Money, Credit, and Allocation Under Complete Dynamic Contracts and Incomplete Markets

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Abstract

We construct a dynamic heterogeneous-agent model with random uninsurable endowments. Two allocation mechanisms are considered, one with long-term complete credit arrangements under private information, and one with incomplete competitive markets. A role for money arises due to random limited participation. A Friedman rule is optimal in the first economy, and replicates a pure credit arrangement in the second. Computational results show that steady state allocations are quite different under the two arrangments, though the responses to changes in long-run inflation are similar.

*Rao Aiyagari died suddenly in May 1997. Rao made important early contributions to the paper, but I bear full responsibility for the completed product. I have received useful comments and suggestions from seminar participants at Johns Hopkins University and the University of Texas at Austin.

1. INTRODUCTION

The primary purpose of this paper is to study the implications of alternative money and credit arrangements for the allocation of consumption and wealth in the steady state. We construct an environment where agents have random uninsurable incomes, and where there is a role for money which arises due to random limited participation in the credit market. Two alternative credit arrangements are considered. First, we allow agents to write long-term contracts, under private information, with financial intermediaries (the private information, or PI model). Second, we consider a competitive incomplete markets equilibrium (the incomplete markets or IM model) where agents trade on a competitive bond market each period, and are subject to borrowing constraints. In each of the two models, we allow for alternative assumptions about commitment. Lack of commitment implies additional defection constraints in the PI model, and additional borrowing constraints in the IM model.

In the literature on dynamic contractual arrangements under private information, the incomplete markets, or permanent income, model has been an important benchmark. Green (1987) and Green and Oh (1991) contrasted the implications of private information setups with incomplete markets models. Atkeson and Lucas (1992) demonstrated that arrangements with competitive markets could not support optimal allocations in a dynamic environment with private information about preference shocks. Essentially, intertemporal marginal rates of substitution are different across agents at the optimum, but they must necessarily be equated across agents in a competitive equilibrium.

Models of dynamic contracts with private information[1] typically rely on assumptions that agents' consumptions can be observed, and that there are no unobservable

[1]See also Spear and Srivastava (1987), Phelan and Townsend (1991), Aiyagari and Alvarez (1995), Atkeson and Lucas (1995), Phelan (1995), and Wang (1995).

intertemporal trades among agents. If these assumptions are relaxed, this affects intertemporal incentives, as there is an attenuation of the rewards and punishments available to induce truth-telling. Indeed, there are some results in the literature showing that, for a limited class of private information environments, the only allocation which can be supported is a competitive equilibrium allocation. In particular, Cole and Kocherlakota (1997) show that, under some conditions, the efficient allocation in an environment with unobserved incomes and hidden storage can be supported by a competitive bond market. Similarly, Allen (1985) considers a repeated principal agent model with unobserved lending and borrowing where the efficient outcome is identical to the Walrasian allocation.

Thus, whether a PI or IM model is appropriate for a particular application would seem to depend on the plausibility of assumptions regarding the observability or non-observability of asset holdings in the context of the problem at hand. However, one might ask whether, quantitatively, it makes any difference. To draw an analogy to the asset pricing literature (see Kocherlakota 1996), there are conditions under which incomplete markets allocations with heterogeneous agents are very close to equilibrium allocations with complete markets. A key question we wish to address here is whether a similar result holds in dynamic private information models. We consider an environment with a continuum of infinite-lived consumers and unobservable incomes, similar to that of Green (1987), except that consumption is constrained to be non-negative, the interest rate is endogenous, and there is a role for money.[2] We compute steady state solutions given PI and IM arrangements, compare the properties of these solutions, and compare the responses to changes in long-run monetary policy.

In the PI model, credit takes the form of long-term contracts between consumers and financial intermediaries. A consumer will hold money, in general, because she

[2] A similar environment is constructed in Aiyagari and Williamson (1997b), and Williamson (1997) examines a related random matching framework.

may contact the financial intermediary at a time when she does not yet know her current income. Due to this random limited participation problem, which is similar in spirit to what occurs in the liquidity effect models studied by Lucas (1990) and Fuerst (1992), money is useful for insurance reasons. During any period, a consumer may defect from her contract with the financial intermediary. If defection occurs, the consumer can not interact with financial intermediaries again, and is restricted to trading on the competitive money market in each succeeding period. The IM model is similar to the PI model, except that credit arrangements take the form of lending and borrowing on a competitive bond market. Consumers are subject to borrowing constraints, as in Aiyagari (1994). As well, each consumer can repudiate her debt in any period, which implies additional defection constraints, as in the PI model. If a consumer repudiates her debt then she can not trade on the bond market again, but can trade in the competitive money market forever.

Without defection constraints, the allocation in the PI model is efficient if the monetary authority follows a Friedman rule. A Friedman rule is not optimal in the IM economy, as no money growth rate achieves efficiency, but the Friedman rule implies that the equilibrium allocation is equivalent to the equilibrium allocation in an economy with a perfect credit system. When the defection constraints are not imposed, the PI and IM allocations, holding constant the rate of money growth, are quite different. The variability of consumption and of expected utilities across the population are markedly higher in the IM economy than in the PI economy. As the money growth rate increases, the mean level of expected utility and the variability in expected utility tend to fall, and the variability in consumption rises. Except for the effects on the variability of consumption, these effects are small however, so that the welfare effects of inflation are small. The effects of these policy experiments are very similar in the IM economy.

With defection constraints imposed, the results change dramatically. Now, an increase in the inflation rate decreases the value of defecting from credit arrangements,[3] and this effect is quantitatively large. In the PI model, the most efficient arrangement is to do away with fiat currency entirely, and to have all transactions done through the centralized credit system. Again, the allocation of consumption and wealth across the population, given the inflation rate, is quite different in the PI and IM economies, but the response to policy is similar.

In Section 2 the model economies are constructed. Section 3 (4) looks at equilibrium allocations in the PI (IM) model. Section 5 contains a discussion of pure credit economies as a benchmark, and optimal monetary policy is derived. Section 6 discusses calibration and the computational methods, and Section 7 contains a presentation of the computational results. Section 8 is a conclusion.

2. THE MODEL

There is a continuum of consumers with unit mass, each with preferences given by

$$E_0(1-\beta)\sum_{t=0}^{\infty}\beta^t u(c_t),$$

where $0<\beta<1$, c_t is consumption, and $u(\cdot)$ is the period utility function. Here, $u(\cdot)$ is strictly increasing and strictly concave with $u(0)=0$. We have $c_t \geq 0$ for all t. In each period t, a consumer receives a random endowment y_t, where $y_t \in \{y_0, y_1\}$ with $0 \leq y_0 < y_1$. Here, $\Pr[y_t = y_1] = \pi$, where $0<\pi<1$. Endowments are i.i.d. over time and across consumers, and they are private information.

The Private-Information Economy

We wish to consider two alternative allocation structures, which differ mainly according to how the credit mechanism works. The first setup is in the spirit of the

[3] Corbae and Blume (1995) also consider a limited commitment problem where inflation affects the value of defection.

allocation mechanisms in dynamic private information models (e.g. Green 1987 or Atkeson and Lucas 1992, 1995), though a novelty here is that we allow a role for money as in Aiyagari and Williamson (1997b) and Williamson (1997).

At $t = 0$, consumers form financial intermediary coalitions. In general, we will permit defection from these coalitions, the details of which will be discussed below. We assume that the intermediary is able to observe consumers' assets (here, their money balances) at the beginning of each period. Let consumers be indexed by $i \in [0, 1]$. During any period, there are two modes of interaction between consumer i and the financial intermediary, dictated by the realization of the random variable $s_t^i \in \{0, 1\}$, which is public information at the beginning of the period. Assume that s_t^i is i.i.d. over time and across consumers, with $\Pr[s_t^i = 1] = \rho$, where $0 \leq \rho \leq 1$. If $s_t^i = 1$, then the consumer receives her current endowment y_t at the beginning of the period, and then makes a report $z_t^i \in \{y_0, y_1\}$ to the financial intermediary concerning her endowment, following which the intermediary makes a transfer of consumption goods τ_t to the consumer. This transfer can be interpreted as a deposit (if the transfer is negative) or a withdrawal (if it is positive). Alternatively, if $s_t^i = 0$, then at the beginning of period t the consumer receives y_0 units of the consumption good (in all states of the world), then obtains the goods transfer from the financial intermediary, and then finally receives $y_1 - y_0$ units of the consumption good with probability π and zero units with probability $1 - \pi$. Note that in the second case the financial intermediary can not make the transfer contingent on the total endowment the consumer receives during the period.

Consumer i enters period t with M_t^i units of fiat money, where M_0^i is given. After receiving transfers from the financial intermediary, consumers can trade money for consumption goods on a competitive market, where the price of consumption goods in terms of money is p_t. Also, at this point the consumer can defect from the long-term contract with the financial intermediary. If defection occurs, the consumer is not permitted to interact with the financial intermediary (or any other intermediary) in any succeeding period, but the consumer may still trade on the money market. Thus, on defection the consumer faces the same opportunities for smoothing consumption as an agent in a Bewley economy where the only asset is money (see Bewley 1980, 1983).

The monetary authority makes a transfer of T_t units of fiat money per consumer to financial intermediaries at the beginning of the period. Letting $\bar{M}_t$ denote the per capita stock of fiat money at the end of the period, the monetary authority must meet the constraint

$$\frac{(\bar{M}_t - \bar{M}_{t-1})}{p_t} = \frac{T_t}{p_t},$$

or, defining $\bar{m}_t \equiv \frac{\bar{M}_t}{p_t}$, $\omega_t \equiv \frac{T_t}{p_t}$ and $\gamma_t \equiv \frac{p_t}{p_{t-1}}$,

$$\bar{m}_t - \frac{\bar{m}_{t-1}}{\gamma_t} = \omega_t. \tag{1}$$

That is, current per capita real balances minus per capita real balances in the previous period multiplied by the gross rate of return on money must equal the per capita money transfer to consumers in units of consumption goods.

The Incomplete Markets Economy

The setup here is identical to the PI economy except for the following. Instead of interacting with a financial intermediary, consumers trade in each period on a bond market, where the one-period real interest rate is r_t, and $q_t = \frac{1}{1+r_t}$ is the price of a bond in period t which matures in period $t+1$. As in the private information economy, a consumer can be in one of two modes, determined by the realization of s_t^i. If $s_t^i = 1$, then the consumer trades on the bond and money markets after receiving her endowment y_t. However, if $s_t^i = 0$, then the consumer receives y_0 units of the consumption good, then trades on the bond market, following which $y_1 - y_0$ units of the consumption good are received with probability π and zero units with probability

$1-\pi$. The consumer then goes to the money market. After trading on the bond market, the consumer has the option of defaulting on her debt. If defection occurs, she may not trade on the bond market in any succeeding period.

There is a government which can issue currency, the supply of which is $\bar{M}_t$. Also, the government can issue one-period real bonds on the bond market. One real bond issued in period t is a claim to one unit of the consumption good in period $t+1$. Let $\bar{B}_t$ denote the quantity of government bonds issued in period t. Currency is injected through open market operations, i.e. swaps of currency for government bonds. The government budget constraint then takes the form

$$\frac{\bar{M}_t - \bar{M}_{t-1}}{p_t} = q_t \bar{B}_t - \bar{B}_{t-1} \tag{2}$$

3. EQUILIBRIUM ALLOCATIONS IN THE PRIVATE INFORMATION ECONOMY

We suppose that financial intermediaries can trade on a bond market, facing the sequence of "efficiency prices" $\{q_t\}_{t=0}^{\infty}$, that is, there is trade in one period bonds which sell at the price $\frac{q_t}{1-q_t}$ in period t and pay off $\frac{1}{1-q_{t+1}}$ units of consumption in period $t+1$. An intermediary also treats the sequence of money prices of consumption goods $\{p_t\}_{t=0}^{\infty}$ parametrically. Defining $m_t \equiv \frac{M_t}{p_t}$ to be the real balances of a consumer, we suppose that the consumers who are members of the financial intermediary agree on an initial distribution $\psi_0(w_0, m_0)$, i.e. a distribution of expected utility entitlements w_0 and real money balances m_0 across the members of the financial intermediary. Here, the initial distribution of money balances across consumers is given, and expected utility entitlements are to be met through the design by the financial intermediary of a history-contingent transfer policy and a specification of history-contingent trading by consumers on the money market.

Along the lines of Atkeson and Lucas (1992, 1995), Aiyagari and Alvarez (1995), Aiyagari and Williamson (1997a, 1997b), or Williamson (1997), we think of the financial intermediary as solving a set of component problems to determine the optimal contractual arrangements with each of its members. Specifically, there is a separate cost minimization problem that the intermediary solves for each initial (w_0, m_0) facing $\{q_t\}_{t=0}^{\infty}$ and $\{p_t\}_{t=0}^{\infty}$. We confine attention to steady states, where $q_t = q$, $\gamma_t = \gamma$, and $\omega_t = \omega$ for all t, where q, γ, and ω are constants. Also, the distribution of expected utilities and real money balances across consumers, $\psi(w, m)$, is constant. Given that any intermediary has a positive measure of consumers, each intermediary faces the same steady state distribution $\psi(w, m)$, and we can analyze this economy as if there were only one representative financial intermediary.

The component planning problems can be specified in recursive form by treating w, the consumer's expected utility, and m, her real money balances, as state variables, and applying Green's (1987) notion of temporary incentive compatibility.. Letting $v(w, m)$ denote the cost to the intermediary of delivering a level of expected utility w at the current date to a consumer holding m units of real balances at that date, the intermediary's problem can be formulated in terms of the following Bellman equation.

$$v(w,m) = \min \left\{ \begin{array}{c} (1-q)[\rho\pi\tau_1(w,m) + \rho(1-\pi)\tau_0(w,m) + (1-\rho)\tau(w,m)] \\ +q \left[\begin{array}{c} \rho\pi v[w_{11}(w,m), m_{11}(w,m)] \\ +\rho(1-\pi)v[w_{10}(w,m), m_{10}(w,m)] \\ +(1-\rho)\pi v[w_{01}(w,m), m_{01}(w,m)] \\ +(1-\rho)(1-\pi)v[w_{00}(w,m), m_{00}(w,m)] \end{array} \right] \end{array} \right\} \tag{3}$$

subject to

$$w = (1-\beta) \left[\begin{array}{c} \rho\pi u(y_1 + m + \tau_1(w,m) - \gamma m_{11}(w,m)) \\ +\rho(1-\pi)u(y_0 + m + \tau_0(w,m) - \gamma m_{10}(w,m)) \\ +(1-\rho)\pi u(y_1 + m + \tau(w,m) - \gamma m_{01}(w,m)) \\ +(1-\rho)(1-\pi)u(y_0 + m + \tau_0(w,m) - \gamma m_{00}(w,m)) \end{array} \right] \tag{4}$$

$$+\beta\left[\begin{array}{c}\rho\pi w_{11}(w,m)+\rho(1-\pi)w_{10}(w,m)\\ +(1-\rho)\pi w_{01}(w,m)+(1-\rho)(1-\pi)w_{00}(w,m)\end{array}\right]$$

$$\begin{aligned}&(1-\beta)u(y_i+m+\tau_i(w,m)-\gamma m_{1i}(w,m))+\beta w_{1i}(w,m)\\ \geq\;&(1-\beta)u(y_i+m+\tau_j(w,m)-\gamma m_{1j}(w,m))+\beta w_{1j}(w,m),\;(i,j)=(1,0),(0,1),\end{aligned} \tag{5}$$

$$\begin{aligned}&(1-\beta)u(y_i+m+\tau(w,m)-\gamma m_{0i}(w,m))+\beta w_{0i}(w,m)\\ \geq\;&(1-\beta)u(y_i+m+\tau(w,m)-\gamma m_{0j}(w,m))+\beta w_{0j}(w,m),\;(i,j)=(1,0),(0,1),\end{aligned} \tag{6}$$

$$\begin{aligned}&(1-\beta)u(y_i+m+\tau_i(w,m)-\gamma m_{1i}(w,m))+\beta w_{1i}(w,m)\\ \geq\;&\delta(y_i+m+\tau_i(w,m)),\;i=0,1,\end{aligned} \tag{7}$$

$$\begin{aligned}&(1-\beta)u(y_i+m+\tau(w,m)-\gamma m_{0i}(w,m))+\beta w_{0i}(w,m)\\ \geq\;&\delta(y_i+m+\tau(w,m)),\;i=0,1,\end{aligned} \tag{8}$$

$$\begin{aligned}y_i+m+\tau_i(w,m)-\gamma m_{1i}(w,m)\;&\geq\;0,\;i=0,1\\ y_i+m+\tau(w,m)-\gamma m_{0i}(w,m)\;&\geq\;0,\;i=0,1\end{aligned} \tag{9}$$

$$m_{ij}(w,m)\geq 0,\;i,j=0,1. \tag{10}$$

Here, the transfer when the endowment is high [low] and $s_t^i=1$ is $\tau_1(w,m)$ [$\tau_0(w,m)$], while the transfer when $s_t^i=0$ is $\tau(w,m)$. The consumer is assigned an expected utility for the following period, which is $w_{jk}(w,m)$ when $s_t^i=j$ and the endowment is y_k. The intermediary also recommends a quantity of real balances that the consumer should hold at the beginning of the next period, $m_{jk}(w,m)$, where the subscripts have the same meaning as for the expected utility assignment. Recommended real balances then imply a recommended transaction for the consumer on the money market.

The financial intermediary minimizes the present discounted value of goods transfers to the consumer. The first constraint in the problem above, (4), is the promise-keeping constraint, which states that contingent transfers, continuation expected utilities, and recommend future money balances are consistent with the consumer receiving current expected utility w. Constraints (5)-(6) are incentive compatibility constraints, which state that it not be in the consumer's interest to misreport her endowment to the financial intermediary. The constraints (7) and (8) are defection constraints. That is, in each period, given the consumer's initial money balances, endowment, and transfer from the financial intermediary, it should not be in her interest to defect from the long-term contract with the intermediary in favor of trading in the current and subsequent periods on the money market. Here, $\delta(y)$ is the value of defecting with assets y, defined by the functional equation

$$\delta(y) = \max_{m'} \left\{(1-\beta)u(y-\gamma m') + \beta\left[\pi\delta(y_1+m') + (1-\pi)\delta(y_0+m')\right]\right\} \tag{11}$$

subject to

$$y - \gamma m' \geq 0, \tag{12}$$

$$m' \geq 0. \tag{13}$$

Here, m' is the quantity of real balances that the consumer would take into the next period if she defected from the contract with the financial intermediary, (12) is a nonnegativity constraint on consumption, and (13) is a nonnegativity constraint on real balances. Finally, in the financial intermediary's problem, (9) and (10) are nonnegativity constraints on consumption and money balances, respectively.

We require that the aggregate resource constraint be satisfied in the steady state, so that

$$\int\int \left[\begin{array}{c} \rho\pi \left[m + \tau_1(w,m) - \gamma m_{11}(w,m) \right] \\ +\rho(1-\pi) \left[m + \tau_0(w,m) - \gamma m_{10}(w,m) \right] \\ +(1-\rho)\pi \left[m + \tau(w,m) - \gamma m_{01}(w,m) \right] \\ +(1-\rho)(1-\pi) \left[m + \tau(w,m) - \gamma m_{00}(w,m) \right] \end{array} \right] d\psi(w,m) = 0, \tag{14}$$

i.e. per capita transfers, through the financial intermediary and through money market exchange, equal zero in the steady state.

Note that money plays two roles here. First, money balances allow consumers to self-insure against the event that they cannot receive a contingent transfer from the financial intermediary. This can be interpreted as a transactions role for money, as in Bewley (1980, 1983). Second, current money balances communicate to the intermediary the endowment shock of the consumer in the previous period, so that money acts as a record-keeping device. The record-keeping role of money is studied in Townsend (1987, 1989), and also in Kocherlakota and Wallace (1997).

Efficient allocations should have the property that $q \leq \gamma$, as otherwise (14) would not hold. Money will thus be dominated in rate of return by bonds. In the case where $q < \gamma$ (i.e. money is strictly dominated in rate of return), we will have $m_{ij}(m,w) = 0$ for $i = 1$ and for $(i,j) = (0,0)$. To see this, suppose that $m_{ij}(m,w) > 0$ for $i = 1$ and for some j. Then, when $s_t^i = 1$ and $y_t = y_j$, the consumer trades off claims to current consumption for claims to future consumption by holding money balances. However, this can be done more efficiently by the financial intermediary, which can reduce the consumer's transfers today and increase future transfers, while facing a higher interest rate. The intermediary can thus meet all the constraints in the above optimization problem while reducing the value of the objective function, so it cannot be optimal to have $m_{ij}(m,w) > 0$ for $i = 1$. A similar argument holds for $(i,j) = (0,0)$, given that incentive compatibility requires that $m_{01}(m,w) \geq m_{00}(m,w)$.

While the problem (3) subject to (4)-(10) may appear formidable, it is possible to simplify it considerably. First, suppose that we perform a change of variables by letting $\tau_i^*(w) = \tau_i(m, w) - m$, for $i = 0, 1$, with $w_{ij}^*(w) = w_{ij}(w, m)$ and $m_{ij}^*(w) = m_{ij}(w, m)$ for $i, j = 0, 1$. Now, we can write the cost function as

$$v(w, m) = -(1 - q)m + \theta(w),$$

and the choice variables $\tau_i^*(w)$, $i = 0, 1$, $w_{ij}^*(w)$, and $m_{ij}^*(w)$, $i, j = 0, 1$, are independent of m. Thus, the current consumption allocation, future money balances, and future expected utility entitlement of the consumer are determined only by the current expected utility entitlement and the current endowment. This simplification of the problem is very important for computational purposes, as the problem collapses to a one-state-variable problem instead of a two-state-variable problem.

4. EQUILIBRIUM ALLOCATIONS WITH INCOMPLETE MARKETS

Here, as in Section 3, we confine attention to steady state allocations. The state variable for a consumer's optimization problem in the IM model is total assets, a, where $a = m + b$. Here, b denotes the quantity of bonds held at the beginning of the period, and m denotes real balances. Let $\phi(a)$ denote the value function associated with the consumer's problem, which is defined by the Bellman equation

$$\begin{aligned} \phi(a) \;=\; & \rho\pi \max_{m_{11}, b_{11}} \left[(1-\beta)u(y_1 + a - qb_{11} - \gamma m_{11}) + \beta\phi(b_{11} + m_{11})\right] \\ & +\rho(1-\pi) \max_{m_{10}, b_{10}} \left[(1-\beta)u(y_0 + a - qb_{10} - \gamma m_{10}) + \beta\phi(b_{10} + m_{10})\right] \\ & +(1-\rho)\max_{b_0} \left\{ \begin{array}{c} \pi \max_{m_{01}} \left[\begin{array}{c} (1-\beta)u(y_1 + a - qb_0 - \gamma m_{01}) \\ +\beta\phi(b_0 + m_{01}) \end{array} \right] \\ +(1-\pi)\max_{m_{00}} \left[\begin{array}{c} (1-\beta)u(y_0 + a - qb_0 - \gamma m_{00}) \\ +\beta\phi(b_0 + m_{00}) \end{array} \right] \end{array} \right\}. \end{aligned}$$

subject to

$$y_k + a - qb_{jk} - \gamma m_{jk} \geq 0, \text{ for } j = 1, \; k = 0, 1, \tag{15}$$

$$y_k + a - qb_0 - \gamma m_{jk} \geq 0, \text{ for } j = 0, \; k = 0, 1, \tag{16}$$

$$m_{jk} \geq 0, \text{ for } j, k = 0, 1, \tag{17}$$

$$b_{jk} + m_{jk} \geq -\frac{y_0}{(1-q)}, \text{ for } j = 1 \text{ and } k = 0, 1, \tag{18}$$

$$b_0 + m_{0k} \geq -\frac{y_0}{(1-q)}, \text{ for } k = 0, 1. \tag{19}$$

$$(1-\beta)u(y_i + a - qb_{1i} - \gamma m_{1i}) + \beta\phi(b_{1i} + m_{1i}) \geq \chi(y_i + a - qb_{1i}), \; i = 0, 1, \tag{20}$$

$$(1-\beta)u(y_i + a - qb_0 - \gamma m_{0i}) + \beta\phi(b_0 + m_{0i}) \geq \chi(y_i + a - qb_0), \; i = 0, 1. \tag{21}$$

Here, b_{jk} is the value of the bonds the consumer chooses to have in period $t+1$ when $s_t^i = j$ and $y_t = y_k$, for $j = 1$ and $k = 0, 1$, while b_0 is the value of bonds the consumer chooses to have in period $t+1$ when $s_t^i = 0$. The steady state one-period interest rate on bonds is $\frac{1}{q} - 1$. Inequalities (15) and (16) are nonnegativity constraints on consumption, (17) are nonnegativity constraints on money balances, and (18) and (19) are borrowing constraints. The latter state that a consumer can borrow at most the present discounted value of the low endowment received for the indefinite future. That is, the borrowing constraints are implied by present value budget balance (see Aiyagari 1994). Constraints (20) and (21) state that the consumer can not borrow an amount that would be in her interest to repudiate. The value of repudiating debt b when income is y and beginning-of-period assets are a is given by $\chi(y + a - qb)$. Note again, that if the consumer repudiates her debt, then she is excluded from the bond market at all future dates and can only trade on the money market. The function $\chi(\cdot)$ is determined by the functional equation

$$\chi(x) = \max_{m'} \{(1-\beta)u(x - \gamma m') + \beta\left[\pi\chi(y_1 + m') + (1-\pi)\chi(y_0 + m')\right]\}$$

subject to

$$x - \gamma m' \geq 0, \tag{22}$$

$$m' \geq 0, \tag{23}$$

where m' is the quantity of real money balances the consumer holds in the succeeding period if repudiation occurs, (22) is a nonnegativity constraint on consumption, and (23) is a nonnegativity constraint on real cash balances.

Clearly, we must have $q \leq \gamma$ in a steady state, so that rate-of-return dominance is a feature of this environment as for the PI model. Also, similar to the PI model, we will have $m_{jk} = 0$ for $j = 1$, and for $(j, k) = (0, 0)$. Consumers will end the period with positive money balances only when $s_t^i = 0$ and they receive a high endowment. Here, when $s_t^i = 0$, the consumer cannot use the credit market to smooth consumption in the face of idiosyncratic income shocks, and must use money instead.

In the steady state, the government budget constraint (2) can be written

$$\bar{m}(1 - \frac{1}{\gamma}) = \bar{B}(q - 1). \tag{24}$$

Let $\sigma(a)$ denote the steady state distribution of assets. Then, market clearing in the steady state gives

$$\int \gamma \left[\rho\pi m_{11}(a) + \rho(1-\pi)m_{10}(a) + (1-\rho)\pi m_{01}(a) + (1-\rho)(1-\pi)m_{00}(a)\right] d\sigma(a) = \bar{m}$$

(the money market clears), and

$$\int \left[\rho\pi b_{11}(a) + \rho(1-\pi)b_{10}(a) + (1-\rho)b_0(a)\right] d\sigma(a) = \bar{B}$$

(the bond market clears).

5. PURE CREDIT ECONOMIES

In this section we wish to consider special cases of the above PI and IM models where $\rho = 1$, so that money is not valued in equilibrium in either economy. We can think of these special cases as "pure credit" economies. This will serve as a convenient benchmark, particularly since the equilibrium allocation for the PI model when $\rho = 1$ is efficient, as we will show.

Private Information

The financial intermediary's problem when $\rho = 1$ reduces to a problem which is similar to the one considered by Green (1987), except here we have a nonnegativity constraint on consumption, the interest rate is endogenous, and there are defection constraints. The problem is

$$z(w) = \min \left\{ \begin{array}{c} (1-q)[\pi\tau_1(w) + (1-\pi)\tau_0(w)] \\ +q\left[\pi z(w_1(w)) + (1-\pi)z(w_0(w))\right] \end{array} \right\} \tag{25}$$

subject to

$$\begin{aligned} w &= (1-\beta)\left[\pi u(y_1 + \tau_1(w)) + (1-\pi)u(y_0 + \tau_0(w)) \right] \\ &\quad +\beta\left[\pi w_1(w) + (1-\pi)w_0(w)\right] \end{aligned} \tag{26}$$

$$\begin{aligned} &(1-\beta)u(y_i + \tau_i(w)) + \beta w_i(w) \\ \geq\ &(1-\beta)u(y_i + \tau_j(w)) + \beta w_j(w),\ (i,j) = (1,0),(0,1), \end{aligned} \tag{27}$$

$$w_i(w) \geq \pi u(y_1) + (1-\pi)u(y_0),\ i = 0,1, \tag{28}$$

$$y_i + \tau_i(w) \geq 0,\ \text{for } i = 0,1. \tag{29}$$

Here, $z(w)$ is the cost function, $\tau_1(w)$ $[\tau_0(w)]$ is the transfer in the high (low) endowment state, and $w_1(w)$ $[w_0(w)]$ is the expected utility entitlement in the following period when the current endowment is high (low). Equation (26) is the promise-keeping constraint, (27) are the incentive constraints, (28) are the defection constraints (i.e. the expected utility entitlement for next period cannot be less than the expected utility in autarky), and (29) are nonnegativity constraints on consumption.

In the steady state, transfers must sum to zero across consumers. Thus, if $\psi(w)$ denotes the steady state distribution of expected utility entitlements, we must have

$$\int [\pi\tau_1(w) + (1-\pi)\tau_0(w)]d\psi(w) = 0. \tag{30}$$

As in Atkeson and Lucas (1995), Aiyagari and Alvarez (1995), and Aiyagari and Williamson (1997a), the steady state allocation which is the solution to (25)-(30) is efficient. We will next show that there are conditions under which any steady state allocation with $\rho \neq 1$ is equivalent to the above pure credit allocation.

Proposition 1: With no defection constraints, i.e. deleting (7), (8), and (28), when $\rho \neq 1$, $\gamma = q$ is optimal.

Proof. First, conjecture that when $\gamma = q$, a solution to (3) subject to (4)-(10) (absent (7) and (8)) is $v(w,m) = z(w) - (1-q)m$, $\tau_1(w,m) = \tau_1(w) - m$, $\tau_0(w,m) = \tau_0(w) - m$, $\tau(w,m) = \tau_0(w) - m$, $m_{01}(w,m) = \frac{\tau_0(w)-\tau_1(w)}{q}$, $m_{ij}(w,m) = 0$ for $(i,j) = (1,1), (1,0), (0,0)$, $w_{ij}(w,m) = w_i(w)$ for $i,j = 0,1$, where $z(w)$, $\tau_1(w)$, $\tau_0(w)$, $w_1(w)$, $w_0(w)$ is the solution to the pure credit problem, (25) subject to (26)-(29) (absent (28)). We have already shown that $m_{ij}(w,m) = 0$ for $(i,j) = (1,1), (1,0), (0,0)$. Now, substituting in the Bellman equation (3), we obtain

$$z(w) - (1-q)m = \min \left\{ \begin{array}{c} (1-q)[\rho\pi\tau_1(w) + \rho(1-\pi)\tau_0(w) + (1-\rho)\tau_0(w) - m] \\ +q \left[\begin{array}{c} \pi z[w_1(w)] + (1-\pi)z[w_0(w)] \\ -(1-\rho)\pi\frac{(1-q)[\tau_0(w)-\tau_1(w)]}{q} \end{array} \right] \end{array} \right\}.$$

Simplifying, we get

$$z(w) = \min \left\{ \begin{array}{c} (1-q)[\pi\tau_1(w) + (1-\pi)\tau_0(w)] \\ +q\left[\pi z[w_1(w)] + (1-\pi)z[w_0(w)]\right] \end{array} \right\}.$$

Similarly, substituting in the constraints (4)-(9), we obtain the constraints (26), (27), and (29), and constraint (10) is also satisfied. Thus, the solution to the financial intermediary's problem is the same for any ρ when $\gamma = q$. Now, it remains to be shown that in the steady state the aggregate resource constraint, (14), is satisfied. Substituting in (14), and given (30), it is straightforward to show that this is the case.□

Thus, a Friedman rule is optimal if there are no defection constraints. That is, at the optimum the real rates of return on money and on bonds are equivalent, and the nominal interest rate is zero. However, it will typically be the case that, in a pure credit equilibrium, $q > \beta$ (see Atkeson and Lucas 1994 and Aiyagari and Williamson 1997a), i.e. the real interest rate is less than the rate of time preference. In Friedman (1969), the real interest rate is equal to the rate of time preference at the optimum.

Incomplete Markets

Here, when $\rho = 1$ the Bellman equation associated with the consumer's problem in the steady state is

$$\begin{aligned} \eta(a) &= \pi \max_{a_1} \left[(1-\beta)u(y_1 + a - qa_1) + \beta\eta(a_1)\right] \\ &\quad +(1-\pi) \max_{a_0} \left[(1-\beta)u(y_0 + a - qa_0) + \beta\eta(a_0)\right] \end{aligned} \tag{31}$$

subject to

$$y_i + a - qa_i \geq 0, \text{ for } i = 0, 1, \tag{32}$$

$$a_i \geq -\frac{y_0}{(1-q)}, \text{ for } i = 0, 1, \tag{33}$$

$$\eta(a_i) \geq \pi u(y_1) + (1-\pi)u(y_0), \; i = 0, 1, \tag{34}$$

where a now denotes the quantity of bonds the consumer enters the period with, and a_i denotes bonds acquired when the endowment is y_i, for $i = 0, 1$. In the above Bellman equation, $\eta(a)$ is the value function, (32) is a nonnegativity constraint on consumption, (33) is the borrowing constraint, and (34) is the defection constraint.

The market-clearing condition is

$$\int \left[\pi a_1(a) + (1-\pi)a_0(a)\right] d\kappa(a) = 0.$$

That is, the bond market clears, given the steady state distribution of assets $\kappa(a)$.

It is straightforward to show that, when $\gamma = q$, and $\rho \neq 1$, there exists a steady state equilibrium allocation for the incomplete markets economy that replicates the steady state equilibrium allocation when $\rho = 1$. That is, given that rates of return are identical on money and bonds when $\gamma = q$, consumers will make the same portfolio choices when $\rho \neq 1$ as in the pure credit economy with $\rho = 1$. This then implies that we will obtain $\kappa(a) = \sigma(a)$ (the steady state asset distributions are identical in the two economies), and the money balances held by consumers will be backed by government loans to the same consumers, which net out on the consolidated balance sheet of the private sector.

Though there is a similarity here to Proposition 1 for the private-information constrained economy, in terms of the equivalence between money/credit economies and pure credit economies when $\gamma = q$, we cannot say that $\gamma = q$ is optimal in any sense, since the pure credit economy with incomplete markets is in general not efficient. That is, the efficient allocation for this environment is given by the solution to (25)-(30) for which, as in Atkeson and Lucas (1992), intertemporal marginal rates of substitution are not equated, in general, across agents. However, intertemporal marginal rates of substitution are equated across agents in the pure credit competitive equilibrium for the incomplete markets economy, so that the pure credit PI and IM allocations differ. Thus, the IM allocation is in general not efficient, even if $\gamma = q$.

6. CALIBRATION AND COMPUTATION

We use the PI economy as a benchmark, setting parameters so that the steady state allocation matches observed features of the U.S. economy. We interpret a period as one quarter, and set y_0, y_1, and π so as to match the variability in quarterly household income. Using PSID data, Aiyagari (1994) argues that a first-order autoregression closely matches the time series properties of annual earnings, with a range of .23 to .53 for the first-order serial correlation coefficient, and a coefficient of variation in unconditional earnings of 20 to 40 percent. Since it is not tractable to introduce serial correlation in endowments in the private information model, we must do the best we can to fit an i.i.d. endowment shock in the model to the data. This is not too problematic, as the estimated serial correlation in annual data is low, and serial correlation for quarterly data would then be even lower. If we take the coefficient of variation to be 30 percent for annual data, then if quarterly income is i.i.d., the coefficient of variation for quarterly data would be 60 percent. Thus, we set $\pi = .5$, $y_0 = 1 - \epsilon$, and $y_1 = 1 + \epsilon$, with $\epsilon = .6$. The utility function we use is $u(c) = 1 - e^{-\alpha c}$, with $\alpha = 1$, which implies a coefficient of relative risk aversion of unity at the mean endowment. The constant relative risk aversion utility function is convenient as it is bounded. The remaining parameters, ρ, β, and γ, were set so as to produce a steady state equilibrium allocation which would match observed average real interest rates, inflation rates, and the observed fraction of transactions for which currency was used. From the real business cycle literature (Prescott 1986), the real interest rate is taken to be 1% per quarter, so in an efficient steady state we want $q = .99$. A survey of households by the Federal Reserve (Avery, Elliehausen, Kennickell, and Spindt 1987), conducted in 1984, finds that 24% of the current value of transactions is carried out in currency. In the model, the steady state quantity of currency transactions is

$$\frac{1}{2}\int\int\left\{\left[1-(1-\rho)\pi\right](m+\omega)+(1-\rho)\pi\mid m+\omega+-\gamma m_{01}(m,w)\mid\right\}d\psi(m,w),$$

and the steady state quantity of credit transactions is

$$\frac{1}{2}\int\int\left[\rho\pi\mid\tau_1(m,w)\mid+\rho(1-\pi)\mid\tau_0(m,w)\mid+(1-\rho)\mid\tau(m,w)\mid\right]d\psi(m,w).$$

When the Federal Reserve survey was done, the inflation rate was approximately 1% per quarter, so we set $\gamma = 1.01$ for calibration purposes.

Solutions were computed for the PI economy as follows. First, grids were chosen for the two state variables, w and m. With no defection constraints, the lower bound on expected utility, w, is $\pi u(y_1 - y_0) + (1-\pi)u(0)$, which is the minimum incentive compatible level of expected utility that can be imposed on a consumer, and the lower bound on m is zero. Since choice variables in the financial intermediary's problem are independent of m, it is only necessary to solve the problem at each point along the w grid, and for a single value for m, say $m = 0$, and then use this solution to determine the solution for all points on the grid. We start with an initial guess for $q \in (\beta, 1)$, and make an initial guess for the function $\theta(w)$. Then, value iteration is used to arrive at the solution for $\theta(w)$ given q. At each iteration, $\theta(w)$ is updated by fitting a third-order Chebychev polynomial (plus an additional term, $\frac{1}{1-w}$, which performed well in fitting the true cost function), to the values computed for the cost function at points on the grid on the previous iteration. When convergence is achieved given q, then the decision rules are interpolated across a finer grid, and a matrix of Markov transition probabilities for the state w is constructed as an approximation using a lottery over the two closest grid points. A limiting distribution over w is computed, the analogue of the left-hand side of (14) is evaluated, and q is updated according to a bisection method. Then value-iteration is performed again, etc. For the incomplete markets economy, the computational procedure is similar, except that the state variable is now total assets, a, and we iterate on the value function rather than a cost function.

To match the observed real interest rate and the evidence from the Federal Reserve survey on household transactions, we set $\beta = .99$ and $\rho = .81$, in computing the allocation for the PI economy. This implies that $q = .99$ (slightly greater than β, but the difference is on the order of 10^{-5}) and that currency accounts for 24% of the value of transactions.

After calibrating the PI model to U.S. data, we compute solutions for the PI and IM economies, varying γ, the gross inflation rate. We can thus compare allocations and the response to policy in the two models. One difficulty here is in evaluating the effects of policy on welfare. In solving for a steady state in either model, we determine the steady state distribution of expected utilities across the population. One approach to evaluating the welfare effects of a change in policy would be to use a Hicksian welfare measure, whereby we would convert welfare changes into consumption-equivalents, and then simply add these across the population. However, in this context this may give nonsensical answers. To take an extreme example, consider the setup in Atkeson and Lucas (1992) which is a model where there is private information about preference shocks. Atkeson and Lucas's model has the property that, with efficient allocation, a vanishing fraction of the population consumes all output in the limit. This is true, for example, with log utility. However, one incentive compatible (but inefficient) allocation is for agents to consume equal shares of total output each period. According to the Hicksian welfare measure, this steady state allocation dominates the efficient allocation.

For our environment, we know that the pure credit allocation for the PI model is efficient. Our approach here is to use a measure of the distance of the limiting distribution of expected utilities from the efficient distribution as a measure of the welfare cost of inflation. Since for the welfare measure we use it is necessary that limiting probabilities be bounded away from zero, we use a kernel estimation technique (Silverman 1986). That is, we estimate the density function $f(x)$ associated with the limiting distribution of expected utilities by

$$f(x) = \frac{1}{h}\sum_{i=1}^{n}\kappa_i K(\frac{x-w_i}{h}),$$

where h is the "window width," κ_i is the limiting probability associated with expected utility level w_i, n is the number of grid points for expected utilities, and $K(\cdot)$ is a probability density function. For $K(\cdot)$, the standard normal density function provided a good fit of the estimated density function to the limiting probabilities. Choosing a grid for x, we let $f_j = f(x_j)$ for $j = 1, 2, ..., \ell$, and then compute the "welfare loss,"

$Z(g, f)$ associated with the limiting distribution $f(x)$, by

$$Z(g, f) = \sum_{j=1}^{\ell} hg_j(\log g_j - \log f_j),$$

where $g(x)$ denotes the limiting distribution with pure credit ($\rho = 1$). This distance measure is adapted from an approach in information theory used by Kullback (1959).

7. COMPUTATIONAL RESULTS

We first consider results for pure credit allocations in Tables 1 and 2, and in Figures 1-3. Table 1 shows, for the PI economy, the mean level of expected utility, standard deviation of expected utility, unconditional standard deviation of consumption, and the cost of inflation (the distance measure discussed in the previous section), for annual inflation rates running from the Friedman rule rate (-3.94% per annum) to an (incipient) inflation rate sufficiently high that money is not held. The entries in Table 2 are similar, but for the IM economy.

Table 1: No Defection Constraints, Private Information

Annual Inflation Rate	Mean E.U.	S.D. of E.U.	S.D. of Cons.	Welfare Cost
-3.94%	.6270	.0519	.1508	0
10%	.6270	.0515	.1549	0.07
100%	.6262	.0500	.1908	0.04
1500%	.6173	.0452	.2964	2.58
>non-mon. threshold	.6156	.0450	.3066	5.47

First, comparing the first row of Table 1 with the first row of Table 2, note that mean expected utility is lower in the IM economy than in the PI economy, and that the dispersion in expected utilities is higher in the IM economy. Figure 1 shows plots of the distributions of expected utilities in each case. We would expect the average level of expected utility to be lower in the IM economy, as risk sharing is less efficient.

Dispersion in expected utilities is higher because, with less risk sharing, there is also less sluggishness in expected utilities. That is, the IM allocation is incentive compatible, but the incentive constraints do not bind, so that expected utilities move from period to period more than is necessary to meet the incentive constraints. This will tend to result in greater dispersion in expected utilities in the steady state. The standard deviation of consumption is higher in the IM economy for two reasons. First, since risk sharing is not as efficient in the IM economy, consumption will tend to be more variable conditional on the level of expected utility. Second, the variability of expected utilities is higher in the IM economy, and consumption increases with expected utility, so that the unconditional variability in consumption in the IM economy will tend to be higher as a result. Figures 2 and 3 show consumption as a function of expected utility in the steady state for the PI and IM economies, respectively. Here, $c0$ ($c1$) denotes consumption in the low (high) income state. Note that risk-sharing is quite good in both cases, but that variability in consumption, conditional on expected utility, is higher in the IM economy than for the PI economy.

Table 2: No Defection Constraints, Incomplete Markets

Annual Inflation Rate	Mean E.U.	S.D. of E.U.	S.D. of Cons.	Welfare Cost
-3.94%	.6167	.0825	.2218	43.56
10%	.6165	.0825	.2477	35.49
100%	.6156	.0825	.1984	34.93
1500%	.6059	.0824	.3284	34.61
>non-mon. threshold	.6042	.0824	.3567	34.58

Though Tables 1 and 2 show a substantial difference between the PI and IM economies in terms of the distributions of expected utilities and consumptions, the response to inflation is quite similar in the two economies. Table 1 shows a small response of the distribution of expected utilities to inflation. Note that, in Figure

4, there is only a small difference in the distributions of expected utilities for the Friedman rule rate of inflation and for the case where money is not held. That is, the number 5.47 in the last row and last column of Table 1, denoting the distance between the two distributions in Figure 4, is a small number.[4] The critical rate of inflation at which money is just driven out of circulation is approximately 2,000 per cent per annum. Table 2 and Figure 5 show similar results for the IM economy. The effects of inflation on the distribution of expected utilities is small. Note, in Table 2, that the costs of inflation are relative to the Friedman rule inflation rate for the PI economy, since this yields the efficient steady state distribution of expected utilities. From Table 2, the IM economy which is most efficient, i.e. yields the steady state distribution of expected utilities closest to efficiency, is the one without currency. However, the costs of inflation do not change much as the inflation rate changes, and the IM economy without currency is still far from being efficient.

Though the effects of inflation on the distribution of expected utilities are small in the PI and IM economies, with mean expected utility and the standard deviation of expected utility falling as inflation rises, the effects on the distribution of consumption are large. With higher inflation, consumers decrease holdings of cash balances, so that they are less capable of insuring against income shocks in the state where they are subject to the limited participation problem. Figures 6 and 7, which should be compared to Figures 2 and 3, respectively, show consumption in the high-income state ($c1$) and low-income state ($c0$) for the PI and IM economies, respectively, given an inflation rate of 10% per annum. In Table 1, the standard deviation of consumption increases by a factor of about 2 as the inflation rate increases from the Friedman rule rate to an incipient rate sufficiently high to drive out currency. In Table 2, the effect is of a similar magnitude, but note that the standard deviation of consumption is

[4]The Hicksian welfare loss for the average consumer of having to do without currency, relative to the Friedman rule, for the PI economy, is approximately 3% of average consumption.

nonmonotonic in the inflation rate in the IM economy.

The results where defection constraints are imposed are reported in Tables 3 and 4, and in Figures 8-11. Here, a Friedman rule is no longer optimal, and inflation will have two effects. First, as in the case with no defection constraints, real balances decrease with inflation, consumers can not smooth consumption as effectively, and welfare will tend to decrease. Second, as inflation increases, the value of defection, either from intermediary contracts (PI economy) or bond market trading (IM economy), decreases, as there is a greater tax on money holdings if defection occurs. This second effect will tend to increase welfare. That is, in the PI economy, the incentives open to the financial intermediary will be better with higher inflation, and in the IM economy higher inflation implies that the lower bound on borrowing decreases, so the consumer is better able to smooth consumption. Table 3 shows that the second effect dominates the first in the PI economy. That is, in the last column of Table 3, the welfare cost of inflation falls as the inflation rate rises, and the distribution of expected utilities which is closest to efficiency is the one where there is no currency in circulation. Note that the two distributions in Figure 8 are much further apart than the two in Figure 10.

Table 3: Defection Constraints, Private Information

Annual Inflation Rate	Mn. E.U.	SD(EU)	SD(Cons)	Int. Rate	Welfare Cost
-1.37%	.6308	.0035	.0783	1.37%	478.56
10%	.6313	.0056	.0635	3.35%	373.65
100%	.6298	.0114	.0952	3.72%	130.54
1500%	.6209	.0136	.2609	3.89%	60.87
>non-mon. threshold	.6166	.0243	.2813	3.82%	12.58

Further, in the second and third columns of Table 3, note that mean expected utility falls while the standard deviation of expected utility rises as inflation increases (since inflation relaxes the defection constraint). In column 4 of Table 3, the standard deviation of consumption increases with inflation, though not monotonically. The interest rate increases (see column 5 of Table 4) as the effect of inflation is to permit lower levels of consumption through the relaxation of the defection constraint (note that consumption increases monotonically with expected utility). For the bond market to clear, the interest rate must then be higher, as this will encourage agents to postpone consumption, and they will then tend to consume more in the steady state.

Table 4: Defection Constraints, Incomplete Markets

Annual Inflation Rate	Mean E.U.	S.D. of E.U.	S.D. of Cons.	Int. Rate	Welfare Cost
-3.16%	.6379	.0086	.2477	3.27%	305.26
10%	.6299	.0131	.1130	3.64%	127.86
100%	.6257	.0251	.0291	3.93%	39.19
1500%	.6102	.0341	.2225	3.93%	56.59
>non-mon. threshold	.6093	.0392	.2873	3.89%	66.87

Most of the effects of inflation in the IM economy (in Table 4) are similar to those for the PI economy, with two exceptions. First, it is not optimal to drive currency out of the IM economy. The closest expected utility distribution to the efficient one is achieved with an inflation rate of 100%. Also, note in column 5 of Table 4 that interest rates are lower as compared to Table 3 for low rates of inflation. That is, defection constraints have a smaller "precautionary savings" effect in the IM economy than in the PI economy at a low rate of inflation. Figures 9 and 11 are the analogues of Figures 8 and 10.

Clearly, it makes a difference for the distribution of consumption and wealth whether we study this environment using the theory of dynamic contracts under private information, or using an incomplete markets competitive equilibrium approach.

However, the choice appears to matter less for the policy implications in this context. That is, at the margin, inflation alters the allocation of consumption and wealth in much the same way in either case. This is not to say that the two models will yield the same policy implications under all circumstances. One could imagine policy interventions, for example government lending programs or unemployment insurance, where the choice of a complete contracts approach (the PI model) versus an incomplete markets approach should make a big difference for the conclusions.

8. CONCLUSION

In the environment considered here, there is a continuum of infinite-lived agents each of whom receive a stream of unobservable random endowments. We consider two arrangements for sharing risk; in the first (the PI economy) there are complete long-term contracts between consumers and financial intermediaries, and in the second (the IM economy) there is a competitive bond market. In both models there is a role for money arising from random limited participation in credit markets. The two models produce steady state distributions of expected utilities and consumption across agents that are very different. In general, the variability in expected utilities and in consumption is higher in the IM economy. However, the effects of inflation in the two models are similar. In the absence of defection constraints, higher inflation has small effects on welfare, with the distribution of expected utilities changing only slightly, but the variability in consumption tends to increase by a large amount. With defection constraints, inflation is good for incentives in that it decreases the value of defecting in the PI economy. In the IM economy, inflation relaxes borrowing constraints. These effects are large, and they imply that eliminating currency transactions is optimal in the PI economy, and a high rate of inflation is optimal in the IM economy.

How seriously should we take these results concerning the optimality of high inflation, or eliminating government-supplied currency altogether? Clearly, our model leaves out some of the important costs of inflation, such as distortions introduced in labor supply decisions, and effects on growth, capital accumulation, and financial intermediation, some of which are captured in work by Cooley and Hansen (1989), Dotsey and Ireland (1996), and Lacker and Schreft (1996). However, there is something important captured in our model that is omitted in the literature on the costs of inflation. That is, government-supplied currency permits transactions and wealth to be hidden, these hidden transactions and wealth may be socially costly, and taxing them through inflation or outright elimination of government-supplied currency might be an efficient thing to do. There may be an important role for physical media of exchange, but such a role might be efficiently played by non-anonymous privately-supplied smart cards.

We hope that this paper provides some help in evaluating when complete contracting private information approaches are useful relative to incomplete markets approaches. The incomplete markets approach has some advantage in general in terms of computational ease (though the advantage is not very big in this context), but otherwise the private information model seems preferable, unless one does not believe the assumptions on asset observability required to make the approach tractable. Of additional interest is the novel approach to modeling money here, which is tractable in both models, and potentially applicable to a wide variety of problems.

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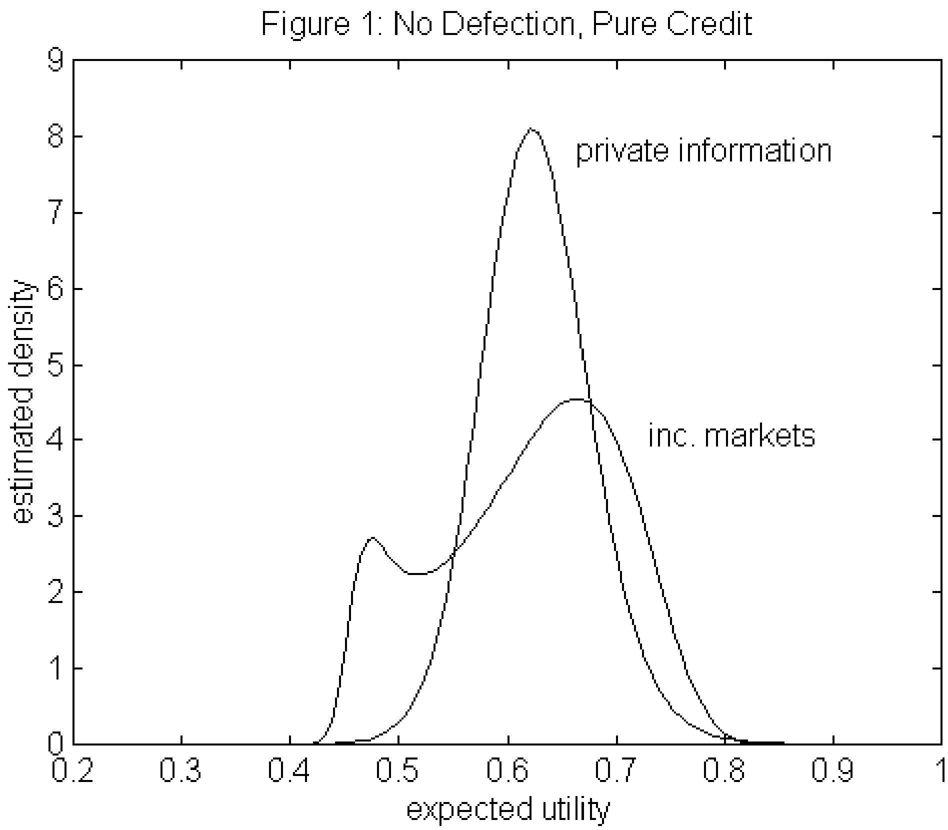
Figure 1: No Defection, Pure Credit
estimated density
expected utility
private information
inc. markets
0
1
2
3
4
5
6
7
8
9
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig. 1.

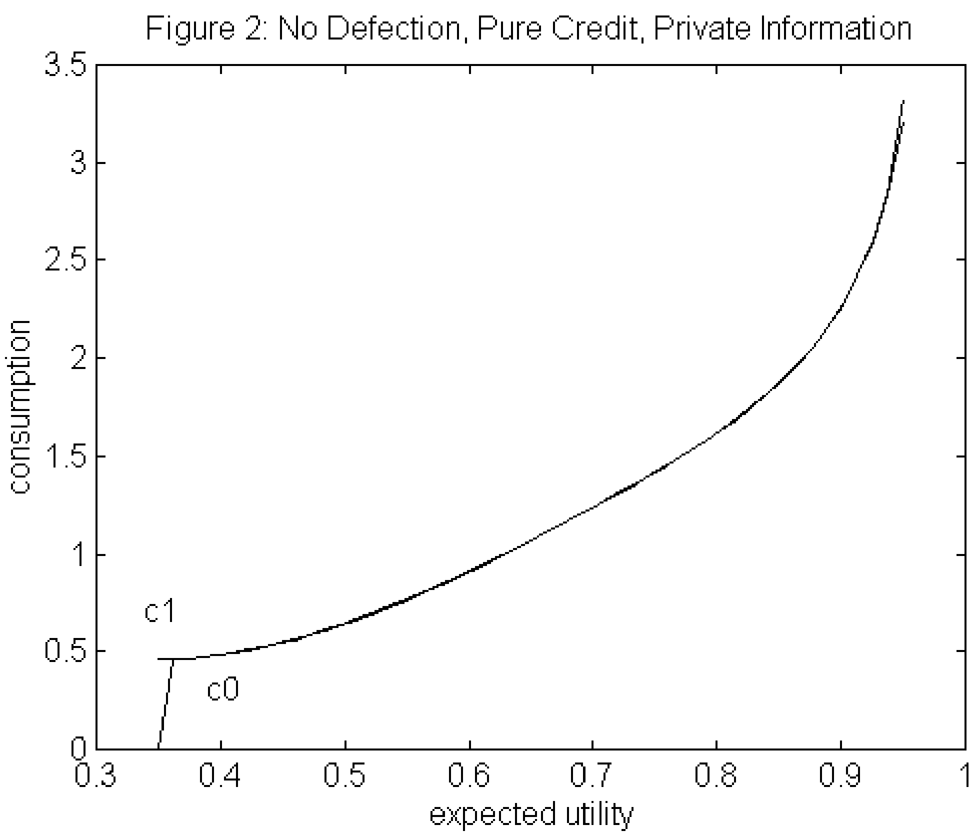
Figure 2: No Defection, Pure Credit, Private Information
consumption
expected utility
c1
c0
3.5
3
2.5
2
1.5
1
0.5
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig. 2.

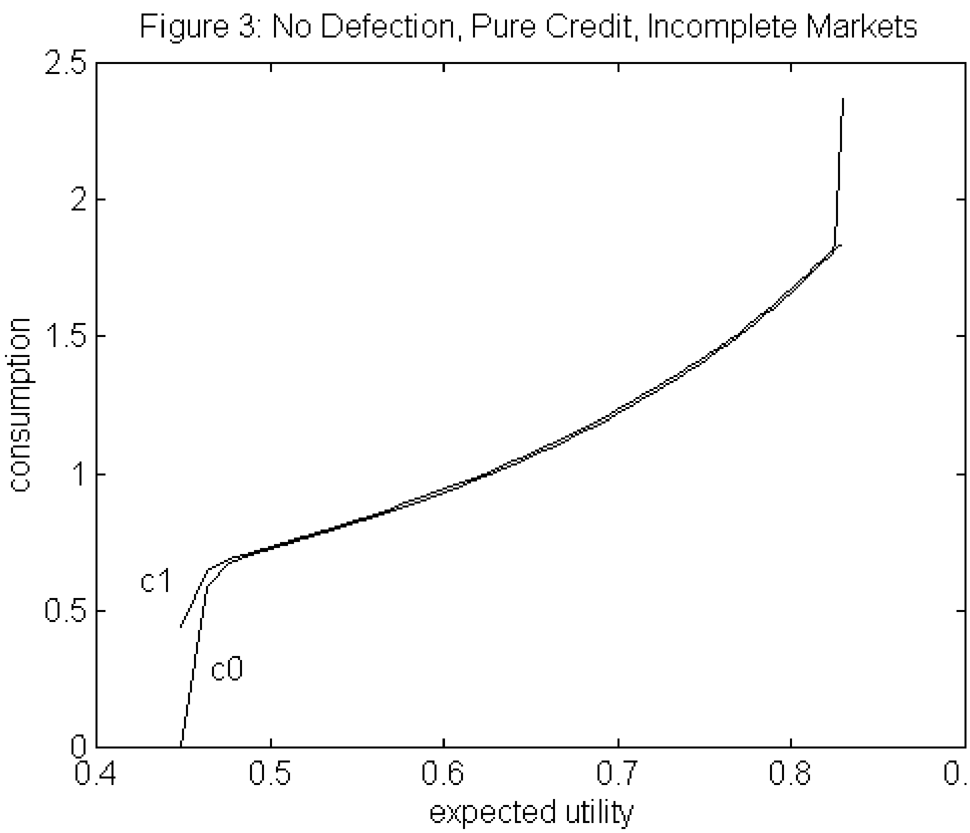
Figure 3: No Defection, Pure Credit, Incomplete Markets
2.5
2
1.5
1
0.5
0
consumption
0.4
0.5
0.6
0.7
0.8
0.9
expected utility
c1
c0

FIG. 3.

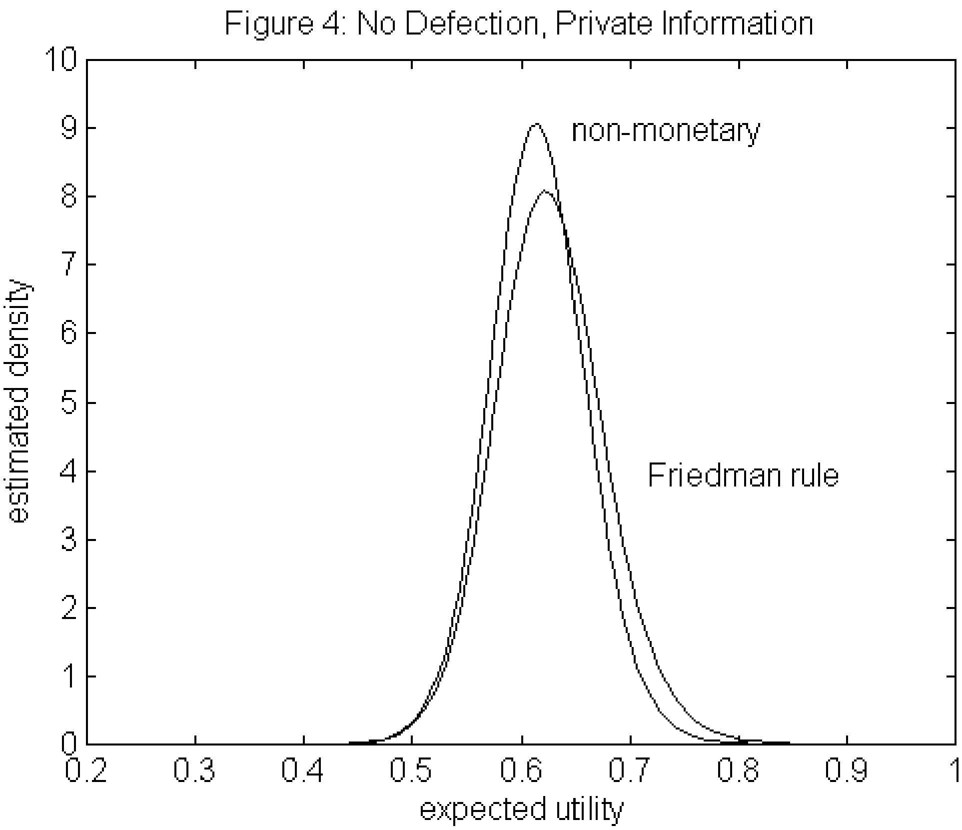
Figure 4: No Defection, Private Information
non-monetary
Friedman rule
estimated density
expected utility
10
9
8
7
6
5
4
3
2
1
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig. 4.

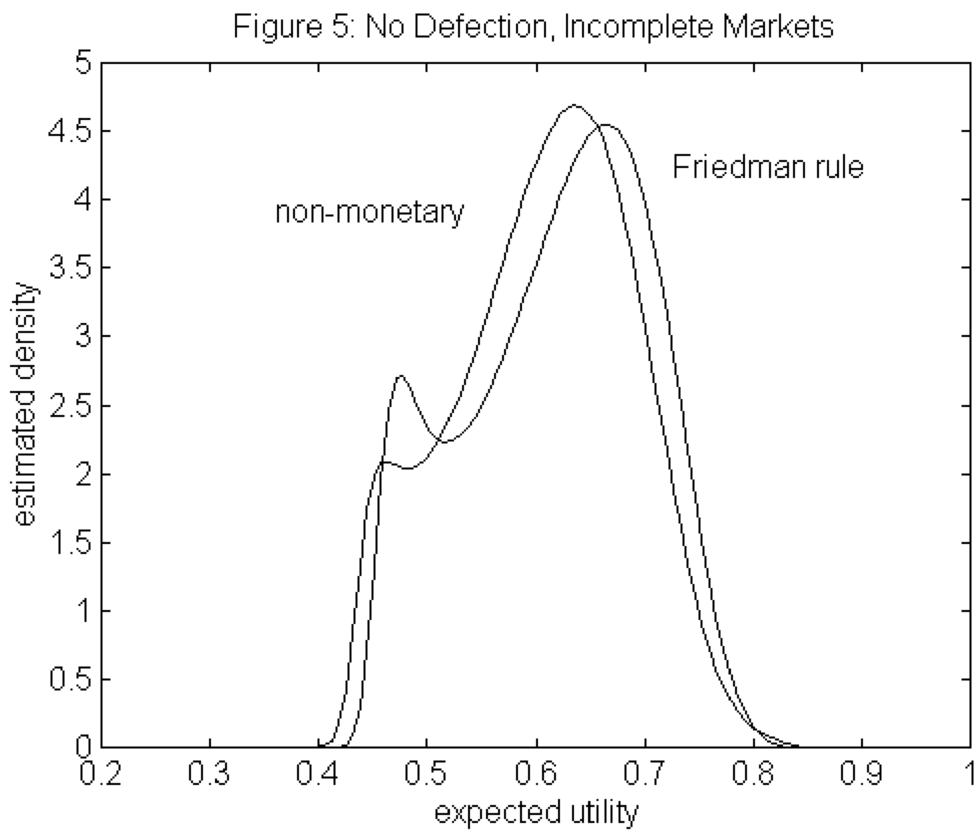
Figure 5: No Defection, Incomplete Markets
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
estimated density
expected utility
non-monetary
Friedman rule

FIG. 5.

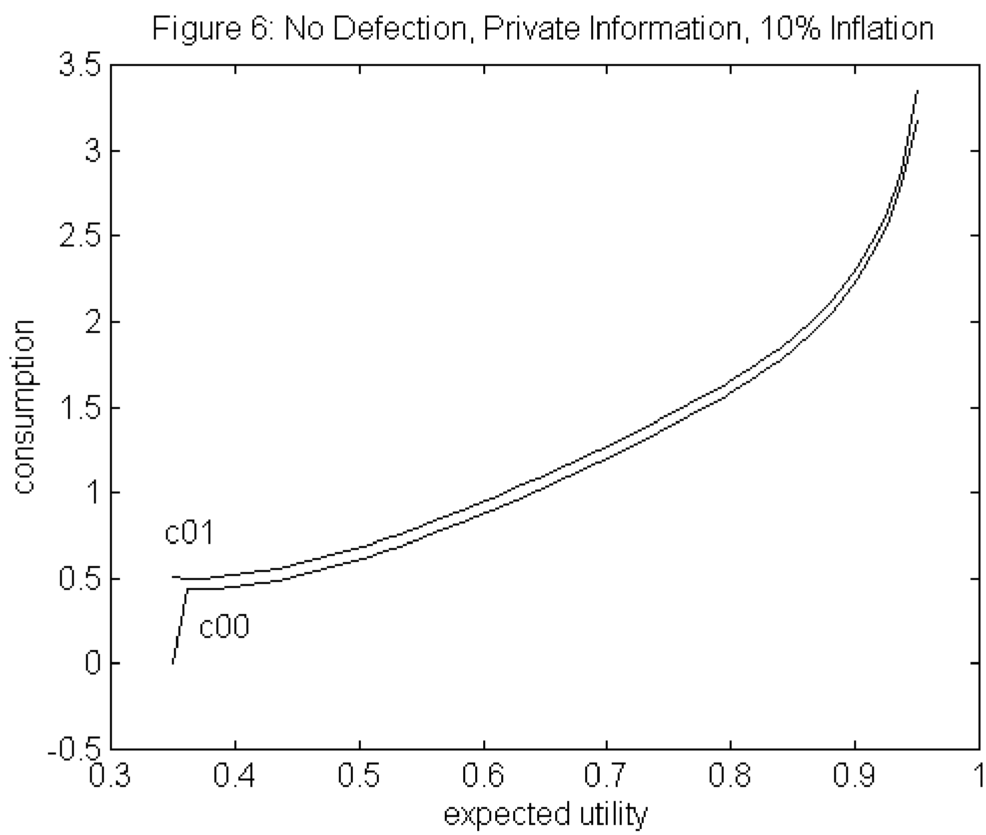
Figure 6: No Defection, Private Information, 10% Inflation
consumption
expected utility
c01
c00
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

FIG. 6.

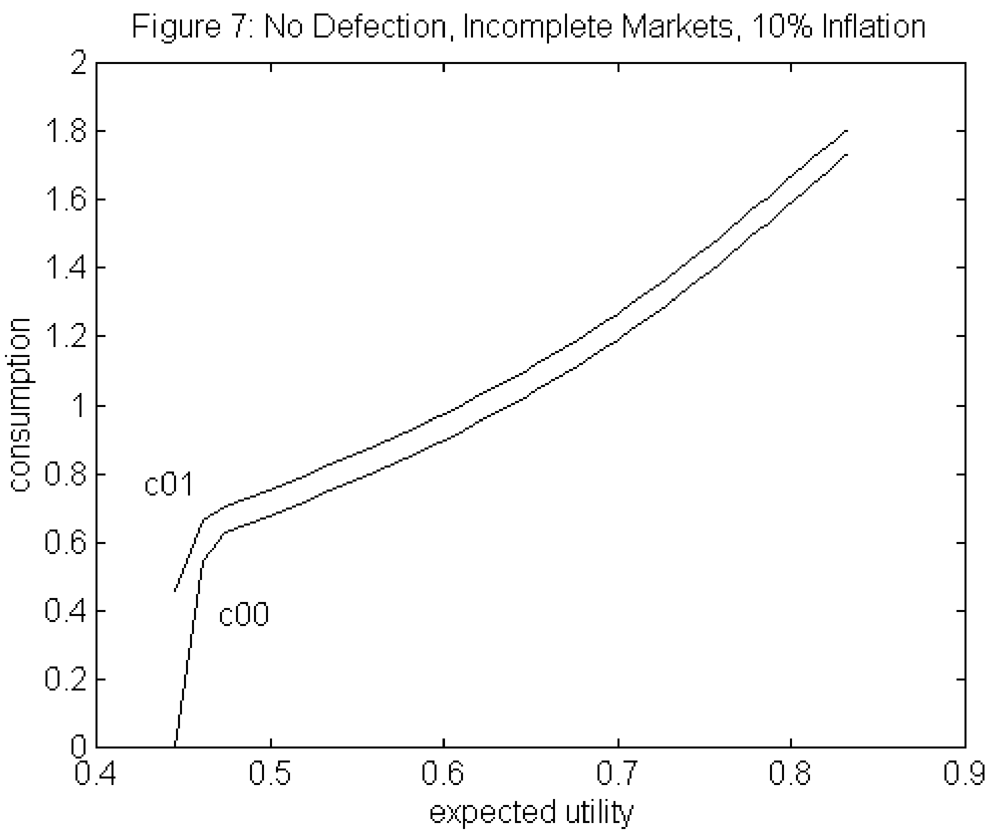
Figure 7: No Defection, Incomplete Markets, 10% Inflation
consumption
expected utility
c01
c00
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0.4
0.5
0.6
0.7
0.8
0.9

Fig. 7.

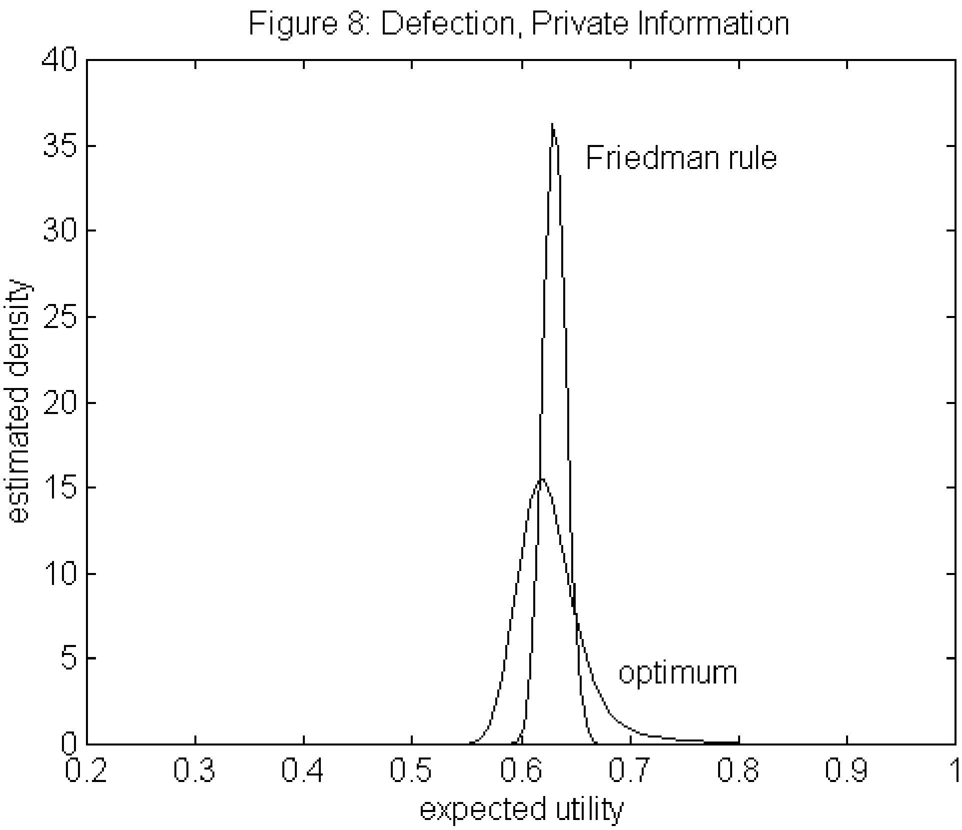
Figure 8: Defection, Private Information
estimated density
expected utility
Friedman rule
optimum
40
35
30
25
20
15
10
5
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig. 8.

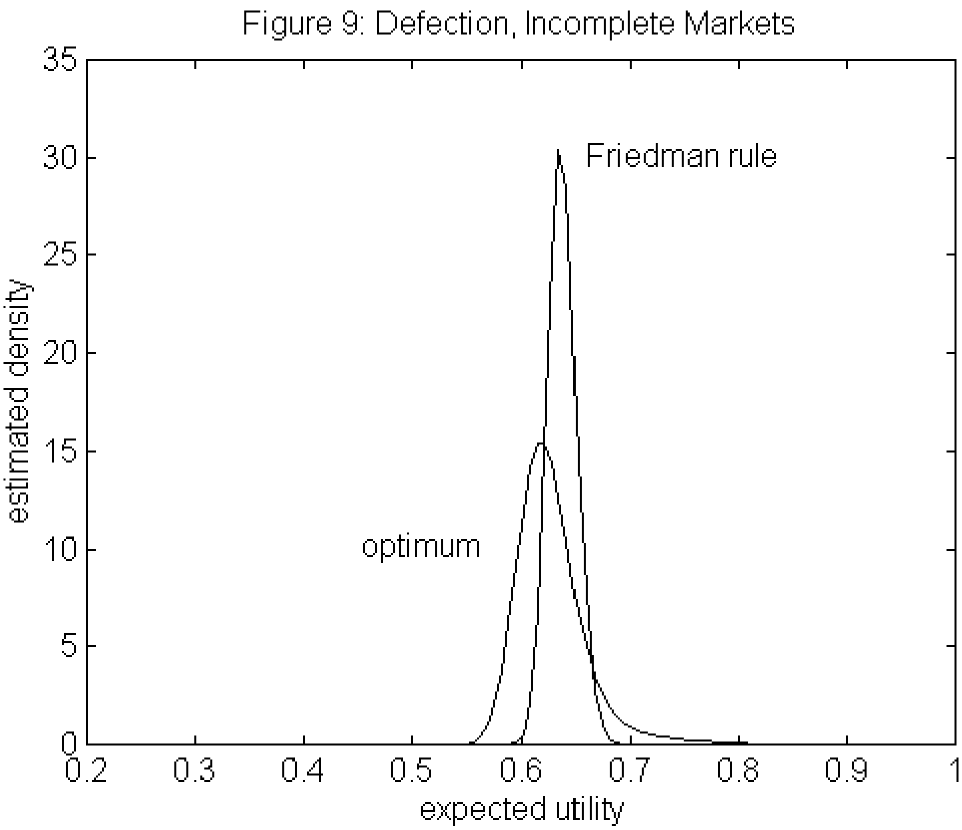

Figure 9: Defection, Incomplete Markets
Friedman rule
optimum
estimated density
expected utility
35
30
25
20
15
10
5
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

FIG. 9.

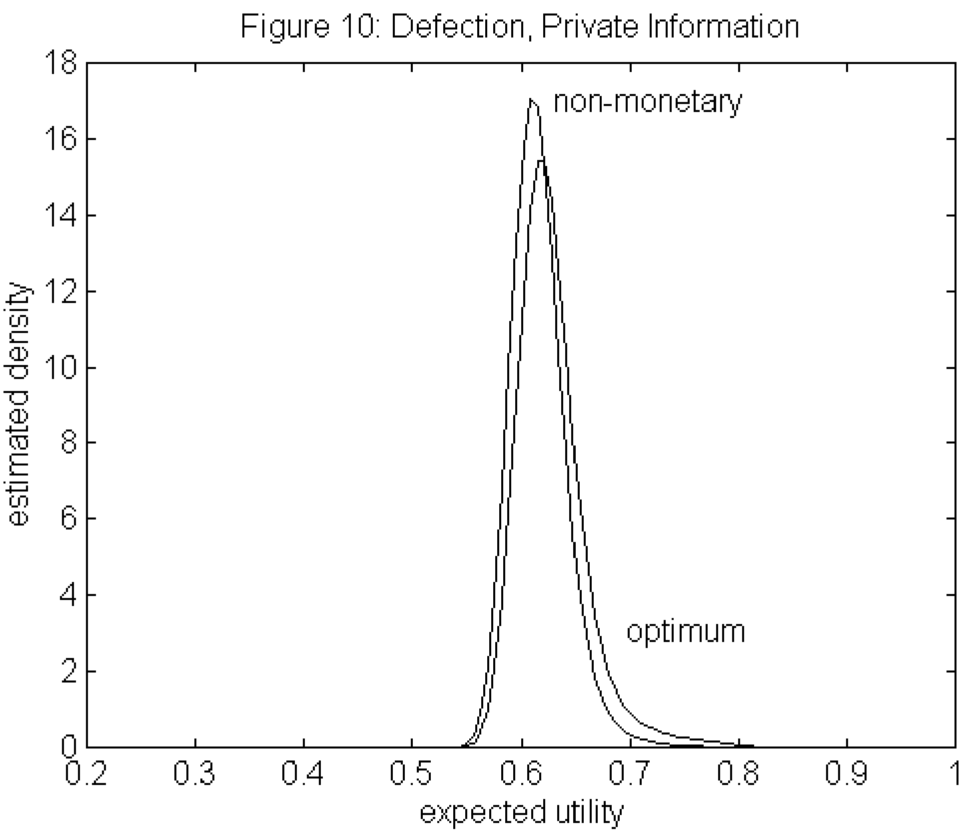

Figure 10: Defection, Private Information
non-monetary
optimum
estimated density
expected utility
0
2
4
6
8
10
12
14
16
18
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig. 10.

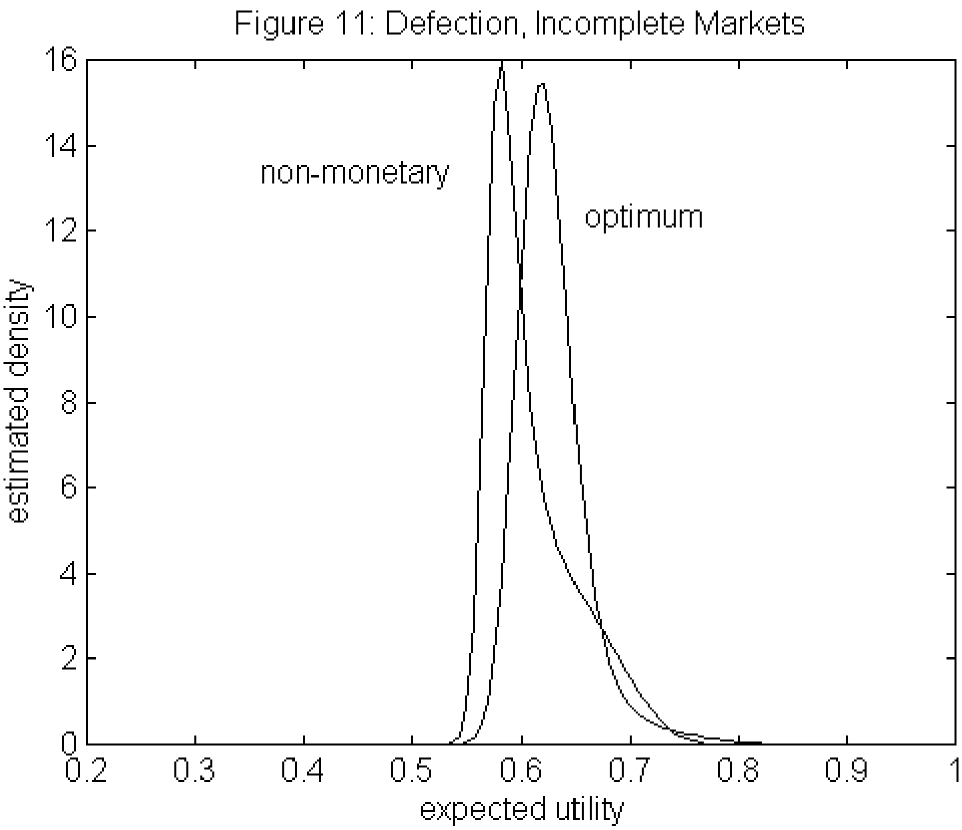
Figure 11: Defection, Incomplete Markets
16
14
12
10
8
6
4
2
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
non-monetary
optimum
estimated density
expected utility

FIG. 11.