Joint Outside Options

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Abstract

Several contractual situations are such that the parties may 'step out' of negotiations and take up outside opportunities only if there is mutual consent to do so. Examples include some forms of employer-employee negotiations, divorce procedures and arbitration. To analyse such cases we develop the general concept of a 'joint outside option' and study its effect in the standard bargaining game. Examples from the economics of divorce and theory of the firm are considered in some depth.

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1 Introduction

In this paper we consider a category of contractual situations, of which the following are examples:

- Employer and employee are locked in a long-term contract and are disputing over working conditions. 'Early retirement', requiring the consent of both parties, offers an alternative termination of the dispute.
- Landlord and tenant are involved in a dispute. In case of a deadlock, they can at any time agree to have their dispute resolved by an arbitrator.
- After marrying, both partners learn of attractive outside alternatives. Unilaterally filing for divorce is not viewed as a practicable (or even legal) option, but a less costly consensual decision to split is[1].

These very diverse situations have a feature in common. Both parties have outside opportunities. However, walking out unilaterally to take up the outside opportunity either has high costs or is forbidden by the law. In other words, we are interested in situations in which outside options *only have net value when they are taken consensually.* The parties must, as it were, agree to disagree.

To be more precise about the previous examples, consensuality of the will to divorce is or was until recently legally required in several countries. In essence, this is the case with fault divorce (see Clarke (1998)). No-fault divorce itself does not necessarily rule out mutual consent (this is explicitly the case, for instance, in some states in the U.S., like Arkansas and Delaware; see for instance Friedberg (1998) and Gray (1998)). Also, unilateral divorce can be legal but very costly compared to an agreed upon divorce: for instance the Scotland Divorce Act of 1976 requires that proof of at least two years of separation if both partners consent, and five years separation in case of unilateral divorce[2]. Early retirement has, for example, been widely used in UK academic institutions

[1] Being partners in a couple as well as coauthors, we should declare a special interest in this particular application of our line of research.

[2] See Clark (1998).

as a strategy to increase research productivity. Conceptually similar consensual ways of terminating negotiations are relevant in several other employer-employee negotiations (e.g. switching from full time to part-time employment). Again, the crucial point here is that although unilateral actions to terminate negotiations (through firing or quitting) may not be legally impossible, they are sometimes less viable/relevant than consensual decisions because of their cost. In several countries, going into arbitration to resolve disputes is a decision that parties must agree upon. Examples include procedures in UK labour disputes, in which final offer arbitration can be triggered only with the consent of both sides; and common civil law disputes in Italy[3].

We believe that the theory of non-cooperative bargaining (as started by Stahl (1972) and Rubinstein (1982)) provides a powerful tool to to analyse this type of situations. However, the standard theory admits essentially only two kinds of alternatives to agreement:

(1) there is perpetual disagreement (either because of an exogenous breakdown or because negotiations never end); or

(2) a *unilateral* action is taken by one of the players to terminate the negotiations.

The unilateral action in *(2)* results in the players taking their 'outside options': payoffs that can be obtained in the next best activity. The literature has explored many interesting variations of the exact meaning of 'taking a unilateral action', which depend on answering questions such as: When can a player opt out? (e.g. Shaked and Sutton (1984), Bester (1988)). Do both players have this opportunity? (Ponsatì and Sàkovics (1998)). But common examples such as those we have mentioned, where the effective power of 'unlocking' bargaining is a *joint* prerogative of both players, simply cannot be framed in any of these variations[4]. In this paper we introduce and analyse the concept of

[3]For instance, in the UK, "[Arbitration] is method of resolving disputes between parties without recourse to the law. It is voluntary, and the procedures have to be agreed by both parties beforehand" (see Kennerley (1994)).

[4]There have also been interesting explorations on what exactly happens in case of disagreement: Does disagreement mean a breakdown of negotiations or a continued failure to agree? (Binmore et al. (1986)). Can a player harm the other? If so when? (Haller and Holden (1990), Manzini (1996), Avery and Zemsky (1994),Fernandez and Glazer (1991),Busch et al. (1998)). Is the disagreement payoff determined by the equilibrium of a game? (Busch and Wen (1995)). Again, we find that such variations do not capture the

a '**joint outside option**' (henceforth, a **joo**) in the theory of bilateral bargaining.

A joo is just like a standard outside option[5] (that is, a pair of payoffs that can be obtained by the players if they stop bargaining), with the single difference that it can only be obtained if *both* players consent to take it.

How do joos affect bargaining? The most interesting feature of the notion of a joo is that it seems at first sight that it should not matter at all! If the joo is efficient (that is, it takes the players on the frontier of the feasible set) then exactly one player stands to lose from terminating the negotiations (with respect to the normal bargaining outcome). But in our examples that player can simply 'veto' this possibility. On the other hand, if the joo is not efficient it may be the case that both players stand to lose with respect to the normal bargaining outcome. *A fortiori*, it is not clear that the presence of a joo should have any effect.

We will demonstrate that joos *do* affect the bargaining outcome in the sense that they are taken, in equilibrium, at out-of-equilibrium decision nodes. Given this fact, an obvious second conjecture is that then bargaining with joos has the same features of bargaining when standard outside options exist. Recall the general pricinciples we have learned regarding outside options. In the case in which only one player has an outside option, and can take it when rejecting an offer, the so called 'Outside Option Principle' (OOP, Binmore et al. (1989)) applies. The exit opportunity has no effect on bargaining unless it yields a payoff larger than the *equilibrium* payoff. When the outside option has an effect, it always acts as a lower bound for the player's payoff. On the other hand, it has been recently pointed out that when *both* players can (unilaterally!) take outside options after a rejection, there is typically a continuum of equilibria (Ponsatì and Sàkovics (1998)). It turns out that the 'obvious' extensions of neither of these two standard cases can model the situation we consider, although, interestingly, the principles just mentioned emerge as particular cases. The basic facts we find are :

(a) The presence of a joo *always* affects the equilibrium set of outcomes of bargaining, provided its inefficiency is not too high relative to bargaining frictions. Bargaining with a

kind of situation we are interested in in this paper.

[5] Of the 'bazaar' type (see Shaked (1994)).

joo has starkly different implications from those of the standard bargaining model, even when the latter is enriched to include some outside option(s). This proves to be relevant in several economic applications such as arbitration, divorce laws and the theory of the firm.

(b) If the joo is 'not too distant' from the standard bargaining outcome (but possibly inefficient), then the equilibrium is *unique*, and is determined according to the OOP. In this case, the veto power conferred to a player to block unfavourable outside options proves to be completely empty; the other player will always be able, in equilibrium, to force an outcome whose value is bounded below by his outside option. Again, this is relevant to judge the practical effectiveness of institutions such as consensual divorce or arbitration.

(c) If the joo is not in the region for which *(b)* applies, then there may exist a continuum of equilibria reminiscent of those determined by double-sided outside options. This opens the door to inefficiency in the bargaining outcome.

A final introductory note concerns the *interpretation* of a joo. Some readers may feel that our model oversimplifies the situations we have mentioned at the outset. For example, the outcome of 'agreeing to split' in a marriage can hardly be viewed as 'parameter': it is itself a strategic situation. Ditto for the outcome of arbitration. However, there is no analytical difficulty in this respect in interpreting the joo as an *equilibrium* outcome of another game the players may decide to play. Our simplified model can deal in principle with very complex extensive forms, such that at any time while bargaining within a given extensive form game, the players can *jointly change the contractual situation they are involved in*, thus generating a new extensive form.

2 The Model

Two players must reach an agreement on an outcome in a set A. A breakdown of negotiations is a distinct outcome denoted b. Players may also agree to terminate the negotiations (with the rules described below): this outcome is denoted t, and is interpreted as the situation in which a joo is taken. To simplify notation, we will work directly in utility space and assume that the set A maps onto a compact convex feasible set of utility pairs,

$S \subset \mathcal{R}^2$. Utilities are viewed as von Neumann-Morgenstern representations of players' risk preferences. Players prefer any agreement in A to b. From now on, we call S the set of *alternatives*. It is convenient to assume that S can be described in the following way. Normalise the utilities of b at 0, and normalise the maximum feasible utility for each player to 1. The utility pair obtained if players agree to trigger outcome t is denoted $m \in S$. Suppose that for, each player, 0 is the utility of the worst alternative in S,and that the worst outcome for i corresponds to the best outcome for j. Let the continuous strictly decreasing concave function $f_i : [0,1] \to [0,1]$ denote the maximum utility that player i can get for any given level of player j's utility, $i, j = 1, 2$. That is, $f_i(x) = \{\max s_i | (s_i, s_j) \in S$ and $s_j = x\}$. So $S = \{s | 0 \leq s_i \leq 1$ and $0 \leq s_j \leq f_j(s_i)\} = \{s | 0 \leq s_j \leq 1$ and $0 \leq s_j \leq f_i(s_j)\}$. For future reference note that obviously we have the identity $f_i(f_j(x)) = x$, $x \in [0,1]$, $i, j = 1, 2$.

There is an unbounded number of potential rounds, $r = 0, 1, ...$, and at any round r there is an exogenous probability $p \in (0,1)$ that negotiations continue to round $r+1$ and a probability $1-p$ that they break down. Thus, at round 0, the expected value of an agreement yielding x in round r is xp^r. As in the standard alternating offers model, players alternate in proposing an alternative (in utility space), which can be either accepted, ending the game, or rejected. In addition, when rejecting a proposal at round r the responding player, say i, can either follow with a counter-offer in round $r+1$; or propose to take the joo. In this latter case player j has to decide whether to accept, in which case the game terminates with the players receiving the alternative (m_1, m_2); or to decline and propose an alternative in round $r+1$. The game continues in this way until a proposal is accepted.

We denote by G^i any subgame starting with the proposal of an alternative by player i. The game begins at round $r = 0$ with a subgame G^1: player 1 proposes the alternative $g^1 = (g_1^1, g_2^1)$, which player 2 can either accept - ending the game-, reject and make a counteroffer, or reject and propose to take the joint outside option.

We denote by D^i any subgame in which player i has to decide whether to agree on taking the joo. If player 2 rejects an alternative and proposes to take the joo, the game moves to a subgame D^1. If the joo is accepted, players' payoff pair is (m_1, m_2); if it is declined, play moves to a subgame of type G^1, in round $r = 1$. Alternatively, if player 2 rejects an alternative to make a counteroffer to player 1, the game moves to the following round $r = 1$, entering a subgame of type G^2. Symmetrically, player 1 can either accept -ending the game-, reject and make a counteroffer (entering a subgame of type G^1 in the following period), or reject and propose to take the joo, in which case the game moves to a subgame of type D^2. Now player 2 can either accept taking the joo, yielding (m_1, m_2); or decline and make a counteroffer in the following round, so that play moves to a subgame of type G^2, and so on.

Furthermore, define the 'Rubinstein' alternatives $\rho^1, \rho^2 \in S$ as the (unique[6]) solution pair to $f_i\left(\rho^j_j\right) = p\rho^i_i$, $i, j = 1, 2$; thus:

$$\rho^i_i = f_i\left(pf_j\left(p\rho^i_i\right)\right)$$

Note that $\rho^i_j = p\rho^j_j$, $i, j = 1, 2$ and that ρ^i and ρ^j lie on the same hyperbola.

In the next propositions we will completely characterise the solution to this game.

First of all notice that all subgames of the same kind starting with a move by player i are identical and have the same set of equilibrium payoffs. Then, let $\overline{g}^i_j$ and $\underline{g}^i_j$ (with $i, j = 1, 2$) be the supremum and the infimum payoffs, respectively, for player j in any subgame perfect equilibrium of subgames of type G^i where player i makes an offer. Similarly, let $\overline{d}^i_j$ and $\underline{d}^i_j$, respectively, be the supremum and infimum equilibrium payoff to player j in any subgame perfect equilibrium of subgames of type D^i where player i has to decide whether to accept taking the joo, or to decline it and make a counteroffer in the next round.

Subgame perfection requires that the following hold:

$$\overline{g}^i_i \leq f_i(\max\left[p\underline{g}^j_j, \underline{d}^i_j\right]),\ \underline{g}^i_i \geq f_i(\max\left[p\overline{g}^i_i, \overline{d}^i_j\right])\ i, j = 1, 2 \tag{1}$$

$$\overline{d}^i_i \leq max\left[m_i, p\overline{g}^i_i\right],\ \underline{d}^i_i \geq max\left[m_i, p\underline{g}^i_i\right]\ i, j = 1, 2 \tag{2}$$

[6]This is verified by standard arguments. A fixed point of $F(x) \equiv f_i(pf_j(px))$, $x \in [0, 1]$ exists by continuity of f_i and f_j. To show that it is unique, check first that F is increasing and **convex.** Then, it suffices to show that neither $x = 0$ nor $x = 1$ are fixed points. We have $0 < F(0)$, since $f_j(0) = 1$ and therefore $F(0) = f_i(p) > 0$ $(p < 1)$. Similarly, $1 > F(1)$, since $f_j(p) > 0$ $(p < 1)$ and $F(1) = f_i(pf_j(p)) < 1$ $(p > 0)$.

3 A Simple Example

In this section we show, in a simple setting, how the presence of a joo dramatically alters the equilibrium structure of the standard bargaining game.

Adam and Eve have a two months old child, and they have to decide how many times a month[7] each of them does the shopping (S) and who babysits (B). Assuming that both Adam and Eve prefer babysitting to shopping - provided the other partner takes care of the other task, payoffs on each occasion could be as below:

		Eve: S	Eve: B
Adam	S	$-4, -6$	$1, 9$
	B	$16, 4$	$-4, -6$

Formally, they must choose any of the payoff alternatives resulting from a correlated strategy pair. Adam makes the first proposal and negotiations follow the pattern described in the previous section. Negotiations break down after every round with probability $\frac{2}{15} = 0.1\overline{3}$, in which case Adam's and Eve's payoffs are zero, i.e. the expected payoff from playing (non cooperatively) the unique mixed strategy Nash equilibrium[8]. The corresponding bargaining set is depicted in figure 1.

The Rubinstein bargaining model yields the equilibrium agreement on the payoff alternative $\left(15, \frac{13}{3}\right)$ which can be obtained if Adam goes Shopping and Eve Babysits twice a month[9], exchanging roles for the rest of the time. Suppose now that the social custom prescribes that the mother should always look after the baby. Then, at any point in the negotiations Adam and Eve can jointly agree to follow such social norm. This convention yields the payoff alternative $(1, 9)$. However in this democratic couple, Eve is powerless to unilaterally impose the (female-biased) convention on Adam, who has to agree to do the shopping. This norm is thus a joo in the sense discussed above. Still, Eve can adopt a *bargaining strategy* consisting of always proposing the alternative $(1, 9)$, rejecting any

[7] We take the average number of days in a month to be 30.

[8] Such strategies are $\left(\frac{2}{5}S, \frac{1}{5}S\right)$for Adam and Eve, respectively.

[9] That is, a fraction $\frac{1}{15}$ of the time.

offer made by Adam yielding her less than 9, always accepting to implement the convention when Adam proposes so, and proposing to allocate tasks in the manner indicated by the convention. Suppose that Adam's strategy prescribes to accept Eve's proposal to implement the convention, to propose the alternative $(1, 9)$ himself, and to propose to implement the convention when rejecting an offer yielding less than 1. Then this pair of strategies is an s.p.e., yielding Eve the non-Rubinstenian payoff 9. Adam cannot deviate profitably at any node, since by doing so he would only either incur the risk of breakdown (when either rejecting the implementation of the convention or failing to propose it), or simply trigger the convention (when making a disequilibrium offer).

In this model the Rubinstein outcome still survives, supported by the usual strategies and simply ignoring the social norm. However, what may be called a 'meta-convention' (namely the *bargaining* convention to uphold the joo of always playing SB) destroys the uniqueness of the Rubinstein outcome.

An interesting twist is obtained if there is some inefficiency in the enjoyment of the joo. Suppose that implementing the convention results in a slight sense of guilt on the part of Eve (not too much, for after all they will have *agreed* to follow the convention!). So the convention yields the inefficient payoff alternative $(1, 8.95)$ (point m in figure 1) Now, *Adam will be better off because of the inefficiency* when a meta-convention like the one described above is played. In other words, he can insist on, and obtain, the alternative $(1.15, 8.95)$ (point a in figure 1) in the next round. Strategies analogous to those described above can be used, the only difference being that in this case both Adam and Eve propose not the 'convention-equivalent' payoff alternative, but its efficient counterpart (that is, a for Adam and e for Eve[10]), which the other spouse will accept. Furthermore, Adam will never propose to implement the convention when rejecting Eve's offer. This strategy profile is optimal as long as for each spouse accepting the inefficient norm is not worse than obtaining the higher payoff with the risk of breakdown - corresponding to either a or e - in the next round. This condition translates into $8.95 \geq \frac{13}{15}9$ for Eve, and $1 \geq \frac{13}{15}1.15$ for Adam, which are both verified.

Finally, suppose now that the convention was that Adam goes shopping four days a

[10]The coordinates of these points are $a = (1.15, 8.95)$ and $e = (1, 9)$.

month[11]. Well, in this case not only is playing according to the convention *an* equilibrium: it is *the unique* equilibrium. In particular, the Rubinstein equilibrium disappears because, if in the face of Adam to insisting on shopping just twice a month, Eve could propose to implement the convention with the certainty that Adam would give in: at that point, insisting would yield him only $\frac{13}{15} \cdot 15 = 13$ rather than a payoff of 14.

4 Results

Observe that the ratios $\frac{m_i}{f_i(m_j)} \in [0, 1]$ provide a measure of how inefficient (distant from the boundary) the joo is. It is the comparison between this inefficiency measure with the bargaining frictions, as measured by the probability of breakdown $(1 - p)$, which will prove crucial to determine the structure of the equilibrium set in the game. Our first proposition establishes that for *any* probability of breakdown there exists an s.p.e. where the negotiated agreement is entirely determined by the joo, provided it is not too inefficient relative to bargaining frictions.

Proposition 1 *For any probability of breakdown $(1-p)$, and for any joint outside option pair of utilities $m \in S$ such that $m_i \geq pf_i(m_j)$, $i, j = 1, 2$, there exists a s.p.e. in which agreement is reached immediately on the alternative $(f_1(m_2), m_2)$. In particular, if m is efficient ($f_1(m_2) = m_1$) then for any value of p there exists a s.p.e. in which agreement is reached immediately on the alternative (m_1, m_2).*

Proof. Denote

$$g^i = \left(g_i^i, g_j^i\right) = \left(f_i(m_j), m_j\right)$$

The following strategies for player $i = 1, 2$ support (g_1^1, g_2^1)as an s.p.e. negotiated alternative:

(i) propose the alternative g^i;

(ii) reject any alternative which yields him any $x < g_i^j$ and accept any $x \geq g_i^j$;

(iii) always accept the joo when player j proposes it;

[11]That is, $\frac{2}{15}$ of the time.

(iv) always propose to take the joo when rejecting an offer.

Checking for subgame perfection is straightforward. Notice that the condition in the statement $m_i \geq pf_i(m_j)\,\forall i,j$ ensures optimality of *(iii)* and *(iv)*. For *(iii)*, by parts *(i)* and *(ii)* of the equilibrium strategies it must be the case that in subgames of type D^i player i prefers the joo to the expected continuation payoff g_i^i. Similarly, for *(iv)* to be optimal at r, it must be the case that the payoff that player i gets when rejecting an offer in G^i and proposing to take the joo is not lower than the expected payoff from inducing G^i in $r+1$ instead. Given part *(iii)* of j's equilibrium strategy, the former payoff is m_i. Therefore this leads to the same inequality considered for *(iii)*. ■

In the equilibrium described in Proposition 1 there is no waste even when the joo is not efficient. The first proposer appropriates all the benefit obtained by not resorting to an inefficient joo.

The next proposition will show that the type of equilibrium analysed above is unique when the joo is not exceedingly favourable to one of the bargainers, where the definition of 'favourable' depends on the probability of continuation. We say that a joo is *balanced* if

$$m_i \in \left(p\rho_i^i, \rho_i^i\right) \forall i$$

For later proofs, it is useful to note the following two facts about balancedness:

- m is balanced if and only if $m_i > p\rho_i^i \forall i$ (this is immediately verified).
- if m is balanced, then

$$m_i > pf_i(m_j)\,\forall i,j$$

To chek this latter statement, note that since f_i is monotonically decreasing, $m_j > p\rho_j^j \Leftrightarrow f_i(m_j) < f_i\left(p\rho_j^j\right) = \rho_i^i$, where the equality derives from the definition of ρ^i and ρ^j as the solution of $f_j(\rho_i^i) = p\rho_j^j$ for $i,j = 1,2$, by applying $f_i(\cdot)$ on both sides. Thus, multiplying both sides of $\rho_i^i > f_i(m_j)$ by p and coupling with $m_i > p\rho_i^i$ yields $m_i > p\rho_i^i > pf_i(m_j)$.

Our next result establishes that when the joo is balanced only the 'joo-type' equilibrium can obtain:

Proposition 2 *If the joo is balanced, then:*

- *If* $m_1 < f_1(m_2)$ *the unique s.p.e. is immediate agreement on the alternative* $(f_1(m_2), m_2)$.
- *If* $m_1 = f_1(m_2)$, *then the unique s.p.e. payoff pair is* (m_1, m_2). *This can be obtained either in an equilibrium identical to the previous case, or by jointly agreeing to take the joo at the first occasion in which this is possible.*

Proof. See Appendix.

In the next result we complete proposition 2 by establishing that lack of balancedness for the joo is both necessary and sufficient for the existence of standard Rubinstenian equilibria.

Proposition 3 *If and only if the joo is not balanced, there exists an s.p.e. in which agreement is reached immediately on the alternative* ρ^1.

Proof. For the 'if' part, we describe strategies supporting the s.p.e.. For player $i = 1, 2$:

(i) propose ρ^i;

(ii) accept only alternatives which yield at least $p\rho_i^i$;

(iii) do not propose to take the joo when rejecting;

(iv) reject the joo when $m_i \leq p\rho_i^i$, and do not reject it otherwise.

The optimality of these strategies is checked easily. We limit ourselves to note that if m is not balanced, then there exists i such that $m_i \leq p\rho_i^i$, which ensures the optimality of *(iii)*.

For the 'only if' part, recall that when the joo is balanced the only s.p.e. payoff is as described in proposition 2. ■

When the inefficiency associated with the joo is large relative to the probability of breakdown, the presence of a joo will be irrelevant, and the Rubinsteinian equilibrium is unique:

Proposition 4 *If* $m_i \leq p\rho_i^i$ *for some* i *and* $p > p_m \equiv \min\left[\frac{m_1}{f_1(m_2)}, \frac{m_2}{f_2(m_1)}\right]$, *then in the unique s.p.e. agreement is reached immediately on the alternative* ρ^1.

Proof. See Appendix.

Figure 2 depicts the regions of existence of the equilibria characterised in propositions 1-4. Notice that in the portion of the set of alternatives delimited by the frontier and the locus of points $m_i = pf_i(m_j)$ it is true that $m_i > pf_i(m_j)$.

Remark 1 It is useful to compare the equilibrium structures of the unilateral and joint outside option models:

- *The Rubinstein outcome is the unique equilibrium outcome in both models.* This happens when $m_1 \leq \rho_1^1$ and $m_2 \leq p\rho_2^2$ and $m_i < pf_i(m_j)$ for some i.

- *The binding outside option determines the unique equilibrium outcome in both models.* This happens when m is balanced.

- *The Rubinstein outcome is the unique equilibrium outcome in the outside option model; both the Rubinstein outcome and the joo-type outcome are equilibrium outcomes in our model.* This happens when $m_i \geq pf_i(m_j)$,$i, j = 1, 2$, $m_1 \leq \rho_1^1$ and $m_2 \leq p\rho_2^2$.

- *The Rubinstein outcome is the unique equilibrium outcome in our model; the binding outside option determines the unique equilibrium outcome in the outside option model.* This happens when $m_i < pf_i(m_j)$ for some player, and either $m_1 > \rho_1^1$ or $m_2 > p\rho_2^2$.

- *The binding outside option determines the unique equilibrium outcome in the outside option model; both the Rubinstein outcome and the joo-type outcome are equilibrium outcomes in our model.* This happens when $m_i \geq pf_i(m_j)$,$i, j = 1, 2$, and either $m_1 > \rho_1^1$ or $m_2 > p\rho_2^2$.

We can conclude this section by completing the description of the efficient equilibria of our model. When the equilibria are not unique, so that both Rubinstein-type and joo-type equilibria exist, the set of efficient equilibrium payoffs is simply obtained by adding all the intermediate alternatives on the boundary of S to the two 'extreme' equilibrium payoffs.

Proposition 5 *If* $m_i > pf_i(m_j)$, $i, j = 1, 2$, *and* $m_j < p\rho_j^j$ *for some* j, *only and all those alternatives which yield player* i *a payoff* $x^* \in [\rho_i^i, f_i(m_j)]$ *if he is the proposer, or* $f_i(y^*) \in [p\rho_i^i, m_i]$ *if he is the responder can be supported in any efficient equilibrium.*

Proof. See Appendix.

As is standard in this type of literatture, the existence of the two extreme equilibrium payoffs guarantees the existence of equilibria with delayed agreement. Here we characterise them for the case $m_2 < p\rho_2^2$; it is straigthforward (thus omitted) to rephrase the proposition in order to obtain analogous results in the symmetric case for $m_1 < p\rho_1^1$.

Proposition 6 *For any time* T *there exist a probability* $\widehat{p}$ *(depending on* T*), such that for* $p \in [\widehat{p}, 1)$, $m_i > pf_i(m_j)$, $i, j = 1, 2$, *and* $m_2 < p\rho_2^2$ *every alternative which yields player 1 a payoff* $z^* \in \left[\frac{\rho_1^1}{p^{T-2}}, f_1(\frac{m_2}{p^{T-1}})\right]$ *can be supported in an equilibrium in which negotiated agreement is reached at time* T.

Proof. See Appendix.

5 Applications

5.1 Economics of Divorce: do divorce laws matter?

An area where the theory of joos has a natural application is the economics of divorce. The legal rights in divorce are allocated differently in the two broad categories of mutual consent and unilateral divorce. However, traditional theory (e.g. Becker et al. (1977) and Becker (1991)) holds that the allocation of legal rights does not affect divorce decisions and rates: whether a spouse has the right to divorce or to prevent divorce should not matter, because the other spouse can always *buy* these rights if the incentives are sufficientlly large. If Adam thinks he is better off divorced than married but the law requires Eve's consent, then - the argument goes - he can give her a payment in return for the consent to divorce. If the payment Adam is willing to give is not enough to persuade Eve, then it means that there is no *aggregate* gain form divorce: but in this case, divorce would not have happened under a unilateral law either! In fact, should Adam threaten to divorce, Eve would pay him not to.

Recently this view has been challenged in a series of papers by Clark (see Clarke (1998), Clark (1998)). He models marriage as a bargaining problem over a set of alternatives M; in case of divorce the set of alternatives changes irreversibly to D. Although he does not use a formal bargaining model, he provides interesting arguments to support the view that divorce law in fact matters in several cases, depending on the relation between M and D and the agreements reached within them. In particular, legal rights influence the outcome of divorce proceedings only if M and D are *not nested*.

However, our model suggests that even if $D \subseteq M$ divorce law matters. The distinction between mutual consent and unilateral divorce can be neatly translated in our framework as the difference between the concept of a joo and a "standard" outside option. Interpret the equilibrium agreement in D as either an outside option or a joo for the bargaining set M. With unilateral divorce the outside option principle applies, and determines the unique equilibrium outcome. On the other hand, the structure of equilibria with mutual consent divorce is more complex. If the agreement in case of divorce is *balanced*, the outcome is the same as with unilateral divorce (although the mechanisms at work is different). Otherwise, the possibility of divorce may have no effect whatsoever - so that the Rubinstein equilibrium obtains - even when the outside option is binding (see Remark 1 for the full set of possibilities). Alternatively, a whole set of marital agreements - including *inefficient* ones - can result (whereas efficiency is guaranteed in the case of unilateral divorce). In summary, we can conclude that divorce law does not matter only when the outcome of divorce is 'close' to that of marriage (in the sense made precise by the notion of balancedness).

Note that a natural interpretation of the joo in this context is as the equilibrium of a "divorce game" determined by a standard alternating offers bargaining without outside options (once parties have filed for it, divorce is irrevocable).

Similar conclusions to those drawn for divorce apply within the context of an arbitration model for the issue of unilateral versus mutual consent arbitration (see Manzini and Mariotti (1998)). In that case, however, an extended model would be different because of the difference in the interpretation of the joo.

5.2 Incentives to invest in a theory of the firm

So far we have analysed equilibria for a given set of alternatives. However interesting conclusions can be drawn when our model is used to study comparative statics when the set of alternatives changes. An important area where *enlargements* of the set of alternatives are relevant is the theory of the firm developed by Grossman, Hart and Moore (see Grossman and Hart (1986), Hart and Moore (1990), Hart (1995); henceforth GHM). The central role in that theory is played by the incentives to invest on the part of owners of different assets, who then bargain over how to share the gains from trading with each other (in which case the parties have access to all assets). At the analytical level, the effect of investments is simply to enlarge the set of available alternatives.

Asset ownership determines the value of no-trade payoffs. Therefore, asset ownership affects investment incentives to the extent that no–trade payoffs determine agreement payoffs. However, it has been pointed out that the exact interpretation of no-trade payoffs can make a crucial difference. In the original GMH theory they are considered as *inside options*, that is payoffs obtained in case of "temporary" disagreement (one which does not preclude resuming of negotiations). More recently, de Meza and Lockwood (1998)[12] have instead proposed to interpret no-trade payoffs as *outside options*, that is payoffs which accrue to the bargainers due to them abandoning negotiations in favour of other activities.

The implications of these alternative interpretations are substantially different. While the details are complex, the basic intuition is clear. If the bargaining outcome is driven by a (binding) outside option, an enlargement of the set of alternatives bears no advantage to the player with the (favourable) outside option, provided of course that the outside option remains binding: in this case the incentive to invest is absent for that player. On the contrary, in the presence of an inside option enlargements of the set of alternatives will normally benefit both players.

The concept of a joo allows some elaboration on these issues.

As shown in the previous section, the existence of an efficient and unbalanced joo ensures that all alternatives included between this joo and the Rubinstein solution can be supported as equilibrium agreements. Consider the case where the joo payoffs obtain

[12] See also Rasjan and Zingales (1998) and Chui (1999).

in equilibrium, which in this case coincides with the outside option equilibrium described above. At this point, an enlargement of the set of alternatives which did not make the joo too inefficient would still result in a joo type equilibrium, so that the player with the (favourable) joo appropriates the entire extra surplus from investment. This is a diametrically opposed result to the outside option model outlined above, as depicted in figure 3. Here the original bargaining set is delimited by the solid line, whereas the dashed line delimits the after investment set of alternatives; m identifies both the original outside option and the joo; r and r' denote the Rubinstein point before and after the investment, respectively; and p and q refer to the after investment outside option and joo-type equilibrium payoffs, respectively.

If the enlargement of the set of alternatives makes the original joo "too" inefficient, the *unique* after investment equilibrium outcome is the Rubinstein point for the new bargaining set, r' (see figure 4). In the case depicted our model implies that the returns to investment for player 2 are higher than in the outside option model (and player 1 would even *lose* in the final equilibrium because of investement). Note also that, although the after investment equilibrium outcome is the same as in the inside option model, the incentives to investment are very different in the two situations.

Without pursuing this application of our model in further detail, it should be clear how transforming an outside option into a joo creates a rich set of new possibilities in the analysis of investements when asset ownership matters. The joo can in this way be envisaged as a pre-contractual tool to shape investment incentives.

6 Concluding remarks

In this paper we have introduced a new concept in the theory of bilateral bargaining, that of a joint outside option. We hope to have shown that it is a relevant concept, in that several economic situations present a broad structure where the concept is useful to draw interesting conclusions. In particular, *the voluntary nature of the act of taking a joo does not makes it ininfluential on the bargaining outcome*, as one might believe it to be at first blush. For the case we have called 'balanced', the implication is that the

‘Outside Option Principle’ holds independently of whether one of the players has veto power on taking an outside option which is unfavourable to him. For the general case, the distinction between standard and joint outside options sheds light, for example, on the difference between consensual versus unilateral divorce and arbitration, as well as on the incentives to invest in the Grossman-Hart-Moore theory of the firm.

Of course, what we have provided is merely a general framework, which is stripped of many details and which may be useful as a ‘canonical form’. Investigations of specific economic problems will benefit from enriching this basic structure with the relevant details. In our opinion, an interesting step to take even in the abstract, is to consider the case in which the parties are not fully informed, or have differing expectations, regarding the joo. This addition would significantly complicate the analysis but seems worth pursuing.

Appendix

Proof of Proposition 2.

The following lemma gives conditions under which the maximum s.p.e. payoff for a player in subgames where he is the proposer is greater than the maximum s.p.e. payoff to that player in subgames when the opponent is a proposer, and will be needed in the proof of proposition 2.

Lemma 1 *If, for some i, $m_i \leq p\overline{g}_i^i$, then $\overline{g}_i^i \geq \overline{g}_i^j \forall p \in (0,1)$.*

Proof. Suppose, by contradiction, that

$$\overline{g}_i^i < \overline{g}_i^j$$

We will show that in this case i would be willing to accept $\overline{g}_i^j - \eta$ in the subgame G^j for some small η, thus contradicting the definition of $\overline{g}_i^j$.

In fact, suppose that i rejected the outcome yielding him $\overline{g}_i^j - \eta$ and made a counteroffer. Then he would get at most $p\overline{g}_i^i$: but

$$\begin{aligned} p\overline{g}_i^i &< \overline{g}_i^i < \overline{g}_i^j \\ &\Rightarrow p\overline{g}_i^i < \overline{g}_i^j - \eta \end{aligned}$$

for η sufficiently small. Suppose, on the other hand, that i rejected the outcome yielding him $\overline{g}_i^j - \eta$ and proposed the joo. Then, if j accepted, i could obtain at most

$$m_i \leq p\overline{g}_i^i < p\overline{g}_i^j < \overline{g}_i^j - \eta$$

for a small η. If j rejected the joo, i could obtain at most

$$p\overline{g}_i^j < \overline{g}_i^j - \eta$$

for a small η. ■

Proof of proposition 2.

We will show that

$$\overline{g}_i^i \leq f_i(m_j) \leq \underline{g}_i^i \text{ with } i, j = 1, 2,$$

from which the conclusion of the statement follows. We proceed in two steps:

Step 1: $\overline{g}_i^i \leq f_i(m_j)$**.**

Suppose to the contrary that $\overline{g}_i^i > f_i(m_j)$. Distinguish two cases:

1. $m_i > p\overline{g}_i^i$
2. $m_i \leq p\overline{g}_i^i$

We will show that, at an equilibrium, in both cases there is a move by player j which is more profitable than accepting $(\overline{g}_i^i, f_j(\overline{g}_i^i))$.

Case 1. Consider the following action by player j: reject the alternative $(\overline{g}_i^i, f_j(\overline{g}_i^i))$ and propose the joo. Player i would accept this proposal in equilibrium, since by rejecting he can get at most $p\overline{g}_i^i$, while by accepting he gets $m_i > p\overline{g}_i^i$ by assumption. Then by this action player j gets

$$m_j > f_j\left(\overline{g}_i^i\right)$$

since $\overline{g}_i^i > f_i(m_j)$ (by assumption) and $f_j(\cdot)$ is monotonically decreasing, hence

$$f_j\left(\overline{g}_i^i\right) < f_j(f_i(m_j)) = m_j$$

Case 2. Player j can profitably reject and propose the alternative $(p(\overline{g}_i^i + \eta), f_j(p(\overline{g}_i^i + \eta)))$. In equilibrium player i accepts this proposal, since otherwise he could either *(i)* make a counteroffer, obtaining at most

$$p\overline{g}_i^i < p\left(\overline{g}_i^i + \eta\right)$$

or *(ii)* propose to take the joo; then, if player j concedes on taking the joo, player i gets

$$m_i \leq p\overline{g}_i^i < p\left(\overline{g}_i^i + \eta\right)$$

(the first inequality following from the definition of case 2). If the joo is rejected, player i gets at most

$$p\overline{g}_i^j \leq p\overline{g}_i^i < p\left(\overline{g}_i^i + \eta\right)$$

(where the first inequality follows from lemma 1).

Finally, player j profits from this counterproposal iff:

$$pf_j\left(p\left(\overline{g}_i^i+\eta\right)\right) > f_j\left(\overline{g}_i^i\right)$$

that is - given that $f_i(\cdot)$ is monotonically decreasing - iff:

$$f_i\left(pf_j\left(p\left(\overline{g}_i^i+\eta\right)\right)\right) < f_i\left(f_j\left(\overline{g}_i^i\right)\right) = \overline{g}_i^i \tag{*}$$

But by balancedness and the condition defining case 2:

$$p\overline{g}_i^i \geq m_i > p\rho_i^i = pf_i\left(pf_j\left(p\rho_i^i\right)\right) \Rightarrow \overline{g}_i^i > f_i\left(pf_j\left(p\rho_i^i\right)\right) = \rho_i^i$$

This implies (*) for η small, because ρ_i^i is the unique fixed point of $F(x) = f_i(pf_j(px))$, $\rho_i^i > 0$; thus, we must have $x > F(x)\ \forall x > \rho_i^i$.

Step 2: $f_i(m_j) \leq \underline{g}_i^i$.

We start showing that at an equilibrium player j cannot improve on m_j. To see this, suppose first that player j rejected the proposal $(f_i(m_j), m_j)$ and made a counter-offer; then, by Step 1 the maximum he could get by this action is $pf_j(m_i)$. Thus, rejecting pays only if $pf_j(m_i) > m_j$; however, by balancedness, $m_j > p\rho_j^j$, so that these two inequalities together require $pf_j(m_i) > p\rho_j^j \Leftrightarrow f_j(m_i) > \rho_j^j$. But by definition[13] $\rho_j^j = f_j(p\rho_i^i) > f_j(m_i)$, a contradiction, so that player j would not find it profitable to follow this action.

On the other hand, suppose that player j proposed to take up the joo after rejecting $(f_i(m_j), m_j)$. In an s.p.e., player i would accept, since by Step 1 and the above argument he would obtain at most $pf_i(m_j) < m_i$ by rejecting and inducing a game G^i.

This shows that, if the joo is balanced, at an s.p.e. player j will accept any proposal yielding him strictly more than m_j, and will be indifferent between *(a)* accepting an offer m_j and *(b)* rejecting and proposing the joo. However, if $f_i(m_j) > m_i$ there cannot be an s.p.e. where player j follows action *(b)* after player i's proposing the alternative $(f_i(m_j), m_j)$, since then player i could improve on his payoff by proposing $(f_i(m_j+\eta), m_j+\eta)$ instead, which, for small η, would yield him a payoff greater than m_i he would obtain in the joo[14], and which would be accepted. ■

[13]Recall that ρ_j^j and ρ_i^i solve $f_i\left(\rho_j^j\right) = p\rho_i^i \forall i,j$. Thus, applying f_j on both sides yields $\rho_j^j = f_j\left(p\rho_i^i\right)$.

[14]Note that $f_i(m_j) \geq m_i$, with equality holding if m is efficient.

Proof of Proposition 4.

We first need to prove a a result similar to Lemma 1:

Lemma 2 *If* $m_i \leq p\rho_i^i$ *and* $\overline{g}_i^i > \rho_i^i$ *for some* i*, then* $\overline{g}_i^i > \overline{g}_i^j \forall p \in (0,1)$.

Proof. Suppose, by contradiction, that

$$\overline{g}_i^i \leq \overline{g}_i^j$$

Then also

$$\overline{g}_i^j > \rho_i^i$$

We will show that in this case i would be willing to accept $\overline{g}_i^j - \eta$ in the subgame G^j for some small η, thus contradicting the definition of $\overline{g}_i^j$.

In fact, suppose that i rejected the alternative yielding him $\overline{g}_i^j - \eta$ and made a counteroffer. Then he would get at most $p\overline{g}_i^i$: but

$$\begin{aligned} p\overline{g}_i^i &< \overline{g}_i^i \leq \overline{g}_i^j \\ &\Rightarrow p\overline{g}_i^i < \overline{g}_i^j - \eta \end{aligned}$$

for η sufficiently small. Suppose, on the other hand, that i rejected the alternative yielding him $\overline{g}_i^j - \eta$ and proposed to take the joo. Then, if j accepted, i could obtain at most

$$m_i \leq p\rho_i^i < \rho_i^i < \overline{g}_i^j - \eta$$

for a small η. If j declined the joo, i could obtain at most

$$p\overline{g}_i^j < \overline{g}_i^j - \eta$$

for a small η. ■

Proof of Proposition 4

Assuming that p satisfies the condition in the statement, we deal with three intermediary steps.

Step 1: $m_i \leq p\rho_i^i \Rightarrow \overline{g}_i^i \leq \rho_i^i$.

Suppose to the contrary that

$$\overline{g}_i^i > \rho_i^i \Leftrightarrow f_j\left(\overline{g}_i^i\right) < f_j\left(\rho_i^i\right) = \rho_j^i$$

We will show that in an s.p.e. where $\overline{g}_i^i$ satisfy this condition a contradiction results, because j could reject any proposal yielding him less than ρ_j^i and offer to player i the share $p\left(\overline{g}_i^i + \eta\right)$, which would be accepted at an s.p.e. Then j would be better off than by accepting a share less than $p\rho_i^i$ because for η small:

$$\begin{aligned}
\overline{g}_i^i &> \rho_i^i \Rightarrow \\
\overline{g}_i^i &> p\left(\rho_i^i + \eta\right) \\
&\Leftrightarrow f_j\left(\overline{g}_i^i\right) < f_j\left(p\left(\rho_i^i + \eta\right)\right)
\end{aligned}$$

To see that i would accept the proposal at an s.p.e., observe that if he made a counteroffer he could get at most

$$p\overline{g}_i^i < p\left(\overline{g}_i^i + \eta\right)$$

On the other hand, if he proposed to take the joo, either j would accept and i would get

$$m_i < p\left(\overline{g}_i^i + \eta\right)$$

by the assumption $m_i \leq p\rho_i^i < p\overline{g}_i^i$; or j would decline and i would get at most

$$p\overline{g}_i^j < p\left(\overline{g}_i^i + \eta\right)$$

by lemma 2.

Step 2: $m_i \leq p\rho_i^i,\ m_j > p\rho_j^j \Rightarrow p_m = \frac{m_i}{f_i(m_j)}$.

Note first that $p_m = \frac{m_i}{f_i(m_j)} \Leftrightarrow m_i f_j\left(m_i\right) < m_j f_i\left(m_j\right)$. To see that this inequality is implied by the assumptions, we give a simple geometric argument. Consider the point $x = \left(x_i, x_j\right) = \left(p\rho_i^i, p\rho_j^j\right) \in S$. Because ρ^i and ρ^j lie on the same hyperbola, x is such that $x_i f_i\left(x_j\right) = x_j f_j\left(x_i\right)$. Thus any point $y \in S$ with $y_i < x_i$ and $y_j > x_j$ is such that $y_i f_i\left(y_j\right) < x_i f_i\left(x_j\right)$, while $y_j f_j\left(y_i\right) > x_j f_j\left(x_i\right)$ (see figure 5, which depicts the case for $y_1 < x_1$ and $y_2 > x_2$). Therefore the conditions $m_i \leq p\rho_i^i,\ m_j > p\rho_j^j$ imply that $m_i f_i\left(m_j\right) < m_j f_j\left(m_i\right)$.

Step 3: $m_i \leq p\rho_i^i,\ m_j > p\rho_j^j \Rightarrow \underline{g}_i^i \geq \rho_i^i$.

Suppose to the contrary that

$$\underline{g}_i^i < \rho_i^i$$

Then there exists an s.p.e. of some subgame in which j would reject any alternative yielding him ρ_j^i. Thus, suppose it was optimal for j to reject and induce a subgame G^j. By so doing he would get at most

$$f_j\left(\max[p\underline{g}_i^i, \underline{d}_i^i]\right) \leq f_j\left(p\underline{g}_i^i\right)$$

in the next round, yielding the contradiction

$$\begin{aligned} \underline{g}_i^i &\geq f_i\left(pf_j\left(p\underline{g}_i^i\right)\right) \\ &\Leftrightarrow \underline{g}_i^i \geq \rho_i^i \end{aligned}$$

where the last equivalence follows from the argument in Step 1 of Case 2 of Proposition 2.

Suppose then that it was optimal for j to reject and propose to take the joo. If i declined, then j would get at most $pf_j\left(\underline{g}_i^i\right)$, so that

$$\underline{g}_i^i \geq f_i\left(pf_j\left(\underline{g}_i^i\right)\right) = F\left(\frac{\underline{g}_i^i}{p}\right) > F\left(\underline{g}_i^i\right)$$

But this is a contradiction by a now familiar argument. Therefore, if it was optimal for j to reject at an s.p.e., i would concede to take the joo. This can only be the case if

$$m_i \geq p\underline{g}_i^i \Leftrightarrow \frac{m_i}{p} \geq \underline{g}_i^i$$

Thus, when rejecting optimally j gets exactly m_j, so that

$$\underline{g}_i^i \geq f_i\left(m_j\right) \geq m_i$$

The last two inequalities are compatible only if

$$\frac{m_i}{p} \geq f_i\left(m_j\right) \Leftrightarrow p \leq \frac{m_i}{f_i\left(m_j\right)}$$

If it is the case that $p_m = \frac{m_i}{f_i(m_j)}$, then the above contradicts the assumption that $p > p_m$. But by step 2 it must indeed be $p_m = \frac{m_i}{f_i(m_j)}$. This concludes the proof of step 3.

To conclude the proof, observe that only two cases can hold:

a) $m_i \leq p\rho_i^i$, $m_j \leq p\rho_j^j$

b) $m_i \leq p\rho_i^i$, $m_j > p\rho_j^j$

In case *(a)*, step 1 yields

$\overline{g}_i^i \leq \rho_i^i$ and $\overline{g}_j^j \leq \rho_j^j$

In case *(b)*, step3 yields

$\overline{g}_i^i \leq \rho_i^i \leq \underline{g}_i^i$

In both cases, it follows from standard reasoning that $\rho_i^i = \underline{g}_i^i = \overline{g}_i^i$ and $\rho_j^j = \overline{g}_j^j = \underline{g}_j^j$. ■

Proof of Proposition 5.

To see that the intervals described in the statement are not empty, recall that $m_j < p\rho_j^j \Rightarrow f_i(m_j) > f_i\left(p\rho_j^j\right) = \rho_i^i$, where the equality derives from the definition $f_j(\rho_i^i) = p\rho_j^j$, $i, j = 1, 2$, by applying f_i on both sides. Furthermore, since $m_i > pf_i(m_j)$, the fact just proved that $f_i(m_j) > \rho_i^i$ implies $m_i > p\rho_i^i$.

The rest of the proof is divided into two parts. In the first part we show that no alternative which yields player i a payoff outside the ranges specified in the proposition can be supported in equilibrium. In part II we describe a pair of strategies which support the s.p.e. outcome introduced above.

Part I: Payoff bounds

Step 1: $\overline{g}_i^i \leq f_i(m_j)$.

The proof of this step is identical to the proof of Step 1 in Proposition 2. Note that the conditions in the statements of both the present proposition and Proposition 2 are the same, except that in the present case we are not assuming balancedness. However, we have proved above that the inequality $m_i > p\rho_i^i$ holds, and it is easy to check that it is only this part of the balancedness condition which is used in the proof of Step 1 of Proposition 2.

Step 2: $\underline{g}_i^i \geq \rho_i^i$.

We show that player j would receive a payoff of at most $\rho_j^i = p\rho_j^j$ when responding in G^i. When rejecting, player j can either *(i)* propose to take the joo, or *(ii)* make a counteroffer. If *(i)*, player i would accept, since in this way his payoff exceeds the highest payoff he could otherwise obtain by declining the joo to make a counteroffer, given that

$$m_i > pf_i(m_j) \geq p\overline{g}_i^i$$

where the weak inequality follows from step 1. Thus, j would obtain $m_j < p\rho_j^j$. Alternatively, if *(ii)*, j would induce a subgame G^j. By so doing he would get at most

$$f_j\left(\max[p\underline{g}_i^i, \underline{d}_i^i]\right) \leq f_j\left(p\underline{g}_i^i\right)$$

in the next round, yielding

$$\begin{aligned} \underline{g}_i^i &\geq f_i\left(pf_j\left(p\underline{g}_i^i\right)\right) \\ &\Leftrightarrow \underline{g}_i^i \geq \rho_i^i \end{aligned}$$

where the last equivalence follows from the argument given in Step 1 of Case 2 of Proposition 2.This shows that player j's payoff is not greater than $p\rho_j^j$, as desired.

Step 3: $\underline{g}_i^j \geq p\rho_i^i$.

Suppose to the contrary:

$$\underline{g}_i^j < p\rho_i^i \Rightarrow f_j\left(\underline{g}_i^j\right) > f_j\left(p\rho_i^i\right) \Leftrightarrow \overline{g}_j^j > \rho_j^j$$

Then player i could reject and propose in the following round an alternative yielding player j a payoff of $p\left(\overline{g}_j^j + \eta\right) > p\rho_j^j$. Player j will accept, because by rejecting he can obtain either $p\overline{g}_j^j$, by making a counterproposal in the subsequent round; or $m_j < p\rho_j^j < p\left(\overline{g}_j^j + \eta\right)$ if he proposes to take the joo (which is accepted by player i, since, as argued in the previous step, $m_i > pf_i(m_j) \geq p\overline{g}_i^i$). Such a deviation is profitable for player i, if

$$pf_i\left(p\left(\overline{g}_j^j + \eta\right)\right) > \underline{g}_i^j = f_i\left(\overline{g}_j^j\right)$$

or, applying f_j on both sides,

$$f_j\left(pf_i\left(p\left(\overline{g}_j^j + \eta\right)\right)\right) < \overline{g}_j^j$$

which is true, for η small enough, if $\overline{g}_j^j > \rho_j^j$, as implied by our contradiction assumption.

Step 4: $\overline{g}_i^j \leq m_i$.

Player i cannot improve by rejecting an alternative yielding him m_i. Rejecting and making a counteroffer yields not more than $p\overline{g}_i^i \leq pf_i(m_j) < m_i$. Rejecting and proposing to take the joo does not improve on m_i either, since: if player j accepts the joo, i gets m_i; if player j rejects the joo, player i can obtain at most $\max[p\overline{g}_i^i, m_i] = m_i$ in the following round.

<u>Part II</u>: **Equilibrium strategies.**

Let $y^* \in \left[f_j(m_i), \rho_j^j\right]$. The arguments are standard and we just sketch the strategies supporting the equilibrium. For player i (player j, respectively): to claim x^* (y^*) for himself, to accept any proposal which yields him at least $f_i(y^*)$ ($f_j(x^*)$) and to reject any other proposal. In case of a deviation, play reverts to the strategies which yield the worst payoff for the deviator, that is either those supporting the Rubinstenian s.p.e. payoff as specified in Proposition 3; or those supporting the joo-type equilibrium, as specified in Proposition 1.

To check that the strategies sketched above constitute an s.p.e., consider first subgames in which player i makes an offer. If he proposed any alternative yielding him a payoff $x \neq x^*$, given his strategy player j would reject, and both players would revert to the Rubinstenian equilibrium play: player j would propose the alternative ρ^j, which player i would accept, obtaining a lower expected payoff than the lowest possible value of x^* (i.e. $p\rho_i^j = p^2\rho_i^i < \rho_i^i$). Player j cannot improve on his payoff by accepting an alternative which yields a payoff different from $f_j(x^*)$, as by rejecting a disequilibrium offer and switching to the Rubinstenian play he can secure a payoff whose expected value is $p\rho_j^j = f_j(\rho_i^i) \geq f_j(x^*)$ (where the inequality follows by applying f_j to both sides of $x^* \geq \rho_i^i$). Consider now subgames where player j is the proposer, so that agreement is reached on the alternative $(f_i(y^*), y^*)$. Player j cannot improve on his payoff by making a different offer, as given his strategy, player i would reject and then revert to play the joo-type equilibrium strategies: he would propose to take the joo, which player j will accept, obtaining a payoff of $m_j \leq f_j(m_i) \leq y^*$. Finally, player i cannot improve on his payoff by accepting an alternative which yields a payoff different from $f_i(y^*)$, as by rejecting a disequilibrium offer and switching to the joo-type play he can secure a payoff of $m_i > f_i(y^*)$. ■

Proof of Proposition 6.

First of all, recall that conditions in the statement ensure that both the Rubinstein and the joo-type equilibria can obtain. Then, strategies that support this equilibrium are: At time t and until time T, player 1 proposes the alternative $\left(f_1\left(p^{T-t}f_2\left(z^*\right)\right);p^{T-t}f_2\left(z^*\right)\right)$ and rejects any offer which yields less than m_1, while player 2 proposes the alternative $\left(p^{T-t}f_1\left(z^*\right);f_2\left(p^{T-t}f_1\left(z^*\right)\right)\right)$ and rejects any offer which yields less than $p\rho_2^2$; both players never propose to take the joo and always decline it when it is called for by their opponent; at time T, player i proposes an alternative yielding player 1 a payoff of z^*, which is accepted. Both players punish deviations by reverting play to the worst equilibrium for the deviator (i.e. either those supporting the Rubinstein s.p.e. payoff, as specified in Proposition 3, or those supporting the joo-type equilibrium, as specified in Proposition 1, as explained in the proof of Proposition 5). We now show that the payoffs specified above can be supported in equilibrium.

1. *Deviations by Player 1.* Consider first a deviation by player 1 in the **first round** (at time $t=0$); there can actually be two such deviations:

   (i) *Deviant offer.* Suppose that player 1's proposed alternative yields his opponent a payoff $p^{T-t}f_2\left(z^*\right)-\eta$ with $\eta>0$. Then player 2 rejects, and play reverts to the Rubinstein strategy profile, yielding player 1 a payoff of $p\rho_1^1$ in the following round. On the other hand, by conforming to his equilibrium strategy player 1 could have obtained a payoff of z^* at time T; consequently, player 1 cannot profit from a deviation in the first round as long as:

   $$p^2\rho_1^1 \leq p^T z^* \Rightarrow z^* \geq \frac{\rho_1^1}{p^{T-2}} \tag{C1}$$

   Similarly, in even periods other than the first ($t>0$), a deviant offer by player 1 will not be profitable as long as $z^* \geq \frac{\rho_1^1}{p^{T-(t+2)}}$; however, $\frac{\rho_1^1}{p^{T-2}} > \frac{\rho_1^1}{p^{T-(t+2)}}$, so that condition 3 above encompasses all deviant offers in periods other than the first.

   (ii) *Response to a deviant proposals to take the joo.* Suppose that, after rejecting player 1's offer, player 2 unexpectedly proposes to take the joo. By accepting it, player 1 would end up with a payoff of m_1 immediately. Consequently, declining the joo cannot be improved upon as long as $m_1 \leq pf_1(m_2)$, which holds always. Similarly one can verify that in odd periods other than the first a deviant response to the proposal to take the joo will not be profitable as long as the above condition holds.

Consider now a deviation by player 1 in the **second round** (at time $t = 1$). He cannot increase his payoff by accepting player 2's offer of $p^{T-1}z^*$, as this exactly matches his equilibrium payoff. Moreover, player 1 cannot profitably propose to take the joo when rejecting, since given his strategy player 2 would decline, and play would then move to the Rubinstein equilibrium (with player 2 as first mover), yielding player 1 a payoff of $p\rho_1^1$; consequently, for such deviation not to be profitable it must be that

$$p^{T-1}z^* \geq p^2\rho_1^1 \Leftrightarrow z^* \geq \frac{\rho_1^1}{p^{T-3}}$$

which is implied by condition 3. Similarly one can check that any deviation by player 1 in any odd period t is going to be prevented by $z^* \geq \frac{\rho_1^1}{p^{T-(t+2)}}$, which is always a less stringent condition than $z^* \geq \frac{\rho_1^1}{p^{T-2}}$, which is in turn encompassed by $z^* \geq p^{-T}m_1$; thus 3 imposes a lower bound on z^* which prevents profitable deviations by player 1.

2. *Deviations by Player 2.* Consider first a deviation by player 2 in the **first round**. Similarly to the above, player 2 cannot increase his payoff by accepting an offer of $p^T f_2(z^*)$, since it exactly matches his equilibrium payoff. Next, suppose player 2 proposes to take the joo after rejecting his opponent's offer. In this case player 1 rejects, and play reverts to the joo-type equilibrium, yielding player 2 a payoff of m_2 in the following round; consequently, the above deviation is not profitable if

$$pm_2 \leq p^T f_2(z^*) \Leftrightarrow z^* \leq f_1\left(\frac{m_2}{p^{T-1}}\right) \tag{C2}$$

Similarly, one can rule out deviations in any odd period t other than the first if $z^* \leq f_1\left(\frac{m_2}{p^{T-(t+1)}}\right)$. However, $f_1\left(\frac{m_2}{p^{T-1}}\right) < f_1\left(\frac{m_2}{p^{T-(t+1)}}\right)$, so that condition 3 above is enough to guarantee the suboptimality of all odd period deviations by player 2.

Consider now a deviation by player 2 in the **second round** (at time $t = 1$). Symmetrically to the case of player 1, there can be two types of deviations:

(i') *Deviant offer.* Suppose player 2 proposed an alternative yielding player 1 a payoff of $p^{T-1}z^* - \eta$, with $\eta > 0$. Then player 1 would reject, and play would revert to the joo-type equilibrium, yielding player 2 a payoff of m_2 one round later. Thus such a deviation is not profitable for player 2 if

$$pm_2 \leq p^{T-1}f_2(z^*) \Leftrightarrow z^* \leq f_1\left(\frac{m_2}{p^{T-2}}\right)$$

More generally, a deviant offer at any even period t is not going to be profitable for player 2 as long as $z^* \leq f_1\left(\frac{m_2}{p^{T-(t+1)}}\right)$, which is a weaker requirement and is the same as the one derived for deviations in odd periods other than the first.

(ii') *Response to deviant proposals to take the joo.* Suppose, after rejecting player 2's offer, player 1 unexpectedly proposes to take the joo. If player 2 accepted, he would obtain a payoff of m_2, so that such a deviation cannot increase player 2's payoff if $m_2 < p\rho_2^2$, which is always true. Similarly for deviations in even periods other than the second.

To conclude the proof, notice that the interval $\left[\frac{\rho_1^1}{p^{T-2}}, f_1\left(\frac{m_2}{p^{T-1}}\right)\right]$ is not empty if

$$f_1\left(\frac{m_2}{p^{T-1}}\right) - \frac{\rho_1^1}{p^{T-2}} \geq 0$$

Now when $p \to 1$ the above expression tends to:

$$f_1(m_2) - \rho_1^1 > 0$$

Thus for any T, in the neighbourhood of 1 it will always be possible to find a value $\widehat{p}$ of the discount factor such that when $p \in [\widehat{p}, 1)$ the interval $\left[\frac{\rho_1^1}{p^{T-2}}, f_1\left(\frac{m_2}{p^{T-1}}\right)\right]$ is not empty. ■

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