b_2$.

Proposition 3 shows that whenever the more loyal expert is willing to reveal some information on his own, the addition of a second expert with opposing bias, regardless of how extreme, creates the possibility of an equilibrium that is strictly preferred by the decision maker and both of the experts.

Are there any informational gains from having a cabinet of extremists? In other words, is there any value to having an extreme diversity of opinion in the cabinet?

5.3 Extremists and the "Crossfire Effect"

Suppose, without loss of generality, that $b_1 < 0 < b_2$, then

Proposition 4 (Crossfire Effect) *If both experts are extremists, no information is transmitted in any monotonic PBE.*

Notice that since extremists never reveal any information when consulted alone, the Crossfire Effect also applies to the case of like biased experts.[8] Proposition 4 highlights the limits to the gains from multiple opposing experts illustrated in the example above. Recall that in the example the presence of expert j led expert i to reveal more information that he was willing to reveal on his own and vice-versa. Proposition 4 shows that this does not hold if both experts are sufficiently extreme in their biases. The key intuition in deriving this result is the importance of a "disagreement" action for expert 2. When experts are extremists, an appropriately constructed "disagreement" action exceeds the highest rationalizable action (namely, $y^*(1)$) on the part of the decision maker. Thus, there is no set of beliefs that the decision maker could announce that would lead expert 2 to anticipate such extreme actions being taken. As a consequence, the perfection refinement, this time as applied to the decision maker, constrains the informativeness of equilibrium.

Proposition 3 shows that balancing opposing non-extremists may result in each conveying more information than each would singly. In contrast, the message of Proposition 4 is that combining the advice of the two experts is of no value when both experts are extremists. Finally, we show that pairing an extremist with a non-extremist can lead to an interval in which the state is completely revealed only if the non-extremist is consulted second.

Example 3 Consider the uniform-quadratic case when $b_1 = -\frac{1}{2}$, $b_2 = \frac{1}{3}$. Then the semi-revealing PBE constructed earlier is preferred by all to consulting either expert singly.

Now suppose that the order of polling is reversed. This is equivalent to setting $b_1 = -\frac{1}{3}$ and $b_2 = \frac{1}{2}$. In this case, the unique monotonic equilibrium is babbling. To see this, temporarily suppose that all monotonic PBE consisted of finite equilibrium actions, then one can show that one of the experts must be indifferent at points of discontinuity. Then, since neither expert will reveal any information when polled alone, it follows that there can be no points of discontinuity where one of the experts is indifferent. We can use the first

[8]The television talk show *Crossfire* regularly pits an avowed right wing extremist against an avowed left wing extremist. The debate is singularly uninformative.

part of the proof of Proposition 4 to show that a continuum (or countably infinite number) of actions cannot occur. Finally, suppose that we are in the like bias case, that is $b_1 = \frac{1}{2}$, $b_2 = \frac{1}{3}$. From Proposition 2, we know that babbling is the only monotonic equilibrium with this cabinet composition.

Thus, we have shown that both the composition of the cabinet as well as the order of polling can have a profound impact on the information content of the most informative equilibrium.

6 Extensions

In this section, we indicate some possible extensions to our basic model.

Our assumption that both experts are perfectly informed about θ and that biases are commonly known ensures that any improvement in information from combining the advice of the experts arises solely from the strategic interaction. In our model, the introduction of a second expert may be useful, not because his information augments that held by the first expert, but because of the strategic interaction between them. Indeed, were the first expert simply to disclose his information honestly, introducing a second experts would obviously have no value.

Of course, in practice the information of experts is neither perfect nor identical. Hence, in addition to the strategic motives highlighted in this paper, familiar information aggregation motives are also likely to influence the optimal number and composition of experts. Thus, our model should be thought of as only a partial description of the problem of choosing a cabinet. Incorporating both motives would obviously enhance the realism of the model but at an increase in complexity that takes it beyond the scope of the present analysis. Such an extension also obscures circumstances in which the pure strategic interaction effect of the two experts is helpful and when it is not.

We also assumed that the two experts offer advice *sequentially* and speak exactly once. Obviously this is a departure from the reality of give-and-take discussions between decision makers and experts. One possibility that captures this "conversational" flavor of consulting experts is to model message sending in a manner analogous to continuous time bargaining games where, at each instant, any one of the parties is free to send a message. Obviously, this complicates equilibrium characterization significantly and remains for future research.

Another alternative to our extensive form is to model the messages of the experts as occurring simultaneously. It is straightforward to show that the most informative equilibrium under such an extensive form is full revelation. Thus, the introduction of a second expert has a dramatic effect on information transmission. This equilibrium, however, is not robust to a number of perturbations of the information structure and the extensive form of the game. For instance, were we to assume that instead of observing θ perfectly, the experts instead received noisy signals of θ, then full revelation is no longer an equilibrium even as the noise term becomes arbitrarily small. Sequential moves has the same effect, but is much more analytically tractable and preserves the original CS analysis as a special case.

We have relied in a significant way on the specific extensive form, in particular that the order in which the experts are polled is fixed. One might reasonably argue that equilibria should be robust to reversing the order of polling. Our conclusions about the like bias case are robust to this perturbation. One can also show that the opposite bias results are likewise robust. In particular, it is possible to construct equilibria in which (a) the strategies of the experts are independent of the order of polling; and (b) the information is superior to consulting either expert alone.[9]

Finally, our analysis only concerns itself with cabinets consisting of two experts. Obviously, the sequential framework we adopt is not particularly conducive to exercises where more and more experts are added. Nonetheless, we believe that the basic intuition that satisfying the incentive constraints of the most loyal agent leads to the most informative equilibrium in the like bias case will carry over into the n agent case. In the case of opposite bias, again it is the most loyal agent who determines the length of the revealing interval in our construction of a semi-revealing equilibrium. Thus, we expect that our construction would continue to be an equilibrium provided that the most loyal expert does not speak first. Whether this can be improved upon by combining the information of more experts remains an open question.

7 Conclusion

Self interested experts influence the decision process by strategically withholding information that they possess. In a single expert setting, less biased experts offer more precise information. However, we show that when there are multiple experts with like biases, the combining of advice from both experts is detrimental. Naturally, the less loyal expert offers less precise information. While this by itself is harmful to the decision process, it has a secondary effect. It also causes the more loyal expert to strategically respond by reducing the precision of the information he conveys. The presence of the disloyal advisor contaminates the advice of the loyal advisor. With like biases, a kind of “Gresham’s Law” operates: Bad advice drives out good advice. Thus, a single advisor is superior to a cabinet composed of advisors from the same side of the spectrum.

When experts have opposing biases, the situation is different. Now unless the more loyal expert is an extremist, combining his advice with that of a second advisor, even if less loyal, is always beneficial. The disloyal expert, even if of little use by himself, can be used to discipline the more loyal expert. With opposing biases, even bad advice can enhance good advice. Thus, a cabinet composed of advisors from opposite sides of the spectrum is superior to a single advisor. But there are limits to how much additional information may be garnered from a cabinet. Full revelation is still not possible. Moreover, if the cabinet consists of opposing extremists, no information is conveyed.

[9]The equilibrium construction is available upon request from the authors.

A Proofs

Proof of Proposition 1. Suppose not. Then there is a PBE in which the state is fully revealed and thus for all θ, the equilibrium action $Y(\theta) = y^*(\theta)$. We first consider the case of opposing biases.

Case 1: Opposing biases

First, consider the sub-case where $b_1 < 0 < b_2$.

Let $\overline{\theta} < 1$ be such that $y^*\left(\overline{\theta}, b_2\right) = y^*(1)$.

Let $\theta \in \left(\overline{\theta}, 1\right)$. Since $b_1 < 0$ we have that $y^*(\theta, b_1) < y^*(\theta)$. Choose a $\theta' > \theta$ close enough to θ so that $y^*\left(\theta', b_1\right) < y^*(\theta)$. Suppose that m_1 and m_2 are the equilibrium messages in state θ. Since the equilibrium is fully revealing, $y(m_1, m_2) = y^*(\theta)$.

Let $m_2' = \mu_2\left(\theta', m_1\right)$ be expert 2's best response to the message m_1 in state θ'. Then by definition, $U\left(y(m_1, m_2'), \theta', b_2\right) \geq U\left(y(m_1, m_2), \theta', b_2\right)$ and since $y(m_1, m_2) = y^*(\theta) < y^*(\theta, b_2) < y^*\left(\theta', b_2\right)$, $U_1\left(y(m_1, m_2), \theta', b_2\right) > 0$ and so $y(m_1, m_2') \geq y^*(\theta)$.

Next observe that $y(m_1, m_2') \geq y^*\left(\theta'\right)$. Suppose that $y(m_1, m_2') < y^*\left(\theta'\right)$. Then by sending the message m_1 in state θ' expert 1 can induce the action $y(m_1, m_2')$ and since $y^*\left(\theta', b_1\right) < y^*(\theta) \leq y(m_1, m_2') < y^*\left(\theta'\right)$ this is a profitable deviation for 1. This is a contradiction and so $y(m_1, m_2') \geq y^*\left(\theta'\right) > y^*(\theta)$.

By the definition of a PBE, it must be the case that the out of equilibrium action $y(m_1, m_2') \leq y^*(1) = y^*\left(\overline{\theta}, b_2\right)$. Now since $\overline{\theta} < \theta$, we also have $y(m_1, m_2') < y^*(\theta, b_2)$.

Thus we have deduced that $y^*(\theta) < y(m_1, m_2') < y^*(\theta, b_2)$. Since $b_2 > 0$, this implies that $U(y^*(\theta), \theta, b_2) < U(y(m_1, m_2'), \theta, b_2)$. But this contradicts the assumption that $y^*(\theta)$ is an equilibrium action in state θ. Thus full revelation cannot be an equilibrium.

The sub-case where $b_2 < 0 < b_1$ is treated similarly.

Case 2: Like Biases

The proof for the case of like biases is analogous. We omit the proof because, in the case of like biases, the conclusion also follows from a more general result to come (Lemma 2). ■

Proof of Lemma 1. In order to economize on notation, in what follows, we will denote $\theta - \varepsilon$ by θ^- and $\theta + \varepsilon$ by θ^+.

Case 1. $b_1 \leq b_2$.

To establish (4), suppose the contrary, that is, suppose $U(y^-, \theta, b_1) < U(y^+, \theta, b_1)$. Then by continuity, for all $\varepsilon > 0$ small enough,

$$U\left(Y\left(\theta^-\right), \theta^-, b_1\right) < U\left(Y\left(\theta^+\right), \theta^-, b_1\right). \tag{6}$$

Now suppose that in state θ^-, expert 1 were to send the message $m_1^+ = \mu_1\left(\theta^+\right)$ and let m_2 be expert 2's best response to this off-equilibrium message in state θ^- so that:

$$U\left(y\left(m_1^+, m_2\right), \theta^-, b_2\right) \geq U\left(y\left(m_1^+, m_2^+\right), \theta^-, b_2\right).$$

This implies that $y\left(m_1^+, m_2\right) \leq y\left(m_1^+, m_2^+\right)$ since otherwise we would have that $U\left(y\left(m_1^+, m_2\right), \theta^+, b_2\right) > U\left(y\left(m_1^+, m_2^+\right), \theta^+, b_2\right)$ contradicting the fact that $Y\left(\theta^+\right) = y\left(m_1^+, m_2^+\right)$ is the equilibrium action in state θ^+.

But now since $y\left(m_1^+, m_2\right) \leq y\left(m_1^+, m_2^+\right)$ and expert 2 weakly prefers the former in state θ^-, the fact that $b_1 \leq b_2$ implies that expert 1 also weakly prefers the former. Thus $U\left(y\left(m_1^+, m_2\right), \theta^-, b_1\right) \geq U\left(Y\left(\theta^+\right), \theta^-, b_1\right)$ and hence by (6)

$$U\left(y\left(m_1^+, m_2\right), \theta^-, b_1\right) > U\left(Y\left(\theta^-\right), \theta^-, b_1\right).$$

Thus by sending the message m_1^+ in state θ^- expert 1 can induce an action that he prefers to the equilibrium action. This is a contradiction and thus (4) holds.

To establish (5), again suppose the contrary, that is, $U\left(y^-, \theta, b_2\right) > U\left(y^+, \theta, b_2\right)$. Then since $b_1 \leq b_2$, $U\left(y^-, \theta, b_1\right) > U\left(y^+, \theta, b_1\right)$.

Then by continuity, for small enough $\varepsilon > 0$,

$$U\left(Y\left(\theta^-\right), \theta^+, b_1\right) > U\left(Y\left(\theta^+\right), \theta^+, b_1\right)$$

and

$$U\left(Y\left(\theta^-\right), \theta^+, b_2\right) > U\left(Y\left(\theta^+\right), \theta^+, b_2\right).$$

Hence if in state θ^+, expert 1 were to send the message $m_1^- = \mu_1\left(\theta^-\right)$ expert 2 will induce an action $y\left(m_1^-, m_2\right)$ that is strictly lower than $Y\left(\theta^+\right)$. This is a profitable deviation for 1 and hence a contradiction. Thus (5) holds.

Case 2. $b_1 \geq b_2$.

The proof for this case is similar. If either (4) or (5) does not hold then expert 1 has a profitable deviation. ■

Proof of Lemma 2. Let $\varepsilon = \min_j \min_\theta \left[y^*\left(\theta, b_j\right) - y^*\left(\theta\right)\right] > 0$.

Suppose $\theta' < \theta''$ are two states such that $Y\left(\theta'\right) \equiv y' < y'' \equiv Y\left(\theta''\right)$. Then there exist m_1', m_2' satisfying $m_1' = \mu_1\left(\theta'\right)$, $m_2' = \mu_2\left(\theta', m_1'\right)$ and $y\left(m_1', m_2'\right) = y'$ and similarly for the double primes. We will argue that $y'' - y' \geq \varepsilon$.

Suppose that $y'' - y' < \varepsilon$.

Let σ', σ'' be such that $y^*\left(\sigma'\right) = y'$ and $y^*\left(\sigma''\right) = y''$. Then clearly $\sigma' < \sigma''$.

CLAIM. $\sigma' \in Y^{-1}\left(y'\right)$ and $\sigma'' \in Y^{-1}\left(y''\right)$.

PROOF OF CLAIM. Let $\underline{\theta} = \min Y^{-1}\left(y'\right)$ and $\overline{\theta} = \max Y^{-1}\left(y'\right)$. Then $y^*\left(\underline{\theta}\right) \leq y' \leq y^*\left(\overline{\theta}\right)$. If $y' < y^*\left(\underline{\theta}\right)$ then $U\left(y', \underline{\theta}\right) < U\left(y^*\left(\underline{\theta}\right), \underline{\theta}\right)$ and since $U_{12} > 0$, for all $t \in \left[\underline{\theta}, \overline{\theta}\right]$, $U\left(y', t\right) < U\left(y^*\left(\underline{\theta}\right), t\right)$. If $y' > y^*\left(\overline{\theta}\right)$ a similar argument holds.

Now since $y^*\left(\cdot\right)$ is increasing, $\underline{\theta} \leq \sigma \leq \overline{\theta}$ and $Y\left(\cdot\right)$ is monotonic, $\sigma' \in Y^{-1}\left(y'\right)$.

This establishes the claim. □

Now since $U_1\left(y', \sigma'\right) = 0$, $U_{13} > 0$ implies that for $j = 1, 2$, $U_1\left(y', \sigma', b_j\right) > 0$ and since $y'' - y' < \varepsilon$, $U_1\left(y'', \sigma', b_j\right) > 0$ also. Similarly, since $U_1\left(y'', \sigma''\right) = 0, U_{13} > 0$ implies that $U_1\left(y'', \sigma'', b_j\right) > 0$ and since $y' < y''$, $U_1\left(y', \sigma'', b_j\right) > 0$ also.

Now let z' solve

$$U\left(y'', \sigma', b_2\right) = U\left(z', \sigma', b_2\right)$$

and let z'' solve

$$U\left(y'', \sigma'', b_2\right) = U\left(z'', \sigma'', b_2\right).$$

Since $U_1(y'', \sigma'', b_2) > 0$ and $U_{11} < 0$, $U_1(z'', \sigma'', b_2) < 0$ and so $z'' > y''$. Next since $U_{12} > 0$, $U(y'', \sigma'', b_2) < U(z', \sigma'', b_2)$ and so $z'' > z'$.

Now in state σ', if expert 1 sent the message m_1'' in lieu of m_1', then we claim that expert 2 could do no better than sending message m_2'' resulting in action y''. This is because all actions in the interval (y'', z'') cannot be induced by expert 2 following m_1'' that is, there does not exist an m_2 such that $y(m_1'', m_2) \in (y'', z'')$. If there were such a message then y'' would not be the equilibrium action in state σ''. Thus, following m_1'', no action greater than y'' is preferred by expert 2 to y''. Thus if expert 1 sends the message m_1'' in state σ', expert 2 will respond by sending the message m_2'', thereby resulting in action y''. This deviation is then profitable for expert 1. ■

Proof of Proposition 2. Suppose $a_1, a_2, ..., a_{n-1}$ are points where the function Y is discontinuous. Let $c = \min\{b_1, b_2\}$. Lemma 1 implies that these points satisfy the following system of inequalities

$$\begin{aligned}
(a_1 + c) - \frac{a_1}{2} &\leq \frac{a_1 + a_2}{2} - (a_1 + c) \\
(a_2 + c) - \frac{a_1 + a_2}{2} &\leq \frac{a_2 + a_3}{2} - (a_2 + c) \\
&\vdots \\
(a_n + c) - \frac{a_{n-1} + a_n}{2} &\leq \frac{a_n + a_{n+1}}{2} - (a_n + c) \\
&\vdots \\
(a_{N-1} + c) - \frac{a_{N-2} + a_{N-1}}{2} &\leq \frac{a_{N-1} + 1}{2} - (a_{N-1} + c)
\end{aligned}$$

This system is equivalent to

$$\begin{aligned}
a_1 &\leq \frac{a_2}{2} - 2c \\
a_2 &\leq \frac{a_1 + a_3}{2} - 2c \\
&\vdots \\
a_n &\leq \frac{a_{n-1} + a_{n+1}}{2} - 2c \\
&\vdots \\
a_{N-1} &\leq \frac{a_{N-2} + 1}{2} - 2c
\end{aligned}$$

which results in the following recursive system:

$$\begin{aligned}
a_1 &\leq \frac{1}{2} a_2 - 2c \\
a_2 &\leq \frac{2}{3} a_3 - 4c
\end{aligned}$$

$$
\begin{aligned}
&\vdots \\
a_n &\leq \frac{n}{n+1}a_{n+1} - 2nc \\
&\vdots \\
a_{N-1} &\leq \frac{N-1}{N} - 2(N-1)c
\end{aligned}
$$

Now let $\overline{a}_1, \overline{a}_2, ..., \overline{a}_{N-1}$ be the solution to the corresponding system of equations. Then clearly we have that $a_1 \leq \overline{a}_1, a_2 \leq \overline{a}_2, ..., a_{N-1} \leq \overline{a}_{N-1}$. We can now directly apply Theorem 4 of CS. This implies that the single expert equilibrium is informationally superior. ■

Proof of Proposition 4. We first show that a continuum of equilibrium actions cannot occur. We then establish that the only monotonic PBE with finite equilibrium actions involves babbling.

Consider an interval of states (σ, τ) such that for all $\theta \in (\sigma, \tau)$, $Y(\theta) = y^*(\theta)$ so that the state is completely revealed in this interval. Since expert 2 is an extremist, for all $\theta \in (\sigma, \tau)$, $U(y^*(\theta), \theta, b_2) < U(y^*(1), \theta, b_2)$. Hence, $y^*(\theta)$ must be the highest action inducible by expert 2 following $\mu_1(\theta)$.

Since $b_1 < 0$, for small $\varepsilon > 0$, $U(y^*(\theta), \theta, b_1) < U(y^*(\theta - \varepsilon), \theta, b_1)$. Hence, in state θ, if expert 1 plays $\mu_1(\theta - \varepsilon)$, expert 2 can do no better than to induce $y^*(\theta - \varepsilon)$ by playing $\mu_2(\theta - \varepsilon, \mu_1(\theta - \varepsilon))$, but this is a profitable deviation for expert 1.

Hence, no interval of the form (σ, τ) can exist and hence there cannot be a continuum of equilibrium actions. Essentially the same argument also rules out that there are a countable infinity of equilibrium actions.

Thus there must be a finite number of equilibrium actions.

Suppose Y has an upward jump at θ. Then since $b_1 < b_2$, Proposition 1 implies that $U(y^-, \theta, b_1) \geq U(y^+, \theta, b_1)$ and $U(y^-, \theta, b_2) \leq U(y^+, \theta, b_2)$.

CLAIM. If $\lim_{\varepsilon \downarrow 0} Y(\theta - \varepsilon) = y^- < \lim_{\varepsilon \downarrow 0} Y(\theta + \varepsilon) = y^+$,then

$$
\text{either } U\left(y^-, \theta, b_1\right) = U\left(y^+, \theta, b_1\right) \text{ or } U\left(y^-, \theta, b_2\right) = U\left(y^+, \theta, b_2\right).
$$

PROOF OF CLAIM. Suppose neither is an equality. Since for all small $\varepsilon > 0$, $Y(\theta - \varepsilon) < y^*(\theta - \varepsilon)$ the fact that expert 2 is an extremist then implies that $U(Y(\theta - \varepsilon), \theta - \varepsilon, b_2) < U(y^*(1), \theta - \varepsilon, b_2)$. Therefore the highest action that 2 can induce following the message $\mu_1(\theta - \varepsilon)$ is $Y(\theta - \varepsilon)$. Now in some state $\theta + \varepsilon$ if 1 were to send the message $\mu_1(\theta - \varepsilon)$ this would result in the action $Y(\theta - \varepsilon)$ and would be a profitable deviation for 1. Since this is a contradiction, the claim is established. □

Thus we have argued that at every point of discontinuity at least one expert is indifferent between y^- and y^+.

Finally, we establish that points of discontinuity where one of the experts is indifferent cannot occur in any monotonic PBE. Suppose that the contrary is true, then there exist at least three break points, a_{n-1}, a_n and a_{n+1}, such that

$$
U(\bar{y}(a_{n-1}, a_n), a_n, b_i) = U(\bar{y}(a_n, a_{n+1}), a_n, b_i) \tag{7}
$$

where $\bar{y}(\sigma, \tau)$ is the action that maximizes $E[U(y, \theta) | \theta \in (\sigma, \tau)]$

First, we show that $a_{n-1} > 0$ and $a_{n+1} < 1$. Suppose that $a_{n-1} = 0$, then we have that $U(\bar{y}(0, a_n), a_n, b_i) = U(\bar{y}(a_n, a_{n+1}), a_n, b_i)$.

For $\alpha \in (0,1)$ define $\beta(\alpha)$ by

$$U(\bar{y}(0, \alpha), \alpha, b_i) = U(\bar{y}(\alpha, \beta(\alpha)), \alpha, b_i).$$

But since the most informative equilibrium with expert i alone involves no information revelation we know that for all $\alpha \in (0,1)$, $\beta(\alpha) > 1$ contradicting (7). Thus, $a_{n-1} > 0$. A similar argument shows that $a_{n+1} < 1$.

Now for $\varepsilon > 0$ define $\phi(\varepsilon)$ by

$$U(\bar{y}(a_{n-1} - \phi(\varepsilon), a_n), a_n, b_i) = U(\bar{y}(a_n, a_{n+1} + \varepsilon), a_n, b_i).$$

Note that $\phi(\varepsilon)$ is well-defined since U is concave and $\bar{y}$ is increasing in both arguments. Furthermore, ϕ is increasing. Let $\bar{\varepsilon} = \min\left\{1 - a_{n+1}, \phi^{-1}(a_{n-1})\right\}$. Thus either $a_{n+1} + \bar{\varepsilon} = 1$ or $a_{n-1} - \phi(\bar{\varepsilon}) = 0$. This contradicts the first observation.

Hence, there are no points of discontinuity at which one of the experts is indifferent. The only remaining monotonic PBE is babbling by both experts. ■

B Non-monotonic Equilibria

Throughout, we have restricted attention to the case where equilibria were monotonic. Indeed, our results on welfare analysis for like biased experts relied essentially on this. We now study some features of non-monotonic equilibria and provide some sufficient conditions for all equilibria to be monotonic for like biased experts in the uniform-quadratic case.

We begin by presenting an explicit example of a non-monotonic equilibrium.

B.1 Example of Non-monotonic Equilibria

Example 4 Once again, consider the uniform quadratic case. Suppose that $b_1 = \frac{11}{160}$ and $b_2 = \frac{3}{20}$ are the biases of the two experts. A PBE for this game is depicted in Figure 3, where the states $a_1 = .1$, $a_2 = .28$, $a_3 = .34$ and the actions $y_1 = .1475$, $y_2 = .19$ and $y_3 = .67$.

The outcome function Y associated with this equilibrium is depicted above and is clearly non-monotonic. Notice that in state a_1 expert 1 is indifferent between y_1 and y_2; hence, for all $\theta > a_1$, expert 1 prefers y_2 to y_1. Likewise, in state a_3, expert 1 is indifferent between y_1 and y_3. Finally, in state a_2 expert 2 is indifferent between y_2 and y_3.

To induce action y_2, expert 1 must suggest the action $m_1 = y_2$, and expert 2 must agree. If, on the other hand, expert 2 disagrees, action y_3 is induced. Since expert 2 (weakly) prefers y_2 to y_3 if and only if $\theta \leq a_2$, then when expert 1 suggests y_2, expert 2 will "agree"

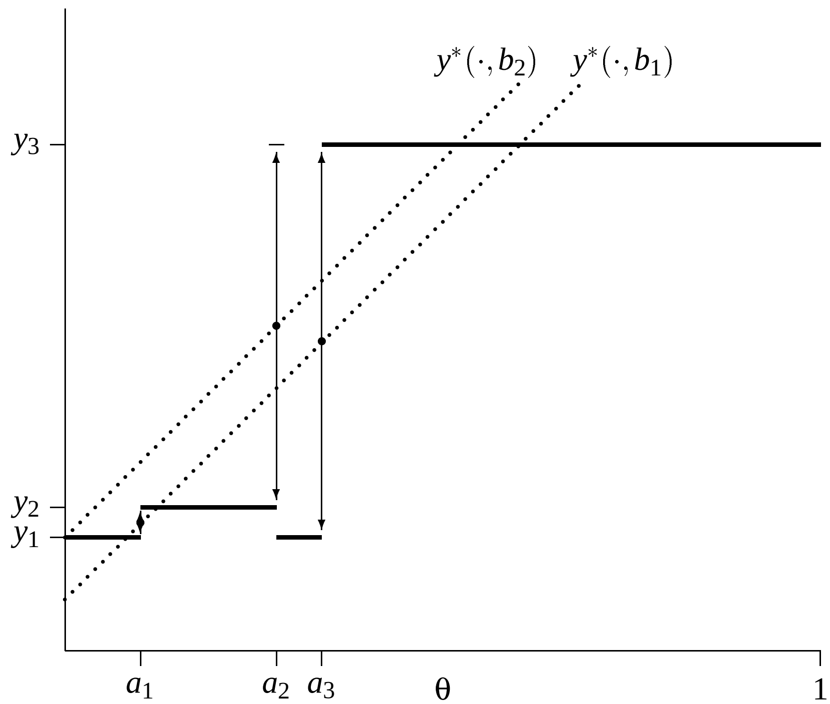

Figure 3: A Non-monotonic PBE

only $\theta \leq a_2$. Thus, despite the fact that both experts prefer y_2 to y_1 for $\theta \in (a_2, a_3)$, expert 1 cannot obtain y_2 since expert 2 will then "disagree" and induce the higher action y_3 that he prefers to y_2. It is the threat of "overshooting" by the more biased expert 2 that sustains the downward jump in the outcome function despite the fact that both experts and the decision maker prefer the higher action y_2 to y_1 when $\theta > a_2$.

B.2 Sufficient Conditions for Monotonicity

We now turn to establishing sufficient conditions for equilibria to be monotonic when experts have like biases.

Lemma 3 *There exists a $\overline{\theta} < 1$ such that $Y(\cdot)$ is monotone over $\left[\overline{\theta}, 1\right]$.*

Proof. Observe that since $U_1(y^*(1), 1) = 0$, and $U_{13} > 0$, $U_1(y^*(1), 1, b_i) > 0$. Let $\overline{\theta} = \inf\{\theta : U_1(y^*(1), \theta, \min\{b_1, b_2\}) > 0\}$. Since $U_1(y^*(1), 1, b_i) > 0$ for $i = 1, 2$, it follows that $\overline{\theta} < 1$. Then for all $\theta > \overline{\theta}$ and all $y \leq y^*(1)$, $U_1(y, \theta, b_i) > 0$ for $i = 1, 2$.

Suppose there exist states θ', θ'' satisfying $\overline{\theta} < \theta' < \theta''$ such that $Y(\theta') = y' > Y(\theta'') = y''$. Suppose (m_1', m_2') is sent in state θ' and $y' = y(m_1', m_2')$. Similarly, suppose (m_1'', m_2'') is sent in state θ'' and $y'' = y(m_1'', m_2'')$. Observe that since y' and y'' are equilibrium actions $y' \leq y^*(1)$ and $y'' \leq y^*(1)$.

Now suppose that expert 1 deviates and sends message m_1' in θ''. Now if expert 2 sends m_2', then action y' occurs and, since $\theta'' > \overline{\theta}$, it follows that

$$U\left(y'', \theta'', b_i\right) < U\left(y', \theta'', b_i\right)$$

for $i = 1, 2$ since $y' \leq y^*(1)$. Thus expert 2's best response to m_1' in state θ'' must yield him at least $U\left(y', \theta'', b_2\right)$. Since $U_1\left(y', \theta'', b_2\right) > 0$ this best response, say m_2''', cannot result in an action $y(m_1', m_2''') < y'$. Thus $y(m_1', m_2''') > y'$ and since $U_1\left(y', \theta'', b_1\right) > 0$,

$$U\left(y\left(m_1', m_2'''\right), \theta'', b_1\right) > U\left(y', \theta'', b_1\right).$$

Thus it is profitable for 1 to deviate to m_1' in state θ'' and we have obtained a contradiction. ■

For the uniform-quadratic case, we can extend Lemma 2 so that the assumption of monotonicity is unnecessary.

Lemma 4 *In the uniform-quadratic case with like biased experts, there are a finite number of equilibrium actions in any PBE.*

The proof is tedious and offers no new insights on the problem, so it is omitted. It is available upon request from the authors.

Lemma 5 *In the uniform-quadratic case, suppose that Y has a downward jump at θ, that is,* $\lim_{\varepsilon\downarrow 0} Y(\theta - \varepsilon) = y^- > \lim_{\varepsilon\downarrow 0} Y(\theta + \varepsilon) = y^+$. *Then*

$$y^- - y^+ \leq 2\left|b_2 - b_1\right|.$$

Proof. Let θ be the largest state at which there is a downward jump in Y. Such a θ exists since Y is eventually monotone (Lemma 3) and there are only a finite number of equilibrium actions (Lemma 4).

Suppose $b_1 < b_2$. As before, in what follows, we will denote $\theta - \varepsilon$ by θ^- and $\theta + \varepsilon$ by θ^+.

First observe that there does not exist a state $\sigma > \theta$ such that $Y(\sigma) = y^-$. To see this note that if $\tau = \sup\{t : Y(t) = y^+\}$ then by Lemma 1 $U(y^+, \tau, b_1) \geq U(y^-, \tau, b_1)$ (where we use the fact that the conclusion of Lemma 1 holds as long as Y is monotonic on the interval $[\theta, 1]$). Hence there is an $\varepsilon > 0$ small enough so that in state θ^-, $U\left(y^+, \theta^-, b_1\right) > U\left(y^-, \theta^-, b_1\right)$. If in state θ^-, expert 1 were to send the message $m_1 = \mu_1\left(\theta^+\right)$ then expert 2 cannot do better than to send the message $m_2 = \mu_2\left(\theta^+, \mu_1\left(\theta^+\right)\right)$. This is a profitable deviation for 1.

Thus we know that $y^- \leq y^*(\theta)$ since otherwise the action y^- could not be a best response to any beliefs held by the decision maker. Thus, $U(y^-, \theta, b_1) > U(y^+, \theta, b_1)$.

Suppose in some state θ^+ expert 1 were to send the message $m_1 = \mu_1\left(\theta^-\right)$. Then there must be a message m_2 such that $y(m_1, m_2) = z$ (say) is such that $U\left(z, \theta^+, b_1\right) < U\left(y^+, \theta^+, b_1\right)$ and so by continuity $U(z, \theta, b_1) \leq U(y^+, \theta, b_1)$. In the uniform-quadratic case this reduces to

$$z - (\theta + b_1) \geq (\theta + b_1) - y^+ \tag{8}$$

But for m_2 to be a best response for expert 2 in state θ^+ requires that $U\left(z, \theta^+, b_2\right) \geq U\left(y^-, \theta^+, b_2\right)$. On the other hand, the equilibrium condition implies $U\left(z, \theta^-, b_2\right) \leq U\left(y^-, \theta^-, b_2\right)$. Thus, $U(z, \theta, b_2) = U(y^-, \theta, b_2)$. In the uniform-quadratic case, this reduces to

$$z = 2(\theta + b_2) - y^- \tag{9}$$

Combining (8) and (9) yields the required inequality.

The proof for the case when $b_1 > b_2$ is similar. ■

Using Lemma 4, we are now in a position to state a sufficient condition for monotonicity of all PBE in case of like biases. We show that if the first expert is more biased than is the second, *all* equilibria are monotonic.

Proposition 5 *In the uniform-quadratic case, if $b_1 \geq b_2 > 0$ then all PBE are monotonic.*

Proof. Let θ be the largest state at which there is a downward jump in Y. Such a θ exists since Y is eventually monotone (Lemma 3) and there are only a finite number of equilibrium actions (Lemma 4). Define y^- and y^+ as follows:

$$\lim_{\varepsilon\uparrow 0} Y(\theta - \varepsilon) = y^- > \lim_{\varepsilon\downarrow 0} Y(\theta + \varepsilon) = y^+$$

As before, in what follows, we will denote $\theta - \varepsilon$ by θ^- and $\theta + \varepsilon$ by θ^+.

There are two cases to consider. First, suppose that $U(y^-, \theta, b_1) \geq U(y^+, \theta, b_1)$. Then for $\varepsilon > 0$ small enough, $U\left(y^-, \theta^+, b_1\right) > U\left(y^+, \theta^+, b_1\right)$. In state θ^+ if 1 sends the message $m_1 = \mu_1\left(\theta^-\right)$, then 2 cannot do better than to send $m_2 = \mu_2\left(\theta^-, \mu_1\left(\theta^-\right)\right)$ resulting in y^-. This is a profitable deviation for 1.

Next, suppose that $U(y^-, \theta, b_1) < U(y^+, \theta, b_1)$. Define $\tau = \sup\{\sigma : Y(\sigma) = y^+\}$. Notice that $\tau < 1$ and there exists a $\sigma > \tau$ such that $Y(\sigma) = y^-$. If $U(y^-, \tau, b_1) > U(y^+, \tau, b_1)$ then sending message $\mu_1\left(\theta^-\right)$ in state τ induces action $Y\left(\theta^-\right) = y^-$ which is a profitable deviation. If $U(y^-, \tau, b_1) \leq U(y^+, \tau, b_1)$, then since $y^*(\tau) \geq y^+$ and thus $y^*(\tau, b_1) > y^+$, we have in the uniform-quadratic case

$$y^- - y^+ \geq 2b_1$$

But this contradicts Lemma 5 and thus Y has no downward jumps. ■

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