

# Market Crashes Without External Shocks<sup>1</sup>

Sergiu Hart<sup>2</sup>

Yair Tauman<sup>3</sup>

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<sup>2</sup>Center for Rationality and Interactive Decision Theory; Department of Economics; and Department of Mathematics; The Hebrew University of Jerusalem, 91904 Jerusalem, Israel. e-mail: hart@math.huji.ac.il. <http://www.ma.huji.ac.il/~hart>. Research partially supported by grants of the U.S.-Israel Binational Science Foundation and of the Israel Academy of Sciences and Humanities.

<sup>3</sup>Recanati School of Business, Tel Aviv University, 69978 Tel Aviv, Israel and Department of Economics, SUNY at Stony Brook, Stony Brook, NY 11794, U.S.A.

### **Abstract**

It is shown here that market crashes and bubbles can arise without external shocks. Sudden changes in behavior coming after a long period of stationarity may be the result of *endogenous* information processing. Except for the daily observation of the market, there is no new information, no communication and no coordination among the participants.

# 1 Introduction

Market crash is usually considered an indication that the fundamentals of the economy have changed and recession is around the corner. This however need not be so. For instance, in October 1987 Wall Street lost over 20% of its value in one day, but this was not followed by a recession. Moreover, in the days preceding the crash, there were no significant external events or “bad news” that could justify the dramatic price fall.

We argue here that market crashes (and, similarly, market bubbles) may well be the result of information processing by the participants — and *nothing else*. Moreover, in terms of market observables, it looks as if nothing is really changing. Still, underneath the surface, there is a gradual updating of information by the participants. Then, at a certain point in time, this causes a sudden change of behavior.

Specifically, the phenomenon we describe here has to do with the step-by-step advance in levels of “mutual knowledge” (what one knows about what the other knows ...). Each trading day increases this level through the daily market observables — prices, quantities traded, etc. — which are common knowledge. However, the behavior does not necessarily change with the level of mutual knowledge. The behavior may be constant for all levels up to a certain critical level — where a jump occurs. Such a phenomenon has been exhibited in Geanakoplos and Polemarchakis [1982].<sup>1</sup> In their paper two agents communicate information to each other by repeatedly announcing and revising their posteriors. It is shown there that the agents can repeat exactly the same opinions, yet still manage to communicate relevant information. Here, we fit their example to our model and show that there is no need for any direct communication between the traders; observing the market suffices.

We emphasize, first, that the phenomenon exhibited here is completely endogenous. Except for the daily observation of the market, there is no new information — whether pertinent or irrelevant (like “sunspots”); also, there is no communication nor any coordination among the participants — whether expressed or

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<sup>1</sup>For other situations with similar behavior, like the well-known puzzle of “the 40 cheating wives and the missionary,” see the survey of Geanakoplos [1994].

tacit. And second, the stationary unchanging behavior of the market for arbitrarily long periods of time is no sign that nothing is happening. Underneath the surface, completely unobservable, information is being processed by the participants — which ultimately leads to a sudden change of behavior.

The behavior of the agents in the example will turn out to resemble rules actually used by traders in the market. For instance: “I am buying every day; but, if others keep selling every day, then at some point I will start selling too.” Some of the so-called “technical analysis” is indeed of this kind. Intuitively, if others are willing to sell all the time, then the buyer will at some time have to take this fact into account: perhaps his assessment is incorrect after all. The framework of this paper allows to make such arguments precise.

We present here a simple example. However, the phenomenon we highlight is robust and general. One may change almost every feature of the example (like the state space, the probabilities, the number of traders, the daily rules of behavior, the prices, and so on), without qualitatively affecting the result.

To summarize: No exogenous “shocks” are needed to explain sudden departures from stationary behavior; these may well be due to the participants updating their information, based on the market observables. This information processing is however not observable — there is no change in behavior — up to the point where it generates an abrupt switch of behavior: a “crash.”

## 2 The Example

There are two traders. Day after day they keep trading — one is selling and the other is buying. Then, at some time, the updating of information leads one of them to reverse his behavior — from buying to selling. This happens in the absence of any exogenous influence.

Let  $I$  be the set of all states of the world. Assume  $I$  contains nine states, say,  $I = \{1, 2, \dots, 9\}$ . We assume a common prior probability assessment,<sup>2</sup> namely the uniform distribution  $P(w) = 1/9$  for all  $w \in I$ .

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<sup>2</sup>This “common priors” assumption plays no role here; it is used for simplicity only. See Remark (5) in Section 3 below.

The private information of each trader is described as usual<sup>3</sup> by a partition of the state space: Two states belong to the same block if and only if the trader cannot distinguish between them. Call the two traders Alice and Bob. Alice’s partition is

$$A_1 = \{1, 2, 3\}, \quad A_2 = \{4, 5, 6\}, \quad A_3 = \{7, 8, 9\},$$

and Bob’s partition is

$$B_1 = \{1, 2, 3, 4\}, \quad B_2 = \{5, 6, 7, 8\}, \quad B_3 = \{9\}.$$

The interpretation is as follows: Trader Bob, for example, cannot distinguish between states 1, 2, 3 and 4, nor between 5, 6, 7 and 8. If state 9 is the true state, then Bob will know that for sure. If, however, state 1 is the true state, then Bob will only know that it is either 1 or 2 or 3 or 4 (but not 5, 6, 7, 8, 9).

Consider the event  $E = \{1, 5, 9\}$ . For instance, assume that  $E$  is the event of a “bad” outcome (e.g., the company earnings will go down). Suppose that each one of the two traders behaves each day according to the following rule:

$$\begin{cases} \text{SELL,} & \text{if the probability of } E \text{ is 0.3 or more;} \\ \text{BUY,} & \text{if the probability of } E \text{ is less than 0.3.} \end{cases}$$

Of course, the relevant probability is always computed given the current information.

The manner in which the cutoff point of 0.3 (or, for that matter, the whole policy) is determined, is irrelevant to the analysis here; in particular, we abstract away from the stock price and the quantities.<sup>4</sup> Also, we note that it does not matter whether both traders use the same behavior strategy.

Assume that the true state of the world is  $w_0 = 1$ . Initially, Alice assesses the probability of  $E$  to be  $1/3$  (since at  $w_0 = 1$  she knows that the true state is either 1 [which belongs to  $E$ ], or 2 or 3 [which do not belong to  $E$ ]) — therefore she

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<sup>3</sup>For a formal treatment, see Aumann [1995].

<sup>4</sup>For concreteness, one may assume that “sell” actually means “sell the quantity  $q$  for the price  $p$ ,” and “buy” means “buy the quantity  $q$  for the price  $p$ ” (where  $q$  and  $p$  are the same for both decisions). Since, as we will see below, every day (up to the “crash”) Alice will sell and Bob will buy, the price need not change. If so desired, varying prices and quantities can be added to the model. This will make the analysis more complex, but—once the traders’ policies are defined appropriately—will not affect the phenomenon we exhibit.

gives an order to sell; Bob assesses this probability to be  $1/4$  (he knows the state is 1, 2, 3 or 4) — therefore he gives an order to buy. So a transaction takes place.

We will show that this will happen not only on the first day, but also on each one of the first four days: the assessments of the two traders for the probability of  $E$  remain  $1/3$  and  $1/4$ , respectively. On the fifth day, however, there is a sudden and major change: Both assessments become  $1/3$  and both traders give orders to sell. So, a “crash” occurs after four seemingly “quiet and normal” days.

Let us see this in detail (see Table 1). Let  $t = 1, 2, \dots$  denote the “days,” and  $I^t$  the common knowledge information at time  $t$ , before the traders choose their actions.

On day  $t = 1$ , we have  $I^1 = I$ . The assessments of Alice are

$$P(E | A_i \cap I^1) = P(E | A_i) = 1/3, \quad i = 1, 2, 3,$$

and those of Bob are

$$P(E | B_j \cap I^1) = P(E | B_j) = \begin{cases} 1/4, & \text{if } j = 1, 2, \\ 1, & \text{if } j = 3. \end{cases}$$

Since  $w_0 = 1$ , the current information is  $A_1$  for Alice and  $B_1$  for Bob, so Alice sells and Bob buys. Note that Bob only computes  $P(E | B_1)$  (he knows that the true state is in  $B_1$ ). However, for the sequel, one also needs to know what Bob would have computed — and done — in the other states as well.

On day  $t = 2$ , it is common knowledge that Bob bought on the previous day, therefore it is common knowledge that  $B_3 = \{9\}$  did not occur.<sup>5</sup> Therefore, the new common knowledge information is  $I^2 = \{1, 2, \dots, 8\}$ . The new assessments are:

$$P(E | A_i \cap I^2) = \begin{cases} 1/3, & \text{if } i = 1, 2, \\ 0, & \text{if } i = 3, \end{cases}$$

for Alice, and

$$P(E | B_j \cap I^2) = 1/4, \quad j = 1, 2,$$

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<sup>5</sup>Note that, at  $w_0 = 1$ , both traders initially knew that the state is not 9 (so this was *mutually known*); however, it was **not commonly known**. To see it, let  $F = \{1, 2, \dots, 8\}$  and let  $K_A F$  be the event “Alice knows  $F$ .” Similarly,  $K_B K_A F$  is the event “Bob knows that Alice knows  $F$ ,” etc. Then  $K_A F = \{1, 2, \dots, 6\}$ ,  $K_B K_A F = \{1, 2, 3, 4\}$ ,  $K_A K_B K_A F = \{1, 2, 3\}$  and  $K_B K_A K_B K_A F = \emptyset$ . Thus, it is never the case that Bob knows that Alice knows that Bob knows that Alice knows that the state is not 9. Hence, in no state of the world — in particular, at  $w_0 = 1$  — is  $F$  common knowledge.

for Bob. Again, at  $w_0 = 1$  a transaction takes place: Alice sells and Bob buys.

The new common knowledge information on day  $t = 3$  is  $I^3 = \{1, 2, \dots, 6\}$  (since Alice would not have sold at  $t = 2$  if the state was in  $A_3$ ), and we have on day 3:

$$P(E | A_i \cap I^3) = 1/3, \quad i = 1, 2,$$

and

$$P(E | B_j \cap I^3) = \begin{cases} 1/4, & \text{if } j = 1, \\ 1/2, & \text{if } j = 2. \end{cases}$$

Thus there is a transaction on day 3, and on day 4 it is common knowledge that the state is in  $I^4 = \{1, 2, 3, 4\}$  (since  $B_2$  is commonly ruled out), and thus

$$P(E | A_i \cap I^4) = \begin{cases} 1/3, & \text{if } i = 1, \\ 0, & \text{if } i = 2, \end{cases}$$

and

$$P(E | B_j \cap I^4) = 1/4, \quad j = 1.$$

Finally, on day 5 we get  $I^5 = \{1, 2, 3\}$  (since  $A_2$  is commonly ruled out),

$$P(E | A_i \cap I^5) = 1/3, \quad i = 1,$$

and

$$P(E | B_j \cap I^5) = 1/3, \quad j = 1.$$

But now both Alice and Bob send orders to sell and a “crash” occurs.

What is happening in this example is the following. Initially, both Alice and Bob know that the state is<sup>6</sup> 1, 2, 3 or 4. However, this fact is *not* common knowledge among them. For example, from Bob’s point of view, the state could well be 4, in which case Alice would have known that it is either 4, 5 or 6. So Bob does *not* know that Alice knows that it is 1, 2, 3 or 4. As time goes by, the trading increases the common knowledge (see the last column of Table 1), until, on day 5, it reaches its conclusion: it is common knowledge that  $w = 1, 2$  or 3.

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<sup>6</sup>Alice even knows more: She knows that the state is not 4.

### 3 Remarks

We conclude with a number of remarks.

(1) We have assumed throughout that  $w_0 = 1$ ; i.e., a “bad” state (in  $E$ ) is the true state. In the end (day 5), both traders indeed want to sell (the two traders, even by pooling their information, cannot distinguish between states 1, 2 and 3; they both sell since the probability assessment of  $E$  ends up being  $1/3$ ). However, exactly the same behavior would have resulted if the true state were  $w_0 = 2$  or  $w_0 = 3$  — which are “good” states (not in  $E$ ). Also, note that if  $w_0 = 4$ , then a “bubble” occurs (i.e., both buy) at  $t = 4$ .

(2) A similar example consisting of  $(n + 1)^2$  (instead of 9) states will yield  $2n$  days where transactions occur, and a “crash” on day  $2n + 1$  (see Aumann’s example, at the bottom of page 97 in Geanakoplos and Polemarchakis [1982]). So a “crash” can be preceded by an arbitrarily long period of time where trading occurs normally and nothing seems to change.

(3) If the state space is finite (or, more precisely, if the two information partitions are finite), then there can be only finitely many instances of information updating, after which the two assessments necessarily agree (the latter, under the assumption of common priors; this is the “Agreement Theorem” of Aumann [1976]).

(4) We have made the example as simple as possible; in particular, there are only two traders. One may of course deal with more traders; one easy way is to have two types of traders, A (like Alice) and B (like Bob).

(5) It is important to emphasize that, even though the example seems to be very “special,” it is not. For instance, any small changes in the prior probabilities — including making these priors different for the two traders — will not affect it. What matters is only that the posterior probabilities computed by the two traders remain above and below 0.3, respectively.<sup>7</sup>

(6) One may add to the model the stock price (changing according to market demand and supply), allow for “limit” orders, varying quantities, and so on. Defining trading policies accordingly — in such a manner that the information deduced remains the same — will however preserve the phenomenon we exhibit

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<sup>7</sup>Actually, even the cutoff point of 0.3 may change with time (by adjusting the trading policies).

(recall also Footnote 4 above).

(7) In our example, Alice is initially better informed at  $w_0 = 1$  than Bob: She knows that Bob's information is  $B_1$ . Therefore she can construct ahead of time the whole process as in Table 1. This can easily be remedied by taking  $w_0 = 5$ , or, if one wants the same number of periods before the crash, a bigger example as in (2) above.

(8) The only way for an order to be executed in our example is for the two traders to give opposite orders. When this is taken into account by the traders, their decisions may be affected. For example, at  $t = 4$ , Bob reasons as follows: The only case where my "buy" order will be executed is when Alice sells; but that happens only when the state is 1, 2 or 3 — and then I should sell, not buy! This is correct when Alice and Bob are the only two traders in the market. Assume therefore that, besides the two "major" informed traders Alice and Bob, there are additional traders, and transactions do not necessarily take place between the two. Still, each one observes the moves of the other (they are, after all, the major players in this market). Then the phenomenon in the example works as presented.

## References

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