On the Strategic Advantage of Negatively Interdependent Preferences[*]

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Abstract

We study certain classes of supermodular and submodular games which are symmetric with respect to material payoffs but in which not all players seek to maximize their material payoffs. Specifically, a subset of players have negatively interdependent preferences and care not only about their own material payoffs but also about their payoffs relative to others. We identify sufficient conditions under which members of the latter group have a strategic advantage in the following sense: at all intragroup symmetric equilibria of the game, they earn strictly higher material payoffs than do players who seek to maximize their material payoffs. We show that these conditions are satisfied by a number of games of economic importance, and discuss the implications of these findings for the evolutionary theory of preference formation and the theory of Cournot competition.

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1 Introduction

A fundamental ingredient of most economic models is the hypothesis of independent preferences: agents choose their actions with the sole purpose of maximizing their own material payoffs regardless of how their actions affect the payoffs of other individuals. While this postulate is seldom given explicit justification, it appears to be based on the intuition that those individuals who are willing to make material sacrifices to affect the payoffs of others will lose wealth relative to those who are unwilling to do so, with the eventual consequence that the latter will come to dominate the economy. In this case, the maximization of one's own material payoffs would simply be a pre-condition for survival in an environment where a competitive selection process is at work. While this intuition may be persuasive in the context of perfectly competitive environments, it can be seriously misleading when applied to strategic settings, for it is not generally true in such environments that agents who pursue the maximization of their own material payoffs will obtain higher material payoffs in equilibrium than symmetrically placed individuals who maximize other objective functions. Indeed, at least in some strategic environments, the reverse may be true.

This last point has been demonstrated in the literature mostly by means of particular specifications of Cournot oligopoly models. For instance, Vickers (1984) and Fershtman and Judd (1987) have shown in the context of such models (with linear demand and cost functions) that a firm whose objective function gives a positive weight to its relative profits or sales will outperform the absolute profit maximizers in terms of absolute profits. It can also be shown that similar results obtain in some other strategic environments, such as common pool resource and public good games (Koçkesen et al., 1997), in which agents with negatively interdependent preferences (that is, those who care about both absolute and relative payoffs) may well obtain greater absolute payoffs in equilibrium than do symmetrically placed absolute payoff maximizers. In such environments, interdependent preferences may be said to yield a strategic advantage to those who possess them.

While it is useful to know that interdependent preferences yield a strategic advantage in the particular examples that have been considered in the literature to date, it is difficult to judge the broader significance of such findings without some assessment of the extent to which such results can be generalized. Our purpose in this paper is to provide a fairly general analysis of the issue, and to show that the cases in which negatively interdependent preferences yield an unambiguous strategic advantage over independent preferences are far more common than one might at first expect. We consider the general classes of supermodular and submodular games in which only a subset of players have independent objective functions whereas the rest have negatively interdependent preferences. Informally stated, we identify several sets of sufficient conditions under which the members of the latter group have a strategic advantage in the following sense: at all intragroup symmetric equilibria, the interdependent individuals earn higher material payoffs than do players who seek to maximize their own material payoffs. We also show that there are simple symmetric games in which players with independent preferences unambiguously outperform those with interdependent preferences in terms of absolute payoffs.

These findings may be considered interesting in their own right, since they relate the familiar notions of strategic complementarities and substitutabilities to the possibility that an envious concern with the payoffs of others may lead one to have greater absolute payoffs in equilibrium than those obtained by (absolute) payoff maximizers. Moreover, these results achieve a useful level of generality, for it turns out that our sufficiency conditions are satisfied by a number of games which play central roles in various branches of economic theory, including the Cournot oligopoly, input and public good games, search models and arms races.

The main findings of this paper find immediate application in at least two contexts. Our first application concerns the theory of preference formation. Evolutionary models of preference formation are typically based on the assumption that the selection dynamics are payoff monotonic: the population share of those endowed with preferences that are more highly rewarded materially increases relative to the population share of those who are less highly rewarded (see, among others, Rogers, 1994, Bergstrom, 1995 and Robson, 1996). In the presence of such payoff monotonic selection dynamics, our results enable us to identify the evolutionary stability properties of absolute payoff maximizing behavior. Specifically, with respect to the economic environments studied here, we are able to show that the long run population composition cannot be a monomorphic one composed only of absolute payoff maximizers. While this does not preclude the persistent presence of independent agents in the population, it calls into question the common practice of modeling economic agents exclusively as absolute payoff maximizers.

Secondly, our findings have some interesting implications for the analysis of oligopolistic industries. This stems from the fact that executive managers may in some circumstances either choose or be given incentives by owners to incorporate relative profit (or market share) concerns into their decision making. For example, when shareholders cannot calculate the potential profits of the firm under different action choices, managers may well seek to occupy a higher rank in the industry distribution of profits so as to demonstrate their success. Alternatively, as advanced by several authors, there may exist informational reasons for owners to provide incentives to their managers that cause the latter to be concerned with the firm's performance relative to those of similarly situated competing firms. In particular, if managers' efforts are unobservable by owners and there is some common uncertainty affecting all firms in the industry, owners may benefit from making their managers' compensation contingent upon relative as well as absolute profits (Holmström, 1982, and Nalebuff and

Stiglitz, 1983). As a corollary of our results, we find here that such contracts may yield an unplanned strategic advantage to a firm in terms of its absolute profits. Since this advantage is not based on technological or marketing superiority, it helps provide a novel explanation for the evident inequality of market shares even in industries which are composed of firms operating with very similar technologies.

The paper is organized as follows. In Section 2 we introduce our general framework and formalize the nature of the present inquiry. Section 3 contains our main results which identify certain classes of supermodular and submodular games in which interdependent agents have a strategic advantage over independent agents in all intragroup symmetric equilibria. Examples of several commonly studied games that belong to these classes are also presented in this section. Finally, in Section 4 we elaborate on the implications of our main findings for the theories of preference formation and Cournot competition. We conclude with a discussion of directions for future research, and an appendix containing the proofs of our results.

2 The Framework

Since our ultimate aim is to compare the performances of different preference structures in terms of monetary outcomes, we shall concentrate on games in strategic form in which no player has an *a priori* advantage in terms of the primitives of the game. Consequently, our focus will be exclusively on symmetric games. Given any integers $n \geq 2$ and $\ell \geq 1$; we let Γ stand for a symmetric n-person normal form game with an ℓ-dimensional action space. That is,

$$\Gamma = (X, \{\pi_r\}_{r=1,...,n})$$

where $X \subseteq \mathbb{R}^\ell$ and $\pi_r : X^n \to \mathbb{R}$ are the action space and the absolute payoff function of player r; and where we have

$$\pi_r(x_1, ..., x_n) = \pi_{\sigma(r)}(x_{\sigma(1)}, ..., x_{\sigma(n)}) \quad \forall x_r \in X \text{ and } r = 1, ..., n \tag{1}$$

for any swap operator σ on $\{1, ..., n\}$.[1] As is usually done in applied and experimental game theory, we interpret π_r as the material payoff function of player r. Moreover, we assume that Γ satisfies the following nonnegativity conditions:

$$\pi_r(x) \geq 0 \quad \forall x \in X^n \text{ and } r = 1, ..., n. \tag{2}$$

These conditions allow us interpret the notion of "relative payoffs" in the usual sense.[2] Finally, we denote the class of all Γ that satisfy (2) by $\mathcal{G}$, and let $N(\Gamma)$ stand for the set of

[1] A swap operator σ on $\{1, ..., n\}$ is a permutation on $\{1, ..., n\}$ such that $|\{r : \sigma(r) \neq r\}| = 2$.

[2] When π_r is bounded from below, the requirement in (2) is not restrictive. For by adding $|\inf_{x \in X} \pi_r(x)|$ to π_r, we obtain a new game which satisfies (2) and which is strategically equivalent to the original game; see the examples given in Section 3.3.

all Nash equilibria of $\Gamma \in \mathcal{G}$.

In what follows, we endow $\mathbb{R}^\ell$ with a partial order $\succsim$ to obtain an ordered vector space.[3] In fact, we shall often take X to be a chain, i.e., assume that $\succsim$ linearly (completely) orders X. We note that the linearity of the order $\succsim$ does not turn out to be demanding as a structural assumption in many applications. Indeed, in numerous economic contexts ℓ is taken to be 1; i.e. $X \subseteq \mathbb{R}$; in which case there is a natural linear order on X. Therefore, insofar as such games are concerned, the assumption that X is completely ordered is without loss of generality. On the other hand, when $\ell > 1$, depending on the economic context, one may linearly order X by means of any completion of the coordinatewise ordering such as the lexicographic, leximin or leximax orderings.

Throughout this paper we assume that the set of players consists of two different types, namely, independent and (negatively) interdependent players. The independent players are those who are absolute payoff maximizers in the usual sense; the objective function of an independent player i is precisely her own monetary payoff function π_i. On the other hand, (negatively) interdependent players are concerned not only with their absolute payoffs, but also with how their absolute payoffs compare with the average payoff in the game. Let $\bar{\pi} \equiv \frac{1}{n} \sum \pi_r$ denote the average (absolute) payoff function on X^n, and define the relative payoff of player j as follows:[4]

$$\rho_j = \begin{cases} \dfrac{\pi_j}{\bar{\pi}}, & \text{if } \pi_j > 0 \\ 0, & \text{if } \pi_j = 0. \end{cases}$$

The objective function of an interdependent player j is given by $x \mapsto F(\pi_j, \rho_j)$ where F is an arbitrary strictly increasing real function on $\mathbb{R}^2_+$. This particular way of representing negatively interdependent preferences has recently been proposed and axiomatically characterized by Ok and Koçkesen (1997). In particular, when Γ is played between individuals (as opposed to, say, firms), the preferences represented in this form can be interpreted as a compromise between the standard case where one is assumed to care only about her monetary earnings π_j, and the extreme case where she is concerned exclusively with her relative payoff in the game, i.e., with ρ_j (the latter case corresponds to Duesenberry's relative income hypothesis.)[5] If, on the other hand, Γ is an oligopoly game, then an interdependent player

[3] A partial order $\succsim$ on $\mathbb{R}^\ell$ is a relation on $\mathbb{R}^\ell$ which is reflexive, transitive and antisymmetric. (As usual, we write $\succ$ for $\succsim \setminus =$.) We say that $(\mathbb{R}^\ell, \succsim)$ is an ordered vector (or Riesz) space, if $\succsim$ is a partial order, and if, for any $x, y \in \mathbb{R}^\ell$ with $x \succsim y$, we have $x + z \succsim y + z$ and $\lambda x \succsim \lambda y$ for all $z \in \mathbb{R}^\ell$ and $\lambda \geq 0$.

[4] We use the convention of setting $\rho_j(x) = 0$ whenever $\pi_j(x) = 0$ to avoid the difficulty of evaluating the indeterminate form $0/0$.

[5] Special cases of this representation of interdependent preferences are utilized in numerous economic contexts ranging from models of optimal income taxation to experimental bargaining games. We refer the reader to the references cited in Ok and Koçkesen (1997) and Koçkesen et al. (1997).

with such an objective function can be thought of as a firm (or manager) who targets the maximization of not only its profits but also its profit share in the industry.

Let us then assume that precisely $k \in \{1, \ldots, n-1\}$ many players in $\Gamma \in \mathcal{G}$ are independent. We define the n-person normal form game

$$\Gamma_F(k) = (X, \{p_r\}_{r=1,\ldots,n})$$

with

$$p_r \equiv \begin{cases} \pi_r, & \text{if } r \in I_k \\ F(\pi_r, \rho_r), & \text{if } r \in J_k \end{cases} \tag{3}$$

where

$$I_k \equiv \{1, \ldots, k\} \quad \text{and} \quad J_k \equiv \{k+1, \ldots, n\},$$

and $F : \mathbb{R}^2_+ \to \mathbb{R}$ is any strictly increasing function.[6] Clearly, in $\Gamma_F(k)$, the set of all independent players is I_k, and the set of all interdependent players is J_k. (By definition, there are $|I_k| = k$ many independent players, and $|J_k| = n - k$ many interdependent players in $\Gamma_F(k)$.) The crucial interpretation is that, while we only observe the payoffs associated with the game Γ as outsiders, the associated players actually engage in playing $\Gamma_F(k)$ instead of Γ.

In this paper, we wish to analyze the nature of Nash equilibrium of an arbitrary $\Gamma_F(k)$. However, it must be noted at the outset that there are two immediate difficulties that we shall often assume away in the general analysis that follows. First, the existence of a Nash equilibrium of $\Gamma_F(k)$ is rather difficult to establish in general. Even if we posit the standard requirement of quasiconcavity of π_r in x_r for all r (along with continuity of π_r, and compactness and convexity of X), the payoff function p_j, $j \in J_k$, need not inherit this property. Even the deeper existence theorems established in the literature (such as those of Topkis, 1979, Nishimura and Friedman, 1981, and Dasgupta and Maskin, 1986) are generally not helpful in settling this existence problem. It appears that the best strategy at this stage is to ignore this problem, and search for some qualitative properties of the equilibria of $\Gamma_F(k)$, when it exists. In fact, in many examples of economic interest (such as Cournot and Bertrand oligopolies, common pool resource and public good games, arms races, etc.) one can directly verify that the set of equilibria of $\Gamma_F(k)$ is nonempty, and hence our line of attack turns out to be fruitful.

The second difficulty is the analytical intractability of certain non–symmetric equilibria of an arbitrary $\Gamma_F(k)$. The analysis is greatly simplified when we focus instead on the

[6] As the proofs given in the appendix will readily reveal, our entire development remains intact if each p_j was defined in terms of a strictly increasing F_j for all $j \in J_k$ where $F_{j_1} \neq F_{j_2}$ was allowed for any $j_1 \neq j_2$.

intragroup symmetric Nash equilibria of a given game $\Gamma_F(k)$, denoted $N_{sym}(\Gamma_F(k))$, which is defined as

$$N_{sym}(\Gamma_F(k)) \equiv \{([a]_k, [b]_{n-k}) \in N(\Gamma_F(k)) : a, b \in X\}$$

where $[t]_l$ denotes the $l-$replication of the object t.[7] One could, of course, advance a "focal point" argument to justify interest in $N_{sym}(\Gamma_F(k))$. Perhaps more importantly, we shall observe that in most of the economic examples considered below, we actually have $N(\Gamma_F(k)) = N_{sym}(\Gamma_F(k))$ so a focus on intragroup symmetric equilibria is unrestrictive. This is trivially the case in all two person games.

Finally, let us clarify what we mean by "studying the nature of $N_{sym}(\Gamma_F(k))$" given a $\Gamma \in \mathcal{G}$. Put precisely, we are interested in identifying some general subclasses of $\mathcal{G}$ where interdependent players have a definite strategic advantage over the independent players in terms of monetary payoffs, that is, where

$$\pi_j(\omega) > \pi_i(\omega) \quad \forall (i, j) \in I_k \times J_k \text{ and } \omega \in N_{sym}(\Gamma_F(k)). \tag{4}$$

There are several concrete economic motivations behind this inquiry as already hinted in the previous section. Indeed, whether or not interdependent players (who do not directly maximize their absolute payoffs) obtain higher (or strictly higher) absolute payoffs than all independent players (who do target the maximization of their absolute payoffs) is a question of great interest in evolutionary theories of preference formation. The same inquiry also turns out to be quite relevant with respect to some recent approaches in oligopoly theory where the distinction between managerial incentives and the objectives of the firm is explicitly modeled. Such applications of our basic analysis will be discussed in some detail in Section 4.

3 Main Results

3.1 Supermodular Games

An n-person normal form game $\Gamma \in \mathcal{G}$ is said to be **supermodular** whenever X is a sublattice of $\mathbb{R}^\ell$ and

$$\pi_r(x \vee y) + \pi_r(x \wedge y) \geq \pi_r(x) + \pi_r(y) \quad \forall x, y \in X^n \text{ and } r = 1, ..., n,$$

[7] By an immediate application of the symmetry condition (1) we have

$$\pi_{i_1}(\omega) = \pi_{i_2}(\omega) \quad \forall i_1, i_2 \in I_k, \text{ and } \pi_{j_1}(\omega) = \pi_{j_2}(\omega) \quad \forall j_1, j_2 \in J_k$$

whenever $\omega \in N_{sym}(\Gamma_F(k))$; this observation will prove extremely useful in what follows.

where $x \vee y$ is the lowest upper bound of $\{x, y\}$ in X (with respect to $\succsim$) and $x \wedge y$ is the greatest lower bound of $\{x, y\}$ in X. We say that Γ is **strictly supermodular** if the above inequality holds strictly for all r and $x, y \in X^n$ such that $\{x \vee y, x \wedge y\} \neq \{x, y\}$.

Supermodular games correspond to games in which the actions of two distinct players are strategic complements in the sense that the best response correspondences of the players are increasing (Bulow et al., 1985). It is well known that if π_r is C^2, then Γ is supermodular if and only if $\partial^2\pi_r/\partial x_r \partial x_q \geq 0$ for all $r \neq q$ (Topkis, 1978). Moreover, any supermodular $\Gamma \in \mathcal{G}$ has at least one symmetric equilibrium, provided that X is compact, and $\pi_r(\cdot, x_{-r})$ is upper semicontinuous for all $x_{-r} \in X^{n-1}$ (Topkis, 1979, and Vives, 1990).[8,9]

We next introduce another interesting subclass of $\mathcal{G}$ that we will work with.

Definition. An n-person normal form game $\Gamma \in \mathcal{G}$ is said to be **positively (negatively) action-monotonic** if, for all $x \in X^n$,

$$x_r \succ (\prec)\ x_q \quad \text{implies} \quad \pi_r(x) > \pi_q(x).$$

Action-monotonicity is a curious property that requires a tight connection (a certain kind of isotonicity) between payoffs and actions. While it is not a standard condition for normal form games, action-monotonicity is nevertheless satisfied by a wide variety of symmetric games. In general, any $\Gamma \in \mathcal{G}$ with

$$\pi_r(x) = \Psi(x_r, \psi(x)), \quad r = 1, ..., n$$

where $\Psi : X \times \mathbb{R} \to \mathbb{R}_+$ is strictly increasing (decreasing) in the first component and $\psi : X^n \to \mathbb{R}$ is symmetric, is positively (negatively) action-monotonic. We shall observe below that several widely studied symmetric games, including common pool resource extraction and public good games, and Cournot oligopolies with constant average costs, are special cases of this general formulation.

The first main result of this section provides an answer to the general question stated in the previous section within the broad class of all action-monotonic strictly supermodular games:

Theorem 1. Let $k \in \{1, ..., n-1\}$ and take any strictly increasing $F : \mathbb{R}^2_+ \to \mathbb{R}$. Let $\Gamma \in \mathcal{G}$ be strictly supermodular with X being any chain. If Γ is action-monotonic, then for any $\hat{x} \in N_{sym}(\Gamma_F(k))$ with $\hat{x}_1 \neq \hat{x}_n$, we have

$$\pi_j(\hat{x}) > \pi_i(\hat{x}) \quad \forall (i, j) \in I_k \times J_k.$$

[8] Unless otherwise is explicitly stated, all references to topological properties are to be considered in terms of the Euclidean topology throughout this paper.

[9] $\pi_r : X \to \mathbb{R}$ is said to be upper (lower) semicontinuous, if, for any $x, x^m \in X$, $m = 1, ...,$ $\lim x^m = x$ implies that $\limsup \pi_r(x^m) \leq \pi_r(x)$ ($\liminf \pi_r(x^m) \geq \pi_r(x)$, resp.)

Theorem 1 states that at any intragroup symmetric equilibrium of an action monotonic strictly supermodular game, the absolute payoffs to interdependent players are strictly greater than those to independent players, unless both groups take the same equilibrium action. There are indeed examples of commonly studied games which satisfy the requirements of the theorem and in which strict inequality of payoffs obtains; a number of these are discussed in Section 3.3 below.

Remark 1. Milgrom and Shannon (1994) have introduced a weakening of the supermodularity concept, namely quasisupermodularity, which is nevertheless strong enough to allow fruitful analysis (especially when X is a chain): an n-person normal form game $\Gamma \in \mathcal{G}$ is said to be **quasisupermodular**, if $\pi_r(x) \geq (>)\, \pi_r(x \wedge y)$ implies $\pi_r(x \vee y) \geq (>)\, \pi_r(y)$, for all $x, y \in X^n$ and $r = 1, ..., n$. We note that Theorem 1 remains valid when we replace strict supermodularity with quasisupermodularity, provided that $k \in \{1, ..., n-2\}$. That this claim is true becomes apparent upon close inspection of the proof of Theorem 1 which is given in the appendix. $\|$

The significance of Theorem 1 is limited, however, by the fact that it deals only with intragroup symmetric equilibria. The following corollary, in contrast, keeps the hypotheses of linear ordering and action-monotonicity of Theorem 1, but deals with all Nash equilibria for the special case when there is only one interdependent player.

Corollary 1. Let $F : \mathbb{R}^2_+ \to \mathbb{R}$ be any strictly increasing function. For any action-monotonic and strictly supermodular $\Gamma \in \mathcal{G}$ where X is any chain, and for any $x^* \in N(\Gamma_F(n-1))$ with $x^*_i \neq x^*_n$ for some $i \in I_{n-1}$,

$$\pi_n(x^*) > \frac{1}{n-1} \sum_{i=1}^{n-1} \pi_i(x^*).$$

Corollary 1 states that in the special case when only one of the players has interdependent preferences, this player earns a payoff that is at least as high as the average payoff earned by the remaining (independent) players. Unless all players choose the same action, moreover, the interdependent player earns a strictly higher level of absolute payoffs than the population average. As we shall see in the next section, this is important from an evolutionary perspective, since it implies that the extinction of players with interdependent preferences cannot occur under any payoff monotonic evolutionary selection dynamics.

Remark 2. (a) Corollary 1 remains valid if we take X to be any lattice in $\mathbb{R}^\ell$, but assume that $\max\{x^*_i : i \in I_{n-1}\} \neq \emptyset$ for all $x^* \in N(\Gamma_F(n-1))$.

(b) The following generalization of Corollary 1 is also true: Let $k \in \{1, ..., n-1\}$, let X be any chain, take any strictly increasing $F : \mathbb{R}^2_+ \to \mathbb{R}$, and let $\Gamma \in \mathcal{G}$ be action-monotonic and strictly supermodular. If $\hat{x} \in N(\Gamma_F(k))$ and $\hat{x}_{k+1} = \cdots = \hat{x}_n$ with $\hat{x}_i \neq \hat{x}_n$ for some $i \in I_k$; then

$$\frac{1}{n-k}\sum_{j=k+1}\pi_j(\hat{x}) > \frac{1}{k}\sum_{i=1}\pi_i(\hat{x}):$$

As with Corollary 1, this observation (the proof of which is given in the appendix) has implications in the context of evolutionary preference formation dynamics; see Section 4.1 below. ‖

Theorem 1 and its corollary rely on the property of action monotonicity, which a number of important supermodular games do not satisfy. Fortunately, this requirement can also be relaxed for the case in which only one of the players has interdependent preferences.

Theorem 2. Let $F : \mathbb{R}^2_+ \to \mathbb{R}$ be any strictly increasing function. For any strictly supermodular $\Gamma \in \mathcal{G}$ where X is any lattice, and for any $\hat{x} \in N_{sym}(\Gamma_F(n-1))$ with $\hat{x}_1 \neq \hat{x}_n$;

$$\pi_n(\hat{x}) > \pi_i(\hat{x}) \quad \forall i \in I_{n-1}:$$

It is worth noting that Theorem 2 dispenses with two restrictive hypotheses of Theorem 1. Indeed, X need not be linearly ordered for Theorem 2 to work; any partial order on $\mathbb{R}^\ell$ (like the familiar vector dominance) would do. Moreover, this result covers all strictly supermodular games, including those that violate action-monotonicity. In particular, Theorem 2 shows that the interdependent player unambiguously holds the upper hand in any strictly supermodular two-person game $\Gamma \in \mathcal{G}$ with X being any lattice in $\mathbb{R}^\ell$; for $N_{sym}(\Gamma_F(1)) = N(\Gamma_F(1))$. Finally, we note that while Theorem 2 refers only to the properties of intragroup symmetric equilibria, this is not restrictive in models where independent players always choose the same equilibrium action. The examples given in Subsection 3.3 will show that this observation is at times quite useful.

We conclude this section by demonstrating that action-monotonicity alone is not sufficient to yield any of the above results. In particular, the following example illustrates the crucial role played by supermodularity in Theorems 1 and 2.

Example. Consider the two-person normal form game $\Gamma \in \mathcal{G}$ represented by the bimatrix

$$\begin{bmatrix} (1;1) & (1;1/2) & (3;2) \\ (1/2;1) & (1;1) & (2;0) \\ (2;3) & (0;2) & (2;2) \end{bmatrix}:$$

Here the strategy space of each agent is the chain $\{1;2;3\}$: This game is easily checked to be (negatively) action monotonic but not supermodular. It has three Nash equilibria, $N(\Gamma) = \{(1;3);(3;1);(2;2)\}$. Taking $F(z_1;z_2) = z_1 z_2$ for all $z_1;z_2 \geq 0$; and adopting the convention of treating the column player as player 2, the game $\Gamma_F(1)$ is represented by the bimatrix

$$\begin{bmatrix} (1,1) & (1,1/3) & (3,8/5) \\ (1/2,4/3) & (1,1) & (2,0) \\ (2,18/5) & (0,4) & (2,2) \end{bmatrix}.$$

Clearly, $N(\Gamma_F(1)) = \{(1,3)\}$, and $\pi_1(1,3) = 3 > 2 = \pi_2(1,3)$. We therefore conclude that in this game the player with interdependent behavior is subject to a strategic disadvantage. $\|$

Example 1 illustrates that action monotonicity is in fact consistent with the possibility of interdependent players having a strategic disadvantage against independent players. It should thus be formally clear that our main inquiry (that is, determining a general subclass of $\mathcal{G}$ the members of which satisfy (4)) is not a trivial one. On the other hand, we know from Theorem 1 that action monotonicity together with strict supermodularity is sufficient for interdependent players to have a strategic advantage over the independent players. Alternative sufficient conditions for the strategic advantage of interdependent players may be found, however, that do not rely on supermodularity. The following section deals with the case of submodular games.

3.2 Submodular Games with Spillovers

As explored by Cooper and John (1988), among others, a wide variety of economically interesting games exhibit a negative (or positive) spillover effect. In such games, an increase in the level of action taken by a player decreases (or increases) the absolute payoffs of all other players. The strong form of this property is, however, too demanding, for it is not satisfied by games in which players have at least one potential action which would nullify the influence of other players. For instance, in the classical Cournot model of oligopoly, a firm may completely annihilate the effect of quantity choices of other firms on its profits simply by choosing to shut down. For this reason, we shall work here with a slightly weaker notion of the spillover effect (which will later be seen to be present in the Cournot game).

Definition. Let

$$A \equiv \{x \in N_{sym}(\Gamma_F(k)) : F : \mathbb{R}^2_+ \to \mathbb{R} \text{ is strictly increasing and } k = 1, \ldots, n-1\}.$$

An n-person normal form game $\Gamma \in \mathcal{G}$ is said to have **negative spillovers**, if for any $x \in A$,

$$t^1 \succ x_r \succ t^2 \quad \text{implies} \quad \pi_q(x_{-r}, t^1) < \pi_q(x) < \pi_q(x_{-r}, t^2)$$

for all r and $q \neq r$. Games with **positive spillovers** are defined dually.

It turns out that in games with negative spillovers, there is a tight connection between action monotonicity and the possibility of $\pi_j(x^*) \geq \pi_i(x^*)$ holding for all $i \in I_k$ and all $j \in J_k$. The following proposition aims to drive this point home.

Lemma 1. Let $k \in \{1, \ldots, n-1\}$ and take any strictly increasing $F : \mathbb{R}^2_+ \to \mathbb{R}$. For any $\Gamma \in \mathcal{G}$ with negative spillovers and any $\hat{x} \in N_{sym}(\Gamma_F(k))$,

$$\pi_j(\hat{x}) \geq (>)\, \pi_i(\hat{x}) \quad \forall (i,j) \in I_k \times J_k$$

holds only if $\hat{x}_j \succsim (\succ)\, \hat{x}_i$ for all $(i,j) \in I_k \times J_k$. Moreover, if $k = n-1$, then, for any $i \in I_{n-1}$,

$$\pi_n(\hat{x}) \geq (>)\, \pi_i(\hat{x}) \quad \text{if and only if} \quad \hat{x}_n \succsim (\succ)\, \hat{x}_i.$$

This lemma shows that positive action-monotonicity at the equilibrium action profile is essentially a necessary condition for (4) to hold in the case of games with negative spillovers. It can be shown similarly that negative action monotonicity at the equilibrium action profile is a necessary condition for (4) to hold for games with positive spillovers.

Given that X is a sublattice of $\mathbb{R}^\ell$, an n-person normal form game $\Gamma \in \mathcal{G}$ is said to be **submodular** if

$$\pi_r(x \vee y) + \pi_r(x \wedge y) \leq \pi_r(x) + \pi_r(y) \quad \forall x, y \in X^n \text{ and } r = 1, \ldots, n.$$

We say that Γ is **strictly submodular** if the above inequality holds strictly for all r and $x, y \in X^n$ such that $\{x \vee y, x \wedge y\} \neq \{x, y\}$. In contrast with supermodular games, submodular games are those in which actions of any two players are strategic substitutes in the sense that the best response maps of all players are decreasing (Bulow et al., 1985).

Finally, we shall need the following concept for the analysis of this subsection.

Definition. An n-person normal form game $\Gamma \in \mathcal{G}$ is called **symmetric in equilibrium** if it does not possess an asymmetric Nash equilibrium, i.e. $\hat{x} \in N(\Gamma)$ implies that $\hat{x}_i = \hat{x}_j$ for all $i \neq j$.

While symmetry in equilibrium is admittedly a demanding property, it is satisfied by a variety of commonly studied symmetric games such as the stag hunt game, prisoner's dilemma, the common pool resource game, many symmetric Cournot and Bertrand oligopoly models, and public good games. In fact, for strictly submodular games, this property is nothing other than the requirement of uniqueness of equilibrium:

Lemma 2. Let $\Gamma \in \mathcal{G}$ be a strictly submodular game such that $N(\Gamma) \neq \emptyset$. Then, Γ is symmetric in equilibrium if, and only if, it has a unique equilibrium.

Our main result takes as primitives those games in $\mathcal{G}$ where the common strategy set of the players is convex and compact, and the payoff function of the rth player is continuous and quasiconcave in x_r, for all r.[10] Denoting the class of all such games by $\mathcal{G}_0$, we are now ready to state

Theorem 3. Let $\Gamma \in \mathcal{G}_0$, $k \in \{1, ..., n-1\}$ and take any strictly increasing $F : \mathbb{R}^2_+ \to \mathbb{R}$. If Γ is a positively (negatively) action monotonic and strictly submodular game with negative (positive) spillovers, and is symmetric in equilibrium, then, for any $\hat{x} \in N_{sym}(\Gamma_F(k))$ such that $\hat{x}_1 \neq \hat{x}_n$ and $\pi_r(\hat{x}) > 0$ for all r, we have

$$\pi_j(\hat{x}) > \pi_i(\hat{x}) \quad \forall (i, j) \in I_k \times J_k.$$

In words, if a positively (negatively) action monotonic strictly submodular game with negative (positive) spillovers satisfies symmetry in equilibrium, then the interdependent players have a strategic advantage over the independent players in that at any intragroup symmetric equilibrium of the game they earn strictly greater absolute payoffs than independent players, provided that both groups take distinct equilibrium actions. The examples of the next subsection will illustrate the power of this observation.

Remark 3. Lemma 1 has demonstrated the necessity of action monotonicity for the conclusion of Theorem 3 to hold. Since we think of submodular games with spillovers as primitives in the above analysis, the only question about the tightness of this result concerns the relaxation of the symmetry in equilibrium condition. To see that this condition too cannot be completely relaxed in Theorem 3, consider the following "hawk-dove" game represented by the bimatrix

$$\begin{bmatrix} (10, 10) & (5, 15) \\ (15, 5) & (1, 1) \end{bmatrix}$$

and define Γ as its mixed strategy extension. One can easily verify that Γ satisfies all the hypotheses of Theorem 3 except for symmetry in equilibrium, and that $((1, 0), (0, 1)) \in N_{sym}(\Gamma_F(1))$ where $F(z_1, z_2) = z_1 z_2$ for all $z_1, z_2 \geq 0$. Hence there exists an equilibrium in which the player with interdependent preferences obtains a strictly lower absolute payoff. ∥

[10] We note that assuming compactness of X is often less restrictive than assuming completeness of X as a chain (i.e. postulating that $\{\sup Y, \inf Y\} \subset X$ for all $Y \subseteq X$), provided that $\succsim$ is a completion of the coordinatewise ordering. For instance, since the order topology derived from the lexicographic ordering is finer than the Euclidean topology, and since a chain is compact in its order topology iff it is complete (Birkhoff, 1963, p.242, Theorem 12), every complete chain w.r.t. the lexicographic ordering must be compact in the Euclidean topology. It is also worth noting that if X is a complete lattice with respect to the coordinatewise ordering, then, and only then, it is compact in the Euclidean topology (see Frink, 1942, Theorems 5 and 9).

In closing, we note that one can again relax the requirement of action-monotonicity when there is only one interdependent player in the game. The following is then a counterpart to Theorem 2.

Theorem 4. Let $F : \mathbb{R}^2_+ \to \mathbb{R}$ be any strictly increasing function. If Γ is a strictly submodular game with negative or positive spillovers, and is symmetric in equilibrium, then, for any $\hat{x} \in N_{sym}(\Gamma_F(n-1))$ such that $\hat{x}_1 \neq \hat{x}_n$ and $\pi_r(\hat{x}) > 0$ for all r, we have

$$\pi_n(\hat{x}) > \pi_i(\hat{x}) \quad \forall i \in I_{n-1}.$$

Theorem 4 provides sufficient conditions for the single player with interdependent preferences to have a strategic advantage with respect to the remaining players in submodular games. As with Theorem 2, this result has evolutionary implications. For any game which satisfies the conditions of the Theorem, a population consisting exclusively of players with independent preferences will be vulnerable to invasion by the emergence of a single player with negatively interdependent preferences, under any payoff monotonic evolutionary selection dynamics. This intuition will be formalized in Section 4.1 below.

3.3 Examples

The usefulness of the results presented in the previous section hinges on the degree to which they may be applied to games of economic interest. In this subsection we present three classes of such examples.

I. Input and Public Good Games. Consider an n-person game $\Gamma \in \mathcal{G}$ with $X = [0, \omega]$, $0 < \omega < \infty$, where the absolute payoff function of player r is additively separable and defined on X^n as

$$\pi_r(x) = u(\omega - x_r) + v\left(\sum_{q=1}^{n} x_q\right), \quad r = 1, \ldots, n.$$

We assume that u and v are strictly positive-valued, strictly increasing C^2 functions on $[0, \omega]$ and $[0, n\omega]$ respectively, with $u'' < 0$, $u'(0+) = \infty$ and $u'(\omega) < v'(0)$. The last two conditions guarantee that the symmetric Nash equilibria of Γ are interior. We show below that all equilibria of Γ are in fact symmetric.

Here, ω stands for the private endowment of agent r, and the action x_r is interpreted as her input (effort) supply to a shared production process or her contribution to the provision of a public good. In turn, the function u is thought of as the utility provided by privately consuming own input (or the utility of private consumption good), and the function v as the utility of jointly produced good (or the public good, respectively). In accordance with the general analysis presented in the previous subsections, we assume in what follows that players $k+1$ to n maximize an objective function of the form (3) for some differentiable F with $F_1; F_2 > 0$:

It is easy to see that Γ is negatively action monotonic and has positive spillovers. To see that Γ is symmetric in equilibrium, take any $\hat{x} \in N(\Gamma)$ and let $\hat{x}_r < \hat{x}_q$ for some players r and q. We must then have $\partial\pi_r(\hat{x})/\partial x_r \leq 0 \leq \partial\pi_q(\hat{x})/\partial x_q$ while $u'' < 0$ implies that

$$\frac{\partial\pi_r(\hat{x})}{\partial x_r} = -u'(\omega - \hat{x}_r) + v'\left(\sum_{s=1} \hat{x}_s\right) > -u'(\omega - \hat{x}_q) + v'\left(\sum_{s=1} \hat{x}_s\right) = \frac{\partial\pi_q(\hat{x})}{\partial x_q}:$$

Hence Γ is symmetric in equilibrium, and the conditions $u'(0+) = \infty$ and $u'(\omega) < v'(0)$ ensure that any equilibrium $\hat{x}$ is interior, i.e., $\hat{x} \in (0; \omega)^n$ for any $\hat{x} \in N(\Gamma)$. The same reasoning also shows that $\hat{x}_{i_1} = \hat{x}_{i_2}$ for all $i_1; i_2 \in I_k$ and all $\hat{x} \in N(\Gamma_F(k))$. (Notice that the last observation implies that $N_{sym}(\Gamma_F(n-1)) = N(\Gamma_F(n-1))$). Finally, we claim that the equilibrium actions of independent and interdependent agents are different from each other, i.e., $\hat{x}_i \neq \hat{x}_j$ for any $(i; j) \in I_k \times J_k$ and any $\hat{x} \in N_{sym}(\Gamma_F(k))$: Note that $\hat{x}_i \in [0; \omega)$ for all $i \in I_k$ since $u'(0+) = \infty$: Now suppose that for each player $i \in I_k$, we have $\hat{x}_i \in (0; \omega)$ so that $\partial\pi_i(\hat{x})/\partial x_i = 0$: If there was an $(i; j) \in I_k \times J_k$ such that $\hat{x}_i = \hat{x}_j$; we would have $\partial\pi_i(\hat{x})/\partial x_i = \partial\pi_j(\hat{x})/\partial x_j = 0$: But this cannot hold, for otherwise

$$\frac{\partial p_j(\hat{x})}{\partial x_j} = -\frac{\pi_j F_2}{\sum \pi_q}\left(\sum_{q \neq j} \frac{\partial\pi_q}{\partial x_j}\right)\Bigg|_{x=\hat{x}} < 0;$$

while we must have $\partial p_j(\hat{x})/\partial x_j = 0$ since $\hat{x}_j = \hat{x}_i \in (0; \omega)$: Hence, when independent players take an interior action, $\hat{x}_i \neq \hat{x}_j$ for any $(i; j) \in I_k \times J_k$ and any $\hat{x} \in N(\Gamma_F(k))$: Finally suppose that for each player $i \in I_k$, we have $\hat{x}_i = 0$. If it were also the case that for each player $j \in J_k$, we have $\hat{x}_j = 0$, then $\hat{x}$ could not be an equilibrium since $u'(\omega) < v'(0)$ and independent players could benefit from a unilateral deviation. Hence $\hat{x}_i \neq \hat{x}_j$ for any $(i; j) \in I_k \times J_k$ and any $\hat{x} \in N_{sym}(\Gamma_F(k))$:

Now, if v is strictly convex (concave), then the game is strictly supermodular (submodular) and since we have established above that $\hat{x}_1 \neq \hat{x}_n$ for any $\hat{x} \in N_{sym}(\Gamma_F(k))$; we may use Theorem 1 (Theorem 3) to conclude that at any intragroup symmetric equilibrium the agents with interdependent preferences obtain strictly higher absolute payoffs than do agents with independent preferences. Moreover, since we have found above that $N_{sym}(\Gamma_F(n-1)) = N(\Gamma_F(n-1))$, if there is only one interdependent player in the game, it follows from Theorem 2 (Theorem 4, resp.) that this player receives a strictly higher absolute payoff than any independent agent at any Nash equilibrium of $\Gamma_F(n-1)$: ∥

II. Diamond-type Search Models. Here we consider a standard search model (cf. Diamond, 1982, and Milgrom and Roberts, 1990) that is characterized by a game $\Gamma \in G$ in which

$X = [0;\xi];\ 0 < \xi < \infty$; and $x_r \in X$ is interpreted as the search effort by player r. The absolute payoff function of player r on X^n is given as

$$\pi_r(x) = x_r f\left(\sum_{q \neq r} x_q\right) - C(x_r) + K; \quad r = 1,\ldots,n$$

where K is a constant, and f and C are twice differentiable functions with

$$f' > 0;\ C' > 0;\ C'' > 0;\ C'(\xi) > f((n-1)\xi);\ f(0) > C'(0) \text{ and } K > C(\xi).[11]$$

As usual, we interpret C as standing for the private cost of search effort.

It is easy to verify that while Γ is supermodular, it is not action monotonic, and therefore, Theorem 1 does not apply to this game. Yet, from Theorem 2 we can deduce the following: if there is only one interdependent player in Γ, then she earns strictly higher absolute payoffs than any other independent player at any intragroup symmetric equilibrium. In fact, this applies to any equilibrium since we again have $N_{sym}(\Gamma_F(n-1)) = N(\Gamma_F(n-1))$ for any strictly increasing F. To see this, suppose for contradiction that $\hat{x}_{i_1} > \hat{x}_{i_2}$ for any $i_1, i_2 \in I_{n-1}$ where $\hat{x} \in N(\Gamma_F(n-1))$. But $C'(\xi) > f((n-1)\xi)$ and $f(0) > C'(0)$ together imply that $(\hat{x}_{i_1}, \hat{x}_{i_2}) \in (0,\xi)^2$ so that we must have

$$f\left(\sum_{r \neq i_1} \hat{x}_r\right) - C'(\hat{x}_{i_1}) = 0 \quad \text{and} \quad f\left(\sum_{r \neq i_2} \hat{x}_r\right) - C'(\hat{x}_{i_2}) = 0.$$

But these equations cannot hold simultaneously, for, given that $f' > 0$ and $C'' > 0$, the hypothesis $\hat{x}_{i_1} > \hat{x}_{i_2}$ implies that $f(\sum_{r \neq i_1} \hat{x}_r) < f(\sum_{r \neq i_2} \hat{x}_r)$ and $C'(\hat{x}_{i_1}) > C'(\hat{x}_{i_2})$. Furthermore, by a reasoning similar to the one used in the previous example we can show that the interdependent agent's equilibrium action is different from that of any of the independent agents. We thus obtain the following result: when there is only one interdependent player in the population, at any equilibrium of the Diamond-type search model considered above, the interdependent player obtains strictly higher payoffs than everybody else. ∥

III. Arms Races.[12] Two countries are engaged in an arms race. The associated game Γ is assumed symmetric with $X = [0;\zeta],\ 0 < \zeta < \infty$, and

$$\pi_r(x) = B(x_r - x_{-r}) - C(x_r) \quad \forall x \in X^2 \text{ and } r = 1,2.$$

An action for a country is a level of military expenditure. We assume that $B : [-\zeta,\zeta] \to \mathbb{R}$ and $C : X \to \mathbb{R}_+$ are twice continuously differentiable and satisfy:

$$B' > 0;\ B'' < 0;\ C' > 0;\ C'' > 0 \text{ and } C'(\zeta) > B'(0) > C'(0).$$

[11] The constant $K > C(\xi)$ is introduced into the payoff function just to ensure that $\pi_r(x) > 0$ for all $x \in [0,\xi]^n$ and $r = 1,\ldots,n$. Obviously, this does not alter the strategic structure of Γ in any way.

[12] The present formulation of this example is again taken from Milgrom and Roberts (1990).

In addition, assume that for any pair of actions $(x_1; x_2)$ in X^2, $B(x_1 - x_2) - C(x_1) > 0$ (this is simply to ensure that payoffs are positive at all admissible action profiles). Now suppose that country 2 decides to act aggressively and thus aims at maximizing an objective function of the type (3) for some differentiable F with $F_1; F_2 > 0$. Country 1 simply maximizes its own material payoff π_1.

It is easy to see that strict concavity of B guarantees that Γ is strictly supermodular. Therefore, while Γ need not be action-monotonic (so that Theorem 1 need not apply), by using Theorem 2 we may conclude that $\pi_2(\hat{x}) > \pi_1(\hat{x})$ for all $\hat{x} \in N(\Gamma_F(1))$ with $\hat{x}_1 \neq \hat{x}_2$: However, since $C'(\zeta) > B'(0) > C'(0)$; we have $\hat{x}_1 \in (0; \zeta)$ so that, by a similar reasoning used in the analysis of input games above, we can show that no $\hat{x} \in N(\Gamma_F(1))$ can satisfy $\hat{x}_1 = \hat{x}_2$. Hence $\pi_2(\hat{x}) > \pi_1(\hat{x})$ holds for all $\hat{x} \in N(\Gamma_F(1))$. In other words, the country with the interdependent preferences has a strictly higher payoff at any equilibrium. Since the present game is with negative spillovers, Lemma 1 allows us to say more: the country with the interdependent objective function will be the one that is more heavily armed at any equilibrium. This is certainly in line with the intuition that country 2 is a more aggressive player than country 1. $\|$

4 Economic Applications

4.1 Theory of Preference Formation

An important question in the theory of individual preferences that has recently received some attention is whether individuals are indeed absolute payoff maximizers as is usually assumed in conventional economic models (see Frank, 1987, Bolton, 1991, Cole et al., 1992, Bisin and Verdier, 1996, and Koçkesen et al., 1997, among others.) The alternative hypothesis is that an individual's well-being is determined not only by the intrinsic utility of her material consumption, but also her relative material standing in the society; the so-called relative income hypothesis (or keeping up with the Joneses effect). There is now substantial evidence suggesting that the "relative standing" concern of individuals is indeed a fact of life (see Frank, 1987, Clark and Oswald, 1996, and references cited therein).

One way in which the nature of individual preferences can be explained is by an appeal to evolutionary arguments. The evolutionary theory of preference formation is based on the premise that individual preferences come to being as a result of an unplanned process of transmission in which children inherit the preferences of their parents or peers either by genetic transmission, imitation or socialization. The population composition is typically assumed to evolve according to an (absolute) payoff monotonic evolutionary selection dynamic: those behaviors which yield the highest material rewards are replicated with greatest frequency from one generation to the next. If one type of behavior persistently outperforms all others, it will be the sole surviving behavior in the long run.[13] Indeed, Friedman (1953) justified the independence of preferences by claiming that a pattern of behavior that does not maximize one's own material payoffs will eventually be driven to extinction. Based on the results of the previous section, the validity of this evolutionary argument is clearly questionable in a great many strategic environments. In this section, by a straightforward application of our previous results, we shall demonstrate formally that evolution may well favor the emergence of at least some individuals with interdependent preferences as opposed to a homogeneous population of absolute payoff maximizers.

Consider a discrete time overlapping generations scenario in which each person lives for two periods and asexually produces a finite number of children in the second period of her life. In period τ; there are n_τ many individuals in the society who are in the first period of their lives. These individuals interact with each other through playing an n_τ-person game $\Gamma^\tau \in \mathcal{G}^\tau$, where $\mathcal{G}^\tau$ is the set of all n_τ-person symmetric games that have ℓ-dimensional action spaces and that satisfy (2).[14] Suppose that at some period τ_0, all individuals in the population have independent preferences, that is, they seek to maximize their absolute payoffs. Suppose further that in the subsequent period $\tau_0 + 1$, one of the young individuals turns out (for instance by random mutation) to have negatively interdependent preferences. The question that we wish to address is the following: if the population composition evolves under pressure of differential material payoffs, as is normally assumed, will it tend to return to it's original monomorphic state or diverge further from it? In other words, is a monomorphic population of absolute payoff maximizers locally stable under payoff monotonic evolutionary selection dynamics if the emergence of individuals with negatively interdependent preferences is permitted? The answer, of course, depends on the particular structure of $\{\Gamma^\tau\}_{\tau=1}^{\infty}$ and the postulated selection dynamics. The results of the previous section, however, allows us to give a negative answer to this question under considerably general circumstances.

The idea that a monomorphic population of absolute payoff maximizers is locally stable under all payoff monotonic selection dynamics is captured by the following stability concept.

Definition. Let $\Gamma^\tau \in \mathcal{G}^\tau$; $\tau \in \mathbb{N}$; and take any strictly increasing $F : \mathbb{R}^2_+ \to \mathbb{R}$. We say that independent preferences are **evolutionarily F-stable** if, for all τ, there exists some

[13] There are several applications of this approach in the economics literature, including the evolution of risk aversion, altruism among kin, and systematic expectational biases; see Koçkesen et al. (1997) for references.

[14] The evolutionary scenario considered here is one in which each individual interacts with each other member of the population in each period (the "playing the field" model). An alternative would be to consider interaction that occurs in randomly matched subgroups drawn from the population in each period. The case of pairwise random matching is explored in Koçkesen et al. (1997).

$x \in N(\Gamma_F^\tau(n_\tau - 1))$ such that

$$\pi_{n_\tau}(x) < \frac{1}{n_\tau - 1}\sum_{i=1}^{n_\tau - 1}\pi_i(x).$$

They are said to be **evolutionarily unstable** if they are not evolutionarily F-stable for any strictly increasing F.

This particular notion of evolutionary stability is a straightforward reflection of the corresponding finite population stability concepts advanced by Schaffer (1989) and Vega-Redondo (1996, p. 32) in our framework. It says that independent preferences are evolutionarily unstable if an originally monomorphic population composed only of independent agents does not stand a chance of expelling any mutant negatively interdependent behavior.[15] Needless to say, given a particular game sequence $\{\Gamma^\tau\}_{\tau=1}^\infty$; if independent preferences were evolutionarily unstable in the sense defined above, then they would be unstable under any deterministic (absolute) payoff monotonic selection dynamics (such as the replicator dynamics) which require that the share of independent agents in the population grows if and only if they obtain, on average, higher absolute payoffs than interdependent agents in the stage game. Our notion of evolutionary instability is therefore quite a general one subsuming most of the instability concepts used in evolutionary game theory. Moreover, it does not require us to address the problem of equilibrium selection: if independent preferences are evolutionarily unstable, then, regardless of which equilibrium is selected in each period, there will be no pressure on the population composition to return to its initial monomorphic state once a player with negatively interdependent preferences of any kind has emerged.

Before stating the main result of this section, we introduce the following refinement of G^τ:

$$G_*^\tau \equiv \{\Gamma^\tau \in G^\tau : x_1 = \cdots = x_{n_\tau - 1} \text{ for all } x \in N(\Gamma_F^\tau(n_\tau - 1)) \text{ and all strictly increasing } F\},$$

for all $\tau \in \mathbb{N}$. The class G_*^τ is a refinement of G^τ which requires that all independent agents take the same action at any equilibrium of $\Gamma_F^\tau(n_\tau - 1)$. This requirement is completely unrestrictive in most of the economic applications discussed in this paper. For instance, in all the n_τ-person games considered in Section 3.3 (along with the Cournot oligopoly discussed next), all independent agents take the same equilibrium actions no matter how many interdependent agents are in the population. All these games are thus members of G_*^τ.

We are now ready to state the following

Proposition 1. Let $\Gamma^\tau \in G_*^\tau$ for all $\tau \in \mathbb{N}$; and assume that the action space of Γ^τ is a lattice. Independent preferences are evolutionarily unstable if, for all τ,

[15] Since the population is finite in our framework, we follow Schaffer, 1989, in formalizing the notion of "a small deviation from independent behavior" by the mutation of a single agent.

(i) Γ^τ is strictly supermodular; or

(ii) Γ^τ belongs to G_0^τ, is strictly submodular and symmetric in equilibrium, and has negative or positive spillovers.[16]

Proof. Since $\Gamma^\tau \in G_*^\tau$, we have $N_{sym}(\Gamma_F^\tau(n-1)) = N(\Gamma_F^\tau(n-1))$ for all $\tau, n \in \mathbb{N}$ and all strictly increasing F. The claims are thus established upon applying Theorems 2 and 4, respectively. Q.E.D.

Proposition 1 shows that in a great variety of economic circumstances (which include all of the games presented in Section 3.3), there are evolutionary reasons to believe that the population will not be composed only of absolute payoff maximizers in the long run; one should expect the presence of at least some individuals with negatively interdependent preferences.[17] Moreover, even when preferences are acquired as a result of the deliberate socialization efforts of parents who seek to inculcate preferences in their children with a view to providing them with greater material payoffs in their adult lives, the resulting population dynamics will be payoff monotonic and the above result applies. At the very least, this calls into question the almost universal practice of modeling economic agents as absolute payoff maximizers.

4.2 The Cournot Oligopoly

Objective functions which incorporate relative payoff concerns are particularly easy to justify in the case of firms which separate management from ownership. In the presence of some common uncertainty which affects all firms within an industry, the performance of other firms may provide valuable information about a manager's ability or effort which is otherwise unobservable to the owners. Owners may therefore benefit from writhing contracts with managers in which the compensation of the latter is based, in part, on the performance of their firm relative to that of other firms, or relative to some industry average.[18] This, in turn, would provide an incentive for managers (even if they are themselves absolute payoff maximizers) to pursue the maximization of objective functions which have the form

[16] By Corollary 1, the same conclusion also holds for all strictly supermodular and action monotonic $\Gamma^\tau \in G^\tau \setminus G_*^\tau$ where the related action spaces are arbitrary chains in $\mathbb{R}^\ell$.

[17] In fact, in the context of certain specific games, more can be said. For instance, if the individuals are playing standard commons or public goods games (not necessarily with additively separable payoff functions), then one can show that the unique stable population composition with respect to any absolute payoff monotonic dynamics is monomorphic with all individuals being interdependent. For a proof and various extensions of this result, see Koçkesen et al. (1997).

[18] See, among others, Holmström (1979, 1982) and Nalebuff and Stiglitz (1983). Gibbons and Murphy (1990) provide supportive empirical evidence.

of interdependent preferences.[19] The results of Section 3 can thus be used to show that such contracts may have the unplanned effect of yielding a strategic advantage to a firm, enabling it to achieve a higher level of profitability than its profit-maximizing competitors.

Consider an oligopolistic industry composed of n firms with identical cost structures producing a homogenous product. Firm r chooses an output level $x_r \in X = [0, \bar{Q}]$, $0 < \bar{Q} < \infty$, where $\bar{Q}$ is interpreted as the capacity limit on a firm's output level. The profit function of firm r is given by

$$\pi_r(x) = x_r P\left(\sum_{q=1} x_q\right) - C(x_r), \quad r = 1, \ldots, n,$$

where the inverse demand function P is a strictly positive and twice differentiable function on $[0, n\bar{Q}]$ and the cost function C is a nonnegative, twice differentiable function on $[0, \bar{Q}]$. We make the standard assumptions that demand is downward sloping and average cost is non-decreasing:

$$P' < 0, \; C(0) = 0, \; C' > 0, \; C'' \geq 0.$$

We also assume that the game is strictly submodular:

$$P'\left(\sum_{q=1} x_q\right) + x_r P''\left(\sum_{q=1} x_q\right) < 0 \quad \forall x \in [0, \bar{Q}]^n. \tag{5}$$

Note that concavity of P would imply (5). Denote the resulting Cournot game by $\Gamma^C \in \mathcal{G}$, and make the additional assumptions that $P(n\bar{Q}) > C(\bar{Q})/\bar{Q}$, to ensure positive profits for each firm at any output profile, and $P(0) > C'(0)$, to ensure that each firm produces a strictly positive amount at any Cournot equilibrium. The latter condition also guarantees that Γ^C has the negative spillovers property. Moreover, by means of an argument similar to that used in the case of input games above, one can show that this game is symmetric in equilibrium. Since it is easy to verify the existence of an equilibrium, we may conclude by Lemma 2 that Γ^C has in fact a unique equilibrium.

Our first result shows that, in the case of a duopoly, one can obtain a particularly strong result regarding the relative performance of an interdependent firm in competition with an independent firm.

Proposition 2. Take any Cournot duopoly Γ^C in which the firms produce below full capacity in equilibrium and let $F : \mathbb{R}^2_+ \to \mathbb{R}$ be strictly increasing. Then, at any $\hat{x} \in$

[19] Another reason why interdependent preferences could be of interest for the theory of industrial organization is the close connection between the relative profits and the market share of a firm. For instance, provided that average costs are constant and relative profits are well-defined, we have $\pi_r(x)/\bar{\pi}(x) = x_r/\bar{x}$ for all x. Therefore, negatively interdependent preferences in the context of Cournot competition with constant average costs encompasses the case of sales or market share maximization on the part of managers.

$N(\Gamma_F^C(1))$; we have $\pi_1(x^*) < \pi_2(x^*)$; i.e., the firm with interdependent preferences obtains a strictly higher profit than does the independent firm.

Proof. First we show that at any $x^* \in N(\Gamma_F^C(1))$, $x_1^* \neq x_2^*$. Suppose, by way of contradiction, that $x_1^* = x_2^* = a$. If $a = \bar{Q}$; then by symmetry of Γ^C; we must have $N(\Gamma^C) = \{(\bar{Q}; \bar{Q})\}$ which is outlawed by hypothesis. If, on the other hand, $a = 0$; then $N(\Gamma^C) = \{(0; 0)\}$ which contradicts $P(0) > C'(0)$: Hence, $a \in (0; \bar{Q})$. But then we must have $\partial\pi_1(x^*)/\partial x_1 = 0$ and $\partial p_2(x^*)/\partial x_2 = 0$; while these two equations cannot hold simultaneously as can be verified by a reasoning similar to the one used in the input games example of Section 3.3. Hence $x_1^* \neq x_2^*$ and, since Γ^C is strictly submodular with negative spillovers, and is symmetric in equilibrium, the result follows from Theorem 4. Q.E.D.

A similar result can be established for the Cournot oligopoly model with n firms which operate under constant marginal costs. In this case it is easily verified that Γ^C is positively action monotonic and, moreover, that all equilibria of $\Gamma_F^C(k)$ are such that all independent players take the same action. As long as at least one firm produces under its capacity in any Cournot equilibrium of Γ^C, it can be shown that at any $x^* \in N_{sym}(\Gamma_F^C(k))$, $x_i^* \neq x_n^*$ for all $i \in I_k$, so that the application of Theorem 3 immediately yields $\pi_i(x^*) < \pi_n(x^*)$ for all $i \in I_k$. Furthermore, in the special case when $k = n - 1$, all Nash equilibria of $\Gamma_F^C(k)$ are intragroup symmetric. Hence, if a single firm in an industry has an objective function that places some weight on relative profits, it will be more profitable in any equilibrium than all of its profit maximizing competitors.

For even a more general class of Cournot oligopoly models in which we relax the assumption of linear cost functions, obtaining similar results requires one additional condition to ensure that the game is action monotonic.

Proposition 3. Take any Cournot oligopoly Γ^C in which the firms produce below full capacity in equilibrium[20] and let $F : \mathbb{R}_+^2 \to \mathbb{R}$ be strictly increasing. Then, if $P(n\bar{Q}) > C'(\bar{Q})$, $\pi_i(x^*) < \pi_j(x^*)$ holds for all $(i; j) \in I_k \times J_k$ and all $x^* \in N_{sym}(\Gamma_F^C(k))$.

Proof. The condition $P(n\bar{Q}) > C'(\bar{Q})$ guarantees that Γ^C is positively action-monotonic. The same reasoning used in the Proof of Proposition 2 can be used to demonstrate that $x_1^* \neq x_n^*$ at all $x^* \in N_{sym}(\Gamma_F^C(k))$. Since Γ^C action monotonic, strictly submodular with negative spillovers, and is symmetric in equilibrium, the result follows from Theorem 3. Q.E.D.

To summarize, in many Cournot oligopoly models, managers who include relative profit considerations in their decision making process (say, due to incentive contracts) will obtain

[20] A sufficient condition for this is $P(n\bar{Q}) + \bar{Q}P'(n\bar{Q}) - C'(\bar{Q}) < 0$.

higher pro…ts in equilibrium than the industry average.[21] In view of Lemma 1, which in the present context links the pro…ts of a …rm to its production, the results of this section provide an explanation for the anecdotal observation that some industries (such as the personal computer industry) are characterized by very unequal markets shares and a high degree of concentration despite the fact that the constituent …rms operate under remarkably similar cost structures. This explanation is somewhat novel, since is does not rely on the di¤erentiation of products and advertising activities of …rms which are usually claimed to account for this phenomenon.

Finally, the …ndings of this section have immediate implications for the long run survival of …rms within an industry if the entry and exit of …rms occurs on the basis of pro…tability. The idea of Darwinian selection in industrial dynamics is not a new one; several authors have argued that competition in an industry resembles biological competition in that more pro…table practices are replicated more rapidly (see, for instance, Alchian, 1950, Friedman, 1953, and Sha¤er, 1989). Along these lines, our results imply that the …rm behavior that corresponds to an interdependent objective function will thrive at the expense of absolute pro…t maximizing behavior in the long run.

5 Concluding Remarks

In this paper we have tried to uncover the generality of the statement that negatively interdependent preferences provide one with a strategic advantage over agents who are motivated exclusively by a concern with their own material payo¤s. It turns out that there is a broad class of strategic environments in which such an advantage is found to exist, and that there is a close connection between this phenomenon and the properties of strategic complementarity and substitutability in games. The …nding that those with interdependent preferences earn greater absolute payo¤s than do (absolute) payo¤ maximizers in such environments has direct implications for theories of preference formation and managerial decision making. In light of our theoretical results, the assumption of absolute payo¤ maximizing behavior on the part of individuals or …rms should not be made as routinely as is done in applications of game theory (on this point, see also Frank, 1987, and Bolton, 1991).

There are a number of directions in which the present work may be extended. The results of Koçkesen, et al. (1997) prove that the su¢cient conditions provided here for interdependent players to outperform independent players are not necessary. Therefore, determining precisely the class of all normal-form games such that this phenomenon occurs remains an

[21] See also Vickers (1984), Fershtman and Judd (1987) and Sklivas (1987) for a similar conclusion in the context of closely related models where the separation of the owner and manager incentives are explicitly modeled.

open problem. It also remains to be seen whether results analogous to ours can be obtained in extensive form and dynamic games. Finally, we stress that our entire analysis has been conducted under the hypothesis of complete information. This is certainly a considerable limitation; entertaining the notion that interdependent preferences are a plausible alternative to the standard assumption of independent preferences arguably necessitates that the game at hand should be modeled as an incomplete information game.[22] The incomplete information issue is an important one which we hope to address in future research.

[22] As noted by Bolton (1997, p. 1112) in the context of the ultimatum bargaining environment, "... the marginal rate of substitution between absolute and relative money most likely varies by individual, making utility functions private information."

Appendix: Proofs

Proof of Theorem 1. Assume first that Γ is positively action-monotonic, and take any $\alpha = ([a]_k, [b]_{n-k}) \in N_{sym}(\Gamma_F(k))$. In what follows, we shall show that $a \succeq b$ must hold; this will yield the claim by virtue of positive action-monotonicity. Let us assume for contradiction then that $a \succ b$.

Since $\pi_1 = p_1$, by definition of Nash equilibrium, $\pi_1([a]_k, [b]_{n-k}) \geqslant \pi_1(b, [a]_{k-1}, [b]_{n-k})$ so that, by using (1) with $\sigma(1) = n$, $\sigma(n) = 1$ and $\sigma(r) = r$ for all $r \notin \{1, n\}$, we find

$$\pi_n(b, [a]_{k-1}, [b]_{n-k-1}, a) \geqslant \pi_n(b, [a]_{k-1}, [b]_{n-k}) = \pi_n((b, [a]_{k-1}, [b]_{n-k-1}, a) \wedge ([a]_k, [b]_{n-k})).$$

By strict supermodularity, therefore,

$$\begin{aligned} \pi_n(\alpha_{-n}, a) &= \pi_n([a]_k, [b]_{n-k-1}, a) = \pi_n((b, [a]_{k-1}, [b]_{n-k-1}, a) \vee ([a]_k, [b]_{n-k})) \\ &> \pi_n([a]_k, [b]_{n-k}) = \pi_n(\alpha) \end{aligned} \tag{6}$$

holds. Now suppose $\pi_n(\alpha) = 0$. From (2) and (6) it follows that $\pi_n(\alpha_{-n}, a) > 0$, and since F is strictly increasing,

$$p_n(\alpha_{-n}, a) = F\left(\pi_n(\alpha_{-n}, a), \frac{\pi_n(\alpha_{-n}, a)}{\bar{\pi}(\alpha_{-n}, a)}\right) > F(0, 0) = p_n(\alpha)$$

contradicting that α is a Nash equilibrium. Thus, we assume henceforth that $\pi_n(\alpha) > 0$. But then since $p_n([a]_k, [b]_{n-k}) \geqslant p_n([a]_k, [b]_{n-k-1}, t)$ for all $t \in X$, we have

$$\frac{\pi_n(\alpha)}{\sum \pi_r(\alpha)} > \frac{\pi_n(\alpha_{-n}, a)}{\sum \pi_r(\alpha_{-n}, a)} \tag{7}$$

by (6) and strict monotonicity of F.

By the symmetry of Γ, it is readily observed that

$$\pi_n(\alpha_{-n}, a) = \pi_1([a]_k, [b]_{n-k-1}, a) = \cdots = \pi_k([a]_k, [b]_{n-k-1}, a)$$

and $\pi_{k+1}([a]_k, [b]_{n-k-1}, a) = \cdots = \pi_{n-1}([a]_k, [b]_{n-k-1}, a)$. But by the hypotheses of $a \succ b$ and positive action-monotonicity, we must have $\pi_n(\alpha_{-n}, a) > \pi_{n-1}([a]_k, [b]_{n-k-1}, a)$ so that

$$\sum_{r=1} \pi_r(\alpha_{-n}, a) = (k+1)\pi_n(\alpha_{-n}, a) + (n-k-1)\pi_{n-1}(\alpha_{-n}, a) \leqslant n\pi_n(\alpha_{-n}, a).$$

(This inequality holds as an equality iff $k = n - 1$.) Therefore using the above inequality along with (7), we may conclude that

$$\frac{\pi_n(\alpha)}{\sum \pi_r(\alpha)} > \frac{1}{n}.$$

But by the symmetry of Γ, we have $\sum \pi_r(\alpha) = k\pi_1(\alpha) + (n-k)\pi_n(\alpha)$, and therefore the above inequality yields $\pi_n([a]_k, [b]_{n-k}) > \pi_1([a]_k, [b]_{n-k})$. However, since $a \succ b$, this contradicts positive action-monotonicity of Γ, and we are done.

To complete the proof of the theorem, assume now that Γ is negatively action-monotonic. Define $\Gamma^- = (\{-X\}_{i=1,...,n}, \{\pi_i^-\}_{i=1,...,n}) \in \mathcal{G}$ where $\pi^-(x) = \pi(-x)$ for all $x \in -X$. Since, for all $x \in -X$, $x_i \succ x_j$ implies $-x_i \prec -x_j$,[23] the negative action-monotonicity of Γ yields $\pi_i^-(x) = \pi_i(-x) > \pi_j(-x) = \pi_j^-(x)$ whenever $x_i \succ x_j$. Hence, Γ^- is positively action-monotonic. Therefore, by the first part of the theorem established above, we have

$$\pi_j^-(\alpha) > \pi_i^-(\alpha) \quad \forall (i, j, \alpha) \in I_k \times J_k \times N_{sym}(\Gamma_F^-(k)).$$

But since $N_{sym}(\Gamma_F^-(k)) = -N_{sym}(\Gamma_F(k))$, we then have $\pi_j(-\alpha) > \pi_i(-\alpha)$ for all $i \in I_k$, $j \in J_k$ and all $-\alpha \in N_{sym}(\Gamma_F(k))$. The proof is complete. Q.E.D.

Proof of Corollary 1. We assume that Γ is positively action-monotonic, for the case of negative action-monotonicity can be easily settled as in the proof of Theorem 1. If $\alpha_1 = \cdots = \alpha_{n-1}$, then the claim follows from Theorem 1. So assume that $\alpha_i \neq \alpha_{i'}$ for some $i, i' \in I_{n-1}$, and let α_1 be the maximum of $\{\alpha_1, ..., \alpha_{n-1}\}$ w.r.t. $\succsim$, relabelling if necessary. (Since $\succsim$ is a linear order, $\max\{\alpha_1, ..., \alpha_{n-1}\} \neq \emptyset$.) We now proceed as in the proof of Theorem 1 to eliminate the trivial case of $\pi_n(\alpha) = 0$, and to obtain the corresponding version of (7) in this case with $\pi_n(\alpha) > 0$:

$$\frac{\pi_n(\alpha)}{\sum \pi_r(\alpha)} > \frac{\pi_n(\alpha_{-n}, \alpha_1)}{\sum_{i=1}^{n-1} \pi_i(\alpha_{-n}, \alpha_1) + \pi_n(\alpha_{-n}, \alpha_1)}. \tag{8}$$

(Implicit in this inequality is the fact that $\pi_n(\alpha_{-n}, \alpha_1) > 0$ which is guaranteed by (6).) But since $\alpha_1 \succsim \alpha_i$ for all $i \in I_{n-1}$ and $\alpha_1 \succ \alpha_i$ for some $i \in I_{n-1}$, by positive action-monotonicity, we have

$$(n-1)\pi_n(\alpha_{-n}, \alpha_1) = (n-1)\pi_1(\alpha_{-n}, \alpha_1) > \sum_{i=1}^{n-1} \pi_i(\alpha_{-n}, \alpha_1)$$

so that (8) yields $\pi_n(\alpha)/\sum \pi_r(\alpha) > 1/n$, and the result follows. Q.E.D.

Proof of Remark 2 (b). Assume that Γ is positively action-monotonic, w.l.o.g.. Let $\alpha_1 = \max\{\alpha_i : i \in I_k\}$, and notice that claim is immediate by positive action-monotonicity if $\alpha_n \succsim \alpha_1$. So, let $\alpha_1 \succ \alpha_n$. Since we can eliminate the trivial case of $\pi_n(\alpha) = 0$ as in the proof of Theorem 1, let us assume that $\pi_n(\alpha) > 0$. We then have

$$\begin{aligned}\frac{\pi_n(\alpha)}{\sum \pi_r(\alpha)} &> \frac{\pi_n(\alpha_{-n}, \alpha_1)}{\sum_{i=1}^{k} \pi_i(\alpha_{-n}, \alpha_1) + (n-k-1)\pi_{n-1}(\alpha_{-n}, \alpha_1) + \pi_n(\alpha_{-n}, \alpha_1)} \\ &> \frac{\pi_n(\alpha_{-n}, \alpha_1)}{(k+1)\pi_n(\alpha_{-n}, \alpha_1) + (n-k-1)\pi_{n-1}(\alpha_{-n}, \alpha_1)} > \frac{1}{n}.\end{aligned}$$

[23] Since $(\mathbb{R}^\ell, \succsim)$ is an ordered vector space, we have $x_i \succ x_j \Rightarrow x_i - x_j \succ ([0]_\ell) \Rightarrow -x_j \succ -x_i$.

Here, the first inequality is derived in a way analogous to (8), the second inequality follows from action monotonicity and $\sigma_1 \succsim \sigma_i$ for all $i \in I_k$; and finally, the third inequality follows from action monotonicity and $\sigma_1 \succ \sigma_{k+1} = \cdots = \sigma_n$. Q.E.D.

Proof of Theorem 2. Let $\sigma = ([a]_{n-1}, b) \in N_{sym}(\Gamma_F(n-1))$. If $a \succ b$, we can easily show that $\pi_n(\sigma)$ and $\pi_n(\sigma_{-n}, a) = \pi_n([a]_n)$ are strictly positive as in the proof of Theorem 1, and then recalling that (7) was obtained above without using the action-monotonicity property, we may conclude that

$$\frac{\pi_n([a]_{n-1}, b)}{(n-1)\pi_1([a]_{n-1}, b) + \pi_n([a]_{n-1}, b)} > \frac{\pi_n([a]_n)}{(n-1)\pi_1([a]_n) + \pi_n([a]_n)} = \frac{1}{n} \tag{9}$$

which yields $\pi_n(\sigma) > \pi_1(\sigma)$. Assume now that $b \succ a$, and note that we can again show that $\pi_n(\sigma)$ and $\pi_n([a]_n)$ are strictly positive by using strict supermodularity as in the proof of Theorem 1. On the other hand, since $\pi_1(\sigma) \geqq \pi_1(b, [a]_{n-2}, b)$, we have

$$\pi_n(x) \geqq \pi_n(b, [a]_{n-2}, b) \tag{10}$$

where $x = (b, [a]_{n-1})$. But then $b \succ a$ implies that

$$\pi_n([a]_n) = \pi_n(x \wedge \sigma) > \pi_n(\sigma) \tag{11}$$

for otherwise, by strict supermodularity, we would find

$$\pi_n(b, [a]_{n-2}, b) = \pi_n(x \vee \sigma) > \pi_n(x)$$

which contradicts (10). Yet, (11) and the fact that $p_n(\sigma) \geqq p_n([a]_n)$ yield (9), completing the proof. Q.E.D.

Proof of Lemma 1. Assume that $\sigma = ([a]_k, [b]_{n-k})$ for some $a, b \in X$ with $a \succ b$. Then, by hypothesis, negative spillovers effect, and symmetry of Γ,

$$\pi_1([a]_k, [b]_{n-k}) \leqq \pi_n([a]_k, [b]_{n-k}) < \pi_n(b, [a]_{k-1}, [b]_{n-k}) = \pi_1(b, [a]_{k-1}, [b]_{n-k})$$

and this contradicts that playing a is a best response for player 1 against $([a]_{k-1}, [b]_{n-k})$. The first assertion follows by the completeness and antisymmetry of $\succsim$.

To prove the second assertion, let $k = n-1$ and notice that all we have to show is that $\pi_n(\sigma) > \pi_1(\sigma)$ whenever $b \succ a$. Assume then for contradiction that

$$b \succ a \quad \text{and} \quad \pi_n([a]_{n-1}, b) \leqq \pi_1([a]_{n-1}, b). \tag{12}$$

Since $\alpha \mapsto \alpha/(\tau + \alpha)$ is a strictly increasing mapping in $\alpha \geqq 0$ for any $\tau > 0$, (12) implies that

$$\frac{\pi_n(\sigma)}{(n-1)\pi_1(\sigma) + \pi_n(\sigma)} \leqq \frac{\pi_1(\sigma)}{(n-1)\pi_1(\sigma) + \pi_1(\sigma)} = \frac{1}{n}.$$

Since b is a best response of player n against $[a]_{n-1}$ in $\Gamma_F(n-1)$; we must have $\pi_n([a]_{n-1}; b) \geq \pi_n([a]_n)$: Therefore, by the negative spillovers effect and symmetry of Γ, we have

$$\pi_1([a]_{n-1}; b) < \pi_1([a]_n) = \pi_n([a]_n) \leq \pi_n([a]_{n-1}; b)$$

which contradicts (12). Q.E.D.

Proof of Lemma 2. If $([a]_n); ([b]_n) \in N(\Gamma)$; then

$$\begin{aligned} \pi_1((a; [b]_{n-1}) \wedge (b; [a]_{n-1})) + \pi_1((a; [b]_{n-1}) \vee (b; [a]_{n-1})) &= \pi_1([a]_n) + \pi_1([b]_n) \\ &\geq \pi_1(b; [a]_{n-1}) + \pi_1(a; [b]_{n-1}): \end{aligned}$$

Hence, unless $a = b$; Γ cannot be strictly submodular. The converse statement trivially follows from the symmetry of Γ: Q.E.D.

Proof of Theorem 3. This theorem is an immediate consequence of the following

Lemma A. Let $\Gamma \in G_0$; $k \in \{1; :::; n-1\}$ and take any strictly increasing $F : \mathbb{R}^2_+ \to \mathbb{R}$: If Γ is a strictly submodular game with negative spillovers, and is symmetric in equilibrium, then, for any $\mathbf{x} \in N_{sym}(\Gamma_F(k))$ with $\pi_r(\mathbf{x}) > 0$ for all r; we have

$$\mathbf{x}_j \succsim \mathbf{x}_i \quad \forall (i; j) \in I_k \times J_k:$$

Proof. Let $\mathbf{x} = ([a]_k; [b]_{n-k}) \in N_{sym}(\Gamma_F(k))$ for some $a; b \in X$: Clearly, since a is a best response of player 1 against $([a]_{k-1}; [b]_{n-k})$ in $\Gamma_F(k)$; we have

$$\pi_1([a]_k; [b]_{n-k}) \geq \pi_1(t; [a]_{k-1}; [b]_{n-k}) \quad \forall t \in X: \tag{13}$$

We claim that

$$\pi_n([a]_k; [b]_{n-k}) > \pi_n([a]_k; [b]_{n-k-1}; t) \quad \forall t \in \{t' \in X : t' \succ b\}: \tag{14}$$

To see this, let us assume for contradiction that

$$\pi_n(\mathbf{x}_{-n}; t) = \pi_n([a]_k; [b]_{n-k-1}; t) \geq \pi_n([a]_k; [b]_{n-k}) = \pi_n(\mathbf{x}) > 0 \tag{15}$$

holds for some $t \in X$ with $t \succ b$: Since $\alpha \mapsto \alpha/(\tau + \alpha)$ is a strictly increasing mapping in $\alpha \geq 0$ for any $\tau > 0$; we must then have

$$\frac{\pi_n(\mathbf{x})}{\sum_{r=1}^{n-1} \pi_r(\mathbf{x}) + \pi_n(\mathbf{x})} \leq \frac{\pi_n(\mathbf{x}_{-n}; t)}{\sum_{r=1}^{n-1} \pi_r(\mathbf{x}) + \pi_n(\mathbf{x}_{-n}; t)} \tag{16}$$

But since $t \succ b$; the negative spillovers effect yields $\pi_r(\mathbf{x}) > \pi_r(\mathbf{x}_{-n}; t)$ for all $r \neq n$ so that

$$\frac{\pi_n(\mathbf{x}_{-n}; t)}{\sum_{r=1}^{n-1} \pi_r(\mathbf{x}) + \pi_n(\mathbf{x}_{-n}; t)} < \frac{\pi_n(\mathbf{x}_{-n}; t)}{\sum_{r=1}^{n-1} \pi_r(\mathbf{x}_{-n}; t) + \pi_n(\mathbf{x}_{-n}; t)}:$$

But this inequality, (16) and (15) yield $p_n(\hat{x}_{-n}; t) > p_n(\hat{x})$ which contradicts that $\hat{x}$ is a Nash equilibrium of $\Gamma_F(k)$. Therefore, we must conclude that (14) holds.

Now, for any $\alpha, \beta \in X$, define the correspondences $K_\beta : X \rightrightarrows X$ and $L_\alpha : X \rightrightarrows X$ as

$$K_\beta(\alpha) \equiv \arg\max_{t \in X} \pi_1(t; [\alpha]_{k-1}; [\beta]_{n-k}) \quad \text{and} \quad L_\alpha(\beta) \equiv \arg\max_{t \in X} \pi_n([\alpha]_k; [\beta]_{n-k-1}; t).$$

We define next the double sequence $(a_m; b_m) \in X^2$ recursively as follows:

$$a_0 = a; \quad b_0 = b; \quad a_m \in K_{b_m}(a_m) \quad \text{and} \quad b_m \in L_{a_{m-1}}(b_m); \ m = 1, 2, \ldots$$

Claim 1. $(a_m; b_m)$ is well-defined.

Proof of Claim 1. Fix any $\alpha, \beta \in X$. Since X is a convex compact set, and π_1 and π_n are continuous, K_β and L_α must be nonempty (by Weierstrass' theorem) and must have closed graphs (by Berge's maximum theorem). Moreover, quasiconcavity of π_1 and π_n entail that K_β and L_α are convex-valued. Therefore, by Kakutani's fixed point theorem, there exist fixed points of K_β and L_α. Since α and β were arbitrary in this reasoning, we may conclude that (a_m) and (b_m) are well-defined sequences. ∥

Let $B^r : X^{n-1} \rightrightarrows X$ be the best response correspondence of player r. We note that, for any $\alpha, \beta \in X$,

$$B^i([\alpha]_{k-1}; [\beta]_{n-k}) = K_\beta(\alpha) \quad \forall i \in I_k \tag{17}$$

and

$$B^j([\alpha]_k; [\beta]_{n-k-1}) = L_\alpha(\beta) \quad \forall j \in J_k \tag{18}$$

hold by symmetry of Γ.

Claim 2. If $a \succ b$, then $a_0 \succ a_1 \succ a_2 \succ \cdots$ and $\cdots \succ b_2 \succ b_1 \succ b_0$.

Proof of Claim 2. Let $a \succ b$. We shall first establish that $b_1 \neq b_0$. If $b_1 = b$, then $b_1 \in L_a(b_1)$ implies by (18) that $b \in B^j([a]_k; [b]_{n-k-1})$ for all $j \in J_k$. But then since $a \in B^i([a]_{k-1}; [b]_{n-k})$ for all $i \in I_k$, it follows that $([a]_k; [b]_{n-k}) \in N(\Gamma)$, contradicting that Γ is symmetric in equilibrium. If, on the other hand, $b_1 \succ b$, then (14) yields that

$$\pi_n(\hat{x}) > \pi_n([a]_k; [b]_{n-k-1}; b_1).$$

But then by submodularity of π_n,

$$\pi_n([a]_k; [b_1]_{n-k}) < \pi_n([a]_k; [b_1]_{n-k-1}; b)$$

which, in turn, contradicts that $b_1 \in L_a(b_1)$. We thus conclude that $b \succ b_1$.

Next, we claim that $a_1 \succsim a_0$. But by (13) and the fact that $a_1 \in K_{b_1}(a_1)$, we have $\pi_1([a]_k; [b]_{n-k}) \geq \pi_1(a_1; [a]_{k-1}; [b]_{n-k})$ and $\pi_1([a_1]_k; [b_1]_{n-k}) \geq \pi_1(a; [a_1]_{k-1}; [b_1]_{n-k})$. Clearly, given that $b \succ b_1$; if $a \succ a_1$; the last two inequalities would contradict the strict submodularity of π_1: Therefore, $a_1 \succeq a$ must hold. In fact, $a_1 \neq a$; for otherwise, $a_1 \in K_{b_1}(a_1)$ and $b_1 \in L_a(b_1)$ would yield that $([a]_k; [b_1]_{n-k}) \in N(\Gamma)$; and this would contradict Γ's symmetry in equilibrium since then $a \succ b \succ b_1$ would have to hold. By linearity of $\succeq$; therefore, we have $a_1 \succ a_0$:

Finally, we claim that $b_1 \succ b_2$: (Since we used (14) in establishing that $b_0 \succ b_1$; this step is necessary to be able to complete the proof by induction.) This claim follows from the fact that $b_1 \in L_a(b_1)$ and $b_2 \in L_{a_1}(b_2)$ imply that $\pi_n([a]_k; [b_1]_{n-k}) \geq \pi_n([a]_k; [b_1]_{n-k-1}; b_2)$ and $\pi_n([a_1]_k; [b_2]_{n-k}) \geq \pi_n([a_1]_k; [b_2]_{n-k-1}; b_1)$; respectively. If $b_2 \succ b_1$ held, given that $a_1 \succ a$; these inequalities would contradict the strict submodularity of π_n: Moreover, if $b_1 = b_2$; then $b_1 \in L_{a_1}(b_1)$ holds, and since $a_1 \in K_{b_1}(a_1)$; we obtain $([a_1]_k; [b_1]_{n-k}) \in N(\Gamma)$ contradicting that Γ is symmetric in equilibrium (because $a_1 \succ a \succ b \succ b_1$). We conclude that $b_1 \succ b_2$:

Proof is completed by a straightforward induction argument. ∥

Since X is compact, there exist convergent subsequences (a_{v_m}) and (b_{v_m}) such that $(a_{v_m}; b_{v_m}) \to (a^*; b^*) \in X^2$ as $m \to \infty$. We now claim that $(a^*; b^*) \in N(\Gamma)$: To see this, notice that $a_{v_m} \in K_{b_{v_m}}(a_{v_m})$ implies that

$$a_{v_m} \in B^1([a_{v_m}]_{k-1}; [b_{v_m}]_{n-k}) \quad m = 1; 2; \ldots$$

But since X is compact and π_1 is continuous, B^1 must have a closed graph, and therefore,

$$a^* = \lim_{m\to\infty} a_{v_m} \in B^1\left(\lim_{m\to\infty}([a_{v_m}]_{k-1}; [b_{v_m}]_{n-k})\right) = B^1([a^*]_{k-1}; [b^*]_{n-k}):$$

Moreover, by symmetry of Γ; $a^* \in B^i([a^*]_{k-1}; [b^*]_{n-k})$ for all $i \in I_k$: Similarly, we can show that $b^* \in B^i([a^*]_k; [b^*]_{n-k-1})$ for all $i \in J_k$: We thus conclude that $(a^*; b^*) \in N(\Gamma)$ as is sought. Therefore, if $a \succ b$ held, by Claim 2 there would exist an $(a^*; b^*) \in N(\Gamma)$ with $a^* \succ b^*$; contradicting that Γ is symmetric in equilibrium. Q.E.D.

Remark A. (a) If $n = 2$; we may drop the hypotheses of convexity of X and quasiconcavity of π_rs from the statement of Lemma A, for then one does not need Kakutani's fixed point theorem in proving that $(a_m; b_m)$ is well-defined.

(b) Lemma A remains valid if the chain X is any compact convex subset of a locally convex topological vector space, and π_rs are continuous with respect to the subspace topology. The proof of this claim is essentially identical to that of Lemma A, the only major modification being the use of Tychonoff-Fan fixed point theorem (Berge, 1963, p.251) instead of Kakutani's theorem in proving Claim 1.

(c) Continuity of π_r can be weakened in Lemma A to upper semicontinuity of π_r and lower semicontinuity of $V_r(y) \equiv \max_{a\in X} \pi_r(x_{-r}; a)$ for all $x_{-r} \in X^{n-1}$: (Given that π_r is upper semicontinuous and X is compact, V_r is well-defined on X^{n-1}:) For, we have used continuity of π_r above only in proving that K_β (and L_α) is nonempty, and that B^r has a closed graph. But upper semicontinuity of π_r readily guarantees that K_β is nonempty. To see that B^r has a closed graph under the above hypotheses as well, take any sequence x^m in X^n such that $x^m \to x^*$ as $m \to \infty$; and let $x_r^m \in B^r(x_{-r}^m)$ for all m. If $x_r^* \notin B^r(x_{-r}^*)$, then it must be the case that $V_r(x_{-r}^*) > \pi_r(x_{-r}^*, x_r^*)$. But then using the upper semicontinuity of π_r, the fact that $x_r^m \in B^r(x_{-r}^m)$ for all m, and the lower semicontinuity of V_r, we find

$$\begin{aligned}\pi_r(x_{-r}^*, x_r^*) &\geq \limsup_{m\to\infty} \pi_r(x_{-r}^m, x_r^m) = \limsup_{m\to\infty} V_r(x_{-r}^m) \\ &\geq \liminf_{m\to\infty} \pi_r(x_{-r}^m, x_r^m) \geq V_r(x_{-r}^*) > \pi_r(x_{-r}^*, x_r^*),\end{aligned}$$

contradiction.[24]

Proof of Theorem 4. Immediate from Lemma 1 and Lemma A. Q.E.D.

[24] Dasgupta and Maskin (1986) use an analogous reasoning in proving their Theorem 2.

References

[1] Alchian, A. (1950): "Uncertainty, Evolution and Economic Theory," Journal of Political Economy, 58, 211-221.

[2] Berge, C. (1963): Topological Spaces, Macmillan, New York.

[3] Bergstrom, T. C. (1995): "On the Evolution of Altruistic Ethical Rules for Siblings," American Economic Review, 85, 58-81.

[4] Birkhoff, G. (1967): Lattice Theory, 3rd ed., Providence: AMS.

[5] Bisin, A., and T. Verdier (1996): "On the Cultural Transmission of Preferences for Social Status," mimeo, MIT.

[6] Bolton, G. (1991): "A Comparative Model of Bargaining: Theory and Evidence," American Economic Review, 81, 1096-1136.

[7] Bulow, J. I., J. D. Geanakoplos and P. D. Klemperer (1985): "Multimarket Oligopoly: Strategic Substitutes and Complements," Journal of Political Economy, 93, 488-511.

[8] Clark A. and A. J. Oswald (1996): "Satisfaction and Comparison of Income," Journal of Public Economics, 61, 359-81.

[9] Cole, H, G. J. Mailath and A. Postlewaite (1992): "Social Norms, Savings Behavior, and Growth," Journal of Political Economy, 100, 1092-125.

[10] Cooper, R. and A. John (1988): "Coordinating Coordination Failures in Keynesian Models," Quarterly Journal of Economics, 103, 441-463.

[11] Dasgupta, P. and E. Maskin (1986): "The Existence of Equilibrium in Discontinuous Economic Games I: Theory," Review of Economic Studies, 53, 1-26.

[12] Diamond, P. (1982): "Aggregate Demand Management in Search Equilibrium," Journal of Political Economy, 90, 881-894.

[13] Fershtman, C. and K. L. Judd (1987): "Incentive Equilibrium in Oligopoly," American Economic Review, 77 , 927-40.

[14] Frank, R. H. (1987): Choosing the Right Pond, New York: Oxford University Press.

[15] Friedman, M. (1953): "The Methodology of Positive Economics," in Essays in Positive Economics. Chicago: University of Chicago Press.

[16] Frink, O. (1942): "Topology in Lattices," Transactions of American Mathematical Society, 51, 569-582.

[17] Gibbons, R. and K. J. Murphy (1990): "Relative Performance Evaluation for Chief Executive O¢cers," Industrial and Labor Relations Review, 43, 30S-51S.

[18] Holmström, B. (1979): "Moral Hazard and Observability," Bell Journal of Economics, 10, 74-91.

[19] Holmström, B. (1982): "Moral Hazard in Teams," Bell Journal of Economics, 13, 324-40.

[20] Koçkesen, L., E. A. Ok and R. Sethi (1997). "Interdependent Preference Formation," C.V. Starr Center working paper 97-18, New York University.

[21] Milgrom, P. and J. Roberts (1990): "Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities," Econometrica, 58, 1255-1277.

[22] Milgrom, P. and C. Shannon (1994): "Monotone Comparative Statics," Econometrica, 62, 157-180.

[23] Nalebuff, B. and J. Stiglitz (1983): "Prizes and Incentives: Towards a General Theory of Compensation and Competition," Bell Journal of Economics, August, 21-43.

[24] Nishimura, K. and J. Friedman (1981): "Existence of Nash Equilibrium in n-Person Games Without Quasi-Concavity," International Economic Review, 22, 637-648.

[25] Ok, E. A. and L. Koçkesen (1997): "Negatively Interdependent Preferences," C.V. Starr Center working paper 97-02, New York University.

[26] Robson, A. J. (1996): "The Evolution of Attitudes to risk: Lottery Tickets, and Relative Wealth," Games and Economic Behavior, 14, 190-207.

[27] Rogers, A. R. (1994): "Evolution of Time Preference by Natural Selection," American Economic Review, 84, 460-82.

[28] Shaffer, M. (1989): "Are Pro…t-Maximizers the Best Survivors? A Darwinian Model of Economic Natural Selection," Journal of Economic Behavior and Organization, 12, 29-45.

[29] Sklivas, S. (1987): "The Strategic Choice of Managerial Incentives," Rand Journal of Economics, 18, 452-458.

[30] Topkis, D.M. (1978): "Minimizing a Submodular Function on a Lattice," Operations Research, 26, 305-321.

[31] Topkis, D.M. (1979): "Equilibrium Points in Nonzero-Sum n-Person Submodular Games," SIAM Journal of Control and Optimization, 26, 305-321.

[32] Vives, X. (1990): "Nash Equilibrium with Strategic Complementarities," Journal of Mathematical Economics, 19, 305-321.

[33] Vega-Redondo, F. (1996): Evolution, Games, and Economic Behavior, Oxford: Oxford University Press.

[34] Vickers, J. (1984): "Delegation and the Theory of the Firm," Economic Journal, Supplement, 95, 138-147.