Breaking the Symmetry: Optimal Conventions in Repeated Symmetric Games

V. Bhaskar*
University of St. Andrews

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Abstract

We analyze the problem of coordinating upon asymmetric equilibria in a symmetric game, such as the battle-of-the-sexes. In repeated interaction, asymmetric coordination is possible possible via symmetric repeated game strategies. This requires that players randomize initially and adopt a convention, i.e a (symmetric) rule which maps asymmetric realizations to asymmetric continuation paths. The multiplicity of possible conventions gives rise to a coordination problem at a higher level if the game is one of pure coordination. However, if there is a slight conflict of interest between players, a unique optimal convention often exists. The optimal convention is egalitarian, and thereby increases the probability of coordination.

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1 Introduction

Symmetric games often have asymmetric pure strategy equilibria, which are "better" than the symmetric mixed strategy equilibrium. Prominent examples include the battle-of-the-sexes and the game of chicken. These games represent several important situations, including the entry of firms into a natural monopoly industry (Dixit and Shapiro [4], Farrell [5]), and competition between two animals for a scarce resource (the hawk-dove game of evolutionary biology). Several games of pure coordination (see Schelling [11]), also have this feature - eg. who should ring back when a telephone call is disconnected. Since asymmetric equilibria pose a major problem of coordination, it is more reasonable to focus on symmetric equilibria, even though these may well be inefficient. The question therefore arises, how can agents achieve asymmetric coordination? One approach to this question proceeds by enlarging or augmenting the basic game, eg. by allowing pre-play communication. The point here is that symmetric equilibria of the augmented game may permit a degree of asymmetric coordination in the base game. A typical equilibrium requires players to randomize (symmetrically) between messages. Since the realized message pairs need not symmetric, this can be used as a basis for asymmetric coordination in the subsequent base game. If the players have sufficiently large message sets, coordination can be almost perfect (Warneryd, [13]).

This argument for asymmetric coordination is not entirely satisfactory. The trouble is, there is a multiplicity of symmetric equilibria, all of which are payoff equivalent.[1] Hence the players face a second coordination problem in the augmented game, in choosing between equivalent symmetric equilibria. One solution to this problem is to restrict players to messages from a commonly understood natural language, which would then allow the "focal" selection of an equilibrium. This approach was taken by Farrell [5]. In this case, cheap-talk can facilitate coordination, but to a more limited extent. Its facilitatory role is inversely related to the extent of conflict of interest between the players - it is maximal in a game of pure coordination, it is less in the battle-of-the-sexes, and falls to zero in the hawk-dove game.

This paper analyzes the role of repeated interaction in facilitating asymmetric coordination, when the base game is symmetric. Players must play symmetric strategies in the repeated game, and these will involve randomization in the initial stage. With positive probability, the realized actions of the players will differ, and hence the history from the

[1] This problem arises since talk is cheap, and does not affect payoffs directly. Consider a symmetric equilibrium of the augmented game which requires players to play the asymmetric equilibrium (α, β) after the message pair $(1, 2)$ and (β, α) after $(2, 1)$. There is another symmetric equilibrium which plays (β, α) after $(1, 2)$ and (α, β) after $(2, 1)$. Both these equilibria have the same payoffs.

point of player one differs from the history from the point of view of player two.Players can therefore use this asymmetry to ensure asymmetric coordination (in the base game) for the remaining periods of the repeated game. In other words, players may use a *convention*, i.e. a rule which maps asymmetric histories to continuation paths along which the players use different actions in each period. The problem of course is that there are many such conventions, and the coordination problem recurs unless we are able to make a unique selection. This, we believe, is a problem with the argument of Crawford and Haller [3], who used this approach in the context of games of pure coordination.

This paper shows that a *slight* conflict of interest in the base game is often sufficient to allow us to select one convention in preference to all others. Let the base game be the battle-of-the sexes, with a pair of asymmetric equilibria (α, β) and (β, α), where player one prefers the former and player two the latter. Let the base game be played twice. Any convention in the repeated game is associated with an equilibrium strategy profile, which specifies randomization probabilities in the intial period. These randomization probabilites are sensitive to the choice of convention. Consider first the *bourgeios* convention, where if coordination is achieved in period one upon (α, β), players play the same equilibrium (α, β) in the second period. Under this convention players have a relatively large stake in securing coordination on their own terms, and hence both players play α with high probability. The probability of expost coordination will consequently be relatively low. Compare this with the *egalitarian* convention, where if the players play (β, α) in period two if coordination in period one has taken place on (α, β). This reduces the incentive a player has to try and ensure coordination on her preferred equilibrium. Hence this convention induces the player to play each action with probability close to one-half, so that the probability of expost coordination is higher than under the bourgeios convention - indeed, it will be higher than in mixed equilibrium of the one-shot game. Hence the symmetric equilibrium associated with the egalitarian convention payoff dominates the symmetric equilibrium associated with the bourgeois convention.

This basic idea generalizes when the game is repeated finitely or infinitely often. If there is some conflict of interest in the base game, the probability of coordination is maximized under an egalitarian convention. Such a convention cannot be bourgeios - players must switch between equilibria. The question is, under what conditions is there a unique optimal convention, which maximizes the probability of coordination. In the case of finite repetitions with discounting, the optimal convention is unique for almost all discount factors. With infinitely many repetition, there is a unique optimal way for players to coordinate if they are relatively impatient. However, if players are sufficiently patient, this uniqueness disappears, and many conventions will be optimal. Our basic message is that if players have some conflict of interest, however small, there is usually a unique way in which to minimize this conflict in finite repetitions. With infinite repetitions, if players are sufficiently patient, then the conflict of interest can be eliminated entirely, and there are many ways of doing this. Uniqueness therefore requires that players be sufficiently impatient.

The remainder of this paper is organized as follows. Section 2 considers the twice-repeated battle-of the sexes and coordination games - we discuss the approach of Crawford and Haller, and introduce our alternative approach. Section 3 discusses the case of general finite and infinite repetitions, in the context of the battle-of-the- sexes. Section 4 shows that our results extend to the entire class of symmetric games including the hawk-dove game, albeit in the special case of two repetitions The final section concludes. For the sake of smooth discussion, all propositions are proved in the appendix.

2 The Twice-Repeated Game

	α	β
α	0,0	x,1
β	1,x	0,0

Fig.1. The Game G

Consider the above symmetric two-person game, where $x > 0$, so that (α, β) and (β, α) are the two pure strategy Nash equilibria. Although players may have some conflict of interest over the two equilibria (if $x \neq 1$), they still have an interest in achieving coordination on one of the pure strategy Nash equilibria, since the payoff to either player is greater than the payoff of zero. Either pure equilibrium also Pareto-dominates the payoff in the mixed strategy equilibrium. Nevertheless, the *coordination problem* that the players face is not trivial - it is not clear which of these two equilibria they should expect to be played, since they are indistinguishable - any argument which one can offer in favour of one can equally well be offered in support of the other. This problem has been noted by a number of authors - eg. Farrell [5], Harsanyi and Selten [7], Warneryd [13] - all of whom restrict attention to symmetric equilibria. More formally, only the symmetric equilibria satisfy Harsanyi and Selten's criterion of symmetry invariance, which requires that the selected equilibrium be invariant to re-labelling of the players or strategies. [2]

While the restriction to symmetric equilibria in symmetric games is, in our view, a necessary restriction, it is far from sufficient. To see this let $x = 1$ in the game G of Fig. 1, so that the resulting game is one of pure coordination. By re-labelling player 2's strategies, so that α becomes β and β becomes α, we get the game G' below:

[2] The equilibrium (α, β) (where player 1 plays α and player 2 plays β) can be transformed into the other equilibrium, (β, α),by exchanging the labels of the players, and hence is not symmetry invariant.

	α	β
α	1,1	0,0
β	0,0	1,1

G'

G' has two symmetric pure strategy equilibria, (α, α) and (β, β). Nevertheless, players have no means of selecting one of these equilibria rather than the other, and hence the coordination problem referred to earlier applies here as well.[3] Hence requiring symmetry of equilibria, is a necessary but not sufficient condition.

What constitutes a sufficient condition for singling out an equilibrium in a symmetric game? Suppose that there is a single symmetric equilibrium which gives players higher payoffs than any other symmetric equilibrium. We shall call such an equilibrium *uniquely optimal,* and assume that the uniquely optimal equilibrium will be played. For example, if we modify the payoffs in G' so that the payoffs to (β, β) are 2 to each player (the other payoffs remaining unchanged), (β, β) becomes uniquely optimal, and we shall assume that players will be able to coordinate on playing this equilibrium. In the remainder of the paper, we shall be concerned with uniquely optimal equilibria when the game G is repeated. Our focus is on the ability of players to use repetitions in order to coordinate upon pure equilibria of the stage game.

2.1 The Crawford-Haller Argument

The role of repeated interaction in solving the coordination problem was analyzed by Crawford and Haller [3] (CH henceforth). They focus on games of pure coordination, without conflict of interest. Since CH assume that labelling of strategies in the base game is of no consequence, we may as well discuss their argument in the context of the game, G, with $x = 1$. CH assume that the players are fully (as opposed to boundedly) rational, and play the coordination game G infinitely often. Their essential point relates to the optimal use of past realizations, and is best understood by considering the simplest case where the game is repeated only twice and players maximize the sum of stage game payofs. Players can clearly randomize in each period, choosing each pure strategy with probability 1/2 (i.e. play the mixed strategy equilibrium of the stage game in each period). However CH argue that the players can ensure additional coordination, in the following way. In period one, players play the mixed equilibrium of the stage game. With positive probability the players realized actions will differ, eg. for example with player 1 choosing α and player 2 choosing β. In this event, CH argue that the players will be able to coordinate with probability one in period 2. For example, the two players could adopt the *bourgeios*

[3] Note that the process of re-labelling can be reversed, so that once again any pure equilibrium will not be symmetry invariant.

convention, whereby they play the same actions in period two in the event that their first period actions are coordinated, and play the mixed equilibrium otherwise. The strategy profile so defined is symmetric and gives rise to a higher probability of coordination in period two (3/4) than the strategy which ignores history and simply randomizes.

However, players could also adopt the *egalitarian convention*, whereby each player chooses a different action from the one she chose in period one (i.e. the players choose (β,α) in period two if their realized first period actions are (α,β). This also gives rise to a higher probability of second period coordination. Indeed, if one writes down the payoffs of the the two supergame strategies which adopt the respective conventions, one has the following payoff matrix:

	Bourgeois	*Egalitarian*
Bourgeios	1.5,1.5	0.75,0.75
Egalitarian	0.75,0.75	1.5,1.5

On comparing this payoff matrix with the matrix in G', it is clear that there is no basis for choosing the strategy adopting one convention over another. Hence players face a coordination problem in the repeated game one level higher than the coordination problem in the one-shot game. Just as the mixed strategy equilibrium is the only reasonable equilibrium in the one shot game, one should expect to randomize between the strategies using these two conventions with equal probability. I.e. players should play the mixed strategy of the repeated game which randomizes between the bourgeiois and egalitarian strategies. However, this mixed strategy is behaviorally equivalent to the strategy which ignores history and plays α and β with equal probability after each history!

Our approach in this paper is relatively orthodox, since we assume that if two strategies profiles cannot be distinguished on the basis of payoffs, players will not be able to select between them. However, it seems unlikely that CH approach can be defended even if we allow players to coordinate upon equilibria based on commonly perceived labels. Such theories have been by advanced by Bacharach [2] (see also Janssen [8] and Sugden [11]), who discusses an example where four of the five pure strategy equilibria of a pure coordination game are labelled with a green colour, with the remaining equilibrium is labelled red. Bacharach argues that if the players perceive the colours in the same way, they will be able to coordinate upon the red equilibrium.[4] Observe that this argument does not apply if one has only two pure strategy equilibria, with one being labelled red and the other green. In the CH context, first period actions have the effect of labelling action profiles. However, with two equilibria, one cannot single out one without simultaneously singling out the other.[5] Note that CH do not suggest that one convention is more

[4]If one of the players is colour blind, this is not possible.

[5]We note that the Bacharach-Janssen approach may provide a defense for CH's analysis of 3×3

persuasive than the other. To quote, they argue that players can use "their perfect recall and knowledge of the structure of the game to maintain coordination forever once they locate a pair of coordinated actions. They can do this *either* by repeating those actions *or* by alternating deterministically between them and the other coordinated pair." (CH, p. 575, emphasis added).The critique of CH that we have offered is by no means original. Goyal and Janssen [6] make a similar point, and also offer a more detailed discussion of this issue.

2.2 A Little Conflict Helps

The basic point of this paper is that the *slightest* conflict of interest between the two players destroys the payoff-equivalence between the two conventions, thus allowing us to select one. Consider the stage game G, and assume that $x \neq 1$. This game is hence the battle-of-the-sexes. Let $p = 1/(1+x)$, and observe that the symmetric equilibrium of the stage game G is where each player chooses action α with probability p, and β with probability $(1-p)$. Hence the probability that the players acheive expost coordination while playing the symmetric equilibrium of the one-shot game equals

$$2p(1-p) = 2x/(1+x)^2$$

This probability is maximum, at 1/2, when $x = 1$. As x diverges from 1, the conflict of interest between the two players increases, and the probability of coordination decreases, and tends to zero as $x \to 0$ or as $x \to \infty$.

We now show that if $x \neq 1$, the players are no longer indifferent between the two conventions - players prefer to coordinate on the *egalitarian convention*, and play the symmetric equilibrirum associated with this convention since this gives them higher payoffs than the equilibrium assoicated with the *bourgeois convention.* The egalitarian convention players minimizes the conflict of interest between the players, and each player has less incentive to try and ensure coordination on her preferred equilibrium. On the other hand, the bourgeois convention increases the conflict of interest between players, and each tries to achieve coordination on her own terms, thus reducing the probability of coordination.

Our focus is on symmetric equilibria of the twice repeated game, where players maximize the discounted sum of stage game payoffs. $\delta \in (0,1]$ is the discount rate. A strategy in the two period game is $\sigma = (\sigma_1, \sigma_2)$, where $\sigma_1 \in \Delta(A)$ is the player's first period strategy. Let $a_1, b_1 \in \{\alpha, \beta\}$ be the actions taken by player 1 and player 2 respectively in period 1. The history at stage 2 from the point of view of player 2 is $h(2) = (a_1, b_1)$; the

coordination games. Although there are several conventions which are all payoff equivalent, there is a single convention which optimally utilizes the labels provided by first period actions. Kramarz [9] also discusses endogenous labelling in $n \times n$ coordination games.

history from the point of view of player 2, $\eta(h(2))$, is a permutation of $h(2)$, i.e. (b_1, a_1). Consider the second stage of this game. Clearly any symmetric equilibrium must a) play an equilibrium of the stage game after any pair of first period actions[6]b) prescribe identical mixed actions for both players if the history is symmetric. Hence in any symmetric equilibrium of the two period game, both players must play the mixed equilibrium of G after the histories (α, α) and (β, β).

Consider now the history (α, β) where player 1 has played α and player 2 has played β. In this case, the history from the point of view of player 1 (which is (α, β)) differs from the history from the point of view of player 2 (which is (β, α)). Hence the strategy profile induced in period 2 can well be asymmetric. A rule which achieves asymmetric coordination by conditioning upon history is called *convention:* more precisely, in this context a convention is such a rule which which could be consistent with a repeated game symmetry equilibrium. Hence we have two possible conventions, the *bourgeois convention* and the *egalitarian convention.* Our key point is that different conventions generate different incentives for first period actions. To see this, write down payoff matrix to any player for first period actions under each of these conventions:

	α	β
α	$\delta x/(1+x)$	$(1+\delta)x$
β	$1+\delta$	$\delta x/(1+x)$

First period payoffs under Bourgeios Convention

	α	β
α	$\delta x/(1+x)$	$x+\delta$
β	$1+\delta x$	$\delta x/(1+x)$

First period payoffs under Egalitarian Convention

Observe that both the above payoff matrices are qualitatively similar to the payoff matrix of the game $G-$ there are two asymmetric pure strategy equilibria, with a single symmetric equilibrium is in mixed strategies. Let σ_1^b (resp. σ_1^e) be the probability with which α is played in the symmetric equilibrium of the payoff matrix of associated with the bourgeios convention (resp. egalitarian convention). These are the mixing probabilities in the symmetric equilibrium strategies associated with these two conventions. To compare these mixing probabilities, note that the diagonal payoffs (i.e. payoffs to (α, α) and (β, β)) are the same in both these matrices. Regardless of whether $x > 1$ or $x < 1$, the payoff difference between (α, β) and (β, α) is greater under the bourgeios convention (equalling $(1+\delta)(x-1)$) than under the egalitarian convention ($(1-\delta)(x-1)$)). Hence σ_1^e is closer to 1/2 than σ_1^b, implying that the probability of expost coordination in period one is greatest under the egalitarian convention. Indeed, σ_1^e is closer to 1/2 than the probability

[6]This is always true if we restrict attention to subgame-perfect equilibrium. It is also true for Nash equilibrium if the first period action is truly mixed, which will indeed be the case.

with which α is played in the one shot game. Since the probability of coordination in period two is increasing in the probability of period one coordination, this implies that the egalitarian convention ensures optimal coordination, and higher payoffs than the bourgeois convention.

Observe that if $\delta = 1$, $\sigma_1^e = 1/2$, so that the players can completely eliminate the conflict of interest in the twice repeated game by using the egalitarian strategy, so that the repeated game becomes a game of pure coordination.

3 Many Repetitions

We now turn to a more general analysis, by considering the optimal choice of conventions in finitely and infinitely repeated games. This requires some additional definitions and notation. Let G be a symmetric two player game, where players 1 and 2 choose actions from a common set A, and each player's payoff is given by a common function $u(a, b)$, where a is the player's own action and b is her opponent's action. The payoffs in G are given as before in Fig. 1. Let $\mathrm{G}^T(\delta)$ be the T-fold repetition of G, with discount rate δ. $\delta \in (0, 1]$ if T is finite, and $\delta \in (0, 1)$ if T=∞. Payoffs in the repeated game are given by the disounted sum of stage game payoffs.

A player's history at period t, $h(t)$, is the sequence $[(a(1), b(1)), ..., (a(t-1), b(t-1))]$. Let $H(t)$ denote the space of all t-period histories. For each sequence of play generates two histories, one for each player. For each $h(t) \in H(t)$, let $\eta(h(t))$ denote the corresponding history for the other player, i.e. $\eta(h(t))$ is obtained by permuting $a(\tau)$ and $b(\tau)$ for each τ. A behavior strategy for a player in $G^T(\delta)$ is a sequence of functions $(s_t)_{t=1}^T$ where $s_1 \in \Delta A$ is the player's first period strategy and for $t > 1, s_t : H_t \to \Delta A$. Let Σ be the set of behavior strategies. Write $V(\sigma, \theta)$ for the payoff of strategy σ against strategy θ in $G^T(\delta)$. A strategy profile (σ, θ) is a Nash equilibrium if $V(\sigma, \theta) \geq V(\sigma', \theta) \forall \sigma' \in \Sigma$ and $V(\theta, \sigma) \geq V(\theta', \sigma) \forall \theta' \in \Sigma$. A Nash equilibrium (σ, θ) is *symmetric* if $\sigma = \theta$. Let $N^S = \{\sigma \in \Sigma : (\sigma, \sigma)$ is a Nash equilibrium of $G^T(\delta)\}$ be the set of symmetric Nash equilibria of the repeated game.

Optimal coordination requires that players optimally exploit the asymmetries generated when (symmetric) randomization occurs. Recall that each sequence of play generates two histories, one for each player. However, from our (the analyst's) point of view, it will be convenient to fix the perspective, and consider histories from player 1's point of view. A history $h(t)$ is *symmetric* if $h(t) = \eta(h(t))$. Let $H^s(t)$ be the set of symmetric histories. Clearly a history is symmetric only if both players have chosen the same action in each period, i.e. if $a(\tau) = b(\tau)$,$1 \leq \tau < t$. Given a history $h(t)$, and a strategy σ, write $\sigma(h\,(t))$ for the continuation strategy induced by $h(t)$. Observe that given a symmetric strategy profile (σ, σ), the continuation strategy profile given a history $h(t)$, $(\sigma(h(t)), \sigma(\eta(h(t))))$

is also symmetric if $h(t)$ is symmetric. I.e. if $h(t) = \eta(h(t))$, then $\sigma(h(t)) = \sigma(\eta(h(t)))$. A history $h(t)$ is *asymmetric* if it is not symmetric. If $h(t)$ is an asymmetric history, then the continuation strategy profile need not be symmetric, since the history from the point of view of player 1 differs from the history from the point of view of player 2. An asymmetric history $h(t)$ will be called *first asymmetric* if $a(t-1)$ differs from $b(t-1)$, but $a(\tau) = b(\tau) \forall \tau < t-1$. In other words a first asymmetric history provides the first opportunity for players to coordinate their subsequent actions while playing symmetric strategies in the overall game.

We shall conduct our analysis by using the notion of paths and continuation paths, introduced by Abreu [1]. This is equivalent to an analysis in terms of strategies, but is more convenient in the present context. However, Abreu restricts attention to pure strategy equilibria, and hence we have to modify Abreu's framework somewhat. Given any period t,a *path* from date t is a sequence of (pure) action combinations for the remaining periods of the game. A *coordinated path* from date t is a sequence $(\gamma_\tau)_{\tau=t}^T$, where $\gamma_\tau \in \{(\alpha,\beta),(\beta,\alpha)\} \forall \tau$. Our focus is on coordinated paths which are part of an equilibrium. For a coordinated path to be an equilibrium path (i.e. part of an equilibrium) one needs to verify that both players have an incentive to comply with it once started. To do this we need to define continuation paths if a player deviates from a coordinated path. This is easy since players are playing a Nash equilibrium of the stage game everywhere along a coordinated path. We can define the path after any deviation by simply ignoring deviations - i.e. once a coordinated path $(\gamma_\tau)_{\tau=t}^T$has started from period t, in periods $\tau > t$, players play γ_τ irrespective of whether players have chosen $\gamma_{t'}$ in periods t' such that $t \leq t' < \tau$.

Given a coordinated path, $(\gamma_\tau)_{\tau=t}^T$ write $\eta((\gamma_\tau)_{\tau=t}^T)$ for the coordinated path obtained by permuting the actions in γ_τ, for each $\tau, t \leq \tau \leq T$. The set of all coordinated paths is $\Gamma_{tT} = (\{(\alpha,\beta),(\beta,\alpha)\})^{(T-t)}$. A *convention* is a symmetric map, ξ_{tT}, from the set $\{(\alpha,\beta),(\beta,\alpha)\}$ to the set of coordinated paths, Γ_{tT}. Symmetry of the map implies that the image of (β,α) under this map is the permutation of the image of (α,β) under this map.Let $h(t)$ be a first asymmetric history. Any convention defines a continuation path (which is coordinated), which is the image of $a(t-1)$, the last component of $h(t)$.[7]

Observe that the set of possible conventions depends upon the number of periods remaining in the game, $T-t$. Hence this set depends upon t if T is finite, but not if $T=\infty$. Consider first the latter case, when the game is infinitely repeated.

[7]We have assumed that a convention conditions the continuation path only upon the last (asymmetric) actions taken in period $t-1$, and not upon the symmetric actions taken in earlier periods. This assumption is justified since our focus is on the existence of a uniquely optimal convention. If the optimal convention is unique, then clearly it will be optimal to adopt the same convention after all histories which differ only in the identity of the symmetric actions in periods earlier than $t-1$. If there non-uniqueness, this in itself is a problem - relaxing our assumption only increases the multiplicity of optimal conventions but not the qualitative nature of the problem.

We consider the class Σ^* of *conventional strategies* in the repeated game, which are defined as follows. If the game is infinitely repeated, a conventional strategy σ is a sequence $(\sigma_t, \xi_{t+1\infty})_{t=1}^{\infty}$. If the game is finitely repeated, σ consists of a sequence $(\sigma_t, \xi_{t+1T})_{t=1}^{T-1}$ and σ_T. These are defined as follows:

ia) σ_1 is the intial randomization probability, specifiying the probabilities with which α and β are chosen in period one.

ib) ξ_{2T} is a convention which maps asymmetric realizations in period one to coordinated paths beginning period two.

iia) For $t > 1, \sigma_t$ specifies the probabilities with which α and β are chosen in period t, given that $h(t)$ is symmetric.

iib) If $t < T$, ξ_{t+1T} is a convention which maps asymmetric realizations in period t to coordinated paths beginning period $t + 1$.

Observe that any conventional strategy defines a behavior strategy, i.e. it defines a player's actions after every possible history. Hence $\Sigma^* \subset \Sigma$. Henceforth, we by strategy we shall always mean a conventional strategy.

Our focus is on symmetric strategy profiles which are Nash equilibria. By construction, players find it optimal to conform to the coordinated paths which are dictated by a convention once an asymmetric realization occurs. Hence if the conventions are chosen, the only remaining criterion for equilibrium is that the randomization probabilities, $(\sigma_t)_{t=1}^{T}$, be chosen to be equilibria.

A strategy $\sigma \in \Sigma^*$ is *optimal* if it is a symmetric Nash equilibrium and the associated payoff is weakly greater than the payoff at any other symmetric equilibrium in Σ^* i.e. $V(\sigma, \sigma) \geq V(\sigma', \sigma') \forall \sigma' \in N^S \cap \Sigma^*$. σ is *uniquely optimal* if it is optimal and no other strategy is optimal. Our focus in this paper is on the conditions under which a uniquely optimal strategy exists. If a uniquely optimal strategy exists, then the coordination problem is solved, i.e. there is a uniquely optimal way in which players should coordinate. We call the conventions associated with optimal strategies optimal conventions.

3.1 Finitely Many Repetitions

We now consider the general case of T-repetitions, $T > 2$. Our focus is on optimal strategies in $G^T(\delta)$, and on the conditions under which this optimum is unique. We have shown that for any $\delta \in (0, 1]$,a uniquely optimal strategy exists for T=1 (the symmetric mixed equilibrium), and for T=2 (the egalitarian strategy).

Suppose that an optimal strategy exists in $G^{T-1}(\delta)$. Call this strategy σ^{T-1}, and let $V(\sigma^{T-1}, \sigma^{T-1})$ be the payoffs of the players under this strategy. Consider now the game $G^T(\delta)$. Suppose that in period one of this game, the realized actions of the players are identical. The continuation game is now equivalent to $G^{T-1}(\delta)$, and hence σ^{T-1} is an optimal strategy in this continuation game. Hence, to find an optimal strategy in $G^T(\delta)$, σ^T, it suffices to define σ_1, the initial randomization probability, and ξ_{2T}, the convention which maps asymmetric realizations in period one to a coordinated path from period two onwards. Given any such convention, let $\xi_{2T}(\alpha, \beta)$ be the path when $h(1) = (\alpha, \beta)$. For $1 < t \leq T$, let $\Delta_t = 1$ if (α, β) is played along this path and $\Delta_t = 0$ if (β, α) is played along this path. Hence the total discounted sum of payoffs to player one conditional on the realized actions being (α, β) are given by

$$x + \sum_{t=2}^{T} \delta^{t-1}[\Delta_t x + (1 - \Delta_t)1] \tag{1}$$

The payoffs to player one conditional on the realized actions being (β, α) are

$$1 + \sum_{t=2}^{T} \delta^{t-1}[\Delta_t 1 + (1 - \Delta_t)x] \tag{2}$$

The payoffs to player one conditional on realized actions being either (α, α) or (β, β) are the same, and are given by $\delta V(\sigma^{T-1}, \sigma^{T-1})$. Hence the payoffs to first period action combinations are given by the following matrix:

	α	β
α	$\delta V(\sigma^{T-1}, \sigma^{T-1})$	$x + \sum_{t=2}^{T} \delta^{t-1}[\Delta_t x + (1 - \Delta_t)1]$
β	$1 + \sum_{t=2}^{T} \delta^{t-1}[\Delta_t 1 + (1 - \Delta_t)x]$	$\delta V(\sigma^{T-1}, \sigma^{T-1})$

Hence an optimal convention requires us to minimize the absolute size of the difference in payoffs between (1) and (2), since this maximizes the probability of expost coordination in period 1. This is given by

$$(x - 1)\left[1 + \sum_{t=2}^{T} \delta^{t-1}[(2\Delta_t - 1)\right] \tag{3}$$

Since our concern is to choose a sequence $< \Delta_t >_{t=2}^{T}$ which minimizes the absolute value of the (3), this does choice does not depend upon the sign or magnitude of $(x - 1)$. Define the sequence $z_t = 2\Delta_t - 1$. Hence $z_t \in \{-1, 1\} \forall t, 1 < t \leq T$, i.e. $< z_t >_{t=2}^{T}$ is a finite sequence with range {-1,1}. Let $Z^T = \{< z_t >: z_t \in \{-1, 1\} \forall t, 1 < t \leq T\}$ be the set of all such sequences, with generic element denoted by z^T. Define the function $F : Z^T \to [0, \infty)$ as follows

$$F(z^T) = \left|[1 + \sum_{t=2}^{T} \delta^t z_t]\right|$$

Hence the problem of finding an optimal strategy and an optimal convention reduces to the problem of minimizing F over the set Z^T.Since there are only finitely many conventions, an optimal convention clearly exists. Consider now the question of uniqueness.

Clearly, for an optimal strategy to be unique in the T-period game, there must also be a unique optimal strategy in the T-1 period game. Hence we assume that an optimal strategy is unique in $G^{T-1}(\delta)$, and investigate the conditions under which uniqueness is ensured in $G^T(\delta)$.

Note firstly that uniqueness cannot be ensured if $\delta = 1$. To see this, let T be any even number greater than two, and suppose that realized actions in period one are (α, β). Consider first the convention which plays (α, β) in periods $2, ..T/2$, and (β, α) in periods $T/2 + 1, ...T$. This convention results in equal payoffs to the realized first period actions (α, β) and (β, α) and hence players will choose both actions with equal probability in period one. However, the convention which alternates between (α, β) and (β, α) also does the same, and hence both these conventions are optimal.

This problem does not arise for generic values of δ, where $\delta < 1$. When players are impatient, no convention will be able to completely equalize the payoffs to first period action profiles (α, β) and (β, α). Also, for generic values of the discount factor, no two distinct conventions, which yield distinct streams of future payoffs, will have the same present value. This ensures that there is a unique optimal convention .

Proposition 1 *$G^T(\delta)$ has a unique optimal strategy for almost all values of δ.*

A second question which arises is the extent to which a uniquely optimal convention reduces the conflict of interest between players, so that randomization probabilities are close to one-half. There are two situations in which the conflict of interest is to a large extent substantially eliminated. The first case is when T is an even number, and the players are very patient, so that δ is close to 1. In this case, it is clear that if any convention induces (α, β) in a$T/2$ periods and (β, α) in $T/2$ periods, then the payoff difference between realized first action profiles (α, β) and (β, α) is of order $(1 - \delta)$ and hence tends to zero as δ tends to one. Notice that conflict of interest remains if T is an odd number, which is not too large, whereas δ is also large. The second case is when T is very large, and $\delta > 1$. In this case, the oddness of T does not matter, since δ^T will be small. This discussion is made precise in the following proposition.

Proposition 2 *Let $\delta > 1/2$ be given. Given $\epsilon > 0, \exists T : \sigma_1$, the probability with which α is chosen in an optimal strategy in $G^T(\delta)$, satisfies $\mid \sigma_1 - 1/2 \mid < \epsilon$.*

3.2 Infinitely Many Repetitions

We now analyze the case of $G^\infty(\delta)$, where G is infinitely repeated and players discount payoffs at rate δ. Much of the framework and analysis for the case of finite repetitions also extends to this case, provided that we n note that $T = \infty$. In particular, the expressions for first period action combinations, set out in the case of finite repetitions, remain unaltered, with the caveat the payoff to an optimal strategy after any symmetric history does not depend upon the length of the history. Specifically, let σ be an optimal strategy in $G^{\infty}(\delta)$, and let $V(\sigma, \sigma)$ be the payoff when σ plays itself. Then it follows that the payoff to action combinations (α, α) and (β, β) is also $V(\sigma, \sigma)$. This does not change any of the analysis that follows.

The problem of selecting an optimal convention hence reduces to that of choosing an infinite sequence $< z_t >$ is an infinite sequence with range {-1,1}. Let $Z = \{< z_t >: z_t \in \{-1, 1\} \forall t, 1 < t < \infty\}$ be the set of all such sequences, with generic element z. Define the function $F : Z \rightarrow [0, \infty)$ as follows

$$F(z) = \left| [1 + \sum_{t=2}^{\infty} \delta^t z_t] \right|$$

Hence the problem of finding an optimal convention reduces to the problem of minimizing F over the set Z.We now show that there is a unique solution to this problem if $\delta \leq 1/2$, and that there are infinitely many solutions if $\delta > 1/2$.

If $\delta \leq 1/2$, F is minimized when $z_t = -1 \forall t$. This equals zero if $\delta = 1/2$, and is strictly greater than zero if $\delta > 1/2$. In this case there is a unique optimal convention in the infinitely repeated game. This strategy involves the convention that players randomize between α and β until their realized actions differ. If the realized actions are (α, β), the players play (β, α) in every period thereafter, whereas if their realized actions are (β, α), the players play (α, β) in every period thereafter. The randomization probability is stationary as long as history is symmetric, and equals 1/2 if $\delta = 1/2$. If $\delta < 1/2$, the players will play α (resp. β) with probability more than 1/2 if $x > 1$ (resp. $x < 1$).

Consider now the case where $\delta > 1/2$. In this case there are infinitely many optimal conventions, which completely eliminate the conflict of interest between the players. One optimal convention proceeds by keeping track of the discounted sum of payoffs to the players at each date. To exposit this, let $x > 1$ so that player one's preferred equilibrium is (α, β). If the realized actions in period one are (α, β), the players play (β, α) in succeeding periods until the total discounted payoffs of player two exceed player one's total discounted payoffs. At this point players switch to playing (α, β) until player one's total payoffs exceeds player two's, and so on. Under this convention, the value of the infinite stream of discounted payoffs of both player is equalized, and hence this convention makes each player indifferent between achieving coordination on (α, β) and achieving coordination on (β, α). This ensures that it is optimal for players to choose both actions with equal probability in pre-coordination phase. However, there are infinitely many conventions which also equalize the payoffs to the two players, and hence the coordination problem that players face is a formidable one. We summarize this discussion in the following proposition, which is proved in the appendix.

Proposition 3 *In the infinitely repeated game, there is a unique optimal convention if $\delta \leq 1/2$. There are infinitely many optimal conventions if $\delta > 1/2$.*

Our analysis highlights a rather sharp distinction between finitely repeated and infinitely repeated games, which is quite different from the usual distinction which arises from the fact that backward induction applies in the first case but not the second. In the finitely repeated game, the conflict of interest between the players can be reduced substantially, but is generically never completely eliminated. In consequence, there is a unique optimal convention which minimizes this conflict. In infinitely repeated games, the conflict between players can be completely eliminated if the players are patient, and hence non-uniqueness follows. This is also related to the nature of the Folk theorems for the two types of games. In the infinite case, any individually rational payoff can be exactly achieved if players are sufficiently patient, whereas in the finite case, this payoff can only by approached by taking a sequence of games of increasingly longer length.

The previous remarks suggest that our positive results, on the existence of a uniquely optimal strategy, hinge upon the failure of an "exact" Folk theorem in these specific contexts. This in turn is related to the fact that mixed strategies are unobservable. The key to an optimal strategy lies in players randomizing initially with probability as close to one-half as possible. If mixed strategies were observable (as, for example, in Abreu [1]), it would be easy to enforce randomization with probability one-half, even in a finitely repeated game. For example, any relatively egalitarian convention could be coupled with punishments- eg. if one player deviates from this randomization, players could play the other player's preferred equilibrium thereafter. Since many conventions could be enforced in this way, non-uniqueness would follow. Hence the unobservability of randomized actions plays an important positive role in our analysis.

4 General Symmetric Games

Our analysis has been conducted for a rather special case, the battle-of-the-sexes. We now discuss the extent to which our qualitative results extend to more general symmetric games, while restricting attention to the case of two repetitions. In the first instance we consider the class of symmetric 2×2 games with payoff matrix given below, and show later that the results extend to $n \times n$ symmetric games.

	α	β
α	0,0	x,1
β	1,x	y,y

The Game Γ

Given our focus on asymmetric coordination, we assume that $x > y$. Observe that Γ encompasses the entire class of 2×2 symmetric games with a pair of asymmetric equilibria - the payoffs 0 and 1 are simply normalizations given that these are von Neumann-Morgenstern utilities.

In the mixed equilibrium of Γ, α is played with probablity

$$p = \frac{x - y}{1 + (x - y)}$$

The payoff in this mixed equilibrium is

$$\bar{u} = \frac{x}{1 + (x - y)}$$

We shall assume henceforth that $x > 1$, so that (α, β) is the preferred equilibrium for player 1 - we focus on this case to simplify exposition, since similar arguments extend when $x < 1$. If $y \leq 1$, the game is still the battle-of-the sexes, but if $y > 1$, we have the game of chicken - unlike the battle of the sexes, players no longer have a common interest in achieving expost coordination- the payoff to player 1 at (β, β) and the payoff in the mixed equilibrium both exceed the payoff at (α, β). Nevertheless, there are still gains to coordination, as we shall see shortly.

A second point to be noted is that although (α, β) is the preferred equilibrium, α may now be the more "risky" strategy. Note that p, the probability with which α is played in the mixed equilibrium, will be less than 1/2 if $x - y < 1$ (a possibility which can arise in the battle-of-the-sexes as well as chicken). If this is the case, in the twice-repeated game, a bourgeios strategy will raise the probability of expost coordination in period 1 (by raising the payoff to playing α) while an egalitarian strategy will reduce the relative payoff to playing α, hence reducing the probability of expost coordination. Hence it might be thought that the bourgeois strategy will be preferred in this case. However, one cannot simply identify optimality with the probability of coordination when the payoffs to non-coordinated actions differ (i.e. y differs from 0).

We now define an optimal strategy in the twice repeated game.$\sigma \in N^S$ (the set of symmetric Nash equilibrium strategies) is optimal strategy if $V(\sigma, \sigma) \geq V(\sigma', \sigma')$ $\forall \sigma' \in N^S$.σ is uniquely optimal if the inequality in this defintion is strict. As before, we shall analyze optimal strategies in terms of a convention coupled with an intial randomization probability.

We now consider the payoffs to action combinations under the bourgeios and egalitarian conventions. First period payoffs to action combinations under the two conventions are given by:

	α	β
α	$\delta\bar{u}$	$(1{+}\delta)x$
β	$1+\delta$	$\delta\bar{u}+y$

First period payoffs under Bourgeios Convention

	α	β
α	$\delta\bar{u}$	$x+\delta$
β	$1+\delta x$	$\delta\bar{u}+y$

First period payoffs under Egalitarian Convention

The associated randomization probabilites in the first period are given by

$$\sigma_1^b = \frac{x-y+\delta(x-\bar{u})}{1+x-y+\delta(x-\bar{u})+\delta(1-\bar{u})} \tag{4}$$

$$\sigma_1^e = \frac{x-y+\delta(1-\bar{u})}{1+x-y+\delta(x-\bar{u})+\delta(1-\bar{u})} \tag{5}$$

Since $x-\bar{u} > 1-\bar{u}$, $\sigma_1^b > \sigma_1^e$, and hence if p and $\sigma_1^b < 1/2$, the probability of coordination will be smaller under the egalitarian strategy than the bourgeois strategy. However, to consider optimality, we need to consider payoffs under the two strategies. The payoff in the symmetric equilibrium with the bourgeios convention is given by

$$V(\sigma^b,\sigma^b) = \frac{\delta\bar{u}[(1+\delta)x-\delta\bar{u}-y]+(1+\delta)x[1+\delta-\delta\bar{u}]}{[(1+\delta)x)-\delta\bar{u}-y]+[1+\delta-\delta\bar{u}]} \tag{6}$$

$$V(\sigma^e,\sigma^e) = \frac{\delta\bar{u}[x+\delta-\delta\bar{u}-y]+(x+\delta)[1+\delta x-\delta\bar{u}]}{[(1+\delta)x)-\delta\bar{u}-y]+[1+\delta-\delta\bar{u}]} \tag{7}$$

The difference in payoffs is given by

$$V(\sigma^e,\sigma^e)-V(\sigma^b,\sigma^b) = \frac{\delta(x-1)^2}{[(1+\delta)x)-\delta\bar{u}-y]+[1+\delta-\delta\bar{u}]} \geq 0 \tag{8}$$

This shows that the egalitarian convention is always uniquely optimal, except in the case that $x=1$. This is regardless of the values of y. Hence the optimal convention is not necessarily the one that increases the probability that players coordinate upon an asymmetric equilibrium, if y is different from zero. Indeed, if $y > x-1$, so that α is the more risky strategy, the egalitarian convention is optimal even though it reduces the probability of coordination as compared to the bourgeois convention.

Some intuition for this difference can be found by asking, what is the payoff maximizing symmetric mixed strategy in the one shot game? Let π be an arbitrary mixed strategy, and note that the derivative of $u(\pi,\pi)$ with respect to π is given by

$$\frac{\partial u(\pi,\pi)}{\partial \pi} = (1-2\pi)(1+x)-2(1-\pi)y \tag{9}$$

This implies that if $y \in (-\frac{1+x}{2}, \frac{1+x}{2})$, the payoff maximizing value of π is given by

$$\pi^* = \frac{1+x-2y}{2(1+x-y)} \tag{10}$$

with $\pi^* = 1$ if $y \geq \frac{1+x}{2}$, and $\pi^* = 0$ if y$\leq -\frac{1+x}{2}$. The difference between the payoff maximizing strategy and the equilibrium strategy of the one shot game is given by

$$\pi^* - p = \frac{1-x}{2(1+x-y)} \tag{11}$$

Hence if $x > 1$, the players payoffs always increase if reduce the probability with which α is played as compared to the equilibrium (p), even if this reduces the probability of expost coordination.

We now consider $n \times n$ symmetric games. Such a game Γ' is defined by a single $n \times n$ payoff matrix $U = [u_{ij}]$, where u_{ij} specifies the payoff to a player choosing action i when his opponent chooses action j. We shall assume that if $j \neq i$, $u_{ij} \neq u_{ji}$ - this ensures that players always get different payoffs at any asymmetric equilibrium of Γ'. Γ' always has a uniquely optimal symmetry invariant equilibrium (Harsanyi and Selten [7]), where the players earn a payoff $\bar{u}$,and hence we assume that this equilibrium will be played in the one-shot game. Consider now the twice-repeated game. The players can earn a payoff of $\bar{u}(1+\delta)$ by playing the symmetry invariant equilibrium of the one-shot game in both periods. However, if the game has an asymmetric equilibrium (α, β), the players can randomize between α and β in period one; if there expost coordination, players adopt the egalitarian convention in period 2. The only difference as compared to the 2×2 case is that if realized actions are identical, players do not necessarily randomize between α and β in period two, but play the uniquely optimal symmetry invariant equilibrium, with payoff $\bar{u}$. Let $\bar{v}(\alpha, \beta)$ be the payoff associated with this repeated game equilibrium - this is defined by equation (7). Let (γ, θ) be any other asymmetric equilibrium of Γ'. Let $\bar{v}(\gamma, \theta)$ be the payoff in the repeated game equilibrium where players randomize between γ and θ in period one, and adopt the egalitarian convention. If the payoff matrix U is generic, $\bar{v}(\gamma, \theta)$ and $\bar{v}(\alpha, \beta)$ will be different numbers, and will also differ from $\bar{u}(1+\delta)$. Hence there will be a uniquely optimal convention in the twice repeated game, provided that the payoff matrix of the stage game is generic.

Our results may be contrasted with the cheap talk literature, discussed in the introduction. By assuming that messages have a focal meaning, Farrell [5] shows that cheap talk can facilitate coordination if the base game is the battle-of-the-sexes. The extent to which it does so is inversely related to the conflict of interest between the players. In the game of chicken, cheap talk is completely ineffective. In contrast, repetition is always effective provided that there is a slight conflict of interest between the players.

5 Conclusions

The focus of this paper has been on the effects of different conventions upon the incentives to coordinate when there is some conflict of interest between the players. Bourgeios conventions, where property rights are entrenched by past precedent, have poor incentive effects. Egalitarian conventions minimize the conflict of interest and provide good incentives for coordination. While the analysis of this paper focuses on repeated games, this point is more general and applies to a variety of contexts such as the allocation of property rights. For example, pollution permits are often based on "grandfathering", where entitlements are related to past emissions. In a world where property rights are not immutable, such criteria cleary have bad incentive effects, by providing a dynamic incentive to increase pollution.

The approach of this paper is based on the assumption that players are rational and will coordinate upon a symmetric payoff dominant equilibrium if this is unique. This assumption is clearly at variance with much of the recent literature where boundedly rational players play a game repeatedly, and use past observations to forecast future behavior. Indeed, this paper was motivated by the observation that in games of pure coordination, rational players found it more difficult to coordinate than boundedly rational players. However, our analysis suggests that with some conflict of interest, players who boundedly rational but also far-seeing, will find it more difficult to coordinate. Bounded rationality may ensure that only bourgeios conventions can be played. Realizing this, players have a greater incentive to try and ensure coordination on their terms, thus reducing the probability of coordination.

This last point also relates to the hawk-dove game of evolutionary biology, concerning a conflict between two animals over a resource . While the symmetric equilibrium of this game is inefficient, Maynard Smith [10] noted that the conflict can be resolved without a fight by conditioning upon some asymmetry between the competing animals, even if this asymmetry is payoff irrelevant. In this context, he discusses the distinction between an owner and an intruder. This distinction is exogenously given, and hence both the bourgeios strategy, where the owner fights and the intruder retreats, and the paradoxical strategy, where the intruder fights and the owner retreats, are evolutionarily stable. Our analysis considers the situation where the asymmetry is not exogenously given, but arises endogenously, in the first stage of the two-stage game. It is not difficult to see that both bourgeios and paradoxical strategies are evolutionarily stable; however, the latter is better from the point of view of the species.

Finally, we note that our approach assumes that players have complete information about payoffs, so that they do not learn anything in repeated interaction. An alternative approach would be to follow Harsanyi, and model the underlying stage game as a game with a small amount of incomplete information about payoffs. In this case, a player's action in period one reveals some private information. This may allow one to select an equilibrium in period two based on risk considerations, rather than a selection in the overall game based on payoff dominance considerations. We leave this for future work.

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6 Appendix

Proof: of Proposition 1

Since T is finite, there are finitely many conventions. Hence there clearly exists an optimal convention, and an associated sequence z^T which minimizes

$$F(z^T) = \left|1 + \sum_{t=2}^{T} \delta^{t-1} z_t\right|$$

If two distinct conventions z^T, y^T, are both optimal, we have that either $[1+\sum_{t=2}^{T} \delta^{t-1} z_t] = [1 + \sum_{t=2}^{T} \delta^{t-1} y_t]$ or $[1 + \sum_{t=2}^{T} \delta^{t-1} z_t] = -[1 + \sum_{t=2}^{T} \delta^{t-1} y_t]$. In either case we have that $P(\delta) = 0$, where $P(\delta)$ is a non-trivial polynomial of degree T, whose coefficients belong to the set $\{-2, 0, 2\}$, and not all of which are zero.There are finitely many values of δ such that $P(\delta) = 0$. Hence we have a unique optimal convention for all but finitely many values of δ. ■

We prove proposition 3 before proving proposition 2.

Proof: of Proposition 3

Since the case where $\delta \leq 1/2$ is obvious, we prove the proposition for $\delta > 1/2$.We construct a sequence $< z_t >$ such that $F(< z_t >) = 0$. Define $G(\tau)$ as the partial sum

$$G(\tau) = 1 + \sum_{t=2}^{\tau} \delta^t z_t$$

The sequence $< z_t >$ is defined as follows: $z_2 = z_3 = -1$. For $t > 3$, $z_t = z_{t-1}$ if the partial sums the same sign, i.e. if $G(t)G(t-1) \geq 0$. If $G(t)G(t-1) < 0$, then $z_t = -z_{t-1}$.Let $T_1 = \inf\{t > 1 : z_t \neq z_{t+1}\}$. Let $T_k = \inf\{t > T_{k-1} : z_t \neq z_{t+1}\}$

We show first that the sequence $< z_t >$ changes sign infinitely often if $\delta > 1/2$, so that T_k is a natural number for any k. It must clearly change sign once - consider the sequence which has value -1 for all t, and note that the value of F corresponding to this sequence is $1-\delta/(1-\delta) < 0$. Hence T_1 is a natural number. Since $G(T_1) < 0$ and $G(T_1-1) > 0$, we have that $0 > G(T_1) \geq \delta^{T_1}$. Hence $G(T_1) + \delta^{T_1}(\delta/(1-\delta) > 0$, so that T_2 is also a natural number. Extending the same argument, we have that $< z_t >$ changes sign infinitely often.

Since $G(T_k)$ and $G(T_k - 1)$ have opposite signs and $|G(T_k)| = \left|G(T_k - 1) + \delta^{T_k}\right|$, it follows that $|G(T_k)| \leq \delta^{T_k}$.

We now show that $F(< z_t >) = 0$. Given $\epsilon > 0, \exists\ T : t > T \implies \delta^t < \epsilon$. Hence $|G(T_k)| < \epsilon$ if $T_k > T$. Further, for $T_k < t < T_{k+1}$, $|G(t)| < |G(T_k)|$since $G(T_k)$ and $G(t)$ have the same sign, but $G(t)$ has added terms which are of the opposite sign of $G(T_k)$. Hence $|G(t)| < \epsilon \forall t > T$. Hence $F(< z_t >) = 0$.

We now show that there are infinitely many sequences $< z' >$ such that $F(< z'_t >= 0$. To consturct one such sequence, let $T > 1$ be given and let $z_t = z'_t \forall t \leq T$, and let $z'_{T+1} = -z_{T+1}$. Let $w_t = z'_t - z_t$. Hence $w_t \in \{-2, 0, 2\} \forall t$.

$$F(< z'_t >) = F(< z_t >) + \delta^T ||[\sum_{t=T+1}^{\infty} \delta^{t-T} w_t]|| \tag{12}$$

Define w_t as follows: $w_t = 0$ if $t \leq T$. w_{T+1} is already defined - it equals 2 if $z_{T+1} = -1$, and equals -2 if $z_{T+1} = 1$. For $k > 1, w_{T+k} = z_k$. Hence

$$F(< z'_t >) = F(< z_t >) + 2\delta^T z'_T ||[1 + \sum_{t=2}^{\infty} \delta^t z_t]|| = 0 \tag{13}$$

Since T was arbitrarily chosen, there are infinitely many sequences $< z'_t >$ such that $F(< z'_t >) = 0$. ■

Proof: of Proposition 2

Let $< z_t >$ be the infinite sequence constructed in the proof of proposition 3, and recall that if $t > T_k$, then $G(t) \leq \delta^{T_k}$, where T_k can be arbitrarily large. Consider $G^T(\delta)$, where $T > T_k$. Let $< z_t >_{t=2}^T$ be the first T terms of the sequence $< z_t >$. Hence $F(< z_t >_{t=2}^T) \leq \delta^{T_k}$. Hence an optimal convention in $G^T(\delta)$ have a value of F less than δ^{T_k}. Hence if T is sufficiently large, the probability of coordination in the intial period(s) can be made arbitrarily close to 1/2. ■

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