A Model of Bargaining with the Possibility of Arbitration

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Abstract

We consider an alternating offer bargaining model in which the players may agree to call in an arbitrator in case of disagreement. The main message of our study is that the mere presence of an arbitrator - who can only become active with the consent of both parties - in the background of negotiations may entirely drive their outcome. We discuss the implications of this result both for theories of arbitration and for the interpretation of cooperative bargainining solutions.

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1 Introduction

In standard bargaining models the only alternative to agreement on a partition is disagreement. Since ancient times, however, social institutions have offered another possibility for resolving disputes when they come to a deadlock: the parties can call in a third party who is perceived as impartial, the arbitrator. While there exists a huge industrial relation literature on the effects of arbitration on negotiations, and while there are several formal models of the arbitration phase itself (e.g. Brams (1990), Farber and Bazerman (1986)), the link between the arbitration outcome and the bargaining outcome has not received much formal attention in economics [1]. This may be due to the perception that, if agreements have to be self-enforcing (i.e., equilibria), the presence of an arbitrator in the background should not matter, at least in the case in which the consent of both parties is needed to take the dispute to arbitration. In that case, it would appear, the player who stands to lose by calling in the arbitrator - with respect to the standard bargaining outcome - may simply ignore any proposal of arbitration. By contrast, it has been observed how in other cases the presence of an arbitrator can make a difference[2]. In this paper we will show how and why 'arbitration matters' even when no party can enforce arbitration on its own, and even when the arbitrator's services are not used.

Our modelling strategy is very simple. We consider the basic version of the standard alternating offers bargaining model (Rubinstein (1982)) in which two parties negotiate over the division of a pie. The only modification we add is this: at any point in time, the parties can agree to call in the arbitrator. The arbitrator's decision is commonly known and binding; but *both* parties must agree to call in the arbitrator for the dispute to be settled by him. In other words, either party has *power of veto* on the event that the dispute goes to arbitration[3]. While we discuss later the relevance of this particular setting , we anticipate now, in summary, its interesting analytical implications.

The main message of our study is that *the sheer presence of an arbitrator - who can*

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[1] A notable and recent exception is Compte and Jehiel (1995), which we discuss later in the paper.

[2] Consider, for example what Brams and III (1983) term 'the paradox of arbitration'. See also Brams and III (1986) and, more recently, Zeng et al. (1996).

[3] Compte and Jehiel (1995) study the case where either party has the power of calling in the arbitrator.

only become active with the consent of both parties - in the background of negotiations may entirely drive their outcome. More precisely, our first result states that (as long as costs of arbitration are sufficiently small) there exists a subgame perfect equilibrium outcome of the modified standard noncooperative bargaining model which coincides - modulo the arbitration cost - with the arbitrated outcome, *no matter what this outcome is.* So, if for example the arbitrator deems it appropriate to assign almost the entire cake to one of the two players, there will be a subgame perfect equilibrium where the veto power of the disadvantaged player comes to nothing: he will voluntarily accept (and propose) a partition that assigns him a minimal part of the cake. In the limit as the cost of arbitration tends to zero, *any* arbitrated outcome can be exactly supported in equilibrium as a negotiated outcome (for any discount factor).

When the arbitrated partition is "unbalanced" (particularly favourable to one of the two players), the standard Rubinstein outcome can also be supported in equilibrium. We show that, as is usual in this type of models, the presence of two equilibria in fact creates a *continuum* of equilibrium partitions, and even allows for the possibility of equilibrium *delays.* However, we also show that when the arbitrated outcome is sufficiently close (depending on the discount factor) to the fifty-fifty partition, the two equilibria can no longer coexist: the arbitration-driven equilibrium partition becomes the *unique* subgame perfect equilibrium outcome of the game. Note well: because of the Pareto optimality of the standard bargaining outcome, even in the case where the arbitrated outcome is not unbalanced, at least one of the players must be worse off by submitting to arbitration. Yet, he has no alternative in equilibrium but to voluntarily submit.

There is a second viewpoint that has raised our interest in this study of arbitration, which is more methodological in nature. Arbitration is important when it comes to justifying and interpreting *cooperative* bargaining solutions. It is sometimes informally suggested that the outcomes of such solutions can be (a) interpreted as arbitrated outcomes, and (b) used to predict the outcome of noncooperative negotiations when the parties may resort to arbitration. In particular, in his influential textbook Myerson (1991) asserts that:

> "The outcomes of effective negotiations in which the players have equal

> opportunity to participate should be the same as the recommendations that would be made by an impartial arbitrator who knows the information that is common knowledge among the players during the negotiations." (p.374)

Myerson suggests that a "focal arbitrator" may be able to drive the attention of the players to a particular equilibrium. Our results allow one to qualify the line of argument suggested by Myerson. The assertion that, because arbitration can drive the outcome of noncooperative negotiations, an arbitration model can be used to study their outcome is broadly supported. Yet, in our case the role of the arbitrator is quite different from that envisaged by Myerson: it is not to render a particular equilibrium out of a given set focal, but rather to determine the equilibrium set itself.

We believe that our model provides a formal basis to interpret the axioms characterising cooperative bargaining solutions as describing features of the behaviour of an impartial arbitrator, rather than describing features of an unmodeled noncooperative negotiation procedure. This distinction is important: game theorist, in fact, seem to be divided as to which interpretation is the more appropriate. For example, while the axioms characterising the Nash Bargaining Solution (NBS) were certainly thought by Nash himself as summarising the features of a noncooperative game, those same axioms are often seen to be interpreted as "fairness requirements" for "impartial" arbitration[4]. We find that, in general, the link between the axioms characterising a particular cooperative solution and the structure of a noncooperative bargaining procedure implementing that solution is somewhat obscure. Our model, on the other hand, provides an explicit noncooperative structure through which a given partition chosen by an arbitrator can be *implemented*. First, a set of axioms that allows a direct and transparent ethical interpretation can be used to select the appropriate arbitrated outcome. Then, players who play noncooperatively according to the rules of our model will reach an agreement consistent with that arbitrated outcome.

More in general, we hope that our approach can provide a basis for the study of the

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[4] For example, Binmore (1997) argues strongly against the fair arbitration interpretation of the NBS, whereas Luce and Raiffa (1957) promoted the latter interpretation, also upheld recently by Border and Segal (1997).

fascinating phenomenon whereby our notions of "fairness" affect negotiations between selfish rational players.

2 The Model

Two players bargain over how to split between them a surplus normalised to unity. The players' utilities are linear in the share of the surplus which they receive, and they discount utility over time by a factor $\delta \in (0,1)$. As in the standard alternating offers model, each player i, $i = 1, 2$, in turn proposes a partition of the surplus, which his opponent can either accept, ending the game, or reject. In addition, when rejecting a proposal the responding player can either follow with a counter-offer in the subsequent round; or propose to call in an arbitrator to settle the dispute. The outcome of the arbitration is common knowledge, and results in the partition (s_1, s_2), with $s_1, s_2 \geq 0$, and $s_1 + s_2 = 1$ (i.e. the arbitrated outcome is always efficient). However, we assume that arbitration is costly[5], weighing equally[6] on both players; consequently, we assume that the actual payoffs for the players are reduced by the same amount $\varepsilon \leq min[s_1, s_2]$. If arbitration is proposed, player j has to decide whether to accept, in which case the game ends with the players receiving the arbitrated outcome net of costs; or to reject and let player i again propose a partition of the surplus in the following round. The game continues in this away until a proposal is accepted; in case of perpetual disagreement, both players end up with a null payoff. Consequently, we can distinguish two types of subgames:

(a) *subgame* G^i, which starts with the proposal of a partition by player i ; and

(b) *subgame* D^i, which starts with the choice of player i whether to accept the proposal of an arbitration or make a counteroffer in the following round.

Each subgame of type G^i can induce either agreement on a partition, or a subgame of type G^j, or a subgame of type D^i; and each subgame of type D^i can induce either

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[5] All our results go through when arbitration costs are zero; however we include positive arbitration costs since they appear to be an important feature of arbitration in practice (e.g. Milner (1993)).

[6] We stick to this assumption for notational simplicity, as allowing for different arbitration costs for the two players would not affect the quality of the results.

agreement on an arbitrated outcome, or a subgame of type G^i, as shown in Figure 1.

Figure 1: Relationship among subgames

The detailed structure of the game is depicted in Figure 2; it begins at time $t = 0$ with negotiations between the two players: player 1 proposes the partition[7] $g^1 = (g_1^1, g_2^1)$, which player 2 can either accept (**A**) - ending the game-, reject and make a counteroffer (**R**), or reject and propose to call an arbitrator(**S**) (subgame of type G^1).

If player 2 rejects and proposes arbitration, the game moves to a subgame of type D^1, in which player 1 has to decide whether to agree on going to arbitration (**y**), in which case players' payoff pair is $(s_1 - \varepsilon, s_2 - \varepsilon)$; or to reject and enter again a subgame of type G^1, at time $t = 1$, in which he is once again proposer. Alternatively, if player 2 rejects to make a counteroffer to player 1, the game moves to the following round, at time $t = 1$, entering a subgame of type G^2. Symmetrically, player 1 can either accept (**A**) - ending the game-, reject and make a counteroffer (**R**) (entering a subgame of type G^1 in the following period), or reject and propose arbitration (**S**, for "Solomon"), in which case the game moves to a subgame of type D^2. Now player 2 can either accept arbitration (**y**), yielding $(s_1 - \varepsilon, s_2 - \varepsilon)$; or reject (**n**) and make a counteroffer in the following round, so that play moves to a subgame of type G^2, and so on.

In the next propositions we will completely characterise the solution to this game.

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[7] Proposed partitions are denoted by ${}_t g^i = ({}_t g_1^i, {}_t\, g_2^i)$ in Figure 2; in the text we omit the time subscript for expositional clarity.

Figure 2: Structure of the game

First of all notice that all subgames of the same kind starting with a move by player i are identical and have the same set of equilibrium payoffs. Then, let $\overline{g}_j^i$ and $\underline{g}_j^i$ (with $i, j = 1, 2$) be the supremum and the infimum payoffs, respectively, for player j in any subgame perfect equilibrium of subgames of type G^i where player i makes an offer. Similarly, let $\overline{d}_j^i$ and $\underline{d}_j^i$, respectively, be the supremum and infimum equilibrium payoff to player j in any subgame perfect equilibrium of subgames of type D^i where player i has to decide whether to accept going to the arbitrator, or to reject arbitration and make a counteroffer in the next round.

By standard arguments, it follows that subgame perfection requires that the following hold:

$$\overline{g}_1^1 \leq 1 - max\left[\delta \underline{g}_2^2, \underline{d}_2^1\right], \ \underline{g}_1^1 \geq 1 - max\left[\delta \overline{g}_2^2, \overline{d}_2^1\right] \tag{1}$$

$$\overline{g}_2^2 \leq 1 - max\left[\delta \underline{g}_1^1, \underline{d}_1^2\right], \ \underline{g}_1^1 \geq 1 - max\left[\delta \overline{g}_2^2, \overline{d}_2^1\right] \tag{2}$$

$$\overline{d}_1^1 \leq max\left[s_1 - \varepsilon, \delta \overline{g}_1^1\right], \ \underline{d}_1^1 \geq max\left[s_1 - \varepsilon, \delta \underline{g}_1^1\right] \tag{3}$$

$$\overline{d}_2^2 \leq max\left[s_2 - \varepsilon, \delta \overline{g}_2^2\right], \ \underline{d}_2^2 \geq max\left[s_1 - \varepsilon, \delta \underline{g}_2^2\right] \tag{4}$$

3 Arbitration-driven Equilibria

Our first proposition establishes that if arbitration costs are not too high there exists an s.p.e. where the negotiated agreement is "almost" determined by the partition that would be obtained by arbitration itself.

Proposition 1 *$\forall (s_1, s_2) \in [0,1] \times [0,1]$,$\forall \delta \in (0,1)$, $\exists \varepsilon_\delta$ such that, $\forall \varepsilon \leq \varepsilon_\delta$, there exists a s.p.e. in which agreement is reached immediately on the partition $(s_1 + \varepsilon, s_2 - \varepsilon)$.*

Proof. Let

$$\left(g_1^1, g_2^1\right) = (s_1 + \varepsilon, s_2 - \varepsilon)$$

and

$$\left(g_1^2, g_2^2\right) = (s_1 - \varepsilon, s_2 + \varepsilon).$$

The following strategies for player i support (g_1^1, g_2^1)as an s.p.e. negotiated partition:

(i) propose the partition (g_1^i, g_2^i);

(ii) reject any partition which yields him any $x < g_i^j$ and accept any $x \geq g_i^j$;

(iii) always accept arbitration when player j proposes it;

(iv) always propose arbitration when rejecting an offer.

Checking for subgame perfection of *(i)* and *(ii)* is straightforward, thus omitted. For *(iii)* to be optimal, by parts *(i)* and *(ii)* of the equilibrium strategies it must be the case that in subgames of type D^i player i prefers the arbitrated partition to g_i^i one period later; or,

$$s_i - \varepsilon \geq \delta \left(s_i + \varepsilon\right) \forall i.$$

This is equivalent to

$$min[s_1, s_2] \geq \frac{1+\delta}{1-\delta}\varepsilon.$$

Therefore, defining

$$\varepsilon_\delta = \frac{(1-\delta)\, min[s_1, s_2]}{(1+\delta)}$$

optimality is immediately verified for $\varepsilon \leq \varepsilon_\delta$.

Finally, for *(iv)* to be optimal, it must be the case that the payoff that player i gets when rejecting an offer in G^i and proposing arbitration is not lower than the payoff he gets by starting G^i one period later instead. Given part *(iii)* of j's equilibrium strategy, the former payoff is $s_i - \varepsilon$. Therefore this leads to the same inequality considered for *(iii)*. ■

In the equilibrium described in Proposition 1 players avoid waste by agreeing immediately and saving the arbitration cost. This gives an advantage to the first proposer, in that he manages to appropriate completely the benefit from not going into arbitration in equilibrium. The above proposition applies, fixing ε and δ, to a wide range of arbitrated outcomes; furthermore, it follows trivially that as the arbitration cost grows smaller and smaller, it is possible to support *exactly* as a negotiated s.p.e. partition *any* arbitrated partition, even one which is unbalanced in the extreme, in the sense that it assigns the whole surplus to just one of the bargainers. In summary:

Corollary 1 *If* $\varepsilon = 0$, *then* $\forall (s_1, s_2) \in [0,1] \times [0,1]$, $\forall \delta \in (0,1)$, *there exists a s.p.e. in which agreement is reached immediately on the partition* (s_1, s_2) .

So, when players may propose arbitration at no cost, even an arbitrarily small cost of delay is sufficient to destroy any bargaining power the player who is disadvantaged by the arbitrator may have in negotiations. This may seem surprising: one conjecture that would seem plausible is that if a player got very little in the arbitrated outcome and the discount factor was very high, he would have been able to reject (at cost near zero) both unfavourable bargaining proposals and proposals of going to arbitration, to counterpropose instead the Rubinsteinian share in the next period. A proposal of arbitration is just like the proposal of a specific partition. So, if - say - $(s_1, s_2) = (1, 0)$ and player 1 proposes arbitration after rejecting an interior partition, why shouldn't player 2 just reject and propose the standard partition, as he would do in the standard bargaining model? The mistake in this argument is that it overlooks the 'commitment effect' which is created by the distinguished nature of the arbitration proposal. Think of why committing to always rejecting any proposal that yields him less than 1 (and always proposing

$(1,0))$ is not a credible strategy for player 1 in the standard bargaining model: if player 2 'called the bluff' and counterproposed, say, $(\delta+\eta\,, 1-\delta-\eta)$ for a small η, then player 1 would be better off by accepting, since the maximum he can get otherwise is δ. But in the model with arbitration, player 1 can credibly commit to rejecting a partition yielding him $(\delta+\eta)$, since the maximum he can get is now closer to 1 (for small ε) than δ. This effect is reminiscent of the one occurring in models with double-sided outside options (see Ponsati and Sakovics (1995)), with the important difference that in our case the value of the outside option is *endogenously* determined in equilibrium (depending on the s.p.e. of the subgames D^i).

The next proposition will show that the type of equilibrium analysed above is unique when the arbitrated partition is not exceedingly favourable to one of the bargainers, where the definition of 'favourable' depends on the discount factor. We say that arbitrated outcome is *balanced* if

$$s_i > \frac{\delta}{1+\delta} + \varepsilon \forall i.$$

Note that the condition of balancedness, together with the efficiency of the arbitrated partition, implies that $s_i < \frac{1}{1+\delta} - \varepsilon \ \forall i$, and therefore it is equivalent to[8]:

$$s_i \in \left(\frac{\delta}{1+\delta} + \varepsilon, \frac{1}{1+\delta} - \varepsilon\right) \forall i.$$

The following lemma gives conditions under which the maximum s.p.e. payoff for a player in subgames where he is the proposer is greater than the maximum s.p.e. payoff to that player in subgames when the opponent is a proposer, and will be useful in the proof of proposition 2.

Lemma 1 *If the arbitrated outcome is balanced and, for some* i, $s_i - \varepsilon \leq \delta \overline{g}_i^i$, *then* $\overline{g}_i^i \geq \overline{g}_i^j \forall \delta \in (0,1)$.

Proof. Suppose, by contradiction, that

$$\overline{g}_i^i < \overline{g}_i^j$$

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[8] Note that $\left(\frac{\delta}{1+\delta} + \varepsilon, \frac{1}{1+\delta} - \varepsilon\right)$ is not empty if $\frac{1}{1+\delta} - \varepsilon \geq \frac{\delta}{1+\delta} + \varepsilon \Leftrightarrow \varepsilon \leq \frac{1-\delta}{2(1+\delta)}$. Since $min[s_1, s_2] \leq \frac{1}{2}$, then $\varepsilon \leq \varepsilon_\delta = \frac{(1-\delta)min[s_1,s_2]}{(1+\delta)} \Rightarrow \varepsilon \leq \frac{1-\delta}{2(1+\delta)}$.

We will show that in this case i would be willing to accept $\overline{g}_i^j - \eta$ in the subgame G^j for some small η, thus contradicting the definition of $\overline{g}_i^j$.

In fact, suppose that i rejected the partition yielding him $\overline{g}_i^j - \eta$ and made a counteroffer. Then he would get at most $\delta\overline{g}_i^i$: but

$$\begin{aligned} \delta\overline{g}_i^i &< \overline{g}_i^i < \overline{g}_i^j \\ &\Rightarrow \delta\overline{g}_i^i < \overline{g}_i^j - \eta \end{aligned}$$

for η sufficiently small. Suppose, on the other hand, that i rejected the partition yielding him $\overline{g}_i^j - \eta$ and proposed arbitration. Then, if j accepted, i could obtain at most

$$s_i - \varepsilon \leq \delta\overline{g}_i^i < \delta\overline{g}_i^j < \overline{g}_i^j - \eta$$

for a small η. If j rejected arbitration, i could obtain at most

$$\delta\overline{g}_i^j < \overline{g}_i^j - \eta$$

for a small η. ■

Proposition 2 *If the arbitrated outcome is balanced, then* $\forall\delta \in (0,1)\ \exists\varepsilon_\delta$ *such that* $\forall\varepsilon < \varepsilon_\delta$ *the unique s.p.e. payoff pair of the game is* $(s_1 + \varepsilon, s_2 - \varepsilon)$.

Proof. We will show that

$$\overline{g}_i^i \leq s_i + \varepsilon \leq \underline{g}_i^i \forall i,$$

(provided ε is sufficiently small) from which the conclusion of the statement follows immediately.

We proceed in two steps: we begin by proving the first inequality, that is, in any subgame G^i in which he is the proposer, player i cannot obtain more than $s_i + \varepsilon$ in equilibrium (step 1). Next, we prove that the second inequality has to hold, that is, in any subgame in which he is the proposer, player i must obtain at least $s_i + \varepsilon$ in equilibrium (step 2).

Step 1: $\overline{g}_i^i \leq s_i + \varepsilon$**.**

Suppose to the contrary that $\overline{g}_i^i > s_i + \varepsilon$. Distinguish two cases:

1. $s_i - \varepsilon > \delta \overline{g}_i^i$

2. $s_i - \varepsilon \leq \delta \overline{g}_i^i$

We will show that, at an equilibrium, in both cases player j will reject the proposal $(\overline{g}_i^i, 1 - \overline{g}_i^i)$. In particular, we will show that there exists a counterproposal by player j which is more profitable to him than accepting, and which will be accepted by player i.

Case 1. Consider the following action by player j: reject the partition $(\overline{g}_i^i, 1 - \overline{g}_i^i)$ and propose arbitration.

Player i will accept this proposal in equilibrium, since by rejecting he can get at most $\delta \overline{g}_i^i$, while by accepting he gets

$$s_i - \varepsilon > \delta \overline{g}_i^i$$

by assumption.

Then by this action player j gets

$$s_j - \varepsilon > 1 - \overline{g}_i^i$$

because

$$\begin{aligned} \overline{g}_i^i &> s_i + \varepsilon \\ &\Leftrightarrow 1 - \overline{g}_i^i < 1 - s_i - \varepsilon = s_j - \varepsilon. \end{aligned}$$

Case 2. Consider the following action by player j: reject and propose the partition $(\delta \overline{g}_i^i + \eta, 1 - \delta \overline{g}_i^i - \eta)$, with η a small strictly positive number to be determined later.

It must be the case that in equilibrium player i accepts this proposal. In fact, if he rejects and proposes a new partition he gets at most

$$\delta \overline{g}_i^i < \delta \overline{g}_i^i + \eta$$

while if he rejects and proposes arbitration, there are two possibilities. If arbitration is accepted by player j, player i gets

$$s_i - \varepsilon \leq \delta \overline{g}_i^i + \eta$$

(the inequality following from the definition of case 2). On the other hand, if arbitration is rejected, a game of type G^j is induced in the next period, so that player i gets at most

$$\delta \overline{g}_i^j \leq \delta \overline{g}_i^i < \delta \overline{g}_i^i + \eta$$

(where the first inequality follows from lemma 1).

We must now verify that it is profitable for player j to make the proposal just described, rather than accept $1 - \overline{g}_i^i$. This will be the case if:

$$\begin{aligned} 1 - \overline{g}_i^i &< \delta(1 - \delta\overline{g}_i^i - \eta) \\ &\Leftrightarrow 1 - \delta < (1 - \delta^2)\overline{g}_i^i - \delta\eta \\ &\Leftrightarrow \overline{g}_i^i > \frac{1}{1+\delta} + \frac{\delta\eta}{1-\delta^2} \end{aligned}$$

Now note that by balancedness and the condition defining case 2 we have:

$$\begin{aligned} \delta\overline{g}_i^i &\geq s_i - \varepsilon > \frac{\delta}{1+\delta} \\ &\Rightarrow \overline{g}_i^i > \frac{1}{1+\delta} \end{aligned}$$

For any δ, this last condition implies

$$\overline{g}_i^i > \frac{1}{1+\delta} + \frac{\delta\eta}{1-\delta^2}$$

provided η is chosen small enough. This shows that it is profitable for player j to make the proposal described in this subcase, and completes the proof of Step 1.

Step 2: $s_i + \varepsilon \leq \underline{g}_i^i$**.**

We will first show that at an equilibrium player j will not be able to improve on the partition $(s_i + \varepsilon, s_j - \varepsilon)$.

To see this, suppose first that player j rejected this proposal and induced a game G^j one period later. By Step 1 the maximum he could get by this action is $s_j + \varepsilon$. As in the proof of proposition 1 define

$$\varepsilon_\delta = \frac{(1-\delta)\, min[s_1, s_2]}{(1+\delta)}.$$

Thus, for $\varepsilon < \varepsilon_\delta$, it must be

$$s_j - \varepsilon > \delta(s_j + \varepsilon),$$

so that player j would not find it profitable to follow this action.

On the other hand, suppose that player j rejected the partition $(s_i + \varepsilon, s_j - \varepsilon)$and proposed arbitration. In an s.p.e., player i would accept as long as $\varepsilon < \varepsilon_\delta$, since by Step 1 he would obtain at most $\delta(s_i + \varepsilon)$ by rejecting and inducing a game G^i.

This shows that, if $\varepsilon < \varepsilon_\delta$, at an s.p.e. player j will accept any proposal yielding him strictly more than $s_j - \varepsilon$, and will be indifferent between *(a)* accepting a share $s_j - \varepsilon$ and *(b)* rejecting and proposing arbitration. However, there cannot be an s.p.e. where player j follows action *(b)* when player i proposes the partition $(s_i + \varepsilon, s_j - \varepsilon)$. For, in that case player i would be better off by proposing $(s_i + \varepsilon - \eta, s_j - \varepsilon + \eta)$ for some $\eta < 2\varepsilon$, which would be accepted. Therefore, at any s.p.e. where player i proposes the partition $(s_i + \varepsilon, s_j - \varepsilon)$, player j will accept. ■

4 Other Equilibria

4.1 "Rubinstenian" Equilibria

In the next proposition we establish conditions for supporting the Rubinstenian equilibrium payoff.

Proposition 3 *If and only if the arbitrated outcome is not balanced (that is,* $s_i \leq \frac{\delta}{1+\delta} + \varepsilon$ *for some* i*), there exists an s.p.e. in which agreement is reached immediately on the partition* $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$.

Proof. For the 'if' part, we describe strategies supporting the s.p.e. Let

$$(g_1^1, g_2^1) = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$$

and

$$(g_1^2, g_2^2) = \left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right).$$

Then the strategies that support this equilibrium are for player i (resp. j):

(i) propose the partition $(g_1^i, g_2^i)((g_1^j, g_2^j))$;

(ii) accept any partition which yields $x \geq g_i^j$ $(x \geq g_j^i)$ and reject any other partition;

(iii) do not propose arbitration when rejecting (do not propose arbitration when rejecting if $s_j \leq \frac{\delta}{1+\delta} + \varepsilon$, and propose arbitration otherwise);

(iv) reject arbitration (reject arbitration when $s_j \leq \frac{\delta}{1+\delta} + \varepsilon$, and do not reject it otherwise).

The optimality of these strategies is checked easily. Regarding *(i)*, player i cannot profit from a deviation, since given his strategy player j would reject and either *(a)* counteroffer $\left(g_1^j, g_2^j\right)$ (if $s_j \leq \frac{\delta}{1+\delta} + \varepsilon$); or *(b)* propose arbitration (if $s_j > \frac{\delta}{1+\delta} + \varepsilon$). If *(a)*, given his strategy i would accept, obtaining g_i^j one period later, or $\delta g_i^j < g_i^i$; if *(b)*, player i would either accept arbitration (if $s_i > \frac{\delta}{1+\delta} + \varepsilon$), obtaining a payoff of $s_i - \varepsilon < \frac{1}{1+\delta}$ (implied by balancedness and efficiency of arbitration); or reject arbitration and make a counteroffer, obtaining g_i^i one period later. Turning to part *(ii)* of the equilibrium strategy, note that it is optimal for player i to reject any partition which yields him a payoff less that g_i^j, since he could otherwise counteroffer (g_1^i, g_2^i) in the following round, which would be accepted and would yield him $\delta g_i^i = g_i^j$. For part *(iii)* of the equilibrium strategy, note that - since $s_i \leq \frac{\delta}{1+\delta} + \varepsilon$ - player i cannot improve on his payoff by accepting arbitration (instead of rejecting and counter-proposing (g_1^i, g_2^i), since such deviation would yield him a payoff

$$s_i - \varepsilon \leq \frac{\delta}{1+\delta} = \delta g_i^i$$

The same applies to player j if $s_j \leq \frac{\delta}{1+\delta} + \varepsilon$. Conversely, when $s_j > \frac{\delta}{1+\delta} + \varepsilon$ player j cannot improve by rejecting arbitration, since the arbitrated outcome yields him

$$s_j - \varepsilon > \frac{\delta}{1+\delta} = \delta g_j^j$$

Similar arguments can be used to check the optimality of part *(iv)* of the equilibrium strategy.

For the 'only if' part, note that when the arbitrated outcome is balanced the only s.p.e. payoff is as described in proposition 2. ■

It is possible to show that for some values of the parameters, the Rubinsteinian equilibrium payoff is unique. First we prove a lemma analogous to lemma1.

Lemma 2 *If* $s_i - \varepsilon \leq \frac{\delta}{1+\delta}$ *and* $\overline{g}_i^i > \frac{1}{1+\delta}$ *for some* i*, then* $\overline{g}_i^i > \overline{g}_i^j \forall \delta \in (0,1)$.

Proof: Suppose, by contradiction, that

$$\overline{g}_i^i \leq \overline{g}_i^j$$

Then also

$$\overline{g}_i^j > \frac{1}{1+\delta}$$

We will show that in this case i would be willing to accept $\overline{g}_i^j - \eta$ in the subgame G^j for some small η, thus contradicting the definition of $\overline{g}_i^j$.

In fact, suppose that i rejected the partition yielding him $\overline{g}_i^j - \eta$ and made a counteroffer. Then he would get at most $\delta \overline{g}_i^i$: but

$$\begin{aligned} \delta \overline{g}_i^i &< \overline{g}_i^i \leq \overline{g}_i^j \\ &\Rightarrow \delta \overline{g}_i^i < \overline{g}_i^j - \eta \end{aligned}$$

for η sufficiently small. Suppose, on the other hand, that i rejected the partition yielding him $\overline{g}_i^j - \eta$ and proposed arbitration. Then, if j accepted, i could obtain at most

$$s_i - \varepsilon \leq \frac{\delta}{1+\delta} < \frac{1}{1+\delta} < \overline{g}_i^j - \eta$$

for a small η. If j rejected arbitration, i could obtain at most

$$\delta \overline{g}_i^j < \overline{g}_i^j - \eta$$

for a small η. ■

Proposition 4 *If* $s_i \leq \frac{\delta}{1+\delta} + \varepsilon$ *for some* i *and* $\varepsilon > \varepsilon_\delta = \frac{(1-\delta)min[s_1,s_2]}{(1+\delta)}$*, then the unique s.p.e. payoff pair of the game is* $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$.

Proof: Assuming that ε satisfies the condition in the statement, we deal with two intermediary steps.

Step 1: $s_i \leq \frac{\delta}{1+\delta} + \varepsilon \Rightarrow \overline{g}_i^i \leq \frac{1}{1+\delta}$.

Suppose to the contrary that

$$\overline{g}_i^i > \frac{1}{1+\delta}$$

We will show that in an s.p.e. where $\overline{g}_i^i$ satisfy such condition a contradiction results, because j could reject any proposal yielding him less than $\frac{\delta}{1+\delta}$ and offer to player i the share $\delta\overline{g}_i^i + \eta$, which would be accepted at an s.p.e. Then j would be better off than by accepting a share less than $\frac{\delta}{1+\delta}$ because for η small:

$$\begin{aligned} \overline{g}_i^i &> \frac{1}{1+\delta} \Rightarrow \\ \overline{g}_i^i &> \frac{1}{1+\delta} + \eta\frac{\delta}{1-\delta^2} \\ &\Leftrightarrow 1 - \overline{g}_i^i < \delta(1 - \delta\overline{g}_i^i - \eta) \end{aligned}$$

To see that i would accept the proposal at an s.p.e., observe that if he made a counteroffer he could get at most

$$\delta\overline{g}_i^i < \delta\overline{g}_i^i + \eta$$

On the other hand, if he proposed arbitration, either j would accept and i would get

$$s_i - \varepsilon < \delta\overline{g}_i^i + \eta$$

by the assumption $s_i - \varepsilon \leq \frac{\delta}{1+\delta} < \delta\ \overline{g}_i^i$; or j would reject and i would get at most

$$\delta\overline{g}_i^j < \delta\overline{g}_i^i + \eta$$

by lemma 2.

Step 2:$s_i \leq \frac{\delta}{1+\delta} + \varepsilon$ and $s_j > \frac{\delta}{1+\delta} + \varepsilon \Rightarrow \underline{g}_i^i \geq \frac{1}{1+\delta}$.

Suppose to the contrary that

$$\underline{g}_i^i < \frac{1}{1+\delta}$$

Then there exists an s.p.e. of some subgame in which j would reject any proposal yielding him $\frac{\delta}{1+\delta}$. Suppose it was optimal for j to reject and induce a subgame G^j. Then he would get at most

$$(1 - max[\delta\ \underline{g}_i^i, \underline{d}_i^j]) \leq (1 - \delta\ \underline{g}_i^i)$$

in the next period, yielding the contradiction

$$\begin{aligned} \underline{g}_i^i &\geq 1 - \delta(1 - \delta\ \underline{g}_i^i) \\ &\Leftrightarrow\ \underline{g}_i^i \geq \frac{1}{1+\delta} \end{aligned}$$

Suppose then that it was optimal for j to reject and propose arbitration. If i rejected, then j would get at most $(1 - \underline{g}_i^i)$ in the next period, yielding the contradiction

$$\begin{aligned} \underline{g}_i^i &\geq 1 - \delta(1 - \ \underline{g}_i^i) \\ &\Leftrightarrow\ \underline{g}_i^i \geq 1 \end{aligned}$$

Therefore, if it was optimal for j to reject at an s.p.e., i would accept arbitration. This can only be the case if

$$\begin{aligned} s_i - \varepsilon &\geq \delta \underline{g}_i^i \\ &\Leftrightarrow\ \frac{s_i - \varepsilon}{\delta} > \underline{g}_i^i \end{aligned}$$

Since when rejecting optimally j gets exactly $s_j - \varepsilon$, so that

$$\underline{g}_i^i \geq 1 - (s_j - \varepsilon) = s_i + \varepsilon$$

The last two inequalities are compatible only if

$$\begin{aligned} \frac{s_i - \varepsilon}{\delta} &> s_i + \varepsilon \\ &\Leftrightarrow\ \varepsilon < \frac{(1-\delta)}{(1+\delta)} s_i \end{aligned}$$

Since by assumption $\varepsilon > \frac{(1-\delta) min[s_1, s_2]}{(1+\delta)}$, it follows that $s_i > s_j$, which is impossible because the assumption in the statement of this step implies $s_j > s_i$. This concludes the proof of step 2.

At this point observe that only two cases can hold:

a) $s_i \leq \dfrac{\delta}{1+\delta} + \varepsilon,\ s_j \leq \dfrac{\delta}{1+\delta} + \varepsilon$

b) $s_i \leq \dfrac{\delta}{1+\delta} + \varepsilon,\ s_j > \dfrac{\delta}{1+\delta} + \varepsilon$

In case *(a)*, step 1 yields

$$\overline{g}_i^i \leq \frac{1}{1+\delta} \text{ and } \overline{g}_j^j \leq \frac{1}{1+\delta}$$

In case *(b)*, step 1 and step 2 yield

$$\overline{g}_i^i \leq \frac{1}{1+\delta} \leq \underline{g}_i^i$$

In both cases, it follows from standard reasoning that $\frac{1}{1+\delta} = \underline{g}_i^i = \overline{g}_i^i = \overline{g}_j^j = \underline{g}_j^j$. ■

4.2 Intermediate Equilibria

As is the case in other bargaining models with non-unique equilibria (see for instance Haller and Holden (1990), Fernandez and Glazer (1991), Avery and Zemsky (1994), Busch and Wen (1995), Busch et al. (1995), Ponsati and Sakovics (1995) and Manzini (1996) among others), in our model as well it is possible to support a whole range of efficient equilibrium payoffs when there exist two extreme equilibrium payoffs, as characterised in the previous propositions. We can thus establish the following:

Proposition 5 *If $\varepsilon < \varepsilon_\delta$ and $s_j < \frac{\delta}{1+\delta} + \varepsilon$ for some j, only and all those partitions which yield player i a payoff $x^* \in \left[\frac{1}{1+\delta}, s_i + \varepsilon\right]$ if he is the proposer, or $1-\ y^* \in \left[\frac{\delta}{1+\delta}, s_i - \varepsilon\right]$ if he is the responder can be supported in any efficient equilibrium.*

Proof: As a preliminary, notice that $s_j < \frac{\delta}{1+\delta} + \varepsilon$ together with efficiency of arbitration implies $s_i > \frac{1}{1+\delta} - \varepsilon$. Furthermore, recall that

$$\begin{aligned} \varepsilon_\delta &= \frac{(1-\delta)\, min[s_1, s_2]}{(1+\delta)} \leq \frac{(1-\delta)}{2(1+\delta)} \\ &\Rightarrow \quad s_j + \varepsilon < \frac{\delta}{1+\delta} + 2\varepsilon < \frac{\delta}{1+\delta} + \frac{1-\delta}{1+\delta} = \frac{1}{1+\delta} \end{aligned}$$

Similarly,

$$s_i - \varepsilon > \frac{1}{1+\delta} - 2\varepsilon > \frac{1}{1+\delta} + \frac{1-\delta}{1+\delta} = \frac{\delta}{1+\delta}$$

The rest of the proof is divided into three parts. In the first two parts we show that no partitions which yield player i a payoff outside the ranges specified in the proposition can be supported in equilibrium. In part III of the proof we introduce a pair of strategies which support the s.p.e. outcome introduced above.

Part I: Subgames in which player i is the proposer.

We proceed in two steps.

Step 1: $\overline{g}_i^i \leq s_i + \varepsilon$.

We prove this by contradiction. Suppose that, on the contrary:

$$\overline{g}_i^i > s_i + \varepsilon$$

Now, consider two possible cases:

Case a: $s_i - \varepsilon > \delta \overline{g}_i^i$.

In this case player j can reject and propose arbitration, which i would accept (by definition of case a). This action is profitable for player j, since now he gets $s_j - \varepsilon > 1 - \overline{g}_i^i$ (recall that $\overline{g}_i^i > s_i + \varepsilon \Leftrightarrow 1 - \overline{g}_i^i < s_j - \varepsilon$).

Case b: $s_i - \varepsilon \leq \delta \overline{g}_i^i$.

In this case, player j could reject and counteroffer the partition which gives player i the share $\delta \overline{g}_i^i + \eta$, which player i would accept. In fact, when rejecting player i could either make a counteroffer, or propose arbitration. If the former, player i could obtain at most $\delta \overline{g}_i^i < \delta \overline{g}_i^i + \eta$. Alternatively, if player i rejected to propose arbitration, he would receive $s_i - \varepsilon < \delta \overline{g}_i^i + \eta$ (if arbitration were accepted); or $\delta^2 \overline{g}_i^i < \delta \overline{g}_i^i + \eta$ if arbitration were rejected (this is so because if player j rejects arbitration then, it must be the case that $s_j - \varepsilon < \delta \underline{g}_j^j$. For $\underline{g}_j^j$ to be in equilibrium, then $1 - \underline{g}_j^j \geq max[s_i - \varepsilon, \delta \overline{g}_i^i] = \delta \overline{g}_i^i$, where the last equality follows from the definition of case b. Therefore if arbitration were rejected player i would end up with a payoff of at most $\delta^2 \overline{g}_i^i < \delta \overline{g}_i^i + \eta$).

Consequently, by offering player i the share $\delta \overline{g}_i^i + \eta$, player j improves on his payoff for η sufficiently small, since

$$\delta(1 - \delta \overline{g}_i^i - \eta) > 1 - \overline{g}_i^i \;\Leftarrow \overline{g}_i^i > s_i + \varepsilon > \frac{1}{1+\delta} + \frac{1}{1-\delta^2}\eta$$

for η sufficiently small.

Step 2: $\underline{g}_i^i \geq \frac{1}{1+\delta}$.

In this case, player j would receive a payoff of at most $\frac{\delta}{1+\delta}$. Suppose now player j were offered exactly $\frac{\delta}{1+\delta}$: is there any strategy which would ensure that he obtains a higher payoff? In case of a rejection, player j can either *(i)* propose arbitration, or *(ii)* make a counteroffer. If *(i)*, player i would surely accept arbitration, since by so doing he obtains a payoff which is in excess of the highest payoff he could achieve if he were to reject arbitration and make a counteroffer in the following round, since

$$s_i - \varepsilon > \delta \overline{g}_i^i = \delta\left(s_i + \varepsilon\right) \text{ if } \varepsilon < \varepsilon_\delta$$

Then, j would end up with a payoff of just

$$s_j - \varepsilon < \frac{\delta}{1+\delta}$$

as verified in the preliminaries to the proof. Alternatively, if *(ii)*, then a condition for an s.p.e. is:

$$1 - \overline{g}_j^j \geq \max[s_i - \varepsilon, \delta \underline{g}_i^i]$$

Now, it must always be the case that

$$s_i - \varepsilon \geq \delta \underline{g}_i^i$$

when player j rejects player's i offer of $\frac{\delta}{1+\delta}$: this follows from the fact that rejection implies

$$\underline{g}_i^i < \frac{1}{1+\delta} \Leftarrow \delta \underline{g}_i^i < \frac{\delta}{1+\delta}$$

and from the preliminaries

$$s_i - \varepsilon > \frac{\delta}{1+\delta}$$

Consequently, player j can obtain at most $1 - (s_i - \varepsilon) = s_j + \varepsilon$ one period later; but

$$\delta\left(s_j + \varepsilon\right) < \frac{\delta}{1+\delta} \Leftarrow s_j + \varepsilon < \frac{1}{1+\delta}$$

as from the preliminaries.

<u>Part II</u>: Subgames in which player i is the responder.

As above, we proceed in two steps.

Step 1: $\underline{g}_i^j \geq \frac{\delta}{1+\delta}$.

We prove this by contradiction. Suppose that, on the contrary,

$$\underline{g}_i^j < \frac{\delta}{1+\delta} \Rightarrow \overline{g}_j^j = 1 - \underline{g}_i^j > \frac{1}{1+\delta}$$

Then player i could reject and counteroffer in the following period a partition yielding player j a payoff of $\delta\overline{g}_j^j + \eta > \frac{\delta}{1+\delta}$, which player j will accept, given that by rejecting he can obtain either $\delta\overline{g}_j^j$, by making a counterproposal in the subsequent round; or $s_j - \varepsilon < \frac{\delta}{1+\delta} < \delta\overline{g}_j^j + \eta$ if he proposes arbitration (which is accepted by player i, since in Part I we showed that $s_i - \varepsilon > \delta\overline{g}_i^i = \delta(s_i + \varepsilon)$ when $\varepsilon < \varepsilon_\delta$). Such a deviation is profitable for player i, since his payoff becomes

$$\delta(1 - \delta\overline{g}_j^j - \eta) = \delta[1 - \delta(1 - \overline{g}_i^j) - \eta] > \underline{g}_i^j \Leftarrow \overline{g}_i^j < \frac{\delta}{1+\delta} - \frac{\delta}{1-\delta^2}\eta$$

Step 2: $\overline{g}_i^j \leq s_i - \varepsilon$.

As for step 2 in part I, could player i improve on his payoff by rejecting a partition which yields him $s_i - \varepsilon$? He can either reject and make a counteroffer in the following round, yielding $\delta\overline{g}_i^i = \delta(s_i + \varepsilon) < s_i - \varepsilon$ when $\varepsilon < \varepsilon_\delta$; or reject and propose arbitration. But this is never optimal for player i, since: *(a)* if $s_j - \varepsilon > \delta\overline{g}_j^j$, then player j accepts arbitration, so that i cannot improve on the initially proposed payoff of $s_i - \varepsilon$, whereas if *(b)* $s_j - \varepsilon \leq \delta\overline{g}_j^j$, then player j could reject arbitration, and offer to player i in the following round at most $max[\delta\overline{g}_i^i, s_i - \varepsilon] = s_i - \varepsilon$.

<u>Part III</u>: Equilibrium strategies.

Let $y^* \in \left[s_j + \varepsilon, \frac{1}{1+\delta}\right]$. Then, using standard arguments, strategies which support the equilibrium of Proposition 5 are for instance for player i (player j, respectively) to claim x^* (y^*) for himself, accept any proposal which yields him at least $1-$ y^* ($1-$ x^*) and reject any other proposal; in case of a deviation, play reverts to the strategies which yield the worst payoff for the deviator[9], that is either those supporting the Rubinstenian

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[9] For the reader's convenience, we here report such strategies. Those which support the Rubinstenian equilibrium are for player i (resp. j): *(i)* propose the partition which give him a payoff of $\frac{1}{1+\delta}$ and of $\frac{\delta}{1+\delta}$ to his opponent; *(ii)* accept any partition which yields $x \geq \frac{\delta}{1+\delta}$ and reject any other partition; *(iii)* do not propose arbitration when rejecting (do not propose arbitration when rejecting if $s_j \leq \frac{\delta}{1+\delta} + \varepsilon$, and

s.p.e. payoff, as specified in Proposition 3; or those supporting the arbitration driven equilibrium, as specified in Proposition 1.

It is easy to check that the strategies sketched above constitute an s.p.e. In fact, suppose $i = 1$, so that player 1 is the first proposer. Then if he proposed any partition yielding him a payoff $x \neq x^*$, given his strategy player 2 would reject, and both players would revert to the Rubinstenian equilibrium play, that is, player 2 would propose the partition $\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right)$, which player 1 would accept. By so doing, however, player 1 obtains a payoff of $\frac{\delta}{1+\delta}$ one period later, which is less than the lowest possible value of x^*. Furthermore, player 2 cannot improve on his payoff by accepting a partition which yields a payoff different from $1 - x^*$, as by rejecting and switching to the Rubinstenian play he can secure a payoff whose present discounted value is $\frac{\delta}{1+\delta} \geq 1 - x^*$. Suppose now $i = 2$, so that equilibrium is reached efficiently on the partition $(y^*, 1 - y^*)$. As above, player $j = 1$ cannot improve on his payoff by making a different offer, as given his strategy, player 2 would reject and then revert to play arbitration driven equilibrium strategies, that is, he would propose arbitration, which would be accepted, yielding player 2 a payoff of $s_1 - \varepsilon \geq 1 - y^*$, and player 1 a payoff equal to $s_2 - \varepsilon \leq s_2 + \varepsilon \leq y^*$. This also shows that player 2 cannot profitably accept a partition yielding a payoff smaller than $1 - y^*$, since by following his equilibrium strategy - that is, by rejecting and proposing arbitration - he can secure a payoff of $s_1 - \varepsilon \geq 1 - y^*$. ■

Our results are summarised in Figure 3, where we represent the combinations of values of s_1 and s_2 (on the horizontal and vertical axis, respectively) for which the various equilibrium regimes can be supported, for any given value of δ and ε, when $\varepsilon < \varepsilon_\delta$. The thick lines represent combinations of s_1 and s_2 which support the Rubinstenian equilibrium. The portion of the -45 degree line in correspondence of the bracket selects pairs of arbitrated outcomes which support the arbitration-driven equilibrium. Where

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propose arbitration otherwise); *(iv)* reject arbitration (reject arbitration when $s_j \leq \frac{\delta}{1+\delta} + \varepsilon$, and do not reject it otherwise).

Strategies which support the arbitration driven equilibrium are for player $i = 1, 2$: *(i)* propose the partition which yields him a payoff of $s_i + \varepsilon$ and $s_j - \varepsilon$ to his opponent; *(ii)* reject any partition which yields him any $x < s_i - \varepsilon$ and accept any $x \geq s_i - \varepsilon$; *(iii)* always accept arbitration when player j proposes it; *(iv)* always propose arbitration when rejecting an offer.

Figure 3: Range of equilibrium payoffs when $\varepsilon < \varepsilon_\delta$.

the bracket overlaps with the thick lines both regimes can be supported, which allows for a multiplicity of equilibrium payoffs to be sustained in equilibrium, as explained in Proposition 5.

So far we have considered only efficient agreements. Similarly to other models of this kind, the existence of two extreme equilibrium payoffs in fact creates the possibility of delayed equilibrium agreements. These sort of inefficiencies are dealt with in the next section.

4.3 Efficiencies and Inefficiencies

One important purpose of (costly) arbitration is to increase the parties' willingness to reach a negotiated agreement instead of resorting to arbitration. The fact that such an institution exists reminds the parties that a settlement to the negotiations can always be achieved, and the cost involved in such a process should discourage lengthy disputes and encourage the bargainers to reach an agreement.

A main feature of our model is that both bargainers know what the outcome would be if an arbitrator were called in, so that they cannot be driven into arbitration by the belief that they would get some better deal. Consequently, it turns out that, as long as arbitration is costly, there can be no equilibrium in which the arbitrated outcome is implemented. This prevents the existence of the family of equilibria which are inefficient because part of the surplus is lost in arbitration costs.

Proposition 6 *Arbitration can never be supported in equilibrium if* $\varepsilon > 0$.

Proof. Suppose not, so that $\varepsilon > 0$ and arbitration can be supported in equilibrium. Then, along the equilibrium path it must be that player i (with $i = 1, 2$) makes a proposal which player j rejects to counteroffer arbitration, which player i accepts. Now let x be player i's claim in the offered partition. Clearly, to be in equilibrium it must be that $1 - x \leq s_j - \varepsilon$, since otherwise player j would be better off by accepting the proposal. That can be rearranged as $x \geq s_i + \varepsilon$. Suppose now that player i proposed instead a partition in which his claim was $x' = s_i + \varepsilon - \eta < x$, leaving $s_j - \varepsilon + \eta$ to player j, where η is a small positive constant satisfying $\eta < 2\varepsilon$. Then it is clearly not optimal for player j to conform to his equilibrium strategy and reject such offer, since accepting player i' proposal gives him a greater payoff. Furthermore, such deviation is profitable for player i, since by so doing he obtains $s_i + \varepsilon - \eta > s_i - \varepsilon$ (given that $\eta < 2\varepsilon$). ■

This shows that incomplete information and heterogeneous beliefs should be an important factor in any story that explains why inefficient arbitration is observed in reality.

However, there are other types of inefficiency which can be supported in equilibrium. They do not arise from arbitration being a costly process; conversely, these inefficiencies exist when arbitration costs are sufficiently small (but they may still be positive), and take the form of delayed (bargained) agreements, as the following proposition illustrates:

Proposition 7 *If* $0 < \varepsilon < \varepsilon_\delta$, *and* $s_2 < \frac{\delta}{1+\delta} + \varepsilon$, *then for any time* T *there exists a discount factor* $\widehat{\delta}$ *(depending on* T*) such that for* $\delta \in \left[\widehat{\delta}, 1\right)$ *every partition which yields player 1 a payoff* $z^* \in \left[\frac{s_1 - \varepsilon}{\delta^T}, 1 - \frac{s_2 - \varepsilon}{\delta^{T-1}}\right]$ *can be supported in an equilibrium in which negotiated agreement is reached at time* T.[10]

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[10] It is possible to find a "twin" proposition which establishes an analogous result for the symmetric

Proof. First of all, recall that when $\varepsilon < \varepsilon_\delta$ both the Rubinstenian and the arbitration-driven equilibria can obtain. Then, strategies that support this equilibrium are: At time t and until time T, player 1 proposes the partition $\left(1-\delta^{T-t}(1-z^*);\delta^{T-t}(1-z^*)\right)$ and rejects any offer which yields less than $s_1-\varepsilon$, while player 2 proposes the partition $\left(\delta^{T-t}z^*;1-\delta^{T-t}z^*\right)$ and rejects any offer which yields less than $\frac{\delta}{1+\delta}$; both players never propose arbitration and never accept arbitration when it is called for by their opponent; at time T, player i offers a partition yielding player 1 a payoff of z^*, which is accepted. Both players punish deviations by reverting play to the worst equilibrium for the deviator (i.e. either those supporting the Rubinstenian s.p.e. payoff, as specified in Proposition 3, or those supporting the arbitration driven equilibrium, as specified in Proposition 1, as explained in the proof of Proposition 5). We now show that the payoffs specified above can be supported in equilibrium.

1. *Deviations by Player 1.* Consider first a deviation by player 1 in the **first round** (at time $t=0$); there can actually be two such deviations:

   (i) *Deviant offer.* Suppose that player 1's proposed partition gives his opponent a share $\delta^T(1-z^*)-\eta$ with $\eta>0$. Then player 2 rejects, and play reverts to the Rubinstenian strategy, yielding player 1 a payoff of $\frac{\delta}{1+\delta}$ in the following round. On the other hand, by conforming to his equilibrium strategy player 1 could have obtained a payoff of z^* at time T; consequently, player 1 cannot profit from a deviation in the first round as long as:

   $$\frac{\delta^2}{1+\delta} \le \delta^T z^* \Rightarrow z^* \ge \frac{1}{\delta^{T-2}(1+\delta)} \tag{C1}$$

   Similarly, in even periods other than the first ($t>0$), a deviant offer by player 1 will not be profitable as long as $z^* \ge \frac{1}{\delta^{T-(t+2)}(1+\delta)}$; however, $\frac{1}{\delta^{T-2}(1+\delta)} > \frac{1}{\delta^{T-(t+2)}(1+\delta)}$, so that condition 5 above actually encompasses all deviant offers in periods other than the first.

   (ii) *Deviant response to arbitration.* Suppose that, after rejecting player 1's offer, player 2 unexpectedly proposes arbitration. By accepting it, player 1 would

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case when $s_1 < \frac{\delta}{1+\delta}+\varepsilon$, rather than s_2. The exact proof and statement can be found along the same lines of Proposition 7, and is left as an exercise to the reader.

end up with a payoff of $s_1 - \varepsilon$ immediately. Consequently, rejecting arbitration cannot be improved upon as long as:

$$s_1 - \varepsilon \leq \delta^T z^* \Rightarrow z^* \geq \delta^{-T}(s_1 - \varepsilon) \tag{C2}$$

Notice that this condition implies 5 if

$$\frac{s_1 - \varepsilon}{\delta^T} > \frac{1}{\delta^{T-2}(1+\delta)} \Rightarrow s_1 > \frac{\delta^2}{1+\delta} + \varepsilon$$

Now, $\varepsilon < \varepsilon_\delta$, $s_2 < \frac{\delta}{1+\delta} + \varepsilon$ and efficiency of arbitration imply that[11]

$$s_1 > \frac{\delta}{1+\delta} + \varepsilon > \frac{\delta^2}{1+\delta} + \varepsilon$$

so that 5 is always more stringent than 5. It is easy to see that in odd periods other than the first a deviant response to arbitration will not be profitable as long as $z^* \geq \delta^{t-T}(s_1 - \varepsilon)$, which is implied by the condition 5.

Consider now a deviation by player 1 in the **second round** (at time $t = 1$). Clearly, he cannot increase his payoff by accepting player 2's offer of $\delta^{T-1}z^*$, as this exactly matches his equilibrium payoff. Moreover, player 1 cannot profitably propose arbitration when rejecting, since given his strategy player 2 would reject, and play would then move to the Rubinstenian equilibrium (with player 2 as first mover), yielding player 1 a payoff of $\frac{\delta}{1+\delta}$; consequently, for such deviation not to be profitable it must be that

$$\delta^{T-1}z^* \geq \frac{\delta^2}{1+\delta} \Rightarrow z^* \geq \frac{1}{\delta^{T-3}(1+\delta)}$$

which is implied by condition 5. From this discussion it is easy to see that any deviation by player 1 in any odd period t is going to be prevented by $z^* \geq \frac{1}{\delta^{T-(t+2)}(1+\delta)}$, which is always a less stringent condition that $z^* \geq \frac{1}{\delta^{T-2}(1+\delta)}$, which is in turn encompassed by $z^* \geq \delta^{-T}(s_1 - \varepsilon)$; thus 5 imposes a lower bound on z^* which prevents profitable deviations by player 1.

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[11] From the preliminaries to the proof of Proposition 5, recall that $s_2 < \frac{\delta}{1+\delta} + \varepsilon$ and efficiency of arbitration yield $s_1 > \frac{1}{1+\delta} - \varepsilon$, which can be rearranged as $s_1 - \varepsilon > \frac{1}{1+\delta} - 2\varepsilon$. But if $\varepsilon < \varepsilon_\delta$, then $\varepsilon < \frac{(1-\delta)min[s_1,s_2]}{1+\delta} \leq \frac{(1-\delta)}{2(1+\delta)}$, where the last inequality follows from $min[s_1, s_2] \leq 1/2$. Consequently, $s_1 - \varepsilon > \frac{1}{1+\delta} - 2\varepsilon > \frac{1}{1+\delta} - \frac{1-\delta}{1+\delta} > \frac{\delta}{1+\delta}$.

2. *Deviations by Player 2.* Let us first consider a deviation by player 2 in the **first round**. Similarly to what we discussed for player 1, player 2 cannot increase his payoff by accepting an offer of $\delta^T(1-z^*)$, since it exactly matches his equilibrium payoff. Next, suppose player 2 proposes arbitration after rejecting his opponent's offer. In this case player 1 rejects, and play reverts to the arbitration driven equilibrium, yielding player 2 a payoff of $s_2-\varepsilon$ in the following period; consequently, the above deviation is not profitable if

$$\delta(s_2-\varepsilon) \leq \delta^T(1-z^*) \Rightarrow z^* \leq 1-\frac{s_2-\varepsilon}{\delta^{T-1}} \tag{C3}$$

Similarly, one can rule out deviations in any odd period t other than the first if $z^* \leq 1-\frac{s_2-\varepsilon}{\delta^{T-(t+1)}}$. However, $1-\frac{s_2-\varepsilon}{\delta^{T-1}} < 1-\frac{s_2-\varepsilon}{\delta^{T-(t+1)}}$, so that condition 5 above is enough to guarantee the suboptimality of all odd period deviations by player 2.

Consider now a deviation by player 2 in the **second round** (at time $t=1$). Symmetrically to the case of player 1, there can be two types of deviations:

(i') *Deviant offer.* Suppose player 2 proposed a partition yielding player 1 a payoff of $\delta^{T-1}z^*-\eta$, with $\eta>0$. Then player 1 would reject, and play would revert to the arbitration driven equilibrium, yielding player 2 a payoff of $s_2-\varepsilon$ one period later. Consequently, such a deviation is not profitable for player 2 if

$$\delta(s_2-\varepsilon) \leq \delta^{T-1}(1-z^*) \Rightarrow z^* \leq 1-\frac{s_2-\varepsilon}{\delta^{T-2}}$$

More generally, a deviant offer at any even period t is not going to be profitable for player 2 as long as $z^* \leq 1-\frac{s_2-\varepsilon}{\delta^{T-(t+1)}}$, which is a weaker requirement and is the same as the one derived for deviations in odd periods other than the first.

(ii') *Deviant response to arbitration.* Suppose, after rejecting player 2's offer, player 1 unexpectedly proposes arbitration. If player 2 accepted, he would obtain a payoff of $s_2-\varepsilon$, so that such a deviation cannot increase player 2's payoff if

$$s_2-\varepsilon \leq \delta^{T-1}(1-z^*) \Rightarrow z^* \leq 1-\frac{s_2-\varepsilon}{\delta^{T-1}}$$

which is simply condition 5. Finally, from the discussion it is clear that any deviation in even periods other than the second will not be profitable if $z^* \leq 1-\frac{s_2-\varepsilon}{\delta^{T-t}}$, which is encompassed by 5.

To conclude the proof, notice that for the interval $\left[\frac{s_1-\varepsilon}{\delta^T}, 1-\frac{s_2-\varepsilon}{\delta^{T-1}}\right]$ to be not empty, it must be the case that

$$1-\frac{s_2-\varepsilon}{\delta^{T-1}}-\frac{s_1-\varepsilon}{\delta^T}\geq 0 \Rightarrow \delta^T-\delta(s_2-\varepsilon)-(s_1-\varepsilon)\geq 0$$

Now when $\delta \to 1$ the above expression tends to

$$1-s_1-s_2+2\varepsilon=2\varepsilon>0$$

where the first equality is obtained because of the efficiency of arbitration. Thus for any T, in the neighbourhood of 1 it will always be possible to find a value $\widehat{\delta}$ of the discount factor such that when $\delta \in \left[\widehat{\delta}, 1\right)$ the interval $\left[\frac{s_1-\varepsilon}{\delta^T}, 1-\frac{s_2-\varepsilon}{\delta^{T-1}}\right]$ is not empty. ■

What we learn from this result is that allowing for the possibility of arbitration, even with complete information, may have a *negative* effect on negotiations, since it creates the scope for delays which were impossible in the classic bargaining model. This rather perverse feature is due to the fact that, although arbitration is never observed *in equilibrium* (Proposition 6), the *out of equilibrium* paths leading to arbitration may be used strategically. The variety of threats thus generated accounts for multiple equilibria which sustain delayed agreements. Note also that - as the proof illustrates - non-stationary strategies are crucial to sustain delays. Arguably, imposing Markov behaviour would thus constitute one plausible way to rule out inefficient equilibria.

4.4 The Role of δ : A Comparison with the Outside Option Model

In this section we contrast our arbitration model with outside option models, by analysing the role of the discount factor. It could be argued that introducing the possibility of resorting to an arbitrator is tantamount to introducing a fixed outside option for the two bargainers which can be taken without delay. However, this is not the case. The crucial difference is that for the arbitrated outcome to obtain it is needed that *both* bargainers agree on it.

Let us consider the simple alternating offers bargaining model in which player 2 can take up his outside option after rejecting player 1's offer, in which case players obtain a

Figure 4: Player 1's equilibrium payoffs in the alternating offers bargaining model with outside option.

payoff of b_1 and b_2, respectively. If we let players' utility be represented by $u_i(x,t) = \delta^t x$, then we know that player 1's equilibrium payoff will be $x_1^* = \frac{1}{1+\delta}$ if $b_2 \leq \frac{\delta}{1+\delta}$, and $1 - b_2$ if $b_2 > \frac{\delta}{1+\delta}$. If we now fix b_2, it is possible to determine a $\widehat{\delta} = \frac{b_2}{1-b_2}$ such that if $\delta \geq \widehat{\delta}$ player 1 obtains the Rubinstenian equilibrium, whereas if $\delta < \widehat{\delta}$ the outside option equilibrium obtains (see Figure 4). Notice that at $\delta = \widehat{\delta}$, $x_1^*(\widehat{\delta}) = 1 - b_2$. Consequently, the equilibrium share for player 1 is *continuous* and *monotonically decreasing* in δ.

Let us now turn to the model with arbitration. According to Proposition 1, when the arbitration outcome is balanced (that is, when $s_i > \frac{\delta}{1+\delta} + \varepsilon \; \forall i$), only the arbitration-driven equilibrium can obtain. Notice that the balancedness condition can be re-written as

$$min[s_1, s_2] > \frac{\delta}{1+\delta} + \varepsilon$$

Consequently, fixing s_1 and s_2, we can rearrange the above expression to determine

$$\overline{\delta} = \frac{min[s_1, s_2] - \varepsilon}{1 - min[s_1, s_2] + \varepsilon}$$

such that for any $\delta \leq \overline{\delta}$ the arbitration-driven equilibrium results. On the other hand, when $\delta > \overline{\delta}$ both the Rubinstenian and the arbitration-driven equilibrium payoff can be sustained, which also implies that all payoffs within such range can be supported in equilibrium, as stated in Proposition 5 above. What happens to the arbitration-driven

Figure 5: Equilibrium payoffs when $s_1 \geq s_2$

and Rubinstenian equilibrium payoffs when $\delta = \overline{\delta}$? The Rubinstenian equilibrium payoff to player 1 is

$$\frac{1}{1+\overline{\delta}} = 1 - min[s_1, s_2] + \varepsilon = max[s_1, s_2] + \varepsilon$$

Consequently, when $s_1 \geq s_2$, at $\delta = \overline{\delta}$ both equilibria yield the same payoff to player 1, whereas if $s_1 < s_2$ the Rubinstenian equilibrium payoff evaluated at $\delta = \overline{\delta}$ is $s_2 + \varepsilon > s_1 + \varepsilon$: in such case, an upper semi-discontinuity in the payoffs arises. This situation is depicted in Figures 5 and 6. Solid lines represent the equilibrium payoff schedule for player 1 in the arbitration-driven equilibrium case $(s_1 + \varepsilon)$ and Rubinstenian equilibrium case $(\frac{1}{1+\overline{\delta}})$; the shaded area highlights the presence of multiple equilibria.

Figure 5 consider the case for $s_1 \geq s_2$, whereas Figure 6 refers to $s_1 < s_2$. Both figures report also the value $\overline{\overline{\delta}}$ of the discount factor; this is defined in correspondence of $\varepsilon \leq \varepsilon_{\overline{\overline{\delta}}} = \frac{(1-\overline{\overline{\delta}})min[s_1,s_2]-\varepsilon}{1+\overline{\overline{\delta}}}$, from which:

$$\overline{\overline{\delta}} = \frac{min\,[s_1, s_2] - \varepsilon}{min\,[s_1, s_2] + \varepsilon}$$

We saw above that when $\delta > \overline{\overline{\delta}}$, the arbitration-driven equilibrium is no longer defined. Notice that $\overline{\overline{\delta}}$ lies always to the right of $\overline{\delta}$, since

$$\overline{\overline{\delta}} = \frac{min\,[s_1, s_2] - \varepsilon}{min\,[s_1, s_2] + \varepsilon} \geq \overline{\delta} = \frac{min[s_1, s_2] - \varepsilon}{1 - min[s_1, s_2] + \varepsilon} = \frac{min[s_1, s_2] - \varepsilon}{max[s_1, s_2] + \varepsilon}$$

Figure 6: Equilibrium payoffs when $s_1 < s_2$

Thus, from Figures 5 and 6 one can retrieve the range of variation of the discount factor which trigger the various equilibrium regimes, keeping s_1, s_2 and ε fixed: if $\delta < \overline{\delta}$ only the arbitration-driven equilibrium occurs, if $\delta > \overline{\overline{\delta}}$ only the Rubinsteinian equilibrium obtains, whereas for $\delta \in [\overline{\delta}, \overline{\overline{\delta}}]$ the range of payoffs which can be supported in equilibrium are as specified in Proposition 5.

This sort of discontinuity would appear if one were to consider a two-sided outside option model, as in Ponsati and Sakovics (1995). However, by contrasting their model with the one presented in this paper it is possible to highlight some important differences. If one were to interpret the arbitrated outcome as an outside option payoff vector ($s_1 - \varepsilon$, $s_2 - \varepsilon$), their main result would imply that whenever the outside option is such that at least one of the players stands to gain by calling in the arbitrator - with respect to negotiated outcome -, only the arbitration driven equilibrium would obtain, whereas if neither of the players preferred the arbitrated outcome to the bargained one, a multiplicity of payoffs could be supported in equilibrium[12]. On the contrary, in our model the fact that arbitration is preferred by at least one of the players is not enough to ensure that the arbitration-driven equilibrium obtains. Indeed, if the arbitrated outcome is not balanced,

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[12] More precisely, their Proposition 1 would imply that if *(i)* $s_i - \varepsilon > \delta(s_i + \varepsilon)$ for some i, only the arbitration driven equilibrium would obtain, whereas if *(ii)* $s_i - \varepsilon \leq \delta(s_i + \varepsilon)$ $\forall i$ a multiplicity of payoff could be supported in equilibrium. Turning to our model, however, *(i)* is compatible with the requirements

the Rubinsteinian equilibrium could result.

Once again, the reasons for this contrast between the arbitration model of this paper and outside option models is to be found in the crucial difference between the two structures, namely the fact that the exploitation of one's outside option is a *unilateral* decision, whereas for the arbitrated outcome to be implemented preference of *both* bargainers for arbitration is needed.

5 Concluding Remarks

A crucial feature of our model is that both players must give their consent before the arbitrator is called in. This assumption, although not universally correct, is consistent with a large number of institutional contexts. In the UK in particular, it is the case that labour disputes are resolved by final offer arbitration (FOA) only with the consent of both parties (e.g. Milner (1993)). This feature has been sometimes overlooked in the literature on the arbitration phase, which is largely influenced by the US institutional setting where in many cases FOA is automatically triggered by law if the negotiations reach a certain stage[13]. For that type of institutional setting, our modelling strategy would essentially yield the bargaining model with outside options (Ponsati and Sakovics (1995)) discussed before, where delayed agreements are possible. Compte and Jehiel (1995) consider a very different model, in which players alternate in making concessions and there is an exogenous probability of breakdown, as well as an exogenous probability that arbitration

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for the Rubinstenian equilibrium to obtain, namely $s_i \leq \frac{\delta}{1+\delta} + \varepsilon$ (see Proposition 3) as long as

$$\delta(s_i + \varepsilon) < s_i - \varepsilon \leq \frac{\delta}{1+\delta} \Rightarrow s_i < \frac{1}{1+\delta} - \varepsilon$$

Consequently under *(i)* our model allows for a continuum of payoffs to be supported in equilibrium. On the contrary, if *(ii)* holds, then it implies $\varepsilon \geq \frac{(1-\delta)s_i}{1+\delta} \forall i$, which requires $\varepsilon \geq \frac{(1-\delta)min[s_1,s_2]}{1+\delta} = \varepsilon_\delta$. But then the arbitration-driven equilibrium can never obtain, so that a continuum of equilibrium payoffs is ruled out.

[13] Moreover, Steve Brams has pointed out to us that even in the United States it did not take a law for both sides to consent to the use of FOA in major league baseball, beginning in 1975. Although the teams have won more cases than the players, there is no movement on the part of the players to revoke FOA. This may be interpreted as players consenting to 'lose' on average.

is triggered automatically. In that case, delayed agreements can be supported as the unique s.p.e. unless the arbitrator is called in immediately.

In addition, although our motivation comes from the interest in the link between arbitration and negotiation, at a more abstract level our model could be interpreted in other ways. The action 'propose arbitration' can in fact be *any* action such that, with the agreement of the other player, leads to a specific, commonly known, partition (possibly in expected terms). For example, this action can be the message 'let's flip a fair coin'. Or, suppose that there exists an obviously focal or "fair" partition s: then the action could be the message 'let's go for s'. In other words, the presence of a third party is not crucial. What is crucial is the presence of a proposal which has the following two features:

a) like an outside option, it involves "stepping out" of the negotiations over the pie;

b) unlike an outside option, it requires the consent of the other party before being implemented.

We hope that with this relatively simple and natural structure we have been able to demonstrate the following facts:

1. Introducing the possibility of arbitration alters fundamentally the standard bargaining model, in a way which is not captured by outside option models.
2. The arbitrated outcome can drive the outcome of negotiations even though arbitration is never observed in equilibrium.
3. Arbitration may generate inefficient delays even when players have complete information about the behaviour of the arbitrator.
4. When arbitration costs are zero, our bargaining procedure can implement exactly *any* given division of the pie between the players as a subgame perfect equilibrium outcome. In addition, it does so uniquely when the division of the pie is sufficiently balanced.

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