A Tragedy Of The Clubs: Excess Entry in Exclusive Coalitions

Paola Manzini* Marco Mariotti†

Queen Mary and Westfield College University of Exeter

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Abstract

We study how social norms and individual rationality in the process of coalition formation sustain a particular form of collective inefficiency, namely excessive entry in the joint production and exploitation of an excludable good. We term this phenomenon the 'tragedy of the clubs'. We model club formation as a non-cooperative game of coalition formation and surplus division, and show that, contrary to common wisdom, the tragedy of the clubs is a pervasive equilibrium phenomenon.

Keywords: Clubs, Coalition Formation, Multiperson Bargaining

*Department of Economics, Queen Mary and Westfield College, Mile End Road, London E1 4NS (tel: 0171-975 5083; e-mail: p.manzini@qmw.ac.uk)

†Department of Economics, University of Exeter, Streatham Court, Rennes Drive, Exeter EX4 4PU, UK.

1 Introduction

The essence of several economic activities is the generation and sharing of a surplus through cooperation by selfish individuals. However, such activities rarely if ever will call for *universal* cooperation. There are almost always limits to the size of a cooperating group, beyond which new entrants in the cooperative activity will not be welcome because they decrease average surplus. When no or limited *exclusion power* can be exercised, excessive entry in the cooperative activity is bound to result. In the limiting case where the surplus is available and is maximum for a single exploiter[1], this phenomenon is called the *tragedy of the commons.* But suppose that groups *do* have the power of excluding the unwanted, so that the cooperating groups have the nature of *clubs.* Does this mean that no excessive entry will result? Although, as we explain below, this appears to be the common wisdom, in this paper we show that the presence of exclusion power generates a surprisingly complex group interaction.

Although we do not yet have a full theory of this interaction, we present a model of group formation and behaviour that yields some interesting insights. In particular, we demonstrate that excess entry in the joint production and exploitation of an *excludable* good by rational and selfish individuals will typically occur. For evident reasons, we term this phenomenon the '*tragedy of the clubs*'. When the tragedy of the clubs occurs, the exclusion power of club members is empty, since it cannot be used in a profitable way.

A simple cost-sharing example will be useful to make concrete the type of issues we address. Any group out of N agents (firms, countries) can set up an R&D joint-venture, which will generate an expected benefit B for *each* member (the benefit itself is thus a non-rival good). The total cost of the R&D project is given by some function $c(n)$, where n is the number of participants, such that $\frac{nB-c(n)}{n}$ attains a maximum at $n^* < N$.

Will a joint-venture form? How large will it be if it forms? In particular, will it have n^* participants?

More in general, define a 'club' as a group of homogeneous agents who can *(1)* jointly generate a divisible and rival net surplus through cooperation, and *(2)* exclude outsiders

[1]This is a limiting case from our perspective because no cooperation is needed.

from the consumption of this joint surplus[2]. Examples range from regional federations and free-trade areas between countries, to joint ventures and cartels between firms, to gangs of thieves.

How is the membership of a club determined? We are interested in the typical situation where, because of congestion, the number of potential members is greater than the membership size which is optimal *from the point of view of the members* (n^* in the example). This is the size that maximises net surplus per capita: would-be members will not be admitted unless they increase the surplus more than they increase the congestion costs they generate[3].

It appears that, since Buchanan (1965)'s pioneering contributions, it is standard in the economics of clubs to sidestep the difficulties of modelling club formation by assuming that club size is determined by efficiency in the exploitation of economies of scale[4]. No excessive entry occurs. We call this the *club-efficiency hypothesis*, and its violation the *tragedy of the clubs.*

In the R&D example, set $N = 3$, $B = 4$, and let $c(n) = n^2 - n + 5$ (or $c(n) = 3 + 2^n$), so that the total costs are:

- 5, if undertaken by a single agent;
- 7, if undertaken by any two agents;
- 11, if undertaken by all three agents.

Given these data, it is not profitable for a single agent to engage in the project. Two agents forming a club will reduce cost per capita to $\frac{7}{2}$ with a net total cooperative surplus

[2]Even if, as in our example, the *good* being produced is non-rival, the *net surplus*, or part of it, will be rival as long as production is costly.

[3]This is sometimes called 'within club optimality' (e.g. Cornes and Sandler (1986)), as opposed to 'total economy' otpimality. The conflict between these two viewpoints is not an issue in the model we present.

[4]Olson (tion) also stresses the central role of economies of scale in group cooperation, but places issues of strategic interaction at the centre of his analysis (albeit in rather informal way). Our approach is very much in that spirit. See Cornes and Sandler (1986) for a very thorough analysis of issues concerning club goods.

of $2\times4-7=1$. The addition of a third agent would further reduce the cost per capita to $\frac{11}{3}$, and ensure positive net surplus per capita; but not enough to increase the net cooperative surplus, which stays put at $4\times3-11=1$ and is now to be divided between three agents rather than two (congestion). The club efficiency hypothesis mandates that the club should comprise two members only (although, given the symmetry between agents, it seems impossible to predict from these data alone *who* the lucky agents will be). But to stress that this conclusion is far from being as trivial as it may seem, just notice that the Core of the cooperative game associated with this situation is empty: *every* joint venture and *every* division of the joint surplus is vulnerable to immediate profitable deviations by other partnerships with other divisions, and is thus seemingly unstable. This reasoning casts doubt on the procedure of singling out two-agent partnerships for special attention, as the club-efficiency hypothesis would recommend. In particular, the grand partnership, even though causing a loss of $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$ per member with respect to the two-person ones, seems on this logic no more unstable than them, despite its (within club) inefficiency.

We take the view that in order to clarify the mechanism at work an *explicit* noncooperative model of club formation should be considered, and its equilibria studied. In this paper, we analyse a natural such model. We distinguish the act of approaching a potential member from that of making a proposal on surplus division once a candidate partnership has been formed. Initially, players costlessly alternate in approaching the partners they would like to form a club with, until the proposed members have agreed to form the club. Once this happens, they bargain over the division of the surplus in a (multi-person) Rubinstein-like fashion. This distinction between coalition formation phase and bargaining phase echoes that found in several recent models of coalition formation, with the major difference that our model allows for a potentially infinite repetition of the two-stage structure[5]. Because the structure of the equilibria is quite complex, here we limit ourselves to analyse the *minimal* case which allows us to capture the effect of first increasing and then decreasing returns to cooperation, namely the three-person case. This allows us to focus on the main issue, and in particular to avoid the thorny problem

[5]We refer the reader to the survey by Carraro and Moriconi (1998) of two-stage models of coalition formation.

of *multiple* club formation.

The results are quite surprising. We begin with the analysis of *stationary* equilibria. This analysis allows us to formulate precisely the issue of the instability of any agreement suggested by the Core logic discussed above, and to tie it up with considerations about technology and time-preferences. If the discount factor is sufficiently high and the returns to scale are sufficiently decreasing, the Core outcome is upheld: no stationary equilibrium exists. If some stationary equilibrium exists (which is always the case for a range of discount factors, depending on the pattern of returns to scale), it is unique, and it generates the tragedy of the clubs. We then consider more complex, non-stationary equilibria. In this case, too, it turns out that the tragedy of the clubs is a pervasive equilibrium phenomenon. Depending on what particular 'social norm' prevails, almost all partitions can be supported as a tragedy of the clubs.

2 The partnership game

In this section we describe our basic model. There is a group of potential partners, or *players*, denoted $I = \{1, 2, 3\}$. Any set of two players can create a net total surplus $S(2)$, provided that they agree on two things: *(1)* forming a partnership and *(2)* a division of the surplus. Similarly, three players can obtain a total net surplus $S(3)$ through cooperation. We assume $S(2) \leq S(3) < \frac{3}{2}S(2)$, so that although the addition of a third partner does not decrease total surplus, average surplus is higher in a two-partner club than in the grand club. The payoff of not being a member is $S(1) < S(2)$. Thus, an agreement can be struck either between two players only (any (i, j) pair - henceforth denoted simply by ij - on a partition (x_i, x_j) of the available surplus, with $x_i + x_j \leq S(2)$, leaving the third player k out with $S(1)$); or among all three players (the triple 123 on a partition (x_1, x_2, x_3), with $\sum_{i \in I} x_i \leq S(3)$). Any particular division of $S(2)$ or $S(3)$ can be interpreted as reflecting a *club membership fee* structure; but, of course, the generality of the setting allows for other interpretations, such as cost-sharing problems of the type considered in the introduction. Using Binmore (1985)'s terminology, this can be called a three-player/four-cake game.

Without loss of generality, from now on we use the normalisations $S(2) = 1$ and

$S(1) = 0$, so that the free parameter is $S(3) \equiv s \in \left[1, \frac{3}{2}\right)$.

The structure of the **partnership game** consists of two phases:

A) Approach phase: In this phase, the **promoter** of an agreement costlessly approaches either only one of the other two players or both of them to solicit entering negotiations on how to share the surplus. The first promoter, at the beginning of the game, is player 1. The other promoters are determined via the following procedure. Each of the **solicited** players can (in the order given by their numbers) either accept the approach, or costlessly reject it. If each and every of the solicited players accepts, we say that a **partnership** is established; the game enters the **bargaining phase B** described below. If, instead, a solicited player i rejects an approach, he becomes the promoter of some different partnership. This phase continues until a partnership is formed. Perpetual disagreement results in a null payoff for all players.

B) Bargaining phase: in this phase, the promoter of an established partnership proposes a partition of the available surplus to the other member(s), who have to accept or reject it in the order given by their numbers. If the proposed partition is *accepted* by every member of the partnership, the surplus is shared accordingly. If, instead, the proposed partition is *rejected* by a member, he can either make a counterproposal within the established partnership, or initiate a new approach phase by becoming the promoter of some different partnership. In either case, a delaying cost is incurred. Perpetual disagreement yields a null payoff in the bargaining phase as well.

Phases **A** and **B** together form a **negotiation round**. The feature we want to capture in the model is that approaching costs - the costs incurred in a 'sounding out' stage - are negligible relative to those of actual bargaining. Rejection of a partition proposal results in the game entering the next round, which entails a cost. Player i's payoff from an agreement struck in round t which yields him a share x_i is given by $\delta^t x_i$, with $\delta \in (0, 1)$ a discount factor representing players' time preferences. The extensive form of the game is depicted in figure 1 below.

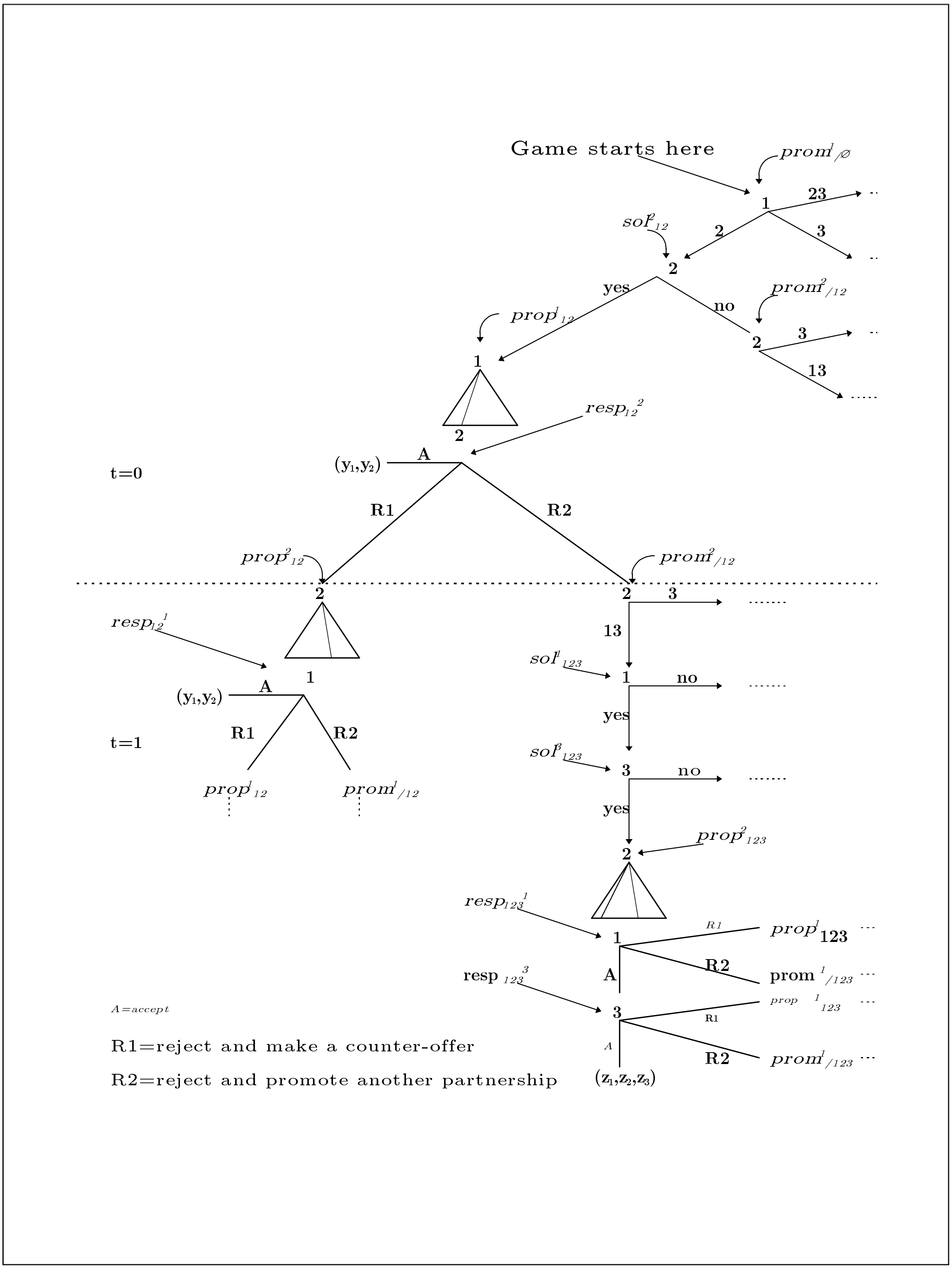

Figure 1: Club formation procedure

As mentioned in the introduction, many recent models of coalition formation exhibit, like ours, a distinction between two phases: one in which the coalition is “chosen” and one in which payoff division is decided upon through some bargaining procedure. Unlike those models, however, ours does not impose the restriction that the game is a *two-stage one*; there is *no exogenous limit* on the number of negotiation rounds that can be played.

For our subsequent formal analysis it is useful to identify the types of decision nodes that can be reached during play. Since there are several of them, as a mnemonic device we have chosen the notation so that: the name of the node suggests the type of decision to be made (e.g. accept or reject a bargaining offer); superscripts indicate which player is making a decision; and subscripts indicate other features of the decision (e.g., the partnership within which a proposal is being made).

Decision nodes in the approach phase:

1. Node of type $prom^i_{/p}$: at this node player i is a promoter who has rejected the approach by the promoter of a partnership p, or has rejected a bargaining offer within partnership p and has not made a bargaining counter-proposal. Player i at this node can promote any partnership different from p. That is, his action set is $\{q \in \{j, k, jk\} | iq \neq p\}$. The *initial node* is considered a special node of type $prom^i_{/p}$, with $p = \emptyset$.

2. Node of type $sol^i_{j,p}$: at this node a solicited player i has been approached by the promoter j of a partnership p. He can either accept or refuse the approach. Nodes belonging to this class can only follow nodes of type $prom^i_{/p}$.

Decision nodes in the bargaining phase:

1. Node of type $prop^i_p$: at this node player i's participation in a partnership p has been established and he has to propose a partition between the members of p. Nodes belonging to this class can only follow the acceptance of an approach by all the solicited players involved.

2. Node of type $resp^i_{j,p}$: at this node player i has to respond to a bargain proposal made by player j within partnership p. He can either accept the proposal, or reject it and move to a node of type $prop^i_p$, or reject it and move to a node of type $prom^i_{/p}$.

3 Equilibria

We analyse the subgame perfect equilibria (s.p.e.) of this game[6]. Each s.p.e. in which agreement is reached will characterise a pair $\left(p^*, x^*_{p^*}\right)$, where $p^* \in \{ij, ik, jk, ijk\}$ is the equilibrium partnership and $x^*_{p^*}$ is the equilibrium vector of shares of the surplus distributed to the partners (recall that the payoff for a player excluded from a partnership is normalised to zero).

There are several s.p.e. in the game, with the tragedy of the clubs featuring prominently, even in the case where the addition of a third partner adds no value at all to the club ($s = 1$)! In particular, we find that, while there exists only one club efficiency equilibrium (with "Rubinstein payoffs"), *almost all* partitions can be supported in equilibrium with the (within club) inefficient grand partnership forming. This type of equilibrium is supported by a "social norm" that punishes partners who try to cheat on the specified agreement. Both the equilibrium strategies supporting this outcome and those supporting the club efficiency outcome are non-stationary.

The complexity of the equilibrium structure is dramatically reduced by restricting attention to *stationary* strategies, a case we will consider first. In this case, the first notable result is that there can *never* exist a club efficiency equilibrium. The Core outcome suggesting the instability of all agreements can be validated when the players are sufficiently *impatient* and/or when the returns to scale decrease sufficiently sharply (s small). But if an equilibrium exists, then it is unique. Moreover, it must involve the tragedy of the clubs. Given s, there is always a range of discount factors such that an equilibrium exists.

3.1 Stationary equilibria

In general, there isn't any strong theoretical justification to confine attention *a priori* to stationary strategies, as is sometimes practice in the bargaining literature. Indeed, in our model the non-stationary equilibria are most interesting. However, a preliminary analysis of stationary equilibria can also be of interest, because it allows us to study in depth exactly how the intrinsic "Core" instability of any agreement in any partnership

[6] In the model presented here, all equilibrium agreements exhaust completely the size of the pie.

limits the scope for "simple" agreements in the club formation process. In this section, therefore, we present a complete characterisation of the stationary equilibria of the game.

It turns out that the instability depends crucially, and in a non-obvious way, on considerations of time preferences and returns to club size. In addition, because stationary equilibria are unique when they exist and never involve club efficiency, they permit us to exhibit in a particularly stark form the tragedy of the clubs phenomenon and thus to strengthen our general point.

The formal proof of our results is very lengthy, so that it is difficult to provide an informal overview which strictly adheres to its logic. Nonetheless, with some amount of hand-waving, we are able to pinpoint the crucial insights.

First, observe that, in order for a stationary equilibrium to exist, s must be high, relative to δ. Otherwise, if there are little gains from extending the partnership and rejection costs are high, the "threat" to form a three person partnership is never credible in two-partner negotiations. In this case, therefore, the only "outside option" available to the players in two-partner negotiations is to form another two-person partnership. This type of three-player negotiations has been studied, for example by Binmore (1985), and it is not too difficult to see that the standard Rubinstein-like logic drives the outcome: the payoffs in any two-person agreement must be the Rubinsteinian ones. But then, because the first proposer has an advantage, an incentive is created to jockey to become the proposer rather than the responder in a two-person partnership. That is, there is an incentive to reject the approach by a player proposing a two-person partnership and turn to the third player instead, to form a two-person partnership with him. So no stable partnership can form and no equilibrium can exist[7]. Note how this logic, although supporting Core outcomes, is different from the one underlying it.

If the surplus generated by the grand partnerships is sufficiently high to guarantee existence, two possible regimes can ensue, both involving the tragedy of the clubs, but differing in terms of the agreements that occur, *off* the equilibrium path, in two-person partnerships. Club efficiency equilibria cannot exist because of the same argument used above to demonstrate non-existence of an equilibrium altogether: *if* they existed, they

[7]We return to this important issue at the end of the next section.

should be Rubinstein-like[8], and this creates instability in the partnership formation phase. So any stationary equilibrium must involve *all three* players and thus a tragedy of the clubs.

Which of the two mentioned regimes prevails depends on whether the responder share in two-person negotiations (off the equilibrium path), is driven by the Rubinstein logic (in which case a rejecting player would make a counterproposal in the same partnership) or by an "outside option" contraint (whereby the rejecting player would form a three-person partnership, so that he must be given the payoff he would gain with this rejection). This in turn depends on the relative values of s and δ. For a responder in a two-person partnership to accept the Rubinstein share it must be the case that his incentive to reject and propose to form the grand partnership is not too great. This imposes, given δ, an *upper bound on* s (or a *lower bound on* δ given s): if the additional surplus attainable with the presence of an extra partner is large relative to the cost of rejection, then it may be better to be the first mover in a three-person partnership rather than the responder in a two-person partnership, even allowing for a period of delay. In this case, the threat to reject and implement a three-person partnership acts as a *binding* outside option constraint faced by proposer in two-person partnerships, and the payoff is determined accordingy.

In the next result we show formally that, for any value of s in the admissible range, the range of discount factors such that a stationary subgame perfect equilibrium (henceforth s.s.p.e.) exists is never empty; that when an s.s.p.s. exists it is unique; and that it must feature the tragedy of the clubs.

Proposition 1 *There exist* $\underline{\delta}_s, \overline{\delta}_s \in (0,1)$ *(which depend on* $s \in \left[1, \frac{3}{2}\right)$*) with* $\underline{\delta}_s < \overline{\delta}_s$ *such that:*

(a) *For all* $\delta \leq \overline{\delta}_s$ *only the partition* $\left(\frac{1}{1+2\delta}s, \frac{\delta}{1+2\delta}s, \frac{\delta}{1+2\delta}s\right)$ *can be supported at an s.s.p.e. where agreement is reached immediately. The off-equilibrium agreements vary according to whether the condition* $\delta \leq \underline{\delta}_s$ *holds;*

(b) *If* $\delta > \overline{\delta}_s$*,there are no stationary s.p.e..*

Proof: The proof of this result is long. Here we only describe its structure, and

[8]For out of equilibrium rejections in two-person partnerships would continue with a two-person partnership.

relegate the actual proof to Appendix 1. The argument follows these steps:

Step 1: Define a stationary strategy profile which is perfect when $\delta \in \left[\underline{\delta}_s, \overline{\delta}_s\right]$ and which supports the equilibrium partition $z_{123} = \left(\frac{1}{1+2\delta}s, \frac{\delta}{1+2\delta}s, \frac{\delta}{1+2\delta}s\right)$.

Step 2: For $\delta \in \left[\underline{\delta}_s, \overline{\delta}_s\right]$, $y_{ij} = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ is the only s.s.p.e. payoff which can be supported in a two-person partnership; however

Step 3: For $\delta \in \left[\underline{\delta}_s, \overline{\delta}_s\right]$, no s.s.p.e. exists which supports agreement in a two-person partnership along the equilibrium path.

Step 4: For $\delta \in \left[\underline{\delta}_s, \overline{\delta}_s\right]$, z_{123} is the only s.s.p.e. payoff which can be supported in a three-person partnership.

Step 5: If $\delta > \overline{\delta}_s$, no s.s.p.e exists.

Step 6: Define a stationary strategy profile (different from that of step 1) which is perfect when $\delta < \underline{\delta}_s$ and which supports the equilibrium partition z_{123}.

Step 7: If $\delta < \underline{\delta}_s$, the only s.s.p.e. payoff that can be supported in an s.s.p.e. is z_{123} in the grand partnership.

Step 8: If $\delta < \underline{\delta}_s$, no s.s.p.e. can exist with agreement in a two person partnership along the equilibrium path. ■

3.2 Club Efficiency Equilibria

One suggestion of the previous section is that not always will "simple" strategies be enough to support an equilibrium. The large scope for deviations which exists in the club formation game may require more complex social arrangements to sustain an equilibrium. This leads us naturally to the study of equilibria in non-stationary strategies. In the sequel of the paper we emphasise in particular the introduction of "punishment mechanisms" for deviators from a given social norm for the formation of a club and the division of the surplus.

From now on we consider the limiting case $S(3) = S(2) = 1$. This notational simplification makes our main results harder to prove: the grand partnership is maximally unattractive, and the underlying characteristic function form game is not even strictly convex. Those results would obviously continue to hold *a fortiori* without the simplification.

We begin by describing strategies that support the formation of a two-person partnership, with Rubinstein shares. This is a useful auxiliary result, which also allows us to compare the workings of our model to those in Binmore (1985), who pioneered the general line of enquiry under exam here.

Lemma 1 *There exists an s.p.e. in which the partnership* $1i$ *is formed in the first round, with* $i = 2$ *or* $i = 3$*, and the partners agree in the first round on the partition* $y_{1i} = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$.

The strategies which support this outcome require player 1 and, say, $i = 2$ to engage in a bilateral Rubinstein bargaining; a "deviant" approach is punished by the solicited player by rejecting and entering a Rubinstein partnership with the remaining player. Any proposal in the grand partnership is also considered deviant and is punished in the same way.

More precisely, the equilibrium strategies that support the partnership 12 are defined in the following manner (the case for the partnership 13 is analogous). We define a set of **states**, on which the action taken by a player at any given node will depend: the **equilibrium state** (which is also the **initial** state), denoted by "E", and the **punishment states**, denoted by "P_i", $i = 1, 2, 3$. State P_i is triggered either by a deviation by player i from the strategies specified for the existing state in the approach phase, or after *any* proposal by player i in the grand partnership 123 (note well: P_i is *not* triggered by a deviation by i in the bargaining phase of a *two*-person partnership).

In state P_i, player i is shut out of the negotiations, and the other two players agree on the Rubinsteinian partition. Denote by y_{ik} the partition which gives players i and j shares $y_i = \frac{1}{1+\delta}$ and $y_k = \frac{\delta}{1+\delta}$, respectively; and by w_{ijk} the partition which gives players i, j and k shares $w_i = \frac{1-\delta}{1+\delta}$ and $w_j = w_k = \frac{\delta}{1+\delta}$, respectively.

Player 1*:*

1. <u>in the approach phase:</u> at nodes $prom^1_{/p}$ approach player 2 in state E and P_3 (note

that in these states $p \neq 12$)[9], and player 3 in state P_2 (in this state $p \neq 13$); at nodes $sol^1_{i,p}$ accept the approach only if $p = 1i$ and the state is not P_i;

2. in the bargaining phase: at nodes $prop^1_p$, in any state, propose y_{1i} if $p = 1i$ and propose w_{1ij} if $p = 1ij$; at nodes $resp^1_{i,p}$, in any state, accept only offers yielding at least $\frac{\delta}{1+\delta}$.

Player $i \neq 1$:

1. in the approach phase: at nodes $prom^i_{/p}$ approach player $k \neq j$ in state P_j (in this state $p \neq ik$); at nodes $sol^i_{j,p}$ accept the approach only if $p = ij$ and the state is not P_j.

2. in the bargaining phase: at nodes $prop^i_p$, in any state, propose y_{ij} if $p = ij$ and propose w_{ijk} if $p = ijk$; at nodes $resp^i_{j,p}$, in any state, accept only offers yielding at least $\frac{\delta}{1+\delta}$.

The equilibrium above can be visualised with the aid of Table 1.

That these strategies constitute an s.p.e. can be checked easily by means of the one-deviation property (which obviously holds in this game since infinite histories yield the worst payoff). Let us consider player 1 first. At all nodes of type $prom^1_{/p}$, deviating from the prescribed strategy triggers the punishment state P_1 which yields player 1 a null payoff in the continuation game; hence there cannot be any profitable deviation. At nodes $sol^1_{i,p}$, the prescribed strategy yields a payoff of either $\frac{1}{1+\delta}$ (if player 1 is responding in P_i to promoter i), or $\frac{\delta}{1+\delta}$ (if player 1 is responding to promoter i in $P_{j\neq i}$). Rejecting an approach that should be accepted yields zero (as the punishment state P_1 is triggered). Accepting an approach that should be rejected yields $\frac{\delta}{1+\delta}$ in the continuation game[10], and is thus not a profitable deviation either. Consider now the bargaining phase. If player 1 were to deviate from his equilibrium bargaining strategy at a node of type $prop^i_p$ and claim

[9]In fact, in general, state P_i must be triggered by an action by player i, while a node of type $prom^j_{/jk}$ can only be reached by player j after an action by player k.

[10]Note the 'united in sin' aspect of this occurrence. This deviation happens when player i is approached by a *deviant* player j and instead of rejecting his approach he enters a partnership.

more than what prescribed by the strategy for himself, the responder(s) would reject, and player 1 would obtain $\frac{\delta}{1+\delta}$ if $p = 1i$ and 0 if $p = 123$, rather than $\frac{1}{1+\delta}$ and $\frac{1-\delta}{1+\delta}$, respectively; hence, no deviation is profitable. Finally, at a node of type $resp^1_{i,p}$ it would be suboptimal for player 1 to accept an offer yielding less than $\frac{\delta}{1+\delta}$, as by rejecting and sticking to his equilibrium strategy he could obtain $\frac{1}{1+\delta}$ in the subsequent round (forming a partnership with just $j \neq i$, which is worth $\frac{\delta}{1+\delta}$ in present discounted value).

The optimality of the strategies prescribed for players 2 and 3 is easily checked in the same way. ■

It is interesting to discuss in a little more detail at this point why club efficiency equilibria and in particular Rubinsteinian equilibria cannot be supported in a "simple" way in our model (we treat this issue more formally in the course of proving proposition 1). The best way to do this is to compare our model to a related one in Binmore (1985) (the "telephone" model). He considers a three-player/three-cake sequential game in which the proposing player chooses which one of the other two potential partners to "phone" and make an offer to. Binmore finds, contrary to us, that the standard Rubinstein equilibria are 'surprisingly resistant to the introduction of a third player': 'although the original players are free to open channel of communication to the third player, it is optimal for them not to do so' (p. 276). The strategies supporting, say, an immediate Rubinstein agreement between players 1 and 2, are simple, and rest on the fact that player 3 is constantly ignored. So, in the course of the negotiations between 1 and 2, either player always makes a counterproposal to the other rather than making a proposal to 3. This is the unique equilibrium in Binmore's model. But such strategies in our model imply that player 2 has an incentive to refuse player 1's *approach* (not offer) and then turn to player 3, obtaining the *first mover* share rather than the responder's share. We believe that the jockeying to be the player with the initiative in bilateral partnerships is an essential feature that emerges in the club formation game under scrutiny. This feature should be modeled, and our "approach phase" captures just that. In order to break the jockeying to be the first mover, we need to construct sufficiently complex social punishment mechanisms, in the form of *exclusion* mechanisms, to support club efficiency equilibria.

			E	P_1	P_2	P_3
player 1	approaches		2		3	2
	rejects approach from		-		2, 23, 32	3, 23, 32
	in two person bargains:	proposes	y_{12}		y_{1k}	y_{1k}
		accepts	-		$x \geq \frac{\delta}{1+\delta}$	$x \geq \frac{\delta}{1+\delta}$
	in three-person bargains:	proposes	-		w_{123}	w_{123}
		accepts	-		$x \geq \frac{\delta}{1+\delta}$	$x \geq \frac{\delta}{1+\delta}$
player 2	approaches		-	3		1
	rejects approach from		-	1, 13, 31		1, 13, 31
	in two person bargains:	proposes	-	y_{2k}		y_{2k}
		accepts	$x \geq \frac{\delta}{1+\delta}$	$x \geq \frac{\delta}{1+\delta}$		$x \geq \frac{\delta}{1+\delta}$
	in three-person bargains:	proposes	-	w_{231}		w_{231}
		accepts	-	$x \geq \frac{\delta}{1+\delta}$		$x \geq \frac{\delta}{1+\delta}$
player 3	approaches		-	2	1	
	rejects approach from		-	1, 12, 21	3, 13, 31	
	in two person bargains:	proposes	-	y_{3k}	y_{3k}	
		accepts	$x \geq \frac{\delta}{1+\delta}$	$x \geq \frac{\delta}{1+\delta}$	$x \geq \frac{\delta}{1+\delta}$	
	in three-person bargains:	proposes	-	w_{312}	w_{312}	
		accepts	-	$x \geq \frac{\delta}{1+\delta}$	$x \geq \frac{\delta}{1+\delta}$	
			Transition In a three-person partnership, or if player i deviates in the approach phase, state changes to P_i.			

Table 1: The perfect equilibrium of lemma 1

3.3 Tragedies of the Club

The club efficiency equilibrium described in lemma 1 can in turn be used to support equilibria in which the tragedy of the clubs occurs. The set of such equilibria is very large, so much so that we are able to establish an "almost folk theorem": almost all divisions (depending on time preferences) of the cooperative surplus in a grand partnership can be supported in equilibrium. What happens is that a social norm is established, whereby any player who attempts to profit from the tragedy of the clubs and to form a club efficient partnership instead is labeled as a "cheater".A cheater in a two-person partnership, faces a credible threat of "social exclusion", with a zero payoff. So, it can never pay to try an break away from the tragedy of the clubs.

Let Δ^k denote the k-dimensional simplex.

Proposition 2 *: Let* $x^*_{123} = (x^*_1, x^*_2, x^*_3) \in \Delta^2$*. There exists* $\delta^* \in (0,1)$ *(which depends on* x^*_{123}*) such that, for all* $\delta \in [\delta^*, 1)$ *the pair* $(123, x^*_{123})$ *is supported in some s.p.e. with immediate agreement.*

Proof. The strategies can be described by means of five states, $*$, PB_1, and P_i $i = 1, 2, 3$, and with the aid of Table 2 below (recall the notation y_{ij} for the partition which gives players i and j in partnership ij shares $y_i = \frac{1}{1+\delta}$ and $y_k = \frac{\delta}{1+\delta}$, respectively). $*$ is the initial state, and PB_1 is the state in which Punishment in the Bargaining phase of $*$ is inflicted upon player 1. The states P_i correspond to the same states in 1.

Along the equilibrium path, in state $*$, player 1 approaches 23, who accept the approach; player 1 proposes x^*_{123}, which is accepted. In PB_1 player 2 at $resp^2_{1,123}$ accepts only proposals $x \in \Delta^2$ such that $x_2, x_3 \geq \frac{\delta}{1+\delta}$, while player 3 accepts only proposals $x \in \Delta^2$ such that $x_3 \geq \frac{\delta}{1+\delta}$. In PB_1, player $i \neq 1$ approaches $j \neq 1$ at $prom^i_{/p \neq ij}$.

Transitions between states work as follows:

- *In state* $*$ a deviation by player i in the approach phase or by player $i \neq 1$ in the bargaining phase triggers a change to state P_i of lemma 1 (see table 1), and then play reverts to the strategies described there. A deviation by player 1 in the bargaining phase triggers instead a change of state to PB_1.

- *In state PB_1* a deviation by player i in the approach phase or in two person bargains changes the state to P_i and play reverts to the strategies described there (a deviation by player i in three-person bargains leaves the state at PB_1).

We claim that the above strategies can support in equilibrium *any* partition x^*_{123} yielding player 1 at least $\frac{1-\delta}{1+\delta}$, which tends to zero for δ tending to one, thus verifying the statement with $\delta^* = \frac{1-x^*_1}{1+x^*_1}$.

Let's check the optimality of the strategies described for $x^*_1 \geq \frac{1-\delta}{1+\delta}$. The optimality at states P_i has been shown in lemma 1.

State $*$:

approach phase: Consider first player 1. If he were to approach i, the state would change to P_1, in which player 1 obtains a null payoff. Consider now player $i \neq 1$; if he rejected at $sol^i_{1,123}$, the state would become P_i, in which player i obtains a null payoff;

bargaining phase: A deviation by player 1 at $prop^1_{123}$ would prompt a change of state to PB_1. This yields player 1 a null payoff if the deviant proposal $x = (x_1, x_2, x_3) \in \Delta^2$ is rejected. For x to be accepted by both players 2 and 3, it must be $x_2, x_3 \geq \frac{\delta}{1+\delta}$: so this is not a profitable deviation by player 1 if $x^*_1 \geq 1 - 2\left(\frac{\delta}{1+\delta}\right) = \frac{1-\delta}{1+\delta}$. Consider next player $i \neq 1$ at $resp^i_{1,123}$. Rejecting the equilibrium offer triggers state P_i, where player i earns a null payoff.

State PB_1:

approach phase: In this phase the only possible deviations at node $prom^i_{/p}$ are by players 2 or 3, which trigger P_i and thus are never profitable. Consider now node $sol^i_{j,ij}$, with $i, j \neq 1$; if player i rejected, state would change to P_i, yielding him a payoff of zero. Notice that this is the only type of $sol^i_{j,p}$ that can occur in state PB_1.

bargaining phase: Player 1 cannot improve by deviating from his acceptance strategy, as this would change the state to P_1, yielding him a payoff of zero. If player $i \neq 1$ accepts a deviant proposal $x \in \Delta^2$, the state remains PB_1. Consider first player

			$*$	PB_1
player 1	approaches		23	
	rejects approach from			
	in two person bargains:	proposes		
		accepts		
	in three-person bargains:	proposes	x^*_{123}	
		accepts		
player 2	approaches			3
	rejects approach from			
	in two person bargains:	proposes		y_{23}
		accepts		
	in three-person bargains:	proposes	-	
		accepts	x^*_{123}	$x_2 \geq \frac{\delta}{1+\delta}$ if $x_3 \geq \frac{\delta}{1+\delta}$
player 3	approaches			2
	rejects approach from			
	in two person bargains:	proposes		y_{32}
		accepts		
	in three-person bargains:	proposes		
		accepts	x^*_{123}	$x_3 \geq \frac{\delta}{1+\delta}$
Transition: see text.				

Table 2: The perfect equilibrium of proposition 2

2, and suppose he is confronted with a deviant proposal $x \in \Delta^2$. If $x_2 < \frac{\delta}{1+\delta}$ or $x_3 < \frac{\delta}{1+\delta}$, the payoff from following the equilibrium strategy and rejecting the deviant offer is $\frac{\delta}{1+\delta}$ (that is, the Rubinstein first mover share, one period later), while the payoff from accepting is $\frac{\delta^2}{1+\delta}$ (that is, the Rubinstein responder share, one period later), since player 3 will subsequently reject and approach player 2. If instead $x_2, x_3 \geq \frac{\delta}{1+\delta}$, the payoff from following the equilibrium strategy and accepting is x_2, since player 3 will also accept. Rejecting, on the other hand, yields $\frac{\delta}{1+\delta}$ (that is, the Rubinstein first mover share one period later). This shows that it is optimal for 2 to reject a deviant proposal x unless $x_2, x_3 \geq \frac{\delta}{1+\delta}$. Consider finally the case when player 3 is confronted with a deviant proposal $x \in \Delta^2$ (which must have been accepted by player 2). The payoff from accepting is x_3, whereas the payoff from rejecting is $\frac{\delta}{1+\delta}$ (that is, the Rubinstein first mover share one period later). If $x_3 \geq \frac{\delta}{1+\delta}$ the equilibrium strategy prescribes to accept, while if $x_3 < \frac{\delta}{1+\delta}$ the equilibrium strategy prescribes to reject. In either case, player 3's equilibrium strategy is optimal. ■

4 Modifications to the basic game

4.1 Approach before any counterproposal

It is important to test the robustness of a bargaining model to modifications of the basic structure. One natural such modification is the following. Instead of allowing a player who rejects an offer in the bargaining phase to make a counteroffer, in the same partnership, in the following round, we could ask that the rejecting player necessarily triggers a new *approach* phase, rather than a new *bargaining* phase, even if he intends to make a counterproposal within the current partnership. In other words, a rejecting player in the bargaining phase would have to "ask for permission" before continuing in the bargaining with the same player(s).

The most interesting change in the results from this alteration is that it would allow us to obtain a full folk theorem, thus strengthening the main message of our paper (results analogous to the others can be obtained). What sustains this result is the social norm acording to which the entire surplus can be extracted from a cheater in a two-person partnership, by means of a credible threat of social exclusion.

Proposition 3 *Let $\delta \geq \frac{1}{2}$; then, for any $x^*_{123} = (x^*_1, x^*_2, x^*_3) \in \Delta^2$ there exists a perfect equilibrium which supports $(123, x^*_{123})$ with immediate agreement.*

Proof. Again, the strategies can be described by means of five states, $*'$, PB'_1, and P_i $i = 1, 2, 3$, and with the aid of Table 3, where $\overline{y}_{ij} = (y_i, y_j) = (1, 0)$. Transitions between states work as in proposition 2, with $*'$ and PB'_1 in place of $*$ and PB_1, respectively.

Along the equilibrium path, in $*'$, player 1 approaches the other two players, who both accept; then, player 1 proposes $x^*_{123} \in \Delta^2$, which is accepted. In PB'_1 player 2 at $resp^2_{1,123}$ accepts only proposals $x \in \Delta^2$ such that $x_2, x_3 \geq \delta$, while player 3 accepts only proposals $x \in \Delta^2$ such that $x_3 \geq \delta$. In PB'_1, player $i \neq 1$ approaches player 1 at $prom^i_{/p}$.

It is helpful to consider what the strategies imply off the equilibrium path. A deviation by player 1 at node $prop^1_{123}$ in state $*'$ triggers his own punishment PB'_1, in which player 2 obtains a unitary payoff in the following round. In PB'_1 Player 1 will have to accept a zero payoff, for otherwise his subsequent approach (in P_1) would be rejected: it is at this point that the change in structure we consider is crucial. Consequently, the reward to player $i \neq 1$ from punishing this type of deviation is worth δ at the time when the deviation is produced (instead of $\frac{\delta}{1+\delta}$ in proposition 2). Thus, in order for player 1 to have his deviant offer accepted, he must concede to the other players at least δ each. But this implies that the most player 1 can squeeze out of a deviation is $1 - 2\delta \leq 0$ if $\delta \geq \frac{1}{2}$, so that such deviant offer is unsustainable.

The optimality of the strategies described can be checked by following steps analogous to those in the proof of proposition 2. This part of the proof is relegated to Appendix 2. ■

4.2 Costly approach

In our basic model, approaches are costless. In fact, all that is needed for the results to go through is that the costs in the approach phase are significantly less than those in the bargaining phase. It is easy to check that all proofs remain valid when there is a cost ε to promote a partnership, provided ε is sufficiently small. That approach costs should

			$*'$	PB'_1
player 1	approaches		23	
	rejects approach from		-	
	in two person bargains:	proposes	-	
		accepts	-	$x_1 \geq 0$
	in three-person bargains:	proposes	x^*_{123}	
		accepts	-	
player 2	approaches		-	1
	rejects approach from		-	
	in two person bargains:	proposes	-	$\overline{y}_{21}$
		accepts	x^*_{123}	
	in three-person bargains:	proposes	-	
		accepts	-	$x_2 \geq \delta$ if $x_3 \geq \delta$
player 3	approaches		-	1
	rejects approach from		-	
	in two person bargains:	proposes	-	$\overline{y}_{31}$
		accepts	x^*_{123}	
	in three-person bargains:	proposes	-	
		accepts	-	$x_3 \geq \delta$
Transition: see text.				

Table 3: The perfect equilibrium of proposition 3

be negligible with respect to bargaining costs is what *defines* the approach phase. This phase should be seen as one in which players are still "sounding out" each other, and no major advantage is gained - unlike in the bargaining phase - by having the initiative. The approach phase is the battleground in which the initiative is sought for the moment when bargaining in earnest starts within a partnership.

5 Concluding Remarks

Club formation when diseconomies of scale justify the exclusion of potential members is an intrinsically unstable process, because there is an immediate incentive to upset every potential agreement. The standard theory of clubs bypasses the difficulty of modelling the strategic complexity thus generated by assuming that *technology* alone (that is, the structure of the economies of scale) determines club membership size at the efficient level. We hope to have shown that even in our simplified model this club efficiency hypothesis does not stand up to scrutiny. When only stationary strategies are allowed, the tragedy of the clubs is the unique equilibrium phenomenon when an equilibrium exists. The existence of a stationary equilibrium depends *both* on time preferences and on the technology of returns to club size. When more complex social arrangements to punish deviations are allowed, an equilibrium always exists, and the tragedy of the clubs is a feature of most equilibria.

Further research is certainly needed. With reference to the theory of club formation alone, one ought to address the issue of larger and/or asymmetric populations and allow for the possibility of *multiple clubs* to come into existence.

In a wider game-theoretic perspective, we believe that our model, which distinguishes the act of approaching potential partners from that of proposing a partition within an established partnership, can be of interest for explaining in general the process of coalition formation and for the analysis of multi-person bargaining. Both models of multiperson bargaining à la Rubinstein (see e.g.Binmore (1985), Sutton (1986), Chae and Yang (1988), Chae and Yang (1994), Krishna and Serrano (1996)) and models of coalitional bargaining in characteristic function or partition function form games (e.g. Selten (1981), Chatterjee et al. (1993), Bloch (1997), Ray and Vohra (1997) and the survey by Carraro and Moriconi (1998)) are somehow related to our model. However, none of those models addresses the crucial issues of *congestion* and *club inefficiency*, which are the focus of our paper.

In work in progress we apply our basic model to the case in which the surplus being produced is a *public*, rather than a club, good. In that case, relevant for example to the formation of environmental coalitions, the central aspect is not the exclusion power but the *free-riding* power of participants.

Appendix 1

Proof of proposition 1: The argument follows these steps:

Step 1: Define a stationary strategy profile which is perfect when $\delta \in \left[\underline{\delta}_s, \overline{\delta}_s\right]$ and which supports the equilibrium partition $z_{123} = \left(\frac{1}{1+2\delta}s, \frac{\delta}{1+2\delta}s, \frac{\delta}{1+2\delta}s\right)$.

Step 2: For $\delta \in \left[\underline{\delta}_s, \overline{\delta}_s\right]$, $y_{ij} = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ is the only s.s.p.e. payoff which can be supported in a two-person partnership; however

Step 3: For $\delta \in \left[\underline{\delta}_s, \overline{\delta}_s\right]$, no s.s.p.e. exists which supports agreement in a two-person partnership along the equilibrium path.

Step 4: For $\delta \in \left[\underline{\delta}_s, \overline{\delta}_s\right]$, z_{123} is the only s.s.p.e. payoff which can be supported in a three-person partnership.

Step 5: If $\delta > \overline{\delta}_s$, no s.s.p.e exists.

Step 6: Define a stationary strategy profile (different from that of step 1) which is perfect when $\delta < \underline{\delta}_s$ and which supports the equilibrium partition z_{123}.

Step 7: If $\delta < \underline{\delta}_s$, the only s.s.p.e. payoff that can be supported in an s.s.p.e. is z_{123} in the grand partnership.

Step 8: If $\delta < \underline{\delta}_s$, no s.s.p.e. can exist with agreement in a two person partnership along the equilibrium path.

Step 1: An s.s.p.e. exists when $\delta \in \left[\underline{\delta}_s, \overline{\delta}_s\right]$.

Denote by y_{ij} the partition which gives players i and j shares $y_i = \frac{1}{1+\delta}$ and $y_j = \frac{\delta}{1+\delta}$, respectively; and by z_{ijk} the partition which gives players i, j and k shares $z_i = \frac{1}{1+2\delta}s$ and $z_j = z_k = \frac{\delta}{1+2\delta}s$, respectively. Strategies which justify part **(a)** of the proposition are for player i:

1. <u>in the approach phase:</u> At nodes $prom^i_{/p\neq ijk}$ approach jk; at nodes $prom^i_{/ijk}$ approach $i+1$ (modulo 3); at nodes $sol^i_{j,ijk}$ accept the approach, and at nodes $sol^i_{j,ij}$ reject the approach;

2. <u>in the bargaining phase:</u> At nodes $prop^i_{ij}$ propose y_{ij} and at nodes $prop^i_{ijk}$, propose z_{ijk}; at nodes $resp^i_{j,ij}$ and $resp^i_{j,ijk}$ accept only offers yielding at least $\frac{\delta}{1+\delta}$ and $\frac{\delta}{1+2\delta}s$, respectively, and move on to node $prop^i_{ij}$ and $prop^i_{ijk}$, respectively, when rejecting.

That these strategies constitute an s.p.e. can be checked easily using the one-deviation property.[11] At all nodes of type $prom^i_{/p\neq ijk}$, the equilibrium payoff $\frac{1}{1+2\delta}s$ cannot be improved upon by a deviation, as any approach to form a two-person partnership is turned down by player $j \neq i$, who then re-approaches the grand coalition, yielding i a payoff of $\frac{\delta}{1+2\delta}s$. At nodes $sol^i_{j,ijk}$, the equilibrium strategy always yields $\frac{\delta}{1+2\delta}s$, as does rejecting. At nodes $sol^i_{j,ij}$, by conforming to his equilibrium strategy player i should reject and obtain the proposer's share of s in a grand coalition, that is $\frac{1}{1+2\delta}s$. On the other hand, by accepting the approach by player j he would obtain $\frac{\delta}{1+\delta}$. Consequently, the optimality of player i's strategy at such a node is ensured as long as $\frac{\delta}{1+\delta} \leq \frac{1}{1+2\delta}s$, that is if[12] $\delta \leq \frac{s-1+\sqrt{s^2+6s+1}}{4} \equiv \overline{\delta}_s$.

Consider now the bargaining phase. If player i were to deviate from his equilibrium strategy at a node of type $prop^i_p$ and claim for himself more than what prescribed by the strategy, the responder(s) would reject. Player i would obtain either $\frac{\delta}{1+\delta}$ or $\frac{\delta}{1+2\delta}s$ one period later, rather than either $\frac{1}{1+\delta}$ or $\frac{1}{1+2\delta}s$, depending on whether $p = ij$ or $p = ijk$, respectively. At a node of type $resp^i_{j,ijk}$ player i cannot profit from accepting an offer

[11] Which, as noted in the text, holds in this game.

[12] $\frac{\delta}{1+\delta} \leq \frac{1}{1+2\delta}s$ can be rearranged as $2\delta^2 - (s-1)\delta - s \leq 0$. The two roots of the polynomial are $\delta_{1,2} = \frac{s-1\pm\sqrt{s^2+6s+1}}{4}$, one of which is negative.

yielding less than $\frac{\delta}{1+2\delta}s$, as by rejecting and conforming to his equilibrium strategy he could obtain $\frac{1}{1+2\delta}s$ in the subsequent round. Furthermore, deviating and moving to a node $prom^i_{/ijk}$ is not profitable either, as the solicited player would reject and approach the grand coalition, where player i would still earn the responder share one period later. Next, consider the optimality of player i's strategy at nodes $resp^i_{j,ij}$. When confronted with a disequilibrium offer, by conforming to his equilibrium strategy he can obtain $\frac{1}{1+\delta}$ in the following round. Alternatively, he could either *(i)* reject and approach player k, or *(ii)* reject and approach the grand coalition. If *(i)*, his approach would be rejected and he would subsequently earn the responder's payoff in the grand partnership. If *(ii)*, such a deviation would be profitable only if $\frac{\delta}{1+\delta} < \delta\left(\frac{1}{1+2\delta}\right)s$, that is if $\delta < \frac{s-1}{2-s} \equiv \underline{\delta}_s$. Thus, $\delta \geq \underline{\delta}_s$ guarantees optimality.

Finally, it remains to notice that

$$\underline{\delta}_s = \frac{s-1}{2-s} < \frac{s-1+\sqrt{s^2+6s+1}}{4} = \overline{\delta}_s$$

To see this, note that:

$$\frac{\partial\left(\frac{s-1}{2-s}\right)}{\partial s} = \frac{1}{(2-s)^2} > 0 \forall s$$

and

$$\frac{\partial\left(\frac{s-1+\sqrt{s^2+6s+1}}{4}\right)}{\partial s} = \frac{1}{4}\left(1+(s+3)\left(s^2+6s+1\right)^{-\frac{1}{2}}\right) > 0 \forall s$$

so that both bounds are monotonically increasing in s. Furthermore, $\lim_{s\to 1}\overline{\delta}_s = \frac{1}{\sqrt{2}} > 0 = \lim_{s\to 1}\underline{\delta}_s$, and $\lim_{s\to\frac{3}{2}}\overline{\delta}_s = 1 = \lim_{s\to\frac{3}{2}}\underline{\delta}_s$. Since

$$\left.\frac{\partial\left(\frac{s-1}{2-s}\right)}{\partial s}\right|_{s=\frac{3}{2}} = 4, \quad \left.\frac{\partial\left(\frac{s-1+\sqrt{s^2+6s+1}}{4}\right)}{\partial s}\right|_{s=\frac{3}{2}} = \frac{47}{56}$$

$\underline{\delta}_s$ approaches $\overline{\delta}_s$ from below. Since the two functions only intersect at $s=\frac{3}{2}$ for $s \in \left[1, \frac{3}{2}\right]$, the interval $\left[\underline{\delta}_s, \overline{\delta}_s\right]$ is always non empty. Therefore, $0 < \underline{\delta}_s < \overline{\delta}_s < 1$ guarantees the existence of a δ which satisfies part **(a)** of the proposition.

Steps 2-8: Notation. Let r denote the role in which a player acts in the bargaining phase of some partnership, where $r = 1, 2, 3$ refers to him being the proposer or the first or the second responder, respectively. Thus, if z_p is some s.s.p.e. payoff vector, $z_i(r)$ indicates the s.s.p.e. payoff to player i in partnership p and role r.

Step 2a: In an s.s.p.e. a player cannot obtain his maximum payoff when acting as a responder.

Let $\overline{I}$ be the maximum s.s.p.e. payoff that player i can achieve in some partnership, and assume to the contrary of the claim that player i can obtain $\overline{I}$ when acting as a responder in some partnership in which some other player, say player j, acts as proposer. Therefore, by rejecting player j's proposal, player i could obtain at most $\delta\overline{I}$. But then, player j could instead propose a partition which gives $\overline{I}-\varepsilon$ to player i (and, in the case of a three-person partnership, his unchanged payoff to player k, who would accept by stationarity), who would accept if $\overline{I}-\varepsilon>\delta\overline{I}$, true for ε sufficiently small.

Step 2b: If $\delta \geq \underline{\delta}_s = \frac{s-1}{2-s}$, in an s.s.p.e. any agreement in a two-person partnership can yield no more than $\frac{1}{1+\delta}$ to the proposer.

Assume to the contrary that the supremum of the s.s.p.e. payoffs that any player, say player i, can obtain as a proposer in a two-person partnership, say ij, is $y_i'(1)=\frac{1}{1+\delta}+\varepsilon$, with $\varepsilon \in \left(0, \frac{\delta}{1+\delta}\right]$. Correspondingly, the infimum of the s.s.p.e. payoffs that any player, say j, can obtain as a responder in a two-person partnership, say ij, is $y_j'(2)=\frac{\delta}{1+\delta}-\varepsilon$. However, player j would be better off by rejecting $y_j'(2)$ and counteroffering (in the same bargain, at node $prop_{ij}^{j}$) a partition y_{ij}'' with $y_i''(2)=\frac{\delta}{1+\delta}+\varepsilon$ and $y_j''(1)=\frac{1}{1+\delta}-\varepsilon$, since $\delta y_j''(1)=\delta\left(\frac{1}{1+\delta}-\varepsilon\right)>\frac{\delta}{1+\delta}-\varepsilon=y_j'(2)$. Player i would accept this offer, since otherwise, he could obtain at most $y_i'(1)$ one period later, with $\delta y_i'(1)=\delta\left(\frac{1}{1+\delta}+\varepsilon\right)<\frac{\delta}{1+\delta}+\varepsilon=y_i''(2)$. We now prove that this assertion is true.

Assume first that the optimal action after i's rejection leads to the formation of a three-person partnership, with player i acting as a proposer, in which he obtains a payoff of $\frac{1}{1+\delta}+\varepsilon+\eta$, $\eta>0$. Let K be the corresponding s.s.p.e. payoff to player k. In order to ensure that this proposal is accepted by player j, it must be the case that there does not exist an x such that the following two inequalities are satisfied:

$$\begin{aligned} \delta x &> s-\frac{1}{1+\delta}-\varepsilon-\eta-K \\ s-x-K &> \delta\left(\frac{1}{1+\delta}+\varepsilon+\eta\right) \end{aligned}$$

Similarly, in order to ensure that this proposal is accepted by player k, it must be the case that there does not exist an x such that the following two inequalities are satisfied:

$$\begin{aligned} \delta x &> K \\ s - x - \left(s - \frac{1}{1+\delta} - \varepsilon - \eta - K\right) &= K - x + \frac{1}{1+\delta} + \varepsilon + \eta > \delta\left(\frac{1}{1+\delta} + \varepsilon + \eta\right) \end{aligned}$$

In both sets of inequalities, the top one ensures that the responding player is better off by rejecting, while the bottom inequality ensures that his deviant proposal is accepted by both other players.

The two sets of inequalities can be written as

$$s - \frac{\delta}{1+\delta} - \delta\varepsilon - \delta\eta - K > x > \frac{1}{\delta}\left(s - \frac{1}{1+\delta} - \varepsilon - \eta - K\right)$$

and

$$K + \left(\frac{1}{1+\delta} + \varepsilon + \eta\right)(1-\delta) > x > \frac{K}{\delta}$$

respectively. They admit a solution in x if

$$0 > s - 1 - (1+\delta)(\varepsilon + \eta) - K$$

and

$$0 < \delta\left(\frac{1}{1+\delta} + \varepsilon + \eta\right) - K$$

hold, respectively. It follows that, in order for *both* deviations to be prevented we need both the above inequalities not to hold, which implies in particular:

$$s \geq \frac{1+2\delta}{1+\delta} + (1+2\delta)(\varepsilon + \eta)$$

But this contradicts (given that $\varepsilon, \eta > 0$) the assumption $s \leq \frac{1+2\delta}{1+\delta} \Leftrightarrow \delta \geq \frac{s-1}{2-s} = \underline{\delta}_s$.

Next, if the optimal action after i's rejection leads to the formation of a two-person partnership with i as a proposer, by definition i cannot get more than $y_i'(1)$ in the following round.

By step 2a we can rule out the cases in which the optimal action after player i's rejection lead to the formation of a partnership in which player i acts as a responder. We can thus conclude that no player can obtain more than $\frac{1}{1+\delta}$ in a two-person partnership when he acts as a proposer.

Step 2c: If $\delta \geq \underline{\delta}_s$, in an s.s.p.e. any agreement in a two-person partnership yields no less than $\frac{1}{1+\delta}$ to the proposer.

Assume to the contrary that in some s.s.p.e. agreement y'_{ij} we have $y'_j(2) = \frac{\delta}{1+\delta} + \varepsilon \Rightarrow y'_i(1) = \frac{1}{1+\delta} - \varepsilon$ (so that $\varepsilon \leq \frac{1}{1+\delta}$). But player i could instead propose $y''_i(1) = \frac{1}{1+\delta} - \delta\varepsilon \Rightarrow y''_j(2) = \frac{\delta}{1+\delta} + \delta\varepsilon$, which player j would accept, since he could obtain at most $y_j(1) = \frac{1}{1+\delta}$ in the following round. This last assertion is true because by rejecting and making a counteroffer in the same partnership, player j obtains at most $\frac{1}{1+\delta}$ (by step 2b), worth $\frac{\delta}{1+\delta}$ in present discounted value.

Similarly, if the optimal action after player i's proposal leads to the formation of a two-person partnership with player j acting as a proposer, by step 2b he obtains at most $\frac{1}{1+\delta}$.

Finally, if the optimal action after player i's proposal leads to the formation of a three-person partnership with player j acting as a proposer, this deviation is profitable only if his payoff is greater than $\frac{1}{1+\delta}$. However, by the argument in step 2b player j cannot achieve an s.s.p.e. payoff in excess of $\frac{1}{1+\delta}$ in a three person partnership.

We can conclude, in turn, that no player obtains less than $\frac{1}{1+\delta}$ in a two-person partnership when he acts as a proposer.

Step 3: If $\delta \geq \underline{\delta}_s$, no s.s.p.e. can exist with agreement in a two person partnership along the equilibrium path.

By steps 2a and 2b, $y_{ij} \equiv \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ is the only s.s.p.e. payoff vector which could be supported in a two-person partnership. However, a strategy supporting the above agreement along the equilibrium path would have to prescribe player i to approach player j at nodes $prom^i_{/p\neq ij}$, and player j to accept the approach at nodes $sol^j_{i,ij}$. But player j would gain by rejecting the approach in order to become (in that same round) the first proposer in a two-person partnership with player k, who would accept the approach. In fact, k cannot promote a partnership with i, who would reject and form a partnership with j (in this way obtaining $\frac{1}{1+\delta}$ instead of $\frac{\delta}{1+\delta}$). The only other alternative is for player k to approach the grand partnership. Here, he has to propose a partition which gives player i at least a payoff of $\frac{1}{1+\delta}$, (otherwise i would reject the approach and form a partnership with j), and player j at least a payoff of $\frac{\delta}{1+\delta}$. To see why player j has to be given at least $\frac{\delta}{1+\delta}$, assume to the contrary that there exists a s.s.p.e. in which player i obtains $\frac{1}{1+\delta}$, player j obtains $\frac{\delta}{1+\delta} - \varepsilon$, and player k obtains $s + \varepsilon$. But then, player j could reject and counteroffer a partition which gives $\frac{\delta}{1+\delta}$ to player i and $s + \varepsilon$ to player k, who would both accept, keeping the residual $\frac{1}{1+\delta} - \varepsilon$ for himself, thereby improving on the payoff he rejected, since $\delta\left(\frac{1}{1+\delta} - \varepsilon\right) > \frac{\delta}{1+\delta} - \varepsilon$. Thus, in the grand partnership player k would obtain a payoff of at most $s - 1$, where $s - 1 < \frac{\delta}{1+\delta} \Leftrightarrow s < \frac{1+2\delta}{1+\delta} \Leftrightarrow \delta > \underline{\delta}_s$.

Step 4a: If $\delta \geq \underline{\delta}_s$, in an s.s.p.e. any agreement in the grand partnership yields a payoff to the proposer which is not less then his payoff when acting as a responder.

Note first that, by step 3 any rejection of a proposal in the grand partnership must eventually lead back to a proposal within that partnership, either immediately with a counteroffer, or after a sequence of approaches. Assume to the contrary that there is an s.s.p.e. in which one player, say player i, obtains payoffs equal to $\underline{I}$ as a proposer, equal to $\overline{I} > \underline{I}$ as a responder to player j's proposal, and equal to $\overline{I}' > \underline{I}$ as a responder to player k's proposal. Consequently, efficiency requires that at least one of the other players, say player j, obtains payoffs $\overline{J}$ and $\underline{J}$, with $\underline{J} < \overline{J}$, when acting as a responder to i and as a proposer, respectively.[13] But then player i could improve by claiming instead a share $\underline{I} + \eta$ for himself and leaving j with a share $\overline{J} - \eta > \delta\overline{J} > \delta\underline{J}$ for η sufficiently small.

Step 4b: If $\delta \geq \underline{\delta}_s$, in an s.s.p.e. any agreement in the grand partnership yields the proposing player a payoff not greater than $\frac{1}{1+2\delta}s$.

Assume in contradiction to the statement that the supremum of the s.s.p.e. payoffs that any player, say player i, can obtain as a proposer in the grand partnership is $z_i' = \frac{1}{1+2\delta}s + \varepsilon$, with $\varepsilon \in \left(0, \frac{2\delta}{1+2\delta}s\right]$. Let $z_j' = \frac{2\delta}{1+2\delta}s - \varepsilon - z_k'$. We show that there cannot be an agreement on z_{123}' since at least one responder could gain by rejecting the proposed share $z_{s\neq i}'$. In fact, for both players j and k *not* to gain by rejecting it must be the case that the following two incompatible inequalities hold (as we explain below):

$$
\begin{aligned}
z_k' &\geq \delta\left(z_k' + \frac{1-\delta}{1+2\delta}s\right) \Leftrightarrow z_k' \geq \frac{\delta}{1+2\delta}s \\
\frac{2\delta}{1+2\delta}s - \varepsilon - z_k' &\geq \delta\left(\frac{1+\delta}{1+2\delta}s - z_k' - \varepsilon\right) \Leftrightarrow z_k' + \varepsilon \leq \frac{\delta}{1+2\delta}s
\end{aligned}
$$

[13] By stationarity, player k obtains the same payoff in three-person partnerships independently of the identity of the proposer.

If the first inequality does not hold, then player k could profitably reject and obtain $z_k'' = z_k' + \frac{1-\delta}{1+2\delta}s$ one period later by proposing the partition z_{123}'' with $z_i'' = \frac{\delta}{1+2\delta}s + \varepsilon$ and $z_j'' = \frac{2\delta}{1+2\delta}s - \varepsilon - z_k' = z_j'$. Player i would accept this offer, since otherwise he could get at most z_i' by making a counteroffer in the following period, worth $\delta z_i' = \delta\left(\frac{1}{1+2\delta}s + \varepsilon\right) < z_i''$. By stationarity, player j would also accept the offer, because it is the same as the one he is assumed to accept it in the s.s.p.e.

If the second inequality does not hold, then player j could profitably reject and obtain $z_j''' = \frac{1+\delta}{1+2\delta}s - z_k' - \varepsilon$ $(= s - \left(\frac{\delta}{1+2\delta}s + \varepsilon\right) - z_k')$ one period later by proposing the partition z_{123}''' with $z_k''' = z_k'$ and $z_i''' = \frac{\delta}{1+2\delta}s + \varepsilon$. Players i and j would accept this offer by the same reasoning of the previous paragraph.

Step 4c: If $\delta \geq \underline{\delta}_s$, in an s.s.p.e. any agreement in the grand partnership yields the proposing player a payoff not less than $\frac{1}{1+2\delta}s$. Assume in contradiction that there is an s.s.p.e. agreement on a partition z_{123}' in which at least one of the responders, say j, gets $z_j' \geq \frac{\delta}{1+2\delta}s + \varepsilon$ with $\varepsilon \in \left(0, \frac{1+\delta}{1+2\delta}s\right]$, so that $z_i' = \frac{1+\delta}{1+2\delta}s - \varepsilon - z_k'$. Then, player i could improve by instead proposing z_{123}'' with $z_j'' = \frac{\delta}{1+2\delta}s + \delta\varepsilon$ and $z_k'' = z_k'$,so that $z_i'' = \frac{1+\delta}{1+2\delta}s - \delta\varepsilon - z_k' > z_i'$. This proposal would be accepted by player k because of stationarity and by player j because of step 4b.

Step 4d: If $\delta \geq \underline{\delta}_s$, in an s.s.p.e. any agreement in the grand partnership yields the responding player a payoff not less than $\frac{\delta}{1+2\delta}s$. This follows from Step 4c, since any responding player would reject any partition yielding a smaller payoff than $\frac{\delta}{1+2\delta}s$ and secure $\delta z_j(1) = \frac{\delta}{1+2\delta}s$ in the following round.

Step 4e: If $\delta \geq \underline{\delta}_s$, in an s.s.p.e. any agreement in the grand partnership yields the responding player a payoff not greater than $\frac{\delta}{1+2\delta}s$. This holds since a responder obtaining more than $\frac{\delta}{1+2\delta}s$ implies that the proposer obtains less than $\frac{1}{1+2\delta}s$, and/or the other responder obtains less than $\frac{\delta}{1+2\delta}s$, contradicting Step 4c and/or step 4d above.

Step 5: No s.s.p.e. exists if $\delta > \overline{\delta}_s$.

As a preliminary, recall that $\underline{\delta}_s$ and $\overline{\delta}_s$ solve $\frac{\delta}{1+\delta} = \delta\left(\frac{1}{1+2\delta}\right)s$ and $\frac{\delta}{1+\delta} = \frac{1}{1+2\delta}s$, respectively. The first condition derives from

$$\frac{\delta}{1+\delta} \geq \delta\left(\frac{1}{1+2\delta}\right)s \Leftrightarrow \delta \geq \underline{\delta}_s \tag{C1}$$

while the second derives from

$$\frac{\delta}{1+\delta} \leq \frac{1}{1+2\delta}s \Leftrightarrow \delta \leq \overline{\delta}_s \tag{C2}$$

If C2 is violated, a player who is solicited in a two person partnership has an incentive to accept. In addition, since $\delta > \overline{\delta}_s$ implies $\delta > \underline{\delta}_s$, condition C1 holds, so that $\frac{1}{1+\delta} \geq \left(\frac{1}{1+2\delta}\right) s$. These two facts imply that a player at node $prom^i_{/p}$ prefers to be the promoter of a two-person rather than of a three-person partnership. Therefore there cannot be an s.s.p.e. agreement in a three-person partnership along the equilibrium path. Joining this to step 3, we can conclude that no s.s.p.e. agreement exists when $\delta > \overline{\delta}_s$.

Step 6: An s.s.p.e. exists when $\delta < \underline{\delta}_s$.

Denote by x_{ij} the partition which gives players i and j shares $x_i = 1 - \frac{\delta}{1+2\delta}s$ and $x_j = \frac{\delta}{1+2\delta}s$, respectively; and by z_{ijk} as above the partition which gives players i, j and k shares $z_i = \frac{1}{1+2\delta}s$ and $z_j = z_k = \frac{\delta}{1+2\delta}s$, respectively. Strategies which justify part **(a)** of the proposition are for player i:

1. in the approach phase: At nodes $prom^i_{/p\neq ijk}$ approach jk; at nodes $prom^i_{/ijk}$ approach $i+1$ (modulo 3); accept the approach at nodes $sol^i_{j,ijk}$ and reject it at nodes $sol^i_{j,ij}$;

2. in the bargaining phase: At nodes $prop^i_{ij}$ propose x_{ij} and at nodes $prop^i_{ijk}$, propose z_{ijk}; at nodes $resp^i_{j,ij}$ and $resp^i_{j,ijk}$ accept only offers yielding at least $\frac{\delta}{1+2\delta}s$, and when rejecting move on to node $prom^i_{/ij}$ and $prop^i_{ijk}$, respectively.

Let's check the optimality of the strategies. At all nodes of type $prom^i_{/p\neq ijk}$, the equilibrium payoff $\frac{1}{1+2\delta}s$ cannot be improved upon by a deviation, as any approach to form a two-person partnership is rejected, eventually yielding i a payoff $\frac{\delta}{1+2\delta}s$ in the three-person pertnership. At nodes $sol^i_{j,ijk}$, the equilibrium strategy yields $\frac{\delta}{1+2\delta}s$, while rejecting the approach yields eventually the same payoff, no matter which player i approaches. At nodes $sol^i_{j,ij}$, by conforming to his equilibrium strategy player i would reject and eventually get the proposer's share in the grand partnership, $\frac{1}{1+2\delta}s$. If instead he accepted he would only obtain the responder's share in the two-person partnership, $\frac{\delta}{1+2\delta}s$.

Consider now the bargaining phase. If player i were to deviate from his equilibrium strategy at a node of type $prop^i_p$ and claim for himself more than what prescribed by the strategy, the responder(s) would reject. Player i would obtain $\frac{\delta}{1+2\delta}s$ in the grand partnership one period later, rather than either $1-\frac{\delta}{1+2\delta}s$ or $\frac{1}{1+2\delta}s$, depending on whether $p=ij$ or $p=ijk$, respectively. So, if $p=ijk$ it is clearly not optimal to deviate. If $p=ij$, there is no gain from deviating if $1-\frac{\delta}{1+2\delta}s \geq \delta\left(\frac{\delta}{1+2\delta}s\right) \Leftrightarrow \delta^2 s-(2-s)\delta-1 \leq 0$. This inequality is satisfied for all δ, s such that $0<\delta<1$ and $1 \leq s \leq \frac{3}{2}$. At a node of type $resp^i_{j,ijk}$ player i cannot profit from accepting an offer yielding less than $\frac{\delta}{1+2\delta}s$, as by rejecting and conforming to his equilibrium strategy he could obtain $\frac{1}{1+2\delta}s$ in the subsequent round. Furthermore, deviating and moving to node $prom^i_{/ijk}$ is not profitable either, as the solicited player would reject and approach the grand coalition, where player i would still earn the responder share. Next, consider the optimality of player i's strategy at nodes $resp^i_{j,ij}$. When confronted with a disequilibrium offer, by conforming to his equilibrium strategy he can obtain $\frac{1}{1+2\delta}s$ in the following round. Alternatively, he could either *(i)* reject and approach player k, or *(ii)* reject and make a counteroffer. If *(i)*, his approach would be rejected and he would subsequently earn the responder's payoff in the grand partnership. If *(ii)*, such a deviation would be profitable only if $\frac{\delta}{1+2\delta}s<\delta\left(1-\frac{\delta}{1+2\delta}s\right)s$, that is if $\delta>\frac{s-1}{2-s} \equiv \underline{\delta}_s$. Thus, $\delta \leq \underline{\delta}_s$ guarantees optimality.

Step 7a: If $\delta<\underline{\delta}_s$, in a two-person partnership any player achieves an s.s.p.e. payoff strictly less than $\frac{1}{1+2\delta}s$.

Assume to the contrary that the supremum of the s.s.p.e. payoffs that any player, say player i, can obtain as a proposer is in a two-person partnership, say ij, and is $x'_i(1) \geq \frac{1}{1+2\delta}s$. Correspondingly, the infimum of the s.s.p.e. payoffs that any player, say j, can obtain as a responder in a two-person partnership, say ij, is $x'_j(2)=1-\frac{1}{1+2\delta}s$. However, player j would be better off by rejecting $x'_j(2)$ and counteroffering (in the same bargain, at node $prop^j_{ij}$) a partition x''_{ij} with $x''_i(2)=\delta x'_i(1)+\varepsilon$, with $\varepsilon>0$, since $\delta x''_j(1)=\delta\left[1-\delta x'_i(1)-\varepsilon\right]>1-x'_i(1)=x'_j(2) \Leftrightarrow x'_i(1)>\frac{1}{1+\delta}+\frac{\delta}{1-\delta^2}\varepsilon$. This inequality holds because $x'_i(1) \geq \frac{1}{1+2\delta}s$ and $\frac{1}{1+2\delta}s>\frac{1}{1+\delta}+\frac{\delta}{1-\delta^2}\varepsilon \Leftrightarrow s>\frac{1+2\delta}{1+\delta}+\frac{\delta(1+2\delta)}{1-\delta^2}\varepsilon$, which is true for ε sufficiently small. Player i would accept this offer, since otherwise, he could obtain at most $x'_i(1)$ one period later, with $\delta x'_i(1)<\delta x'_i(1)+\varepsilon$. Thus, either *(i)* $x'_i(1)<\frac{1}{1+2\delta}s$, or *(ii)* the supremum $x'_i(1)$ is not achieved in a two-person partnership, or *(iii)* both. However, by step 6 we know that $\frac{1}{1+2\delta}s$ is attainable at an s.s.p.e., so that we can rule out cases *(i)* and *(iii)*.

By step 2a we can rule out the cases in which the optimal action after player i's rejection lead to the formation of a partnership in which player i acts as a responder. We can thus conclude that no player can obtain a payoff of at least $\frac{1}{1+2\delta}s$ in a two-person partnership when he acts as a proposer.

Step 7b: If $\delta < \underline{\delta}_s$, in an s.s.p.e. any agreement in the grand partnership yields a payoff to the proposer which is not less then his payoff when acting as a responder.

Assume to the contrary that there is an s.s.p.e. with agreement in the grand partnership in which one player, say player i, obtains payoffs equal to $\underline{I}$ as a proposer, equal to $\overline{I} > \underline{I}$ as a responder to player j's proposal, and equal to $\overline{I}' > \underline{I}$ as a responder to player k's proposal. Consequently, efficiency and stationarity require that at least one of the other players, say player j, obtains payoffs $\overline{J}$ and $\underline{J}$, with $\underline{J} < \overline{J}$, when acting as a responder to i and as a proposer, respectively. If player j were unable to succesfully promote a two person partnership, then the same reasoning as in step 4a would apply. Thus, assume that player j can succesfully constitute a two person partnership with some other player, and denote by X the corresponding s.s.p.e. payoff to player j. If $X \leq \underline{J}$, then player j's continuation payoff when rejecting an offer would be $\underline{J}$, so that the same reasoning as in step 4a applies. Alternatively, if $X > \underline{J}$, then player j would promote a two person partnership when rejecting player i's proposal in a grand partnership. But for $\underline{J}$ to be part of an equilibrium offer when player j makes a counterproposal after rejecting an offer in the grand partnership, it must be that $\delta\underline{J} \geq \delta X$, contradicting $\underline{J} < X$.

Step 7c: In an s.s.p.e. any agreement in the grand partnership is on z_{123}.

This follows by repeating the same arguments used in steps 4b-4e, which hold since by step 7a the maximum s.s.p.e. payoff cannot be achieved in a two person partnership.

Step 8: If $\delta < \underline{\delta}_s$, no s.s.p.e. can exist with agreement in a two person partnership along the equilibrium path.

Assume to the contrary that a two-person partnership, say ij, forms along the equilibrium path. In this bargain, after the rejection of an offer, player i will either either remain in the bargain with player j, or approach k. In fact, given that the maximum s.s.p.e. payoff cannot be achieved in the ij partnership (step 7a), it must be the case that either j or k or both would reject the approach by player i to form a grand partnership (otherwise, it would not have been optimal for player i to approach j to form ij). Similarly, if player j were to reject an offer by player i, it must be the case that either i or k or both would reject the approach by player j to form a grand partnership (otherwise it would have been optimal to reject i's approach to form ij).

We will now consider two cases. First, assume that, when player j rejects an offer in ij he approaches player k and player k accepts the approach. By a reasoning analogous to the above, this means that player k cannot successfully promote the grand partnership. Thus, the grand partnership can never form either on or off the equilibrium path. By standard arguments, the only s.s.p.e. partition that can obtain in a two-person partnership is the rubinsteinian one; and a straightforward adaptation[14] of step 3 shows that a two-person partnership cannot form in an equilibrium, yielding a contradiction.

Next assume that k rejects j's approach; if so, then the optimal action by player k can lead to the formation of either *(i)* some two person partnership, or *(ii)* to a grand partnership where player k is the promoter. If *(i)*, then it must be that player k cannot succesfully promote a three person partnership, so that the same reasoning as in the previous paragraph applies. If *(ii)*, then both players i and j obtain a payoff of $\frac{\delta}{1+2\delta}s$; thus , it must be that the s.s.p.e. payoff to player j in a partnership ij is equal to $\frac{\delta^2}{1+2\delta}s$ (since when player j rejects an offer in ij he ends up with a payoff of $\frac{\delta}{1+2\delta}s$ one period later). Consequently, the payoff to player i in ij must be equal to $1-\frac{\delta^2}{1+2\delta}s$. But in turn this means that z_{123} is not an s.s.p.e. partition, except when the payoff that player i could achieve by forming a partnership ij in the following round is equal to his payoff in the grand partnership promoted by player k, that is when $\delta\left(1-\frac{\delta^2}{1+2\delta}s\right)=\frac{\delta}{1+2\delta}s \Leftrightarrow s=\frac{1+2\delta}{1+\delta^2}$. However, this can never be, since by step 7a the supremum payoff to any player in a two person partnership must be less than $\frac{1}{1+2\delta}s$, which in this case translates into requiring $1-\frac{\delta^2}{1+2\delta}s<\frac{1}{1+2\delta}s \Leftrightarrow s>\frac{1+2\delta}{1+\delta^2}$.

Finally, assume that when player j rejects an offer in ij he makes a counteroffer to

[14] The argument in step 3 holds since now we know that a responder in a three person partnership obtains an s.s.p.e. payoff of $\frac{\delta}{1+2\delta}s$. Given that $\frac{\delta}{1+2\delta}s>\frac{\delta}{1+\delta} \Leftrightarrow s>\frac{1+2\delta}{1+\delta}$, such equilibrium payoff is greater than the payoff $\frac{\delta}{1+\delta}$ required in the proof (i.e., the payoff that player k would have to guarantee to player j in a grand partnership promoted by player k).

player i in the same bargain. By stationarity this implies that the only s.s.p.e. partition that can obtain in this partnership is the rubinsteinian one. Once again, this is easily checked not to be an s.s.p.e. In fact, in such an equilibrium player k would end up with a null payoff, whereby both players i and j would have an incentive to enter negotiations with him. Furthermore, an s.s.p.e. in which player k obtains a null payoff in some two person partnership (off the equilibrium path) cannot be sustained, since - given that as we just showed the grand coalition cannot be supported at an equilibrium - player k would always improve on his (zero) payoff by rejecting the partition and counteroffering for instance a partition yielding δ to his opponent. ■

Appendix 2

Here we check the optimality of the strategies described in proposition 3

State $*'$:

approach phase: As in proposition 2, state $*$.

bargaining phase: A deviation by player 1 at $prop^1_{123}$ prompts a change of state to PB'_1. This yields player 1 a null payoff if the deviant proposal $x = (x_1, x_2, x_3) \in \Delta^2$ is rejected. For x to be accepted by both players 2 and 3, it must be $x_2, x_3 \geq \delta$: this is not feasible if $\delta > \frac{1}{2}$, and it yields player 1 a null payoff if $\delta = \frac{1}{2}$. Consider next player $i \neq 1$ at $resp^i_{1,123}$. Rejecting the equilibrium offer triggers state P_i, where player i earns a null payoff.

State PB'_1:

approach phase: In this phase the only possible deviations at node $prom^i_{/p}$ are by players 2 or 3, which trigger P_i and thus are never profitable. Consider now node $sol^1_{i,1i}$; if player 1 rejected, state would change to P_1, yielding him a payoff of zero. Notice that this is the only type of $sol^i_{j,p}$ that can occur in state PB_1.

bargaining phase: Player 1 cannot improve by deviating from his acceptance strategy, as this would change the state to P_1, yielding him a payoff of zero. If player $i \neq 1$

accepts a deviant proposal $x \in \Delta^2$, the state remains PB_1. Consider first player 2, and suppose he is confronted with a deviant proposal $x \in \Delta^2$. If $x_2 < \delta$ or $x_3 < \delta$, the payoff from following the equilibrium strategy and rejecting the deviant offer is δ (that is, the unitary payoff one period later), while the payoff from accepting is 0, since player 3 will subsequently reject and approach player 1. If instead $x_2, x_3 \geq \delta$, the payoff from following the equilibrium strategy and accepting is x_2, since player 3 will also accept. Rejecting, on the other hand, yields δ (recall that the state remains PB_1', so that player 2 obtains the entire surplus one period later). This shows that it is optimal for 2 to reject a deviant proposal x unless $x_2, x_3 \geq \delta$. Consider finally the case when player 3 is confronted with a deviant proposal $x \in \Delta^2$ (which must have been accepted by player 2). The payoff from accepting is x_3, whereas the payoff from rejecting is δ (that is, the unitary payoff one period later). If $x_3 \geq \delta$ the equilibrium strategy prescribes to accept, while if $x_3 < \delta$ the equilibrium strategy prescribes to reject. In either case, player 3's equilibrium strategy is optimal. ■.

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