Through Trial & Error to Collusion*

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Abstract

In this note we study a very simple trial & error learning process in the context of a Cournot oligopoly. Without any knowledge of the payoff functions players increase, respectively decrease, their quantity by one unit as long as this leads to higher profits. We show that this process converges to a collusive outcome.

JEL– classification numbers: C72, L13.

Very preliminary, but comments welcome!

1 Introduction

In this note we consider a very simple learning process, which we call trial & error learning. The process is simple in two ways. First, it requires a very low cognitive effort of players. And second, it does not require *any* information about the payoff function of the game. It works as follows.

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Players choose their strategies from a finite grid. Consider a strategy set that can be ordered. Typical examples include prices, quantities, expenditures and so on. Everytime players increase or decrease their strategy by one grid point, they check whether this change results in an increase or a decrease in payoffs. If it increases payoffs, the movement in this direction is continued. If it does not, the reverse direction is taken.

We study the consequences of this learning process in the context of a standard Cournot oligopoly. Somewhat surprisingly, it turns out that trial & error learning yields a collusive outcome.[1] We prove this result analytically for the duopoly case. With simulations we demonstrate that the result is robust to the introduction of more firms and slight modifications of the learning rule.

The result is remarkable as there is to our knowledge no other sensible learning rule that would yield collusion in this setting. In fact, almost all known learning processes converge to the (unique) Cournot–Nash equilibrium of the game if they converge.[2] This holds for best reply learning (Cournot, 1838), fictitious play, evolutionary dynamics like the replicator dynamics, gradient learning (Arrow and Hurwicz, 1960), or more generally for the class of adaptive learning processes (Milgrom and Roberts, 1991).

Trial & error learning can be considered a particular form of learning direction theory (Selten and Buchta, 1998). This theory assumes that players have a model which allows them to conclude in which direction better actions can be found. In the absence of information about demand and cost conditions, one interpretation is that the right direction can be found by looking which direction was successful last period.

The remainder of the paper is organized as follows. The next section introduces the learning rule and presents the main theoretical result. Section 3 contains the simulation results. In the concluding Section 4 we discuss some experimental evidence.

2 Trial & error learning

Consider a standard Cournot oligopoly with n firms. Each firm may choose outputs from a finite grid

$$\Gamma := \{0, \delta, 2\delta, ..., v\delta\}$$

[1] By collusive outcome we mean an outcome on the Pareto frontier.

[2] One interesting exception is a simple imitation process, which was shown to converge to the competitive outcome (Vega–Redondo, 1997).

for some arbitrarily small grid size $\delta > 0$ and $v \in \mathbb{N}$ large enough. Inverse demand, $p(Q) \geq 0$, $Q = \sum_{i=1}^{n} q_i$ is assumed to satisfy $p' < 0$ and $p' + 2p''Q < 0$. Note that the latter assumption is weaker than requiring concavity of demand.

Firm i's cost function $C_i(q_i)$ is increasing and weakly convex. Thus, $C_i' > 0$ and $C_i'' \geq 0$. Furthermore, we assume that for all i there exists a finite q such that $p(q) = C_i'(0)$. In particular, this is satisfied if there is a finite q such that price becomes zero. To avoid a monopolized market we assume that each firm's monopoly price is larger than the minimal marginal cost of each other firm, $p(q_i^m) > C_j'(0)$, for all i and j. Let $\Pi_i(q_1, ..., q_n) = p(Q)q_i - C_i(q_i)$ denote profit of firm i.

Next, let us define the set of collusive outcomes (i.e. the Pareto frontier)

$$\left\{ \mathbf{q}^c \in \mathbb{R}_+^n : \mathbf{q}^c = \arg\max \sum_{i=1}^{n} \lambda_i \left(p(Q)q_i - C_i(q_i) \right), \lambda_i \geq 0 \right\}.$$

The joint profit maximum is found by setting $\lambda_i = 1$, for all i.

We assume that players behave according to the following *trial & error learning* process

$$q_i^t = \max \left\{ 0, q_i^{t-1} + \delta s_i^t \right\}, \tag{1}$$

where

$$s_i^t := \text{sign}(q_i^{t-1} - q_i^{t-2}) \times \text{sign}(\pi_i^{t-1} - \pi_i^{t-2})$$

if $\left(q_i^{t-1} - q_i^{t-2}\right)\left(\pi_i^{t-1} - \pi_i^{t-2}\right) \neq 0$. Otherwise $s_i^t = +1, 0$, or -1, each with positive probability. In period $t = 0$ players start with some arbitrary q_i^0 and s_i^0.

The process requires that if profits last period increased due to an increase in quantity, then one would increase quantity again this period. On the other hand, if profits decreased following an increase in quantity, one would decrease quantity this period. If the change in profits or the change in quantities was zero, quantity either remains the same, is increased by one unit or decreased by one unit, each with positive probability. The restriction to movements of one grid point may be justified by convex adjustment cost, which can make it optimal to adjust in very small steps.

Note that neither information about other firms' demand or cost function, nor information about their past actions, nor information about one's *own* demand or cost functions are required for the trial & error process to work.

Each plausible learning process should be subject to some noise as individuals generally either make mistakes in the execution of their strategies or (more or less) systematically try out different actions. We assume therefore that with some small probability $\varepsilon > 0$ each firm independently reverses the direction of change s_i^t. At the cost of some more cumbersome notation we could equally assume that this probability is different for each firm and/or time period or that deviations to quantities which are further away than one grid point are possible.

Our assumptions define a Markov process on the finite state space $\Gamma^n \times \Gamma^n$, where a state is given by $(\mathbf{q}^t, \mathbf{q}^{t-1})$. For $\varepsilon > 0$ the process is irreducible and, therefore, has a unique stationary distribution. Formally, we consider the limit distribution for $\varepsilon \to 0$. For $\varepsilon = 0$ the process may have several absorbing sets.[3] By standard arguments (see e.g. Samuelson, 1994) only the members of absorbing sets of the unperturbed process can appear in the support of the limit distribution. In this paper we will speak of *convergence* to some point $\mathbf{q}$ if all states $\mathbf{q}'$, which have positive probability under the limit distribution, are close to $\mathbf{q}$ in the sense that $\|\mathbf{q} - \mathbf{q}'\|$ is of the order of the grid size δ, i.e. $\|\mathbf{q} - \mathbf{q}'\| = O(\delta)$.

Theorem 1 *For a duopoly the trial & error process converges to a collusive outcome $\mathbf{q}^c$. If cost functions are symmetric, $C_i(\cdot) = C_j(\cdot)$, then it converges to the joint profit maximizing outcome.*

Proof. We use the notation $\downarrow\uparrow$ to indicate that firm 1 decreases its quantity and firm 2 increases its quantity. Equivalently, for $\uparrow\downarrow$,$\uparrow\uparrow$ and $\downarrow\downarrow$. Let us first consider the change in individual profits due to $\uparrow\uparrow$.

$$\Delta\Pi_i(\uparrow\uparrow) = (q_i + \delta)p(q_1 + q_2 + 2\delta) - q_i p(q_1 + q_2) - C_i(q_i + \delta) + C_i(q_i).$$

We can now implicitly define two functions $z_i(q_i)$, $i = 1, 2$ by

$$\Delta\Pi_i(\uparrow\uparrow)(q_i, z_i(q_i)) \equiv 0. \tag{2}$$

We call $z_i(q_i)$ the improvement frontier for firm i since for all $q_j < z_i(q_i)$ ($q_j > z_i(q_i)$) profits of firm i increase (decrease) when both firms raise their quantities. Similarly, we can derive an improvement frontier for $\Delta\Pi_i(\downarrow\downarrow) = 0$, which is one grid point above and to the right of the curve for $\Delta\Pi_i(\uparrow\uparrow) = 0$.

[3] A set of states is called absorbing if there is zero probability to exit the set and a positive probability of moving from any state in the set to any other state in the set in finite time.

Note that for $\delta \to 0$ (2) becomes

$$\frac{\partial \Pi_i(q_i, z_i(q_i))}{\partial q_i} + \frac{\partial \Pi_i(q_i, z_i(q_i))}{\partial q_j} = 0. \tag{3}$$

or equivalently,

$$p(q_i + z_i(q_i)) + 2q_i p'(q_i + z_i(q_i)) - C_i'(q_i) = 0. \tag{4}$$

By implicitly differentiating (4) we find that

$$\frac{dz_i(q_i)}{dq_i} < -1. \tag{5}$$

Our assumption that $p(q_i^m) > C_j'(0)$, for all i and j, guarantees that an intersection, P_1, of the two improvement frontiers z_1 and z_2 in the (q_1,q_2) space exists in the interior. By (5) this intersection is unique (as shown in Figure 1).

We claim that P_1 corresponds to some $\mathbf{q}^c$. To see this note that collusion requires for $i = 1, 2; i \neq j$ and some $\lambda_j \geq 0$

$$\frac{\partial \Pi_i}{\partial q_i} + \lambda_j \frac{\partial \Pi_j}{\partial q_i} = 0. \tag{6}$$

Thus, conditions (3) and (6) are equivalent if

$$\frac{\partial \Pi_i}{\partial q_j} = \lambda_j \frac{\partial \Pi_j}{\partial q_i}, \tag{7}$$

that is, if $q_i p'(Q) = \lambda_j q_j p'(Q)$ or if $q_i = \lambda_j q_j$. Hence, there always exists some λ_j such that both conditions are equivalent. Clearly, if costs are symmetric, the solutions to (3) and (6) are symmetric, and hence (7) holds for $\lambda_j = 1$. Thus, for $\delta \to 0$ the improvement frontiers intersect at the joint profit maximum.

Next, we define curves r_1 and r_2 in the (q_1, q_2) space along which price equals marginal costs for firm i, i.e.

$$p(q_i + r_i(q_i)) - C_i'(q_i) = 0. \tag{8}$$

Some simple facts are immediate. (i) Implicitely differentiating (8) yields

$$\frac{dr_i(q_i)}{dq_i} \leq -1.$$

Table 1: **Movements in** $t+1$

Region	Movement in t			
	$\uparrow\uparrow$	$\uparrow\downarrow$	$\downarrow\uparrow$	$\downarrow\downarrow$
$\mathcal{A}$	$\uparrow\uparrow$	$\uparrow\uparrow$	$\uparrow\uparrow$	$\uparrow\uparrow$
$\mathcal{B}$	$\downarrow\downarrow$	$\uparrow\uparrow$	$\uparrow\uparrow$	$\downarrow\downarrow$
$\mathcal{C}$	$\downarrow\uparrow$	$\uparrow\uparrow$	$\uparrow\uparrow$	$\downarrow\uparrow$
$\mathcal{D}$	$\uparrow\downarrow$	$\uparrow\uparrow$	$\uparrow\uparrow$	$\uparrow\downarrow$
$\mathcal{L}_0$	$\downarrow\downarrow$	$\downarrow\downarrow$	$\downarrow\downarrow$	$\downarrow\downarrow$
$\mathcal{L}_1$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\uparrow\downarrow$
$\mathcal{L}_2$	$\downarrow\downarrow$	$\uparrow\downarrow$	$\uparrow\downarrow$	$\downarrow\downarrow$
$\mathcal{L}_3$	$\downarrow\downarrow$	$\downarrow\uparrow$	$\downarrow\uparrow$	$\downarrow\downarrow$
$\mathcal{L}_4$	$\downarrow\uparrow$	$\downarrow\uparrow$	$\downarrow\uparrow$	$\downarrow\uparrow$

(ii) $r_i(0)$ is finite because we have assumed that there exists a finite q such that $p(q) = C_i'(0)$. (iii) Comparing (4) and (8) shows that $z_i(0) = r_i(0)$. (iv) $r_i(q_i) \geq z_i(q_i)$ because if profits are unchanged when both quantities are increased marginally, they are certainly not lowered if q_i is increased and q_j is decreased. Taken together those facts define the shape of the four curves, r_1, r_2, z_1, z_2. Figure 1 shows a typical constellation. Note that P_2, P_3, and P_4 need not necessarily exist.

We can now define nine subsets of the quantity space (some of which might be empty) as shown in Figure 1. In regions $\mathcal{L}_0$ through $\mathcal{L}_4$ at least one firm makes a loss.

For each region we can determine the transitions as shown in Table 1. For example, in regions $\mathcal{A}$ through $\mathcal{D}$ $\uparrow\downarrow$ and $\downarrow\uparrow$ are followed by $\uparrow\uparrow$ because $\Delta p = 0$. In region $\mathcal{L}_0$ all movements result in $\downarrow\downarrow$ as price is below marginal cost for both firms. In region $\mathcal{L}_4$ all movements result in $\downarrow\uparrow$ since for firm 1 price is below marginal cost, whereas firm 2 is still below its improvement frontier. All other entries in Table 1 can be derived similarly. Note in particular that nowhere in regions $\mathcal{B}$ and $\mathcal{C}$ ($\mathcal{B}$ and $\mathcal{D}$) the movements $\downarrow\uparrow$ ($\uparrow\downarrow$) are possible. Hence, the process in regions $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ always weakly moves in the direction of the diagonal.

Applying Table 1 repeatedly it can be seen that after at most two steps in each region directions are given as indicated by the arrows in Figure 1, namely, always decreasing in regions $\mathcal{B}$ and $\mathcal{L}_0$, and always increasing in region $\mathcal{A}$. In regions $\mathcal{C}$ and $\mathcal{D}$ quantities are zigzagging in the direction of region $\mathcal{B}$. In regions $\mathcal{L}_2$ and $\mathcal{L}_3$ both indicated directions are possible depending on the starting condition.

Considering the dynamics in Figure 1 it is clear that the only candidates for absorbing sets are limit cycles around one of the intersections P_1 through P_4. Especially, it can be checked easily that no cycles of length two between regions $\mathcal{B}$ and $\mathcal{C}$ or between regions $\mathcal{B}$ and $\mathcal{D}$ are possible. Clearly, P_1 is a sink of the dynamics whereas P_2, P_3, and P_4 are saddles. The latter are not stable to noise which can always put the process into region $\mathcal{B}$ and thus in the basin of attraction of P_1. The sink P_1, however, is robust to noise. Thus, for ε small the process will be almost always in an absorbing set around P_1. The maximum distance between any point $\mathbf{q}'$ in such an absorbing set and $\mathbf{q}^c$ is bounded by the fact that after at most two transitions inside each region the directions are as indicated by the arrows in Figure 1. Thus, the distance between $\mathbf{q}^c$ and $\mathbf{q}'$ is of the order of δ. ■

Remark 1 *If the cost functions are linear and the same for both firms, then the intersection points P_2, P_3, and P_4 do not exist since in this case $\frac{dr_i(q_i)}{dq_i} = -1$. Therefore, global convergence to the neighborhood of P_1 is assured even without noise.*

The intuition for Theorem 1 is the following. It is relatively easy to see why firms which are perfectly aligned will move to a collusive outcome. Suppose two symmetric firms start from some output q larger than the collusive outcome. If both decrease their quantity, both increase their profits and will continue to due so until the collusive outcome is reached. This example also shows why the Cournot outcome is not a rest point of our process. Once firms surpass the collusive outcome, profits are lowered by further reductions in quantities. Hence, both firms will turn around and start jumping around the collusive outcome. In fact, the typical cycle which emerges in simulations consists of both firms moving in step $(q^c - \delta, q^c - \delta) \to (q^c, q^c) \to (q^c + \delta, q^c + \delta) \to (q^c, q^c) \to (q^c - \delta, q^c - \delta)$,[4] though more complicated patterns are possible.

The question then arises why firms which start from arbitrary initial quantities and directions of change (s_i^0), could become perfectly aligned. Suppose two firms with different quantities move downwards. They will continue until for one (or both) of them profit decreases. If firms are not too close together, it is always the firm with the smaller output which hits this

[4]This cycle also exists with more than two firms.

boundary first. Thus, while the firm with lower output moves already upwards, the other firm continues downwards decreasing the distance between the firms by 2 grid points. Similarly, when moving upwards the firm with higher output hits the boundary first. Thus, there is a general tendency to equalize quantities. In terms of Figure 1 firms generally move to the diagonal. Once on the diagonal, the argument of the previous paragraph applies.

Finally, consider a parallel to the conjectural variations model (Hicks, 1935). In this model when the conjectural derivative is +1, the equilibrium conditions are identical to our equation (3). Note, however, that conjectural variations models exogenously fix the conjectural derivative to +1, while with our learning process (3) results endogenously.

3 Simulations results

In order to check whether Theorem 1 is simply an artefact of our assumptions, we ran computer simulations to assess its robustness.[5] In particular, we simulated oligopolies with up to 10 firms and various functional forms both, for symmetric and for asymmetric firms. Further, we analyzed two modifications of our learning rule.

3.1 n-firm oligopolies

The most important result from the simulations is that Theorem 1 also holds for more than two firms. We simulated oligopolies with $n = 2, ..., 10$ firms and for three functional forms, linear demand and cost, linear demand and quadratic cost, and quadratic demand and linear cost.[6] In all simulations with symmetric firms, play converged to the joint profit maximizing outcome.[7] With cost asymmetries a Pareto efficient outcome was reached.

[5] The programming was done in Turbo Pascal. The source code is available from the authors upon request.

[6] The functional forms used for symmetric firms were the following: (a) $\Pi_i = (1{:}1\ \Gamma\ q_{1-\mathrm{II}2})q_i\ r\ 0{:}1q_i$; (b) $\Pi_i = (1\ \Gamma\ q_{1-\mathrm{II}2})q_i\ r\ (q_i)^2$; (c) $\Pi_i = (1{:}1\ \Gamma\ (q_{1-\mathrm{II}2})^2)q_i\ r\ 0{:}1q_i$. For asymmetric firms, different cost parameters were employed. The grid size was in all cases ffi$= 0{:}001$. Initial quantities were random, but restricted to $q_i < 1{:}$ The noise was modelled such that there is a probability $" = 1{=}500$ for each firm in each period that the sign of s_i^t is reversed.

[7] Here again, convergence to q means convergence to a limit cycle which is close to q.

Table 2: **Percentage of limit cycles**

Numer of firms	2	3	4	5	6	7	8	9	10
Limit cycles in %	0	2.65	2.69	7.34	7.64	11.59	12.70	17.84	20.16

Result In simulations, the trial & error process with noise converged globally to the collusive outcome $\mathbf{q}^c$ in all cases. When cost functions were symmetric, $C_i(\cdot) = C_j(\cdot)$, it converged to the joint profit maximizing outcome.

As pointed out above without noise the process can get stuck in limit cycles around intersection points P which are far away from Pareto efficient outcomes. To see how large the proportion of such limit cycles is for different numbers of firms we also ran simulations without noise. Table 2 shows the average percentage of limit cycles in 10,000 simulations with random starting quantities for oligopolies with linear demand and cost.

Interestingly, the number of limit cycles increases with the number of firms. Numerical values for quadratic specifications are similar. No matter how large the number of limit cycles, *with* noise play always converged to the collusive outcome.

3.2 Modifications of the learning rule

One property of our trial & error process is that all firms move with the same step size δ. What happens if firms have different step sizes δ_i? Consider for example a symmetric oligopoly with constant marginal cost in which one firm's step size is twice that of the other firms. What happens in simulations is that total output still converges to the joint profit maximum. However, the distribution of output has changed: the firm with the larger step size has twice the output share of the other firms. The intuition is that from the perspective of the remaining firms the first firms moves and behaves like 2 firms which are perfectly aligned. In fact, we obtained this result for up to 10 firms, and with both, linear and quadratic demand. The factor k by which one firm's step size is multiplied is arbitrary — the first firm will end up with k times the shares of the other firms. Even with quadratic cost there is convergence to some Pareto efficient outcome in which firms produce different quantities (also if costs are asymmetric).

A second feature of our learning process is that only the direction of movement but not the step size is influenced by the change in profits. We ran simulations where instead of using constant step size δ we used a variable step size of

$$\delta \left(\frac{\Pi_i^{t-1} + c}{\Pi_i^{t-2} + c} \right)^d .$$

The constant c served to avoid division by zero, and d was chosen to be even in order to avoid changes in sign.[8] Again we found convergence to a collusive outcome.

However, not all modifications leave Theorem 1 unchanged. For example, consider a model in which step size is given by

$$\frac{\Pi_i^{t-1} - \Pi_i^{t-2}}{q_i^{t-1} - q_i^{t-2}}.$$

This can be approximated by the following system of differential equations[9]

$$\begin{aligned} \dot{q}_1 &= \frac{\partial \Pi_1}{\partial q_1} + \frac{\partial \Pi_1}{\partial q_2} \dot{q}_2 \\ \dot{q}_2 &= \frac{\partial \Pi_2}{\partial q_2} + \frac{\partial \Pi_2}{\partial q_1} \dot{q}_1 \end{aligned}$$

For some parametric specifications (linear or quadratic demand, linear or quadratic cost) this process can be solved numerically and it converges globally to the Cournot outcome.

4 Conclusion

In this note we studied a trial & error learning process which is to our knowledge unique in that it converges to a collusive outcome. This result may be somewhat surprising as players in our setting are totally ignorant of the other players; they do even not realize that they are playing a game.

[8] For c we chose values between 0.1 and 1, and for d between 2 and 8.

[9] Note the similarity to the gradient learning process of Arrow and Hurwicz (1960), which is given by $\dot{q}_i = \partial \Pi_i / \partial q_i$. The difference is that with gradient learning the opponent is assumed to stay fixed.

In fact, they do not have to know their own payoff function. Nevertheless, they manage to coordinate on a collusive outcome endogenously.

We believe that the trial & error process has some intuitive appeal. In particular, in situations in which players are not familiar with the payoff structure of the game it seems plausible that they adjust cautiously in a direction that has proved to be successful. Ultimatively, it is an empirical question whether players behave according to such a process or not.

There is some experimental evidence, both on the individual level and on the aggregate level. On the individual level Huck, Normann, and Oechssler (1999) show that the *direction* of change is predicted correctly by the trial & error process for 80% of subjects' choices. However, since the experiment was not designed to test trial & error learning in particular,[10] subjects were not constrained to change their quantity by one grid point only and mostly adjusted by more. Secondly, the theory requires that *all* subjects play according to the rule. If only some subjects violate it occasionally, one cannot expect convergence to the theoretical prediction any more. Hence, it is not surprising that total quantities were not even close to the collusive outcome. On the aggregate level several other studies found substantial support for a tendency toward collusion but mostly in duopoly cases (see Holt, 1995, for an overview). In oligopolies with more than 2 firms, however, collusion seems difficult to achieve.

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