On the Social Stability of

Coalitional Property Rights Regimes

Gerhard Glomm[*] and *Roger Lagunoff*[†]

Revised: September 1, 1996[‡]

Abstract

We present a model of *coalitional property rights (CPR) regimes* — regimes in which ownership of a good is attributable to coalitions of various sizes. Specifically, for each good, we define a legal structure that specifies the legal coalitions of individuals that share a communal claim to that good. Generally, each legal coalition may use *exclusionary rules* to allocate its holdings internally. These rules allow eligible subcoalitions to recontract by expropriating some fraction of the legal coalition's endowment. We then ask: what types of CPR regimes are *socially stable* in the sense of having a nonempty core? We give conditions on the legal structure and the primitives of the economy that achieve social stability in this sense. We emphasize two cases of particular interest.

(I) Unanimity. Unanimity is required for a legal coalition to recontract against (block) the status quo. In this case, the core is nonempty under standard assumptions. Each agent's ability to veto an alternative allocation allows a partial characterization in terms of the economies that are privatized by dividing up the communal endowment among the members of each legal coalition. We show that in some economies collective vs private ownership matters in terms of social stability.

(II) Exclusion. Many eligible subcoalitions can expropriate the legal coalition's entire endowment. An example is the collection of simple majorities. The presence of cycles can easily lead to social instability. We show that if endowment holdings are sufficiently "specialized" and each agent's "veto power" sufficiently large, then stability can be achieved despite the presence of cycles in some goods.

[*]Department of Economics, Michigan State University, East Lansing, MI 48824-1038, USA.

[†]Department of Economics, Georgetown University, Washington DC 20057-1036, USA.

[‡]We have benefitted from the comments and conversations with Francis Bloch, Stefano Fenoaltea, Michael Meurer, Herve Moulin, Peter Streufert, Lin Zhou, and, especially, Roy Radner and Myrna Wooders. We also gratefully acknowledge the helpful suggestions of an anonymous referee who identified errors in an earlier draft. Support from the University of Pennsylvania Research Foundation is acknowledged.

1 Introduction

Formal economic analysis such as that found in the classic General Equilibrium model typically deals with a special class of economies, namely those in which ownership rights for endowments, factors of production and profit shares are assigned to individual consumers (see, for example, Debreu (1959)). Though these models capture some aspects of our current economy quite well, they are limited in scope.

Property rights regimes observed in different historical periods and in different geographic areas often do not fit this framework. According to North (1981),

> A variety of property rights underlay the various types of economic organizations. Initially, exclusive communal rights were established by the first agricultural communities; in some places these gave way to exclusive state-owned property rights and in others to individual property rights.

Examples of diverse property rights structures abound. MacFarlane (1978) traces the origins of private property in England only back to the eleventh century. According to Dahlman (1980) even "in the middle of the eighteenth century, the open field system (in England) is dominant over vast regions." Shanin (1972) documents that in Russia, as late as the early 20th century, land was neither owned by individuals nor all of society, but, rather, by coalitions of intermediate size. Land, workshops, and mills were jointly owned by the peasant commune. Other historical examples are abundant. See, for example, Troost (1990) and Brown (1990) for a discussion of communal ownership in medieval Japan; Ostrom (1990) discusses village ownership of Alp territory; Anderson and Lueck (1990) characterize tribal institutions in North American Indian reservations. Currently, U.S. law recognizes many forms of coalitional property rights. In legal partnerships, for instance, legal resources are jointly owned, while partners have joint claims on the revenue generated from the practice.

There does exist by now a large literature on particular property rights regimes which differ from the private property assumptions of General Equilibium theory. The literature on public goods and club goods considers the resource allocation problem for goods that are nonexcludable or nonrivalrous either globally or locally.[1] While the problems of nonexcludability are certainly relevent to questions of ownership, they are conceptually different from many of the "legal" problems associated with communal ownership; after all, coalitions that own resources often can and do exclude outsiders. There are cases, for instance, in which land that is collectively owned by a village coalition lies adjascent to physically undifferentiated land held by individuals. It would be difficult to argue in this case that one owner's land exhibits nonexcludability characteristics while the other's does not.

We therefore seek to augment the existing literature in two ways. First, we offer a general model of *coalitional property rights (CPR) regimes* that encompass some of the property rights regimes that we have been observed historically. In our framework ownership rights

[1] A survey of this literature can be found in Cornes and Sandler (1986).

to various resources may be held by groups of various sizes rather than by either single individuals or the whole of society. Second, we consider the *social stability* of these general, coalitional property rights regimes. That is, we ask what kinds of property rights regimes give rise to outcomes against which no group in society would attempt to "recontract out;" i.e., give rise to outcomes that are in the *core* of the corresponding CPR regime. The purpose of this paper is to provide characterizations of CPR regimes that are socially stable in the sense of having a nonempty core.

A coalitional property rights (CPR) regime consists of a legal structure imposed on such an economy. This structure specifies, for each commodity, the coalitions of individuals that share a communal claim to that commodity. Specifically, for each commodity, there is a partition of the members of society. Each partition element defines a "legal" coalition that holds joint claims to that commodity. Private property is the special case in which the partition elements are all singletons. However, unlike the specification of typical private property economies, the more general specification allows us to distinguish between the property rights structure and the wealth endowments. For instance, a married couple may be regarded as a legal coalition that holds joint claims to each of their labor incomes even if one or both are currently unemployed.

For simplicity, we consider a standard exchange economy. We do not consider production here. We also completely abstract from public goods aspects of commodities. In this paper we focus on the legal structure rather than on the technological possibilities for exclusion and rivalry. This approach is analogous to that part of the public economics literature (see, for example, Arrow (1971)) which assumes that certain private goods are publically provided and studies how public provision of these goods determines resource allocation. For this reason our paper is similar in spirit, but generally different from the literature on the core and equilibria of economies with local public goods and club goods such as in, say, Greenberg (1983), Scotchmer (1985), and Wooders (1978).

Finally, we do not consider the emergence of joint ownership based on efficiency or other social desiderata such as in Grossman and Hart (1986) or Hart and Moore (1990). We consider a given legal structure and ask whether that legal structure gives rise to socially stable outcomes.

An Example To motivate our model, consider this example. There are three individuals $\{1, 2, 3\}$, that collectively own a single good. All agents' utilities are strictly increasing in the good. If collective decisions are made via simple majority rule then the winning subcoalitions are $\{\{1, 2\},\{1, 3\},\{2, 3\},\{1, 2, 3\}\}$, and any division of the good is unstable since there is a voting cycle among the majority coalitions. No allocation can survive this potential "recontracting out" process and so the CPR regime is socially unstable.

Now suppose that that there are now four commodities, indexed by the set $\{e, f, g, h\}$ and the economy is described by matrix in Table 1.

commodities	coalitions/endowments		
e	{1}	{2}	{3}
	3	0	0
f	{1}	{2}	{3}
	0	3	0
g	{1}	{2}	{3}
	0	0	3
h		{1,2,3}	
		3	

Table 1

In Table 1, the three individuals collectively own commodity h of which they initially hold 3 units. There are private property rights defined on the other three commodities. Person 2, for instance, individually owns 3 units of commodity f while person 1 and 3 own none of that commodity. Each person owns some of one good that no one else has. In this sense, individual holdings are "specialized." The voting rule for $\{1, 2, 3\}$ is majority rule, and we assume that the three individuals have identical, concave, and increasing CES utility functions over the four goods.

In this example, as in the other, there exists a voting cycle in the fourth commodity, h, since any majority can expropriate the entire endowment. However, when one considers each person's outside holdings, there may be allocations that are stable. For example, consider the allocation in which all individuals divide all goods equally. Then it can be shown that if each agent's elasticity of substitution between the four goods is sufficiently low, then this allocation lies in the core of the economy. No coalition can benefit by "recontracting out" against the status quo, because each individual has something that is desired sufficiently by the other two. Hence, this regime is socially stable.

On the other hand, if each agents's elasticity of substitution is high enough so that the four goods are close substitutes, then all agents will simply consume their private endowment and majority voting over the fourth good will produce cycles as in the previous example.

To see what is crucial for stability in the example above we modify the economy so that it is now described by the second matrix in Table 2.

commodities	**coalitions/endowments**		
e	{1}	{2}	{3}
	1	1	1
f	{1}	{2}	{3}
	1	1	1
g	{1}	{2}	{3}
	1	1	1
h		{1,2,3}	
		1	

Table 2

Here, individual agents will simply consume their private endowments since trade in the privately owned goods is not mutually advantageous. Individual holdings in this example are not specialized. Hence, majority voting over the fourth good results in a voting cycle and in instability.

In section 2 a basic model of CPR regimes under unanimity is defined. The model in section 2 is not the most general we have in mind, but it serves as the basic foundation for the subsequent generalization in section 3. In section 2 we look at the core of the CPR regime for the special case in which each coalition is governed by unanimity rule. Though this is restrictive, it allows us to compare the core of this regime to the standard case of private property (which, implicitly, also assumes intracoalitional unanimity in the definition of the core). We show that core allocations of a CPR regime under the unanimity requirement contains the core allocations of a private property economy derived by dividing up the communal endowments among members of each legal coalition. However, the inclusion may be strict. Some CPR core allocations cannot be attained by this natural "privatizion."

Hence, there is a fundamental difference between coalitional stability in the CPR economy and in the newly"privatized" cores. Under unanimity, each individual's ability to veto an alternative allocation gives an added measure of indeterminacy absent in private property regimes.

In section 3 we drop the unanimity assumption and consider different intra-coalitional social choice procedures such as majority voting. We call such regimes *exclusionary* coalitional property rights regimes since strict subsets of the legal coalitions may expropriate up to the entire coalitional endowment from the rest of the coalition.

Section 3.2 evaluates the core-stability consequences of such regimes. Again the CPR core is shown to contain the cores of all economies in which endowments are partially privatized only in those sectors in which individuals have veto power. Unlike the unanimity case, nonemptiness of the core is not automatic. We provide an existence result for *purely* exclusionary CPR regimes that satisfy two conditions. First, individuals hold goods that are not available elsewhere in the economy. In this sense, individuals' holdings are specialized. Second, the utility function of each individual must exhibit sufficient curvature so that other agents' specialized holdings are valued by that individual. Section 4 contains concluding remarks.

2 CPR Regimes and the Unanimity Rule

2.1 The Basic Model

The primitives of the model are as follows:

Agents: $I = \{1, ..., n\}$, is a set of agents that constitute a society. Ths set I has generic element $i \in I$.

Commodities: $K = \{1, ..., m\}$ is a set of commodities. The set K has generic element $k \in K$.

Preferences: Each agent $i \in I$ has a utility function, $u^i : \Re^m_+ \to \Re$ which is assumed strictly concave and strictly increasing in each good. We will assume the normalization $u^i(0, \ldots, 0) = 0$ for all i,

Aggregate Endowments: The total endowment of all commodities is given by the vector $\omega = (\omega_k)_{k \in K}$ where $\omega_k > 0$ for all $k \in K$. We will refer to an *allocation* as a vector $x = (x^i_k)_{k \in K, i \in I}$ that satisfies $\sum_{i \in I} x^i_k \leq \omega_k$ for all k.

Property Rights Structure: For each commodity $k \in K$, let $\mathcal{P}(k)$ denote a partition of I. Each element $p_k \in \mathcal{P}(k)$ is a *legal coalition*. That is, each partition $\mathcal{P}(k)$ defines the coalitions, each of which describes a group of individuals who, together, have joint rights to commodity k. Our notion of coalitional property rights means that these rights cannot necessarily be partitioned into separate claims given to each agent in the communal coalition. This inseparability is a legal notion rather than a technological one as with public goods. Also we allow for there to be different rights structures for different commodities, i.e., it is possible that $\mathcal{P}(k) \neq \mathcal{P}(\hat{k})$ if $k \neq \hat{k}$. In the special case of private property in the Arrow-Debreu world, the $\mathcal{P}(k)$ partitions consist of the singleton sets in I.

Ownership Structure: To complete our specification requires an assignment of physical property to the communal coalitions. We define the functions $w_k : \mathcal{P}(k) \to \Re_+$, for all $k \in K$, which satisfy

$$\sum_{p_k \in \mathcal{P}(k)} w_k(p_k) = \omega_k.$$

The amount $w_k(p_k)$ denotes the endowment of commodity k held collectively by coalition $p_k \in \mathcal{P}(k)$.

We may now define a *Coalitional Property Rights (CPR) Regime* to be a specification of agents' utilities $u \equiv (u^i)_{i \in I}$, total resources, $\omega \equiv (\omega_k)_{k \in K}$, property rights, $\mathcal{P} \equiv (\mathcal{P}(k))_{k \in K}$, and communal endowments, $w \equiv (w_k(\cdot))_{k \in K}$. That is, a CPR regime is given by the tuple,

$$\mathcal{E} \equiv (u, \omega, \mathcal{P}, w).$$

Observe that in this specification, property rights in a regime $\mathcal{E}$ are tied to the index of the commodity, independent of the actual quantity owned. Individuals are bundled together to legally share rights to consumption. Two agents, for example, have joint rights to arable land; as opposed to: these two jointly own a particular piece of land. For this reason, the definition of regime $\mathcal{E}$ is far from a satisfactory notion since it corresponds to a relatively small number of possibilities given the plethora of observed communal regimes.[2] We augment this definition in Section 3 to cover many more possibilities.

For now, we will consider the simple internal rule of unanimity used by the legal coalitions p_k to allocate internal holdings. Furthermore, whatever allocations result from a CPR regime $\mathcal{E}$, it seems reasonable that these allocations should not offer recontracting opportunities for improvements by groups of agents. We define a core concept under the unanimity rule for this economy that is analogous to the standard concept in private property regimes.

Definition 1 *Given a CPR regime, $\mathcal{E}$, an allocation x is in the Core of the CPR Regime under Unanimity Rule, denoted $\mathcal{C}(\mathcal{E})$, if there is no coalition, $C \subseteq I$, and no coalitional allocation, $(y^i)_{i \in C}$, such that*

$$\sum_{i \in C} y_k^i \leq \sum_{p_k \subseteq C} w_k(p_k), \forall k \in K, \tag{1}$$

$$u^i(y^i) > u^i(x^i), \ \forall i \in C. \tag{2}$$

Inequality (2) is the standard "blocking" condition; the members of C cannot be made better off by recontracting out of the status quo. Inequality (1) specifies what is feasible if C attempts to recontract. The structure of a CPR regime puts natural restrictions on the possibilities for coalitional recontracting. In particular, coalition C can only use those endowments to which its members have a legal claim. This legal claim is given by the restriction in (1) that members of C may claim $w_k(p_k)$ if and only if $p_k \subseteq C$. Agents in C whose entire legal coalition p_k is not contained in C must essentially relinquish their rights within p_k. This feature in which all members of p_k must consent in order to claim $w_k(p_k)$ is entirely a property of the unanimity assumption.

[2]However, this notion seems to correspond to the description of the feudal system in Europe by Townsend (1984) as a collection of "Arrow-Debreu social contracts."

2.2 Social Stability under Unanimity

To illustrate the effects of Unanimity Rule on the CPR core, consider the following example. There are three agents, $I = \{1, 2, 3\}$, and two goods y and z. Agents' preferences are given by $u^i(y^i, z^i) = (y^i)^{1/2} + (z^i)^{1/2},\ i \in I$. The property rights arrangement is given by the partitions

$$\mathcal{P}(y) = \{\{1, 2\}, \{3\}\},$$
$$\mathcal{P}(z) = \{\{1\}, \{2, 3\}\}.$$

Actual endowments for the legal coalitions in this property rights structure is given by $w_y(\{1, 2\}) = 4,\ w_y(\{3\}) = 0,\ w_z(\{1\}) = 0,\ w_z(\{2, 3\}) = 4$. This structure is summarized in the matrix below:

commodities	**coalitions/endowments**	
y	{1,2}	{3}
	4	0
z	{1}	{2,3}
	0	4

Table 3

Observe that agent 2 belongs to the legal coalitions that own all the resources. Yet, there is a range of possible core allocations that may deny agent 2 some or all of it. For instance, it is easy to show that the allocation $(y^2, z^2) = (0, 0)$, and $(y^i, z^i) = (2, 2),\ i = 1, 3$, lies in the core. This owes to the fact that the unanimity assumption gives each person (in this case agent 1 and agent 3) the legal right within his coalition to veto an alternative allocation. This is, no doubt, a restrictive requirement. There are, however, historical examples in which unanimity is required to allocate communal shares of resources such as land. Peasant communes in 19th century Russia had such a requirement (see Shanin (1972)). In England during the Tudor years, enclosures occurred under unanimity rules to an extent.

A different sort of enclosure occured in 17th century England. MacFarlane (1978) describes the historical process by which communal endowments at the time were "carved up" and allocated to the members. In this model, these enclosures correspond to: for each legal coalition, p_k, the existing coalitional endowment, $w_k(p_k)$, is divided up among the agents in p_k. Let Λ denote the set of such divisions defined as the set of $\lambda \in \Re^{mn}_{+}$ that satisfies

$$\sum_{i \in p_k} \lambda^i_k = 1,\ \forall p_k, \forall k.$$

Let $\mathcal{E}^\lambda$ denote the standard exchange economy created by privatizing the CPR economy $\mathcal{E}$ using λ. That is, each agent $i \in p_k$ has a privatized endowment, $\lambda^i_k w(p_k)$ of good k since each legal coalition divides up its resources so that each individual receives a private share. Let $\mathcal{C}(\mathcal{E}^\lambda)$ denote the standard core of this private property economy. Standard results show that the core of the private property economy is nonempty when agents' utilities are quasi-concave.

A general comparison can be made between core allocations under CPR regimes with unanimity and core allocations under private property. Under Unanimity Rule it turns out that any core allocation of a private property economy derived from privatizing the communal endowments of the legal coalitions corresponds to a coalitional property rights core allocation. However, interestingly, not all CPR core allocations can result from a privatized economy despite the fact that there is no transfer of wealth across the legal coalitions, p_k.[3]

Proposition 1 *For any CPR Regime $\mathcal{E}$, $\mathcal{C}(\mathcal{E}) \neq \emptyset$. In particular,*

$$\bigcup_{\lambda \in \Lambda} \mathcal{C}(\mathcal{E}^\lambda) \subseteq \mathcal{C}(\mathcal{E})$$

and for some economy $\mathcal{E}$, the inclusion is strict.

Proof of Proposition 1
Establishing the inclusion is straightforward. We show that $\mathcal{C}(\mathcal{E}) \supseteq \bigcup_{\lambda \in \Lambda} \mathcal{C}(\mathcal{E}^\lambda)$. Given $\mathcal{E}$, suppose $x \notin \mathcal{C}(\mathcal{E})$. Let $C \subseteq I$ and $(y^i)_{i \in C}$ satisfy the blocking conditions (1) and (2). Then for any $\lambda \in \Lambda$, and $k \in K$,

$$\sum_{i \in C} y^i_k \leq \sum_{p_k \subseteq C} w_k(p_k) = \sum_{p_k \subseteq C} \sum_{i \in p_k} \lambda^i_k w_k(p_k) \leq \sum_{i \in C} \lambda^i_k w_k(p_k(i)), \tag{3}$$

where $p_k(i)$ denotes the coalition in $\mathcal{P}(k)$ that contains i. From (3), $x \notin \mathcal{C}(\mathcal{E}^\lambda)$.

To show that there are CPR economies for which the set inclusion is strict consider the following simple example. There are two goods, y and z and three individuals, $i = 1, 2, 3$. The property rights structure is given in Table 4 below.

commodities	**coalitions/endowments**		
y	{1,2}		{3}
	3		0
z	{1}	{2}	{3}
	0	0	3

Table 4

[3]We thank the referee for pointing out the strict inclusion.

Preferences of all individuals are given by

$$u^i(y^i, z^i) = (y^i)^{.6} + (z^i)^{.6}, \ \forall i = 1, 2, 3.$$

We first establish that the allocation x with $x^i = (y^i, z^i) = (1, 1), \ i = 1, 2, 3$, yielding $u^i(1, 1) = 2$, lies in the CPR Core, $\mathcal{C}(\mathcal{E})$. Clearly, by the symmetry of preferences this allocation is Pareto optimal, and so it is not blocked by $\{1, 2, 3\}$. Also, the singleton coalitions, $\{i\}, \ i = 1, 2$ cannot block since neither 1 nor 2 has any quantity of endowment by himself. Individual 3 can claim $(0, 3)$, i.e., 3 units of z. Since $3^{.6} > 2$, he does not block alone. Finally, observe that the two-member coalitions can claim no more than $(0, 3)$ which is what individual 3 can claim by himself. Hence, the two member coalitions will not block. Hence, $x \in \mathcal{C}(\mathcal{E})$.

We now show that $x \notin \bigcup_{\lambda \in \Lambda} \mathcal{C}(\mathcal{E}^\lambda)$. Fix an arbitrary λ. Since there is only a communal endowment in good y, it must be the case that either $\lambda_y^1 \geq 1/2$ or $\lambda_y^2 \geq 1/2$. Without loss of generality suppose the former holds. Then coalition $\{1, 3\}$ has the rights to aggregate endowment of at least $(1.5, 3)$. If they block with allocation $(.75, 1.5)$ to each of individuals 1 and 3, then since $u^i(.75, 1.5) = (.75)^{.6} + (1.5)^{.6} \approx 2.11 > 2$, this coalition blocks successfully. That is, $x \notin \mathcal{C}(\mathcal{E}^\lambda)$. However, since the choice of individual 1 or 2 with share $\lambda_y^i \geq 1/2$ was arbitrary, then $x \notin \mathcal{C}(\mathcal{E}^\lambda), \ \forall \lambda \in \Lambda$. □□

Remark Since the set-theoretic inclusion in Proposition 1 is strict, it does matter for some economies whether resources are owned collectively or privately. The example of Table 4 in the Proof is similar to the classic example in which two buyers (individuals 1 and 2) compete for a single seller (individual 3). In the core of this prototype example the buyers compete away their endowments, hence, the core comprises the allocation in which the seller gets everything. In the present example in Table 4, since each of the two "buyers" can still consume his privatized endowment if there is no exchange, the privatized core is empty (with sufficient curvature in the utility function). By contrast, in the example in Table 3 the inclusion may be replaced with equality. That is, the CPR core of that economy is precisely the union of all privatized cores. The crucial difference seems to be that in the Table 3 economy, what each blocking coalition can obtain in all possible privatized economies is not much different than in the CPR regime. However, in the Table 4 economy the coalitional endowment in the CPR regime is distinctly inferior to that of the privatized economies for at least *some* potential blocking coalition. Yet, it is an open question for exactly which economies privatization makes a difference in terms of coalitional stability.

Proposition 1 makes clear that unanimity rules allow for a large number of core allocations. This should not be too surprising since unanimity makes it easy for individuals to veto proposed alternatives. Status quo allocations will often prevail.

3 Exclusionary CPR Regimes

3.1 The Model

In this section we consider more general intra-coalitional social choice procedures. Procedures other than unanimity seem to have been, historically, fairly common. Dahlman (1980) reports, for example, that in villages in England during the late 18th century, "in order to reach a decision to enclose a village, four-fifths and later three-quarters of the votes in a village were required. This was not in numbers, but reckoned in value of arable land." (p.25)

We refer to the more general intra-coalitional social choice procedures as *exclusionary rules* since they specify which subcoalitions can form and, therefore, which agents are excluded from these subcoalitions. It will be shown that under exclusionary rules, the issue of social stability is no longer as straightforward as in Section 2.

To formalize exclusionary rules, we specify for each communal coalition p_k a collection $\mathcal{D}(p_k)$ of subsets $d \subseteq p_k$. Here we interpret $\mathcal{D}(p_k)$ as the collection of "winning" subcoalitions that are eligible to block with good k by voting to leave communal coalition p_k. Therefore, subcoalition d can block with endowment $w_k(p_k)$ iff $d \in \mathcal{D}(p_k)$, i.e., d is a "winning" subcoalition. We assume that $\mathcal{D}(p_k)$ satisfies:

(i) if $d \in \mathcal{D}(p_k)$ and $d \subseteq d' \subseteq p_k$, then $d' \in \mathcal{D}(p_k)$.

(ii) $d \in \mathcal{D}(p_k)$ iff $p_k \backslash d \notin \mathcal{D}(p_k)$

The pair $(p_k, \mathcal{D}(p_k))$ is sometimes called a *simple game*, or a *committee*.[4] Properties (i) and (ii) above are standard assumptions for simple games. Assumption (i) states that supersets of winning subcoalitions in p_k are winning, while (ii) states that relative complements of winning subcoalitions are not winning (and vice versa). One obvious example of the pure exclusion is the case where $\mathcal{D}(p_k)$ defines a Majority Rule. Suppose that $p_k = \{1, 2, 3\}$, then $\mathcal{D}(p_k) = \{\{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$ defines the majority subcoalitions that can expropriate the entire coalitional endowment. Utilizing these assumptions, we define the amount, $\mu(B)$, of ω that eligible subcoalitions can obtain for B if B recontracts out of the status quo by

$$\mu_k(B) = \sum_{\{p_k | \exists d \in \mathcal{D}(p_k), d \subseteq B\}} w_k(p_k), \ \forall k \in K \tag{4}$$

Let the set of all exclusion rule be given by $\mathcal{D} = (\mathcal{D}(p_k))_{p_k \in \mathcal{P}(k),\, k \in K}$. An exclusionary CPR regime will be denoted by $(\mathcal{E}, \mathcal{D})$.

Definition 2 Given $(\mathcal{E}, \mathcal{D})$, an allocation x is in the *Core of the CPR Regime with Exclusion*, denoted $\mathcal{C}(\mathcal{E}, \mathcal{D})$, if there is no $C \subseteq I$ and no allocation $(y^i)_{i \in C}$ such that

[4] See Peleg (1984) for a general treatment of the theory of simple games.

$$\sum_{i \in C} y^i \leq \mu(C) \tag{5}$$

$$u^i(y^i) > u^i(x^i), \forall i \in C. \tag{6}$$

Inequality (5) specifies what is feasible if C attempts to recontract under exclusion. Each coalition C can only use those endowments to which some eligible subcoalition has a legal claim. This legal claim is given by the restriction in (5) that members of C may claim $w_k(p_k)$ if and only if there is an eligible subcoalition d in a legal coalition p_k such that $d \subseteq C$. In this case, agents in C who belong to no eligible subcoalition that is entirely contained in C must essentially relinquish their rights within p_k.

3.2 Social Stability under Exclusionary CPR

With this notion of the CPR Core, the eligible or "winning" subcoalitions have the right to expropriate the entire coalitional endowment. The voting mechanisms are therefore perfectly exclusionary since they deny all remaining individuals the right to veto an alternative allocation. Conditions that give rise to a nonempty core may be characterized by the "degree" to which individuals can be denied veto rights within their legal coalitions.

Define a *veto player* of coalition p_k to be an agent in p_k that belongs to every winning coalition $d \in \mathcal{D}(p_k)$. Let $V(p_k)$ denote the set of veto players of p_k. Note that $V(p_k)$ might be an empty set. In fact, typical voting rules imply that the set of veto players is empty. For example, the 50% majority voting rule in the set $\{1, 2, 3\}$ yields the winning coalitions $\{\{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$. No single agent is included in all of these sets. By contrast, for this same set of players, if the collection of winning subcoalitions is $\{\{1\}, \{1, 2\}, \{1, 2, 3\}\}$, then agent 1 is a veto player. In the case of unanimity modelled in the previous Section, $V(p_k) = p_k$.

Denote the aggregate "veto holdings" of a coalition C by

$$v_k(C) \equiv \sum_{\{p_k | \, V(p_k) \neq \emptyset, \, V(p_k) \subseteq C\}} w_k(p_k), \; k \in K. \tag{7}$$

The existence of veto players somewhere in the economy turns out to be a crucial, necessary condition for social stability. To see this, we first consider a partial privatization in which, unlike the unanimity case, only the endowments for which there are veto players are divided up. Let Θ denote the set of $\theta \in \Re^{mn}_{+}$ that satisfies

$$\sum_{i \in V(p_k)} \theta^i_k = 1, \; \forall p_k \in \mathcal{P}(k), \; \forall k \in K,$$

where the values θ^i_k that apply to coalitions with no veto players are assumed to have unrestricted range in $\Re^{mn}_{+}$. Let $\mathcal{E}^\theta$ denote the "partially privatized" pure exclusion regime in which, for each p_k that has a nonempty set of veto players, each veto player receives a private endowment of $\theta_k^i w_k(p_k)$. The coalitions with no veto players remain "nonprivatized" in $\mathcal{E}^\theta$. The union of cores of all such partially privatized regimes constitutes a subset of the core of the original CPR regime. That is:

Proposition 2 *For any CPR regime with pure exclusion rules, $(\mathcal{E}, \mathcal{D})$,*

$$\bigcup_{\theta \in \Theta} \mathcal{C}(\mathcal{E}^\theta) \subset \mathcal{C}(\mathcal{E}, \mathcal{D}).$$

Proof of Proposition 2:
The proof of mirrors the proof of Proposition 1. Fix θ and suppose that $x \in \mathcal{C}(\mathcal{E}^\theta)$. Suppose also that $x \notin \mathcal{C}(\mathcal{E}, \mathcal{D})$. Let coalition C block x with allocation $(y^i)_{i \in C}$. Then for each $k \in K$, we must have:

$$\begin{aligned} &\textstyle\sum_{i \in C} y_k^i \;\leq\; \mu_k(C) \;\equiv\; \sum_{\{p_k | \exists d \in \mathcal{D}(p_k), d \subseteq B\}} w_k(p_k) \\ &\leq\; \left[\textstyle\sum_{\{p_k |\, V(p_k) = \emptyset,\, \exists d \in \mathcal{D}(p_k),\, d \subseteq B\}} w_k(p_k) \;+\; \sum_{\{p_k |\, V(p_k) \neq \emptyset\}} \sum_{i \in C \cap V(p_k)} \theta_k^i w_k(p_k)\right] \end{aligned}$$

However, this means that $(y^i)_{i \in C}$ is feasible for C in the partially privatized economy $\mathcal{E}^\theta$ which gives the contradiction. □ □

Clearly, the core of a CPR regime under pure exclusion rules is characterized in a manner similar to the regimes under unanimity. However, since all the voting cycles take place in the nonprivatized sector, there is no guarantee, generally, that the core is nonempty.

Proposition 3 *For any $(\mathcal{E}, \mathcal{D})$ such that every coalition p_k has a nonempty set of veto players, then $\mathcal{C}(\mathcal{E}, \mathcal{D}) \neq \emptyset$. If there is no coalition p_k that contains a veto player, then $\mathcal{C}(\mathcal{E}, \mathcal{D}) = \emptyset$.*

Proof of Proposition 3:
If, for each p_k, $V(p_k) \neq \emptyset$, then for any C, $\mu_k(C) = v_k(C)$ since the endowment claimed by C from legal coalitions in which there are no veto players must be zero. Therefore, $v(I) = \mu(I) = \omega$. Now fix $\theta \in \Theta$ such that $\theta_k^i = 0$ whenever $i \notin V(p_k(i))$ (recall that $p_k(i)$ denotes the communal coalition which contains i). Since $V(p_k) \neq \emptyset$ for all $p_k \in \mathcal{P}(k)$, $k \in K$, $\sum_{i \in p_k} \theta_k^i w_k(p_k) = w_k(p_k)$ by construction. Therefore, $\theta \in \Lambda$ and so the economy $\mathcal{E}^\theta$ is a fully privatized, standard exchange economy. Clearly, then, $\mathcal{C}(\mathcal{E}^\theta) \neq \emptyset$. By Proposition 2, $\mathcal{C}(\mathcal{E}, \mathcal{D}) \neq \emptyset$.

As for the second part of the proof, suppose that there are no veto players in any of the p_k coalitions. Suppose that $\mathcal{C}(\mathcal{E}, \mathcal{D}) \neq \emptyset$ and let $x \in \mathcal{C}(\mathcal{E}, \mathcal{D})$. Fix some agent i for which $x^i \neq 0$. Now consider a potential blocking allocation $(y^j)_{j \neq i}$ for coalition $I \backslash \{i\}$. This allocation is constructed as follows:

$$y^j = x^j + \frac{x^i}{n-1}$$

That is, i's allocation is divided up amongst all the remaining agents. By supposition, i is not a veto player in any coalition p_k that contains i. This means that in each such p_k containing i, there is a $d \in \mathcal{D}(p_k)$ with $i \notin d$. Then, $\sum_{j \in I \backslash \{i\}} y_j \leq \mu(I \backslash \{i\})$, and so allocation $(y^j)_{j \neq i}$ is feasible for $I \backslash \{i\}$. Moreover, $u^j(y^j) > u^j(x^j)$ by strict monotonicity of u^j in each good. Hence, $x \notin \mathcal{C}(\mathcal{E}, \mathcal{D})$. □ □

Many if not most of the historical regimes are somewhere between the two extremes spelled out in Proposition 3. Some regimes that we have observed historically had at least a few commodities for which exclusion rules do not allow for veto players, e.g., simple majority rule. Nevertheless, many of these regimes persisted for centuries. Despite the existence of voting cycles in the absence of veto players for *some* commodities, stability is possible when individuals' "external veto power" is considered. As a special case of this veto power, we define an agent i to *uniquely specialized* in good k if $V(p_k) = \{i\}$ for p_k in which $i \in p_k$, and $V(p_k) = \emptyset$ for all p_k for which $i \notin p_k$. Let $\pi(i)$ denote the set of goods in which i is uniquely specialized. Note that possibly $\pi(i) = \emptyset$. We will show that if unique specialization holds for all agents then it is possible to achieve social stability.

Before proceeding with the result, it is first necessary to define a curvature property on utility functions which guarantees that the veto holdings of any coalition is of sufficient value (in utility terms) to agents outside that coalition. To begin, we require the following standard definitions in core theory. The collection $\mathcal{B}$ of subsets of I is *balanced* if there is a tuple of "balancing weights" $(\delta_B)_{B \in \mathcal{B}}$ where $\delta_B \geq 0$ satisfying for each agent i,

$$\sum_{\{B \in \mathcal{B} | i \in B\}} \delta_B \ = \ 1.$$

Definition 3 We will say that utility function u^i for agent i satisfies the *Curvature Property for economy* $\mathcal{E}$ if for any feasible consumption bundle $x^i \in \Re^m_+$ with $x^i \leq \omega$, and any balanced collection $\mathcal{B}$ with no disjoint subsets and balancing weights $(\delta_B)_{B \in \mathcal{B}}$, there exists a positive number $\bar{z} > 0$, such that:
for all tuples $(z^i(B))_{B \in \mathcal{B}} \in \Re^{m|\mathcal{B}|}_+$ such that for each $B \in \mathcal{B}$, $z^i_k(B) > \bar{z}$ whenever both $i \in B$ and $k \in \cup_{j \in B} \pi(j)$, and $z^i_k(B) = 0$ otherwise, then

$$\sum_{\{B \in \mathcal{B} | i \in B\}} \delta_B u^i(x^i + z^i(B)) \ < \ u^i(\sum_{\{B \in \mathcal{B} | i \in B\}} \delta_B z^i(B)) \tag{8}$$

The Curvature Property is a joint restriction on utility functions and an "initial" feasible consumption vector. It states, loosely, that the curvature of each agent's utility function is sufficient that certain sufficiently large transformations $(z^i(B))_{B \in \mathcal{B}}$ to agent i's balanced consumption over coalitions to which he belongs, overwhelms the effect of any "starting point" x. To see what this means, observe that if $x^i = 0$ then (8) is the familiar Jensen's Inequality for strictly concave functions, and so strict concavity of u^i would then imply that (8) is trivially satisfied. Therefore, (8) implies that the effect of the fixed vector x is washed out by a large enough increase in the specialized holdings of coalitions that contain

i. Note that the more restrictive the requirements on the tuple $(z^i(B))$, or the smaller is the initial feasible point x^i, the easier is it for a given utility function to satisfy (8). In Section 3.2.1 which follows Proposition 4 below, we give an example which applies the Curvature Property. We further show how the logic of the result works when the Curvature Property and Specialization are assumed in tandem.

Proposition 4 *Suppose that $(\mathcal{E}, \mathcal{D})$ denotes an exclusionary CPR regime which satisfies (1) each agent is uniquely specialized in exactly one good, and (2) each agent's utility function satisfies the Curvature Property. Then there is a $\bar{z} > 0$ such that if $v_k(I) \geq \bar{z}$ for each $k \in \cup_{i \in I} \pi(i)$, then the core of the CPR regime is nonempty (i.e., $\mathcal{C}(\mathcal{E}, \mathcal{D}) \neq \emptyset$).*

The rough idea of Proposition 4 is that, though an individual may be legally excluded from the holdings of a legal coalition if he is not a veto player in that coalition, if his marginal contribution of other veto holdings is sufficiently valuable to the other members of the coalition, they will *choose* not to exclude him. Hence, CPR regimes in which all agents have a sufficiently large, specialized veto holding of some commodity are socially stable. The specific requirements for the result may be described as follows. It must be shown that each individual has sufficient veto power so as not to be excluded from any blocking coalition. To satisfy this condition, it suffices to consider two properties. First, each individual holds larger and larger amounts of some good that is not available elsewhere in the economy.[5] Second, each individual's utility must display enough curvature so that another person's specialized holdings are valued by this individual.

The proof will show that the cooperative game induced from the associated CPR regime is *balanced* in the sense of Billera (1974).[6] Billera shows that for superadditive NTU (nontransferable utility) games, a balancedness condition is sufficient for the existence of a nonempty core. We state this result before proceeding with the proof of the Proposition.

Let $\mathcal{U} : 2^I \rightarrow \Re^n$ denote a superadditive function that assigns to each subset $B \subseteq I$ a set of utility vectors in $\Re^n$. Intuitively, $\mathcal{U}(B)$ is the set of payoffs that are feasible to B if it recontracts out of the status quo payoff. To every standard exchange economy there corresponds an NTU game derived by taking $\mathcal{U}(B)$ to be the utility payoffs of the total endowment held by B.[7] The NTU game $\mathcal{U}$ corresponding to the CPR regime with pure exclusion, $(\mathcal{E}, \mathcal{D})$, is defined by

$$\mathcal{U}(B) \equiv \Big\{\bar{u} \in \Re^n \Big| \exists (x^i)_{i \in B} \in \Re_+^{m|B|} \text{ s.t. } \textstyle\sum_{i \in B} x^i \leq \mu(B), \text{ and } \bar{u}^i \leq u^i(x^i), \forall i \in B, \text{ and } \bar{u}^i \leq 0, \forall i \notin B\Big\}. \tag{9}$$

[5]Admittedly, assuming that each agent is uniquely specialized in a *single* good is restrictive. However, we felt that this restriction makes the intuition garnered from the examples most lucid. It can weakend by assuming that each agent is specialized in *at least one* good. For a generalization, see Lagunoff (1994).

[6]This paper uses the definition of Shapley and Shubik (1969) and applied by Billera to NTU games rather than the generally weaker, ordinal definition of Scarf (1967).

[7]Since we assume the normalization $u^i(0, \ldots, 0) = 0$ for all i, we asume the characteristic function satisfies $\mathcal{U}(B) \subset \Re^n$ and for any $\bar{u} \in \mathcal{U}(B)$, $\bar{u}^i \leq 0$ if $i \notin B$.

Core Existence Theorem: *The core of any superadditive NTU game* $\mathcal{U}$ *is nonempty if for any balanced collection* $\mathcal{B}$ *with balancing weights* $(\lambda_B)_{B\in\mathcal{B}}$,

$$\sum_{B\in\mathcal{B}} \lambda_B \mathcal{U}(B) \subseteq \mathcal{U}(I) \tag{10}$$

3.2.1 The Logic of Proposition 4: An Example

Before proceeding with the proof, we give an example that demontrates the intuition of the argument.

From a general version of the economy in Table 1 we construct an example which satisfies the dual requirements of the Curvature Condition and unique specialization. Consider the economy in Table 5 below.

commodities	**coalitions/endowments**		
e	{1}	{2}	{3}
	α	0	0
f	{1}	{2}	{3}
	0	α	0
g	{1}	{2}	{3}
	0	0	α
h		{1,2,3}	
		3	

Table 5

Here, $\omega = (\alpha, \alpha, \alpha, 3)$ and good h is allocated via majority rule. Clearly, each agent is uniquely specialized in one good. No individual is specialized in good h. The only relevent balanced collection is the collection $\mathcal{B} = \{\{1,2\},\{1,3\},\{2,3\}\}$ with weights 1/2 for each coalition. The aggregate resources in each balanced coalition is given by

$$\begin{aligned} \mu(\{1,2\}) &= (\alpha, \alpha, 0, 3) \\ \mu(\{1,3\}) &= (\alpha, 0, \alpha, 3) \quad \text{and} \\ \mu(\{2,3\}) &= (0, \alpha, \alpha, 3) \end{aligned}$$

Suppose that each agent i receives fraction $\gamma_k^i(B)$ of good k if B forms and $i \in B$. Consider agent 1, for example. Then 1's allocation in coalitions $\{1,2\}$ and $\{1,3\}$ is

$$\begin{aligned} y^1(\{1,2\}) &= (\gamma_e^1(\{1,2\})\alpha,\ \gamma_f^1(\{1,2\})\alpha,\ 0,\ \gamma_h^1(\{1,2\})3) \quad \text{and} \\ y^1(\{1,3\}) &= (\gamma_e^1(\{1,3\})\alpha,\ 0,\ \gamma_g^1(\{1,3\})\alpha,\ \gamma_h^1(\{1,3\})3) \end{aligned}$$

To apply the Curvature Property to agent 1, observe that if we let $z_k^1(B) = y_k^1(B)$ when $k = e, f, g$, and $z_h^1(B) = 0$ then Inequality (8) becomes in this case,

$$\begin{aligned} &\tfrac{1}{2}u(\gamma_e^1(\{1,2\})\alpha, \gamma_f^1(\{1,2\})\alpha, 0, \gamma_h^1(\{1,2\})3) \ + \ \tfrac{1}{2}u^1(\gamma_e^1(\{1,3\})\alpha, 0, \gamma_g^1(\{1,3\})\alpha, \gamma_h^1(\{1,3\})3) \\ < \ & u^1(\tfrac{1}{2}(\gamma_e^1(\{1,2\}) + \gamma_e^1(\{1,3\}))\alpha, \tfrac{1}{2}\gamma_f^1(\{1,2\})\alpha, \tfrac{1}{2}\gamma_g^1(\{1,3\})\alpha, 0) \end{aligned} \tag{11}$$

We now verify that the certain preferences do indeed satisfy this inequality when α is sufficiently large. Let $u^i(x^i) = \sum_k (x_k^i)^{\frac{1}{2}},\ i = 1,2,3$. For these preferences the Inequality (11) becomes

$$\begin{aligned} 0 \ < \ & \alpha^{1/2}\left[(\tfrac{1}{2})^{1/2}(\gamma_e^1(\{1,2\}) + \gamma_e^1(\{1,3\}))^{1/2} - \tfrac{1}{2}((\gamma_e^1(\{1,2\}))^{1/2} + (\gamma_e^1(\{1,3\}))^{1/2})\right] \\ & + \alpha^{1/2}\left[(\tfrac{1}{2})^{1/2}(\gamma_f^1(\{1,2\}))^{1/2} - \tfrac{1}{2}(\gamma_f^1(\{1,2\}))^{1/2}\right] \\ & + \alpha^{1/2}\left[(\tfrac{1}{2})^{1/2}(\gamma_g^1(\{1,3\}))^{1/2} - \tfrac{1}{2}(\gamma_g^1(\{1,3\}))^{1/2}\right] \\ & - 3^{1/2}\left[\tfrac{1}{2}((\gamma_h^1(\{1,2\}))^{1/2} + (\gamma_g^1(\{1,3\}))^{1/2})\right] \end{aligned} \tag{12}$$

Observe that while all the terms inside square brackets on the right-hand side of the inequality in expression (12) are positive, the first three terms are unboundedly increasing for large enough α. The last term does not vary with α and is therefore fixed. Hence, (12) holds for large enough α, and so these preferences satisfy the Curvature Condition.

The Inequality (12) can now be used to verify (10) for the resulting NTU game. Observe that the agent 1's utility on the left side of Inequality (11) is the first vector component (corresponding to agent 1) in a utility vector $\hat{u} \in \sum_{B\in\mathcal{B}} \lambda_B \mathcal{U}(B)$. This utility vector is component-wise dominated by the utility vector $u \in \Re^3$ which, for each agent, corresponds to the right side of (11). This vector is given by:

$$\begin{aligned} &\Big(u^1(\tfrac{1}{2}(\gamma_e^1(\{1,2\}) + \gamma_e^1(\{1,3\}))\alpha, \tfrac{1}{2}\gamma_f^1(\{1,2\})\alpha, \tfrac{1}{2}\gamma_g^1(\{1,3\})\alpha, 0) \\ &u^2(\tfrac{1}{2}\gamma_e^2(\{1,2\})\alpha, \tfrac{1}{2}(\gamma_f^2(\{1,2\}) + \gamma_f^2(\{2,3\}))\alpha, \gamma_g^2(\{2,3\})\alpha, 0), \\ &u^3(\tfrac{1}{2}(\gamma_e^3(\{1,3\})\alpha, \tfrac{1}{2}\gamma_f^3(\{2,3\})\alpha, \tfrac{1}{2}(\gamma_g^3(\{1,3\}) + \gamma_g^3(\{2,3\}))\alpha, 0)\Big) \end{aligned} \tag{13}$$

Since this vector of utilities of lies in $\mathcal{U}(I)$, the vector $\hat{u}$ must lie in $\mathcal{U}(I)$ as well.

3.2.2 Proof of Proposition 4

Let $(\mathcal{E}, \mathcal{D})$ with $\mathcal{E} = (u, \omega, \mathcal{P}, w, \mathcal{D})$ satisfy the requisite properties. Since for each i, $\pi(i)$ is a singleton set, denote $k(i)$ as the good in which i is uniquely specialized. For simplicity, we relabel the commodities so that $k(i) = i$. Then, we will take $v_i(I)$ sufficiently large to achieve the desired result.

Observe first that $v_i(I\backslash\{i\}) = 0$ by the definition of unique specialization. In particular, we have that $v_i(I) = v_i(B)$ for any coalition B for which $i \in B$. Therefore, there is no confusion by writing v_i to denote i's veto holdings of good i (in any coalition).

Relabeling the agents if necessary, we can rewrite equation (7) so that for each coalition $B \subseteq I$,

$$v(B) = (\overbrace{v_1, v_2, \ldots, v_b}^{|B|}, \overbrace{0 \ldots, 0}^{m-|B|})$$

(recall that there are n agents and $m \geq n$ commodities). Observe that $v_k(B) = 0$ if $k \notin B$ since then $k \notin \cup_{i \in B}\pi(i)$.

It will prove useful to consider the NTU game in (9) as the explicit decomposition between $v(B)$ and the rest of coalition B's endowment. Therefore, we define for each coalition B,

$$\begin{aligned} \mathcal{U}(B) \quad \equiv \quad & \left\{\bar{u} \in \Re^n \middle| \; \exists(x^i)_{i \in B},\ \exists(z^i)_{i \in B}, \text{ where } \textstyle\sum_{i \in B} z^i \leq v(B) \text{ and } \sum_{i \in B} x^i \leq \mu(B) - v(B) \right. \\ & \left. \text{such that} \quad \bar{u}^i \leq u^i(x^i + z^i),\ \forall i \in B, \text{ and } \bar{u}^i \leq 0,\ \forall i \notin B \right\}. \end{aligned} \tag{14}$$

To clarify the definition in (14), coalition B's endowment includes the amounts from each member's uniquely specialized holdings as well as the residual obtained from other sources.

It may be shown that $\mathcal{U}$ is superadditive and each $\mathcal{U}(B)$ is closed and comprehensive (we omit the argument), and so, by the Core Existence Theorem, it suffices to show that for any balanced collection $\mathcal{B}$ with balancing weights (δ_B), if $v(I)$ is sufficiently large in all nonzero components then

$$\sum_{B \in \mathcal{B}} \delta_B \mathcal{U}(B) \subseteq \mathcal{U}(I). \tag{15}$$

Let $\bar{u} \in \sum_{B \in \mathcal{B}} \delta_B \mathcal{U}(B)$ for some balanced collection $\mathcal{B}$ with balancing weights (δ_B). Observe that each $\mathcal{U}(B)$ is closed and contains $-\Re^n_+$. Therefore, $\sum_{B \in \mathcal{B}} \delta_B \mathcal{U}(B)$ is closed.[8] Therefore, without loss of generality, we suppose that $\bar{u}$ lies on the boundary of $\sum_{B \in \mathcal{B}} \delta_B \mathcal{U}(B)$. Moreover, we can take each vector

$$\left((u^i(x^i(B) + z^i(B))_{i \in B}, \overbrace{0 \ldots, 0}^{n-|B|} \right)$$

[8]Note that while the sum of closed sets need not be closed generally, the sum $\sum_{B \in \mathcal{B}} \delta_B \mathcal{U}(B)$ is closed if the asymptotic cones of the sets $\mathcal{U}(B)$ intersect at points other than zero (see Debreu (1959), p. 23). Since all sets $\mathcal{U}(B)$ contain $-\Re^n_+$, their asymptotic cones also contain $-\Re^n_+$.

to be a boundary point of $\mathcal{U}(B)$.

We now establish that $\bar{u} \in \mathcal{U}(I)$ whenever $v(I)$ is sufficiently large in nonzero components. Observe that by the construction in (14) above we can write

$$\bar{u} = \sum_{B} \delta_B u(x(B) + z(B)) \tag{16}$$

where $x(B) = (x^i(B)) \in \Re^{nm}_+$ and $z(B) = (z^i(B)) \in \Re^{nm}_+$ for each $B \in \mathcal{B}$, and we adopt the convention that $x^j(B) = 0$ and $z^j(B) = 0$ if $j \notin B$. This construction also supposes that

$$\sum_{i \in B} z^i(B) = v(B) \quad \text{and} \quad \sum_{i \in B} x^i(B) = \mu(B) - v(B). \tag{17}$$

From equation (17), $x^i_k(B) = 0$ if $k \in B$, while $z^i_k(B) = 0$ if $k \notin B$. Also, $\sum_{i \in B} x^i_k(B) = \mu_k(B)$ if $k \notin B$. It follows that

$$\begin{aligned} \sum_{i \in I} \sum_{B \in \mathcal{B}} \delta_B z^i_k(B) &= \sum_{B \in \mathcal{B}} \delta_B \sum_{i \in I} z^i_k(B) = \sum_{B \in \mathcal{B}} \delta_B \sum_{i \in B} z^i_k(B) = \sum_{B \in \mathcal{B}} \delta_B v_k(B) \\ &= \sum_{\{B \in \mathcal{B}:\, k \in B\}} \delta_B v_k(B) = v_k \end{aligned} \tag{18}$$

which holds for all $k \leq n$ (recall that for $k > n$, $v_k(I) = 0$). For each pair $i, k \in B$ we can write $z^i_k(B) = \gamma^i_k(B) v_k$ where $1 \geq \gamma^i_k(B) \geq 0$. Then we must have $\sum_{i \in B} \gamma^i_k(B) = 1,\ k \in B$.

Case 1. $\gamma^i_k(B) > 0$ for all $B \in \mathcal{B}$ and all $i, k \in B$.

In this construction, since members of coalition B are uniquely specialized in goods $k \in B$, the allocation $(z^i(B))_{i \in B}$ gives to each agent $i \in B$ the strictly positive fraction $\gamma^i_k(B)$ of commodity k. Let $x^i = \sum_{B \in \mathcal{B}} \delta_B x^i(B)$. Clearly $x^i \leq \omega$ since $x^i(B) = 0$ whenever $i \notin B$. By strict concavity of utility function u^i,

$$\bar{u}^i \equiv \sum_{B} \delta_B u^i(x^i(B) + z^i(B)) \ < \ \sum_{B} \delta_B u^i(x^i + z^i(B)) \tag{19}$$

Now we are in a position to apply the conclusion (i.e., Inequality (8)) of the Curvature Condition. Define $\bar{z}^i$ as the infimum over all positive reals z which satisfy: for every B with $i \in B$, $v_k \geq z$ whenever $k \in B$, and

$$\sum_{B} \delta_B u^i(x^i + \gamma^i(B)\alpha) \equiv \sum_{B} \delta_B u^i(x^i + z^i(B)) < u^i(\sum_{B \in \mathcal{B}} \delta_B z^i(B)). \tag{20}$$

By the Curvature Condition, $\bar{z}^i < \infty$. Now define $\bar{z} = \max_{i \in I} \bar{z}^i$ and let $v_k \geq \bar{z}$ for all $k \in I \equiv \cup_{i \in I} \pi(i)$. Since $(u^i(\sum_{B \in \mathcal{B}} \delta_B z^i(B)))_{i \in I} \in \mathcal{U}(I)$ by equation (18), we conclude from (19) and (20) that $\bar{u} \in \mathcal{U}(I)$.

Case 2. $\gamma^i_k(B) = 0$ for at least one $B \in \mathcal{B}$ and one pair i, k with $i, k \leq b$.

This means that agent i holds no amount of good k in coalition B. From Case 1 we have established that any $\bar{u} \in \sum_{B \in \mathcal{B}} \delta_B \mathcal{U}(B)$ generated from strictly positive weights $(\gamma^i_k(B))$ lies in $\mathcal{U}(I)$. Observe then that for any $\hat{u} \in \sum_{B \in \mathcal{B}} \delta_B \mathcal{U}(B)$ generated from weights $(\gamma^i_k(B))$ with $\gamma^i_k(B) = 0$ for at least one B and one pair i, k must lie in the closure of $\mathcal{U}(I)$. However, $\mathcal{U}(I)$ is already closed, and so $\hat{u} \in \mathcal{U}(I)$. This concludes the proof. □□

3.3 On Partial Exclusion

A result similar to Proposition 4 may be shown for the general case of partial or "impure" exclusion. We provide only an illustrative example here.

In a setup more general than the exclusionary CPR environment, ownership is now tied to a fraction of the communal endowment for a subcoalition if it leaves the communal coalition. For example, two agents A and B jointly own land in Florida, while two agents B and C own some land in New Jersey. "Impure" exclusion allows perhaps A and B and/or B and C to expropriate either piece of land. The legal coalition, containing A, B, and C is then just defined to be the smallest coalition that contains all of these overlapping arrangements. An example is given in Table 6 below. It modifies the example of Table 1 so that the property rights structure is:

commodities	**coalitions/endowments**		
e	{1}	{2}	{3}
	1	α	α
f	{1}	{2}	{3}
	α	1	α
g	{1}	{2}	{3}
	α	α	1
h		{1,2,3}	
		β	

Table 6

where $0 < \alpha < 1$ and $\beta > 0$. Let agents' utility functions be given by $u^i(x^i) = \sum_k (x^i_k)^{\frac{1}{\varrho}}$ where $-\infty < \varrho < 1$. Consider a "partial" exclusionary rule given by,

$$\mu(d) = \begin{cases} w_k(p_k) & \text{if } d = \{1,2,3\} \\ \gamma w_k(p_k) & \text{if } d \in \{\{1,2\},\{1,3\},\{2,3\}\}, \\ 0 & \text{if } d \notin \{\{1,2\},\{1,3\},\{2,3\}\} \end{cases}$$

Here, the simple, minimal majority coalitions are awarded the fraction $\gamma \leq 1$ of the coalitional endowment. Other subcoalitions receive nothing. If $\gamma \leq \frac{2}{3}$, a straightforward application of Border (1985, Thm 23.6) establishes that the core is nonempty. If $\frac{2}{3} < \gamma \leq 1$, then the core may still be nonempty, depending on the values of α, β, and elasticity of substitution parameter, ϱ. The values $\gamma = \frac{4}{5}$, $\alpha = \frac{1}{2}$, $\beta = 1$, and $\varrho = \frac{1}{2}$, for example, yield an empty core. The values $\gamma = \frac{4}{5}$, $\alpha = \frac{1}{2}$, $\beta = \frac{3}{4}$ and $\varrho = \frac{1}{2}$, by contrast, yield a core that is nonempty.

In this example, since a majority coalition's liquidation value may be substantially less than the entire endowment, minorities may be endowed with rights that prevent a "tyrrany of the majority."

4 Summary

In this paper, a general model of coalitional property rights regimes is studied. Our main contribution is to provide conditions under which coalitional property rights regimes are socially stable in the sense of having a nonempty core. Our model has abstracted from production and public goods aspects. These factors are relevent to questions of communal ownership and could be incorporated into the model. This is left for future work.

We consider this paper to be a first step in addressing the larger questions: what economic factors (e.g., tastes, technologies, endowments, information structures) determine which kinds of property rights regimes will prevail? When and why do economies switch from one property rights regime to another? Ultimately, answers to such questions depend upon the particular mechanism that governs the choice of property rights regime.

References

[1] Anderson, T. and D. Lueck (1990), "Property Rights in Indian Country: The Impact of Land Tenure on Agriculture." manuscript.

[2] Arrow, K. (1971), "Equality in Public Expenditures." *Quarterly Journal of Economics*, 85, 409-15.

[3] Billera, L. (1974), "On Games without Sidepayments Arising from a General Class of Markets," *Journal of Mathematical Economics*, 1:129-39.

[4] Border, K. (1985), *Fixed Point Theorems with Applications to Economics and Game Theory*, Cambridge: Cambridge University Press.

[5] Brown, P. (1990), "Reallocation of Arable Land Use Rights in Early Modern Japan: Hypothesis on its Origins and Functions." mimeo, Ohio State University.

[6] Cornes, R. and T. Sandler (1986) *The Theory of Externalities, Public Goods, and Club Goods*, Cambridge: Cambridge University Press.

[7] Dahlman, C. (1980), *The Open Field System and Beyond*, Cambridge: Cambridge University Press.

[8] Debreu, G. (1959), *The Theory of Value*. New Haven: Yale University Press.

[9] Greenberg, J. (1983), "Local Public Goods with Mobility: Existence and Optimality of a General Equilibium," *Journal of Economic Theory*, 30, 17-33.

[10] Grossman, S., and O. Hart (1986), "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 94, 691-719.

[11] Hart, O., and J. Moore (1990), "Property Rights and the Nature of the Firm," *Journal of Political Economy*, 98, 1119-58.

[12] Lagunoff, R. (1994), "Sufficiently Specialized Economies Have Nonempty Cores," mimeo, University of Pennsylvania, July.

[13] MacFarlane, A. (1978), *The Origins of English Individualism*, Oxford: Basil Blackwell.

[14] Moulin H. and B. Peleg (1982), "The Core of Effectivity Functions and Implementation Theory," *Journal of Mathematical Economics*, 10, 115-45.

[15] North, D. (1981), *Structure and Change in Economic History*, New York: Norton.

[16] Ostrom, E. (1990), *Governing the Commons: The Evolution of Institutions of Collective Action*, Tucson: University of Arizona Press.

[17] Owen, G. (1982), *Game Theory*, Orlando, FL: Academic Press.

[18] Peleg, B. (1984), *A Game Theoretic Analysis of Voting in Committees*, Cambridge University Press: Cambridge.

[19] Scarf, H. (1967), "The Core of an N Person Game," *Econometrica*, 35, 50-69.

[20] Scotchmer, S. (1985), "Profit Maximizing Clubs," *Journal of Public Economics*, 27, 25-45.

[21] Shanin, T. (1972), *The Awkward Class*, Oxford: Oxford University Press.

[22] Shapley, L. and M. Shubik (1969), "On Market Games," *Journal of Economic Theory*, 1:9-25.

[23] Townsend, R. (1984), "Taking Theory to History," mimeo, University of Chicago.

[24] Troost, K. (1990), "The Formulation of Common Property in Central Japan in the Late Medieval Period, 1300-1600," manuscript.

[25] Wooders, M. (1978), "Equilibria, the Core, and Jurisdiction Structures in Economies with a Local Public Good," *Journal of Economic Theory*, 18, 328-48.