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A Tiebout Theory of Public vs Private Provision of Collective Goods

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ABSTRACT

We study whether "coercive" public provision or voluntary private provision of public goods can survive when individuals who "vote with their feet" can choose between communities that differ in the way that public goods are provided. We obtain the following findings: (*i*) an equilibrium aways exists in which all individuals migrate to the community which uses voluntary provision; (*ii*) under very robust conditions on preferences and income distribution, an equilibrium exists in which all individuals migrate to the community which uses coercive provision; (*iii*) "interior" equilibria in which collections of individuals move to both communities exist when income distribution is sufficiently polarized. Such equilibria are shown to be stratified — richer individuals migrate to the community with voluntary provision while poorer individuals reside in the public provision community. In the case where there are two types of wealth endowments, existence of stratified equilibria seems to require a negative tradeoff between the wealth ratio of the rich to the poor and the numerical ratio of rich to poor in society.

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1 Introduction

Institutions which are used to allocate public resources differ widely. In some economies the lion's share of the resources are allocated through decentralized institutions such as markets or voluntary provision whereas in other economies the centralized public sector assumes a large role. Often it does not seem to be the "public" nature of the good which determines whether or not the good is provided through centralized or decentralized mechanism. Fire protection, for example, is centrally funded through local taxes in some communities, and is provided by voluntary means in others. Funding in some churches take place through donations, and in others, mandatory membership fees are collected.

As a step toward understanding these institutions consider the following scenario. An individual must choose to live in one of several communities. In each community there is a local public good or service such as public education or fire protection. In one community provision of the good is centrally determined via a majority vote. In another, provision is completely decentralized so that the public good is funded by voluntary contributions from the community. Where should this individual choose to live? If there are many individuals facing these choices, which type of community, the one with centralized provison or the one with decentralized provision, is likely to stand up to migratory pressure? Finally, how does the income distribution in society affect the answer?

This paper addresses these questions in a model of Tiebout-like migration between communities that utilize distinct allocation procedures for public goods. Agents make location decisions over communities. The "communities" may be non-spatial entities such as social clubs or religious organizations. We also abstract from explicit market pricing mechanisms. This allows us to focus purely on the different collective decision rules as a sorting device between rich and poor.[1]

To simplify the analysis, there are two locations each with a distinct mechanisms for allocating public goods. The first locations uses a completely decentralized procedure in which contributions toward a local, collective good are given voluntarily by the citizenry or membership of the community. The second uses a centralized scheme by which individuals are taxed proportionately to wealth with the tax rate determined by the median voter. Each mechanism is said to be *viable* if in equilibrium at least one individual locates in the

[1] In two historical examples which we subsequently examine, there was no mention of any explicit pricing, e.g. land prices, entry costs, etc, to account for those particular movements between communities. See footnote 2.

community which utilizes that provision mechanism. While the locations here are exogenous, the viability of the institutions which support the communities are not.

Individuals differ in wealth holdings. We are interested in the effect of wealth dispersion on the spatial patterns across communities. This, in turn, determines the viability of the provision institutions. In some equilibria, only one or the other institution is viable. We also show that "interior" equilibria exist for some parameters. These equilibria have the property that wealth is stratified across communities. When this occurs we find that wealthier individuals tend to prefer more voluntary over compulsory contribution. Poorer individuals tend to prefer the voting mechanism. The reason is that the benefit from overcoming free riding with compulsory contributions must be weighed against the loss from having preferences which diverge from the median voter. This latter cost takes the form of nonoptimal "smoothing" across consumption goods. For wealthier individuals, this cost tends to be greater since the loss of control over one's resources is greater the larger is one's endowment of those resources.

We find that equilibria in which both the centralized and decentralized communities are simultaneously viable seem to require a negative tradeoff between the wealth ratio of the rich to the poor and the numerical ratio of rich to poor in society. That is, if both communites are to be viable, then the higher is the wealth of the rich relative to the poor, the smaller can be the relative numbers of rich to poor in society. For diffuse distributions, or for unimodal distributions, only the two extremal equilibria in which only one or the other community is viable exist. In this case, despite the preference of one group to separate itself from the other, the concentration of wealth in a single community may be large enough to prevent unilateral departures. In the centralized community an individual who's preferences diverge from the pivotal voter is compensated by the aggregation of wealth which provides massive quantity of public goods. In the decentralized community, the aggregate wealth is also large enough to overcome departures.

The influential paper of Tiebout (1956) is among the first to model how individuals "vote with their feet" to determine, indirectly, the types of local public tax and expenditure policies. A small sample of this literature includes Greenberg (1983), Wooders (1989), and Scotchmer and Wooders (1987). The effect of Tiebout-like migration on relative policy outcomes between communities has been examined by Epple and Romer (1992), Fernandez and Rogerson (1994), and Bearse (1994). In Epple and Romer, individuals differ by income and redistribution takes the form of lump sum transfers. The level in each community is determined by a majority vote. Fernandez and Rogerson examine the effect of locational

choice on funding for public education. Here, redistribution is in kind. Finally, Bearse studies how location in criminal activity depends on relative law enforcement activity in the city vs the suburbs. In each of these papers, the equilibrium spatial distribution across communities is characterized.

The difference between the present paper and those cited above is that here the preferences of the citizenry are explicitly aggregated using some kind of institutional procedure which differs across jurisdictions. Each procedure has different equilibrium implications. A number of historical episodes suggest that these institutional differences mattered when individuals made migration decisions. Examples include the English migration to Dutch schools in the 17th century (see Eby (1952)) and the Babtist movement in Colonial Virginia (see Isaac (1982)). In each of these, locational selection toward public provision drastically shrank participation in private provision.[2]

Other papers in which Tiebout-style locational choice has been applied to the issue of selection of mechanisms include Caplin and Nalebuff (1992, 1993) who examine existence issues in a rich class of static models in which communities may be differentiated by the social welfare function used to determine policy outcomes, and Gensemer, Hong, and Kelly (1996), who examine the implementation of a social choice rule that can withstand migration toward other rules. Dynamic issues in Tiebout-like selection of aggregation procedures include evolutive approaches of Lagunoff (1993) and Kollman, Miller, and Page (1995), and the public goods approach of Glomm and Lagunoff (1996). The last is the companion to the present paper.

In Section 2 we first introduce the model. the funding outcome within each region is characterized. Section 3 characterizes the spatial equilibria across the two communities. We also examine the two type case, deriving bounds on relative wealth to show that interior equilibria require a polarized income distribution. Section 4 gives some examples and extensions, and explores various rationale for these results. Finally, the Section 5 Appendix contains the proofs.

[2]In the former, according to some estimates, about 10,000 Englishmen took up residence in Holland during the stay of the pilgrims, paid taxes and took advantage of the common schools (see Eby (1952), p. 124). The Dutch schools were common property of the people and financed by the municipalities. At the same time in England the Poor Law of 1601 declared that education was a charity and not a right. Formal schooling was henceforth carried out by and large in a decentralized fashion in which each family that could afford to do so provided education for their own children. In the case of the latter, in the 1740s in Colonial Virginia a large number of, typically poorer, individuals left the established Church of England which relied on voluntary contributions from its membership. They joined the Babtist movement where decisions were made by unanimity and by later majority rule. See Isaac (1982), pp 164-65.

2 The Model

There is a society $I = \{1, \dots n\}$ of individuals. Each $i \in I$ is distinguished by a wealth endowment z_i. The individuals are ordered so that $z_1 < z_2 < \cdots < z_n$. The aggregate income is given by $Z = \sum_{i \in I} z_i$. The profile of wealth levels, in ascending order, is given by $\boldsymbol{z} = (z_1, \dots, z_n)$.

An individual i's preferences are given by $u(x_i, Y)$, where Y is the quantity of a pure public good produced, and x_i consumption of a private good. The amount of the private contribution to the public good from individual i is given by $y_i = z_i - x_i$. We will specialize to standard CES preferences of the form:

$$u(x_i, Y) = \frac{1}{1-\sigma}[x_i^{1-\sigma} + Y^{1-\sigma}],\ \sigma > 0. \tag{1}$$

If σ is between 0 and 1 then the two goods are gross substitutes with $\sigma = 0$ yielding perfect substitutes. If $\sigma > 1$ then the public and private good are complementary goods with $\sigma \to \infty$ giving the limiting case of perfect complements.

We assume a simple linear technology so that Y is produced with amount $\sum_i y_i$ of the private good collected from the populace. Let $y = (y_1, \dots y_n)$ be the profile of contributions.

Each of n individuals may choose to live in one of two comunities. In community D a public good is produced with voluntary contributions from the membership. In community C the good is provided by a simple proportional tax on wealth determined by a majority vote.[3] For each community let $I_k \subseteq I$ denote the set of individuals located in $k = C, D$. Aggregate income is given by $Z_k = \sum_{i \in I_k} z_i$, and aggregate consumption of the collective good is given by $Y_k = \sum_{i \in I_k} y_i$ for k=C,D. To simplify notation, we will use upper bars, e.g., $\bar{Z}, \bar{Y}$ to denote corresponding aggregates in the private provision community D, and we will use lower bars, e.g., $\underline{Z}, \underline{Y}$ to denote aggregates in the public provision community C. Finally, let $\bar{Z}_{-i}$ and $\underline{Z}_{-i}$ denote, say, the aggregate income in the D and C communities, resp. when individual i is excluded.

The private and public provision mechanisms are as follows. In location D all decisions are completely decentralized and made privately and voluntarily. Each individual jointly consumes $\bar{Y} = \sum_{i \in I_D} y_i$ where y_i is i's voluntary contribution. Location C is assumed to

[3]While other tax schemes might be considered, the proportional tax is a natural first step since it is a tractable income-based tax which, as a class, are commonly used to pay for public goods. In Section 4, we discuss extensions of the model to lump sum taxes.

centrally allocate the nonrivalrous good via majority rule. We assume that a uniform tax rate, τ, is levied in community C and determined by a majority vote. Since the good is nonrivalrous, each individual $i \in I_C$ pays $y_i = \tau z_i$ and jointly consumes $\tau \underline{Z}$. Since each individual may choose which location to belong to, this decision, when aggregated with other individuals' choices of which community live determines whether one or the other or both provision mechanisms survive.

2.1 Private Provision

The analysis of voluntary provision follows Blume, Bergstrom, and Varian (1985) who consider just such an institution with a public good. In location D with the parameterization in (1) the utility for person i may be rewritten as

$$\begin{aligned} u_i(x, \bar{Y}) &= \frac{1}{1-\sigma}\left[x_i^{1-\sigma} + \bar{Y}^{1-\sigma}\right] && (2) \\ &= \frac{1}{1-\sigma}\left[(z_i - y_i)^{1-\sigma} + (y_i + \bar{Y}^{-i})^{1-\sigma}\right], && (3) \end{aligned}$$

where $\bar{Y}^{-i}$ is the amount of the public good provided by the members of community D other than i. The first order condition for an interior solution gives

$$(z_i - y_i)^{-\sigma} = (\bar{Y}^{-i} + y_i)^{-\sigma}$$

Otherwise, when z_i is too low we have $y_i = 0$. This gives the solution

$$\begin{aligned} y_i &= \max\{0, \frac{z_i - \bar{Y}^{-i}}{2}\}, && (4) \\ x_i &= \min\{z_i, \frac{z_i + \bar{Y}^{-i}}{2}\}. && (5) \end{aligned}$$

Let K denote the set of contributors in D, i.e., those with $y_i > 0$. Then, summing over contributors yields

$$\bar{Y} = \sum_{i \in I_D} y_i = \sum_{i \in I_D} \frac{z_i - \bar{Y}^{-i}}{2} = \frac{\bar{Z}}{2} - \frac{1}{2}\sum_{i \in I_D}\sum_{j \neq i} y_j = \frac{\bar{Z} - (K-1)\bar{Y}}{2}.$$

Solving for $\bar{Y}$ gives

$$\bar{Y} = \frac{\bar{Z}}{K+1}. \quad (6)$$

Using equations (4), (5), and (6) together gives

$$y_i = \max\{z_i - \frac{\bar{Z}}{K+1}, 0\}, \tag{7}$$

$$x_i = \min\{z_i, \frac{\bar{Z}}{K+1}\}. \tag{8}$$

In the solutions (7) and (8), if individual i's endowment z_i falls below the lower bound $\frac{\bar{Z}}{K+1}$ then he consumes his endowment and "free rides" completely since $y_i = 0$. If, however, i's endowment is higher than $\frac{\bar{Z}}{K+1}$ then i contributes the difference $z_i - \frac{\bar{Z}}{K+1}$. To clarify terminology, we will distinguish between *contributors* ($y_i > 0$) and *free riders* ($y_i = 0$).

Observe that $\frac{\bar{Z}}{K+1}$ is just below the average contribution of those who contribute. Since the wealth levels of all contributors must exceed this approximate contributor average, there can only be a narrow band of wealth levels that can account for voluntary provision. Since the amount $\bar{Z}$ includes i's contribution if i is a contributor, i is a contributor iff $z_i \geq \frac{\bar{Z}^{-i}}{K^{-i}+1}$ where $\bar{Z}^{-i} = \sum_{j \in I_D \setminus \{i\}} z_j$ and K^{-i} denotes the number of contributors excluding possibly individual i. Observe that individual i's indirect utility in the private provision community is therefore given by

$$V(z_i, I_D) = \begin{cases} \frac{2}{1-\sigma}\left(\frac{\bar{Z}^{-i}+z_i}{K^i+2}\right)^{1-\sigma} & \text{if } z_i \geq \frac{\bar{Z}^{-i}}{K^{-i}+1} \\ \frac{1}{1-\sigma}\left[z_i^{1-\sigma} + \left(\frac{\bar{Z}^{-i}}{K^{-i}+1}\right)^{1-\sigma}\right] & \text{if } z_i < \frac{\bar{Z}^{-i}}{K^{-i}+1} \end{cases} \tag{9}$$

2.2 Public Provision

Recall, that in this community, the public good is allocated via majority rule (median voter), and $\underline{Z}$ denotes the aggregate endowment $\sum_{i \in I_C} z_i$. Let $\tau(\boldsymbol{z}, s)$ the tax rate which is most preferred by the median voter. Tax rate $\tau(\boldsymbol{z}, s)$ is the rate that would prevail against any other in a majority vote. We will often denote this median tax rate by τ when there is no confusion. For each i,

$$y_i = \tau \underline{Z} \tag{10}$$

$$x_i = (1-\tau) z_i \tag{11}$$

the preferences of individual i over tax rates are then given by

$$u((1-\tau)z_i, \tau\underline{Z}) = \frac{1}{1-\sigma}\left[((1-\tau)z_i)^{1-\sigma} + (\tau\underline{Z})^{1-\sigma}\right]$$

Strict concavity of u and continuity of u in τ guarantees that each individual has a unique most preferred tax rate and all preferences over tax rates are single peaked. A standard result by Black guarantees that with single peaked preferences over tax rates a majority voting outcome has a solution — the median type's most preferred tax rate. Monotonicity in wealth of the most preferred tax rate ensures that the person with median wealth is the median voter.

Let $\underline{z}$ denote the median income of those in I_C. The most preferred tax rate for this individual is given by

$$\tau(\boldsymbol{z}, s) = \tau = \frac{1}{1 + (\underline{z}/\underline{Z})^{(1-\sigma)/\sigma}}. \tag{12}$$

Observe that τ is strictly decreasing in $\underline{z}/\underline{Z}$ when $\sigma \in [0, 1)$ (x and y are gross substitutes), and is strictly increasing when $\sigma > 1$ (x and y are complements). In the case of substitutes, relatively wealthier individuals desire lower taxes. When the goods are complements to an individual, then the opposite is true. The indirect utility to i in the Public Provision community I_C is

$$W_i(z_i, I_C) \equiv \frac{1}{1-\sigma}\left[((1 - \tau(\boldsymbol{z}, s))z_i)^{1-\sigma} + (\tau(\boldsymbol{z}, s)\underline{Z})^{1-\sigma}\right] \tag{13}$$

where $\tau(\boldsymbol{z}, s)$ in (13) is defined by (12).

2.3 Equilibrium

An *equilibrium* in this environment is a location profile $(s_i^*)_{i\in I}$ where $s_i^* \in \{C, D\}$ determines i's optimal location decision. We now characterize the equilibria in the static model. Given an equilibrium (x, y, s) we will say that public (resp., private) provision is *viable* if the C (resp., D) region contains a nonempty set of individuals. In any equilibrium the following incentive constraints hold:

$$V(z_i, I_D) \geq W(z_i, I_C \cup \{i\}) \tag{14}$$

and

$$W(z_j, I_C) \geq V(z_j, I_D \cup \{j\}) \tag{15}$$

where z_i is the wealth of a arbitrary individual $i \in D$, while z_j is the wealth of an individual in $j \in C$. The constraints (14) and (15) simply state that individual i as a current member of D has no incentive to change locations to C. Likewise, individual j has no incentive to move to from C to D.

The constraints (14) and (15) shed light on a basic tradeoff in public goods provision. First, there is a common interest problem of overcoming free riding when provision is not coercive. To the extent that individuals value provision of a public good, all may prefer to submit to some coercion given individual incentives in voluntary provision lead to underprovision of the good. However, even if coercion solves the free riding problem, it imposes a conflict of interest problem since individuals will by no means agree on the appropriate coercive tax rate. Public provision better solves the common interest problem since it overcomes free riding. Private provision deals better with conflicts of interest since it does not compel an individual who differs widely from the median voter to pay taxes that allocate resources nonoptimally across the two goods from his point of view. The Tiebout model allows each to make a personal choice depending on how each community is expected to deal with these two problems. The constraints (14) and (15) form the basis for proving results of the next Section.

3 Existence and Characterization

3.1 Interior and Noninterior Equilibria

We will distinguish between two types of equilibria. Equilibria in which individuals locate in both the C and D locations will be referred to as *interior* equilibria. The interior equilibria are distinguished from the two types in which all individuals locate in either the C or the D provision communities. We first characterize conditions under which noninterior equilibria exist.

Proposition 1 *1. An equilibrium in which all individuals locate in D always exists.*

2. *If $\sigma > 1$ (public and private goods are complements) then there is an equilibrium in which all individuals locate in C.*

3. *If $\sigma \in [0, 1]$ (public and private goods are substitutes) and $\frac{\bar{Z}}{z_n} > 2^{\sigma/(1-\sigma)}$ then there is an equilibrium in which all individuals locate in C.*

From the Proposition, there is always an equilbrium in which everyone resides in the community with private provision. The condition $\frac{\bar{Z}}{2^{\sigma/(1-\sigma)}} > z_n$ indicates the existence of an equilibrium in which all locate in the other community as well. Such a condition is ensured

when either the private and public goods are close to perfect substitutes (σ is small) or when no individual is excessively wealthy relative to the aggregate ($\frac{\bar{Z}}{z_n}$ is large). If goods are close substitutes then the ability of the C community to overcome the free rider problem overrides concerns that one is taxed at an undesirable rate. If $\frac{\bar{Z}}{z_n}$ is large enough then no individual's preferred tax rate differs that much from the median voter's.

Proposition 1 shows that a "wealth effect" can overcome either the common interest problem in D or the conflict of interest problem in C. Specifically, according to the wealth effect, someone will not move from a large community to a small one even if the small one is more closely aligned with the individual's preferences. Even if the small community solves, say, the common interest problem with tax rates most preferred by the individual, its aggregate wealth is too low to offer an attraction. The proof involves a straightforward manipulation of incentive constraints in (14) and (15). We leave the remaining details, which consists mostly of the algebra derived from these incentive constraints, to the Appendix.

Proposition 2 *Suppose that s is an interior equilibrium locational profile. Then:*

1. *There is some $\hat{z}$ such that all those with wealth $z < \hat{z}$ belong to the C community, while all those with wealth $z \geq \hat{z}$ belong to the D community.*
2. $\frac{\max\{z_i:\, i \in C\}}{\min\{z_j:\, j \in D\}} \leq 2 - 2^{1-\sigma}$.

By this Proposition, interior equilibria exist only if they have the property that richer individuals utilize private provision, while poorer individuals utilize public provision, and that the spread between those in C and those in D is sufficiently great. Again, the details of the proof are in the Appendix.

By the results above, the only possible interior equilbrium is one in which all individuals richer than some threshold level locate to the D community while all those poorer than the threshold prefer public community. We leave the question of existence to Section 3.4 where the case of two income types is examined.

3.2 The Homogeneous Population Case

This case of homogeneous individuals is an important special case since it will allow us to isolate the effect of wealth heterogeneity in our subsequent results. In this case, the location

problem is reduced to a coordination problem since no conflict of interest arises between voters in region C. To see this, first observe from Proposition 2 that no interior equilibrium exists, and that both types of the extremal equilibria (everyone lcoates in D or everyone locates in C) exist. We compare the payoffs of everyone in C vs everyone in D.

Let $|I| = n$. In D, each contributes the same amount and receives

$$V(z, I_D) = 2(\frac{z + Z_{-i}}{n+1})^{1-\sigma} = 2(\frac{n}{n+1})z^{1-\sigma},$$

while each in C receives

$$W(z, I_C) = [(1-\tau)z]^{1-\sigma} + [\tau n z]^{1-\sigma},$$

where $\tau = 1/(1 + n^{(\sigma-1)/\sigma})$. Using (12) W can be rewritten as

$$\left(\frac{z}{n^{(\sigma-1)/\sigma}+1}\right)^{1-\sigma} + \left(\frac{zn^{1/\sigma}}{n^{(\sigma-1)/\sigma}+1}\right)^{1-\sigma}$$

From these payoffs it turns out that $W \geq V$ iff

$$\frac{1}{1-\sigma} n^{1-\sigma} \left(1 + n^{(\sigma-1)/\sigma}\right) \;\geq\; \frac{2}{1-\sigma} \left(\frac{n}{n+1}\right)^{1-\sigma} \tag{16}$$

For $\sigma > 1$ it is easy to check that the inequality in (16) holds strictly. For $\sigma = 1$ (Cobb-Douglas utility) condition (16) holds with equality. For $\sigma < 1$ we verify that (16) holds when $n \geq 2$. Hence, we have:

Proposition 3 *Let $z_i = z_j$ for all $i, j \in I$. Then an equilibrium in which everyone locates in C exists and Pareto dominates the D equilibrium for almost every σ.*

3.3 The Two Types Case

We are interested in how income distribution influences the type of equilibrium. In particular we would like to know what kind of wealth stratification emerges in an interior equilibrium? We consider this question for the case in which there are two types, rich and poor. The wealth of the rich is given by z_H. The wealth of the poor is denoted $z_L < z_H$. Let r denote the number of rich, while p is the number of poor. By Proposition 2 any interior equilibrium will be completely stratified: all the poor reside in centralized community C while the rich reside in the decentralized community D.

To characterize these equilibria we derive first the median voter's tax rate when all the poor locate in C. This is given by

$$\tau = \frac{1}{1 + (1/p)^{(1-\sigma)/\sigma}}. \tag{17}$$

Then the indirect utility of the rich in location D and the poor in location C are, resp.,

$$V(z_H, I_D) = \frac{2}{1-\sigma}\left(\frac{r}{r+1}z_H\right)^{1-\sigma} \tag{18}$$

$$W(z_L, I_C) = \frac{1}{1-\sigma}\left[p^{1-\sigma}\left(1 + p^{(\sigma-1)/\sigma}\right)^{\sigma}\right] z_L^{1-\sigma}. \tag{19}$$

As for incentive contraints, one can verify that no rich individual will migrate to C if

$$\frac{2}{1-\sigma}\left(\frac{r}{r+1}\right)^{1-\sigma} z_H^{1-\sigma} > \frac{1}{1-\sigma}\left[(1-\tau')^{1-\sigma} z_H^{1-\sigma} + \tau'^{1-\sigma}(z_H + pz_L)^{1-\sigma}\right] \tag{20}$$

where τ', the new tax rate if the rich individuals joins C, is given by

$$\tau' = \frac{1}{1 + (\frac{z_L}{z_H + pz_L})^{(1-\sigma)/\sigma}}. \tag{21}$$

On the other hand, no individual will move from C to D if

$$\frac{1}{1-\sigma}\left[p^{1-\sigma}\left(1 + p^{(\sigma-1)/\sigma}\right)^{\sigma}\right] z_L^{1-\sigma} > \frac{1}{1-\sigma}\left[z_L^{1-\sigma} + \left(\frac{r}{r+1}z_H\right)^{1-\sigma}\right]. \tag{22}$$

An upper bound and lower bound for the ratio of the poor's wealth to that of the rich, z_L/z_H, can be determined by the incentive contraints (22) and (20) under two reasonable conditions. The first is that $z_L < \frac{r}{r+1}z_H$ which is the case when a poor individual would free ride in community D. This inequality is easily verified for large enough r, though typically r need not be all that large. The second is that $z_L < \frac{p-1}{p}z_H$ which again easily holds for large p. We now state the bounds formally.

Proposition 4 *Fix strictly positive values of r and p. If $z_L < \frac{p-1}{p}z_H$ and $z_L < \frac{r}{r+1}z_H$, then an interior equilibrium exists if the ratio of poor to rich income z_L/z_H satisfies*

$$\begin{aligned} & \frac{r}{r+1}\left[p^{1-\sigma}\left(1 + p^{(\sigma-1)/\sigma}\right)^{\sigma} - 1\right]^{-1/(1-\sigma)} \\ < \; & \frac{z_L}{z_H} \\ < \; & \frac{1}{p}\left[2\left(\frac{r}{r+1}\right)^{1-\sigma}(1 + p^{(\sigma-1)/\sigma})^{1-\sigma} - p^{-(1-\sigma)^2/\sigma}\right] - \frac{1}{p} \end{aligned} \tag{23}$$

4 Discussion

Observe that the upper bound in (23) is determined by (20), i.e., the incentive of the rich in D. The lower bound is determined by the incentives of the poor in C which is given by inequality (22). To give an indication of how stringent are the bounds given in (23), the series of graphs in Figures 1 and 2 display the upper and lower bounds of the wealth ratio z_H/z_L as a function of σ for values of p and r. The region where interior equilibria exist — i.e., where the upper bound exceeds the lower bound is indicated with "E" whereas nonexistence is indicated with "NE". Existence holds only for values of σ between 0 and 1, i.e., the case of substitutability between private and public good. In Figure 1 the number of rich is held fixed at $r = 200$ while the number of poor varies. In Figure 2 we let $p = r$ and vary the two together in order to examine scale effects. The tradeoff between the relative numbers of rich and poor and the wealth ratio is striking. In both figures, for comparable number of rich and poor the admissible wealth ratio is very small. More generally, for interior equilibrium to exist seems to require a polarized income distribution in the sense that the ratio z_L/z_H is very small whenever p is large.

[Figures 1 and 2 here]

One explanation for what is happening is that when individuals in C are extremely poor relative to their counterparts in D, they must be sufficiently numerous so that they generate enough of the public good though cumpulsory taxation to prevent each member from leaving and free riding in the D community. The (indirect) preferences of the rich are then sufficiently different to prevent them from joining the poor. While this explanation is correct, it cannot, by itself account for the extremely small admissable wealth ratios. Those come from the incentive constraint of the rich which, recall, determines the upper bound in (23). This means that if z_L/z_H is large enough then the rich prefer to move to C to take advantage of the public good.

Consider, for example, the parameters $\sigma = 1/2$, $r = 1000$, $p = 2000$, $z_H = 15000$, and $z_L = 10$ which can be verified to support a stratified equilibrium. For these parameters, the tax rate chosen by the poor in C is virtually unity. This means that the amount of public good provided is close to 20000, the aggregate income of C. Now observe that the total level of public good in the private provision region D never exceeds z_H due to free riding. Hence, even though the aggregate income is considerably greater in D than in C, no poor individual

would join since the actual amount of public good provided due to free riding is less than 15000, less than the amount provided in C. Moreover, no rich person wishes to move to C since the public and private goods are imperfect substitutes and almost all his private wealth would be taxed away in C. However, suppose that $z_L = 1000$. Then the stratified equilibrium is destroyed since the rich have an incentive to move into the neighborhood of the poor! This differs from migration models in which taxation is for purely redistributive purposes (e.g., see Epple and Romer (1991)). There, taxation redistributes a fraction of *per capita* wealth $\frac{z_H + p z_L}{p+1}$, and it is the poor who then chase the rich which destroys interior equilibria.

Notice that the aggregate quantity of public good is greater in the poor community than in the rich one. This is not only due to solving the free rider problem, but also due to the sheer size of the poor community. As an example, public transportation in inner cities typically exceeds that in richer and smaller suburbs. As another example, in Isaac's (1982) discussion of the Babtist movement in Colonial Virginia, the poor defected from the wealthy Church of England precisely because religious services were underfunded.[4]

In the present model, it is not only the difference between redistribution and public goods provision which matters, but it is also the institutional differences as well. The institutional differences create a tradeoff for each agent between the common interest (coordination) problem in D and the conflict of interest problem in C. Interestingly, while one might think that the common interest problem favors the poor since they have the least to lose from free riding, it actually favors the rich since they are most capable of self financing in the presence of free riding. On the other hand, the conflict of interest problem in C clearly favors the poor since departures from one's optimal mix of private and public goods are less harmful to a poor person. Since only unilateral deviations from equilibrium are considered, the common interest problem in D is severe enough to keep out any *single* poor person, while the conflict of interest problem in C is severe enough to keep out any *single* rich person.[5] By way of comparison, a companion paper (Glomm and Lagunoff (1996)) looks at the dynamics of this model when long-lived individuals' location decisions are made repeatedly and asynchronously. There, the possibility that one's move may be a part of a cumulative process establishes that one or the other community (and its associated provision process) is uniquely viable in the long run despite the multiplicity of equilibria in the static

[4] See Isaac (1982), pp 147-65.

[5] Buchanon and Tullock (1962) derive an optimal voting rule as one that balances these two incentive problems. The difference here is that the present contribution allows the participants themselves to make personal decisions between the mechanisms based on this trade off.

location game. The reason is that the intertemporal interaction overcomes the inability for multilateral departure in the present model.

It is worth asking how sensitive is the results and intuition to certain assumptions of the model. One useful extension is to examine properties of equilibria under mean-preserving spreads in the wealth distribution. For instance, given the parametric example above, consider a relatively small mean-preserving spread of the distribution of poor. We keep the incomes of the rich fixed. Suppose that among the poor, 400 individuals have an income of 6, 400 have income of 8, 400 have income 10, 400 have income 12, and 400 have income 14. In this group average income is still 10 so that this is a mean preserving spread of the degenerate distribution in which all 2000 individuals have income of 10. There remain 1000 rich individuals with income 15000. Starting with the stratified location profile in which rich are in D and poor are in C, it is easy to see that this profile remains an equilibrium. Each poor individual would free ride were he move to D. His incentive constraint is $[(1-\tau)z_i]^{1/2} + [\tau 20000]^{1/2} < z_i^{1/2} + (\frac{1000}{1001}15000)^{1/2}$ where the median voter's tax rate approximately one. This inequality requires that $z_i > 377$ which holds in our example. This small perturbation above in the income distribution leaves the interior equilibrium unaffected. While larger perturbations can be constructed which destroys the stratified equilibrium, there is at least a neighborhood of these parameters in which the stratified equilibrium remains.

Another extension to examine other tax schemes in the centralized community. While the qualitative features of equilibria are unlikely to change under a mildly progressive income tax (such as in the U.S.), interior equilibria may be harder to come by under a head tax. Since head taxes do not punish the rich as much as income taxes (relative to the poor), the conflict of interest problem in C is mitigated to an extent.

Indeed in a richer model that allows for an institutional response, one might expect that communities change their tax schemes as a way to attract a larger tax base. However, this is not so clear. Suppose, for example, that citizens in C can vote on a linear tax system. This includes progressive, regressive, as well as flat tax schemes. One might conjecture that the median voter would, if he could, commit to a regressive tax in order to attract rich into the community. However, since taxation decisions are made subsequent to migration, the median voter's incentives are different. The median voter would generally prefer a progressive tax schedule in order to expropriate the wealth of the rich to finance the public good. Hence, a

regressive tax may well be optimal, but it is not necessarily time consistent.[6]

Finally, the present work considers only *pure* public goods. There is an established literature which has examined various types of provision procedures for public goods in isolation from other procedures. Bergstrom, Blume, and Varian (1986), for instance, examine private provision of a public good. They provide a fairly complete characterization which may be compared directly with established results showing the efficiency of majority voting (see Bergstrom (1979)).[7] We anticipate that when provision of private goods in a spatial model is the issue, the benefit of overcoming the free rider problem dissipates, and so public provision is more difficult to support. Usher (1977) and Besley and Coate (1991) examine models with private goods in which individuals can choose (albeit, not by migration) between the private and public sectors. Usher, for example, finds that with voting, the probability of public provision is larger the larger is income inequality. The reason is that the motive for redistribution rises with inequality. For this reason, we suspect that in our model, when congestion (an impure public good) plays a role, there is a strong motive for the rich to effect an institutional response to keep out the poor.

5 Appendix

proof of Proposition 1 To see that all individuals in D always constitutes an equilibrium, observe that if an individual i with wealth z moves alone to C, then he is the only member in that community and so his most preferred tax rate is $\tau = 1/2$. Hence, he will only leave D if, as a contributor in D,

$$\frac{2}{1-\sigma}\left(\frac{\bar{Z}_{-i}+z_i}{K_{-i}+2}\right)^{1-\sigma} < \frac{2^\sigma}{1-\sigma} z_i^{1-\sigma}. \tag{24}$$

Inequality (24) may be rewritten as $2(\bar{Z}_{-i}/K_{-i}) < z$ which states that i's wealth exceeds twice the average wealth of the remaining contributors. However, it must hold that an individual with the average wealth of the remaining contributors, Z_{-i}/K_{-i}, must himself be a contributor. Therefore, we must have $Z_{-i}/K_{-i} \geq \frac{Z_{-i}+z}{K_{-i}+2}$ which reduces to $2(Z_{-i}/K_{-i}) \geq z$

[6] Caplin and Nalebuff (1993) allow the possibility of precommitment in a similar model. They establish existence but do not discuss many other qualitative features of the model.

[7] More generally, there is a large formal literature on public provision via voting. See Laffont (1988) for references.

thereby contradicting (24). A free rider in D will leave iff

$$\frac{1}{1-\sigma}[z_i^{1-\sigma} + \left(\frac{\bar{Z}_{-i} + z_i}{K_{-i}+1}\right)^{1-\sigma}] < \frac{2^\sigma}{1-\sigma} z^{1-\sigma}. \tag{25}$$

This condition is true if and only if

$$\left(\frac{\bar{Z}_{-i} + z_i}{K_{-i}+1}\right)^{1-\sigma} < (2^\sigma - 1)^{1/(1-\sigma)}\, z_i.$$

For any $\sigma > 0$ this contradicts the condition $z_i < \frac{\bar{Z}_{-i}+z_i}{K_{-i}+1}$ which is required if i is a free rider.

To show part (2) of the Proposition, observe that if everyone locates in C then this is an equilibrium iff

$$2^\sigma z^{1-\sigma} \leq \left[((1-\tau)z)^{1-\sigma} + (\tau \underline{Z})^{1-\sigma}\right]. \tag{26}$$

Let $m = \frac{\underline{Z}}{\max\{z:\, z \in C\}}$. Observe that from (26) it follows that everyone locating in C is an equilibrium if

$$\frac{2^\sigma}{1-\sigma} \leq \frac{1}{1-\sigma}((1-\tau))^{1-\sigma} + (m\tau)^{1-\sigma}. \tag{27}$$

Now observe that the right side of (27) is strictly concave in τ. When $0 \leq \sigma \leq 1$ then τ varies between $1/2$ and 1. Therefore, the right side of (26) is minimized at either $\tau = 1$ or $\tau = 1/2$. If $\tau = 1/2$ then (26) becomes $1 \leq m^{1-\sigma}$ while if $\tau = 1$ then (26) becomes $2^\sigma \leq m^{1-\sigma}$. Taking the more stringent condition $\tau = 1$ we have shown that if $2^\sigma \leq m^{1-\sigma}$ then no individual will leave the C community. When $\sigma > 1$ then τ varies between 0 and $1/2$. Therefore, the right side of (26) is minimized at either $\tau = 0$ or $\tau = 1/2$. In the latter case we have that $1 < m$ which is always true and so no one leaves C. If $\tau = 0$ we have $2^\sigma > 1$ which again always holds. □

proof of Proposition 2 We first show part (1) of the result. Given an interior equilbrium (x, y, s) suppose by contradiction that the interior equilibrium has the property that z_1 is a in D and $z_1 < z_2$ with z_2 in C. The incentive constraints are then given by

$$V(z_1, I_D) \geq W(z_1, I_C \cup \{z_1\}) \tag{28}$$

and

$$W(z_2, I_C) \geq V(z_2, I_D \cup \{z_2\}) \tag{29}$$

It can easily be shown that $V(z_2, I_D \cup \{z_2\}) > V(z_1, I_D)$. That is, a wealthier member cannot do worse than the existing payoff without that individual. This, together with (28) and (29),

implies that $W(z_2, I_C) \geq W(z_1, I_C \cup \{z_1\})$. Let τ' denote the tax rate associated with the C equilibrium, and τ'' denote the tax rate in $I_C \cup \{z_1\}$. Using the definition of W in (13), the inequality $W(z_2, I_C) \geq W(z_1, I_C \cup \{z_1\})$ implies:

$$(1-\tau')^{1-\sigma} z_2^{1-\sigma} + \tau'^{1-\sigma} \underline{Z}^{1-\sigma} \geq (1-\tau'')^{1-\sigma} z_1^{1-\sigma} + \tau''^{1-\sigma} (\underline{Z} + z_1)^{1-\sigma} \tag{30}$$

Observe from the definition of τ' and τ'' in (12), that

$$\tau'' > \tau' \text{ iff } \frac{\underline{z}(z_1)}{\underline{Z} + z_1} < \frac{\underline{z}}{\underline{Z}}$$

where $\underline{z}(z_1)$ denotes the median wealth after the individual with z_1 joins C. There are two cases to consider. First, suppose that z_1 always brings down the median, i.e., $\underline{z}(z_1) \leq \underline{z}$. Then tax rate τ'' is preferred by z_1 to the rate τ'. This means that

$$(1-\tau'')^{1-\sigma} z_1^{1-\sigma} + \tau''^{1-\sigma} (\underline{Z} + z_1)^{1-\sigma} > (1-\tau')^{1-\sigma} z_1^{1-\sigma} + \tau'^{1-\sigma} (\underline{Z} + z_1)^{1-\sigma}. \tag{31}$$

Suppose on the other hand that z_1 increases the median by joining C, i.e., $\underline{z}(z_1) > \underline{z}$. Then it must be the case that individual with income z_2 prefers the new rate τ'' to the original rate τ' since $z_2 > z_1$. In this case we have

$$(1-\tau'')^{1-\sigma} z_2^{1-\sigma} + \tau''^{1-\sigma} \underline{Z}^{1-\sigma} > (1-\tau')^{1-\sigma} z_2^{1-\sigma} + \tau'^{1-\sigma} \underline{Z}^{1-\sigma}. \tag{32}$$

Therefore, in either the case of (31) together with (30) or the case of (32) together with (30) we have

$$(1-\tau)^{1-\sigma} z_2^{1-\sigma} + \tau^{1-\sigma} \underline{Z}^{1-\sigma} > (1-\tau)^{1-\sigma} z_1^{1-\sigma} + \tau^{1-\sigma} (\underline{Z} + z_1)^{1-\sigma} \tag{33}$$

where we let $\tau = \tau'$ in the case of (31) combined with (30), and let $\tau = \tau''$ in the case of (32) combined with (30). Let τ be either τ' or τ'' which is defined to satisfy Inequality (33). Let n satisfy $nz_2 = \underline{Z}$. We can rewrite (33) as

$$[(1-\tau)^{1-\sigma} + (n\tau)^{1-\sigma}] z_2^{1-\sigma} > (1-\tau)^{1-\sigma} z_1^{1-\sigma} + \tau^{1-\sigma} (nz_2 + z_1)^{1-\sigma}. \tag{34}$$

Observe that since z_1 is a contributor in D, it follows that $z_1 \geq \frac{\underline{Z}}{c+1}$ (where c, recall, is the number of contributors). Therefore, an upper bound to z_1's payoff $V(z_1, D)$ is $2z_1^{1-\sigma}$. From (14) we then have

$$2z_1^{1-\sigma} > (1-\tau)^{1-\sigma} z_1^{1-\sigma} + \tau^{1-\sigma} (nz_2 + z_1)^{1-\sigma}$$

which we rewrite as

$$z_1 > \frac{n\tau z_2}{H - \tau} \tag{35}$$

where $H = (2 - (1-\tau)^{1-\sigma})^{1/(1-\sigma)}$. Substituting Inequality (35) into Inequality (34), we obtain

$$(1-\tau)^{1-\sigma} + (n\tau)^{1-\sigma} > (1-\tau)^{1-\sigma}(\frac{n\tau}{H-\tau})^{1-\sigma} + (n\tau)^{1-\sigma}(1+\frac{\tau}{H-\tau})^{1-\sigma} \tag{36}$$

Inequality (36) can, in turn, be rewritten as

$$(H-\tau)^{1-\sigma}((1-\tau)^{1-\sigma} + (n\tau)^{1-\sigma}) > (n\tau)^{1-\sigma}(H^{1-\sigma} + (1-\tau)^{1-\sigma}) \tag{37}$$

However, (37) is contradicted for all $\tau \in [.5, 1]$ and all $\sigma \in [0, 1]$.

We now show part (2) of the Proposition. Suppose $2 - 2^{1-\sigma} \leq \frac{\max\{z:\, z \in C\}}{\min\{z:\, z \in D\}}$. Then we prove there is no interior equilibrium. It suffices by the proof of part (1) to show that if z_1, z_2 denote any two income levels where $z_2 > z_1$ and z_1 belongs to the C community, then z_2 also belongs to the C community. Suppose by contradiction that $z_1 \in C$ and $z_2 \in D$. Then the incentive constraints in this case are

$$V(z_2, I_D) \geq W(z_2, I_C \cup \{z_2\}) \tag{38}$$

and

$$W(z_1, I_C) \geq V(z_1, I_D \cup \{z_1\}). \tag{39}$$

As before, let τ' denote the staus quo tax rate in C and let τ'' denote the rate if individual z_2 joins C. There are two cases to consider. Case (a): if z_1 joins D then he is a contributor; and Case (b): if z_1 joins D then he is a free rider. Suppose first Case (a). Then $z_1 \geq \frac{\bar{Z}}{c+1}$. It is then clear that

$$V(z_1, I_D \cup \{z_1\}) \equiv 2\left(\frac{\bar{Z}_{-1} + z_1}{c+2}\right)^{1-\sigma} \geq 2\left(\frac{\bar{Z}}{c+1}\right)^{1-\sigma} \equiv V(z_2, I_D).$$

This implies that

$$W(z_1, I_C) \geq W(z_2, I_C \cup \{z_2\})$$

which may be rewritten as

$$(1-\tau')^{1-\sigma} z_1^{1-\sigma} + \tau'^{1-\sigma}\underline{Z}^{1-\sigma} \geq (1-\tau'')^{1-\sigma} z_2^{1-\sigma} + \tau''^{1-\sigma}(\underline{Z} + z_2)^{1-\sigma}. \tag{40}$$

Since $z_1 < z_2$, and since $\underline{Z} + z_2 > z_2$ and $\underline{Z} + z_2 > \underline{Z}$, it follows from (40) that $\tau' > \tau''$. Moreover, it must be the case that in order to satisfy the incentive constraint (40), z_1's most preferred tax rate $\tau(z_1) \equiv 1/(1 + (z_1/\underline{Z})^{\frac{\sigma}{1-\sigma}})$ must be greater than τ''. Hence we have

$$\frac{z_1}{\underline{Z}} < \frac{\underline{z}(z_2)}{\underline{Z} + z_2}$$

where $\underline{z}(z_2)$ is the median income after z_2 joins C, and so $z_1 < \underline{z}(z_2)$. However, z_1 is chosen from C arbitrarily, and so $z_1 < \underline{z}(z_2), \forall z_1 \in I_C$. However, this implies that $\underline{z}(z_2) \notin I_C$ which is only the case if $I_C = \{z_1\}$. Since $I_C = \{z_1\}$ cannot be part of an interior equilibrium, Case (a) cannot occur. Notice that Case (a) did not use the fact that $2 - 2^{1-\sigma} \leq \frac{\max\{z:\, z \in I_C\}}{\min\{z:\, z \in I_D\}}$, and so this case is impossible in any circumstance.

Consider now Case (b). Then $z_1 < \frac{\bar{Z}}{c+1}$. The incentive constraints for this case are

$$(1-\tau')^{1-\sigma} z_1^{1-\sigma} + \tau'^{1-\sigma} \underline{Z}^{1-\sigma} \geq z_1^{1-\sigma} + (\frac{\bar{Z}}{c+1})^{1-\sigma} \tag{41}$$

and

$$2(\frac{\bar{Z}}{c+1})^{1-\sigma} \geq (1-\tau'')^{1-\sigma} z_2^{1-\sigma} + \tau''^{1-\sigma} (\underline{Z} + z_2)^{1-\sigma}. \tag{42}$$

Observe from the proof of Case (a) that we cannot have $\tau' > \tau''$. Since $\tau' \leq \tau''$, we have

$$(1-\tau'')^{1-\sigma} z_2^{1-\sigma} + \tau''^{1-\sigma} (\underline{Z} + z_2)^{1-\sigma} \geq (1-\tau')^{1-\sigma} z_1^{1-\sigma} + \tau'^{1-\sigma} \underline{Z}^{1-\sigma}.$$

Therefore,

$$[\text{Left Side of (42)}] - [\text{Right Side of (41)}] > [\text{Right Side of (42)}] - [\text{Left Side of (41)}] \tag{43}$$

If Inequality (43) is satisfied then

$$(\frac{\bar{Z}}{c+1})^{1-\sigma} - z_1^{1-\sigma} > (1-\tau'')^{1-\sigma} [z_2^{1-\sigma} - z_1^{1-\sigma}] + \tau''^{1-\sigma} [(\underline{Z} + z_2)^{1-\sigma} - z_2^{1-\sigma}]. \tag{44}$$

Using the fact that $z_2 > \frac{\bar{Z}}{c+1}$ (44) is rewritten as

$$z_2^{1-\sigma} - z_1^{1-\sigma} > (1-\tau'')^{1-\sigma} [z_2^{1-\sigma} - z_1^{1-\sigma}] + \tau''^{1-\sigma} [(\underline{Z} + z_2)^{1-\sigma} - z_2^{1-\sigma}]. \tag{45}$$

In order to derive the contradiction, it suffices from (45) to show

$$z_2^{1-\sigma} - z_1^{1-\sigma} \not> [(\underline{Z} + z_2)^{1-\sigma} - z_2^{1-\sigma}] \tag{46}$$

or

$$2z_2^{1-\sigma} \not> (\underline{Z} + z_2)^{1-\sigma} + z_1^{1-\sigma} \tag{47}$$

which is implied by

$$2z_2^{1-\sigma} \not> 2z_2^{1-\sigma} + z_1^{1-\sigma} \tag{48}$$

or

$$2 - 2^{1-\sigma} \not> (\frac{z_1}{z_2})^{1-\sigma}$$

and since this must be true for all z_1 and z_2 the condition

$$\frac{\max\{z:\ z\in C\}}{\min\{z:\ z\in D\}}\geq 2-2^{1-\sigma}$$

suffices for nonexistence of this type of equilibrium. □

proof of Proposition 4 The constraint (22) gives the lower bound for z_L/z_H in (23). Meanwhile, using the rate (21) in Inequality (20) yields an unwieldly expression. However, if $z_L<\frac{p-1}{p}z_H$, which holds under our hypothesis, we observe that the tax rate given by (17) is preferred by the rich individual to the one in (17) which he would actually obtain by moving to C. That is,

$$\frac{1}{1-\sigma}\left[(1-\tau')^{1-\sigma}z_H^{1-\sigma}+\tau'^{1-\sigma}(z_H+pz_L)^{1-\sigma}\right]<\frac{1}{1-\sigma}\left[(1-\tau)^{1-\sigma}z_H+\tau^{1-\sigma}(z_H+pz_L)^{1-\sigma}\right]. \tag{49}$$

To see this observe that if $\sigma\in[0,1]$ then

$$\frac{1}{1+(\frac{z_H}{z_H+pz_L})^{(1-\sigma)/\sigma}}<\frac{1}{1+(1/p)^{(1-\sigma)/\sigma}}<\frac{1}{1+(\frac{z_L}{z_H+pz_L})^{(1-\sigma)/\sigma}} \tag{50}$$

where the left-most expression is the most preferred rate of the rich individual when he moves to C, the middle is the pre-existing rate, and the right-most is the actual rate that his move would induce. If $\sigma>1$ then the string of inequalities in (50) is simply reversed. In any case, since preferences over tax rates are single peaked, clearly the pre-existing rate in the middle is preferred to the realized one on the right. Hence, the constraint (20) combined with Inequality (49) determines gives the upper bound in (23) □

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