Loss Aversion Equilibrium *†

Jonathan Shalev ‡

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Abstract

The Nash equilibrium solution concept for games is based on the assumption of expected utility maximization. Reference dependent utility functions (in which utility is determined not only by an outcome, but also by the relationship of the outcome to a reference point) are a better predictor of behavior than expected utility. In particular, loss aversion is an important element of such utility functions.

We extend games to include loss aversion characteristics of the players. We define two types of loss-aversion equilibrium, a solution concept endogenizing reference points. The two types reflect different types of updating of reference points during the game. In equilibrium, reference points emerge as expressions of anticipation which are fulfilled.

We show existence of myopic loss-aversion equilibrium for any extended game, and compare it to Nash equilibrium. Comparative statics show that an increase in loss aversion of one player can affect her and other players' payoffs in different directions.

Keywords: loss aversion, reference dependence, equilibrium.

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1 Introduction

Expected utility dominates the analysis of game-theoretic situations, despite overwhelming evidence that it fails to adequately describe or predict human behavior. Kahneman and Tversky's (1979) prospect theory proposes an alternative to expected utility in which outcomes are

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‡CORE, 34 voie du Roman Pays, B-1348 Louvain-la-Neuve, Belgium. Fax: +32-10-474301, Phone: +32-10-478186, E-mail: SHALEV@CORE.UCL.AC.BE.

evaluated with respect to a reference point. Such *reference dependent* utility functions are successful in explaining many systematic deviations from the maximization of expected utility. Rabin (1996) writes that "reference dependence deserves to be, and is gradually becoming, an important part of economic modeling."

The most striking result of the investigation of reference-dependent utility functions is the existence of loss aversion. Experimental works in both the psychological and the economic literature suggest that people are motivated to minimize losses (relative to a reference point) much more than they are motivated to maximize gain. For example, Fishburn and Kochenberger (1979) empirically assessed utility functions over changes in wealth. They found that the slope of the utility function below the reference point was on average almost five times as steep as the slope above the reference point. Other examples emphasizing the different treatment of losses and gains (and implicitly or explicitly implying reference dependence) are De Dreu, Emans and Van de Vliert (1992), Kahneman and Tversky (1979), Kahneman, Knetsch and Thaler (1990, 1991), Kramer (1989), Taylor (1991), and Tversky and Kaneman (1992)). Gul (1991) axiomatizes disappointment aversion, which is closely related to loss aversion. Gul's formula is that obtained when using a reference point for a lottery which is based on the evaluation of the lottery. This is the path we take in the definition of consistent reference points in Section 2.

The traditional definition of games ignores the possibility of reference dependent utility functions, assigning for each player a single number to represent each possible (pure) outcome resulting from a profile of pure strategies. These numbers are the von Neumann-Morgenstern utilities of the players for the outcome given by the strategy profile. For any pair of lotteries over outcomes, each player is assumed to prefer the lottery giving her a higher expected utility. This embodies the risk-aversion characteristics of the players, but not the loss-aversion characteristics, as can be seen from the following example. Assume that the possible (utility) payoffs are 0, 2, and 4. With expected utility, the player is assumed to be indifferent between a lottery giving probability 0.5 each of the outcomes 0 and 4, and receiving the outcome 2 for sure. Assume the player was indifferent (ex ante) and chose the lottery. If the outcome was 0, ex post the player may "suffer" from the effects of loss-aversion, if her reference point was above 0. The utility of the outcome may therefore be less than it would have been if the same outcome were received, but it was expected for sure, and was not part of a lottery, and therefore the reference point was zero. The loss-aversion characteristics cannot simply be embodied in the payoff numbers of the game. This is true, as the utility of an outcome in a lottery depends on the reference point, which would usually depend on all the possible outcomes of the lottery. Thus an outcome will have possibly different utility values for different lotteries of which it is a component. An example where reference dependence can help is the Allais (1953) paradox, which demonstrates a systematic violation of expected utility maximization. Using a reference dependent utility function with loss aversion provides a robust justification for the modal choices in the Allais paradox, as demonstrated by Example 1 in Section 4.

We extend the analysis of games to incorporate reference dependence and loss aversion. We first give a formula, based on experimental results, that systematically relates outcomes and reference points to utility. We assume an underlying utility function that translates outcomes into values, and a loss aversion coefficient for each player that captures her level of loss aversion. The values of outcomes are modified according to the reference point (and whether they are gains or losses) and the loss-aversion coefficient to give the final utility of the outcome for the player with respect to the reference point. The players' preferences over lotteries are assumed to be represented by the expectation of these final utilities (which depend on the reference point). In our solution concept we implicitly assume that the loss-aversion coefficients of all players are common knowledge. While this may seem unrealistic, it is in fact no more so than the standard assumption that the utilities of all possible outcomes for all players are common knowledge.

Thus, given an *extended game*, which includes a game and a loss aversion coefficient for each player, and given reference points for all the players, we can transform the game into a new (standard) game with final utilities. This new game could be analyzed in the standard fashion. However, this method sidesteps the important question of the significance of the reference points. In experimental situations the reference points are manipulated by the experimenters according to the framing of the outcomes. In contrast, in real life the reference points are "manipulated" by experiences and anticipations. Kahneman (1992) provides a review of much relevant work on reference points. We endogenize the reference points into the solution concept, and in a solution the utility of the reference points will be equal to the utility of the outcomes. This captures the "anticipation" characteristic of the reference points, that they represent beliefs about the outcome. This endogenization evaluates the utility of outcomes with the same function as the local utility function given in Gul (1991), which is a special case of Dekel's (1986) characterization of preferences with a weakening of the independence axiom.

We define a *loss-aversion equilibrium* as a strategy profile in which for each player the expected outcome (using loss aversion evaluation and thus giving higher weight to losses than to gains) is equal to her reference point, and no unilateral deviation of a player from this strategy can increase her utility. We define two types of loss-aversion equilibria, which are myopic loss-aversion equilibrium and non-myopic loss-aversion equilibrium. The term myopia refers to the updating of reference points as situations change. Since in simultaneous games there is no time involved, the two notions are equivalent for such games, as we show in Section 3. In general the two notions can differ in extensive form games, as shown by Examples 6 and 7 in Section 4. The reference points have a dual interpretation in a loss aversion equilibrium: they are used to evaluate payoffs (given values and loss aversion coefficients), and these same reference points are equal to the expectation of the evaluated payoffs for each player.

The following results are proved in Section 3. Proposition 1 is that for simultaneous games, the set of myopic loss-aversion equilibria is equal to the set of non-myopic loss-aversion equilibria. We show in Proposition 2 that any pure strategy Nash equilibrium of a game with perfect information is also a loss aversion equilibrium (both myopic and non-myopic) of any corresponding extended game. We prove existence of myopic loss aversion equilibria for any game (Proposition 3).

If none of the players is loss averse then the loss aversion equilibria of an extended game coincide with the Nash equilibria of the underlying standard game.

Loss aversion equilibrium, similarly to Nash equilibrium, does not select a unique solution for each games, but a set that may contain multiple elements. The number of loss aversion equilibria in an extended game may be either higher, lower, or the same as the number of Nash equilibria of the corresponding game before the extension (i.e. with no loss aversion). Examples of all these cases are given, along with other examples of games and their loss-aversion equilibria, in Section 4.

In Section 5 we perform some comparative statics, and show that an increase in loss aversion of a player can affect both her and other players' payoffs in both directions. We conclude in Section 6 with some thoughts about possible experimental tests of the theoretical results of the paper.

2 The Model

We deal mainly with games in extensive form. A formulation of games in extensive form can be found in Chapter 3 of Fudenberg and Tirole (1991) We denote a game by G. The set of *players* is I, which we take to be a finite set, the *pure-strategy space* is S_i for each player i, and *payoff functions* u_i give player i's von Neumann-Morgenstein utility $u_i(s)$ for each pure strategy profile $s \in S$, where $S = \prod_{i \in I} S_i$. We assume that these utilities $u_i(s)$ are the basic values of the outcomes[1], which should then be modified according to their relation to the reference points and the loss aversion characteristics of the players, as explained in the next paragraphs. The outcomes are assumed to be pure outcomes and not lotteries. This is without loss of generality, as any lottery over pure outcomes can be represented by an additional level in the tree and a move by nature.

An *extended game* $(G, (\lambda_i)_{i \in I})$ has an additional element, the *loss-aversion coefficients* of the players. For player i, $\lambda_i \in \mathbb{R}_+$ specifies the player's degree of loss aversion. Higher values of λ_i represent more loss aversion. A value of $\lambda_i = 0$ characterizes an expected utility maximizer (player i's utility function is not reference dependent if $\lambda_i = 0$). Given a reference point $r_i \in \mathbb{R}$ and a basic utility value $x_i \in \mathbb{R}$, the final (loss-aversion) utility of the player is given by

$$v_i(x_i, r_i) = \begin{cases} x_i & \text{if } x_i \geq r_i \\ x_i - \lambda_i(r_i - x_i) & \text{if } x_i < r_i \end{cases} . \tag{1}$$

The utility function given by (1) is similar to the value function found experimentally by Tversky and Kahneman (1992) for monetary prospects. Tversky and Kahneman found that the value function (when the reference point is zero) has the approximate form x^α for $x \geq 0$ and $-\lambda(-x)^\alpha$ for $x < 0$. They found the median values of α and λ to be 0.88 and 2.25 respectively. By using von Neumann-Morgenstern utility values instead of monetary values and using Formula (1), we retain the loss-aversion aspect of the utility function, which is that the function is steeper for losses (relative to the reference point) than for gains. However, we cannot get the "S"-shaped value function of prospect theory. Risk aversion and risk seeking are both included

[1]The basic value of an outcome is the utility of the outcome when the reference point is equal to the outcome.

as possibilities of our specification, but not as a function of the reference point. We allow λ to vary for different players to reflect the heterogeneity of loss aversion. The existence and range of heterogeneity has not been directly investigated empirically, but as discussed in Section 6 there is evidence to indicate that there exists such heterogeneity.

For exogenously given reference points, we can transform an extended game into a standard game by evaluating the utility of each outcome of the game according to Formula (1). Given an extended game $(G,(\lambda_i)_{i\in I})$ and a vector of reference points $r \in \mathbb{R}^I$, we define the transformation (to a standard game) $L(G,\lambda,r) = (G')$, where G' differs from G only in the utility of the outcomes for the players. The utility of each outcome for each player is transformed according to Formula (1), using the appropriate reference points and loss-aversion coefficients. Thus, if S is the set of pure strategy profiles, then for $s \in S$, $u'_i(s) = v_i(u_i(s), r_i)$.

We extend the utility function to include mixed strategies as follows. Denote by Σ_i the set of player i's mixed strategies. Denote $\Sigma = \prod_{i\in I} \Sigma_i$. If a mixed strategy profile $\sigma \in \Sigma$ gives probability $p_\sigma(s)$ to each pure action profile $s \in S$, then the utility of player i from Σ when he has a reference point r_i is given by

$$w_i(\sigma,r_i) = \sum_{s\in S} p_\sigma(s) v_i(u_i(s),r_i) \tag{2}$$

Note that the payoffs for player i are first defined on the outcomes (as a function of the player's reference point) and then defined for mixtures. This sequence is important for extended games, as the payoffs are not linear in the reference points, and the reference point is used to evaluate each pure outcome, and not the expected value of the outcome. Note also the implicit assumption that the reference point is fixed. This assumption may not be valid when we deal with extensive games that have more than one information set for a player, as the reference points might change as the player receives new information about the actions of the other players (possibly including moves by nature). We do not ignore this matter and we treat it when we discriminate between types of loss aversion equilibria.

Definition 1

We say a reference point is *consistent* with a lottery, if the utility evaluation of the lottery with respect to the reference point is equal to the reference point. Formally, for a lottery x giving outcomes $x^1, \ldots, x^n$ with respective probabilities $p^1, \ldots, p^n$, a reference point r_i is consistent for a player i with loss-aversion coefficient λ_i if

$$r_i = \sum_{j=i}^{n} p^j v_i(u_i(x^j), r_i). \tag{3}$$

The value of a consistent reference point for a lottery is analogous to the utility given by the appropriate disappointment averse utility function as defined in Gul (1991).

For an extended game (G, λ) and a mixed strategy profile $\sigma \in \Sigma$, denote $R_i(\sigma) = \{r_i \in \mathbb{R} \mid w_i(\sigma, r_i) = r_i\}$. This is the set of reference points that are consistent for player i with the lottery over outcomes implied by the strategies σ.

For an extended game (G, λ), define $\overline{r} = \max_{i \in I}\{\max_{s \in S}\{u_i(a)\}\}$ and $\underline{r} = \min_{i \in I}\{\min_{s \in S}\{v_i(u_i(s), \overline{r})\}\}$. Thus, for any lottery over outcomes, if the reference points are all in the interval $[\underline{r}, \overline{r}]$, the evaluated utilities of all players will also be in this interval.

The following lemma shows that for any $i \in I$ and any $\sigma \in \Sigma$, $R_i(\sigma)$ contains a single value:

Lemma 1 *If $(G, (\lambda_i)_{i \in I})$ is an extended game, then for all $i \in I$, and for all $\sigma \in \Sigma$, the correspondence $R_i(\sigma)$ is single-valued, and the value is in the interval $[\underline{r}, \overline{r}]$.*

Proof: Take $i \in I$ and $\sigma \in \Sigma$. $w_i(\sigma, r_i)$, viewed as a function of $r_i \in \mathbb{R}$, is non-increasing and continuous. The following three relations hold, defining $u_i(\sigma) = \sum_{s \in S} p_\sigma(s) u_i(s)$:

$$u_i(\sigma) \in [\underline{r}, \overline{r}] \tag{4}$$

$$w_i(\sigma, \underline{r}) = u_i(\sigma) \geq \underline{r} \tag{5}$$

$$w_i(\sigma, \overline{r}) \leq u_i(\sigma) \leq \overline{r} \tag{6}$$

Therefore, since $w_i(\sigma, \overline{r})$ is non-increasing and continuous in r_i, there exists a unique $r_i^* \in \mathbb{R}$ satisfying $w_i(\sigma, r_i^*) = r_i^*$. Moreover, $r_i^* \in [\underline{r}, \overline{r}]$. ■(Lemma 1)

As a consequence of Lemma 1 we can define $r_i(\sigma)$ as a function with the value of the unique element of $R_i(\sigma)$. This function can be evaluated not just at the root of a game tree, but also at any information set, and will give the consistent reference point for a player at that information set, given his belief that σ is the strategy profile being played.

Definition 2

A strategy profile $\sigma \in \Sigma$ is a *myopic loss aversion equilibrium* of (G, λ) if there exists $r \in \mathbb{R}^I$ such that σ is a Nash equilibrium of the transformed game $L(G, \lambda, r)$, and the payoff to the players from using σ in $L(G, \lambda, r)$ is r.

There are two aspects of myopia in this definition. The first is that all evaluation is done at the root of the tree. The players do not take into account possible changes of the reference point as the game proceeds. This might be a reasonable assumption for situations where reference points adjust slowly relatively to the duration of the game (or the actions are played by agents of the player, who therefore does not update her expectations during the play). The second aspect is that when evaluating a deviation, the player does not change her reference point, even though the distribution of outcomes may change if she deviates. Here too, there are situations where we might consider the reference point to be fixed and not shift in line with contemplated deviations. Kahneman (1992) discusses how multiple reference points might be used, and suggests that an important problem for future research is how multiple reference points compete and combine.

Definition 3

A strategy profile σ is a *non-myopic loss-aversion equilibrium* of (G, λ) if for all $i \in I$, all $\sigma_i' \in \Sigma_i$, and for all information sets μ of player i that are reached with positive probability under σ, the evaluation at μ satifies

$$r_i(\sigma) \geq r_i((\sigma_{-i}, \sigma_i')), \tag{7}$$

where (σ_{-i}, σ_i') signifies tha all players $j \in I \setminus \{i\}$ play σ_j and player i plays σ_i.

We call these non-myopic loss-aversion equilibria, as a player considering a deviation takes into account an appropriate change in her reference point that is consistent with her deviation. Evaluation is done at each information set that might be reached, so all available information is used when evaluating the lottery over outcomes implied by the strategies. Non-myopic loss-aversion equilibria are therefore appropriate for situations where we would expect reference points to adjust swiftly (relative to the duration of the game), and where the players are sophisticated and take into account these future expected changes in the reference point.

Both these definitions endogenize the reference points into the model, and the reference points serve both as comparison values to determine gains and losses, and also as anticipation values which are rational, as they are reached in an equilibrium.

We show in Section 3 that the set of myopic loss-aversion equilibria coincides with the set of non-myopic loss-aversion equilibria for games with only one information set for each player. These are simultaneous games whose form is essentially captured by strategic form representation. The fact that there is no time for adjustment of reference points gives the intuition for this result. In such games, all decisions must be taken before any relevant information about other players' decisions or realizations of chance moves is received. This equivalence result does not hold in general for extensive form games as shown by Example 6 in Section 4.

3 Results

The first proposition we prove shows that non-myopic and myopic loss-aversion equilibrium are identical for games where each player has only one information set, which is always reached. For such games, time is not of the essence. The information available to a player when she has to choose her action is no different from the information she had at the root of the game tree. This proposition shows that the difference between myopic and non-myopic loss-aversion equilibria comes from the differences in timing the updating of reference points, and not from re-evaluating reference points when considering deviations.

Proposition 1 *For any extended game where each player has exactly one information set, which is reached on every path of play (a simultaneous game), the set of myopic loss-aversion equilibria coincides with the set of non-myopic loss-aversion equilibria.*

Proof: Take an extended game (G,λ) satisfying the requirements of the proposition. From Definition 2 and Lemma 1, the set of myopic loss-aversion equilibrium is the set of $\sigma \in \Sigma$ that satisfy

$$w_i(\sigma, r_i(\sigma)) \geq w_i((\sigma_{-i}, \sigma'_i), r_i(\sigma)) \quad \forall i \in I,\ \forall \sigma'_i \in \Sigma_i. \tag{8}$$

From Definition 3 and Lemma 1, the set of non-myopic loss-aversion equilibrium is the set of $\sigma \in \Sigma$ that satisfy

$$w_i(\sigma, r_i(\sigma)) \geq w_i((\sigma_{-i}, \sigma_i'), r_i((\sigma_{-i}, \sigma_i'))) \quad \forall i \in I,\ \forall \sigma_i' \in \Sigma_i. \tag{9}$$

For this inequality we used the fact that each player has only one information set, and it is always reached. Therefore, the information at this point is the same as the player had at the root of the tree. From the definition of the function $r_i(\sigma)$, we have

$$w_i(\sigma, r_i(\sigma)) = r_i(\sigma) \quad \forall i \in I,\ \forall \sigma_i' \in \Sigma_i \tag{10}$$

and

$$w_i((\sigma_{-i}, \sigma_i'), r_i((\sigma_{-i}, \sigma_i'))) = r_i((\sigma_{-i}, \sigma_i')) \forall i \in I,\ \forall \sigma_i' \in \Sigma_i. \tag{11}$$

We first show that (9) implies (8). Inequality (9) together with Equations (10) and (11) implies $r_i(\sigma) \geq r_i((\sigma_{-i}, \sigma_i'))$. Since w_i is continuous and monotonically non-increasing in its second parameter, this implies

$$w_i((\sigma_{-i}, \sigma_i'), r_i((\sigma_{-i}, \sigma_i'))) \geq w_i((\sigma_{-i}, \sigma_i'), r_i(\sigma)) \quad \forall i \in I,\ \forall \sigma_i' \in \Sigma_i. \tag{12}$$

which together with (9) implies (8).

We now show the other direction. Inequality (8) and Equation (10) imply $r(\sigma) \geq w_i((\sigma_{-i}, \sigma_i'), r_i(\sigma))$, for all $i \in I$ and for all $\sigma_i' \in \Sigma_i$, so from continuity and monotonicity of w_i with respect to its second parameter we can conclude that $r_i(\sigma) \geq r_i((\sigma_{-i}, \sigma_i'))$, which implies (9), using Equations (10) and (11). ■(Proposition 1)

The next proposition states that all pure strategy equilibria of an underlying game with perfect information (no moves by nature) are loss-aversion equilibria of any extension of this game.

Proposition 2 *For any game G with perfect information, any pure-strategy Nash equilibrium of G is both a myopic and a non-myopic loss-aversion equilibrium of (G, λ) for any λ.*

Proof: If σ is a pure strategy equilibrium giving payoffs $x = (x_i)_{i \in I}$, then σ is also a pure-strategy equilibrium in $L(G, \lambda, x)$. This is true, since any deviation from σ by a player i in G is not profitable, i.e. it gives no more than x_i. Therefore, in $L(G, \lambda, x)$ it also gives no more than x_i as the payoffs in $L(G, \lambda, x)$ are no higher than in G. Thus, the outcome of σ which also gives x in $L(G, \lambda, x)$ cannot be improved on by a unilateral deviation, and x is therefore a loss-aversion equilibrium. To show that the proposition holds also for non-myopic loss-aversion equilibria, note first that with a profile of pure strategies σ in a game of perfect information, each information set is reached with probability one or probability zero. Exactly one terminal node is reached with probability one, and the payoffs at this node are exactly the consistent reference points of the players for the profile σ. If a player has a deviation with a consistent reference point that gives her more, then the same deviation gives a higher expected payoff (without loss aversion evaluation) than that of σ, in contradiction with the assumption that σ is a Nash equilibrium of G. ■(Proposition 2)

Proposition 3 *For any extended game* (G, λ), *there exists a myopic loss-aversion equilibrium.*

Proof:[2] The proof is by the use of Kakutani's fixed point theorem. Assume an extended game (G, λ). We define the correspondence f from $\Sigma \times [\underline{r}, \bar{r}]^I$ to itself as follows. $(\sigma', r') \in f(\sigma, r)$ if σ'_i is a best response to σ_{-i} in the game $L(G, \lambda, r)$ for all $i \in I$, and r'_i is the payoff to i from (σ'_i, σ_{-i}) in the game $L(G, \lambda, r)$ for all $i \in I$.

To apply Kakutani's fixed point theorem we need to show that the domain is non-empty, compact and convex and that the correspondence is nonempty, convex valued, and has a closed graph.

Both the strategy space and $[\underline{r}, \bar{r}]^I$ are non-empty, compact and convex, and therefore so is their product. The correspondence is non-empty as for every (σ, r) and each i there exists a best response (at least one pure strategy is a best response) σ'_i to σ_{-i} in $L(G, \lambda, r)$, and taking r'_i as the payoff from (σ'_i, σ_{-i}) in $L(G, \lambda, r)$ we have an element (σ', r') in $f(\sigma, r)$. If there is more than one best response for a player i, then all give the same payoff, and so does any convex combination of the best responses, therefore the correspondence is convex valued. The correspondence has a closed graph from the continuity of the payoffs as a function of r and the fact that the best-response function has a closed graph.

[2]This proof was suggested by J-F. Mertens.

Therefore, applying Kakutani's fixed point theorem, there exist σ^* and r^* such that $(\sigma^*, r^*) \in f(\sigma^*, r^*)$. From the definition of f, we have that σ^* is a myopic loss-aversion equilibrium of (G, λ), giving payoffs of r^*. ■(Proposition 3)

The existence of non-myopic loss-aversion equilibrium is not guaranteed for non-simultaneous games. An example of a game with no non-myopic loss-aversion equilibria is Example 7 in Section 4. This might lead us to believe that we have used too restrictive a definition, as non-emptiness is surely a desirable property of any solution concept. The basic cause of the non-existence is as follows. The value of a final outcome that is possible given a strategy profile could be different when evaluated as part of different lotteries (depending on how disappointing the outcome is, relative to the entire lottery). Requiring that the strategy of each player be preferred to any other strategy (taking the strategies of the other players as fixed) at more than one information set can lead to no strategies being preferred at all information sets. Without an appropriate notion of timing of the game, it is difficult to suggest how to balance different preferences at different nodes. This problem is sidestepped with the myopic concept, as time doesn't enter in the evaluation and the reference point is fixed (as a function of the strategy profile) before play starts. Another approach is that of Ferreira, Gilboa and Maschler (1995), which deals with games with preferences that change during the play of the game. They give a solution concept (credible equilibrium) that always exists. However, this concept makes assumptions about the updating of preferences that are not suitable for the reference-dependent utility function we use.

Remark: For any extended game (G, λ) with $\lambda_i = 0$ for all $i \in I$, the set of loss aversion equilbria (either myopic or non-myopic) of (G, λ) coincides with the set of Nash equilibria of G. This is obvious from the definitions, as the evaluation of any lottery for a player with $\lambda_i = 0$ gives the expected utility of this lottery.

4 Examples

This section provides some examples that clarify and exemplify the previous sections. Example 1 demonstrates that the Allais paradox is no longer a paradox when loss aversion is taken into account. Examples 2 and 3 examine two classical games, and shows how different levels of loss aversion affect the equilibria of these games. Examples 4 and 5 are extreme cases showing that the number of Nash equilibria can be either greater or smaller than the number of loss aversion equilibria. In Example 4 there is a unique loss aversion equilibrium and a continuum of Nash equilibria. Example 5 gives the opposite situation, with a unique Nash equilibrium and a continuum of loss aversion equilibria. Example 6 is an extensive form game where there exists a unique non-myopic loss aversion equilbrium and a unique myopic loss-aversion equilbrium, but they are completely different. Except for Examples 6 and 7, all the examples are given in normal form, with the understanding that they represent simultaneous games, where each player has a single information set.

Example 1

We start with a single-player decision problem, the Allais paradox, from Allais (1953), and demonstrate that if we assume subjects are loss-averse there is no paradox. This was done in Gul (1991), with disappointment aversion. Subjects were presented with two pairs of lotteries, and asked to choose one from each pair. The first pair, lotteries A and B, are the following (in Francs):

Lottery A

Outcome	0	100*m*	500*m*
Probability	0	1	0

Lottery B

Outcome	0	100*m*	500*m*
Probability	0.01	0.89	0.1

The second pair, lotteries C and D, are the following:

Lottery C

Outcome	0	100*m*	500*m*
Probability	0.89	0.11	0

Lottery D

Outcome	0	100m	500m
Probability	0.9	0	0.1

The modal choice was lotteries A and D, even though this selection is not consistent with expected utility maximization for any utility values of the outcomes. However, for a subject with a loss-aversion coefficient in the range $(\frac{1}{9}, 10)$, which is the case with virtually all experimental results, this selection of lotteries does not cause any contradictions.

We assume that a subject will evaluate a lottery using a reference point that is consistent with the lottery (which is unique from Lemma 1). Thus, a lottery that has a higher consistent reference point is preferred over a lottery with a lower consistent reference point. If the values of the outcomes are 0 for 0m, 1 for 500m and 0.91 for 100m, then any subject with a loss aversion coefficient above $\frac{1}{9}$ will choose A over B, and any subject with a loss aversion coefficient below 10 will choose D over C.

The remaining examples are of games with more than one player. The elements in each square of the payoff matrices in the following examples are the values of the outcomes of the relevant pure strategies. These values are transformed into final utilities (for given reference points) according to Formula (1).

Example 2 *The Battle of the Sexes*

	Boxing	Ballet
Boxing	2, 1	0, 0
Ballet	0, 0	1, 2

The battle of the sexes has two pure-strategy Nash equilibria, (Boxing,Boxing) and (Ballet,Ballet), and one mixed strategy equilibrium with each player playing the strategy with his/her most preferred outcome with probability $\frac{2}{3}$.

Both pure strategy equilibria are also loss aversion equilibria (Proposition 2). There is also a mixed strategy equilibrium. It can be calculated by solving the following equations, which specify that each player is indifferent between both of his/her strategies, given his/her reference point, and that the reference point is equal to the utility of the expected outcome. p represents the probability that player 1 plays Boxing and q the probability that player 2 plays Boxing.

$$2q+(-\lambda_1 r_1)(1-q) = -\lambda r_1 q+(1-q) = r_1 \tag{13}$$

$$-p\lambda_2 r_2+(1-p) = (-\lambda_2 r_2)p+2(1-p) = r_2 \tag{14}$$

With the restriction that p and q are in $[0,1]$, there is a unique solution to these equations, which is given (for $\lambda \gg 0$) by[3]

$$p = 1-\frac{-3-2\lambda_2+\sqrt{9+8\lambda_2(2+\lambda_2)}}{2\lambda_2} \tag{15}$$

$$q = \frac{-3-2\lambda_1+\sqrt{9+8\lambda_1(2+\lambda_1)}}{2\lambda_1} \tag{16}$$

$$r_i = \frac{-3+\sqrt{9+8\lambda_i(2+\lambda_i)}}{2\lambda_i(2+\lambda_i)}, \quad i=1,2. \tag{17}$$

It can be verified that p is decreasing as a function of λ_2 and q is increasing as a function of λ_1. Each r_i is decreasing as a function of λ_i. This means that a player who becomes more loss averse has a higher probability of receiving his/her preferred outcome in the mixed-strategy equilibrium, but receives a lower utility. A change in a player's loss-aversion coefficient does not affect the other player's payoff in the mixed strategy loss-aversion equilibrium.

Example 3 *Matching Pennies*

	H	T
H	1, 0	0, 1
T	0, 1	1, 0

For any values of λ_1 and λ_2, the only loss-aversion equilibrium strategies in extended matching pennies are for each player to play each pure strategy with probability $\frac{1}{2}$. The payoffs

[3]There is a unique solution also for the case where $\lambda_i = 0$, which is the limit of Equations (15)-(17) as λ_i tends to zero.

and the reference points are $r_i = \frac{1}{2+\lambda_i}$, thus as a player becomes more loss averse, she receives a lower payoff. The payoff of each player is not affected by a change in the loss-aversion coefficient of the other player. We show in Section 5 that neither of these properties holds in general.

We now give two examples to show that the number of loss-aversion equilibria in an extended game can be either higher or lower than the number of Nash equilibria in the underlying game.

Example 4

	L	R
A	1, 0	0, 1
B	0, 1	1, 0
C	2, 0	-1, 1
D	-1, 1	2, 0

This game has a continuum of Nash equilibria. In all of them player 2 mixes his two strategies with probabilities $(\frac{1}{2}, \frac{1}{2})$. Player 1's strategy is of the form $(\alpha, \beta, \frac{1}{2} - \alpha, \frac{1}{2} - \beta)$, where α and β are in $[0, \frac{1}{2}]$. When $\lambda = (1,1)$, there is only one loss-aversion equilibrium. Player 2 still mixes with probabilities $(\frac{1}{2}, \frac{1}{2})$. Player 1 uses the mixed strategy $(\frac{1}{2}, \frac{1}{2}, 0, 0)$.

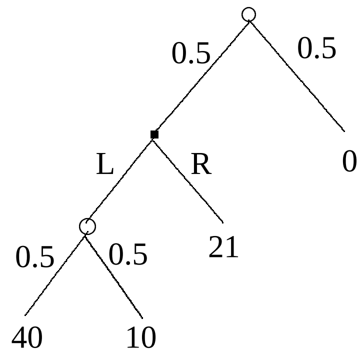
Example 5

	L	R
A	1, 0	0, 1
B	0, 1	1, 0
C	0.4, 2	0.4, 2

This game has a unique Nash equilibrium. Player 2 mixes his two strategies with probabilities $(\frac{1}{2}, \frac{1}{2})$. Player 1's strategy is $(\frac{1}{2}, \frac{1}{2}, 0)$. The extended game with $\lambda = (1,1)$ has a continuum of loss-aversion equilibria. In these, player 2 mixes with probabilities $(\alpha, 1-\alpha)$, where $\frac{3}{7} < \alpha < \frac{4}{7}$, and player 1 plays the pure strategy C.

In Examples 2 and 3 the number of Nash equilibria is equal to the number of loss aversion equilibria in any extension of these games. We therefore see that there is no fixed relationship

Figure 1: The tree for Example 6



between the quantity of Nash equilibria in the underlying basic game and the number of loss aversion equilibria in the extended games.

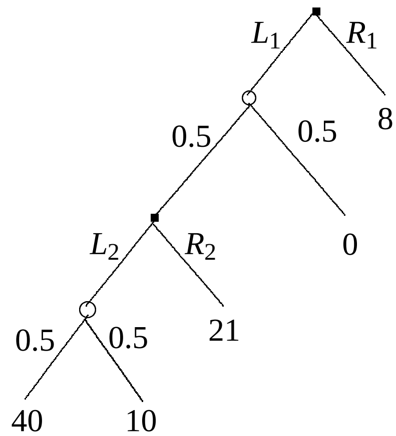
The following is an example of a one-player extensive form game which has different myopic loss-aversion equilibria and non-myopic loss-aversion equilibria.

Example 6

The tree for this example is given in Figure 1.

There is one player, with $\lambda = 1$. There are two nodes belonging to nature (the hollow circles), and one decision node of the player (the solid square). We first calculate the myopic loss-aversion equilibria. If the player chooses R, he faces the lottery giving 0 with probability $\frac{1}{2}$ and 21 with probability $\frac{1}{2}$, i.e. $(0, 0.5;\ 21, 0.5)$. The reference point consistent with this lottery is 7. If the player chooses L she faces the lottery $(0, 0.5;\ 40, 0.25;\ 10, 0.25)$. The consistent rerference point of this lottery is $8\frac{1}{3}$, so choosing L is the unique myopic loss-aversion equilibrium. We now calculate the non-myopic loss-aversion equilibria. Starting at the subtree headed by the decision node, R gives 21 for sure, so has a consistent reference point of 21. L gives $(0.5, 40;\ 0.5, 10)$, with a consistent reference point of 20, so the only non-myopic loss-aversion equilibrium is to choose R. We therefore see that the game has different non-myopic and myopic loss-aversion equilibria. This difference can be understood as follows. In the non-myopic equilibrium, when the player is called on to choose at her decision node, he adusts her expectations to reflect the fact that she will not receive 0, and her reference point takes this

Figure 2: The tree for Example 7



into account. In the myopic equilibrium, the player still has the 0 figuring in the calculation of her reference point when she chooses between L and R, even though she will not receive it. This happens when we have slow adjustment of reference points. Another interpretation could be that a player (the principal) sends agents to play for her at her information sets, and they are given their instructions in advance. If the principal is not sophisticated, she does not take into account that when an agent will be called upon, this might convey information about what has occurred in the game and this leads to myopic behavior. For this example, using a reference point which still considers the possibility of receiving 0 makes the 10 of the second lottery more attractive than it would be with a higher reference point, and therefore the lottery is preferred to the sure 20 for this case.

The following is an example of a one-player extensive form game which has no non-myopic loss-aversion equilibria.

Example 7

The tree for this example is given in Figure 2.

There is one player, with $\lambda = 1$. There are two nodes belonging to nature (the hollow circles), and two decision nodes of the player (the solid squares). Note that if the player starts with L_1 then she is in the situation of Example 6. The following story is behind the decision tree. The first decision is whether to study at Harvard business school (L_1) or to get a job at McDonalds (R_1). If the business school is chosen, with probability $\frac{1}{2}$ she fails in her studies (the right branch) and gets a very low payoff, having spent much time and money. If she succeeds, and is awarded her MBA (the left branch), she now has to decide whether to accept a job on Wall street (R_2) or try for a PhD (L_2), with probability of success equal to $\frac{1}{2}$. The player has 4 pure strategies, which are $\sigma^1 = (L_1, L_2)$, $\sigma^2 = (L_1, R_2)$, $\sigma^3 = (R_1, L_2)$ and $\sigma^4 = (R_1, R_2)$. There are no non-myopic loss-aversion equilibria in this game, according to the following reasoning. At the first decision node, the player strictly prefers σ_1 to all other strategies, as she evaluates this outcome as equivalent to a sure $8\frac{1}{3}$. Therefore, this is the only possible candidate for a non-myopic loss-aversion equilibrium. However, at the second decision node σ^2 is preferred to σ^1 and therefore σ^1 is not a non-myopic loss-aversion equilibrium. The unique *credible* equilibrium of this game (see Ferreira, Gilboa and Maschler, 1995) is σ^4, which is to take the job at McDonalds, as the player is sophisticated enough to know that if she succeeded to get her MBA, she would go for the job on Wall street, and she prefers the sure thing of McDonalds to the 50:50 gamble between Wall street and failure.

5 Comparative Statics

It is not obvious how to compare loss-aversion equilibria of games with varying loss-aversion coefficients, as the correspondence between loss aversion coefficients in extended games and their respective loss-aversion equilibria is not continuous (similar to regular games and Nash equilibria).

However, we can provide a number of examples which have unique loss-aversion equilibria which show that increasing loss aversion of a player can either increase or decrease the payoffs for the player herself and for the other players.

Example 3 (matching pennies) has a unique loss-aversion equilibrium for each value of (λ_1, λ_2). This equilibrium gives payoffs of $(\frac{1}{2+\lambda_1}, \frac{1}{2+\lambda_2})$. Thus, in this example, when a player becomes more loss averse, she receives a lower payoff and the payoff of the other player remains the same. The same happens in the payoffs of the mixed equilibrium of Example 2 (the battle of the sexes).

We now give a 3-player game with a unique loss-aversion equilibrium for most values of λ (there are always unique equilibrium strategies for players 1 and 2, and for almost all values of λ player 3 has a unique equilibrium strategy), and in which when player 1 becomes more loss-averse, her payoff increases and the payoff of another player decreases.

Example 8

	L	R
T	2, 0, 0	0, 1, 1
B	0, 2, 0	1, 0, 1
	F	

	L	R
T	4, 0, 0.45	0, 1, 0.45
B	0, 2, 0.45	2, 0, 0.45
	S	

The strategy sets of this game are as follows: Player 1 chooses either T or B. Player 2 chooses L or R, and player 3 chooses either F (the first matrix) or S (the second one). We now show that the game extended with $\lambda = (1,1,1)$ has a unique loss-aversion equilibrium, and the same applies for the game extended with $\lambda = (2,1,1)$.

There are no pure-strategy equilibria for this game. We calculate the loss-aversion equilibrium as follows: using the calculations for Example 2 (note that the payoffs for players 1 and 2 are similar to those of the battle of the sexes for either choice of player 3), we find that in any loss-aversion equilibrium the probability of player 1 playing T is

$$p = 1 - \frac{-3 - 2\lambda_2 + \sqrt{9 + 8\lambda_2(2 + \lambda_2)}}{2\lambda_2} \tag{18}$$

and the probability of player 2 playing L is

$$q = 1 - \frac{-3 - 2\lambda_1 + \sqrt{9 + 8\lambda_1(2 + \lambda_1)}}{2\lambda_1} \tag{19}$$

Faced with these strategies of players 1 and 2, playing F gives player 3 a payoff of $\frac{1-q}{1+q}$, and playing S gives player 3 a payoff of 0.45. Therefore, if $\frac{1-q}{1+q} \neq 0.45$ there is a unique loss-aversion equilibrium of the game.

For $\lambda = (1,1,1)$, $q = \frac{-5+\sqrt{33}}{2} \approx 0.37228$, so playing F gives player 3 a payoff of approximately 0.457427, which is greater than 0.45. With player 3 playing F, players 1 and 2 also receive a payoff of $\frac{-3+\sqrt{33}}{6} \approx 0.457427$. Therefore, the loss-aversion equilibrium for $\lambda = (1,1,1)$

has $p = q \approx 0.37228$, player 3 choosing F, and the payoffs are approximately 0.457427 for each player.

For $\lambda = (2,1,1)$, p remains unchanged, and $q = \frac{-7+\sqrt{73}}{4} \approx 0.38600$, so playing F gives player 3 a payoff of approximately 0.44300, which is less than 0.45. Therefore, player 3 will play S. With player 3 playing S, players 2 receives a payoff of $\frac{-3+\sqrt{33}}{6} \approx 0.457427$, and player 1 receives $2\frac{-3+\sqrt{73}}{16} \approx 0.69300$. Therefore, the loss-aversion equilibrium for $\lambda = (2,1,1)$ has $p \approx 0.37228$, $q \approx 0.38600$, player 3 choosing S, and the payoffs are approximately (0.457427,0.69300,0.45).

In summary, as player 1's loss-aversion increased, her payoff increased, that of player 3 decreased and that of player 2 remained the same (by multiplying the payoffs of player 2 in the second matrix by a positive constant, we could have her payoff increasing or decreasing). This is not too surprising, recalling that the result is analogous to what occurs in the following pair of three player games, using regular expected utility.

Example 9

Game 1:

	L	R
T	**2**, 0, 0	0, 1, 1
B	0, 2, 0	1, 0, 1
	F	

	L	R
T	**4**, 0, 0.6	0, 1, 0.6
B	0, 2, 0.6	2, 0, 0.6
	S	

Game 2:

	L	R
T	**1**, 0, 0	0, 1, 1
B	0, 2, 0	1, 0, 1
	F	

	L	R
T	**2**, 0, 0.6	0, 1, 0.6
B	0, 2, 0.6	2, 0, 0.6
	S	

These are 3-player games differing only in two of player 1's payoffs which are lower in the second game than in the first. Each game has a unique Nash equilibrium. The payoffs for the Nash equilibrium of game 1 are $(\frac{2}{3}, \frac{2}{3}, \frac{2}{3})$. The payoffs for the Nash equilibrium of game 2 are $(1, \frac{2}{3}, 0.6)$. Player 1's payoff is higher in the equilibrium of the second game (in which she had lower payoffs), while player 3's payoff is lower in the equilibrium of the second game.

6 Directions for future research

One of the goals of game theory is to make predictions. In the previous sections we have made predictions of outcomes of games, when reference dependence and loss aversion are taken into consideration. A first step in testing these results is to measure the loss aversion of individuals. Virtually all work on loss aversion has looked for averages, and not dealt with variations between individuals. Significant differences between averages in different experiments could indicate heterogeneity. A related project is to try to correlate the level of loss aversion of an individual with factors such as age, social status, gender, culture etc. Such work has been done on risk aversion, and experimental and empirical evidence shows that women are more risk averse than men.[4] Since many gambles naturally include outcomes both above and below one's reference point, increased loss aversion would lead to a higher measure of risk aversion. An interesting hypothesis therefore, is that women are more loss averse than men. Once loss aversion has been measured, experimental games could test the predictions of this paper. The games would be such that differing levels of loss aversion would lead to different equilibrium strategies. Since the results are based on the assumption that the loss-aversion characteristics (as well as the payoffs) are common knowledge, there would have to be a stage where the players learn about each other's level of loss aversion.

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[4]For examples of experimental evidence, see Levin, Snyder and Chapman (1988), Hudgens and Fatkin (1985), Zinkhan and Karande (1991), Arch (1993), Kogan and Wallach (1964), Slovic (1964), Maccoby and Jacklin (1974). For empirical evidence see Jianakoplos and Bernasek (1996).

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