Hazardous Facility Siting When Cost Information is Private: An Application of Multidimensional Mechanism Design

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Abstract. The siting of noxious facilities often involves externalities that extend beyond the border of the community selected as a site. Thus, the private information of each community is potentially a vector of costs comprising a cost for each of the possible sites. I characterize the conditions for the existence of a mechanism that is incentive compatible, individual rational, and budget balancing and show that efficient mechanisms under reasonable assumptions will satisfy these conditions. However, incentive compatibility implies a pattern of compensation payments that often conflicts with commonly held views on how communities should be compensated for environmental costs.

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1. Introduction

A group of communities faces the problem of selecting a site for a hazardous facility. Suppose that some of the costs associated with each site are the private information of the individual communities. If the communities as a group would like to use that information in deciding where to site the facility (e.g., to efficiently site the facility), then the group must implement an incentive mechanism that is able to elicit truthful reports of the private information. However, the requirement that communities have an incentive to truthfully report their costs places certain limitations on the outcomes that can be achieved. I characterize the conditions under which a siting policy can be implemented as an incentive compatible, individually rational, and budget balancing mechanism. Under certain reasonable conditions I show that an efficient siting policy can be implemented in this way. However, incentive compatibility can imply that an increase in the cost of a community can lead to a decrease in expected compensation.

The nature of the problem of siting hazardous facilities makes it a natural application for multidimensional mechanism design techniques because of the potential for cross-boundary environmental costs. Suppose that there are five potential sites for a hazardous facility. For each community there could be five different cost levels associated with the hazardous facility, one for each of the potential sites. If that cost information is private, then the community's private information is vector valued and multidimensional mechanism design techniques are required. Hence, an incentive mechanism in such an environment must elicit a truthful report of a vector of information rather than just a scalar value.

A number of researchers have modelled the siting problem as one of inducing communities to reveal hidden cost information, but most of these papers model each community's private information as a scalar value rather than a vector. These papers assume that the only private information possessed by a community is the cost associated with a site within its boundaries. (The costs associated with sites outside the boundaries of a community are assumed to be public information.) Kunreuther and Kleindorfer (1986) and Kunreuther, et.al. (1987) analyze the max-min strategies in a sealed-bid auction-like mechanism, and O'Sullivan (1993) considers the Nash equilibrium of an auction-like mechanism when there are two communities. Richardson and Kunreuther (1993) propose a dynamic mechanism for the siting of hazardous facilities that they show performs well in experiments, but they fail to analyze the properties of its equilibrium. Cramton, Gibbons, and Klemperer (1987) results on dissolving partnerships can be applied to the problem of selecting a site from hazardous facilities. In their scalar mechanism design model they show that a simple bidding mechanism can achieve efficiency while maintaining a balanced budget for the mechanism designer as long as each community who is the host of a potential site has some of its own waste to be disposed of.[1] Ingberman (1995) is critical of most of these papers for not considering the negative effects of a site on neighboring communities as well as the host community. Ingberman argues that a host community has an incentive to shift as much of the costs as possible onto its neighbors by locating the facility near its border.[2] However, a community need not be adjacent to a site to experience the negative effects of a hazardous facility. Cross-border effects can arise from being down-wind or down-stream from a site or from being along the transportation routes for hazardous materials enroute to the site. These circumstances imply a model where each site could have a different effect on a community even if the site is not within the boundaries of the community.

An efficient mechanism must be able to consistently select the site with the lowest costs while still eliciting truthful reports from the communities. O'Sullivan (1993) shows that when cost information is scalar, then auction-like mechanisms can consistently achieve efficiency. Despite the increased complication due to the multidimensional aspect of the problem I show

[1] Applying the results in Cramton, Gibbons, and Klemperer (1987) to the problem of siting hazardous facilities also requires the assumption that there are constant returns to scale in hazardous facilities. That is, if community i's share of the waste is r_i and its cost of hosting the site for everyone's waste is c_i, then its cost of disposing of only its own waste is $r_i c_i$.

[2] Of the papers mentioned Richardson and Kunreuther (1993) is the only one that explicitly considered cross-boundary effects. O'Sullivan (1993) is able to avoid the criticism by considering only two communities.

that efficient siting policies can be implemented with a balanced budget. Hence, incentive mechanisms can be designed that are immune from Ingerman's (1995) criticism of previous models.

The model I present is very closely related to those presented by Jehiel and Moldovanu (1996), Jehiel, Moldovanu, and Stacchetti (1996a and 1996b), and Krishna and Perry (1997) who consider auctions with externalities. Other mechanism design problems where types are multidimensional include Armstrong (1996), Bernheim and Whinston (1986), McAfee and McMillan (1988), McAfee, McMillan, and Whinston (1989), and Rochet (1985).[3]

I present a characterization of incentive compatible direct mechanisms for siting a hazardous facility. I also derive a number of properties of the transfer or payment function that are necessary for incentive compatibility. The expected transfer payment received by a community exhibits an entropy-like property where a community's expected compensation decreases as its costs move away from a vector of equal costs for all sites. I also present a characterization of siting policies that could be implemented as part of an incentive compatible, individually rational, budget balancing mechanism. Makowski and Mezzetti (1994) provide a similar characterization for efficient trading mechanisms. My result generalizes their theorem in that I considers all mechanisms not just efficient mechanisms. I show that even though the requirements of incentive compatibility and individual rationality are more stringent in multidimensional mechanism problems it is likely that efficiency can still be achieved with a balanced budget.

[3] Jehiel, Moldovanu, and Stacchetti (1996b), McAfee and McMillan (1988), Krishna and Perry (1997), and Rochet (1985) provide necessary and sufficient conditions for the implementability of a multidimensional mechanism. Armstrong (1996), Jehiel, Moldovanu, and Stacchetti (1996a and 1996b), Krishna and Perry (1997), McAfee and McMillan (1988), McAfee, McMillan, and Whinston (1989) consider the properties of optimal mechanisms from the standpoint of maximizing the revenue or profits of the designer. Jehiel and Moldovanu (1996) consider the participation decision in first-price auctions. Bernheim and Whinston (1986) show that first-price menu auctions acheive efficiency.

2. The Model

Suppose that a group of communities face a decision problem of where to site a hazardous facility. In addition, the communities may need to decide whether to build the facility at all. It is presumed that the group would like the decision to be based on the costs (and benefits) associated with each of the potential sites. For instance, an efficient siting arrangement would be the one that imposes the lowest total cost on the communities as a group. Alternatively, the group may wish to use the cost information to compensate communities who suffer largest environmental damage as a result of the siting. However, if the cost information is not common knowledge but instead known only to the individual communities, then a siting policy that is cost based will require the use of an incentive mechanism to induce the communities to truthfully reveal their costs either indirectly through their actions or directly by announcing them.

Part of the problem faced by the communities may be to decide whether to build the facility at all. Even if some cost information is private, there may be enough common information for the group to know that they (as a group) are always better off building the facility rather than continuing without one. However, there are likely to be many situations where the decision to build the facility or not must be based on the information that is privately held by individual communities.

Let the $M_0 = \{0, \ldots, m\}$ index the set of potential outcomes. That is, M_0 is the union of the set of potential sites for the hazardous facility (outcomes $M = \{1, \ldots m\}$) and the outcome corresponding to not building the facility (allocation $\{0\}$).

Let $N = \{1, \ldots, n\}$ index the set of communities that could be affected by the siting of the hazardous facility.[4] Let c denote an $m \times n$ matrix of costs where the element c_{ji} is community i's cost associated with site j. I normalize the cost to each community of not

[4] I focus on the preferences of communities and not the individuals within the communities. Thus, the model ignores the possibility [discussed by Sullivan (1990)] that individuals may move between communities to either avoid costs associated with a site or to receive compensation. Sullivan (1992) proposes a lottery siting mechanism that reduces the distortions associated with these effects.

building the facility to zero. c_{ji} is interpreted as the net negative effect on community i from site j. Each community's net cost can be decomposed into three parts.

$$c_{ji} = c^e_{ji} + c^n_{ji} - b_{ji}$$

where $c^e_{ji} \geqslant 0$ and $c^n_{ji} \geqslant 0$ are community i's environmental cost and share of the construction cost (the nonenvironmental cost) for site j and $b_{ji} \geqslant 0$ is i's benefit from being able use site j. I assume that the benefit b_{ji} and construction cost c^n_{ji} are common knowledge, and the environmental cost c^e_{ji} is the private information of community i. Net costs can be negative so that a particular site may provide net benefits to a community. Community i's vector net costs are c_i the ith column of c. While c_i is known to community i, it is considered a random vector by the other communities. I assume that the communities have a common belief regarding the distribution of $(c_1, \dots, c_n)$ where each community's cost vector is believed to be independent of the other communities cost vectors. I assume that for each $i \in N$, the support of c_i, denoted $\Omega_i \subseteq \mathbb{R}^m$, is compact and convex and has a nonempty interior such that every point on the boundary of Ω_i is arbitrarily close to a point in the interior of Ω_i. Let $\Omega = \times_{i \in N} \Omega_i$, $\Omega_{-i} = \times_{j \in N \setminus \{i\}} \Omega_j$, and c_{-i} denote all of the columns of c except the ith. The set of all possible probability distributions over the sites is $\Xi = \{(x_1, \dots, x_m) \in [0,1]^m | \sum_{j \in M} x_j \leqslant 1\}$ The sum of the probabilities over the sites can be less than one to allow for the possibility that the facility is not built.[5]

A mechanism for allocating hazardous facilities selects a site for the hazardous facility and the payments to be made to the participants based on reports from the communities. I assume that the mechanism is committed to prior to the communities reports. However, how the mechanism is selected and committed to is beyond the scope of the paper. Instead

[5] Allocation $\{0\}$ need not be interpreted as the allocation corresponding to not siting the facility. Mathematically it differs from the other allocations only by fact that each community's cost of that allocation is known by the other communities. It is possible to consider a situation where the cost associated with every potential allocation (i.e., all allocations with a nonzero probability of being selected) is private information. To do this simply define the siting policy such that $\Phi_0(c) = 0$ for all c.

I analyze the properties of all direct, deterministic and differentiable mechanisms that are feasible, in the sense that they induce the truthful revelation of the private cost information, individual communities voluntarily participate, and the payments made by the mechanism balance.

In a direct mechanism communities report their cost vectors. By the Revelation Principal any equilibrium outcome of an indirect mechanism can be implemented through a direct mechanism where truthful reporting is the equilibrium strategy that results in the desired outcome. If an outcome cannot be achieved through a direct mechanism, then it is not possible to achieve it by use of an indirect mechanism. Therefore, while direct mechanisms are rarely used in practice, the study of equilibrium outcomes for direct mechanisms is informative about what outcomes can result from indirect mechanisms such as competitive bidding or lotteries.

Definition 1. A *direct siting mechanism* is a triple $\langle \mathbf{T}, \Phi, Q \rangle$, $\mathbf{T} : \Omega \to \mathbb{R}^n$, $\Phi : \Omega \to \Xi$, and $Q = (Q^1, \ldots, Q^n)$ *such that* $Q^i : \Omega_{-i} \to \Xi$.

In a mechanism $\mathbf{T} = (T_1, \ldots, T_n)$ defines the transfer payments, $\Phi = (\Phi_1, \ldots, \Phi_m)$ defines the rule for selecting the site when everyone participates in the mechanism, and $(Q^1, \ldots, Q^n)$ (where for each $i \in N$, $Q^i = (Q^i_1, \ldots, Q^i_m)$) defines the rule for selecting the site when a community does not participate.

The function $\mathbf{T}(c)$ returns the vector of transfer or compensation payments to the communities as a function of the announced cost matrix. $T_i(c)$ is the payment made to community i when the cost matrix $\mathbf{c}$ is announced. While I refer to it as a payment made to community i, T_i can be negative, and thus, the model allows for the possibility that communities make payments rather than receive them.

The function $\Phi_j(c)$ indicates the probability that site j will result when the cost matrix $\mathbf{c}$ is announced. (If the mechanism is deterministic, then $\Phi_j(c) = 1$ or 0.) The probability that no facility is built is $\Phi_0(c) = 1 - \sum_{j \in M} \Phi_j(c)$. I refer to Φ as a siting policy. Let $\phi^i_j(c_i)$

denote the probability that site j is chosen conditional on community i announcing cost vector c_i and the other communities truthfully announcing their costs; since the columns of c are independent $\phi^i_j(c_i) = E_{-i}[\Phi_j(c)]$, where $E_{-i}[\,\cdot\,]$ denotes the expectation with respect the all of the costs except i's. Let $\phi^i = (\phi^i_1, \ldots, \phi^i_m)$. I restrict attention to distributions of c and the siting policies Φ such that the conditional assignment probabilities ϕ^i are continuous vector-valued functions. I use this assumption throughout in order to guarantee that the interim expected payoff functions are differentiable at points in the interior of Ω_i. Let $t^i(c_i) = E_{-i}[T_i(c)]$. Thus, $t^i(c_i)$ is community i's expected transfer payment when it announces cost vector c_i and the other communities announce their true cost.

Let $\pi^i(c'_i, c_i)$ denote community i's expected payoff when it announces cost vector c'_i but actually has cost vector c_i and the other communities announce their true cost. That is, $\pi^i(c'_i, c_i) = t^i(c'_i) - c_i \cdot \phi^i(c'_i)$. Furthermore, define the function $v^i(c_i) = t^i(c_i) - c_i \cdot \phi^i(c_i)$. The function v^i is community i's interim expected payoff.

Definition 2. The siting mechanism $\langle \mathbf{T}, \Phi, Q\rangle$ is *incentive compatible* (IC) if and only if for all $i \in N$ and for all $c, c'_i \in \Omega_i$, $v^i(c_i) \geqslant \pi^i(c'_i, c_i)$ (or equivalently $v^i(c_i) - v^i(c'_i) \geqslant -\phi^i(c'_i) \cdot [c_i - c'_i]$).[6]

The functions Q are the siting policies in the event that a community does not participate in the mechanism. That is, $Q^i(c_{-i}) = (Q^i_1(c_{-i}), \ldots, Q^i_m(c_{-i}))$ where $Q^i_j(c_{-i})$ is the probability that site j is selected when community i does not participate in the siting mechanism and the other communities do participate and announce costs c_{-i}.[7] Thus, at the interim

[6] The second inequality in the definition of incentive compatibility highlights its relationship to subgradient theory (see Rockafellar (1970)). The vector x'_i is a subgradient of a convex function v^i at c'_i if and only if for all $c_i \in \Omega_i$, $v^i(c_i) - v^i(c'_i) \geqslant x'_i \cdot [c_i - c'_i]$. Therefore, the following equivalence is immediate. A mechanism $\langle \mathbf{T}, \Phi, Q\rangle$ is incentive compatible if and only if for each $i \in N$ and all $c_i \in \Omega_i$, the vector $-\phi^i(c_i)$ is a subgradient of the convex function v^i at c_i. (Jehiel, Moldovanu, and Stacchetti (1996b) and Krishna and Perry (1997) also discuss the relationship between incentive compatibility and subgradients.)

[7] I only consider equilibria where everyone participates. Therefore, to check that such participation is indeed part of the Bayesian Nash equilibrium I need only check that every individual is not worse off participating under the Nash conjecture that all of the other communities are participating. A complete description of the game should include the definition of payoffs under any combination of actions (e.g., when two or

stage community i's beliefs regarding the siting of the facility when it does not participate is given by $\rho^i = E_{-i}[Q^i(c_{-i})]$ where $\rho^i = (\rho_1^i, \ldots, \rho_m^i)$. That is, community i expects site j to be chosen with probability ρ_j^i when i chooses not to participate.

If participation in the mechanism is voluntary, then participating communities must be better off participating than not. However, in siting hazardous facilities nonparticipation can potentially mean a number of different things. For instance, nonparticipating communities may be able to block the choice of certain sites over which they exercise some legal control. Or alternatively nonparticipation by one community may not effect the choice set of the participating communities. I assume that the mechanism designer selects each Q^i out of a feasible set $\mathcal{Q}^i$ where $\mathcal{Q}^i$ is the set of all functions from Ω_{-i} to $H_i \subseteq \Xi$. The specific implications of nonparticipation (i.e., rights of control over particular sites) are embodied in the difference between H_i and Ξ. For example, if nonparticipation by community i implies that the other communities cannot select site j, then $Q^i(c_{-i}) \in H_i$ implies $Q_j^i(c_{-i}) = 0$ and thus $\rho_j^i = 0$. Since they are part of the mechanism, at the interim stage of the game $\rho^1, \ldots, \rho^n$ are known to the communities.

When a community does not participate, it may be able to avoid some of the cost associated with a site (e.g., construction costs). It may also be excluded from the benefits from a facility. Hence, a community's expected cost when site j is chosen differs depending on whether or not the community participates. I assume that nonparticipating communities only suffer environmental costs. Hence, community i's net cost from site j when it does not participate is $c_{ji}^e = c_{ji} - s_{ji}$, where $s_{ji} = c_{ji}^n - b_{ji}$. I also assume that given a particular site will be chosen, the total net cost to society is not reduced by having one or more communities fail to participate in the mechanism. Let $s_i = (s_{1i}, \ldots, s_{mi})$. Therefore, community i's interim expected payoff when it does not participate and the other communities truthfully announce

more communities do not particiapte). However, those payoffs are unreached in equilibrium or in Nash deviations from equilbria where everyone participates. Hence, I leave those outcomes undefined except for the assumption that nonparticipation never lowers the net social cost associated with any given outcome.

their cost is $-(c_i - s_i) \cdot \rho^i$. The essential implication of the preceding discussion is that each community's interim expected payoff when it does not participate is a nonincreasing linear function of the difference between its net cost vector when it participates and the net costs that if avoids by not participating.

Definition 3. The siting mechanism $\langle \mathbf{T}, \Phi, Q \rangle$ is *interim individually rational* (IR) if and only if for all $i \in N$ and for all $c_i \in \Omega_i$, $v^i(c_i) \geqslant -(c_i - s_i) \cdot \rho^i$.

For the communities to implement the siting mechanism without the help of an outside party who could make up any budget deficits the sum of the transfer payments must not be positive. So that resources are not wasted within the group I will also assume that the sum of the compensation payments is not negative. Hence, the balanced budget condition.

Definition 4. The siting mechanism $\langle \mathbf{T}, \Phi, Q \rangle$ exhibits *an ex post balanced budget* (BB) if and only if for all $c \in \Omega$, $\sum_{i \in N} T_i(c) = 0$.

3. Incentive Compatibility

In this section I characterize siting policies that are consistent with incentive compatibility using the concept of cyclical monotonicity. The vector-valued function $-\phi^i$ is *cyclically monotone* if and only if

$$-\phi^i(c_i^1) \cdot (c_i^2 - c_i^1) - \phi^i(c_i^2) \cdot (c_i^3 - c_i^2) - \cdots - \phi^i(c_i^k) \cdot (c_i^1 - c_i^k) \leqslant 0,$$

for any finite set of cost vectors $\{c_i^1, \ldots, c_i^k\} \subset \Omega_i$. Cyclical monotonicity implies monotonicity.[8]

Theorem 1. *There exists a transfer function* $\mathbf{T}$ *such that* $\langle \mathbf{T}, \Phi, Q \rangle$ *is incentive compatible if and only if for all* $i \in N$, $-\phi^i$ *is cyclically monotone.*

[8] The function $-\phi^i$ is said to be *monotone* if for any $c_i, c_i' \in \Omega_i$, $-\phi^i(c_i) \cdot (c_i' - c_i) - \phi^i(c_i) \cdot (c_i - c_i') \leqslant 0$.

Proof. (only if): For any finite set of cost vectors $\{c_i^1, c_i^2, \dots, c_i^k\} \subset \Omega_i$, (IC) implies

$$\begin{aligned} v^i(c_i^2) - v^i(c_i^1) &\geqslant -\phi^i(c_i^1) \cdot [c_i^2 - c_i^1] \\ v^i(c_i^3) - v^i(c_i^2) &\geqslant -\phi^i(c_i^2) \cdot [c_i^3 - c_i^2] \\ &\vdots \\ v^i(c_i^1) - v^i(c_i^k) &\geqslant -\phi^i(c_i^k) \cdot [c_i^1 - c_i^k]. \end{aligned}$$

Summing the inequalities above results in

$$0 \geqslant -\phi^i(c_i^1) \cdot [c_i^2 - c_i^1] - \phi^i(c_i^2) \cdot [c_i^3 - c_i^2] - \dots - \phi^i(c_i^k) \cdot [c_i^1 - c_i^k],$$

for any finite set of cost vectors $\{c_i^1, \dots, c_i^k\} \subset \Omega_i$. Hence, $-\phi^i$ is cyclically monotone.

(if): Suppose that the transfer function is defined so that the interim expected payoff function v^i has the property that for all $c_i \in \Omega_i$, $-\phi^i(c_i)$ is a subgradient of v^i at c_i (I define subgradients in Footnote 6). Such a function v^i exists and is convex (by Theorem 24.8 in Rockafellar (1970)). Hence, for arbitrary $c_i', c_i \in \Omega_i$,

$$v^i(c_i) - \pi^i(c_i', c_i) = v^i(c_i) - v^i(c_i') + \phi^i(c_i') \cdot (c_i - c_i') \geqslant 0.$$

where the inequality follows from the convexity of v^i and the fact that $-\phi^i(c_i)$ is its subgradient. Therefore, the mechanism is incentive compatible. *Q.E.D.*

Rochet (1985) presents a similar characterization theorem where an interim expected payoff function v^i is shown to be consistent with incentive compatibility if and only if it is convex. When ϕ^i is differentiable on the interior of Ω_i, the cyclical monotonicity of $-\phi^i$ is equivalent to the symmetry and the negative semi-definiteness of the Jacobian matrix of ϕ^i. The symmetry of the Jacobian is necessary and sufficient for the existence of a solution to the differential equation $\nabla v^i(c_i) = -\phi^i(c_i)$ that follows from the first-order condition.

The negative semi-definiteness of the Jacobian ensures that the mechanism satisfies the second-order condition.

The following Lemma presents necessary and sufficient conditions for incentive compatibility that I use in the proofs of other results.[9]

Lemma 1. *The siting mechanism* $\langle \mathbf{T}, \Phi, Q \rangle$ *is incentive compatible, if and only if for all* $c_i, c_i' \in \Omega_i$,

$$v^i(c_i) - v^i(c_i') = -\int_0^1 \phi^i(\alpha c_i + (1-\alpha)c_i') \cdot (c_i - c_i') d\alpha. \tag{1}$$

and $-\phi^i$ *is cyclically monotone.*

(Proofs not found in the text are presented in the Appendix.)

Besides being used in the proofs of results presented later in the paper, the preceding lemma also has some interesting economic content. The lemma implies that differences in a community's interim expected payoff for different cost vectors depends only on the siting policy. As is also discussed by Krishna and Perry (1997), it is this condition that forms the basis for expected payoff equivalence results when private information is multidimensional. The function v^i must satisfy an initial condition (usually part of the individual rationality condition) and the differential equation $\nabla v^i = -\phi^i$ (which arises from incentive compatibility). Any two functions that satisfy the differential equation on an open convex set can only differ by an additive constant. Therefore, if two mechanisms implement the same siting policy and imply interim expected payoffs of v^i and $\hat{v}^i$ such that for at least one $c_i \in \Omega_i$, $v^i(c_i) = \hat{v}^i(c_i)$, then $v^i(c_i) = \hat{v}^i(c_i)$ for all $c_i \in \Omega_i$.

Clearly, the same property must hold for the expected transfer function t^i since (1) if and only if

$$t^i(c_i) - t^i(c_i') = c_i \cdot \phi^i(c_i) - c_i' \cdot \phi^i(c_i') - \int_0^1 \phi^i(\alpha c_i + (1-\alpha)c_i') \cdot (c_i - c_i') d\alpha.$$

[9] Similar characterization results are presented by Jehiel, Moldovanu, and Stacchetti (1996b), Krishna and Perry (1997), McAfee and McMillan's (1988), and Rochet (1985).

Therefore, once a siting policy has been decided upon, the mechanism designer has no leeway regarding the design of interim expected compensation payments except to add or subtract a constant from the function.

Lemma 1 also implies that the expected payoff function v^i is nonincreasing in a community's costs. In fact, interim expected payoff strictly decreases whenever the cost of a site is raised that has a positive expected probability of being selected. Except in the trivial case where the probability of all sites are zero, in expectation, a community is always worse off when it has higher costs. Therefore, even if the primary goal is to design a mechanism that compensates communities for higher costs, the only siting policy that holds community's harmless when they have higher costs is the policy of never sites the facility.

Define the set $\mathbf{i} = \{x \in \mathbb{R}^m | \, x = (\delta, \ldots, \delta), \, \delta \in \mathbb{R}\}$. (Note that it is possible for $\mathbf{i} \cap \Omega_i = \emptyset$.)

Theorem 2. *If $\langle \mathbf{T}, \Phi, Q \rangle$ satisfies incentive compatibility, then*

(a) $\phi^i(c_i) = \phi^i(c_i')$ *and* $t^i(c_i) = t^i(c_i')$, *if for any* $c_i, c_i' \in \Omega_i$, *such that* $c_i - c_i' \in \mathbf{i}$ *and* $\phi_0^i(c_i) = \phi_0^i(c_i') = 0$.

(b) $t^i(c_i) \geqslant t^i(c_i')$, *for* $c_i, c_i' \in \Omega_i$ *such that* $c_i' = \alpha c_i$ *and* $\alpha > 1$.

The results of Theorem 2 help to provide some intuition regarding the implications of incentive compatibility for the interim expected transfer functions t^i. Part (a) of Theorem 2 states that if the cost vector of a community decreases in such a way that the cost of each allocation decreases by the same amount while at the same time the probability of not siting the facility is zero along this price change path, then the community's expected transfer payment remains unchanged. Geometrically, in the region of Ω_i where ϕ_0^i is zero, the level curves of t^i are parallel to $\mathbf{i}$, the diagonal line of $\mathbb{R}^m$.

Part (b) of the theorem implies that a community can expect a lower payment from costs c_i than a vector of costs on a line between c_i and the origin. Furthermore, an immediate implication of part (b) is that when the origin is in Ω_i, community i maximizes its expected payment by announcing a cost of zero for all sites. That is, a community receives the highest expected payment when its expected cost is zero no matter what it announces, or alternatively, it is indifferent between all of the potential outcomes. The intuition for this result is straightforward. Suppose community i has cost vector $c_i = (0, \ldots, 0) \in \mathbf{i}$. Then community i's expected cost remains the same no matter what costs it announces (i.e., $c_i \cdot \phi^i(c_i') = 0$, for all $c_i' \in \Omega_i$). Therefore, the only way to give community i an incentive to truthfully reveal its costs is to provide it with the highest expected payment when it is indifferent between all of the potential allocations. More formally, incentive compatibility requires that $t^i(c_i) - c_i \cdot \phi^i(c_i) \geqslant t^i(c_i') - c_i \cdot \phi^i(c_i')$, for all $c_i' \in \Omega_i$. However, since $c_i = 0$, $t^i(c_i) \geqslant t^i(c_i')$, for all $c_i' \in \Omega_i$. Geometrically, the system of upper contour sets of t^i could be represented as the intersection of Ω_i and a system of concentric star-shaped sets with respect to the origin.[10]

Theorem 2 and the preceding discussion describe the properties of the interim expected payoff and transfer functions. While it is clear that once the siting policy has been set the designer has very little discretion over the interim expected payoff and transfer functions. However, a given interim expected payoff function can arise from a variety of different ex post transfer functions $\mathbf{T}$. Therefore, it is possible for a mechanism designer to influence the ex post transfers as long as that in expectation they satisfy the conditions necessary for incentive compatibility.

4. Efficient Siting Policies

In the present context an efficient siting policy is one where no matter what the profile of costs are in Ω, the outcome with the lowest overall cost is chosen.

Definition 5. A siting policy Φ is *ex post efficient* (E) if and only if for any other siting policy $\hat{\Phi}$ and for all $c \in \Omega$, $\sum_{i \in N} c_i \cdot \Phi(c) \leqslant \sum_{i \in N} c_i \cdot \hat{\Phi}(c)$.

[10] The set $A \subseteq \mathbb{R}^m$ is star-shaped with respect to the origin if for all $x \in A$ and all $\alpha \in [0, 1]$, $\alpha x \in A$.

An implication of this definition is that if Φ is efficient, then for all $c_i, c_i' \in \Omega_i$, $\sum_{i \in N} c_i \cdot \Phi(c_i) \leqslant \sum_{i \in N} c_i \cdot \Phi(c_i')$. To see this, notice that for any particular pair $c_i, c_i' \in \Omega_i$, define the alternate policy $\hat{\Phi}$ such that $\hat{\Phi}(c_i) = \Phi(c_i')$. It is this fact that underlies the logic of the Groves mechanism. In a Groves mechanism, each community's transfer payment is made equal to the total announced benefit (negative costs) of the other communities from siting the facility plus a function independent of the community's costs. A community has an incentive to make a truthful announcement. While a community may be able to reduce its cost by making a false announcement that causes the selection of an inefficient site that is lower cost for the community than the efficient site, when the siting policy is efficient such a gain is offset by a decrease in the benefits to the other communities and, hence, the community's transfer payment.

For Bayesian mechanisms, which I consider here, D'Aspremont and Gerard-Varet (1979) show that incentive compatible efficient mechanisms are Groves mechanisms in expectation. That is, each community's payoff is equal to the expected total benefits conditional on the community's costs plus a constant. Hence, the following lemma due to D'Aspremont and Gerard-Varet's (1979) (a proof is provided in the Appendix for completeness).

Lemma 2. *An efficient siting mechanism $\langle \mathbf{T}, \Phi, Q \rangle$ is incentive compatible if and only if for all $c_i, c_i' \in \Omega_i$,*

$$v^i(c_i) - v^i(c_i') = E_{-i}\left[\sum_{j \in N \setminus \{i\}} c_j \cdot [\Phi(c_i', c_{-i}) - \Phi(c_i, c_{-i})]\right] + c_i' \cdot \phi^i(c_i') - c_i \cdot \phi^i(c_i). \qquad (2)$$

The result states that a mechanism is incentive compatible if and only if a community's interim expected payoff is equal to a constant term plus the total expected gains from trade conditioned on the community's private information. Equation (2) if and only if

$$t^i(c_i) - t^i(c_i') = E_{-i}\left[\sum_{j\in N\setminus\{i\}} c_j \cdot [\Phi(c_i', c_{-i}) - \Phi(c_i, c_{-i})]\right]. \tag{3}$$

An efficient siting policy is clearly consistent with incentive compatibility and, hence, the efficient siting policy satisfies the conditions of Theorem 1. That is, efficiency implies that $-\phi^i$ is cyclically monotone.

For efficient siting mechanisms the implication of Part (b) of Theorem 2 that a community's expected transfer is maximized at the origin (if the origin is in Ω_i) can be derived directly from (3). By making an announcement of zero cost for all of the sites a community is essentially letting the preferences of the other communities determine the site choice. Hence, that siting choice will minimize the total cost of the other communities.

5. The Participation Constraint and The Balanced Budget Condition

In this multidimensional mechanism problem the individual rationality condition can be simplified in a way similar to scalar problems.

Lemma 3. *The siting mechanism $\langle \mathbf{T}, \Phi, Q\rangle$ is interim individually rational (IR) if and only if for all $i \in N$, $\min_{c_i\in\Omega_i}\left\{v^i(c_i) + (c_i - s_i)\cdot\rho^i\right\} \geqslant 0$.*

In most scalar mechanism problems, individual rationality is shown to hold everywhere if and only if the "worst type" is guarantied at least his reservation utility. In this multidimensional environment the participation constraint is complicated by the fact that a community's so called reservation utility also depends on his type. Thus, the worst type in terms of satisfying individual rationality is not necessarily the type with the lowest interim expected payoff. Define c_i^{*}[11] such that for all $c_i \in \Omega_i$,

$$\int_0^1 \left[\rho^i - \phi^i(\alpha c_i + (1-\alpha)c_i^*)\right] \cdot (c_i - c_i^*) d\alpha \geqslant 0.$$

Notice that c_i^* does not depend on the transfer function. When incentive compatibility is satisfied, c_i^* is the "worst type" in the sense that if individual rationality holds for c_i^*, then individual rationality holds for all types. To see this, notice that

$$\begin{aligned} v^i(c_i) + (c_i - s_i) \cdot \rho^i - \left[v^i(c_i^*) + (c_i^* - s_i) \cdot \rho^i\right] \\ = \int_0^1 \left[\rho^i - \phi^i(\alpha c_i + (1-\alpha)c_i^*)\right] \cdot (c_i - c_i^*) d\alpha \geqslant 0 \end{aligned}$$

where the equality follows from Lemma 1 and the inequality follows from the definition of c_i^*. Therefore, if incentive compatibility is satisfied and $v^i(c_i^*) + (c_i^* - s_i) \cdot \rho^i \geqslant 0$, then individual rationality is satisfied.

Given ρ^i finding c_i^* is straight forward because assuming incentive compatibility $v^i(c_i) + (c_i - s_i) \cdot \rho^i$ is convex in c_i with gradient equal to $\rho^i - \phi^i(c_i)$. Therefore, if $c_i^* \in \text{int}\Omega_i$, then $\rho^i = \phi^i(c_i^*)$, and if there exists $c_i' \in \Omega_i$ such that $\rho^i = \phi^i(c_i')$, then $\phi^i(c_i') = \phi^i(c_i^*)$. If there is no $c_i \in \Omega_i$ such that $\rho^i = \phi^i(c_i)$, then c_i^* is an element of the boundary of Ω_i.

With this simplification of the participation constraint I can now characterize siting policies that can be implemented with a balanced budget.

Theorem 3. *There exists a* $\mathbf{T}$ *such that the siting mechanism* $\langle \mathbf{T}, \Phi, Q \rangle$ *is incentive compatible, interim individual rational, and budget balancing, if and only if for all* $i \in N$,

[11] When $-\phi^i$ is cyclically monotone, c_i^* is well defined. Using Lemma A from the Appendix, cyclical monotonicity implies that there exists a convex function w^i so that the inequality in the definition can be written as $w^i(c_i) + c_i \cdot \rho^i \geqslant w^i(c_i^*) + c_i^* \cdot \rho^i$. Hence, c_i^* exists since $\arg\min_{c_i \in \Omega_i}\{w^i(c_i) + c_i \cdot \rho^i\}$ is nonempty.

$-\phi^i$ *is cyclically monotone and*

$$\sum_{i \in N} \left\{ E\left[\Upsilon^i(c_i, c_i^*)\right] + c_i^* \cdot \left[\phi^i(c_i^*) - \rho^i\right] + s_i \cdot \rho^i \right\} \leqslant 0, \tag{4}$$

where $\Upsilon^i(c_i, c_i') = c_i \cdot \phi^i(c_i) - c_i' \cdot \phi^i(c_i') - \int_0^1 \phi^i(\alpha c_i + (1-\alpha)c_i') \cdot (c_i - c_i') d\alpha.$

A similar result that applies only for efficient mechanisms is presented by Makowski and Mezzetti (1994). The preceding theorem provides a characterization of all policies that can be implemented with a balanced budget. Using Lemma 2, Makowski and Mezzetti's (1994) characterization of the conditions under which efficient policies are implementable with a balanced budget can be derived as a corollary of Theorem 3.[12] The result is stated in the following Corollary. A short proof is provided to demonstrate it relation to the conditions of Theorem 3.

Corollary 1. *There exists a transfer function* $\mathbf{T}$ *such that the efficient siting mechanism* $\langle \mathbf{T}, \Phi, Q \rangle$ *is incentive compatible, interim individually rational, and ex post budget balancing if and only if*

$$\sum_{i \in N} E\left[\sum_{j \in N \setminus \{i\}} c_j \cdot [\Phi(c_i^*, c_{-i}) - \Phi(c_i, c_{-i})] + c_i^* \cdot \left[\phi^i(c_i^*) - \rho^i\right] + s_i \cdot \rho^i\right] \leqslant 0. \tag{5}$$

Proof. When Φ is efficient, for each $i \in N$, $-\phi^i$ is cyclically monotone. Lemmas 1 and 2 and the fact that efficient mechanisms satisfy (IC) imply that when the siting policy is efficient, for all $c_i, c_i' \in \Omega_i$,

$$\Upsilon^i(c_i, c_i') = E_{-i}\left[\sum_{j \in N \setminus \{i\}} c_j \cdot [\Phi(c_i', c_{-i}) - \Phi(c_i, c_{-i})]\right]$$

[12] Strictly speaking Makowshi and Mezzetti's (1994) result is not a special case of Theorem 3 since I assume much more structure on Ω and the cost distributions than they do.

Thus, for efficient siting policies, (4) and (5) are equivalent. *Q.E.D.*

An interpretation of inequality (4) will help to provide some intuition as to when siting policies can be implemented with a balanced budget. I discuss each of the three terms within the summation of (4) in turn.

When $-\phi^i$ is cyclically monotone, $\Upsilon^i(c_i, c_i')$ has the same properties as $t^i(c_i) - t^i(c_i')$. (Recall that for incentive compatible mechanisms, $\Upsilon^i(c_i, c_i') = t^i(c_i) - t^i(c_i')$.) From the definition of Υ^i, it follows immediately that $\Upsilon^i(c_i, c_i') = -\Upsilon^i(c_i', c_i)$, for any $c_i, c_i' \in \Omega_i$. Furthermore, when $-\phi^i$ is cyclically monotone, $\Upsilon^i(c_i, c_i') \geqslant 0$, for $c_i, c_i' \in \Omega_i$ such that $c_i' = \alpha c_i$ and $\alpha > 1$. This fact follows from Part (b) of Theorem 2. Therefore, $\Upsilon^i(c_i, 0) \leqslant 0$, for all $c_i \in \Omega_i$.

For the purpose of the following discussion assume that the origin is in Ω. If for each $i \in N$, $c_i^* = (0 \quad \cdots \quad 0)$ then $\sum_{i \in N} E[\Upsilon^i(c_i, c_i^*)] \leqslant 0$. However, there also exists some point on the boundary of Ω_i such that when c_i^* is equal to that point $\sum_{i \in N} E[\Upsilon^i(c_i, c_i^*)] \geqslant 0$. In addition, $\sum_{i \in N} E[\Upsilon^i(c_i, c_i^*)]$ will decrease if for any $i \in N$, c_i^* is moved towards the origin along a straight line. Therefore, the sign of the first term within the summation of (4) can be either positive or negative. If for each $i \in N$, c_i^* is close to the origin (as it will be when ρ^i is close to $\phi^i(0)$), then $\sum_{i \in N} E[\Upsilon^i(c_i, c_i^*)] \leqslant 0$. Therefore, the closer c_i^* is to the origin the better the chance that the first term within the summation in (4) will be negative.

If for each $i \in N$, there exists a $c_i' \in \Omega_i$ such that $\rho^i = \phi^i(c_i')$, then $\sum_{i \in N} c_i^* \cdot [\phi^i(c_i^*) - \rho^i] = 0$. Therefore, the second term of (4) is zero when for each community i, there is some announcement that implies conditional assignment probabilities that are equal to ρ^i. If each Ω_i is a rectangular set that includes the origin, then $\sum_{i \in N} c_i^* \cdot \left[\phi^i(c_i^*) - \rho^i\right] \geqslant 0$. To see this, recall that $c_i^* \in \arg\min_{c_i \in \Omega_i} \{v^i(c_i) + (c_i - s_i) \cdot \rho^i\}$ when the mechanism is incentive compatible. The gradient of the objective function of this minimization problem is $\rho^i - \phi^i(c_i)$. Therefore, if $\phi_j^i(c_i^*) - \rho_j^i > 0$, then c_i^* must be on the boundary of Ω_i such that its jth element is at the upper limit of its support. Since the upper limit of the cost of site j must be positive, $c^*_{ji}\left[\phi^i_j(c^*_i) - \rho^i_j\right] > 0$. A similar argument can be constructed for the case where $\phi^i_j(c^*_i) - \rho^i_j < 0$. For more generally shaped sets, the term $\sum_{i \in N} c^*_i \cdot \left[\phi^i(c^*_i) - \rho^i\right]$ can be negative.

The sign of the final term in (4) is related to the sign of the s_i. Recall that s_i is the difference between community i's share of the construction cost minus its benefits from the facility (which it can be excluded from enjoying when it does not participate in the mechanism). In the public goods case, nonparticipation implies that a community enjoys the benefits of the good but avoids sharing in its production cost. Hence, in the public goods case, $s_i \geqslant 0$. In the current application it seems reasonable to suppose that a nonparticipant can be excluded prevented from benefitting from the facility. Therefore, when the benefits from the sites is larger than the community's share of the construction costs, $s_i < 0$. If for all $i \in N$, $s_i \leqslant 0$, then $\sum_{i \in N} s_i \cdot \rho^i \leqslant 0$.

The following lemma provides a sufficient condition for the implementation of an siting policy with a balanced budget that is easier to work with than (4).

Lemma 4. *Suppose for each $i \in N$, $s_i \leqslant 0$ and $-\phi^i$ is cyclically monotone. if*

$$\sum_{i \in N} \left\{c^*_i \cdot \left[E[\Phi(c)] - \rho^i\right] + s_i \cdot \rho^i\right\} \leqslant 0, \tag{6}$$

then there exist a $\mathbf{T}$ such that the siting mechanism $\langle \mathbf{T}, \Phi, Q\rangle$ that is incentive compatible, interim individually rational, and ex post budget balancing.

Whether or not an efficient siting policy can be implemented with a balanced budget depends on the implications of nonparticipation. If the mechanism designer can pick and commit to a particular ρ^i (that is, chose any siting policy Q^i in the event of nonparticipation), then as is made precise in the following proposition, it is possible to implement any siting policy that satisfies the cyclical monotonicity condition with a balanced budget.

Proposition 1. *If for each $i \in N$, $s_i \leqslant 0$ and $-\phi^i$ is cyclically monotone, then there a Q and a $\mathbf{T}$ such that the siting mechanism $\langle \mathbf{T}, \Phi, Q \rangle$ is incentive compatible, interim individually rational, and ex post budget balancing.*

Proof. For each $i \in N$, pick $Q^i(c_{-i}) = E[\Phi(c)]$. Then $\rho^i = E_{-i}[Q^i(c_{-i})] = E[\Phi(c)]$. The conclusion of the proposition follows immediately from part (b) of Lemma 4. *Q.E.D.*

Notice that to implement an efficient siting policy with a balanced budge when $s_i \leqslant 0$ it is not necessary to set the default assignment probabilities to place all of the probability weight on a community's least preferred site. As is stated in the proof of Proposition 1, it is sufficient to set the default probabilities equal to the ex ante expected assignment probabilities.

However, the mechanism designer may not be able to commit to a particular distribution over the sites that is independent of the other communities' costs. It may be the case that the mechanism designer must always select the site the minimizes the total cost to participating communities. Therefore, when all communities participate the outcome is efficient. However, when one community does not participate, then the other communities select the outcome that minimizes their total cost (including the nonparticipants portion of the construction costs), ignoring the implications on the nonparticipating community. Assuming $c_i^n \in \Omega_i$ (i.e., a community's environmental cost can take a value equal to the community's benefit) if participating communities always seek to minimize there costs plus the nonparticipant's share of the construction costs, then $Q^i(c_{-i}) = \Phi(c_i^n, c_{-i})$ and $\rho^i = \phi^i(c_i^n)$ where Φ is an efficient siting policy. The mechanism designer cannot commit to any default siting policy other than this one. That is, $\mathcal{Q}^i = \{\Phi(c_i^n, \cdot)\}$. It turns out that when c_i^n is a feasible announcement and close to zero, then the efficient siting policy can be implemented with a balanced budget even when the designer must minimize the total cost to the participating communities.

Proposition 2. *Suppose Φ is an efficient siting policy and for each $i \in N$, $c_i^n \in \Omega_i$ and $Q^i(\cdot) = \Phi(c_i^n, \cdot)$. Then If $\sum_{i \in N} \left\{ c_i^n \cdot E[\Phi^i(c)] - b_i \cdot \phi^i(c_i^n) \right\} \leqslant 0$, then there exists a $\mathbf{T}$ such that the siting mechanism $\langle \mathbf{T}, \Phi, Q \rangle$ is incentive compatible, interim individually rational, and ex post budget balancing.*

Proof. Using the facts that $s_i = c_i^n - b_i$, $c_i^* = c_i^n$,[13] and $\rho^i = \phi^i(c_i^n)$, it is straightforward to show that $\sum_{i \in N} \left\{ c_i^n \cdot E[\Phi^i(c)] - b_i \cdot \phi^i(c_i^n) \right\} \leqslant 0$ implies (6). The conclusion now follows immediately from part (b) of Lemma 4. *Q.E.D.*

Proposition 2 implies that even when the mechanism designer must minimize the cost of participating communities an efficiency siting policy can be implemented with a balanced budget when b_i is sufficiently larger than c_i^n and $\phi_0^i(c_i^n) < 1$. Notice that if construction costs are zero, then the inequality condition in Proposition 2 is satisfied.

6. Conclusion

I have described the siting mechanism as a means of solving an informational problem. The siting mechanism elicits cost information from communities that would otherwise not be available when selecting a site. In this paper I demonstrate that in many cases it is possible to construct efficient mechanisms for siting hazardous facilities that are also budget balancing. Two problems face any authority wishing to use such mechanisms as the basis for their siting decision: (1) recent research seems to indicate that the general public may resist attempts to treat the siting decision as a commodity, (2) It is as of yet not clear what simple mechanism can achieve the efficient outcome described here.

Compensation has been discussed as a means of gaining the support for a siting choice from injured communities. However, as indicated by Theorem 2, the expected compensation associated with incentive compatible mechanisms may be considered unfair since even communities who benefit from a selected site in some cases will be paid compensation. However,

[13] There is no loss in generality in assuming that $c_i^* = c_i^n$ since under the assumptions of the proposition (IC) implies $c_i^n \in \arg\max_{c_i \in \Omega_i} \{ v^i(c_i) + (c_i - s_i) \cdot \rho^i \}$.

as discussed by Frey and Oberholzer-Gee (1996), Frey, Oberholzer-Gee, and Eichenberger (1996), and Kunreuther and Easterling (1996), compensating communities as a means of gaining their acquiescence for a particular siting plan can backfire if the compensation is viewed as a bribe. The fairness of the decision making process seems to be key to the acceptance of the final siting decision. Decentralized mechanisms such as the cost mechanisms described here might provide some legitimacy to the decision.[14] However, the results of Frey and Oberholzer-Gee (1996), Frey, Oberholzer-Gee, and Eichenberger (1996), and Kunreuther and Easterling (1996) may be indicating that the general public do not believe that siting decisions should be a tradeable commodity.

[14] When a decision is made according to a decentralized mechanism, it acquires a legitimacy that might not be present if the decision were made by a corruptible individual or an individual with a personal stake in the outcome. Smith (1989) discusses this and other "noneconomic" aspects of auctions.

Appendix

The following lemma is used in a number of the proofs.

Lemma A. Suppose there exists a convex function $w^i : \Omega_i \to \mathbb{R}^m$ such that for all $c_i, c_i' \in \Omega_i$,

$$-\phi^i(c_i) \cdot (c_i - c_i') \geqslant w^i(c_i) - w^i(c_i') \geqslant -\phi^i(c_i') \cdot (c_i - c_i'). \qquad (7)$$

Then for all $c_i, c_i' \in \Omega_i$, $w^i(c_i) - w^i(c_i') = -\int_0^1 \phi^i(\alpha c_i + (1-\alpha)c_i') \cdot (c_i - c_i') d\alpha$.

Proof. The function w^i is continuous due to the fact that ϕ^i is bounded. To see this, notice that the bounds in (7) approach zero as $c_i - c_i'$ approach zero. Let c_i and c_i' be in the interior of Ω_i such that $c_{ji} - c_{ji}' = \varepsilon$ and for $k \in M \backslash \{j\}$, $c_{ki} - c_{ki}' = 0$. Such cost vectors exist since the interior of Ω_i is assumed to be nonempty. Dividing expression (7) by ε and taking the limit as $\varepsilon \to 0$ yields $\partial w^i(c_i)/\partial c_{ji} = -\phi_j^i(c_i)$. Hence, for $c_i \in \operatorname{int} \Omega_i$, $\nabla w^i(c_i) = -\phi^i(c_i)$. Therefore, $-\phi^i$ has a potential function on the interior of Ω_i and by the fundamental theorem of calculus for line integrals

$$(\forall c_i, c_i' \in \operatorname{int}\Omega_i) \qquad w^i(c_i) - w^i(c_i') = -\int_0^1 \phi^i(\alpha c_i + (1-\alpha)c_i') \cdot (c_i - c_i') d\alpha. \qquad (8)$$

To complete the proof I extend this result to the entire support of i's cost vector. Define the sequence $\{\xi_k\}$ in the interior of Ω_i that converges to $c_i \in \Omega_i$. Such a sequence exists for every cost vector c_i in Ω_i since every boundary point is arbitrarily close to points of the interior of Ω_i. For $c_i \in \Omega_i$ and $c_i' \in \operatorname{int}\Omega_i$,

$$\begin{aligned} w^i(c_i) - w^i(c_i') &= \lim_{k\to\infty} w^i(\xi_k) - w^i(c_i') \\ &= -\lim_{k\to\infty} \int_0^1 \phi^i(\alpha \xi_k + (1-\alpha)c_i') \cdot (\xi_k - c_i') d\alpha \\ &= -\int_0^1 \phi^i(\alpha c_i + (1-\alpha)c_i') \cdot (c_i - c_i') d\alpha. \end{aligned}$$

In the expression above, the first equality follows from the continuity of w^i, the second follows from (8), and the third equality follows from the continuity of ϕ^i. The same set of arguments can be used to show that c_i' can also be in the boundary of Ω_i. *Q.E.D.*

Proof of Lemma 1. (If): (IC) is satisfied if for all $i \in N$ and $c_i, c_i' \in \Omega_i$,

$$v^i(c_i) - v^i(c_i') \geqslant -\phi^i(c_i') \cdot (c_i - c_i').$$

Using (1), the condition above can be written as

$$\int_0^1 \left[\phi^i(c_i') - \phi^i(\alpha c_i + (1-\alpha)c_i')\right] \cdot (c_i - c_i') d\alpha \geqslant 0.$$

The inequality follows from cyclical monotonicity. To see this, notice cyclical monotonicity implies

$$-\phi^i(\alpha c_i + (1-\alpha)c_i') \cdot \left[c_i' - \alpha c_i - (1-\alpha)c_i'\right] - \phi^i(c_i') \cdot \left[\alpha c_i + (1-\alpha)c_i' - c_i'\right] \leqslant 0$$

which implies $\left[\phi^i(c_i') - \phi^i(\alpha c_i + (1-\alpha)c_i')\right] \cdot (c_i - c_i') \geqslant 0$, for $\alpha > 0$.

(only if): (IC) implies cyclical monotonicity by Theorem 1. It remains to show that (1) is implied by (IC). (IC) implies that for all $i \in N$ and $c_i, c_i' \in \Omega_i$,

$$-\phi^i(c_i) \cdot (c_i - c_i') \geqslant v^i(c_i) - v^i(c_i') \geqslant -\phi^i(c_i') \cdot (c_i - c_i').$$

Therefore, the conclusion for this part of the lemma follows from Lemma A. *Q.E.D.*

Proof of Theorem 2. Part (a): Let $c_i - c_i' = \Delta = (\delta, \ldots, \delta)$. (IC) implies $t^i(c_i) - t^i(c_i') \geqslant \left[\phi^i(c_i) - \phi^i(c_i')\right] \cdot c_i$. Therefore,

$$\left[\phi^i(c_i) - \phi^i(c_i')\right] \cdot c_i' \geqslant t^i(c_i) - t^i(c_i') \geqslant \left[\phi^i(c_i) - \phi^i(c_i')\right] \cdot c_i.$$

The bounds in the expression above are equal since

$$\left[\phi^i(c_i) - \phi^i(c_i')\right] \cdot \Delta = \delta \sum_{j\in M} \left[\phi_j^i(c_i) - \phi_j^i(c_i')\right] = 0.$$

The second equality follows from the fact that $\sum_{j\in M} \phi_j^i(c_i) = 1$ for any c_i such that $\phi_0^i(c_i) = 0$. Therefore, $t^i(c_i) - t^i(c_i') = \left[\phi^i(c_i) - \phi^i(c_i')\right] \cdot c_i$.

Using this equality and (1) yields

$$\int_0^1 \left[\phi^i(c_i') - \phi^i(\alpha c_i + (1-\alpha)c_i')\right] \cdot \Delta d\alpha = 0.$$

However, cyclical monotonicity implies $\left[\phi^i(c_i') - \phi^i(\alpha c_i + (1-\alpha)c_i')\right] \cdot \Delta \geqslant 0$, for all $\alpha > 0$ (this is derived in the proof of Lemma 1). Therefore, $\phi^i(c_i') = \phi^i(\alpha c_i + (1-\alpha)c_i')$, for all $\alpha \in (0, 1]$.

Part (b): Since (IC) implies cyclical monotonicity,

$$-\phi^i(c_i) \cdot (\alpha c_i - c_i) - \phi^i(\alpha c_i) \cdot (c_i - \alpha c_i) \leqslant 0.$$

Rearranging and using the fact that $\alpha > 1$ yields $c_i \cdot \left[\phi^i(c_i) - \phi^i(\alpha c_i)\right] \geqslant 0$. The conclusion follows from the fact that (IC) implies $t^i(c_i) - t^i(c_i') \geqslant c_i \cdot \left[\phi^i(c_i) - \phi^i(c_i')\right] \geqslant 0$ where the final inequality follows once αc_i is substituted for c_i'. *Q.E.D.*

Proof of Lemma 2. I prove the lemma by showing that (2) is sufficient for (IC). Once sufficiency is established, its necessity follows from Lemma 1 since for efficient siting policies the sufficiency of (2) implies that the right hand sides of (1) and (2) must be equal. Hence, if (2) is sufficient for (IC), then it must also be necessary for (IC).

Notice that (3) implies that for all $c_i, c_i' \in \Omega_i$,

$$v^i(c_i) - \pi^i(c_i', c_i) \quad = \quad E_{-i}\left[\sum\nolimits_{j\in N\backslash\{i\}} c_j \cdot \left[\Phi(c_i', c_{-i}) - \Phi(c_i, c_{-i})\right]\right] - c_i \cdot \left[\phi^i(c_i) - \phi^i(c_i')\right]$$

$$= \quad E_{-i}\left[\sum\nolimits_{j\in N} c_j \cdot [\Phi(c_i', c_{-i}) - \Phi(c_i, c_{-i})]\right] \geqslant 0.$$

The inequality in the preceding expression implies (IC) and follows from the efficiency of the siting policy. *Q.E.D.*

Proof of Lemma 3. Follows immediately from the definition of a minimum. *Q.E.D.*

Proof of Theorem 3. (Only if): By Theorem 1 the cyclical monotonicity of $-\phi^i(c)$ is necessary for (IC). Notice that (BB) implies $0 = \sum_{i\in N} E[T_i(c)] = \sum_{i\in N} E[t^i(c_i)]$. Therefore,

$$\begin{aligned}
0 &= \sum_{i\in N} E[t^i(c_i)] \\
&= \sum_{i\in N} E[c_i \cdot \phi^i(c_i) + v^i(c_i)] \\
&= \sum_{i\in N} E[c_i \cdot \phi^i(c_i) + v^i(c_i) - v^i(c_i^*) + v^i(c_i^*) - c_i^* \cdot \phi^i(c_i^*) + c_i^* \cdot \phi^i(c_i^*)] \\
&\leqslant \sum_{i\in N} E[c_i \cdot \phi^i(c_i) + v^i(c_i) - v^i(c_i^*) - c_i^* \cdot \phi^i(c_i^*) + c_i^* \cdot [\phi^i(c_i^*) - \rho^i] + s_i \cdot \rho^i] \\
&= \sum_{i\in N} E[\Upsilon^i(c_i, c_i^*) + c_i^* \cdot [\phi^i(c_i^*) - \rho^i] + s_i \cdot \rho^i]
\end{aligned}$$

where the inequality follows from the fact that (IR) implies for all $i \in N$, $v^i(c_i^*) \geqslant -(c_i^* - s_i)\cdot\rho^i$ and the last equality follows from (IC) by Lemma 1.

(If): For each $i \in N$, consider the transfer function

$$\begin{aligned}
T_i(c) &= \Psi_i(c) - (c_i^* - s_i)\cdot\rho^i + a_i \\
&\quad -\frac{1}{n}\sum_{j\in N}\left\{\Psi_j(c) - E_{-i}[\Psi_j(c)] + E_{-(i+1)}[\Psi_j(c)] - E[\Psi_j(c)]\right\}
\end{aligned}$$

where $\Psi^i(c) = c_i \cdot \Phi(c) - \int_0^1 \Phi(z_i, c_{-i}) \cdot dz_i$ and $E_{-i}[\,\cdot\,]$ denotes the expectation with respect to all of the cost vectors except i, $E_{-(n+1)}[\,\cdot\,] = E_{-1}[\,\cdot\,]$, and a_i is a constant with respect to

c. Note that

$$\begin{aligned} t^i(c_i) &= E_{-i}[T_i(c)] \\ &= E_{-i}[\Psi^i(c)] - (c_i^* - s_i) \cdot \rho^i + a_i \\ &= c_i \cdot \phi^i(c_i) - \int_0^1 \phi^i(z_i) \cdot dz_i - (c_i^* - s_i) \cdot \rho^i + a_i. \end{aligned}$$

The expression for t^i can be rearranged to yield

$$v^i(c_i) = -\int_0^1 \phi^i(z_i) \cdot dz_i(\alpha) - (c_i^* - s_i) \cdot \rho^i + a_i$$

and, hence, for all $c_i \in \Omega_i$,

$$v^i(c_i) - v^i(c_i^*) = -\int_0^1 \phi^i(z_i) \cdot dz_i.$$

Cyclical monotonicity implies the premise of Lemma A (by Theorem 24.8 in Rockafellar (1970)). Therefore, for all $c_i, c_i' \in \Omega_i$,

$$\begin{aligned} v^i(c_i) - v^i(c_i') &= \left[v^i(c_i) - v^i(c_i^*)\right] - \left[v^i(c_i') - v^i(c_i^*)\right] \\ &= \left[w^i(c_i) - w^i(c_i^*)\right] - \left[w^i(c_i') - w^i(c_i^*)\right] \\ &= w^i(c_i) - w^i(c_i') \\ &= -\int_0^1 \phi^i(\alpha c_i + (1-\alpha)c_i') \cdot (c_i - c_i')d\alpha, \end{aligned}$$

where w^i is defined as in Lemma A and the second and forth equalities follow from the conclusion of Lemma A. Hence, by Lemma 1, the mechanism is incentive compatible (by Lemma 1).

Interim individual rationality is satisfied if $a_i \geqslant 0$. To see this, notice that the transfer function defined above implies $v^i(c_i^*) = -(c_i^* - s_i) \cdot \rho^i + a_i$. If $a_i \geqslant 0$, then $v^i(c_i^*) + (c_i^* - s^i) \cdot \rho^i \geqslant$

0 and (IR) is satisfied.

It remains to show that there exists $(a_1, \ldots, a_n) \in \mathbb{R}^n_+$ that are consistent with an ex post balanced budget. Let $a_i = \frac{1}{n} \sum_{j \in N} \Big\{ (c_j^* - s_j) \cdot \rho^j - E[\Psi^j(c)] \Big\}$. The fact that $a_i \geqslant 0$ follows from (4), and the mechanism has an ex post balanced budget since for any $c \in \Omega$,

$$\sum_{i \in N} T_i(c) = \sum_{i \in N} \big\{ E[\Psi^i(c)] - (c_i^* - s_i) \cdot \rho^i + a_i \big\} = 0.$$

Q.E.D.

Proof of Lemma 4. By Theorem 3, (4) is sufficient for (IC), (IR), and (BB) when for each $i \in N$, $-\phi^i$ is cyclically monotone. Cyclical monotonicity implies that for any $c_i \in \Omega_i$,

$$-\phi^i(c_i) \cdot (\alpha c_i + (1-\alpha)c_i^* - c_i) - \phi^i(\alpha c_i + (1-\alpha)c_i^*) \cdot (c_i - \alpha c_i + (1-\alpha)c_i^*) \leqslant 0,$$

and hence, for $\alpha < 1$, $\big[\phi^i(c_i) - \phi^i(\alpha c_i + (1-\alpha)c_i^*)\big] \cdot (c_i - c_i^*) \leqslant 0$. Therefore, cyclical monotonicity implies that for any $c_i \in \Omega_i$,

$$\int_0^1 \big[\phi^i(c_i) - \phi^i(\alpha c_i + (1-\alpha)c_i^*)\big] \cdot (c_i - c_i^*) d\alpha \leqslant 0. \tag{9}$$

Rearranging (4) yields

$$\sum_{i \in N} \left\{ E\left[\int_0^1 \big[\phi^i(c_i) - \phi^i(\alpha c_i + (1-\alpha)c_i^*)\big] \cdot (c_i - c_i^*) d\alpha + c_i^* \cdot \big[\phi^i(c_i) - \rho^i\big] \right] + s_i \cdot \rho^i \right\}$$
$$\leqslant \sum_{i \in N} \big\{ c_i^* \cdot \big[E[\phi^i(c_i)] - \rho^i\big] + s_i \cdot \rho^i \big\} \leqslant 0.$$

In the expression above the first inequality follows from cyclical monotonicity by (9) and the second inequality follows from the assumption of the lemma. Therefore,

$$\sum_{i \in N} \big\{ c_i^* \cdot \big[E[\Phi(c)] - \rho^i\big] + s_i \cdot \rho^i \big\} \leqslant 0$$

is sufficient for (4) and, hence, is also sufficient for the existence of a mechanism that satisfies (IC), (IR), and (BB). *Q.E.D.*

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