Money and Dynamic Credit Arrangements with Private Information

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Abstract

We construct a model with private information in which consumers write dynamic contracts with financial intermediaries. A role for money arises due to random limited participation of consumers in the financial market. Without defection constraints, a Friedman rule is optimal, the mean and variability of wealth tend to fall in the steady state, and the welfare effects of inflation are very small. With defection constraints, it is optimal to eliminate currency

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entirely, the variability of wealth tends to rise with inflation, and the welfare effects of inflation are large.

1. INTRODUCTION

The main goal of this paper is to show how the theory of dynamic contracts under private information can be extended to address issues in monetary economics. We construct a dynamic risk-sharing model in which endowments are private information, and where there are roles for money and for long-term credit arrangements. The interaction between money and credit is analyzed, and we examine the implications of the model for long-run optimal monetary policy. Solutions are computed, and we measure the welfare effects of inflation.

It is clear that credit instruments currently play an important role in the payments systems of developed economies. Evidence from a Federal Reserve survey of households (Avery, Elliehausen, Kennickell and Spindt 1987) indicates that non-cash transactions accounted for 76% of the value of household transactions in the United States in 1984. More recent evidence, compiled by the Bank For International Settlements (1996), shows that, in the United States between 1991 and 1995, the nominal value of payments by credit card increased by 81%, and the nominal value of transactions over CHIPS and FedWire (electronic interbank transactions mechanisms) increased by 30.1%.

A typical feature of most observed credit relationships is that they are long-term. That is, it is usual for consumers and firms to establish enduring relationships with banks and other financial intermediaries, and there are good reasons for this. In particular, long-term contractual arrangements permit the use of dynamic incentives, which in turn promote efficiency. In this paper, we construct a model where dynamic credit arrangements and fiat currency are both useful in carrying out transactions, and where credit can potentially be as important as it appears to be currently in the U.S. economy.

We start with a credit paradigm, using it to derive a role for money, with money and credit coexisting. In particular, the environment we consider is in the spirit of the literature on dynamic private information (Green 1987, Spear and Srivastava 1987, Phelan and Townsend 1991, Aiyagari and Alvarez 1995, Wang 1995, Aiyagari and Williamson 1997). The basic model builds on Green (1987) in that there is no aggregate risk, but individuals face random endowment shocks which are private information. Here, a nonnegativity constraint on consumption and an endogenous interest rate guarantee that a limiting distribution of wealth with mobility will exist.[1]

Consumers make long-term credit arrangements with financial intermediaries, and they also have the opportunity to trade on a competitive money market in each period. There is a transactions role for money which arises due to random limited participation in the financial market. That is, timing within the period is such that a consumer may contact the financial intermediary before full information on her current income is available. It may then be desirable to transact on the money market in order to smooth consumption. The notion of limited participation is similar to the constructs used by Lucas (1990) and Fuerst (1992), though here this limited participation occurs at random, and it is idiosyncratic; in each period some fraction of consumers is subject to limited participation while the remaining fraction is not.

An important feature of the model is the defection constraints that arise in the dynamic contracting problem of a financial intermediary. In any period, each consumer has the option of defecting from her long-term contract with the financial intermediary, and trading in each succeeding period on the competitive money market. Effectively, the outside option for each consumer in defecting from the contract with the intermediary, is to become a Bewley incomplete markets consumer, as in Bewley (1980, 1983).

[1]In Green's model, in the limit a vanishing fraction of the population receives all of the aggregate endowment. Limiting distributions with mobility are obtained in Atkeson and Lucas (1994) with exogenous lower bounds on expected utilities, and in Phelan (1995), with endogenous lower bounds.

It is certainly not new to consider defection constraints in models with private information and dynamic contracts. For example, Atkeson and Lucas (1994) construct a model with arbitrary defection constraints, while Phelan works with an environment where endogenous defection constraints arise due to the outside option of defecting to an alternative long-term contract. What is new here is that the value of the outside option in our model depends on monetary policy. Specifically, the higher the inflation rate, the lower the value of defecting to money market trading only. This creates an interesting tension between two effects of monetary policy. First, higher money growth and inflation produce a standard effect, in that consumers economize too much on real money balances, they are then less able to insure against income risk, and the gap between the steady state allocation and an efficient allocation widens. Second, since higher money growth and inflation reduces the value of the outside option to a consumer, the efficiency of long-term contracts with intermediaries tends to increase. That is, since more severe penalties are open to a financial intermediary with higher inflation, incentives tend to improve and there is a gain in efficiency. Whether the first or second effect dominates is a quantitative question.

As a benchmark, we first consider a pure credit model. This is a special case where consumers are never subject to limited participation in the financial market, and so money is not valued and defection is equivalent to going to autarky. We then show that, in a version of the model without defection constraints, a version of the Friedman rule holds. In particular, it is optimal (in that the allocation is identical to the one for the pure credit allocation) for the money supply to grow at a rate that equates the real rate of return on money and the real interest rate, but this money growth rate does not equate the rate of return on money with the rate of time preference.

We compute solutions, again starting with the benchmark pure credit economy. Steady state allocations have the property that there is a well-defined limiting distribution with mobility, which is certainly not the case in all dynamic private information contracting models (see for example Green 1987, Thomas and Worrall 1990, or Atkeson and Lucas 1992). The critical features of the environment that give rise to a limiting distribution with mobility are that the interest rate is endogenous, consumption is nonnegative, and that the economy has random endowments rather than random preferences (see Aiyagari and Alvarez 1995). The defection constraints are not important for this particular result, as we obtain limiting distributions with mobility without these constraints.

Dropping defection constraints, in which case a Friedman rule is optimal, we examine the quantitative effects of inflation on welfare and the distribution of consumption and wealth (taking an agent's expected utility as a proxy for wealth). Higher inflation tends to reduce the variance of consumption and wealth for the population, though the variance of consumption conditional on wealth tends to rise since the money market is less useful for smoothing consumption at high rates of inflation. However, these effects are quantitatively small. For sufficiently high money growth rates, money will cease to be held in this model, but the cost of eliminating currency altogether is small. Money will not be held for inflation rates in excess of about 1500 percent per annum.

Now, for the economy with defection constraints on intermediary contracts, the results are quite different. Here, higher inflation tends to increase (rather than decrease) the variability of wealth and consumption for the population as a whole. That is, the effect of inflation on the endogenous defection constraints dominates the standard effects of money growth. The quantitative effects of inflation are large, and there is a sense in which the economy without currency yields a steady state allocation closest to the efficient allocation.

This is not the first paper to examine money and distribution in a dynamic model with private information. Lucas (1992) compares the distributional implications of cash-in-advance vs. an Atkeson-Lucas (1992) setup. Also, Taub (1994) studies a model with dynamic private information and risk neutrality where the optimal allocation can be supported with money or with a bond market. Other work which studies money and credit-type arrangements in models where memory is important are Kocherlakota (1996) and Kocheralakota and Wallace (1997).

In Section 2 the model economy is constructed, and Section 3 looks at equilibrium allocations. Section 4 contains a discussion of a pure credit economy as a benchmark, and optimal monetary policy is derived. Section 5 discusses calibration and computational methods, and Section 6 contains a presentation of the computational results. Section 7 is a conclusion.

2. THE MODEL

There is a continuum of consumers with unit mass, each with preferences given by

$$E_0(1-\beta)\sum_{t=0}^{\infty}\beta^t u(c_t),$$

where $0<\beta<1$, c_t is consumption, and $u(\cdot)$ is the period utility function, with $u(\cdot)$ strictly increasing and strictly concave and $u(0)=0$. We have $c_t \geq 0$ for all t. In each period t, a consumer receives a random endowment y_t, where $y_t \in \{y_0, y_1\}$ with $0 \leq y_0 < y_1$. Here, $\Pr[y_t = y_1] = \pi$, where $0<\pi<1$. Endowments are i.i.d. over time and across consumers, and they are private information.

At $t=0$, consumers form coalitions, which we will denote financial intermediaries. We assume that the intermediary is able to observe consumers' assets (here, their money balances) at the beginning of each period. Let consumers be indexed by $i \in [0,1]$. During any period, there are two possible modes of interaction between consumer i and the financial intermediary, dictated by the realization of the idiosyncratic random variable $s_t^i \in \{0,1\}$, which is public information at the beginning of the period. Assume that s_t^i is i.i.d. over time and across consumers, with $\Pr[s_t^i = 1] = \rho$, where $0 \leq \rho \leq 1$. If $s_t^i = 1$, then the consumer receives his/her current endowment

y_t at the beginning of the period, and then makes a report $z_t^i \in \{y_0, y_1\}$ to the financial intermediary concerning his/her endowment, following which the intermediary makes a transfer of consumption goods τ_t to the consumer. Alternatively, if $s_t^i = 0$, then at the beginning of period t the consumer receives y_0 units of the consumption good (in all states of the world), then obtains the goods transfer from the financial intermediary, and then finally receives $y_1 - y_0$ units of the consumption good with probability π and zero units with probability $1 - \pi$. Note that in the second case the financial intermediary can not make the transfer contingent on the total endowment the consumer receives during the period. A positive transfer can be viewed as a withdrawal from the consumer's account with the financial intermediary, while a negative transfer can be interpreted as a deposit.

The way in which consumers interact with financial intermediaries in this model is similar to the limited participation structures in Lucas (1990) and Fuerst (1992). For example, in Lucas's model the member of the Lucas household who purchases consumption goods is not able to access the financial market after a random open market operation occurs in the financial market during the period. In our model, limited participation occurs at random for individuals, and a constant fraction of the population, $1 - \rho$, is subject to limited participation in any given period. Random limited participation is here intended to capture the idea that money is held to insure against random circumstances where it is very costly or impossible to access the technology which would allow a credit transaction.

Consumer i enters period t with M_t^i units of fiat money, where M_0^i is given. After receiving transfers from the financial intermediary, consumers can trade money for consumption goods on a competitive market, where the price of consumption goods in terms of money is p_t. The monetary authority makes a transfer of T_t units of fiat money per consumer to financial intermediaries at the beginning of the period.

In any period, after contacting the financial intermediary the consumer may abandon the long-term credit contract. The alternative is to trade in the current period, and each subsequent period, on the competitive money market, i.e. the consumer then behaves in the same way as an agent in a Bewley-type incomplete markets economy where money is the only asset (see Bewley 1980, 1983). On defection, it is not possible for the consumer to negotiate another contract with another intermediary; intermediary contracts can be agreed to only at the first date. The possibility of defection from the long-term contract leads to a set of defection constraints that the contract must satisfy. In the subsequent analysis, we will determine steady state equilibrium allocations with and without these defection constraints.

3. EQUILIBRIUM ALLOCATIONS

Letting $\bar{M}_t$ denote the per capita stock of fiat money at the end of the period, the monetary authority must meet the constraint

$$\frac{(\bar{M}_t - \bar{M}_{t-1})}{p_t} = \frac{T_t}{p_t},$$

or, defining $\bar{m}_t \equiv \frac{\bar{M}_t}{p_t}$, $\omega_t \equiv \frac{T_t}{p_t}$ and $\gamma_t \equiv \frac{p_t}{p_{t-1}}$,

$$\bar{m}_t - \frac{\bar{m}_{t-1}}{\gamma_t} = \omega_t. \tag{1}$$

That is, current per capita real balances minus per capita real balances in the previous period divided by the gross rate of return on money must equal the per capita money transfer to consumers in units of consumption goods.

We suppose that each intermediary can trade on a bond market, facing the sequence of prices $\{q_t\}_{t=0}^{\infty}$ and the sequence of money prices of consumption goods $\{p_t\}_{t=0}^{\infty}$. We can think of the bond market involving trade in one-period bonds, which sell at price $\frac{q_t}{1-q_t}$ in period t, and pay off $\frac{1}{1-q_{t+1}}$ units of the consumption good in period $t+1$. Consumers can not trade on the bond market. Defining $m_t \equiv \frac{M_t}{p_t}$, the real balances of a consumer, we suppose that the consumers who are members of the financial intermediary agree on an initial distribution $\psi_0(w_0, m_0)$, i.e. a distribution of expected utility entitlements w_0 and real money balances m_0 across the members of the financial intermediary. Here, the initial distribution of money balances across consumers is given, and expected utility entitlements are to be met through the design by the financial intermediary of a transfer policy and a specification of trading by consumers on the money market.

Along the lines of Atkeson and Lucas (1992, 1994), Aiyagari and Alvarez (1995), or Aiyagari and Williamson (1997), we think of the financial intermediary as solving a set of component problems to determine the optimal contractual arrangements with each of its members. Specifically, there is a separate cost minimization problem that the intermediary solves for each initial (w_0, m_0), facing $\{q_t\}_{t=0}^{\infty}$ and $\{p_t\}_{t=0}^{\infty}$.We confine attention to steady states, where $q_t = q$, $\gamma_t = \gamma$, and $\omega_t = \omega$ for all t, where q, γ, and ω are constants. Also, the distribution of expected utilities and real money balances across consumers, $\psi(w, m)$, is constant. Given that any intermediary has a positive measure of consumers, each intermediary faces the same steady state distribution $\psi(w, m)$, and we can analyze this economy as if there were only one representative financial intermediary.

The component planning problems can be specified in recursive form by treating w, the consumer's expected utility, and m, his/her real money balances, as state variables, and applying Green's (1987) notion of temporary incentive compatibility. Letting $v(w, m)$ denote the expected discounted cost to the intermediary of delivering a level of expected utility w at the current date to a consumer holding m units of real balances at that date, the intermediary's problem can be formulated in terms of the following Bellman equation.

$$v(w,m) = \min \left\{ \begin{array}{c} (1-q)[\rho\pi\tau_1(w,m) + \rho(1-\pi)\tau_0(w,m) + (1-\rho)\tau(w,m)] \\ +q \left[\begin{array}{c} \rho\pi v[w_{11}(w,m), m_{11}(w,m)] \\ +\rho(1-\pi)v[w_{10}(w,m), m_{10}(w,m)] \\ +(1-\rho)\pi v[w_{01}(w,m), m_{01}(w,m)] \\ +(1-\rho)(1-\pi)v[w_{00}(w,m), m_{00}(w,m)] \end{array} \right] \end{array} \right\} \quad (2)$$

subject to

$$\begin{aligned} w \;=\; & (1-\beta) \left[\begin{array}{c} \rho\pi u(y_1 + m + \tau_1(w,m) - \gamma m_{11}(w,m)) \\ +\rho(1-\pi)u(y_0 + m + \tau_0(w,m) - \gamma m_{10}(w,m)) \\ (1-\rho)\pi u(y_1 + m + \tau(w,m) - \gamma m_{01}(w,m)) \\ (1-\rho)(1-\pi)u(y_0 + m + \tau(w,m) - \gamma m_{00}(w,m)) \end{array} \right] \quad (3) \\ & +\beta \left[\begin{array}{c} \rho\pi w_{11}(w,m) + \rho(1-\pi)w_{10}(w,m) \\ +(1-\rho)\pi w_{01}(w,m) + (1-\rho)(1-\pi)w_{00}(w,m) \end{array} \right] \end{aligned}$$

$$\begin{aligned} & (1-\beta)u(y_i + m + \tau_i(w,m) - \gamma m_{1i}(w,m)) + \beta w_{1i}(w,m) \quad (4) \\ \geq \; & (1-\beta)u(y_i + m + \tau_j(w,m) - \gamma m_{1j}(w,m)) + \beta w_{1j}(w,m), \; (i,j) = (1,0),(0,1), \end{aligned}$$

$$\begin{aligned} & (1-\beta)u(y_i + m + \tau(w,m) - \gamma m_{0i}(w,m)) + \beta w_{0i}(w,m) \quad (5) \\ \geq \; & (1-\beta)u(y_i + m + \tau(w,m) - \gamma m_{0j}(w,m)) + \beta w_{0j}(w,m), \; (i,j) = (1,0),(0,1), \end{aligned}$$

$$\begin{aligned} & (1-\beta)u(y_i + m + \tau_i(w,m) - \gamma m_{1i}(w,m)) + \beta w_{1i}(w,m) \quad (6) \\ \geq \; & \delta(y_i + m + \tau_i(w,m)), \; i = 0,1, \end{aligned}$$

$$\begin{aligned} & (1-\beta)u(y_i + m + \tau(w,m) - \gamma m_{0i}(w,m)) + \beta w_{0i}(w,m) \quad (7) \\ \geq \; & \delta(y_i + m + \tau(w,m)), \; i = 0,1, \end{aligned}$$

$$y_i + m + \tau_i(w,m) - \gamma m_{1i}(w,m) \quad \geq \quad 0, \; i = 0,1 \tag{8}$$

$$y_i + m + \tau(w,m) - \gamma m_{0i}(w,m) \quad \geq \quad 0, \; i = 0,1$$

$$m_{ij}(w,m) \geq 0, \; i,j = 0,1. \tag{9}$$

Here, the transfer when the endowment is high [low] and $s_t^i = 1$ is $\tau_1(w,m)$ $[\tau_0(w,m)]$, while the transfer when $s_t^i = 0$ is $\tau(w,m)$. The consumer is assigned an expected utility for the following period, which is $w_{jk}(w,m)$ when $s_t^i = j$ and $y_t = y_k$. The intermediary also recommends a quantity of real balances that the consumer is to carry into the next period, $m_{jk}(w,m)$, where the subscripts have the same meaning as for the expected utility assignment. Recommended real balances then imply a recommended transaction for the consumer on the money market.

The financial intermediary minimizes the present discounted value of goods transfers to the consumer. The first constraint in the problem above, (3), is the promise-keeping constraint, which states that contingent transfers, continuation expected utilities, and recommend future money balances are consistent with the consumer receiving expected utility w in the current period. Constraints (4) and (5) are incentive compatibility constraints, which state that it not be in the consumer's interest to misreport his/her endowment to the financial intermediary. The constraints (6) and (7) are defection constraints. That is, in each period, given the consumer's initial money balances, endowment, and transfer from the financial intermediary, it should not be in the consumer's interest to defect from the long-term contract with the financial intermediary and trade in each subsequent period on the money market. The function $\delta(y)$ is the value of defecting with assets y, and is defined by the functional equation

$$\delta(y) = \max_{m'} \left\{ (1-\beta)u(y - \gamma m') + \beta \left[\pi\delta(y_1 + m') + (1-\pi)\delta(y_0 + m') \right] \right\} \tag{10}$$

subject to

$$y - \gamma m' \geq 0, \tag{11}$$

$$m' \geq 0. \tag{12}$$

Here, m' is the quantity of real balances that the consumer would take into the next period if she defected from the contract with the financial intermediary, (11) is a nonnegativity constraint on consumption in the current period, and (12) is a nonnegativity constraint on real balances. Finally, in the financial intermediary's problem, (8) and (9) are nonnegativity constraints on consumption and money balances, respectively.

Note that the expected utility the consumer receives should she defect from the contract with the financial intermediary is essentially expected utility in a Bewley-type economy (see Bewley (1980, 1983)). That is, the long-term contract must guarantee the consumer a path for expected utility that never falls below what could be achieved in the Bewley allocation. It is immediately apparent from (10) that the expected utility the consumer receives by defecting is decreasing in the gross inflation rate, γ. That is, $\delta(y)$ is decreasing in γ, so that an increase in γ will tend to relax the defection constraints (6) and (7).

We require that the aggregate resource constraint be satisfied in the steady state, so that

$$\int\int \left[\begin{array}{c} \rho\pi \left[m + \tau_1(w,m) - \gamma m_{11}(w,m)\right] \\ +\rho(1-\pi)\left[m + \tau_0(w,m) - \gamma m_{10}(w,m)\right] \\ +(1-\rho)\pi\left[m + \tau(w,m) - \gamma m_{01}(w,m)\right] \\ +(1-\rho)(1-\pi)\left[m + \tau(w,m) - \gamma m_{00}(w,m)\right] \end{array} \right] d\psi(w,m) = 0, \tag{13}$$

i.e. net transfers among consumers through financial intermediaries and through the money market must equal zero.

Note that money plays two roles here. First, money balances allow consumers to self-insure against random limited participation in the financial market. This can be interpreted as a transactions role for money. Second, current money balances communicate to the intermediary the endowment shock of the consumer in the previous period, so that money acts as a record-keeping device.[2]

Equilibrium allocations will have the property that $q \leq \gamma$, as otherwise (13) would not hold. That is, if $q > \gamma$, financial intermediaries could make infinite profits by borrowing on the bond market and having consumers acquire higher-yielding money on the competitive money market. In the case where $q < \gamma$ (i.e. money is strictly dominated in rate of return), we will have $m_{ij}(m, w) = 0$ for $i = 1$ and for $(i, j) = (0, 0)$. To see this, suppose that $m_{ij}(m, w) > 0$ for $i = 1$ and for some j. Then, when $s_t^i = 1$ and $y_t = y_j$, the consumer trades off claims to current consumption for claims to future consumption by holding money balances. However, this can be done more efficiently by the financial intermediary, which can reduce the consumer's transfers today and increase future transfers, while facing a higher interest rate. The intermediary can thus meet all the constraints in the above optimization problem while reducing the value of the objective function, so it cannot be optimal to have $m_{ij}(m, w) > 0$ for $i = 1$. A similar argument holds for $(i, j) = (0, 0)$, given that incentive compatibility requires that $m_{01}(m, w) \geq m_{00}(m, w)$.

While the problem (2) subject to (3)-(9) may appear formidable, it is possible to simplify it considerably. First, suppose that we perform a change of variables by letting $\tau_i^*(w) = \tau_i(m, w) - m$, for $i = 0, 1$, with $w_{ij}^*(w) = w_{ij}(w, m)$ and $m_{ij}^*(w) = m_{ij}(w, m)$ for $i, j = 0, 1$. Now, we can write the cost function as

$$v(w, m) = -(1 - q)m + \theta(w),$$

where $\theta(\cdot)$ is a function, and the choice variables $\tau_i^*(w)$, $i = 0, 1$, $w_{ij}^*(w)$, and $m_{ij}^*(w)$, $i, j = 0, 1$, are independent of m. Thus, the current consumption allocation, future money balances, and future expected utility entitlement of the consumer are determined only by the current expected utility entitlement and the current endowment.

[2] The record-keeping role of money has been explored by Townsend (1987, 1989) in two-period environments with spatial separation. See also Kocherlakota and Wallace (1997).

For computational purposes, this simplification is very important, as it allows us to solve the financial intermediary's problem as a one-state-variable problem rather than as a two-state-variable problem.

4. A PURE CREDIT ECONOMY

In this section we wish to consider the special case where $\rho = 1$, so that money is not valued in equilibrium. That is, money has value here only to the extent that consumers require it to self-insure against random limited participation. If $\rho = 1$ and money has no value, the expected utility from deviating from the long-term contract with the financial intermediary is the current utility from consuming in the present plus the discounted expected utility from autarky, i.e. $\delta(y) = (1-\beta)u(y)+\beta[\pi u(y_1)+(1-\pi)u(y_0)]$ for all y. We can think of this special case as a pure credit economy.

The financial intermediary's problem when $\rho = 1$ reduces to a problem which is identical to the one considered by Green (1987), except that we have a nonnegativity constraint on consumption, the interest rate is endogenous, and we impose defection constraints. The problem is

$$z(w) = \min \left\{ \begin{array}{c} (1-q)[\pi\tau_1(w) + (1-\pi)\tau_0(w)] \\ +q\left[\pi z(w_1(w)) + (1-\pi)z(w_0(w))\right] \end{array} \right\} \tag{14}$$

subject to

$$\begin{aligned} w &= (1-\beta)\left[\ \pi u(y_1+\tau_1(w)) + (1-\pi)u(y_0+\tau_0(w))\ \right] \\ &\quad +\beta\left[\pi w_1(w) + (1-\pi)w_0(w)\right] \end{aligned} \tag{15}$$

$$(1-\beta)u(y_1+\tau_1(w)) + \beta w_1(w) \geq (1-\beta)u(y_1+\tau_0(w)) + \beta w_0(w) \tag{16}$$

$$(1-\beta)u(y_0+\tau_0(w)) + \beta w_0(w) \geq (1-\beta)u(y_0+\tau_1(w)) + \beta w_1(w) \tag{17}$$

$$w_1(w), w_0(w) \geq \pi u(y_1) + (1-\pi)u(y_0) \tag{18}$$

$$y_i + \tau_i(w) \geq 0, \text{ for } i = 0, 1. \tag{19}$$

Here, $z(w)$ is the cost function, $\tau_1(w)$ $[\tau_0(w)]$ is the transfer in the high (low) endowment state, and $w_1(w)$ $[w_0(w)]$ is the expected utility entitlement in the following period when the current endowment is high (low). Equation (15) is the promise-keeping constraint, inequalities (16) and (17) are the incentive constraints, (18) are the defection constraints, and (19) are nonnegativity constraints on consumption.

In the steady state, transfers must sum to zero across consumers. Thus, if $\psi(w)$ denotes the steady state distribution of expected utility entitlements, we must have

$$\int [\pi\tau_1(w) + (1-\pi)\tau_0(w)]d\psi(w) = 0. \tag{20}$$

The pure credit economy is a useful benchmark, as we know that the steady state allocation for this economy is efficient, and dominates any other allocation with $\rho \neq 1$. That is, if $\rho \neq 1$ then random limited participation implies first, that consumption must sometimes be smoothed inefficiently through the money market (or not at all if money is not valued) rather than through the intertemporal credit mechanism offered by the financial intermediary. Second, since for any $\gamma > 0$ the value of defection will weakly dominate autarky, and strictly dominate in the case where the consumer would choose to hold positive cash balances in some future state of the world if defection occurs, the defection constraints will be more severe when $\rho \neq 1$. The first source of inefficiency is a standard type of distortion caused by inflation, which will be reflected in a suboptimal "quantity of money," and a positive nominal interest rate. The second source of inefficiency is more unconventional. Because consumers have an alternative to the long-term contract with the financial intermediary, this puts constraints on the types of contracts that intermediaries can offer. Further, since the alternative involves trading on the money market, inflation will affect the value of this outside option.[3]

[3]Corbae and Blume (1995) have explored the idea that an outside option of trading on a com-

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As a step towards evaluating the welfare effects of inflation in this model, we first consider efficient allocations in the absence of constraints (6) and (7). We will show that a Friedman-rule result holds in this case.

We start by observing that, dropping (6) and (7) from the financial intermediary's problem, if we can find a value for γ such that an economy with $\rho \neq 1$ replicates the steady state allocation with $\rho = 1$ (and here dropping constraint (18)), we can say that the money growth rate $\gamma - 1$ is optimal. We have the following proposition.

Proposition 1: When $\rho \neq 1$, $\gamma = q$ is optimal.

Proof. First, conjecture that when $\gamma = q$, a solution to (2) subject to (3)-(9) (absent (6) and (7)) is $v(w,m) = z(w) - (1-q)m$, $\tau_1(w,m) = \tau_1(w) - m$, $\tau_0(w,m) = \tau_0(w) - m$, $\tau(w,m) = \tau_0(w) - m$, $m_{01}(w,m) = \frac{\tau_0(w)-\tau_1(w)}{q}$, $m_{ij}(w,m) = 0$ for $(i,j) = (1,1),(1,0),(0,0)$, $w_{ij}(w,m) = w_j(w)$ for $i,j = 0,1$, where $[z(w), \tau_1(w), \tau_0(w), w_1(w), w_0(w)]$ is the solution to the pure credit problem, (14) subject to (15)-(19), dropping (18). We have already shown that $m_{ij}(w,m) = 0$ for $(i,j) = (1,1),(1,0),(0,0)$. Now, substituting in the Bellman equation (2), we obtain

$$z(w) - (1-q)m = \min \left\{ \begin{array}{c} (1-q)[\rho\pi\tau_1(w) + \rho(1-\pi)\tau_0(w) + (1-\rho)\tau_0(w) - m] \\ +q \left[\begin{array}{c} \pi z[w_1(w)] + (1-\pi)z[w_0(w)] \\ -(1-\rho)\pi\frac{(1-q)[\tau_0(w)-\tau_1(w)]}{q} \end{array} \right] \end{array} \right\}.$$

Simplifying, we get

$$z(w) = \min \left\{ \begin{array}{c} (1-q)[\pi\tau_1(w) + (1-\pi)\tau_0(w)] \\ +q\left[\pi z[w_1(w)] + (1-\pi)z[w_0(w)]\right] \end{array} \right\}.$$

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petitive money market can affect the efficiency of long-term contracts in an environment without private information. They show that money can be valued for no other reason than that it provides this outside option, but the fact that it provides it implies that valued fiat money is inefficient.

Similarly, substituting in the constraints (3)-(8), we obtain the constraints (15)-(19), and constraint (9) is also satisfied. Thus, the solution to the financial intermediary's problem is the same for any ρ when $\gamma = q$. Now, it remains to be shown that in the steady state the aggregate resource constraint, (13), is satisfied. Substituting in (13), and given (20), it is straightforward to show that this is the case.□

Thus, a Friedman rule is optimal here when there are no defection constraints. That is, since the only monetary distortion in this special case is the standard type of intertemporal distortion caused by inflation which exists in many monetary models (cash-in-advance, and money-in-the-utility-function models, for example), it would be surprising if a Friedman rule were not optimal. However, note that this Friedman rule is modified somewhat from its form in Friedman (1969). Here, it is optimal to equate the real returns on the bond market (accessible only to financial intermediaries) and the money market, as is consistent with Friedman (1969), but this need not involve equating the rate of time preference of consumers with this rate of return. In general, it will be the case that $q > \beta$ for the efficient pure credit allocation, as in Atkeson and Lucas (1994) and Aiyagari and Williamson (1997).[4]

5. CALIBRATION AND COMPUTATION

We use the economy without defection constraints as a benchmark, setting parameters so that the steady state allocation matches observed features of the U.S. economy. We interpret a period as one quarter, and set y_0, y_1, and π so as to match the variability in quarterly household income. Using PSID data, Aiyagari (1994) argues that a first-order autoregression closely matches the time series properties of

[4]Note that transfers and real money balances are not uniquely determined when $\gamma = q$, since the financial intermediary is indifferent when trading off current for future consumption, between giving the consumer less transfers in the present and more transfers in the future, and requiring the agent to acquire money balances in the present and spend them in the future.

annual earnings, with a range of .23 to .53 for the first-order serial correlation coefficient, and a coefficient of variation in unconditional earnings of 20 to 40 percent. Since it is not tractable to introduce serial correlation in endowments in this model, we must do the best we can to fit an i.i.d. endowment shock in the model to the data. This is not too problematic, as the estimated serial correlation in annual data is low, and serial correlation for quarterly data would then be even lower. If we take the coefficient of variation to be 30 percent for annual data, then if quarterly income is i.i.d., the coefficient of variation for quarterly data would be 60 percent. Thus, we set $\pi = .5$, $y_0 = 1 - \epsilon$, and $y_1 = 1 + \epsilon$, with $\epsilon = .6$. The utility function we use is $u(c) = 1 - e^{-\alpha c}$, with $\alpha = 1$, which implies a coefficient of relative risk aversion of unity at the mean endowment. The constant relative risk aversion utility function is convenient for computational purposes here as it is bounded. The remaining parameters, ρ, β, and γ, were set so as to produce an equilibrium steady state allocation which would match observed average real interest rates, inflation rates, and the observed use of currency in transactions. From the real business cycle literature (Prescott 1986), the real interest rate is taken to be 1% per quarter, so in a steady state we want $q = .99$. A survey of households by the Federal Reserve (Avery, Elliehausen, Kennickell, and Spindt 1987), conducted in 1984, finds that 24% of the current value of household transactions is carried out in currency. In the model, the steady state quantity of currency transactions is

$$\frac{1}{2}\int\int\{[1-(1-\rho)\pi]\,(m+\omega)+(1-\rho)\pi\mid m+\omega+-\gamma m_{01}(m,w)\mid\}\,d\psi(m,w),$$

and the steady state quantity of credit transactions is

$$\frac{1}{2}\int\int[\rho\pi\mid\tau_1(m,w)\mid+\rho(1-\pi)\mid\tau_0(m,w)\mid+(1-\rho)\mid\tau(m,w)\mid]\,d\psi(m,w).$$

When the Federal Reserve survey was done, the inflation rate was approximately 1% per quarter, so we set $\gamma = 1.01$ for calibration purposes.

Solutions were computed for the economy without defection constraints as follows. First, grids were chosen for the two state variables, w and m. The lower bound on expected utility, w, is $\pi u(y_1 - y_0) + (1-\pi)u(0)$, which is the minimum incentive compatible level of expected utility that can be imposed on a consumer, and the lower bound on m is zero. Since choice variables in the financial intermediary's problem are independent of m, it is only necessary to solve the problem at each point along the w grid, and for a single value for m, say $m = 0$, and then use this solution to determine what the solution is for all points on the grid. We start with an initial guess for $q \in (\beta, 1)$, and make an initial guess for the function $\theta(w)$. Then, value iteration is used to arrive at the solution for $\theta(w)$ given q. At each iteration, $\theta(w)$ is updated by fitting a third-order Chebychev polynomial (plus an additional term, $\frac{1}{1-w}$, which performed well in fitting the points), to the values computed for the cost function at points on the grid on the previous iteration. When convergence is achieved given q, then the decision rules are interpolated across a finer grid, and a matrix of Markov transition probabilities for the state w is constructed as an approximation using a lottery over the two closest grid points. A limiting distribution over w is computed, the analogue of the left-hand side of (13) is evaluated, and q is updated according to a bisection method. Then value-iteration is performed again, etc.

To match the observed real interest rate and the evidence from the Federal Reserve survey on household transactions, we set $\beta = .99$ and $\rho = .81$, in computing the allocation for the economy without defection constraints. This implies that $q = .99$ (actually slightly greater than β, but the difference is on the order of 10^{-5}) and that currency accounts for 24% of the value of transactions.

The procedure for computing solutions with defection constraints is similar, but first we need to use the functional equation (10) to determine the value function $\delta(y)$ (again, using value iteration, and interpolation of the value function with a Chebychev polynomial). Then, the lower bound for expected utilities for intermediary contracts is given by

$$\underline{w} = \pi\delta(y_1 - y_0) + (1-\pi)\delta(0).$$

Note that $\underline{w} \geq \pi u(y_1) + (1-\pi)u(y_0) > \pi u(y_1 - y_0) + (1-\pi)u(0)$, the lower bound on expected utilities without defection constraints. That is, without defection constraints the worst incentive compatible treatment a consumer could receive is to give up y_0 (the low endowment) each period. With defection constraints, at worst the intermediary could force the agent to give up y_0 in the current period, with the consumer defecting to the money market in each succeeding period. Once the consumer abandons the long-term contract, she can do at least as well as autarky, which is better than giving up y_0 in each state at each date.

6. COMPUTATIONAL RESULTS

We first consider the pure credit case (i.e. $\rho = 1$) as a benchmark, noting that the allocations here will be identical to those with $\rho \neq 1$ and $\gamma = q$ (a Friedman rule) when there are no defection constraints. We then compare these benchmark allocations to allocations with various inflation rates greater than the inflation rate for the Friedman rule allocation, for the case with no defection constraints. Finally, we consider the case with defection constraints, and look at similar inflationary experiments.

Pure Credit Economy

To get an idea of some of the general features of the solutions, it helps to look at allocations with low discount factors. We first consider an economy with $\rho = 1$ (the pure credit case) and $\beta = .5$, with the other parameters set as discussed in Section 5. The results are in Figures 1-3. Figure 1 shows $w_1(w)$ ($w1$ in the figure) and $w_0(w)$ ($w0$ in the figure). Here, note that, for low levels of w, future expected utility remains at the lower bound on expected utilities, $\pi u(y_1 - y_0)$, when the consumer receives a low endowment in the current period. In general, given the binding incentive constraint [(16) binds at the optimum, but (17) does not], expected utility rises when the consumer receives a high endowment, and falls when there is a low endowment. Figure 2 shows the limiting distribution of consumers across expected utility entitlements. Here, note that a significant fraction of consumers, over 14%, are effectively credit-constrained in that they are at the lower bound on expected utilities. Half of these consumers, those with low endowments, will consume zero, as we see in Figure 3. Note in Figure 3 that the gap between consumption in the good state ($c1$) and consumption in the bad state ($c0$) is larger at the low end of the distribution, where the binding lower bound on expected utilities mitigates the incentives available to induce truthful reporting on the part of consumers. Due to the low discount factor, intertemporal incentives are not very good, and there is on average a fairly large gap between consumption in the high-endowment state and consumption in the low-endowment state.

We next consider pure credit economies which are calibrated to the data. That is, we use the same parameters as for the previous example, except that $\beta = .99$. Note again that the allocations here will be identical to what we would get with $\rho \neq 1$ and a Friedman rule in place. For the results, see Figures 4 and 5. We do not show plots of $w_1(w)$ and $w_0(w)$ (as in Figure 1) as these functions are very close to the 45-degree line. Figure 4 shows efficient steady state distributions of expected utilities (w) across the population, for the cases where the defection constraints (18) are imposed and where they are not. Note that the dispersion in expected utilities is higher without the defection constraints. That is, the defection constraints impose a lower bound on expected utilities, which eliminates the lower tail of the distribution. Since the market-clearing condition (20) must hold, and since average consumption (across endowment states) increases with w (see Figure 5), steady state q will tend to adjust to cut off the upper tail of the distribution as well, so that the imposition of defection constraints must reduce variability in w in the steady state. In Figure 4, we see that there are essentially no credit-constrained consumers in the steady state either with or without defection constraints. This is due to the fact that, with a high discount factor, the range of expected utilities for which a low endowment will imply that next period's expected utility is at the lower bound, is negligible, in contrast to Figure 1. In Figure 5, where $c0$ ($c1$) denotes consumption in the low (high) endowment state when there are no defection constraints, there is very little variability in consumption, conditional on expected utility, i.e. in this sense the solution is very close to perfect insurance. This reflects the fact that, since consumers do not discount the future much, intertemporal incentives work very well. However, note in Figure 4 that the variability in expected utilities is substantial. Therefore, since the slopes of the consumption profiles in Figure 5 are quite steep, this will lead to high variability in consumption across the population. In the steady state, the unconditional coefficient of variation of consumption is about 15%, while the unconditional coefficient of variation of income is 60%, when there are no defection constraints. With defection constraints, the unconditional coefficient of variation of consumption is about 8%.

Money and Credit With No Defection Constraints

Here, we examine the effects of inflation for the calibrated economy with no defection constraints. We consider equilibrium allocations for inflation rates running from the Friedman rule inflation rate ($\gamma = q$) to an (incipient) inflation rate sufficiently high to rule out a steady state monetary equilibrium. This latter equilibrium is essentially an inefficient pure credit allocation where, with probability $1 - \rho$, the transfer to the consumer from the financial intermediary is noncontingent on income.

Table 1: $\rho = .81$, **No Defection Constraints**

Annual Inflation Rate	Mean E.U.	S.D. of E.U.	S.D. of Cons.	Cost of Inflation
-3.94%	.6270	.0519	.1508	0
10%	.6270	.0515	.1549	0.07
100%	.6262	.0500	.1908	0.04
1500%	.6173	.0452	.2964	2.58
>non-mon. threshold	.6156	.0450	.3066	5.47

The results are summarized in Table 1. Note that mean expected utility falls with inflation, the standard deviation of expected utility falls, and the standard deviation of consumption rises substantially. The last row of the table consists of results for an inflation rate in excess of what is required to drive money out of the system, a rate which is greater than 1500% per annum, but not by much. These results reflect the fact that, as the inflation rate rises, money becomes a less efficient store of value, and is therefore used less for insurance purposes. There is then less consumption smoothing, the variability of consumption rises, and welfare falls. Figure 6 shows consumption profiles with 10% inflation in the state where the consumer is subject to the limited participation problem, and cannot get insurance through the financial intermediary ($c01$ is consumption when income is high, and $c00$ is consumption when income is low). Here, note the difference in the variability in consumption, conditional on expected utility, relative to Figure 5. Due to the fact that there is less insurance, intertemporal incentives are used less, and this tends to reduce the variability in expected utilities across the population.

Evaluating the welfare effects of inflation is somewhat problematic here, as we need to deal with the whole distribution of expected utilities. We might consider looking at a particular moment of the distribution, but this can lead to results that do not make sense. For example, consider the optimal distributions in Figure 4, with and without defection constraints, and suppose that we focus only on the mean of the distribution. Mean expected utility without defection constraints is .6270, while mean expected utility with defection constraints is .6281, which might lead us to conclude that the allocation with defection constraints dominates. However, we know that the distribution without defection constraints and with $\rho = 1$ is the steady state distribution for an efficient allocation, so using the mean of the distribution would be inappropriate. Our approach here is to use a measure of the distance of the limiting distribution of expected utilities from the optimal distribution as a measure of the welfare cost of inflation. Since for the welfare measure we use it is necessary that limiting probabilities be bounded away from zero, we use a kernel estimation technique (Silverman 1986). That is, we estimate the density function $f(x)$ associated with the limiting distribution of expected utilities by

$$f(x) = \frac{1}{h} \sum_{i=1}^{n} \kappa_i K(\frac{x - w_i}{h}),$$

where h is the "window width," κ_i is the limiting probability associated with expected utility level w_i, n is the number of grid points for expected utilities, and $K(\cdot)$ is a probability density function. For $K(\cdot)$, the standard normal density function provided a good fit of the estimated density function to the limiting probabilities. Choosing a grid for x, we let $f_j = f(x_j)$ for $j = 1, 2, ..., \ell$, and then compute the "welfare loss," $Z(g, f)$ associated with the limiting distribution $f(x)$, by

$$Z(g, f) = \sum_{j=1}^{\ell} h g_j (\log g_j - \log f_j),$$

where $g(x)$ denotes the limiting distribution with pure credit ($\rho = 1$). This distance measure is adapted from an approach in information theory used by Kullback (1959).

The last column of Table 1 shows the welfare costs of inflation (using the above distance measure). Note that these costs do not increase monotonically with inflation (though at low levels of inflation and given the small effects, this may just be computational error), but they are small, in general. In fact, an examination of Figure 7 shows that the distribution of expected utilities in the economy where currency is driven out is not that different from the distribution under a Friedman rule (the optimum).

Money and Credit With Defection Constraints

Table 2 shows results for the case with defection constraints imposed. Here, note the differences with Table 1. With defection constraints, the variability in expected utility (column 3) and in consumption (column 4) is considerably smaller. In the examples, the participation constraint binds only at the lower bound on expected utilities, which limits the incentives that financial intermediaries can give to consumers, especially near this lower bound. Since part of the effect of the participation constraint is to eliminate the lower tail of the limiting distribution of expected utilities, and since consumption increases with the level of expected utility, the interest rate must fall in order for the bond market to clear in the steady state. That is, a lower interest rate induces consumers (through intermediary contracts), to initially trade away claims to future consumption, so that consumption tends to be lower in the steady state. Note that, for low rates of inflation, the annualized interest rate (column 5 in Table 2) is considerably less than 4%, which is the interest rate for any inflation rate when there are no defection constraints.

Table 2: $\rho = .81$, **With Defection Constraints**

Annual Inflation Rate	Mn. E.U.	SD(EU)	SD(Cons)	Int. Rate	Welfare Cost
-1.37%	.6308	.0035	.0783	1.37%	478.56
10%	.6313	.0056	.0635	3.35%	373.65
100%	.6298	.0114	.0952	3.72%	130.54
1500%	.6209	.0136	.2609	3.89%	60.87
>non-mon. threshold	.6166	.0243	.2813	3.82%	12.58

Note also in Table 2 that, in contrast to Table 1, the mean level of expected utility falls and the variability in expected utilities rises with the inflation rate. Variability rises due to the fact that the value of abandoning the intermediary contract for any consumer falls with the inflation rate, which increases the opportunities for punishing consumers for misreporting their incomes, increases the steady state interest rate, and therefore increases opportunities for rewarding consumers as well.

With defection constraints, there are two effects of inflation which have opposing implications for welfare. First, as is the case when there are no defection constraints, higher inflation tends to reduce real cash balances, which then reduces consumers' ability to insure against income shocks in the state where they are subject to limited participation. The variability of consumption conditional on expected utility goes up, and the steady state allocation is therefore less efficient. Second, higher inflation causes the value of the outside option (trading on the money market forever) to go down, which is good for incentives, and therefore good for efficiency. Quantitatively, we see in Table 2 that the second effect is much larger than the first. That is, welfare costs fall with inflation, since the steady state distribution gets closer to the optimum as the inflation rate increases. The distribution which is closest to the optimum is the one where currency is driven out of the system, i.e. where the economy operates with credit only. This is a striking result, which is pictured in Figures 8 and 9. Figure 8 shows the Friedman rule distribution relative the optimum, and Figure 9 shows the "no currency" distribution relative to the optimum.

The result from the economy with participation constraints that it would be efficient to eliminate the use of government-supplied currency, is something that we take seriously. The benefits from the use of Federal Reserve notes in the U.S. economy are small, as currency is used in a small fraction of the total value of transactions. The cost of the use of currency is that there is little or no record-keeping associated with currency transactions. Alternatively, when transactions take place through the check-clearing system or by credit card, the record-keeping involved permits the use of intertemporal incentive devices by financial intermediaries, which makes the payments system work more efficiently. There may be a need in some transactions for a mechanism which allows for decentralized transfer of wealth, but we currently have a technology that permits this through privately-supplied smart cards.

Inflation in an Economy With A Very Inefficient Credit System

The key results of the previous sections are first, that without defection constraints the effects of inflation are small, and with defection constraints the effects are large. Second, when there are no defection constraints a Friedman rule is optimal, so that the economy could operate efficiently with currency only, while with defection constraints the economy operates most efficiently with credit only. Of course, these results were obtained for a model which was calibrated to the U.S. economy, where currency accounts for a relatively small fraction of the total value of transactions. One might suspect that some of these results would change significantly if we computed solutions given parameter values which make money much more important in transactions. In this subsection, we repeat the computations of the previous two subsections for an economy where credit is as inefficient as possible, and money is as useful as it can be, i.e. the case where $\rho = 0$.

Table 3: $\rho = 0$, No Defection Constraints

Annual Inflation Rate	Mn. E.U.	SD(EU)	SD(Cons)	Welfare Cost
-3.94%	.6270	.0519	.1508	0
10%	.6277	.0499	.1732	2.63
100%	.6239	.0416	.2244	2.31
>non-mon. threshold	.5639	0	.6000	-

We set all parameters, except ρ, at their previous values. Table 3 shows solutions for the case where there are no defection constraints. We did not compute results for high rates of inflation (e.g. 1500% inflation as in Table 1), as this led to non-convergence problems. Note that, as one would expect, the effects of inflation in Table 3 are much larger than in Table 1. Given the inflation rate, the standard deviation of expected utility is lower, the standard deviation of consumption is higher, and the welfare cost of inflation is higher with $\rho = 0$. For the nonmonetary economy, in the last row of Table 3, the steady state distribution of expected utilities is indeterminate. That is, if currency is not used, then there is no means for insuring against income shocks, so that an individual's expected utility will remain constant over time, with the level of expected utility determined by the transfer that the consumer receives from the financial intermediary each period. One steady state distribution is for each consumer to receive a transfer of zero each period, so that everyone consumes autarkically, and this is the distribution used in calculating the statistics in the last row of Table 3.

Table 4: $\rho = 0$,**With Defection Constraints**

Annual Inflation Rate	Mn. E.U.	SD(EU)	SD(Cons)	Int. Rate	Welfare Cost
-1.37%	.6308	.0035	.0783	1.37%	487.56
10%	.6308	.0058	.0781	2.09%	385.03
100%	.6261	.0093	.1802	3.79%	122.25
>non-mon. threshold	.5639	0	.6000	-	-

Table 4 shows results for $\rho = 0$ when defection constraints are imposed. Here, as compared to Table 2, the effects of inflation are not necessarily larger when $\rho = 0$. The standard deviation of consumption is higher for each inflation rate as, conditional on the level of expected utility the probability of non-participation is much higher (1 as compared to .19 in Table 2), and the variability of consumption is much higher conditional on non-participation, for an inflation rate above the Friedman rule rate. The welfare cost of inflation is higher for 10% inflation in Table 4, and higher for 100% inflation in Table 2. Further, the standard deviation of expected utilities is higher for 10% inflation in Table 4, and higher for 100% inflation in Table 2. In terms of the welfare costs of inflation, these costs are not minimized in Table 4 at a rate of inflation below 100%, so some of the flavor of the results for the model calibrated to U.S. data remain. The key difference here is that an economy without currency (the last row in Table 4) can not be more efficient than having some inflation rate such that currency is held, as the no-currency economy is autarkic. The optimal inflation rate, according to the welfare metric we are using here, would be something between 100% and approximately 1700% per annum. Again, we could not get a precise estimate of this optimal inflation rate because of non-convergence problems at high inflation rates.

7. CONCLUSION

We have constructed a model of money and credit with dynamic private information, and examined the implications of this model for the effects of inflation on welfare, the distribution of consumption, and the distribution of wealth. Consumers have random unobservable endowments and write long-term contracts with financial intermediaries. A role for money arises due to random limited participation. That is, contact with financial intermediaries is such that consumers are not always able to smooth consumption adequately with credit, and instead must sometimes resort to currency transactions.

The key findings relate to how defection constraints, arising from the option consumers have of abandoning contracts with financial intermediaries in favor of spot trading on the money market, alter the effects of changes in the money growth rate in the steady state. In particular, in the absence of defection constraints a Friedman rule is optimal, and increases in the money growth rate and inflation tend to reduce the mean level of expected utility and the variability in expected utility across the population and to increase the variability in consumption. The effects of inflation are small. Alternatively, with defection constraints the results change dramatically. In general, defection constraints tend to reduce the variability in expected utility across the population. Higher inflation tends to increase the variability in expected utilities, while (according to the metric used) pushing the steady state allocation closer to the optimum. In this case, it is efficient to eliminate the use of currency altogether, as this improves incentives in financial contracts, and the resulting welfare benefits outweigh the costs resulting from less efficient risk sharing in the absence of money.

Our results are of course sensitive to the information structure we have assumed. First, if we assumed that there was private information concerning preference shocks rather than endowments, as for example in Atkeson and Lucas (1992), then we would not obtain a steady state limiting distribution with mobility in the case without defection constraints. However, the defection constraint case would yield qualitatively similar results with preference shocks. What makes preference shocks unattractive for this application is that a preference shock model will yield allocations where consumption is more variable than income (assuming fixed incomes), which is at odds with observation. Preference shocks do have the advantage, however, that it is more plausible that they be unobservable than that incomes be unobservable.

Second, it is clearly important that we have assumed that money balances are observable. This assumption is key to the model's tractability, but it lacks superficial plausibility. In practice individual cash balances would seem to be quite difficult to observe. Further, some results in the literature (e.g. Cole and Kocherlakota 1997) suggest that, if assets can be hidden, then efficient allocations are identical to what can be achieved in an incomplete markets economy where all trade in contingent claims is shut down. These results, though limited to fixed-interest-rate economies, could be interpreted as having negative consequences for private information theory. That is, we might ask why we should go to the work of analyzing a complicated private information economy rather than saving some trouble and analyzing a related, and much simpler, incomplete markets setup. In defense of our model, it might be possible to consider a setup where money can be hidden, but it is in everyone's interest to reveal their true cash balances (note that allocations have the property that higher cash balances are always associated with a higher expected utility entitlement), and where the allocation does not collapse to the incomplete markets allocation. This approach, however, is beyond what we know how to do. Of course, assets in practice are neither perfectly observable or perfectly unobservable, so the issue of what works better for particular applications, incomplete markets models or private information models, will depend on the question we have in mind, and the quantitative factors that play a role in answering the question. We intend to explore this idea further in future work.

Given the novel approach to modeling money and credit here, there are many possible extensions. It would be interesting to explore issues related to capital accumulation,[5] an extension that would be relatively straightforward, or to investigate business cycle phenomena and cyclical monetary policy, which would require some additional theoretical advances.

[5]Aiyagari and Williamson (1997) is a private information model of dynamic contracts with capital accumulation but without monetary exchange.

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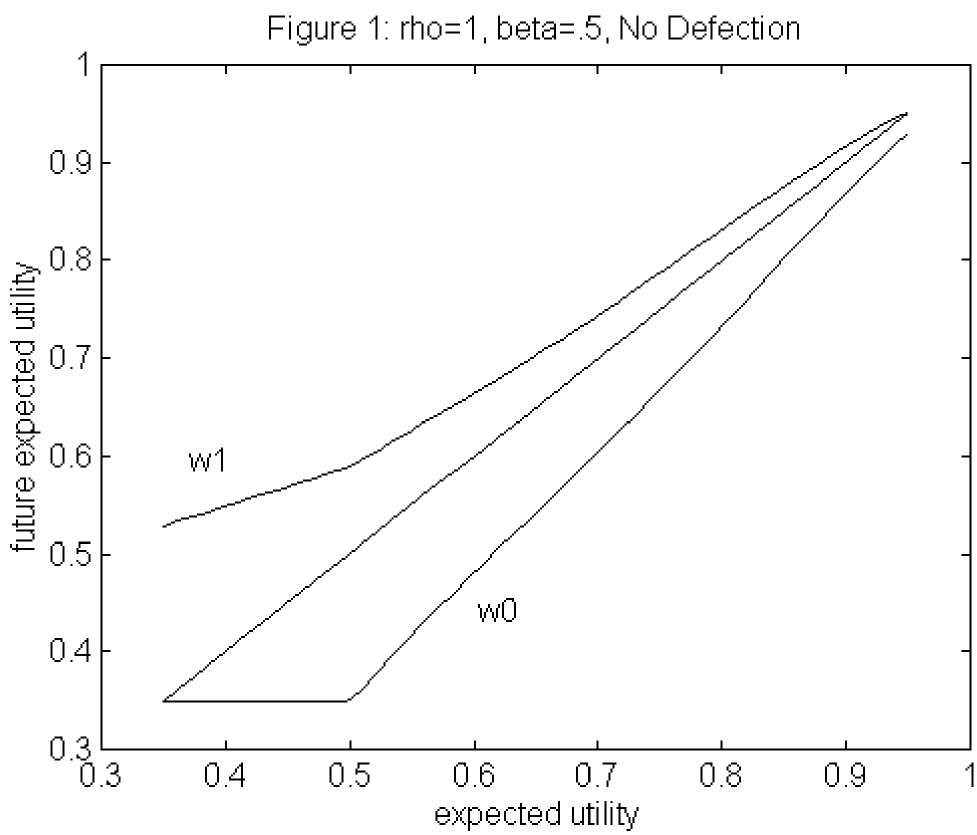
Figure 1: rho=1, beta=.5, No Defection
future expected utility
expected utility
w1
w0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig. 1.

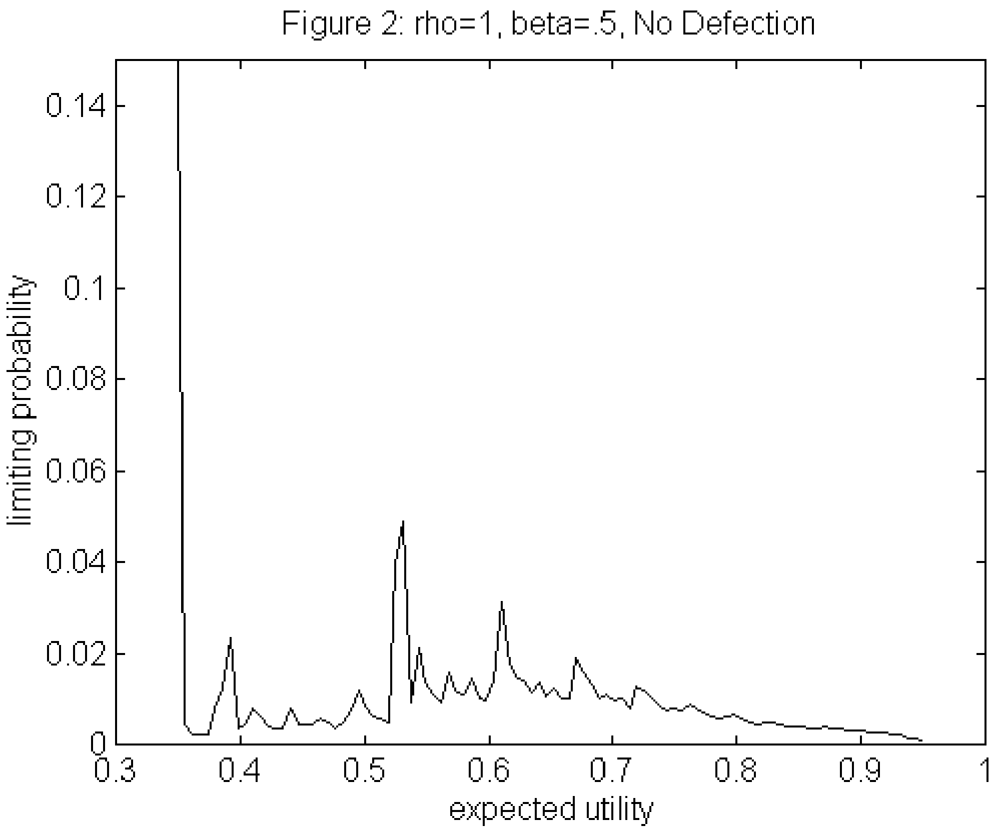
Figure 2: rho=1, beta=.5, No Defection
limiting probability
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
expected utility

Fig. 2.

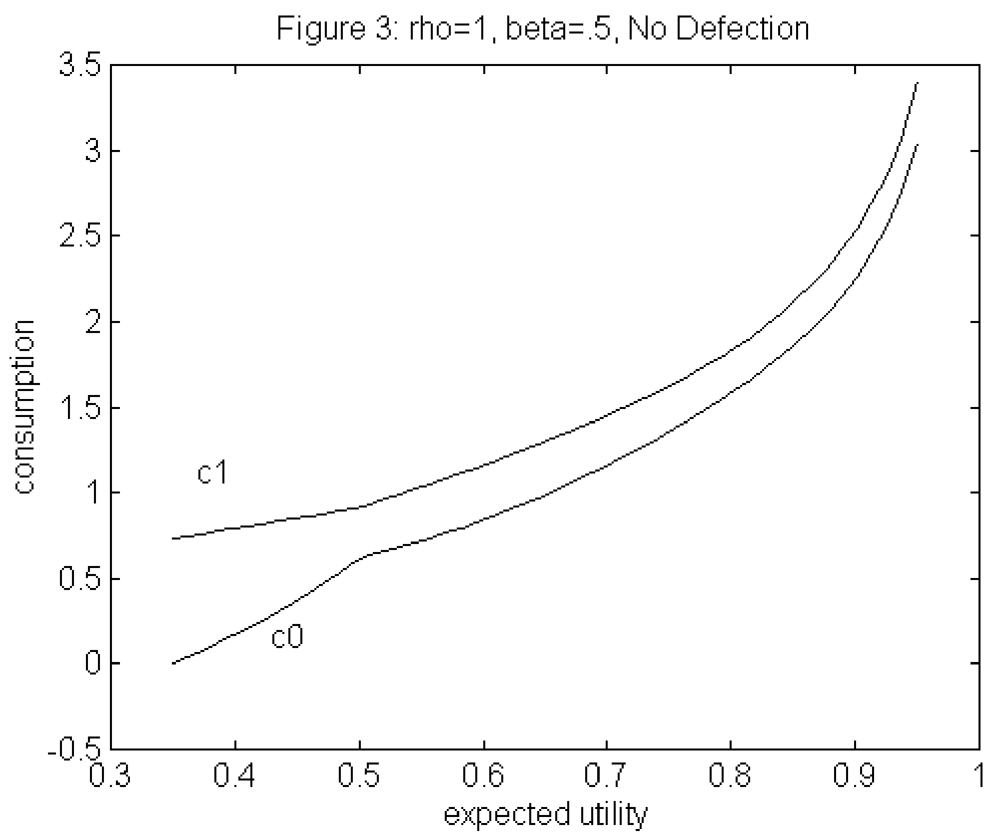
Figure 3: rho=1, beta=.5, No Defection
consumption
expected utility
c1
c0
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

FIG. 3.

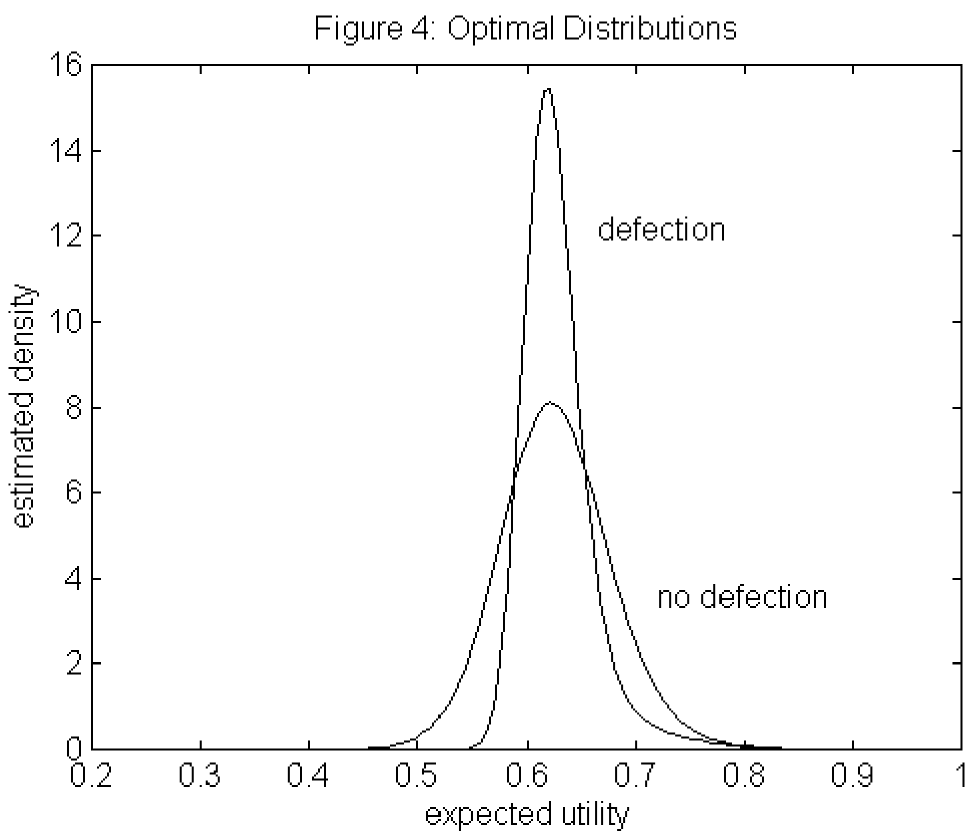

Figure 4: Optimal Distributions
16
14
12
10
8
6
4
2
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
estimated density
expected utility
defection
no defection

FIG. 4.

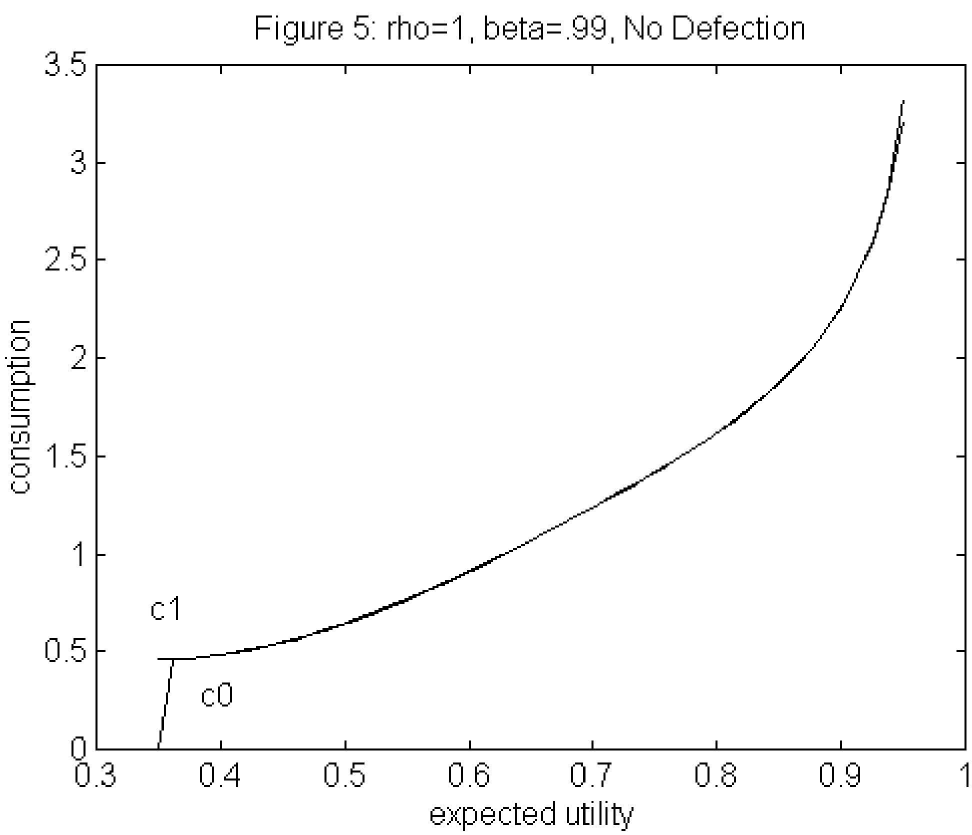
Figure 5: rho=1, beta=.99, No Defection
consumption
expected utility
c1
c0
3.5
3
2.5
2
1.5
1
0.5
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

FIG. 5.

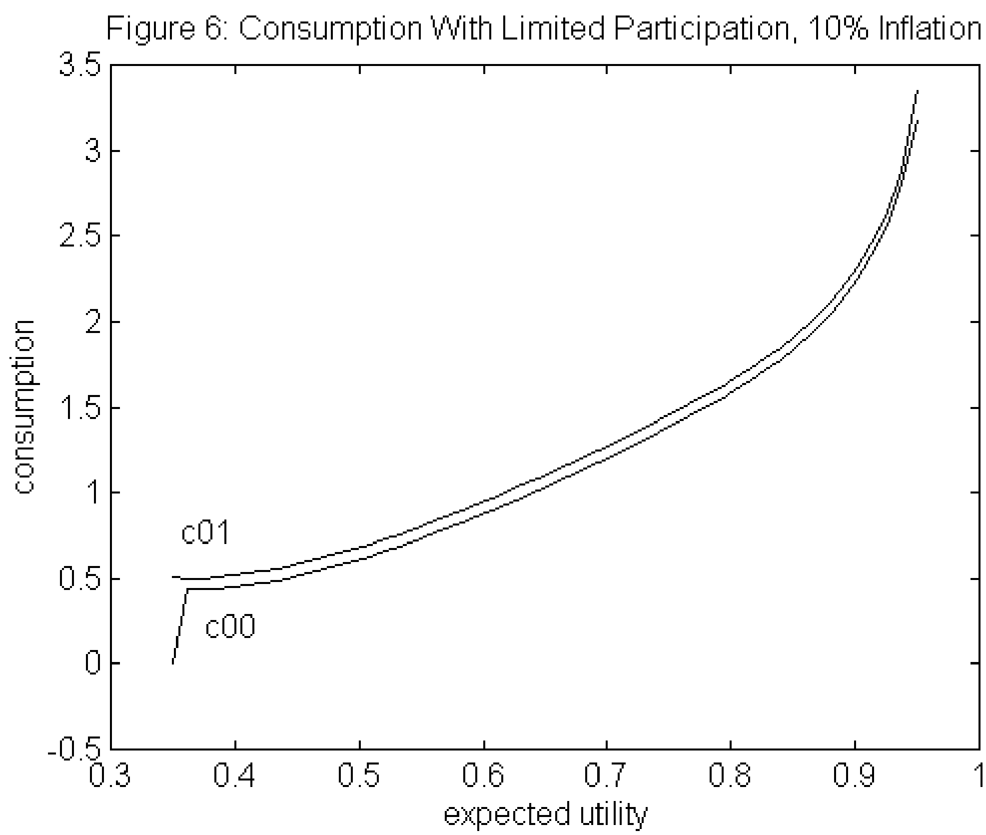
Figure 6: Consumption With Limited Participation, 10% Inflation
consumption
expected utility
c01
c00
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig. 6.

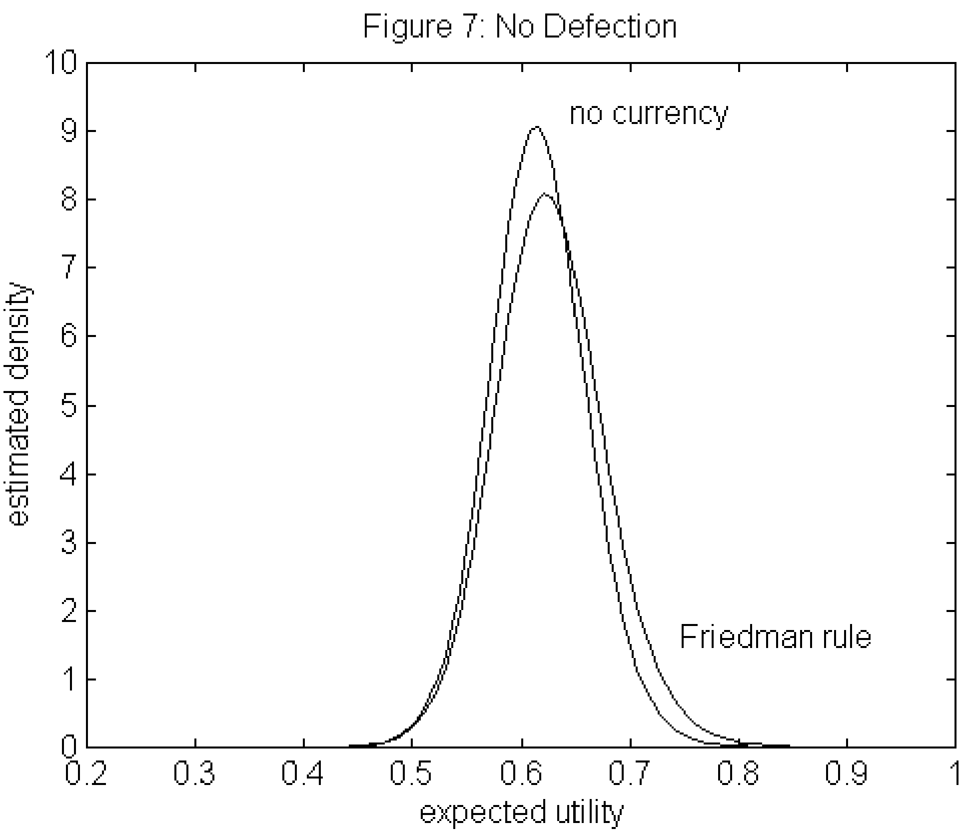

Figure 7: No Defection
no currency
Friedman rule
estimated density
expected utility
0
1
2
3
4
5
6
7
8
9
10
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

FIG. 7.

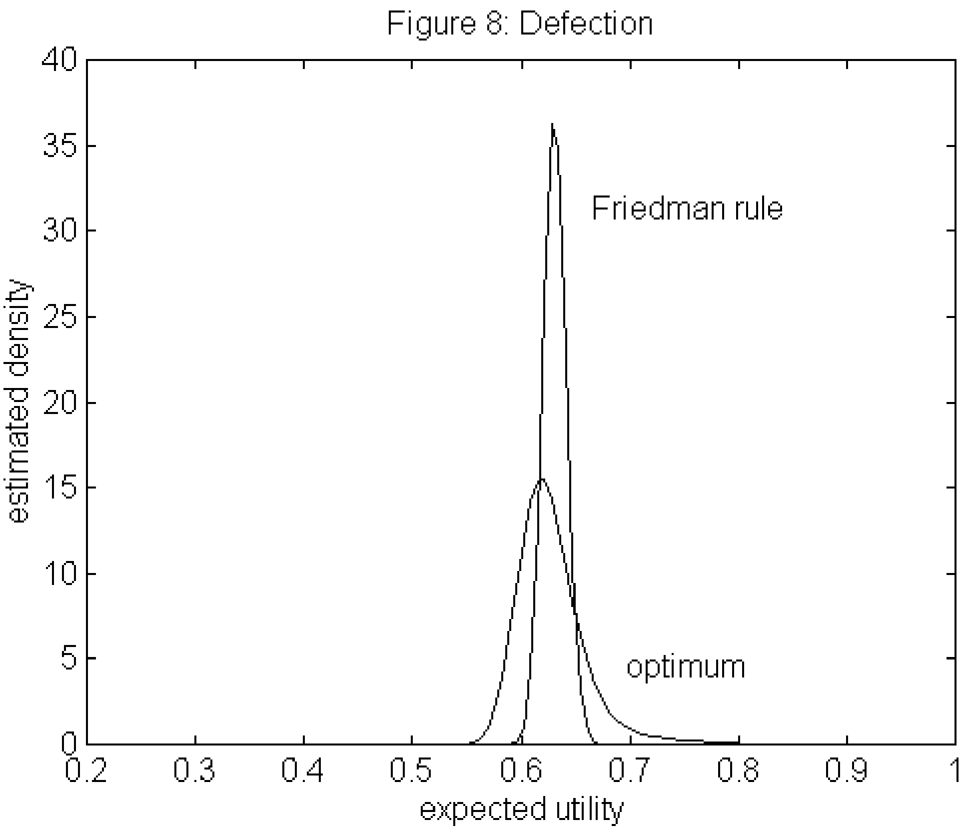
Figure 8: Defection
estimated density
expected utility
Friedman rule
optimum
0
5
10
15
20
25
30
35
40
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

FIG. 8.

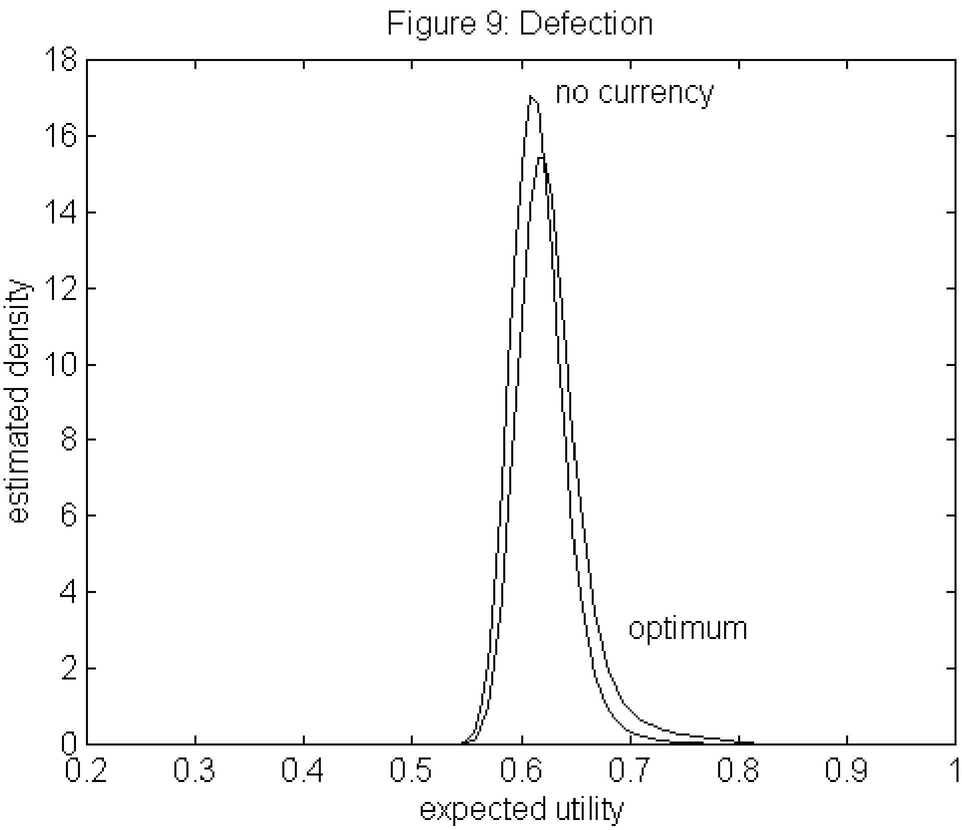
Figure 9: Defection
no currency
optimum
estimated density
expected utility
0
2
4
6
8
10
12
14
16
18
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig. 9.