A Dynamic Tiebout Theory of Voluntary versus Involuntary Provision of Public Goods

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ABSTRACT

This paper considers a dynamic model of Tiebout-like migration between communities that utilize distinct allocation procedures for public goods. At issue is whether voluntary or compulsory procedures are more likely to prevail over time. We model infinitely lived individuals who make repeated, sequential location decisions over one of two communities. Each community uses a distinct mechanism for allocating public goods. The first is one in which contributions are given voluntarily by the citizenry of the community. The second is a compulsory scheme by which individuals are taxed proportionately to wealth with the tax determined by a majority vote. Opportunities to accumulate wealth exist via accumulation of public capital.

The Markov Perfect equilibria of the dynamic game are studied. Our main result shows that when accumulated wealth converges to a steady state, individuals' locational choices eventually "select" the involuntary provision mechanism. This holds despite the fact that unanimous location in the voluntary provision community may in many cases *remain as a Nash equilibrium of the static game each period*. We also describe conditions under which voluntary provision survives. These conditions require that accumulation of capital fails to decrease wealth dispersion over time. The results are shown to be consistent with findings relating inequality to school choice.

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1 Introduction

There are two major problems associated with public goods provision. One is the standard free rider problem. This problem exists when provision is voluntary rather than coercive. Since individual incentives lead typically to underprovision of the good, individuals may prefer to submit to some degree of centralized coercive provision. A common way to accomplish this is through involuntary taxation which is decided by some voting rule. However, this leads to a second potential problem. Even if free riding is be overcome, heterogeneous individuals will by no means agree on the appropriate output and contributions. Hence, there is a problem of conflicting interests. Individuals in different income groups may have widely varying preferences on the appropriate tax rate. An individual may be compelled to contribute at, what is for him, a highly undesirable tax rate.

Given these problems, the tradeoff between voluntary provision and coercive/involuntary provision is clear. Involuntary provision may overcome free riding, but creates potential conflicts in choosing tax rates. By contrast, voluntary provision does not compel anyone to contribute, but it cannot prevent free riding. In either case, if an individual does nor like the current provision level and contribution, he can "vote with his feet" by choosing to move elsewhere. This paper is concerned with how migration or "voting with one's feet" affects the viability of these two mechanisms.

Since the influential paper of Tiebout (1956),[1] models of local public goods entail that individuals migrate to determine the types and spatial distributions of local public tax and expenditure policies offered in society. It is often assumed that each community has a local "social planner" that responds optimally to migratory pressures given the policies offered by other communities. Moreover, institutions for provision of public goods are modelled as being homogeneous across communities and clubs. However, this is not necessarily or even typically the case. In the U.S., most small communities and suburbs have a voluntary fire department; at the same time, in larger cities fire protection is funded from the tax base. Public schools rely on property or income taxes, while private school tuition is subsidized by charitable donations. Some churches rely on member donations while others impose income tax (i.e., means-tested membership fees) on the membership.

The magnitudes of funding from these different provision institutions are large. Volun-

[1]See also, for example, Greenberg (1983), Wooders (1978, 1988, 1989), and Scotchmer and Wooders (1987).

tary money donations in the U.S. measured 96 billion dollars in 1989 with approximately the same amount constituting the value of donated time. The total measures almost 2% of GDP.[2] These contributions go to fund public monuments, political parties, homeless shelters, after-school programs for children, free medical clinics, art institutes, community orchestras, neighborhood crime-watch organizations, and day care and pre-schools programs. By contrast, state and local purchases (excluding transfers) constitutes approximately 12% of GDP.[3] For purposes of comparison, in the consolidated government in the US, defense and other purchases are 52% of the entire budget.[4] Since government's share of GDP is about 35%, purchases of goods and services constitute 18.5% of GDP. We take this number as an upper bound on the amount of public goods provided by compulsory means. Notice that while involuntary provision of public goods is larger than voluntary provision, voluntary provision constitutes a sizeable fraction of GDP.

Evidence that differentiation in provision matters in mobility decisions can be found in White (1980) and Clark (1986). According to Clark: "The *public good* component is especially important in choosing a destination city among several alternatives in the same region" (italics ours).[5] From examining patterns of residential migration as part of the U.S. Census, White (1980) observed: "The early 20th century model of the metropolis in which political, economic, and residential systems were coextensive has been largely replaced by a 'vote with your feet' residential system where the choice of neighborhood of residence not only implies a certain composition of the housing stock and social composition of the neighbors, but also participation in a *differentiated political structure*." (p. 263. Italics ours)

This paper therefore introduces a theory of spatial migration between communities which are distinguished by the political structures they use to provide these goods. We are less interested in what these mechanisms do for migration than what migration does for the mechanisms. The question we address is: what kinds of provision mechanisms stand up to migratory forces? Are voluntary or involuntary procedures likely to prevail over time? In other words, we want a theory which can explain the relative sizes of voluntary and involuntary contributions to public goods within an economy. Finally, what is the role of dynamic forces such as capital accumulation in the "selection" process?

To simplify the analysis we examine migration between two communities, each endowed

[2]See Schiff (1990).

[3]See Barro (1993).

[4]See Auerbach and Kotlikoff (1995).

[5]p.67. Clark notes that, generally, detailed survey data on mobility decisions are scarce.

with a distinct mechanism for allocating public goods. In the first, contributions toward a local public good are given voluntarily by the citizenry or membership of the community. In the second, individuals are taxed proportionately to wealth with the tax determined by the majority voting. Individuals differ only by having different wealth endowments. We allow for the possibility that congestion within a community diminishes the public good. Section 2 first introduces the spatial model.

A companion paper (Glomm and Lagunoff (1997)) characterizes equilibria of the static version of this dual location scenario. There it is shown that multiple equilibria typically exist, each supporting one or the other provision mechanism.[6] A primary reason for the multiplicity is that the static model cannot preclude simple coordination problems. For example, a person will not move from a large community to a small one even if the smaller community is more closely aligned with the individual's preferences. Even if the small community solves, say, the free rider problem with desireable tax rates, its aggregate wealth is too low to offer an attraction. This is due to the nonrivalrousness of the good.

To address this problem, the present paper examines an intertemporal model of migration. This model is introduced in Section 3. Individuals repeatedly make sequential location decisions over communities. Opportunities to accumulate wealth exist through accumulation of public capital which is also consumed. Human capital or infrastructure are examples that have this feature. We examine Markov Perfect equilibria of the dynamic game. Which mechanisms survive depend on the degree of conflict of interests generated by the accumulation process over time. Our main result shows that if congestion effects are not too strong, and if capital accumulation in a community converges to a steady state level, then involuntary provision *survives uniquely*. This means that all individuals in society eventually move to the community with involuntary provision and remain there permanently.

The mechanics of the model show how societies without large inequities in income could agglomerate eventually to communities with involuntary rather than voluntary procedures. We also provide examples in which the *opposite* is true if the accumulation process does not converge. Namely, when the accumulation technology admits sufficient income dispersion, then voluntary provision may be better suited to those whose preferences differ widely from that of the median voter. In Section 4 we relate these results to some recent findings on wealth distribution and private versus public school choice. While private schooling also has a strong private component, the local externality generated by one's education generates

[6]Stratified equilibria between the two mechanisms also exist under certain conditions but were shown to be comparatively rare.

potential free frider problems similar to other voluntary mechanisms. The results are roughly consistent with these findings, bearing in mind the imperfect link between schooling choices and choices between mechanisms examined in the model.

Of course, the actual mechanics are meant to be more illustrative than predictive. To understand why these mechanics work and how the multiplicity issue is resolved, recall the tradeoff between the two mechanisms. Free riding occurs in the voluntary community; conflicting interests occur in the coercive community. However, the accumulation process mitigates the problem of conflicting interests because individual wealth levels converge. Specifically, there is a maximal steady state wealth level which is attained aymptotically by all individuals if they unanimously locate in the taxation community. But with identical wealth levels, all individuals' preferences over tax rates coincide, and so unanimous location in the taxation community comes to Pareto dominate location in the voluntary one. Despite this, unanimous location in the voluntary community often remains a Nash equilibrium in the static game — a potential coordination failure. The dynamics overcome this pitfall because decisions are sequential rather than synchronous. In the model, no two individuals can perfectly coordinate the timing of their actions. With asynchronously repeated decisions, if everyone locates in the community with involuntary provision, then no one wishes to be the first to depart. Yet, if from that point onward no one leave this community, then sufficiently patient individuals will sequentially move there, anticipating that unanimous location in the involuntary provision community is an "absorbing state" (backward induction). Unlike the behavior, Markovian or otherwise, in standard repeated/dynamic games, an individual will not choose an inferior location simply because he expects that others will do the same.

We caution that the analysis is not intended to suggest that migration is exclusively or even primarily driven by public goods provision. Rather it is meant to demonstrate how migration clearly exposes strengths and weaknesses in each of the mechanisms. Migration weakens support for the voting mechanism when there are widely varying preferences over the appropriate tax rate. Migration weakens support for the voluntary mechanism when the free rider problem is most intense. Changing wealth distribution alters the relative strengths and weaknesses of each mechanism. In Section 5 we discuss related literature. Finally, Section 6 contains an Appendix with the proofs of the main results.

2 The Model

There is a finite number of individuals denoted by the set $I = \{1, \dots n\}$. Individual $i \in I$ is distinguished by a wealth endowment z_i. The individuals are ordered so that $z_1 < z_2 < \cdots < z_n$. The aggregate income is given by $Z = \sum_{i \in I} z_i$. The vector of wealth levels, in ascending order is given by $\boldsymbol{z} = (z_1, \dots, z_n)$.

Each individual consumes two goods. One of the goods is private or "fully rivalrous." The other is a local public good. An individual i's preferences are given by $u(z_i - y_i, G)$ where G is the quantity of the public good produced, and y_i is the amount of the voluntary contribution to the public good from individual i. The utility function u is assumed to be strictly increasing and concave.

There are two communities, C and D, in which individuals in this society can reside. In community D the public good is produced with voluntary contributions from the membership. In community C the good is provided by a simple proportional tax on wealth determined by a majority vote. These mechanisms are defined formally below. For each community let $I_k \subseteq I$ denote the set of individuals located in $k = C, D$. For each $k = C, D$, aggregate income is given by $Z_k = \sum_{i \in I_k} z_i$, and the aggregate contributions in k are given by $Y_k = \sum_{i \in I_k} y_i$.

In each community the public good G_k is produced according to technology $G_k = F(Y_k, |I_k|)$ where $|I_k|$ is the number of citizens in location k. We allow for the possibility that the good is not completely non-rivalrous. Congestion may diminish the amount of public good available for consumption. Specifically, we assume:

(A1) F is increasing and concave in Y,

(A2) F is weakly decreasing in $|I_k|$,

(A3) for each $y > 0$, $F(my, m)$ is increasing in m.

The first assumption is standard. The second reflects the potential for congestion. Larger population sizes reduce output (e.g., fire protection is more costly to provide in larger cities). The final assumption limits the extent of congestion. Proportional scaling up of the public good still outweighs the effect of congestion. In particular, this assumption prevents the public good from merely becoming a private good in large populations. Hence, we rule out the case of pure redistribution, i.e., $G = Y/n$ since we wish maintain increasing returns from having others in a community. A special case that conforms to our assumptions is the case of a pure public good, i.e., $G = F(Y, n) = F(Y, m)$ for all m, n.

The voluntary and involuntary provision mechanisms are formally specified as follows. In the location D all decisions are completely decentralized and made privately and voluntarily. Each individual jointly consumes $G_D = F(\sum_{i \in I_D} y_i, |I_D|)$ where y_i is i's voluntary contribution. The second location is assumed to allocate the public good via majority rule. We assume that a uniform tax rate, τ, is levied in community C and determined by a majority vote. Each individual $i \in I_C$ pays $y_i = \tau z_i$ and jointly consumes $G_C = F(\tau Z_C, |I_C|)$. Clearly, the restriction to these two mechanisms puts us in a "second best" world. Still, they are both commonly observed in local public goods provision. Examples include neighborhood watch groups versus police protection, and voluntary versus municipal fire departments, and charitable versus city-run homeless shelters. While these "mechanisms" often coexist, we focus attention on the polar case in which only or the other is used in a community.

We assume that each individual may choose which community to join. This decision, when aggregated with other individuals' choices of which community join, ultimately determines whether one or the other or both provision mechanisms survive. The location decision of individual i is given by $s_i \in \{D, C\}$. The *location profile* s is defined by $s = (s_1, \dots s_n)$ with $s \backslash \hat{s}_i \equiv ((s_j)_{j \neq i}, \hat{s}_i)$. Define the indirect utility $\bar{u}$ of a location profile s given wealth profile $\boldsymbol{z}$ by

$$\bar{u}_i(s; \boldsymbol{z}) = \begin{cases} V_i(s; \boldsymbol{z}) \equiv \max_{y_i \leq z_i} u_i(z_i - y_i, F(\sum_{j \in I_D} y_j, |I_D|)\ ) & if \quad s_i = D \\ W_i(s; \boldsymbol{z}) \equiv u_i((1 - \tau(\boldsymbol{z}, s)) z_i, F(\tau(\boldsymbol{z}, s) Z_C, |I_C|)\ ) & if \quad s_i = C \end{cases} \tag{1}$$

where $\tau(s, \boldsymbol{z})$ is the tax rate which is most preferred by the median voter or, more precisely, the voter with the median income in C. The tax rate $\tau(s, \boldsymbol{z})$ is the rate which would be chosen by majority rule. Strict concavity of u and continuity of u in τ guarantee that each individual has a unique most preferred tax rate and all preferences over tax rates are single peaked. A standard result by Black guarantees that with single peaked preferences over tax rates a majority voting outcome has a solution — the median type's most preferred tax rate. Monotonicity in wealth of the most preferred tax rate ensures that the person with the median wealth is the median voter. Functions V and W denote the indirect utilities of consumption in voluntary and involuntary communities, resp.

A *(static) Nash equilibrium in locations* is a location profile s such that

$$W_i(s; \boldsymbol{z}) \geq V_i(s \backslash D; \boldsymbol{z}),\ \forall i \in I_C. \tag{2}$$

and

$$V_i(s; \boldsymbol{z}) \geq W_i(s \backslash C; \boldsymbol{z}),\ \forall i \in I_D. \tag{3}$$

Generally, Nash equilibria in locations need not exist.[7] However, if congestion is not excessive, then there is always an equilibrium in which everyone locates in the voluntary region. To see why, observe that unanimous location in D is an equilibrium if $V_i(D, \ldots, D; \boldsymbol{z}) \geq W_i((D, \ldots, D) \backslash C; \boldsymbol{z})$. Given definitions of V and W this becomes

$$\begin{aligned} V_i(D, \ldots, D; \boldsymbol{z}) \;&=\; \max_y u(z - y, F(\sum_{j \neq i} y_j + y, n)) \\ > \;\max_y u(z - y, F(y, 1)) \;&=\; \max_\tau u((1-\tau)z, F(\tau z, 1)) \;=\; W_i((D, \ldots, D) \backslash C, s_i; \boldsymbol{z}) \end{aligned} \tag{4}$$

Let y^* be the optimal contribution for the indivdual if he moves to C by himself. Let $\bar{y}$ be his voluntary constribution in D. One can verify that $\sum_{j \neq i} y_j + \bar{y} \geq y^*$. Intuitively, the reason is that since an individual can always replicate in community D the contribution which he would tax himself in C, the aggregate constributions Y_D in D must exceed y^*. Hence, if $F(Y_D, n) \geq F(y, 1)$ (i.e., if congestion is not excessive) then inequality (4) holds. One can easily verify that this equilibrium exists, for example, in the case of pure public goods, i.e., $F(Y, n) = F(Y, m)$ for all m, n.

Homogeneous Individuals

An important special case is one in which the population is homogeneous: $z_i = z_j = z,\ \forall i, j$. In this case, the strategic environment reduces to a *coordination game*. A coordination game is a normal form game with possibly multiple, Pareto ranked equilibria, one of which Pareto dominates every other payoff in the game. With a homogeneous population, there is always an equilibrium in which all individuals locate in C. To see this observe

$$\begin{aligned} W_i(C, \ldots, C; \boldsymbol{z}) \;&=\; \max\nolimits_\tau u((1-\tau)z, F(\tau Z_n, n)) \\ > \;\max\nolimits_\tau u((1-\tau)z, F(\tau z, 1)) \;&=\; \max\nolimits_y u(z - y, F(y, 1)) \;=\; V_i((C, \ldots, C) \backslash D; \boldsymbol{z}) \end{aligned} \tag{5}$$

where the inequality follows from assumption (A3). Clearly, in a homogeneous society, every individual's preferred tax rate coincides with that of the median voter. Note that choosing

[7] We exclude mixed strategies.

an optimal tax rate is equivalent to choosing a contribution simultaneously for all members of society. It also turns out that the profile $(C,\ldots,C)$ is unanimously preferred to any other profile. To see why, notice that by (A3),

$$\max_{\tau} u((1-\tau)z, F(\tau Z_n, n)) \;>\; \max_{\tau} u((1-\tau)z, F(\tau Z_m, m)) \tag{6}$$

for any $m < n$, implying that unanimous location in C is preferable to joining a community C with fewer members. Also,

$$\max_{\tau} u((1-\tau)z, F(\tau Z_n, n)) = \max_{y} u(z-y, F(ny, n)) \;>\; \max_{y} u(z-y, F((m-1)\bar{y}_m + y, m)) \tag{7}$$

for any $m \leq n$ and where $\bar{y}_m$ is the symmetric equilibrium contribution toward the public good when m identical individuals belong to community D. This condition states that unanimous location in C is unanimously preferred to any payoff from locating in D. Intuitively, the reason is that the marginal effect of one's preferred contribution in C is multiplied n-fold. This is not so in community D where a classic free rider problem exists.

Observe that the inequalities in (5)-(7) are strict. This is a crucial point because it means that these properties hold if all indiviuduals' wealth levels are *approximately* identical. We state this formally.

Proposition 1 *There exists $\epsilon > 0$ such that if $|z_i - z_j| < \epsilon$ for all $i, j \in I$, then (1) unanimous location in C is a Nash equilibrium of the location game, and (2) unanimous location in C Pareto dominates every other location profile.*

To summarize from prior analysis, if congestion is not excessive, then there are at most two equilibria with homogeneous population. There is always where everyone locates in C. If (4) holds then there is also an equilibrium in which everyone moves to D. The former equilibrium Pareto dominates the latter.

3 A Dynamic Model

Here we consider a dynamic extension of the static analysis in which infinitely lived individuals may repeatedly revise their location decisions. Time is discrete and indexed $t = 1, 2, \ldots,$. All variables, i.e. z_i, s_i, etc. now have a time subscript t. Let $\beta > 0$ denote the common discount factor. We demonstrate how two key dynamic features, capital accumulation and sequential decision making, may systematically push society toward one or the other communities.

Accumulation of Public Capital

Consider a capital accumulation technology available to each agent and for which y_{it} is an input. It is given by

$$z_{it+1} = h_i(z_{it}, Y_{kt}) \tag{8}$$

where $k = D, C$. In equation (8), z_{it} is the capital of individual i at time t. Here it is assumed that capital can be turned into contemporaneous wealth one for one, so that wealth enters directly as an input. In this model Y_{tk} may be interpreted as some type of aggregate capital in community k so that it enters into both the accumulation technology and, as in the static model, the production of the public good. As an example, infrastructure yields utility directly but also enters as an input in investment technologies. Another example is public education which generates a consumption externality and affects wealth creation in the subsequent period. In either case, the public component to human capital accumulation may or may not be large. A special case of (8) is one where z_{it+1} varies only with z_{it} and y_{it}. In such a case, there is no externality in capital accumulation. Our interest in the specification (8) is not the existence of growth, per se. Rather, it is in the way in which growth asymptotically affects wealth distribution.

Let $\boldsymbol{z}_t$ denote the wealth profile and $s_{it} \in \{C, D\}$ the location choice of individual i at time t. Then the wealth profile of society evolves according to

$$\boldsymbol{z}_{t+1} = h(\boldsymbol{z}_t, Y_t) \equiv (h_i(z_{it}, Y_{t\, s_{it}})_{i \in I})$$

We consider accumulation technologies with a steady state wealth level. Specifically, we assume that within each community, fixing the population, incomes converge to some common level. Many standard technologies have this property. One typical example is the Cobb Douglas technology $z_{it+1} = z_{it}^{\gamma} Y_{t\, s_{it}}^{\delta}$ with $\gamma + \delta < 1$. Notice that accumulation may occur at different rates across communities.

Finally, for simplicity we will restrict attention to economies with initial wealth holdings that are lower than this steady state level.[8] Let $\bar{z}$ denote the steady state wealth of

[8]The restriction on initial wealth holding may be relaxed with further assumptions on the technology. See the discussion in Section 3.1.

an individual when everyone locates in C. By assumption (A3) and Proposition 1, it is straightforward to show that for any path $s_t,\ t = 0, 1, \ldots,$ the corresponding wealth levels satisfy $z_{it} \leq \bar{z}$ for all i and all t. That is, the maximal Nash equilibrium accumulation occurs if everyone resides in community C.

Sequential Choice

Proposition 1 demonstrates that the profile in which all individuals reside in D is Pareto inferior to one in which all individuals reside in C if wealth inequality is sufficiently small. Consequently, there is a possibility of a "coordination failure" as wealth accumulates. This occurs in the Tiebout model because each player remains in an inferior location only because he expects that all others will do the same. If all players move at once, no player can unilaterally signal his intent to do otherwise. This problem persists if the dynamics are modelled as an iterated stage game where each iteration has synchronized moves. Yet, it seems at least as natural in this context that inertia in individuals' decision making breaks up the simultaneity.

We examine a timing structure in which individuals make location decisions asynchronously. We use the same structure as the oligopoly models of Maskin and Tirole (1988a,b). They consider a duopoly in which each firm sets its capacity given the temporarily fixed capacity set by its rival.[9] Firms alternate in moves, and the Markov Perfect equilibria of the repeated/dynamic game are studied. As with capacity decisions, location decisions often entail some adjustment cost or lags. We therefore model individuals who move sequentially, with one individual having the opportunity to change his action in a period. We order the individuals $1, \ldots n \pmod{n}$ so that if individual j had a decision opportunity in period t then individual $j + 1$ has a decision opportunity in period $t + 1$, and individual 1 has the opportunity if $j = n$. This ordering need not coincide with the natural ordering in which higher indices have higher wealth levels.

Naturally there are many possible models of asynchronous choice. Others include the (stationary) stochastic move model in which each individual i has a move with probability p_i where $\sum_j p_j = 1$. The results do not appear to depend criticically on the precise details of the sequential move model.[10] Finally, the timing structure is more general than it first appears since the length of a period may be small. Indeed, our main result will be shown to

[9] For a study of a general family of repeated strategic settings with asynchronous choice see Lagunoff and Matsui (1997).

[10] We conjecture that the results hold for many other timing structures as well

hold when, equivalently, individuals are sufficiently patient.

If the current location profile at time t is s and if i has a move and chooses $\hat{s}_i$ at time $t+1$ then the new profile becomes $s \backslash \hat{s}_i$. Inductively, we can define the path from an initial profile s as $s \backslash \hat{s}_i \backslash \hat{s}_{i+1} \backslash \hat{s}_{i+2} \cdots$. Given the ordering, $s_{i+n} = s_i$.

Markovian Behavior

We examine Perfect equilibria in which individuals' strategies are Markovian. A strategy for an individual is a *Markovian strategy* if it varies only across states that are payoff relevent. In this context this also means that behavior does not treat unequally individuals who are otherwise equal. Formally, a *Markovian strategy* for i is a function $f_i : (s; \boldsymbol{z}) \mapsto \hat{s}_i$. We will limit the analysis to Markovian strategies which satisfy these properties:

(B1) For each s and i, $f_i(s; \boldsymbol{z})$ is continuous in $\boldsymbol{z}$ almost everywhere.[11]

(B2) For any pair $i, j \in I$, consider any pair of states $(s; \boldsymbol{z})$ and $(s; \boldsymbol{z}')$ such that $z_i = z'_j$ and $z'_i = z_j$ ($\boldsymbol{z}$ and $\boldsymbol{z}'$ permute i and j's wealth) and which are otherwise identical. Then $f_i(s; \boldsymbol{z}) = f_j(s; \boldsymbol{z}')$.

Since location decisions are discrete, assumption (B1) implies that for almost every $\boldsymbol{z}$ there is a neighborhood of $\boldsymbol{z}$ such that $f_i(s, \hat{\boldsymbol{z}}) = f_i(s, \boldsymbol{z})$ for all $\hat{\boldsymbol{z}}$ in that neighborhood. In particular, this means that f_i is locally constant near the limit of the accumulation process. Assumption (B2) is an asymptotic symmetry condition. It implies that any two individuals who are otherwise identical and face the same state choose the same strategy. This assumption is vacuous along the accumulation path since individuals are generally endowed with different incomes except in the limit. Together, however, they imply symmetric behavior in a small neighborhood of the limit.[12]

As in single agent dynamic programming problems, there is a tractable recursive formulation for the utility of an agent in the dynamic game. Recall that $\bar{u}_i(s; \boldsymbol{z})$ denotes the per period utility of the location profile s given income distribution $\boldsymbol{z}$. Define $U_i(\ f \mid s; \boldsymbol{z}, j')$ to be the value to individual i of the profile of Markovian strategies $f = (f_j)_{j \in I}$, given the current location profile s, the current income profile $\boldsymbol{z}$, and the current mover $j \in I$. U_i has

[11] Here, "almost everywhere" means "except on a set of Lebesgue measure zero."

[12] Additionally, the assumptions help to ensure that dynamic behavior does not depend crucially on the particular order of the sequential decisions.

the recursive representation,

$$U_i(f \mid s; \boldsymbol{z}, j) \;=\; (1-\beta)\bar{u}_i(s; \boldsymbol{z}) \;+\; \beta\, U_i(f \mid , s \backslash f_{j+1}; h(\boldsymbol{z}, Y), j+1) \tag{9}$$

where payoffs are normalized by multiplying through by $(1-\beta)$. A *Markov Perfect equilibrium (MPE)* is a a tuple $f = (f_j)$ such that for any (possibly non-Markovian) strategy $\hat{f}_i$, for all s, all $\boldsymbol{z}$, and all $j \in I$.

$$U_i(f \mid s; \boldsymbol{z}, j) \geq U_i(f \backslash \hat{f}_i \mid s; \boldsymbol{z}, j).$$

It is assumed here that dynamic choices are limited to location, rather than location and public contributions. The reason is that in a sufficiently large society, individuals may reasonably neglect the strategic effect of their own contribution/voting decisions on others. There is little to be gained by choosing a contribution or vote that does not maximize one's temporary period payoff. By contrast, location decisions do have long term consequences since one's own locational decision may be fixed for some time. Hence, one's optimal location depends on the long term forecast of where he thinks others will be, instead of where they currently reside.[13]

3.1 Main Result: Public Provision Survives Uniquely

Our main result states that for any initial state, if individuals are sufficiently patient then they all eventually reside in the C location permanently. That is, *only involuntary provision is viable in the long run.* What is surprising about this result is that, unlike in the standard repeated game setup, certain static equilibria cannot arise here as Markov Perfect equilibria. One notable profile which does not arise is one in which all individuals remain permanently in D. This is due primarily to the asynchronicity by which location decisions are made.

The result is broken into two Propositions. The first states that, independently of the discount factor, if incomes are close enough to their steady state levels, then if all individuals have unanimously reached the public provision region C, no one ever departs thereafter.

[13] A further problem with assuming that contribution and voting decisions are long run is technical. While aggregation via majority vote as specified here is standard in the literature, the explicit process by which individuals actually vote is not formally defined. Hence it is not a formally defined game. Presumably, the choice of tax rate is one that would survive a majority vote against any alternative. However, the process by which alternative tax rates "arrive" or are placed on the agenda is often not modelled. Neither do we model it here.

Proposition 2 *Let f be any Markov Perfect equilibrium satisfying (B1) and (B2). Then there is a $\epsilon > 0$ and a neighborhood, $N_\epsilon(\bar{\boldsymbol{z}})$ of $\bar{\boldsymbol{z}}$, such that for all i and all $\boldsymbol{z} \in N_\epsilon(\bar{\boldsymbol{z}})$, $f_i(C, \ldots, C; \boldsymbol{z}) = C$.*

The second part of the result asserts that, starting from any profile, unanimous location in C is reached eventually if individuals are patient enough.

Proposition 3 *There is a $\bar{\beta}$ such that for all $\beta \geq \bar{\beta}$, for any Markov Perfect equilibrium f satisfying (B1) and (B2), there exists a time $\bar{t}$ such that $s_t = (C, \ldots, C)$ for all $t \geq \bar{t}$.*

The Logic of the Result

The proofs are in the Appendix, but the basic idea behind the two results is not complicated. Beginning with Proposition 1, if incomes have approximately converged then there are at most two static Nash equilibria of the stage game in each period. In one, everyone locates in C, in the other all locate in D. The former Pareto dominates the latter. Since choice is sequential, someone must unilaterally initiate a departure from C. But since C is Pareto dominant, the individual must expect that sufficiently many others will follow his move to D thereafter. More than half of society must depart in order for him to benefit from a move to D. Therefore, more than $n/2$ periods must pass before his departure is beneficial. Since, the individual has a decision opportunity every n periods he would always prefer to wait until the others have already departed. However, if everyone waits for everyone else to depart, then no one leaves. Hence, departure from C is not consistent with Markov Perfection.

The role of sequential choice in this logic is clear. Since the free rider problem is solved in community C, even under the "worst" expectation it is preferable to wait until others have moved sequentially to the voluntary community. Note that the symmetry assumption prevents the model from being overly sensitive to the order of moves when a neighborhood of the steady state is reached. However, it is easy to construction symmetric equilibria in which individuals depart from community C if play is either nonsequential or non-Markovian.[14]

The idea of Proposition 2 is that if individuals are patient enough, then the waiting cost of reaching C is negligible. The logic utilizes a standard backward induction argument: if i

[14] If choices are not sequential, then Pareto inferior static equilibria are always equilibria of the infinite game even with the Markovian restriction. If behavior is not Markovian, then Folk Theorems exist even in (generic) sequential games (see Dutta (1995)).

is the only one who is not in C then he moves there; if i and j are the only ones who have not moved to C then i moves there knowing then that j will move there, etc. An individual may therefore move to a small community if he expects others to follow. Hence, transitions from inferior static equilibria must occur. Note that the backward induction argument may break down if initial wealth holdings lie above the steady state. In such a case, if this steady state is not the global optimum, there may be incentives for wealthy individuals to "delay" the transition indefinitely by remaining in the voluntary community D.[15] The logic is not unlike results of Rubinstein and Wolinski (1994) and Lagunoff and Matsui (1997) who prove "anti-Folk Theorems" in repeated pure coordination games with asynchronous choice.

Nonconvergence

If the accumulation technology does not converge globally, then the result need not hold. The intuition is this: if wealth levels of different individuals do not converge then conflicting interests persist. The Pareto advantage of living with involuntary provision is no longer inevitable. Hence, despite the ever present free rider problems in the voluntary community, some might prefer D to an undesirable tax rate in C. This gives voluntary provision the base of support it lacked under convergence.

Generally, the wealthiest individual z_n is the most likely to prefer to remain in D even if it means locating there by himself. A sufficient condition for community D to maintain a population is that individual n will never prefer to leave D in any equilibrium scenario. If this were not the case, then one could always contruct a MPE in which individual n rationally anticipates the scenario in which he is better off in C. A sufficient condition holds if for all t, for all s with $s_n = C$,

$$V_n(s \backslash D; \boldsymbol{z}_t) > W_n(s; \boldsymbol{z}_t) \tag{10}$$

If D is to be more than just a Robinson Crusoe community, it suffices that individual $n-1$, the next wealthiest member, never prefers to leave D *given* that individual n always remains in D. If this holds, then it suffices for individual $n-2$ to prefer to remain in D given that individuals $n, n-1$ remain in D, and so forth... This kind of locational cascade is sufficient for community D to survive *uniquely.*

However, such a cascade could not exist without sufficient wealth dispersion. To see this,

[15] This delay might be ruled out if the technology satisfies decreasing returns to scale everywhere, as in the Cobb Douglas example with $\gamma + \delta < 1$. We conjecture then that the argument will work for any initial wealth profile. However, the proof would be considerably more complicated.

observe that Inequality (10) can be rewritten as

$$\max_{\tau} u((1-\tau)z_{nt}, F(\tau z_{nt},\ 1)\ ) \ >\ u((1-\tau(s;\boldsymbol{z}_t)z_{nt}, F(\tau(s;\boldsymbol{z}_t)z_{nt} + \tau(s;\boldsymbol{z}_t)(Z_t - z_{nt}),\ n)\ )$$

Note that in the left side of the inequality, a lone individual's choice of tax rate imposed upon himself is equivalent to his choice of contribution. Putting aside congestion issues, this inequality can only hold if individual n prefers his own tax rate to that of community C even if the tax base in D consists only of his own income. This requires that his preferred tax rate τ is far from $\tau(s;\boldsymbol{z}_t)$, and/or that $Z_t - z_{nt}$ is small. In either case, he must be sufficiently wealthier than everyone else.

Hence, for the locational cascade to hold requires that each individual i be sufficiently wealthier than $1, 2, \ldots, i-1$. Of course, the resulting wealth profile would be extreme. Less extreme, however, is the possibility that voluntary provision can be supported by a small but wealthy elite. Such support could perpetuate if accumulation follows a balanced growth path which maintains the wealth inequality.

4 Some Examples

The first examples relating wealth/income inequality to voluntary and involuntary provision of a public good come from education. James (1993) assembles data on the fraction of private school enrollments in elementary and secondary levels for 40 countries. At the secondary level, for instance, this fraction varies between 1% and 89%. Regressing these private enrollments on the fraction of incomes accruing to the bottom decile yields negative coefficients, i.e, higher equality is associated with smaller private school enrollments (with t-statistic around -1.4). We ran this regression twice, once for elementary level and once for secondary level education. These results are robust to inclusion of GDP as a right hand side variable.

From the Digest of Education Statistics we obtain the fraction of students in the 50 states enrolled in private school in the U.S. Braun (1988) calculates Gini coefficients for family income distribution. Regressing private enrollments on these Gini coefficients yields positive regression coefficients (with t-statistics of 1.3). Again, higher inequality is associated with higher private enrollments. Similar results are obtained by Hamilton and Macauley (1991) and Schmidt (1992).[16]

[16] There are other potential explanations for the correlation between inequality and private provision. In

Note that the data is cross-sectional, and so there is no associated time series on migration or explicit evolution of wealth inequality. Nevertheless, as stylized facts they are consistent with the theory: low inequality is associated with public provision. There are conceptual issues as well. First, in many instances taxation for schooling cannot be avoided. In particular, a family whose child attends a private school usually cannot opt out of the state's taxation system. Yet, the inability to opt out of public school taxation mitigates, but does not invalidate the logic of how location determines the relative strengths and weaknesses of the two institutions. Indeed, voluntary provision becomes even less desirable if one must also pay taxes. It is worth noting, moreover, that a few states in the U.S., notably Wisconsin, offer tuition tax credits for private enrollees. This amounts to a limited option to exit the cumpulsory system. Current policy debates on education have often focussed on expanding these tax breaks at a federal level. The present model becomes a useful analytical tool for evaluating these proposals.

Second, private schooling is not the same as voluntary provision. Education is certainly not a pure public good. Tuition payments confer substantial private returns. On the other hand, many economists believe that there are substantial external returns with education which, in turn, create classic free rider problems associated with voluntary provision. According to Friedman (1962), "the gain from the education of a child accrues not only to the child or his parents, but also to other members of society. The education of my child contributes to your welfare by promoting a stable, democratic society. It is not feasible to identify the particular individuals (or families) benefitted, and so charge for services rendered" (p.86).[17]

The logic of the argument is more powerful at the local level since one's education arguably has greater effect on those in greater proximity, particularly within the school itself. Weisbrod (1962) argues that external effects associated with education accrue at the local level: education affects one's neighbors "by inculcating acceptable social values and behavior norms in the community children." Furthermore, education benefits neighbors by lowering crime and thus law enforcement costs. Haveman and Wolf (1984) establish a taxonomy of non-market effects of education. They include crime reduction and social cohesion as local externalities. In empirical work, Henderson, Mieszkowski, and Sauvageau (1978), Link and Mulligan (1991), and Summers and Wolf (1977) find externalities which operate within the

models of public provision of goods in which there is a private alternative within the same jurisdiction such as the models of Hamilton and Macauley (1991), Epple and Romano (1996), and Glomm and Ravikumar (1998), higher inequality implies a thicker upper tail of the income distribution. The thicker upper tail implies, in turn, that more people choose the private alternative over the compulsory one.

[17] Weisbrod (1962) and Cohn and Geske (1990) make similar arguments.

classrom, while Brown and Saks (1975) find externalities within the school district.

Clearly, the model, unlike contemporary data, assumes that a single type of institution exists in any community. There are good tractability reasons for the assumption. Existence problems are known to arise for equilibria in majority voting problems when voluntary alternatives to public provision exist (see, for example, Stiglitz (1974)). Nevertheless, the assumption is restrictive since no such strict separation typically exists in contemporary communities. The same community that uses tax revenue to build roads and fund a police force relies on charitable donations to fund public monuments and the community orchestra.

For this reason, it easier to find historically older examples that conform more closely and neatly to the two mechanism scenario in the model. For example, Eby (1952) reports that English migration to Dutch schools took place in the 17th century took place largely because Dutch schools were financed via compulsory taxation by the municipalities. At the same time, in England the Poor Law of 1601 declared that education was a charity and not a right. Isaac (1982) documents in 18th century migration in Colonial Virginia from the established Church of England to the new Babtist Church. The former relied on voluntary contributions from its membership, while the latter required unanimity and later by majority voting.

We do not have data for the relevant time periods in these cases to determine whether increasing equality drove the relocations. On the other hand the nature of the status quos in each instance suggests that large, favorable transformations in wealth distribution took place along side these relocations. In England, the emergence of a merchant middle class during this time is well documented. In Colonial America, Isaac (1982) notes that the new migrants were increasingly wealthy common farmers, seeking escape from the gentry-driven voluntarism of the established Church of England (p.164).

5 Summary and Related Literature

There is an established literature which has examined various types of provision procedures for public goods in isolation from other procedures. Bergstrom, Blume, and Varian (1986), for instance, examine private provision of a public good. They provide a fairly complete characterization that may be compared directly with established results on public provision via majority voting which can sometimes be shown to supply public goods efficiently (see

Bergstrom (1979)). An early seminal contribution from this literature which examines explicitly the trade off between problems of conflicting interests and the problems of free riding in public goods provision is Buchanon and Tullock (1962). They derive the optimal voting rule as one that balances these two incentive problems. By way of comparison, the present contribution allows the participants themselves to make personal decisions between the mechanisms based on this trade off.

On this score, a large literature has emerged which concerns the link between Tiebout-style locational choice and policy outcomes. For example, Epple, Filimon, and Romer (1984), Epple and Romer (1992), Fernandez and Rogerson (1994, 1996) study redistributive policies as a consequence of voting and migration. Bearse (1994) studies location of criminal activity as a consequence of law enforcement activity. De Bartolome (1990), and Oates and Schwab (1991) examine the effects of peer groups and neighborhood composition on locational choices. While the aformentioned papers are static, Benabou (1993,1996a, 1996b) and Durlauf (1996) examine dynamic multi-jurisdictional models. The former studies the determinants and consequences of segregation, while the latter shows that wealth inequality causes poverty traps.

All these papers assume a coercive mechanism which is homogeneous across communities. They all address the question: what is the influence of the mechanism on migration and the evolution of the wealth distribution? The present paper differs from these by taking this evoluton as, in a sense, given and asks how migration and the evolution of wealth inequality influences the spatial selection of mechanisms.

More recently, Tiebout-style locational choice has been applied to spatial selection of social choice mechanisms. An example is Caplin and Nalebuff (1992, 1997). They examine existence issues in a rich class of static models in which communities may be differentiated by the social welfare function used to determine policy outcomes. They examine the potential for communities to respond to migrational pressure by changing their aggregation rule.

The only two papers that we are aware of in which migration occurs over real time to determine viability of social choice procedures are Kollman, Miller and Page (KMP) (1997) and Lagunoff (1997). The first studies a computational model in which boundedly rational agents can move between locations differentiated by voting procedures that determine an abstract social choice outcome. The latter uses evolutionary game theoretic tools to consider Tiebout choice between public and private provision of a public project. The main difference between these two papers and the present one is that the present one considers a standard equilibrium analysis in a neoclassical environment for public goods provision. Here, fully optimizing agents face a concave production technology for a public good.

Yet, the present model is far from definitive. It only suggests some of the differences between long run viability of voluntary and involuntary provision. Many issues have yet to be examined. For example, the dynamic model should be generalized eventually to consider many communities and many different provision mechanisms. The difficulty of such a model is that complicated locational cycles are likely to appear even between communities offering the same voting procedure. Ultimately, a computational route such as the one taken by KMP may be the most fruitful for the more general setup.

Another important missing element is the presence of idiosyncratic shocks. In the model with convergence, the presence of shocks slows, and may well stop the asymptotic out-migration from the voluntary provision community. The latter could occur if the magnitude of the shocks is large and the variability is stationary. In that case individuals with perfect foresight choose to remain in the voluntary community since the expected distance between their ideal contribution and that of the median voter is large.[18]

In order for stochastic versions of Propositions 2 and 3 to hold, the shocks must be small and sufficiently infrequent. In that case, $N_\epsilon(\bar{z})$ defines a basin of attraction in community C from which no individual, in the absence of an income shock, would depart. The potential problem comes from the backward induction logic if a number of individuals realize similar shocks to wealth that put them above the steady state before they move. These individuals may have incentives to remain permanently in D unless the new steady state reached in community D eventually falls below $\bar{z}$.

Finally, our comparison has been limited to provision of *public goods* so far. When provision of private goods is the issue, the benefit of overcoming the free rider problem dissipates, and so involuntary provision is more difficult to support.[19]

6 Appendix

For all the results in this Appendix, we fix a Markov Perfect equilibrium f satisfying

[18]We thank an anonymous referee for raising this point.

[19]Besley and Coate (1991) examine this type of problem when individuals can choose between the private and public sectors.

(B1) and (B2). Then we let $s^* = (\overbrace{C, \ldots C}^{n})$ denote the location profile in which everyone locates in C. That is, s^* satisfies $s_i^* = C$ for all i.

The proof of Proposition 2 will utilize the following four lemmata.

Lemma 1 *Suppose that $\boldsymbol{z} = \bar{\boldsymbol{z}}$. Then $f_i = f_j$ for each i, j whenever $s_{-i} = s_{-j}$ and f_i varies only over $|I_C|$, the number of individuals in C*

Proof: The fact that $f_i = f_j$ for each i, j clearly follows from the symmetry assumption, (B2). Moreover, since the Markovian restriction implies that f_i varies only over payoff relevant information, f_i varies only over $|I_C|$, the number of individuals in C (clearly, $|I_D|$ can be inferred from $|I_C|$). □

The next Lemma asserts that $W_i(s^*; \bar{\boldsymbol{z}})$ is the uniquely globally maximal stage game payoff of the dynamic game.

Lemma 2 *For all i, $W_i(s^*; \bar{\boldsymbol{z}}) > \bar{u}_i(s; \boldsymbol{z})$ for each s and each $\boldsymbol{z}$.*

Proof Recall that $\bar{u}_i$ describes generally the stage game payoff in the dynamic game. By Proposition 1, $W_i(s^*; \bar{\boldsymbol{z}}) > \bar{u}_i(s; \bar{\boldsymbol{z}})$ for all s. Now by assumption (A3), and the assumption that given to the steady state $\bar{\boldsymbol{z}}$ in location profile s^*, it follows that $\bar{z} \geq z_{it}$ for each i and each t. Since every individual has wealth $\bar{z}$ in the distribution $\bar{\boldsymbol{z}}$, we therefore have $\bar{u}_i(s; \bar{\boldsymbol{z}}) \geq \bar{u}_i(s; \boldsymbol{z})$ for all $\boldsymbol{z}$ and s. To summarize, we have shown $W_i(s^*; \bar{\boldsymbol{z}}) > \bar{u}_i(s; \bar{\boldsymbol{z}}) \geq \bar{u}_i(s; \boldsymbol{z})$ for all $\boldsymbol{z}$ and s. □

The next Lemma asserts that in $\bar{\boldsymbol{z}}$, every mover is better off for n periods by remaining in s^* even if every subsequent mover moves to D.

Lemma 3 *Suppose that $\boldsymbol{z} = \bar{\boldsymbol{z}}$. Suppose that the current profile is s^*, and the current mover is i. Then, for any subsequent path $s^* \backslash s_{i+1} \backslash \cdots \backslash s_{i+n-1}$ from s^*, we have*

$$\sum_{t=0}^{n-1} \beta^t W_i(s^* \backslash C \backslash \cdots \backslash s_{i+t}; \bar{\boldsymbol{z}}) \; > \; \sum_{t=0}^{n-1} \beta^t V_i(s^* \backslash D \backslash \cdots \backslash s_{i+t}; \bar{\boldsymbol{z}})$$

Proof. Recall from Proposition 1 that unanimous location in C under $\bar{\boldsymbol{z}}$ is a globally dominant payoff. Observe that the worst n-period payoff to i if he remains in C is obtained if all subsequent $n-1$ individuals choose D. Hence, for any sequence $\{s_t\}_{t=0}^{n-1}$ with $s_0 = C$,

$$\sum_{t=0}^{n-1} \beta^t W_i(s^* \backslash C \backslash \cdots \backslash s_{i+t}, \bar{\boldsymbol{z}}) \geq \sum_{t=0}^{n-1} \beta^t W_i(C \backslash \overbrace{D \backslash \cdots \backslash D}^{t}; \bar{\boldsymbol{z}}) \tag{11}$$

On the other hand, the best n-period payoff to i if he moves to D is obtained if all subsequent $n-1$ individuals also choose D. Hence, for any sequence $\{s_t\}_{t=0}^{n-1}$ with $s_0 = D$,

$$\sum_{t=0}^{n-1} \beta^t V_i(s^* \backslash D \backslash \cdots \backslash s_{i+t}; \bar{\boldsymbol{z}}) \leq \sum_{t=0}^{n-1} \beta^t V_i(D \backslash \overbrace{D \backslash \cdots \backslash D}^{t}; \bar{\boldsymbol{z}}) \tag{12}$$

It suffices to show

$$\sum_{t=0}^{n-1} \beta^t \left[W_i(C \backslash \overbrace{D \backslash \cdots \backslash D}^{t}; \bar{\boldsymbol{z}}) \; - \; V_i(D \backslash \overbrace{D \backslash \cdots \backslash D}^{t}; \bar{\boldsymbol{z}}) \right] \; > \; 0 \tag{13}$$

To simplify notation, let us write $W_i(C \backslash \overbrace{D \backslash \cdots \backslash D}^{t}; \bar{\boldsymbol{z}}) \equiv W(I \backslash \{i+1, \ldots, i+t\})$ where $I \backslash \{i+1, \ldots, i+t\}$ is the set of individuals remaining in C. Similarly, we write $V_i(D \backslash \overbrace{D \backslash \cdots \backslash D}^{t}; \bar{\boldsymbol{z}}) \equiv V(\{i, i+1, \ldots, i+t\})$ where $\{i, i+1, \ldots, i+t\}$ is the set of people who choose D.

Observe first that more than $n/2$ individuals must have left C before $|I_D| > |I_C|$. In addition, $W(I') > V(I')$ for any subcollection $I' \subseteq I$ of society. Therefore, the only time periods for which $W(I_C) < V(I_D)$ is possible are the last $n/2$ periods which are discounted more heavily than the first $n/2$ periods. Finally, observe that $W(I') - V(I'') > V(I') - W(I'')$ for any two disjoint collections I', I'' with $I' \cup I'' = I$. To verify this, one need only collect W terms on one side of the inequality and V terms on the other. Pareto dominance of C over D for equal populations then finishes the job. Putting these facts together, observe then that for each difference $W(I \backslash \{i+1, \ldots, i+t\}) - V(\{i, i+1, \ldots, i+t\})$ in the sum in (13) there is a symmetric difference $V(\{i, i+1, \ldots, i+n-t\}) - W(I \backslash \{i+1, \ldots, i+n-t\})$ in the sum. Rather, each depends only on the cardinality of the sets I_D and I_C. Hence we have that

$$W(I \backslash \{, i+1, \ldots, i+t\}) - V(\{i, i+1, \ldots, i+t\}) > 0$$

whenever $t \leq n/2$ and for any $t = 1, \ldots n$,

$$\begin{aligned} & W(I \backslash \{i+1, \ldots, i+t\}) - V(\{i, i+1, \ldots, i+t\}) \\ > \; & V(\{i, i+1, \ldots, i+n-t\}) - W(I \backslash \{i+1, \ldots, i+n-t\}). \end{aligned} \tag{14}$$

Inequality (14) comes from the symmetric inequality $W(z, I') - V(z, I'') > V(z, I') - W(z, I'')$ proved above. From these last two inequalities, we conclude that the sum in the right side of (13) is strictly positive. Hence the Lemma is proved. $\square$

The next Lemma asserts that i's n-period forward continuation payoff from choosing location C is no worse than if he had chosen D.

Lemma 4 *Let s^* be the location profile and suppose that $\boldsymbol{z} = \bar{\boldsymbol{z}}$. Let $s\backslash C\backslash f^n$ and $s\backslash D\backslash f^n$ denote the n period equilibrium paths from s^* when the current mover chooses either C or D, respectively. Then*

$$U_i(f|\ s^*\backslash C\backslash f^n; \bar{\boldsymbol{z}}, i) \geq U_i(f|\ s^*\backslash D\backslash f^n; \bar{\boldsymbol{z}}, i).$$

Proof: Suppose first that the next mover, $i+1$, chooses $f_{i+1}(s^*\backslash C) = C$. Then, by Lemma 1, each subsequent j also chooses C when faced with profile s^*, and so s^* is an absorbing state. In this case, the Lemma follows from Lemma 2.

Suppose then that $f_{i+1}(s^*\backslash C) = D$. Let s_i^D (resp. s_i^C) denote the n-period forward location chosen taken by i in period $t+n$ when i chooses location D (resp. location C) currently. The two alternative scenarios are

$$s^*\backslash C\backslash \overbrace{D\backslash \ldots \backslash D}^{q} \backslash \ldots \backslash s_i^C$$

and

$$s^*\backslash \overbrace{D\backslash \ldots \backslash D}^{q} \backslash \ldots \backslash s_i^D$$

Now observe that $s_i^D = C$ cannot be a best response for i since some previous mover j chose location D when facing $q-1$ other members of D. By Lemma 1, it is therefore optimal for mover i to make the same location decision.

Hence, suppose $s_i^D = D$. Then i can do no worse by choosing C in period t, then choosing D in period $t+n$, since he then joins at least q, rather than $q-1$ other citizens in community D and, by (A3), production of the public good is higher. That is, $U_i(f|\ s^*\backslash C\backslash f^{n-1}\backslash D; \bar{\boldsymbol{z}}, i) \geq U_i(f|\ s^*\backslash D\backslash f^{n-1}\backslash D; \bar{\boldsymbol{z}}, i)$. Again the conclusion follows. $\square$

Proof of Proposition 2 Putting Lemmas 2-4 together, observe that at each time t,

$$\begin{aligned} U_i(f|\, s^*\backslash C, \bar{\boldsymbol{z}}, i) \;&=\; \sum_{\tau=t}^{t+n-1}(1-\beta)\beta^{\tau-t}W_i(f(s_{t-1},\bar{\boldsymbol{z}});\bar{\boldsymbol{z}}) \;+\; \beta^n U_i(f|\, s^*\backslash C\backslash f^n;\bar{\boldsymbol{z}}, i) \\ >\; \sum_{\tau=t}^{t+n-1}(1-\beta)\beta^{\tau-t}V_i(f(s_{t-1},\bar{\boldsymbol{z}});\bar{\boldsymbol{z}}) &+ \beta^n U_i(f|\, s^*\backslash D\backslash f^n;\bar{\boldsymbol{z}}, i) = U_i(f|\, s^*\backslash D, \bar{\boldsymbol{z}}, i). \end{aligned} \tag{15}$$

Therefore, at profile $\bar{\boldsymbol{z}}$, remaining in s^* is a sequential best response for every individual.

By assumption (B1) there is some neighborhood of $N_\epsilon(\bar{\boldsymbol{z}})$ of $\bar{\boldsymbol{z}}$ with $\epsilon > 0$, such that all individuals' strategic behavior is constant over $N_\epsilon(\bar{\boldsymbol{z}})$. Hence, strategies only vary across location profiles s. By the Convergence assumption, there is, for each location, some time T_ϵ such that if the profile within the location has not changed for T_ϵ periods, then incomes within the location are in neighborhood $N_\epsilon(\bar{\boldsymbol{z}})$. Moreover, by making ϵ small enough, by Proposition 1 unanimous location in C gives a globally dominant payoff. Suppose then that s^* has been the current profile for T_ϵ periods. Since everyone resides in the same location, all incomes are in $N_\epsilon(\bar{\boldsymbol{z}})$. By continuity assumption (B1), (15) holds for all $\boldsymbol{z}$ in the neighborhood $N_\epsilon(\bar{\boldsymbol{z}})$. This concludes the proof. □ □

The next two lemmata will be utilized to prove Proposition 3.

The next Lemma states that s^* is an absorbing location if individuals are patient enough.

Lemma 5 *Let $(s^*, \boldsymbol{z}, j)$ be any state in which location profile is s^*. Then there is a $\bar{\beta} < 1$ such that if $\beta \geq \bar{\beta}$ then $f_i(s^*;\boldsymbol{z}) = s_i^*$ for all i.*

<u>Proof</u> The proof involves a backward induction argument. Let $\epsilon > 0$ denote the size of the neighborhood $N_\epsilon(\bar{\boldsymbol{z}})$ in which behavior coincides with that on $\bar{\boldsymbol{z}}$. By the convergence assumption, from any state $(s^*, \boldsymbol{z}, j)$ there is some time T_ϵ after which each individual i has $|z_i - \bar{z}| < \epsilon$. To see why, observe that with a finite number of individuals even the individual with the slowest accumulation eventually enters $N_\epsilon(\bar{z})$. Then, by Proposition 2, no individual leaves community C if s^* has occurred for T_ϵ periods. As a normalization, let T_ϵ coincide with calendar time. We can also choose ϵ small enough so that, using the Lemma 2 there is some $\eta < \epsilon$ such that

$$W_j(s^*;\boldsymbol{z}_{T_\epsilon}) > \bar{u}_j(s;\boldsymbol{z}) + \eta \tag{16}$$

for all s and all $\boldsymbol{z} \notin N_\epsilon(\bar{\boldsymbol{z}})$. This implies

$$W_j(s^*;\boldsymbol{z}_{T_\epsilon}) > U_j(f|\, s^*\backslash D;\boldsymbol{z}_{T_\epsilon}, j) + \eta \tag{17}$$

Hence, let $\beta(T_\epsilon)$ be large enough so that for all $\beta \geq \beta(T_\epsilon)$,

$$\begin{aligned} & (1-\beta)W_j(s^*;\boldsymbol{z}_{T_\epsilon - 1}) \;+\; \beta \sum_{t=T_\epsilon}^{\infty} (1-\beta)\beta^{t-T_\epsilon} W_j(s^*;\boldsymbol{z}_t) \\ > \;\; & (1-\beta)V_j(s^* \backslash D;\boldsymbol{z}_{T_\epsilon}) \;+\; \beta U_j(f|\; s^* \backslash D \backslash f_{j+1};\boldsymbol{z}_{T_\epsilon + 1}, j) \end{aligned} \tag{18}$$

But according to the inequality in (17), the choice $s_j = C$ is a best response for individual j at time $t = T_\epsilon - 1$. Hence, if $\beta \geq \beta(T_\epsilon)$, then s^* is absorbing at time $T_\epsilon - 1$. This argument can now be repeated for time $t = T_\epsilon - 2$: there is some discount factor $\beta(T_\epsilon - 1) > \beta(T_\epsilon)$ such that choosing community C is a best response at time $T_\epsilon - 2$. To see this we need only substitute $\boldsymbol{z}_{T_\epsilon - 1}$ for $\boldsymbol{z}_{T_\epsilon}$ in the inequalities (16)-(18). Repeating this argument for each time $t = 0, 1, \ldots, T_\epsilon - 1$, we obtain a sequence $\beta(1), \beta(2), \ldots, \beta(T_\epsilon)$ of discount factors with $\beta(1) > \beta(2) > \cdots > \beta(T_\epsilon)$, so that if $\beta \geq \beta(1)$, then $f_j(s^*;\boldsymbol{z}_t) = s_j^*$ is a best response for all j. Letting $\bar{\beta} = \beta(1)$, the proof is complete. □

The last Lemma asserts that, starting from the initial period, sufficiently patient individuals will move in sequence to community C, thereby reaching s^* after n periods.

Lemma 6 *Let $\boldsymbol{z}_0$ denote the initial wealth endowment. Then there is a $\hat{\beta} < 1$ such that if $\beta \geq \hat{\beta}$ then $f_t(s^*;\boldsymbol{z}_t) = s_t^*$ for $t = i = 1, \ldots, n$.*

Proof As in the previous Lemma, we utilize a backward induction argument. Without loss of generality suppose that $s(0) = (D, \ldots, D)$. Suppose that $s(n) = s^* \backslash D$ so that in period n, all individuals except person n have located in community C. Let $\beta(1), \beta(2), \ldots, \beta(T_\epsilon)$ be the sequence as constructed in the proof of Lemma 5. The logic of that Lemma easily extends to person n: let $\beta(0) > \beta(1)$ be such that for all $\beta \geq \beta(0)$,

$$\begin{aligned} & (1-\beta)W_j(s^*;\boldsymbol{z}_n) \;+\; \beta \sum_{t=n+1}^{\infty} (1-\beta)\beta^{t-n-1} W_j(s^*;\boldsymbol{z}_t) \\ > \;\; & (1-\beta)V_j(s^* \backslash D;\boldsymbol{z}_n) \;+\; \beta U_j(f|\; s^* \backslash D \backslash f_{j+1};\boldsymbol{z}_{n+1}, j) \end{aligned} \tag{19}$$

and so person n's best response at time $t = n$ is to choose $s_n = C$.

To complete the induction step, suppose that individual k has a move at time k given a profile $s(k)$ in which individuals $i = 1, \ldots, k-1$ successively chose community C in periods $1, \ldots k-1$, and individuals $i = k+1, \ldots, n$ will choose C if person k does so. Let β_{k+1}

denote the requisite discount factor for individual $k+1$. Then define $\beta_k > \beta_{k+1}$ so that for all $\beta \geq \beta_k$,

$$
\begin{aligned}
& (1-\beta)\sum_{t=k}^{n-1}\beta^{t-k}W_j(s(t);\boldsymbol{z}_t) \;+\; \beta\sum_{t=n+1}^{\infty}(1-\beta)\beta^{t-n-1}W_j(s^*;\boldsymbol{z}_t) \\
> \;\; & (1-\beta)V_j(s(k)\backslash D;\boldsymbol{z}_k) + \beta U_j(f|\; s(k)\backslash D\backslash f_{k+1};\boldsymbol{z}_{k+1},j)
\end{aligned}
\tag{20}
$$

Continuing this argument for all k, we obtain a sequence

$$\beta_1 > \beta_2 > \cdots > \beta_n \equiv \beta(0) > \beta(1)\cdots > \beta(T_\epsilon)$$

Hence, for all $\beta > \beta_1$, each individual moves in sequence to s^* starting from the initial mover. Letting $\hat{\beta} = \beta_1$, the proof is complete. □

Proof of Proposition 3 Proposition 3 now follows directly from Lemmata 5-6. □ □

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