

Informational Cascades with Continuous Action Spaces*

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Abstract

The recent literature on informational cascades has nurtured the impression that cascades can occur only if the action space is coarser than the signal space. In particular, it is sometimes claimed that with continuous action spaces cascades are impossible. In this note we present a simple counter-example showing that this conclusion is not necessarily warranted.

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1 Introduction

The recent literature on informational cascades has nurtured the impression that cascades can occur only if the action space is coarser than the signal space. In particular, it is sometimes claimed that with continuous action spaces cascades are impossible. For example, Gul and Lundholm (1995, p. 1057) write “Informational cascades can be eliminated by enriching the setting in a way that allows prior agents’ information to be transmitted; a finer set of action choices eliminates statistical cascades,... .” Similarly, Vives (1996, p. 591) claims that “With continuous action spaces ... and agents being rewarded according to the proximity of their action to the full-information optimal action convergence to the latter obtains.” In this note we present a simple counter-example showing that this conclusion is not necessarily warranted.¹

2 The example

Consider a continuous action space A which consists of all points on the unit-circumference of a circle. A priori the unique optimal action $a^* \in A$ is uniformly distributed on A . Agents are rewarded according to the quadratic distance to a^* . Let $d(a) = |a - a^*|$ denote the shortest arc length between a and a^* . Utility for all agents is then given by $u(a) = -d(a)^2$.

Agents are ordered according to some fixed sequence $n = 1, 2, 3, \dots$ and choose an action $a \in A$ which maximizes their expected utility given their beliefs about a^* . Each agent observes all the actions taken by others before him. Beliefs are updated according to the observed actions and an independent private signal each agents receives before deciding.

There are two possible signals, s_1 and s_2 , where s_i means that a^* is in the set S_i with probability $p \in (\frac{1}{2}, 1)$. The (convex) sets S_1 and S_2 partition A into two arcs of length $r > \frac{1}{2}$ and $1 - r$, respectively. That is, p measures the reliability of the signal, and reliability is assumed to be the same for

¹Banerjee (1992) obtains herding with a continuous action space but his payoffs are degenerate (payoffs are zero unless the optimal action is chosen exactly).

both signals,²

$$p := \text{prob}(a^* \in S_i | s_i).$$

For reasons that become clear later we assume that $p > r$.

We say that an informational cascade occurs if agent n ignores his signal, i.e. if he chooses the same action regardless of his signal. Note, that all agents following n will also choose to ignore their signals as no additional information is revealed by agent n 's action.

Let q denote the posterior probability that $a^* \in S_1$

$$q := \text{prob}(a^* \in S_1 | h, s_i),$$

which depends on the current signal s_i and on h , the history consisting of all actions taken so far. Let a_i denote the midpoint of the arc S_i . It is easy to see that depending on (h, s_i) either a_1 (the midpoint of arc S_1 which has length r) or a_2 (the midpoint of S_2) is the optimal action. In case of indifference, *any* action in A is optimal. An agent will prefer a_1 to a_2 if

$$\begin{aligned} & q \int_0^{r/2} a^2 \frac{1}{r} da + (1-q) \int_{r/2}^{1/2} a^2 \frac{1}{1-r} da \\ & > q \int_{\frac{1-r}{2}}^{1/2} a^2 \frac{1}{r} da + (1-q) \int_0^{\frac{1-r}{2}} a^2 \frac{1}{1-r} da \end{aligned}$$

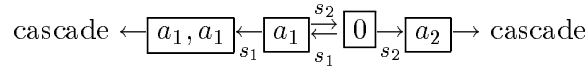
that is, if and only if $q > r$. Note that for the first agent $q = p$. As we have assumed that $p > r$, the first agent will follow his signal. Later agents care only about the *reduced history* which consists of the difference in the frequencies of a_1 and a_2 . Neither the order in which those actions were taken nor the frequencies matter, only the difference in frequencies. To see this note that as long as no cascade has formed all actions are fully revealing of the signal. Therefore, if e.g. k_1 times a_1 and $k_2 - 1$ times a_2 has been observed and the current signal is s_2 , then q is given as

$$\begin{aligned} \text{prob}(a^* \in S_1 | k_1 \times s_1, k_2 \times s_2) &= \frac{p^{k_1}(1-p)^{k_2}}{p^{k_1}(1-p)^{k_2} + p^{k_2}(1-p)^{k_1}} \\ &= \frac{p^{k_1-k_2}}{p^{k_1-k_2} + (1-p)^{k_1-k_2}} \\ &= \text{prob}(a^* \in S_1 | (k_1 - k_2) \times s_1). \end{aligned}$$

²Alternatively, signal technologies can be defined by specifying the probability of a signal conditional on the state. Equal reliability of both signals in our sense requires $\frac{\text{prob}(s_1|a^* \in S_1)(1 - \text{prob}(s_1|a^* \in S_1))}{\text{prob}(s_2|a^* \in S_2)(1 - \text{prob}(s_2|a^* \in S_2))} = \frac{(1-r)^2}{r^2}$.

It is now straightforward to determine the histories for which a cascade must begin. Consider a reduced history consisting only of one a_2 . What will the next agent do? Clearly, he will choose a_2 in case of s_2 . In case of receiving s_1 both signals cancel out and his posterior belief will be $q = \frac{1}{2}$. Given this belief a_2 is the unique optimal action. Therefore, an agent facing a reduced history consisting of only one action s_2 will ignore his signal and so will all other agents following him. Likewise, reduced histories consisting of (a_1, a_1) trigger a cascade as a signal of s_2 yields $q = p$, and hence, a_1 is optimal.

Noting that an s_2 -signal takes an a_1 -history back to the null history the dynamics of the system can be described by the following figure.



Result Since all transitions occur with strictly positive probability, the process enters into a cascade with probability 1 eventually.

In fact, generally a cascade will start very soon. Of course it is interesting whether the right or the wrong cascade is being triggered. To answer this question we need to calculate the transition probabilities of the Markov chain shown in the figure. Those transition probabilities depend only on the probability of receiving a certain signal which, in turn, depend on the true state. Note first that the unconditional probability of signal s_1 is given by

$$\text{prob}(s_1) = \frac{r + p - 1}{2p - 1}.$$

Using this expression we can now state the transition probabilities of the Markov chain as follows:

$$t_1 := \text{prob}(s_1 | a^* \in S_1) = 1 - \text{prob}(s_2 | a^* \in S_1) = \frac{p}{r} \text{prob}(s_1)$$

$$t_2 := \text{prob}(s_1 | a^* \in S_2) = 1 - \text{prob}(s_2 | a^* \in S_2) = \frac{1 - p}{1 - r} \text{prob}(s_1).$$

Since the transitions probabilities define a standard gambler's ruin problem, the probability for an a_1 -cascade if $a^* \in S_i$ is

$$\frac{\left(\frac{t_i}{1-t_i}\right)^3 - \left(\frac{t_i}{1-t_i}\right)^2}{\left(\frac{t_i}{1-t_i}\right)^3 - 1}$$

$a^* \in$	r	p	P
S_1	.51	.57	.53
S_1	.51	.90	.90
S_1	.80	.90	.98
S_2	.51	.57	.66
S_2	.51	.90	.99
S_2	.80	.90	.75

Table 1: Probabilities P for correct cascades.

(see e.g. Karlin and Taylor, 1975). An a_2 -cascade occurs with the remaining probability. Table 1 shows the probability, P , for ending up in a correct cascade for a number of possible parameter values.

If the arcs S_1 and S_2 are of almost equal size and if the signal's reliability is not too good, the probability for the wrong cascade can approach $\frac{1}{2}$.

3 Discussion

We provide an example refuting the frequently stated conjecture that informational cascades will not occur when the action space is continuous and the payoff function non-degenerate. This result is driven by the assumption that agents choose actions on a circle, that there are only two signals and that agents want to minimize the quadratic distance to some a priori uniformly distributed optimal action. This leads agents rather soon to ignore their signal. Cascades start with probability one in finite time. Cascades in which agents herd on the wrong action may be quite frequent if the signal has a low reliability.

One may argue that the example is due to fact that only two actions can be optimal actions implying that the example is no different from the binary choice case. But note that the restriction to two optimal actions is endogenously rather than exogenously imposed and depends in an important way on the signal space. For example, with three signals (corresponding to three arcs partitioning the circle) cascades would not occur. In fact, in this case agents would always reveal their signals by moving to points other than the midpoints of the arcs. In case of four arcs cascades may occur for a short while (signals may 'cancel out') but will eventually be stopped. This shows the crucial role of the signal and action spaces and their structure. It would be interesting to know whether other spaces (e.g. a torus) would

yield similar behavior.

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